Exam #1 Review

Wednesday 2/04/2025

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Supplemental Instruction

(Do not review strikethroughed slides)

Q0.) *Math is…?*

- Fill in the blank: "Math is FUN"

Important question, highly relevant to **exam content**!

*Q1.) Find the Following Limits

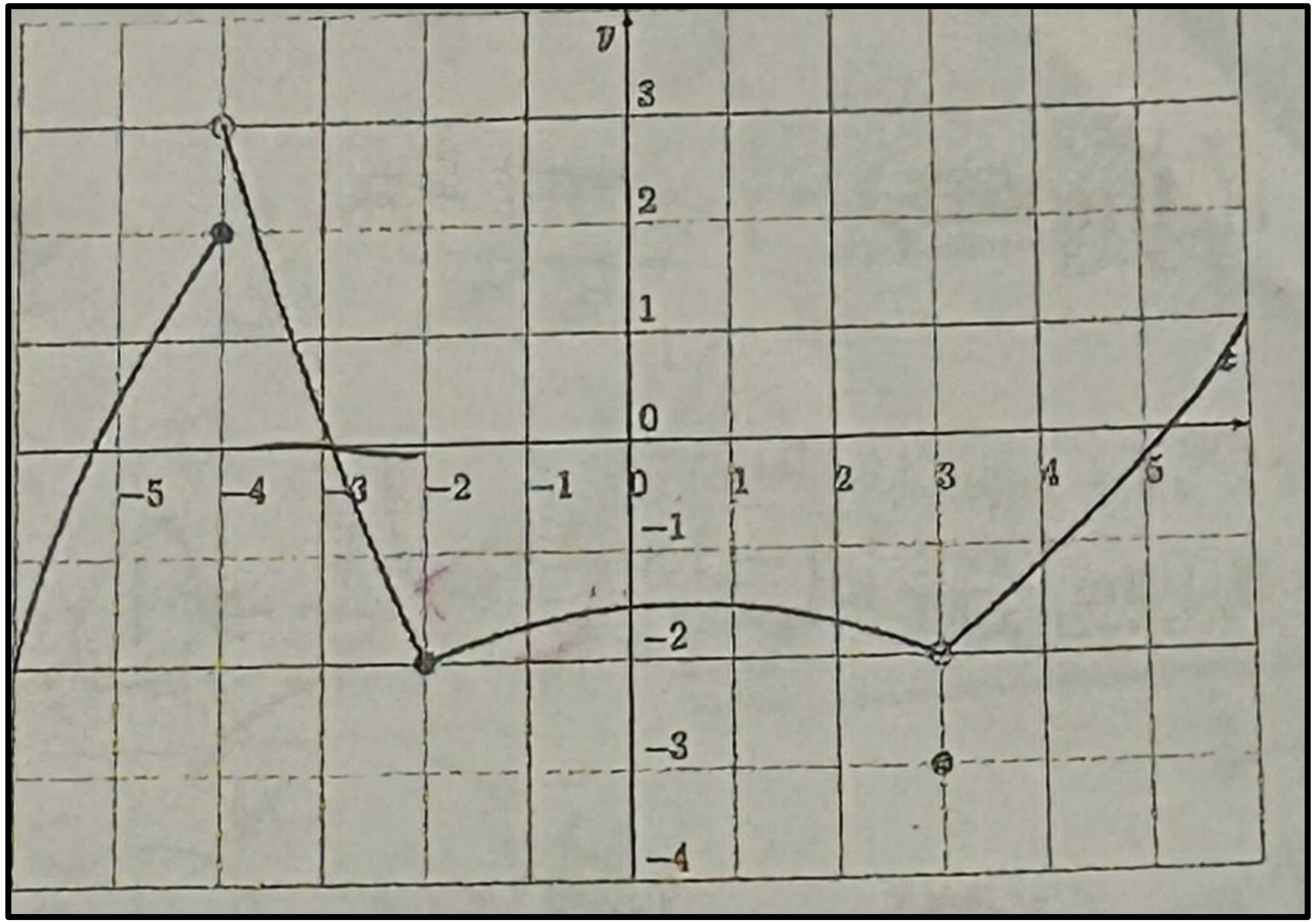
a.) $\lim_{x \to -2} g(x)$ = -2

b.) $\lim_{x \to 3} g(x)$ = -2

c.) $\lim_{x \to -4^-} g(x)$ = 2

d.) $\lim_{x \to -4^+} g(x)$ = 3

e.) $\lim_{x \to -4} g(x)$ = DNE



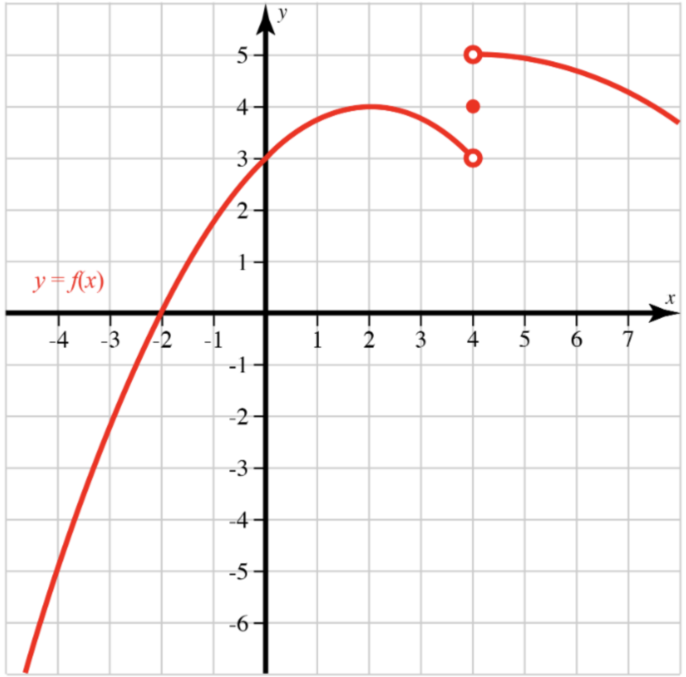
Function g(x) is graphed above

d.) Is g(x) continuous at x=3? If not, what is the discontinuity? No; removable discontinuity.

e.) Is g(x) defined at x=3? If so, what is the output value? Yes; g(3)=-3

Achieve Study Guide - #8

Use the graph of f to determine the value of $\lim_{x \to 4^-} f(x)$.



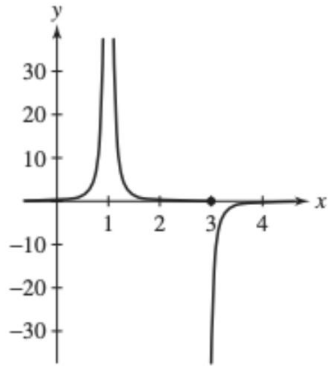
$\lim_{x \to 4^-} f(x) =$ 3

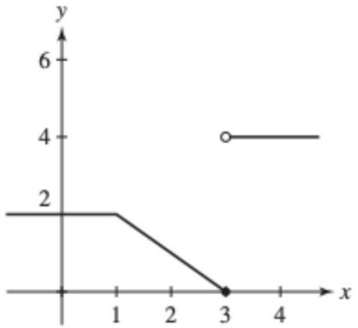
Achieve Study Guide - #1

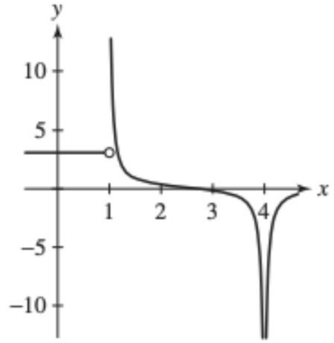
Identify the graph with the following properties.

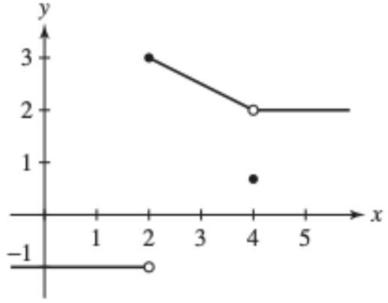
$$\lim_{x\to 1^+} f(x) = \infty, \quad \lim_{x\to 1^-} f(x) = 3, \quad \lim_{x\to 4} f(x) = -\infty$$

Choose the corresponding graph.









Achieve Study Guide - #3

Evaluate the limit by calculating the left-hand and the right-hand limits at $x = 7$.

If the limits are infinite, enter ∞ or $-\infty$.

$$\lim_{x \to 7^+} \frac{2x}{x - 7} = \infty$$

$$\lim_{x \to 7^-} \frac{2x}{x - 7} = -\infty$$

Given the calculations of the right-hand and the left-hand limits, select the correct conclusion about $\lim_{x \to 7} \frac{2x}{x - 7}$.

- (●) The limit does not exist because $\lim_{x \to 7} \frac{2x}{x - 7}$ is undefined.
- ( ) The limit exists because $\lim_{x \to 7} \frac{2x}{x - 7} = L$, for some real number L.
- ( ) The limit does not exist and the function decreases without bound as x approaches 7.
- ( ) The limit does not exist and the function increases without bound as x approaches 7.

*Q2.) Find Average Rate of Change

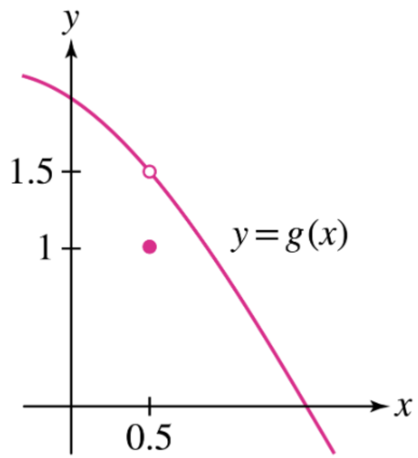
- Find the average rate of change for the function p(x)= $\frac{3x^2-6}{x}$ over the interval [3, 7].

Avg. rate of change = slope (m) $= \frac{y_2-y_1}{x_2-x_1} = \frac{p(7)-p(3)}{7-3}$

$$= \frac{\frac{141}{7} - 7}{4} = \frac{\frac{92}{7}}{4} = \frac{92}{28} = \boxed{\frac{23}{7}}$$

Achieve Study Guide - #9

Determine $\lim_{x \to 0.5} g(x)$ for g as in the figure.



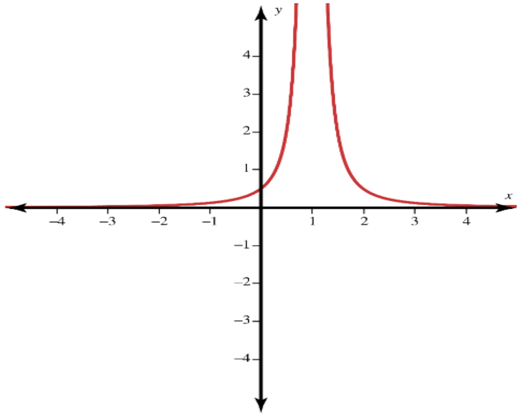
(Give an exact answer. Use symbolic notation and fractions where needed. Enter DNE if the limit does not exist.)

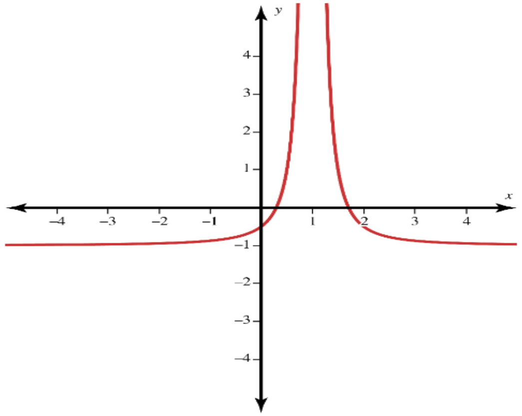
$\lim_{x \to 0.5} g(x) =$ 1.5

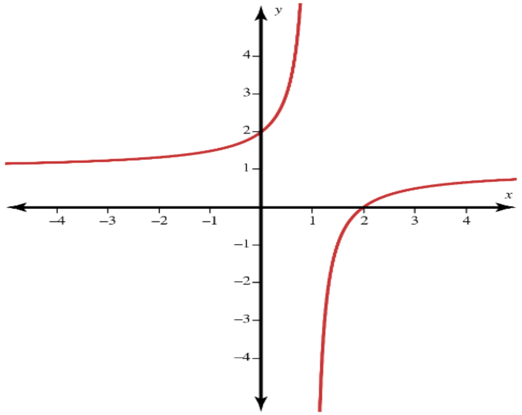
Achieve Study Guide - #4

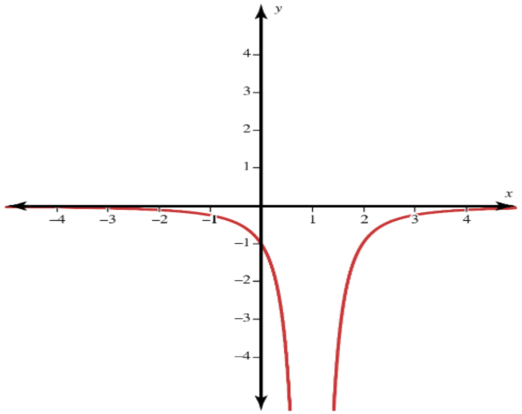
$\lim_{x \to 1} f(x) = \infty$

Select the graphs of f that exhibit the given infinite limit.









*Q3.) Find derivative of the function.

- Find the derivative of R(z)= $\frac{5}{z}$, using the limit definition of the derivative.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{\frac{5}{z+h}-\frac{5}{z}}{h}=\frac{\frac{5z-5(z+h)}{z(z+h)}}{h}$$

$$=\frac{-5h}{z(z+h)h}=\frac{-5}{z(z+h)}=\frac{-5}{z(z+0)}=\boxed{\frac{-5}{z^2}}$$

Achieve Study Guide - #11

Consider the following limit.

$$\lim_{z \to 0} \frac{\cos(5z) - 1}{z}$$

Does the limit of the function as z approaches 0 exist?

- ○ No, the limit does not exist.
- ● Yes, the limit exists.

If the limit exists, what is its value? If the limit does not exist, leave the following blank.

$$\lim_{z \to 0} \frac{\cos(5z) - 1}{z} = \lim_{z \to 0} 5 \cdot \frac{\cos(5z) - 1}{5z} = 5 \cdot 0 = 0$$

Achieve Study Guide - #12

Question 12 of 34

Calculate the limit.

$$\lim_{x \to -5} \frac{x^2 - 25}{x^2 + 1x - 20} = \lim_{x \to -5} \frac{(x-5)(x+5)}{(x-4)(x+5)} = \frac{x-5}{x-4} = \boxed{\frac{10}{9}}$$

Select the correct value of the limit.

- $\frac{10}{9}$
- The limit does not exist.
- $\frac{x-5}{x-4}$
- 1
- 0

Achieve Study Guide - #13

Question 13 of 34

Find $\lim\limits_{x \to 4} f(x)$ for the given piecewise function.

$$f(x) = \begin{cases} 2x + 5 & \text{if } x < 4 \\ 6 & \text{if } x = 4 \\ -\frac{1}{2}x + 15 & \text{if } x > 4 \end{cases}$$

$$\lim_{x \to 4^-} f(x) = 2(4) + 5 = 13$$

$$\lim_{x \to 4^+} f(x) = -\frac{1}{2}(4) + 15 = 13$$

If the limit does not exist, enter DNE.

$\lim\limits_{x \to 4} f(x) =$ 13

*Q4.) Horizontal Asymptotes.

- Find and identify, if they exist, the horizontal asymptote(s) of $f(x) = \frac{5+\frac{3}{5x^{-4}}}{1+\frac{2}{7x^{-5}}}$. **You must use calculus for your work.**

$$\lim_{x\to\pm\infty} f(x) = \frac{5+\left(\frac{3}{5}\right)x^4}{1+\left(\frac{2}{7}\right)x^5} = \frac{x^{-5}}{x^{-5}} \cdot \frac{5+\left(\frac{3}{5}\right)x^4}{1+\left(\frac{2}{7}\right)x^5} = \frac{\left(5x^{-5}+\left(\frac{3}{5}\right)x^{-1}\right)}{\left(1x^{-5}+\left(\frac{2}{7}\right)\right)} = \frac{0}{\frac{2}{7}} = 0; \boxed{\text{H. A at y} = 0}$$

Achieve Study Guide - #25

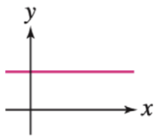
Find the equation of the tangent line to the graph of $f(x) = \dfrac{3}{\sqrt{x}}$ at $x = 49$.

(Express numbers in exact form. Use symbolic notation and fractions where needed. Let $y = f(x)$ and give the equation in terms of y and x.)

equation:

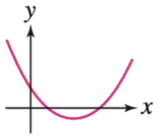
Achieve Study Guide - #29

Three graphs of derivatives are given. Match each of the four function graphs with the graph of its derivative.



(I)

B



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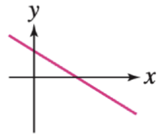
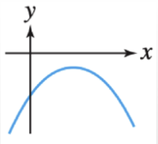


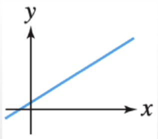
(III)

D, A

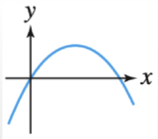
Answer Bank



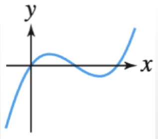
(D)



(B)



(A)

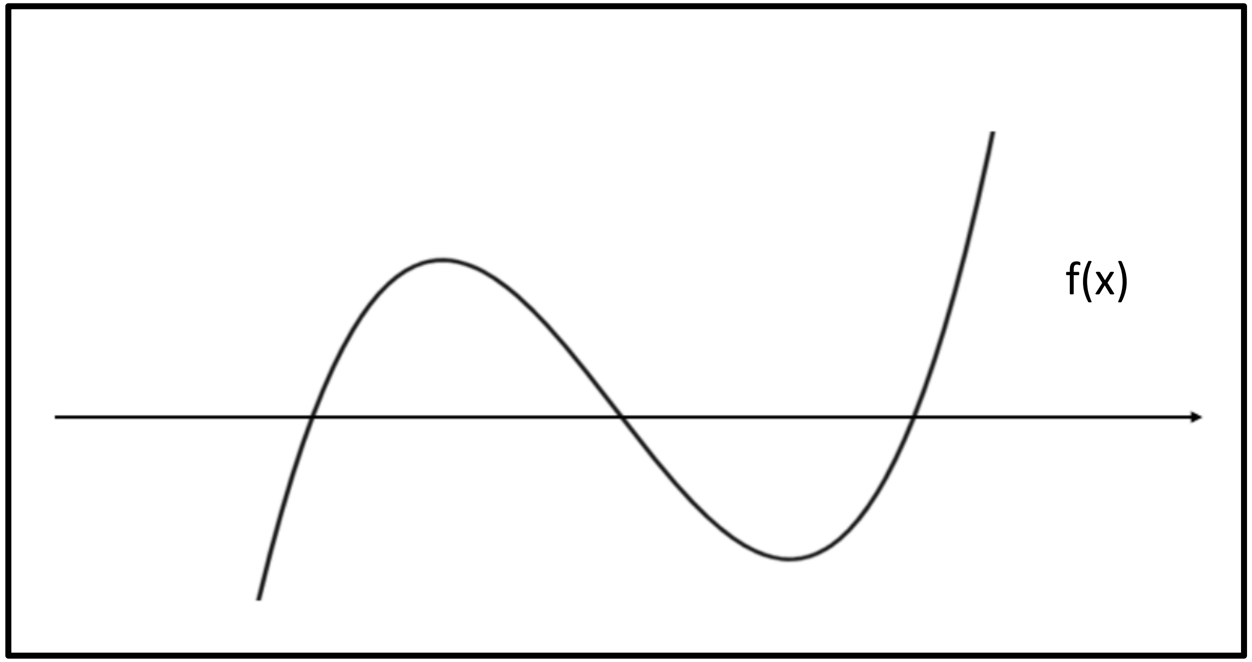


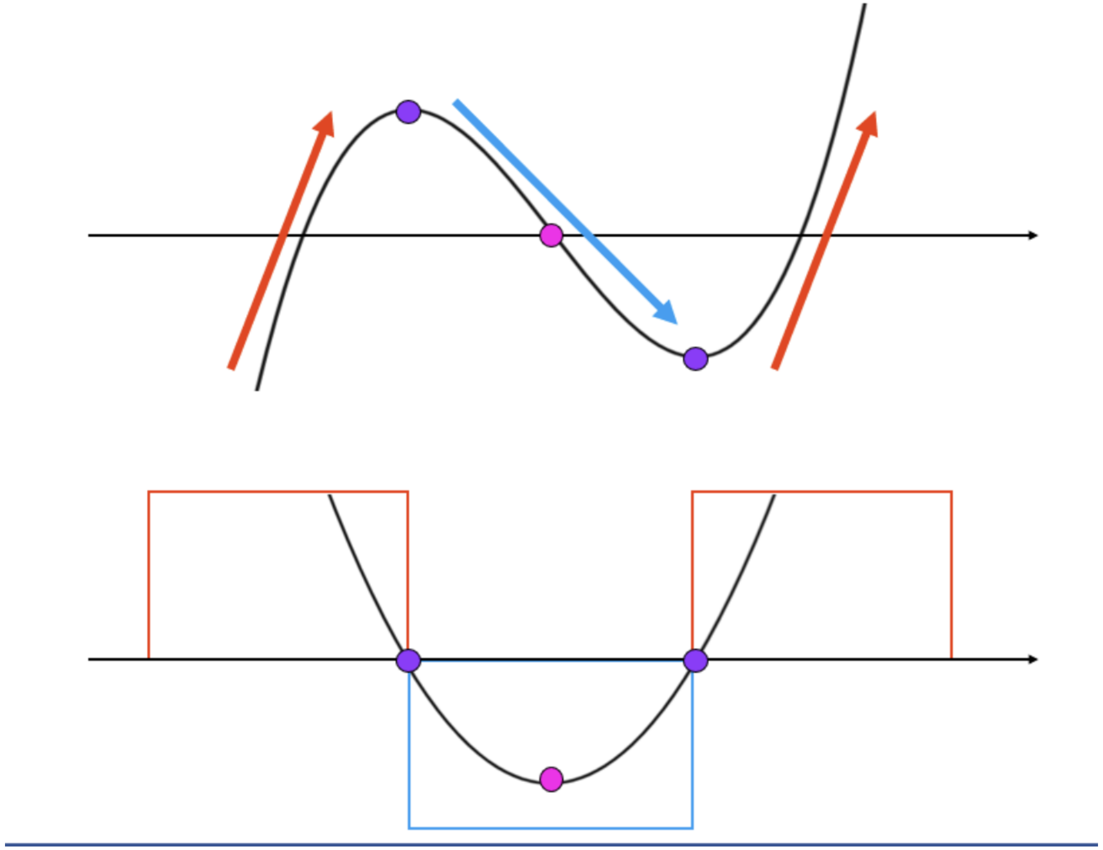
(C)

Identify why two graphs have the same derivative graph.

- ○ The derivative at a particular value of x does not change if the graph is scaled horizontally by a factor of $|k|$.
- ● The derivative at a particular value of x does not change if the graph is shifted up or down.
- ○ The derivative at a particular value of x does not change if the graph is shifted left or right.
- ○ The derivative at a particular value of x does not change if the graph is scaled vertically by a factor of $|k|$.

*Q5.) Sketch f'(x) from f(x)





~~Achieve Study Guide - #31~~

$$\text{Product Rule} = f'g + g'f = \boxed{\frac{16x(x^4+4)-(4x^3)8x^2}{(x^4+4)^2}\cdot\frac{x^3+3}{x+1}+\frac{3x^2(x+1)-1(x^3+3)}{(x+1)^2}\cdot\frac{8x^2}{x^4+4} = h'(x)}$$

< Question 31 of 34 >

Use the product rule to differentiate h.

$$h(x) = \frac{8x^2}{x^4+4}\cdot\frac{x^3+3}{x+1}$$

$h'(x) =$

~~Achieve Study Guide - #33~~

Consider the functions p and q.

$$p(x) = \frac{3x}{5x+3}$$

$$q(x) = 8x - 1$$

Calculate r' if $r(x) = \frac{p(x)}{q(x)}$.

$r' =$

*Q6.) Determine existence of solution for f(x)

- Does f(x) $= 2x^3 - 5x + 2$ have a solution over (0, 1)?

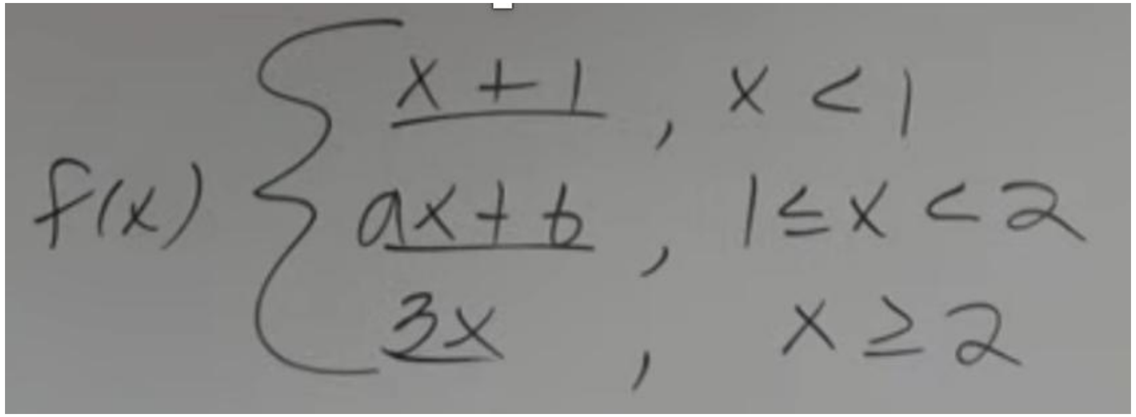
$$f(0) = 2(0) - 5(0) + 2 = 2$$
$$f(1) = 2(1) - 5(1) + 2 = -1$$

"Thus, since f(0)>0 and f(1)<0, f(x) has a solution by the conclusion of the IVT"

- Does f(x)$= \frac{3-\sin(x)}{2}$ have a solution?

"No, the maximum output of sin(x) is 1, so there is ***no*** *x where f(x)<0 and thus by the conclusion of the IVT, f(x) must not have a solution"*

Solve a SoE to Make a Piecewise Function Continuous



$ax + b = 1 + 1 \; at \; x = 1$
$ax + b = 3(2) \; at \; x = 2$

$\mathrm{a} + \mathrm{b} = 2 \rightarrow b = 2 - a$
$2a + b = 6 \rightarrow 2\mathrm{a} + 2 - \mathrm{a} = 6$
$a + 2 = 6$
$\boxed{a = 4}$
$b = 2 - a = 2 - 4$
$\boxed{b = -2}$

Achieve Study Guide - #19

Question 19 of 34

Assume that if $\lim_{x\to a} f(x) = L$, then $\lim_{x\to a} \cos(f(x)) = \cos(L)$. Evaluate the following limits.

(Give exact answers. Use symbolic notation and fractions where needed. Enter DNE if the limit does not exist.)

(a) $\lim_{x\to 0} \cos\left(\frac{3x}{1-3x}\right) =$

(b) $\lim_{x\to \pi/2} \frac{\cos(x)}{x} =$

(c) $\lim_{x\to 1} \left(x^3 \cos(1-x)\right) =$

(d) $\lim_{x\to 0} \frac{1-x^3}{1-\cos(x^3)} =$

*Q7.) Limit Questions

a.) $\lim_{x\to 7} \frac{x^2-10x+21}{x-7} = \frac{(x-7)(x-3)}{(x-7)} = x-3 = \boxed{4}$

b.) $\lim_{x\to 5} \frac{\sqrt{x+4}-3}{x-5} \cdot \frac{\sqrt{x+4}+3}{\sqrt{x+4}+3} = \frac{x+4-9}{(x-5)\left(\sqrt{x+4}+3\right)}$

$= \frac{(x-5)}{(x-5)\left(\sqrt{x+4}+3\right)} = \frac{1}{\sqrt{x+4}+3} = \boxed{\frac{1}{6}}$

c.) $\lim_{x\to 3^+} \frac{|x+3|}{x+3}$

$= \frac{3+3}{3+3} = \boxed{1}$

d.) $\lim_{x\to 0} \frac{\sin 5x+\tan 7x}{3x}$

$= \frac{\sin(5x)}{3x} + \frac{\sin(7x)}{3x} \cdot \frac{1}{\cos(7x)}$

$= \frac{1}{3} \cdot \frac{5}{1} \cdot \frac{\sin(5x)}{5(x)} + \frac{1}{3} \cdot \frac{7}{1} \cdot \frac{\sin(7x)}{7x}$

$\cdot \frac{1}{\cos(7x)}$

$= \frac{5}{3} \cdot 1 + \frac{7}{3} \cdot 1 \cdot 1 = \frac{5}{3} + \frac{7}{3} = \boxed{4}$

e.) $\lim_{x\to\infty} \ln x = \ln(\infty) = \boxed{\infty}$

Some Final Review Questions…

- Does continuity imply differentiability, why or why not?
- If f(x)=3π^2, what is f'(x)?
- If f(x)=4ln(π^2)^3, what is f'(x)?
- ~~Find $\frac{d}{dt}$ of $\frac{2t^2+t^2-5}{t}$, using only the power rule.~~
- ~~Is $f(x) = 2x^3 + \frac{300}{x^3} + 4$ increasing, decreasing or not changing at $x = -2$?~~
- If f'(x)=k for some constant k, what do we know about f(x)?
- The limit of f(x)=k (for some constant k) as x approaches 7 is what?

Some Final Review Questions…

- The instantaneous velocity of a function is the _____, also known as the _____.
- Find f'(x) and the limit as x approaches 0 for f(x) below:

$$f(x) = \frac{21\pi^{e}(5 + \ln(294 + \pi))^{\pi}}{3\ln(2) - \sqrt{e + 56\pi - 17.2}}$$

- What is the derivative of f(x)=|x|?
- $\lim\limits_{x \to 3} \frac{\ln(x^3) + \ln(x^3)}{3}$

Final Reminders!!

- If you still need support, ask for help in the GroupMe so your classmates or I can assist!
- If you are unable to solve a question, make sure to at least write the formulas/setup (def. of derivative, special trig limit) for the problem, this will help **maximize partial credit!**
- If variables are different from the norm, e.g t instead of x, use the original variable in the problem!
- If using a theorem to prove your answer, i.e IVT, cite it in your final answer.
  - Ex: "Since $f(5) > 0$ and $f(2) < 0$, then by the IVT [or 'conclusion of the IVT'], $f(x)$ has a solution"
- This PowerPoint contains comprehensive review material, try to **at least do the red-starred questions**! These will help you **prepare the most for your exam**.

Good Luck!!

- **Math is fun!!**
- **How a certified math enjoyer sleeps**