Preferences Resulting From Weighted Sums

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1 Motivation

Suppose students are told to rank different schools they would like to get into. The preferences of the students are likely correlated in some way due to an inherent notion of the quality of different schools. One way to define such a correlation is to assume there is some underlying space of quality along different attributes (e.g. STEM education vs liberal arts education) and the students' preference are determined by these attributes. The simplest instance of this is for each student to rank schools according to a weighted sum of the different attributes of the school.

We want to study the inherent complexity of the collection of preferences that result from these procedure, as a function of the number of attributes the schools have. In other words, what sort of correlation arises in the preferences of students in this model?

A Figures

We can prove the following structural impossibility results on the set of preferences we consider.

$$x >_a y >_a z$$
$$y >_b z >_b x$$
$$z >_c x >_c y$$

Figure 1: This preference set is impossible if $d = 2$.

$$x >_a y >_a z$$
$$x >_b z >_b y$$
$$y >_c z >_c x$$
$$z >_d y >_d x$$

Figure 2: This preference set is impossible if $d = 2$.

$$w >_a x >_a y >_a z$$
$$x >_b y >_b z >_b w$$
$$y >_c z >_c w >_c x$$
$$z >_d w >_d x >_d y$$

Figure 3: This preference set is impossible if $d = 3$.

The following table gives approximate upper bounds for the number of preferences possible on n options in d dimensions. The pattern seems like it might be something kinda like $O(n^{d-1})$, which is much less than the full set of preferences, which has size $n!$.

n	d=2	d=3
3	4	6
4	7	18
5	10	41
6	15	87
7	20	121
8	24	
9	28	