

Multi-Class and Multi-Task Strategies for Neural Directed Link Prediction



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Executive Summary

- 1) Naively porting ULP testing techniques to DLP is not sound → need for a **“three-way” testing setup**.
- 2) Naively porting ULP training techniques to DLP leads to suboptimal performances → gap
- 3) We fill this gap by **developing three DLP training strategies** perform better in the three-way testing setup

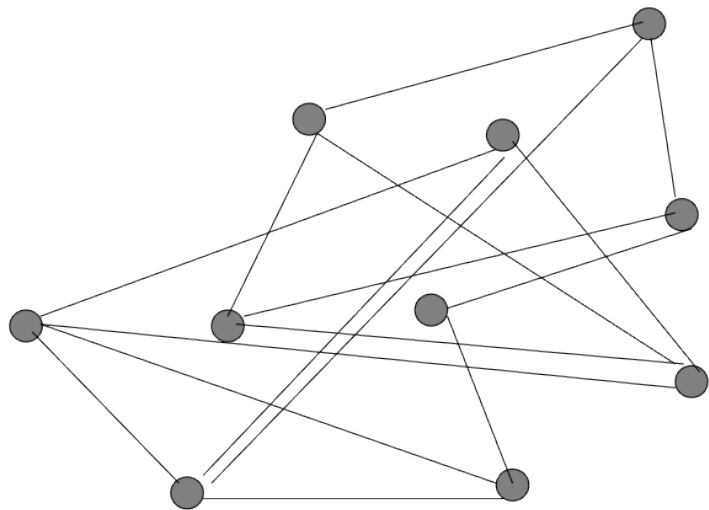
Outline

- 1) Undirected Link Prediction
- 2) Naive Directed Link Prediction
- 3) New Training Strategies & Results

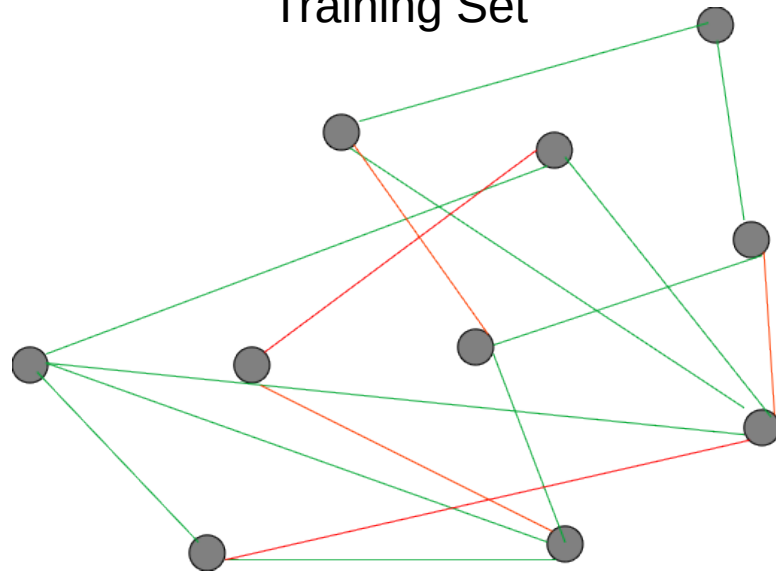
Undirected Link Prediction

Training

Whole Graph



Training Set

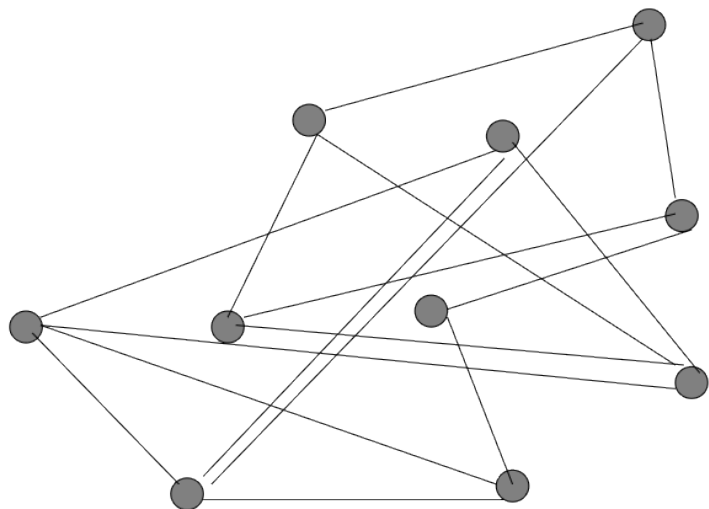


$$\mathcal{L}(\Theta) = \sum_{e \in E_{s+n}} w_p y_e \ln(\hat{p}_{\Theta}(e)) + (1 - y_e) \ln(1 - \hat{p}_{\Theta}(e))$$

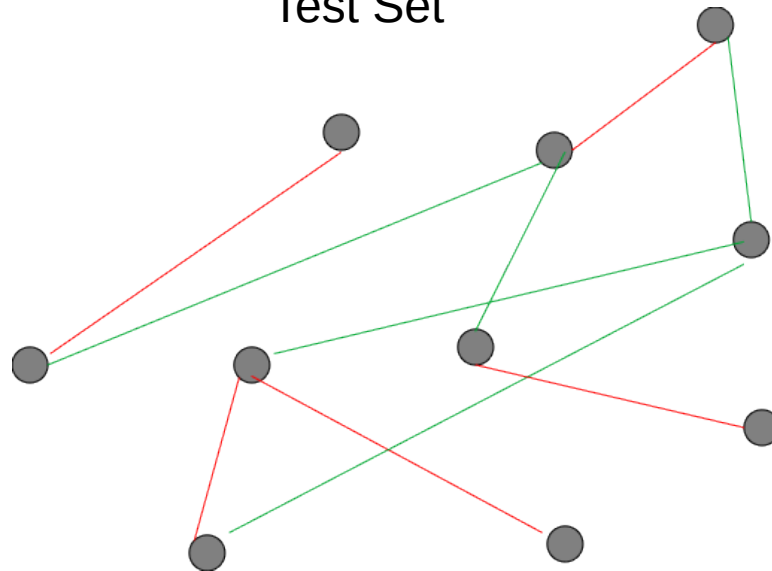
Undirected Link Prediction

Testing

Whole Graph



Test Set

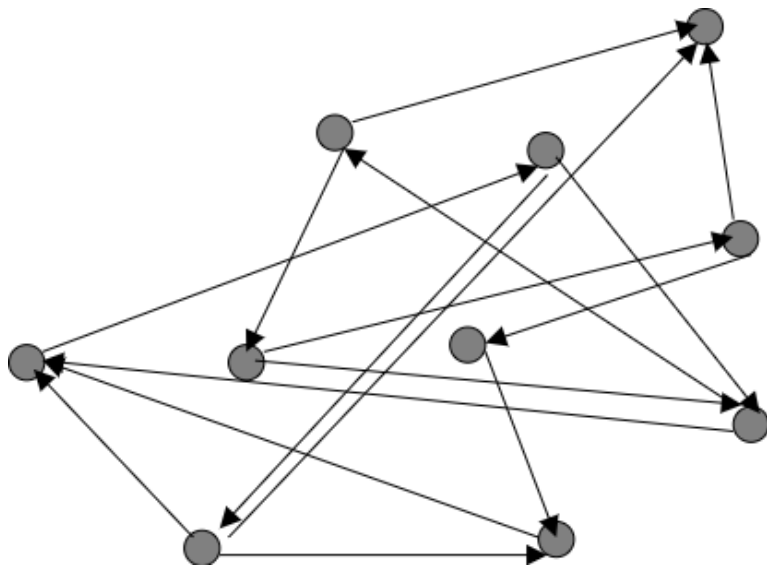


AUC, AP, Hits@K, MRR

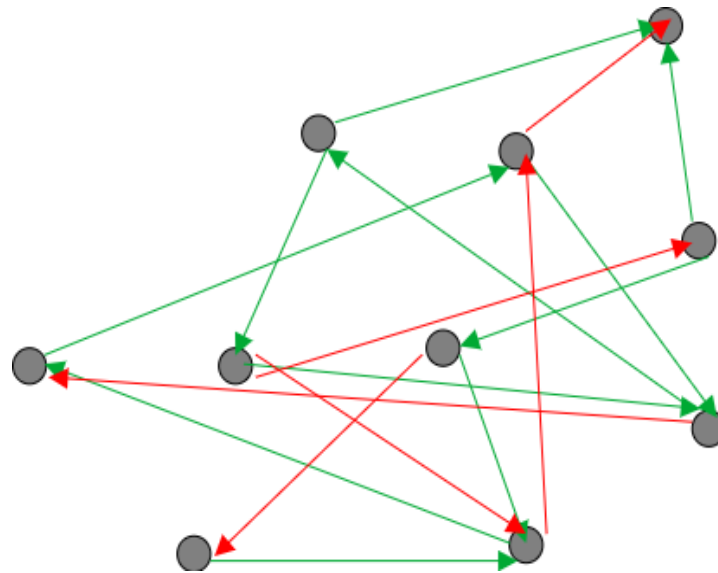
Directed Link Prediction

Naive Training

Whole Graph



Training Set

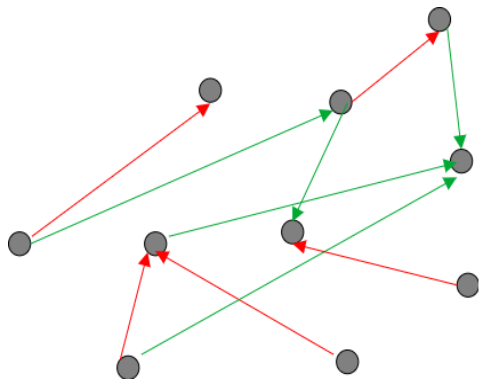


$$\mathcal{L}(\Theta) = \sum_{e \in E_{s+n}} w_p y_e \ln(\hat{p}_{\Theta}(e)) + (1 - y_e) \ln(1 - \hat{p}_{\Theta}(e))$$

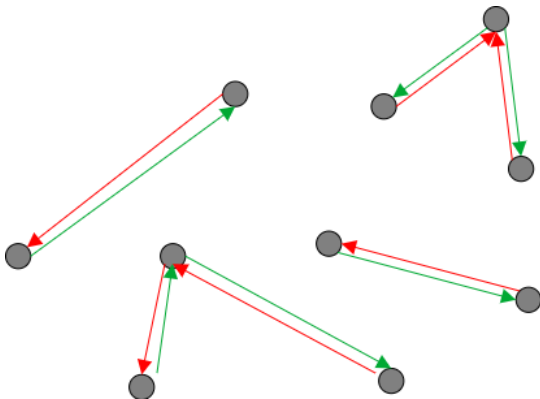
Directed Link Prediction

Testing

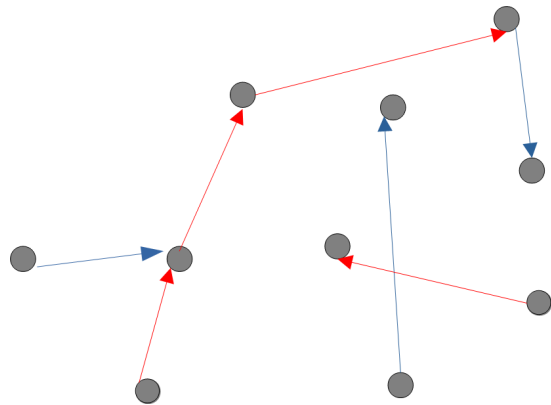
General Test Set



Directional Test Set



Bidirectional Test Set



AUC, AP, Hits@K, MRR

Directed Link Prediction

Results with Naive Training

model	decoder	GENERAL		DIRECTIONAL		BIDIRECTIONAL	
		ROC-AUC	AUPRC	ROC-AUC	AUPRC	ROC-AUC	AUPRC
GAE	$\vec{z}_i \cdot \vec{z}_j$	84.6	88.6	50.0	50.0	62.4	64.0
GR-GAE	$\sigma(\vec{z}_v[0] - \lambda \ln(\ \vec{z}_u[1:] - \vec{z}_v[1:]\ _2^2))$	89.2	92.4	63.4	61.5	69.1	66.5
ST-GAE	$\sigma(\vec{z}_v[:\frac{L}{2}] \cdot \vec{z}_u[\frac{L}{2}:])$	87.8	90.1	60.8	64.5	74.6	74.1
DiGAE	$\sigma(\vec{z}_v^S \cdot \vec{z}_v^I)$	80.4	85.3	57.5	63.0	70.4	68.6
MLP-GAE	$\text{MLP}(\vec{z}_v \parallel \vec{z}_u)$	77.1	78.2	90.7	90.7	69.9	69.7
MAGNET	MLP-like	75.2	77.8	90.4	89.8	71.9	70.4
dMPLP	MLP-like	86.1	88.0	75.7	76.8	81.1	82.2

Directed Link Prediction

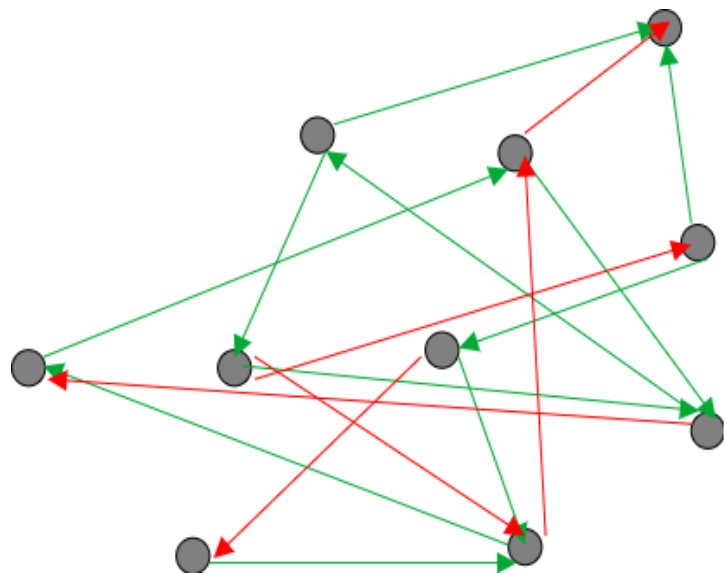
Existing Solutions

- Naive Training [Kollias et al. 2022](#)
- Train one model per task [Salha et al. 2019](#)
- Ad-hoc architectures [Zhang et al. 2021](#)

We wish to develop techniques that allow any NDLP-capable architecture to achieve good performance on all three tasks simultaneously

New Training Strategies

Multi Class



We need to **balance the loss w.r.t. positives vs negatives AND unidirectionals vs bidirectionals SIMULTANEOUSLY!**

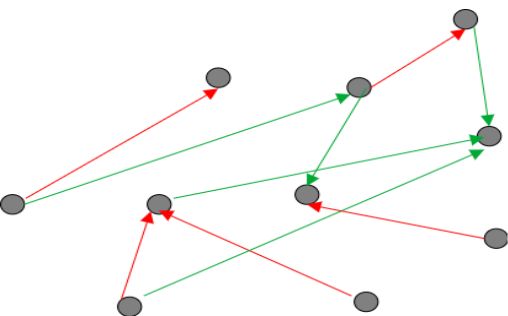
$$[\hat{p}_{uv}^{nb}, \hat{p}_{uv}^{nu}, \hat{p}_{uv}^{pu}, \hat{p}_{uv}^{pb}] = [(1 - \hat{p}_{uv})(1 - \hat{p}_{vu}), \\ (1 - \hat{p}_{uv})\hat{p}_{vu}, \\ \hat{p}_{uv}(1 - \hat{p}_{vu}), \\ \hat{p}_{uv}\hat{p}_{vu}]$$

$$\mathcal{L}_{MC-NDLP}(\Theta) = - \sum_{c \in C} \sum_{uv \in T} w_{y_{uv}} \mathbb{I}(y_{uv} = c) \log(\hat{p}_{uv}^{y_{uv}}), \quad w_{y_{uv}} = \frac{n_x}{n_{y_{uv}}}$$

New Training Strategies

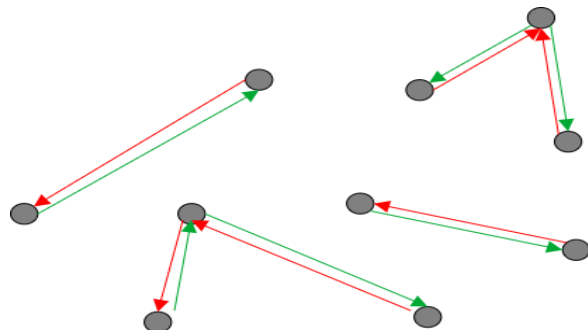
Multi Task - MGDA

General Training Set



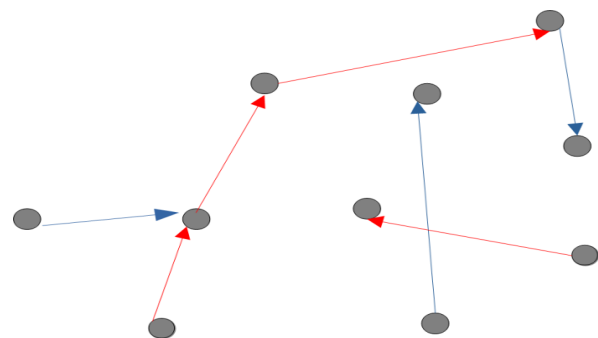
$\nabla \mathcal{L}_G$

Directional Training Set



$\nabla \mathcal{L}_D$

Bidirectional Training Set



$\nabla \mathcal{L}_B$

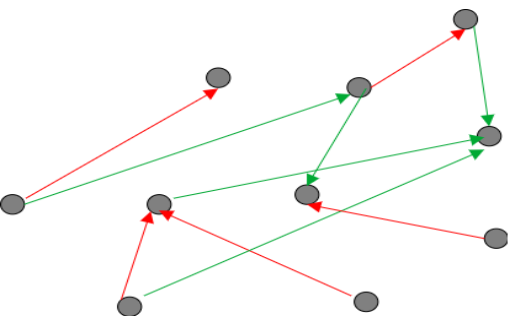
$$\vec{g}_{\text{MGDA}} = \min_{\|\sum_i \alpha_i \nabla \mathcal{L}_i\|_2} \sum_i \alpha_i \nabla \mathcal{L}_i \quad \text{s.t.} \quad \sum_i \alpha_i = 1$$

$$\theta' = \theta - \eta \vec{g}_{\text{MGDA}}$$

New Training Strategies

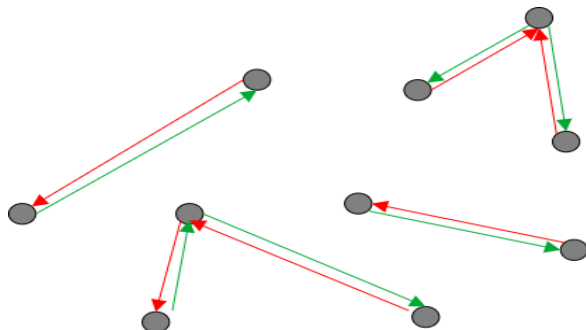
Multi Task - Scalarization

General Training Set



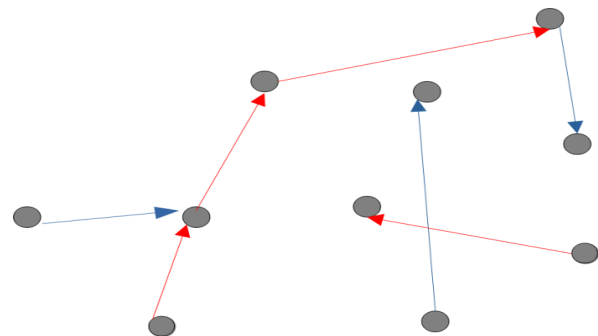
$\nabla \mathcal{L}_G$

Directional Training Set



$\nabla \mathcal{L}_D$

Bidirectional Training Set



$\nabla \mathcal{L}_B$

$$\vec{g}_S = \sum_i \alpha_i \nabla \mathcal{L}_i$$

$$\theta' = \theta - \eta \vec{g}_S$$

New Training Strategies

Results - Cora

model	strategy	GENERAL		DIRECTIONAL		BIDIRECTIONAL	
		ROC-AUC	AUPRC	ROC-AUC	AUPRC	ROC-AUC	AUPRC
GAE	BASELINE	84.6 \pm 0.4	88.6 \pm 0.3	50.0 \pm 0.0	50.0 \pm 0.0	62.4 \pm 3.0	64.0 \pm 3.1
GR-GAE	BASELINE	89.2 \pm 0.4	92.4 \pm 0.2	63.4 \pm 2.5	61.5 \pm 2.7	69.1 \pm 3.1	66.5 \pm 3.3
	MO-NDLP	84.5 \pm 1.1	86.3 \pm 1.1	80.6 \pm 0.7	80.2 \pm 0.9	79.6 \pm 4.3	84.6 \pm 3.5
	MC-NDLP	88.6 \pm 0.4	90.0 \pm 0.4	82.1 \pm 0.5	81.8 \pm 0.7	77.3 \pm 2.2	76.3 \pm 1.7
	S-NDLP	87.8 \pm 0.6	89.5 \pm 0.5	82.3 \pm 0.5	81.6 \pm 0.4	89.6 \pm 1.6	92.4 \pm 1.1
ST-GAE	BASELINE	87.8 \pm 0.7	90.1 \pm 0.5	60.8 \pm 0.5	64.5 \pm 0.6	74.6 \pm 1.8	74.1 \pm 2.2
	MO-NDLP	86.3 \pm 0.5	86.2 \pm 0.4	79.3 \pm 1.0	80.0 \pm 0.9	79.3 \pm 0.5	79.5 \pm 1.9
	MC-NDLP	80.7 \pm 2.0	80.1 \pm 2.1	79.0 \pm 2.3	81.6 \pm 1.9	70.3 \pm 3.0	68.1 \pm 2.1
	S-NDLP	84.5 \pm 0.4	84.9 \pm 0.7	75.8 \pm 1.0	78.4 \pm 0.9	81.1 \pm 0.9	80.4 \pm 1.6
DiGAE	BASELINE	80.4 \pm 1.1	85.3 \pm 0.8	57.5 \pm 1.3	63.0 \pm 1.4	70.4 \pm 2.2	68.6 \pm 1.2
	MO-NDLP	70.2 \pm 3.8	72.6 \pm 3.6	73.6 \pm 5.4	76.0 \pm 4.2	67.3 \pm 4.6	69.6 \pm 4.1
	MC-NDLP	75.4 \pm 0.9	77.4 \pm 1.0	84.3 \pm 0.6	85.4 \pm 0.8	68.9 \pm 1.5	69.3 \pm 1.1
	S-NDLP	72.5 \pm 4.0	77.4 \pm 4.4	61.6 \pm 1.3	69.2 \pm 1.4	72.1 \pm 5.6	74.4 \pm 5.7
MLP-GAE	BASELINE	77.1 \pm 0.9	78.2 \pm 0.6	90.7 \pm 0.6	90.7 \pm 0.6	69.9 \pm 3.2	69.7 \pm 3.7
	MO-NDLP	76.0 \pm 0.8	76.4 \pm 0.7	93.4 \pm 0.6	93.5 \pm 0.6	80.7 \pm 1.6	79.2 \pm 2.4
	MC-NDLP	74.5 \pm 0.7	75.6 \pm 0.7	94.3 \pm 0.6	94.4 \pm 0.5	71.7 \pm 2.4	65.7 \pm 1.8
	S-NDLP	74.7 \pm 1.0	74.9 \pm 0.9	90.5 \pm 0.7	90.0 \pm 0.9	72.0 \pm 2.6	70.5 \pm 2.9
MAGNET	BASELINE	75.2 \pm 1.4	77.8 \pm 1.0	90.4 \pm 0.9	89.8 \pm 0.8	71.9 \pm 2.3	70.4 \pm 2.8
	MO-NDLP	74.4 \pm 1.4	77.4 \pm 1.1	91.3 \pm 1.0	90.9 \pm 1.0	70.6 \pm 2.7	68.6 \pm 2.7
	MC-NDLP	74.4 \pm 1.0	77.4 \pm 1.0	92.1 \pm 0.7	91.6 \pm 0.7	71.8 \pm 2.6	70.0 \pm 2.6
	S-NDLP	74.6 \pm 1.3	77.5 \pm 1.1	91.0 \pm 1.0	90.4 \pm 1.0	71.8 \pm 2.8	70.2 \pm 2.9
dMPLP	BASELINE	86.1 \pm 0.5	88.0 \pm 0.9	75.7 \pm 2.2	76.8 \pm 1.6	81.1 \pm 3.6	82.2 \pm 5.3
	MO-NDLP	83.5 \pm 0.6	85.1 \pm 0.6	89.1 \pm 1.7	89.0 \pm 2.1	85.8 \pm 3.3	89.3 \pm 2.5
	MC-NDLP	81.4 \pm 1.7	82.0 \pm 1.5	83.7 \pm 4.2	83.7 \pm 3.6	70.0 \pm 4.3	71.5 \pm 3.9
	S-NDLP	85.6 \pm 1.0	86.9 \pm 1.0	84.8 \pm 2.7	86.3 \pm 2.3	83.6 \pm 4.7	87.0 \pm 4.3

Thanks!



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