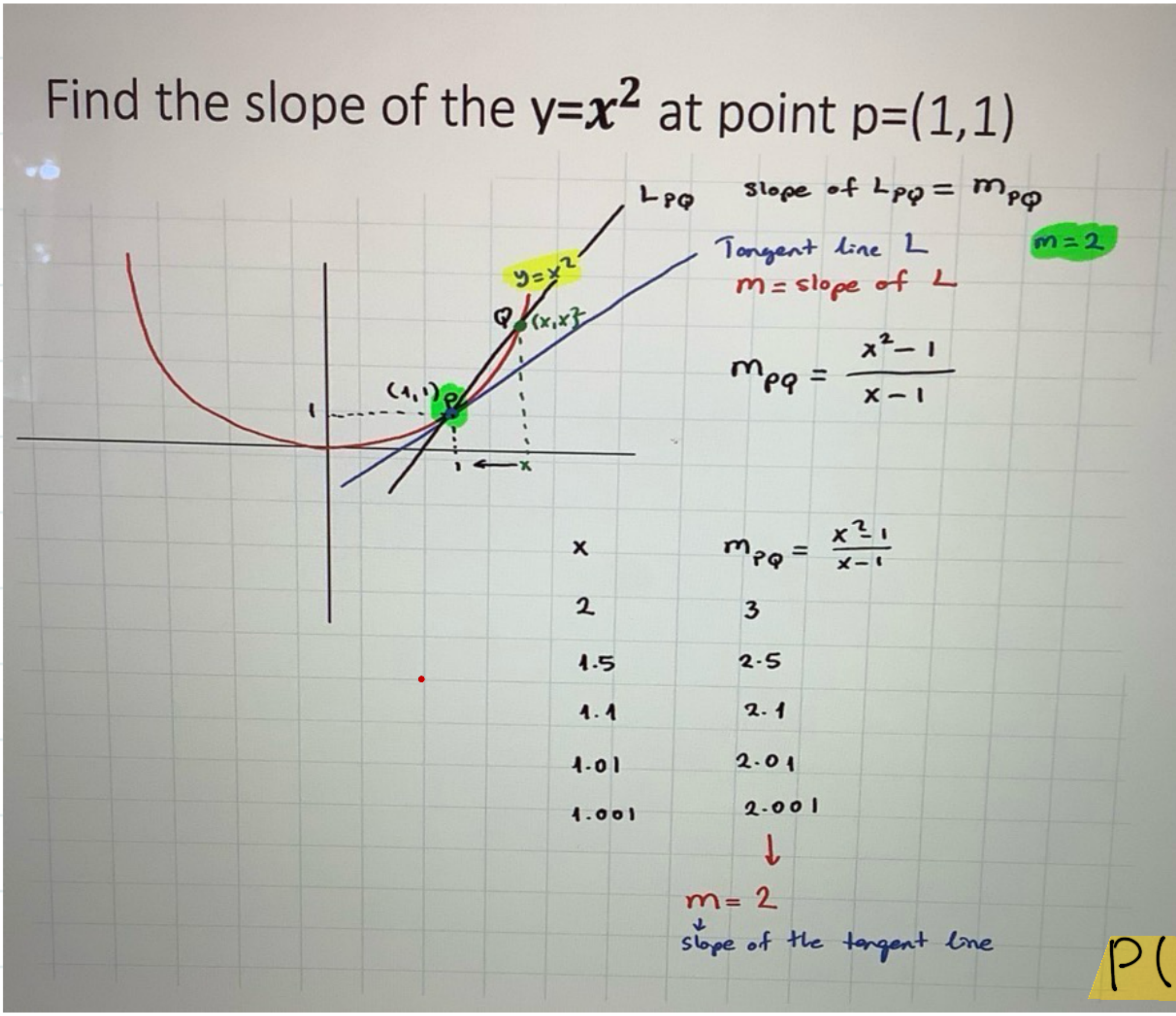
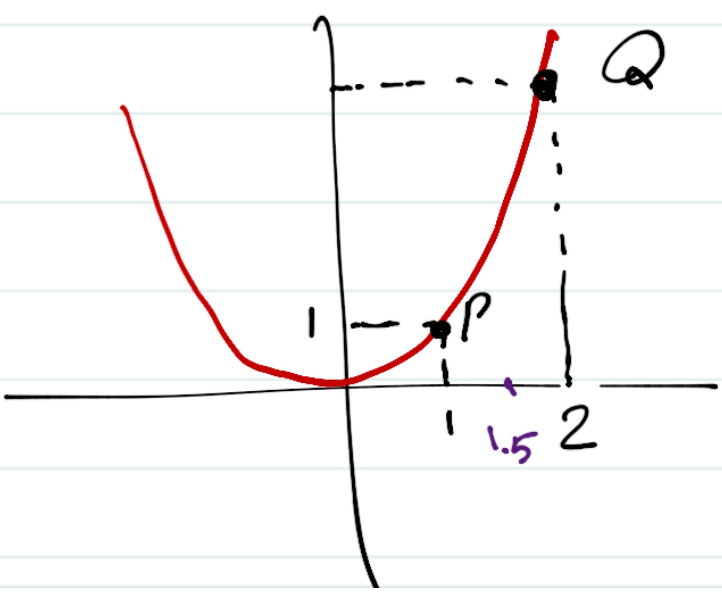
Ex 3: Find the slope of $y = x^2$ at point $P = (1,1)$



P(1,1)

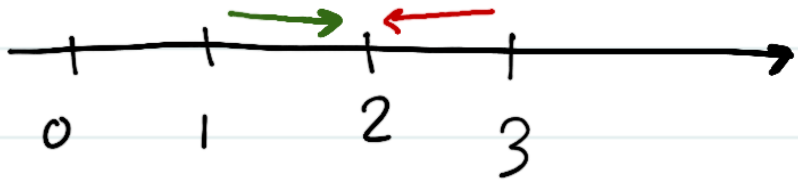


$$\frac{y_2 - y_1}{x_2 - x_1} = m_{PQ}$$

x	m_{PQ}
2	$\frac{4-1}{2-1} = 3$
1.5	$\frac{2.25-1}{1.5-1} = 2.5$
1.1	2.1
1.01	2.01
1.001	2.001

$m = 2$

2.2 $x \to a$



$x \to 2^-$

x approches to 2 from Left

$x \to 2^+$

x approches to 2 from right

$x \to -3^+$

$x \to -3^-$

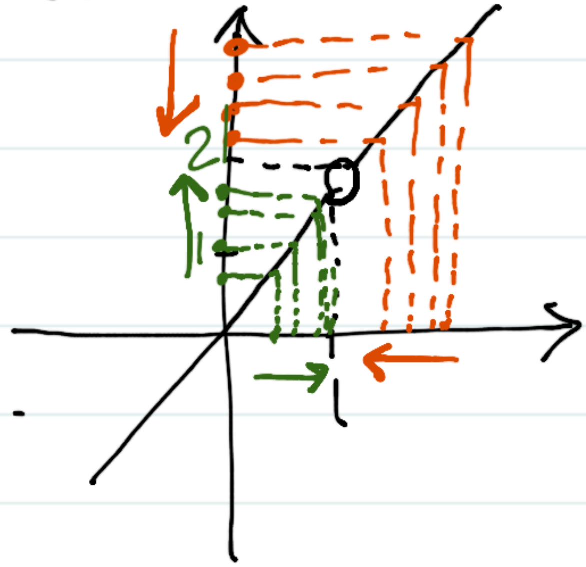
$x \to 2^-$

$x \to 2^+$

$x \to a^+$ means x approches to "a" from Right

$x \to a^-$ means x approches to "a" from Left

Ex1: Find the limit of function below at point $x=1$



$\lim_{x \to 1} f(x) = ?$

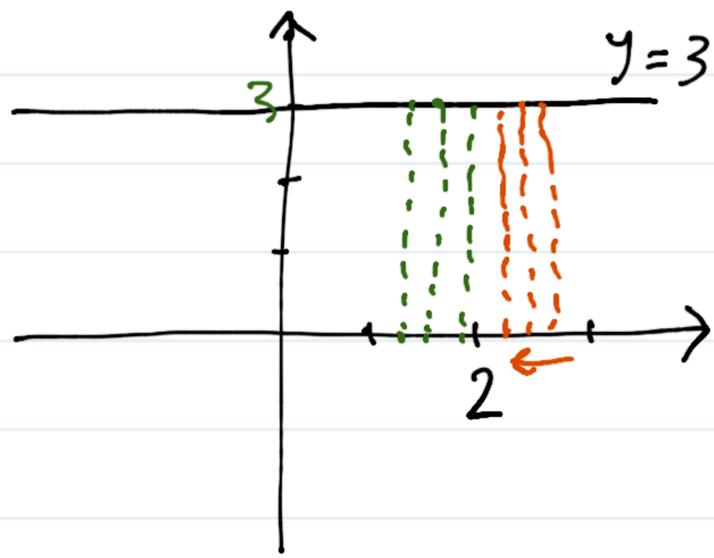
$\lim_{x \to 1^-} f(x) = 2$

$\Rightarrow \boxed{\lim_{x \to 1} f(x) = 2}$

$\lim_{x \to 1^+} f(x) = 2$

Since $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 2 \Rightarrow \lim_{x \to 1} f(x) = 2$

Ex 2: $\lim_{x \to 2} f(x) = ?$



$\lim_{x \to 2^-} f(x) = 3$

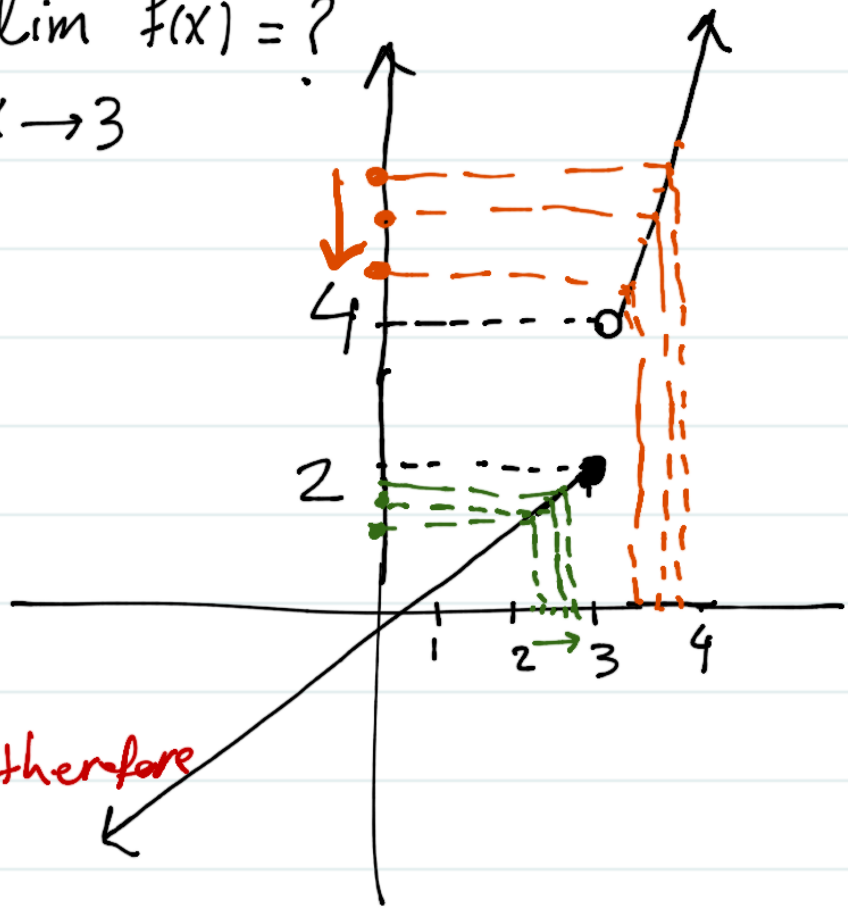
$\Rightarrow \boxed{\lim_{x \to 2} f(x) = 3}$

$\lim_{x \to 2^+} f(x) = 3$

Ex. 3: Suppose $f(x) = 13$ Find:

$\lim_{x \to 0} f(x) = 13$ $\qquad \lim_{x \to 7} f(x) = 13$ $\qquad \lim_{x \to -5} f(x) = 13$

Ex 4: Find $\lim_{x \to 3} f(x) = ?$



$\lim_{x \to 3^-} f(x) = 2$

$\lim_{x \to 3^+} f(x) = 4$

Since $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$ therefore

$\lim_{x \to 3} f(x)$ Does Not Exist. (DNE)

Find the limit of functions:

a) $\lim_{x \to -3} (5x+2) = ?$ $5(-3)+2 = -15+2 = -13$

b) $\lim_{x \to 1} \frac{x+2}{x+5} = ?$ $\frac{1+2}{1+5} = \frac{\cancel{3}}{\cancel{6}} = \frac{1}{2}$

c) $\lim_{x \to 0} \frac{|x|}{x} = ?$ DNE

$\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1$

$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = \lim_{x \to 0^-} -1 = -1$

Since $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x) \Rightarrow$ DNE

d) $\lim_{h \to 0} \dfrac{5}{\sqrt{3h+1}+2} = ?$ $\dfrac{5}{\sqrt{\cancel{3(0)}+1}+2} =$

$\dfrac{5}{\cancel{\sqrt{1}}\,1+2} = \dfrac{5}{3}$ ✓

$=$

e) $\lim_{h \to 0} \dfrac{\sqrt{3h+1}-1}{h} = ?$ $\dfrac{\sqrt{3(0)+1}-1}{0} = \dfrac{1-1}{0} = \dfrac{0}{0}$

$\lim_{h\to 0} \dfrac{\sqrt{3h+1}-1}{h} \cdot \dfrac{\sqrt{3h+1}+1}{\sqrt{3h+1}+1} = \lim_{h\to 0} \dfrac{\cancel{\sqrt{3h+1}}^{\cancel{2}}-1^2}{h(\sqrt{3h+1}+1)}$

$\lim_{h\to 0} \dfrac{3h+\cancel{1}-\cancel{1}}{h(\sqrt{3h+1}+1)} = \lim_{h\to 0} \dfrac{3\cancel{h}}{\cancel{h}(\sqrt{3h+1}+1)}$

$= \lim_{h\to 0} \dfrac{3}{\sqrt{3h+1}+1} = \dfrac{3}{\sqrt{3(0)+1}+1} = \dfrac{3}{1+1} = \boxed{\dfrac{3}{2}}$

f) $\lim_{x \to 5} \dfrac{x-5}{x^2-25} = ?$ $\dfrac{5-5}{5^2-25} = \dfrac{0}{0}$

$\lim_{x\to 5} \dfrac{\cancel{x-5}}{(\cancel{x-5})(x+5)} = \lim_{x\to 5} \dfrac{1}{x+5} = \dfrac{1}{5+5} = \boxed{\dfrac{1}{10}}$ ✓

9) $\lim\limits_{x\to 8} \dfrac{x^2-4x-32}{x-8} \overset{?}{=} \dfrac{8^2-4(8)-32}{8-8} = \dfrac{0}{0}$

-32

$-8 \quad 4$

-4

$= \lim\limits_{x\to 8} \dfrac{\cancel{(x-8)}(x+4)}{\cancel{x-8}}$

$= \lim\limits_{x\to 8} x+4 = 8+4 = \boxed{12}$ ✓

$\lim\limits_{x\to 5^+} \dfrac{-3}{x-5} = \dfrac{-3}{5^+-5} = \dfrac{-3}{0^+} = -\infty$

$\lim\limits_{x\to 5^-} \dfrac{-3}{x-5} = \dfrac{-3}{5^--5} = \dfrac{-3}{0^-} = +\infty$

$$\lim_{x \to 2^+} \frac{-7}{2-x} = ? \quad \frac{-7}{2-2^+} = \frac{-7}{0^-}$$

$$= +\infty$$