For each of the following functions, find two functions $f(x)$ and $g(x)$ so that $F(x) = (f \circ g)(x)$. Do not choose $f(x) = x$ or $g(x) = x$.

(a) $F(x) = \ln(x^2 - 3)$

(b) $F(x) = \sin^3 x$

(c) $F(x) = \sin(x^3)$

(d) $F(x) = \sqrt{3e^x - 4}$

(e) $F(x) = \dfrac{1}{\tan^2 x + 1}$

Solutions on the next page

(a) $F(x) = \ln(x^2 - 3): \quad f(x) = \ln x,\ g(x) = x^2 - 3$

Note that the "outside" function is the natural log function ($F(x)$ is "the natural log of something"), and the "inside" function is $x^2 - 3$ (that is, $x^2 - 3$ is the "something" we are taking the natural log of).

(b) $F(x) = \sin^3 x: \quad f(x) = x^3,\ g(x) = \sin x$

It's helpful here to rewrite $F(x) = (\sin x)^3$ – remember that what the notation $sin^3 x$ means. Now we can see that the cube function is the outside function ($F(x)$ is "something cubed") and $\sin x$ is the inside function.

(c) $F(x) = \sin(x^3): \quad f(x) = \sin x,\ g(x) = x^3$

Contrast this with the previous problem. Here, the outside function is $\sin x$ (that is, $F(x)$ is "sine of something"), and the inside function is x^3.

(d) $F(x) = \sqrt{3e^x - 4}: \quad f(x) = \sqrt{x},\ g(x) = 3e^x - 4$

The outside function here is the root function, and the inside function is whatever is under the root.

(e) $F(x) = \dfrac{1}{\tan^2 x + 1}: \quad f(x) = \dfrac{1}{x^2 + 1},\ g(x) = \tan x$

Remember $\tan^2 x = (\tan x)^2$. There are other possibilities here, such as $f(x) = \frac{1}{x+1}$ and $g(x) = \tan^2 x$, or $f(x) = \frac{1}{x}$ and $g(x) = \tan^2 +1$, but the answer above has the advantage of breaking the function into its "algebraic" piece and its "trigonometric" piece.