Calculus - Chain Rule

Chain Rule is a differentiation rule used when finding the derivative of a composite function.

- Differentiation- The process of finding the derivative of a function.
- Composite Function- A function that is formed by applying one function to the result of another ($f(g(x))$).

Chain Rule: $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$ or ~~$\frac{d}{dx}[f(g(x))]$~~

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

- $f'(g(x))$ - take the derivative of the outer function and apply the inner function (unchanged) to it.
- $g'(x)$ - derivative of inner function.

Imagine y changes as u changes, and u changes as x changes.

$$y = f(u), \quad u = g(x)$$

Then, the rate at which y changes with respect to x is:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

1. Find the derivative of $y = (3x^2+2)^5$.

$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$ $\quad f(u) = u^5, \; u = 3x^2+2$

$f'(u) = 5u^4, \; u' = 6x$

$5(3x^2+2)^4 \cdot 6x$

$= 30x(3x^2+2)^4$

Calculus: Chain Rule pt. 2

2. Differentiate $y = (3x+2)^5$

$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$

$u = 3x+2$, $f(u) = u^5$

$u' = 3$, $f'(u) = 5u^4$

$5(3x+2)^4 \cdot 3$

$= \boxed{15(3x+2)^4}$

3. Differentiate $y = \sin(4x)$

$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$

$u = 4x$ $\quad u' = 4$

$f(u) = \sin(x)$ $\quad f' = \cos(x)$

$\cos(4x) \cdot 4$

$= \boxed{4\cos(4x)}$

4. $y = e^{2x^3+x}$

$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$

$f = e^u$ $\quad f' = e^u$

$u = 2x^3+x$ $\quad u' = 6x^2+1$

$\boxed{e^{(2x^3+x)} \cdot 6x^2+1}$ or $\boxed{e^{(2x^3+x)}(6x^2+1)}$

5. $y = \cos^3(5x^2)$

$\frac{dy}{dx} = f'(g(x)) \cdot g(x)$

Outer: $\cos^3(x) = u^3 = f$, $f' = 3u^2$

Middle: $\cos(5x^2) \rightarrow -\sin(5x^2)$

Inner: $5x^2 = u$, $u' = 10x$

$3(\cos(5x^2))^2 \cdot (-\sin(5x^2)) \cdot 10x$

$\boxed{y' = -30x\cos^2(5x^2)\sin(5x^2)}$