Find the domain of each function:

(a) $Q(x) = x^3 + 4x - 17$

(b) $f(x) = \dfrac{x^2 - 1}{\ln(x+1)}$

(c) $g(x) = \tan x + \sin x$

(d) $h(x) = \dfrac{\sqrt[3]{x+7}}{e^x - 1}$

(e) $P(x) = \sqrt{x+1} + \sqrt{6-x}$

Solutions on the next page

(a) $Q(x) = x^3 + 4x - 17$ D: $(-\infty, \infty)$

Note that $Q(x)$ is a polynomial, and the domain of polynomials is all real numbers.

(b) $f(x) = \dfrac{x^2 - 1}{\ln(x+1)}$ D: $(-1, 0) \cup (0, \infty)$, i.e. $(-1, \infty)$ except $x = 0$

The numerator is a polynomial, so it gives no restrictions. The denominator is a log function, and requires that the argument be positive, so $x + 1 > 0$, which implies $x > -1$. Further, we must ensure that the denominator is not 0, so we must exclude $\ln(x + 1) = 0$. Since $\ln 1 = 0$, we must exclude $x + 1 = 1$, or $x = 0$.

(c) $g(x) = \tan x + \sin x$ D: $(-\infty, \infty)$ except odd integer multiples of $\pi/2$

The domain of the sine function is all real numbers, so it provides no restrictions. The domain of the tangent function is the only restriction, and the tangent function is undefined when $\cos x = 0$, so at $\pi/2, 3\pi/2, 5\pi/2$, etc.

(d) $h(x) = \dfrac{\sqrt[3]{x+7}}{e^x - 1}$ D: $(-\infty,) \cup (0, \infty)$, or all real numbers except $x = 0$

Since the root in the numerator is odd, it gives no restrictions. Similarly, the domain of $y = e^x$ is all real numbers. However, we must ensure that the denominator is not 0. Solving $e^x - 1 = 0$ yields $e^x = 1$, or $x = 0$. So $x = 0$ must be eliminated from the domain.

(e) $P(x) = \sqrt{x+1} + \sqrt{6-x}$ D: $[-1, 6]$

Each square root must have a nonnegative argument, so we need $x + 1 \geq 0$ and $6 - x \geq 0$. (Equality is allowed, since $\sqrt{0} = 0$.) Together, we must have both $x \geq -1$ and $x \leq 6$ satisfied.