

Inverse Trig Functions

Remember: $\sin^{-1} x = y$ means $\sin y = x$

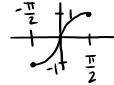
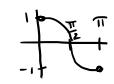
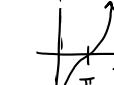
↑ ratio ↓ angle

angle ↗ ratio

For example: $\sin^{-1} \frac{1}{2} = \frac{\pi}{6}$, because $\sin \frac{\pi}{6} = \frac{1}{2}$

$$\frac{1}{2} \xrightarrow[\sin]{\sin^{-1}} \frac{\pi}{6}$$

Read " $\sin^{-1} \frac{1}{2}$ " as "the angle (between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$) whose sine is $\frac{1}{2}$ "

Inverse Function	Domain	Range	Possible Quadrants	Graph of associated trig function w/ restricted domain
$y = \sin^{-1} x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$	$+ \begin{matrix} * \\ * \end{matrix}$	
$y = \cos^{-1} x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$	$\begin{matrix} * \\ + \end{matrix} *$	
$y = \tan^{-1} x$	all reals	$-\frac{\pi}{2} < y < \frac{\pi}{2}$	$+ \begin{matrix} * \\ * \end{matrix}$	
$y = \csc^{-1} x$	$x \leq -1$ or $x \geq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ $y \neq 0$	$+ \begin{matrix} * \\ * \end{matrix}$	
$y = \sec^{-1} x$	$x \leq -1$ or $x \geq 1$	$0 \leq y \leq \pi$ $y \neq \frac{\pi}{2}$	$\begin{matrix} * \\ + \end{matrix} *$	
$y = \cot^{-1} x$	all reals	$0 < y < \pi$	$+ \begin{matrix} * \\ * \end{matrix}$	

Remember: ① $\sin^{-1} x \neq \frac{1}{\sin x}$ (but $\frac{1}{\sin x} = \csc x$)

② $\sin^{-1} x$ and $\arcsin x$ mean the same thing

③ $\sin(\sin^{-1} x) = x$, but $\sin^{-1}(\sin x) \neq x$ if x is outside the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$