

Station 1: Rational Expressions — Combine & Reduce

Goal: Practice rational simplification by factoring and reducing

Simplify each of the following rational expressions

$$1. \frac{x^2-9}{x^2-x-6} = \frac{(x+3)(x-3)}{(x-3)(x+2)} = \frac{x+3}{x+2}$$

$$2. \frac{x^2-4x}{x^2+5x} = \frac{x(x-4)}{x(x+5)} = \frac{x-4}{x+5}$$

$$\begin{aligned} 3. \frac{x^4-16}{x+2} &= \frac{(x^2-4)(x^2+4)}{x+2} \\ &= \frac{(x+2)(x-2)(x^2+4)}{x+2} \\ &= (x-2)(x^2+4) \end{aligned}$$

Station 2: Factor Then Simplify

Goal: Recognize factoring opportunities inside larger expressions

Simplify each of the following expressions

$$1. \frac{x^2 - 9x}{x} + 9 = \frac{\cancel{x}(x-9)}{\cancel{x}} + 9 = x-9+9 \\ = x$$

$$2. \frac{x^2 + 5x + 6}{x^2 - x - 6} \cdot \frac{x^2 - 4x - 12}{x^2 + 7x + 12} \\ = \frac{(x+2)(x+3)}{(x-3)(x+2)} \cdot \frac{(x-6)(x+2)}{(x+3)(x+4)} = \frac{(x+2)(x-6)}{(x-3)(x+4)}$$

$$3. \frac{4x^2 - 9}{2x+3} = \frac{(2x-3)(2x+3)}{2x+3} = 2x-3$$

Station 3: Basic Exponent Rules — Multiply, Divide, Power of a Power

Goal: Reinforce foundational exponent rules.

Simplify each of the following using exponent properties

$$1. \frac{x^2y^4}{(2xy)^3} = \frac{x^2y^4}{8x^3y^3} = \frac{y}{8x}$$

$$2. \left(\frac{x^{-3}}{3}\right)^{-2} = \left(\frac{1}{3x^3}\right)^{-2} = \left(\frac{3x^3}{1}\right)^2 = 3^2(x^3)^2 \\ = 9x^6$$

$$3. \sqrt{x^3} \cdot \sqrt[3]{x^2} = x^{3/2} \cdot x^{2/3} = x^{3/2 + 2/3} \\ = x^{13/6}$$

$$4. \left(\frac{\sqrt{x}}{x^{1/4}}\right)^3 = \left(\frac{x^{1/2}}{x^{1/4}}\right)^3 = \left(x^{\frac{1}{2} - \frac{1}{4}}\right)^3 \\ = (x^{1/4})^3 \\ = x^{3/4}$$

Station 4: Complex Fractions — Clean It Up

Goal: Simplify complex (fractions within fractions) expressions by identifying least common denominators and multiplying strategically.

Simplify each of the complex fractions

$$1. \quad \left(\frac{3+\frac{2}{x}}{\frac{1}{1-x}} \right) \left(\frac{x}{x} \right) = \frac{3x+2}{x-1}$$

\uparrow
 $LCM = x$

$$2. \quad \frac{\frac{6}{x+5} - \frac{1}{x}}{\frac{-2}{x}} \left(\frac{x(x+5)}{x(x+5)} \right) = \frac{6x - (x+5)}{-2(x+5)}$$

\uparrow
 $LCM = x(x+5)$

$$= \frac{6x - x - 5}{-2(x+5)}$$
$$= \frac{5x - 5}{-2(x+5)}$$
$$= \frac{-5(x-1)}{2(x+5)}$$

Station 5: Expressions You'll Likely See in Calculus

Goal: Work with expressions resembling those in Calculus

Simplify each of the following expressions

$$\begin{aligned} 1. \quad \frac{(x+h)^2 - x^2}{h} &= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} \\ &= \frac{2xh + h^2}{h} \\ &= \frac{h(2x + h)}{h} \\ &= 2x + h \end{aligned}$$

$$\begin{aligned} 2. \quad \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \left(\frac{x(x+h)}{x(x+h)} \right) &= \frac{x - (x+h)}{h x (x+h)} \\ \text{LCD} = x(x+h) \quad &= \frac{-h}{h x (x+h)} \\ &= \frac{-1}{x(x+h)} \end{aligned}$$

