A Geometry

Question 1. Reviewing lines:

(a) Find an equation for a horizontal line through the point $(2, -5)$.

of the form $y = b$

y-coordinate

$\boxed{y = -5}$

(b) Find an equation for a line through the point $(2, -5)$ that is parallel to the line $2x - 4y = 3$.

(x_1, y_1)

same slope

$-4y = -2x + 3$

$y = \frac{1}{2}x - \frac{3}{4}$

Point-slope form: $y - y_1 = m(x - x_1)$

$m = \text{slope} = \frac{1}{2}$

$y - (-5) = \frac{1}{2}(x - 2)$

$\boxed{y = \frac{1}{2}x - 6}$

Question 2. What is the area of a circle with radius 3? What is the circumference?

$A = \pi r^2 = \pi(3^2) = \boxed{9\pi \text{ square units}}$

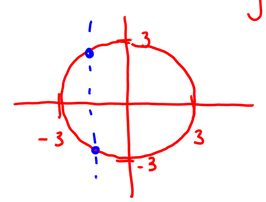
$C = 2\pi r = 2\pi \cdot 3 = \boxed{6\pi \text{ units}}$

Question 3. Sketch the graph of the equation $x^2 + y^2 = 9$. Is it the graph of a function?

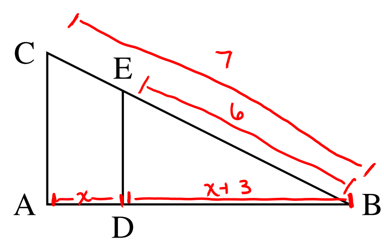
$x^2 + y^2 = r^2$, so $r = 3$

centered at the origin

Not a function:
Fails the vertical line test



Question 4. In the picture below, find the length of AB if DB is $x + 3$ cm, AD is x cm, CB is 7 cm, and EB is 6 cm. Assume that AC and DE are parallel.



Similar triangles

$\frac{EB}{CB} = \frac{DB}{AB}$

$AB = AD + DB$
$= x + (x+3) = 2x + 3$

$\frac{6}{7} = \frac{x+3}{2x+3}$

$6(2x+3) = 7(x+3)$
$12x + 18 = 7x + 21$
$5x = 3$
$x = \frac{3}{5}$

$AB = 2x + 3$
$= 2(\frac{3}{5}) + 3$
$= \frac{6}{5} + \frac{15}{5}$
$\boxed{= \frac{21}{5} \text{ cm.}}$

B Trigonometry

Question 5. Write in terms of sine and cosine:

(a) $\tan x = \dfrac{\sin x}{\cos x}$

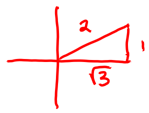
(b) $\cot x = \dfrac{\cos x}{\sin x}$

(c) $\sec x = \dfrac{1}{\cos x}$

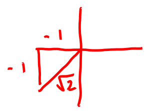
(d) $\csc x = \dfrac{1}{\sin x}$

Question 6. Evaluate (if possible):

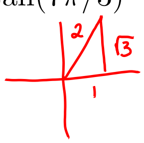
(a) $\sin\left(\dfrac{\pi}{6}\right) = \dfrac{1}{2}$



(b) $\cos\left(-\dfrac{3\pi}{4}\right) = -\dfrac{1}{\sqrt{2}}$



(c) $\tan(7\pi/3) = \sqrt{3}$



(d) $\cot(-\pi/3) = -\dfrac{1}{\sqrt{3}}$

(e) $\sec\left(\dfrac{\pi}{2}\right) = \dfrac{1}{\cos\frac{\pi}{2}} = \cancel{\dfrac{1}{0}}$

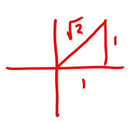
undefined

(f) $\csc \pi = \dfrac{1}{\sin \pi} = \cancel{\dfrac{1}{0}}$

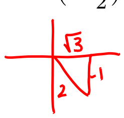
undefined

Question 7. Evaluate:

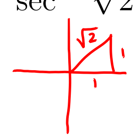
(a) $\tan^{-1} 1 = \dfrac{\pi}{4}$



(b) $\arcsin(-\frac{1}{2}) = -\dfrac{\pi}{6}$



(c) $\sec^{-1} \sqrt{2} = \cos^{-1} \dfrac{1}{\sqrt{2}} = \dfrac{\pi}{4}$



(d) $\arctan 0 = 0$

$\tan 0 = 0$

Question 8. Simplify:

(a) $2\sin^2 x + 2\cos^2 x - 1$

$= 2\underbrace{(\sin^2 x + \cos^2 x)}_{\text{identity}} - 1$

$= 2(1) - 1$

$\boxed{= 1}$

(b) $\dfrac{\sec x}{\tan x} = \dfrac{\frac{1}{\cancel{\cos x}}}{\frac{\sin x}{\cancel{\cos x}}} \cdot \dfrac{\frac{\cancel{\cos x}}{1}}{\frac{\cancel{\cos x}}{1}}$

$= \dfrac{1}{\sin x}$

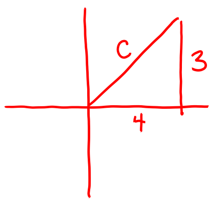
$\boxed{= \csc x}$

Video solution in D2L

Question 9. Use right triangles to solve the following:

(a) If $\tan\theta = \frac{3}{4}$ and $-\pi/2 \leq \theta \leq \pi/2$, find $\cos\theta$.

positive, so $0 \leq \theta \leq \frac{\pi}{2}$



$\tan\theta = \frac{O}{A}$

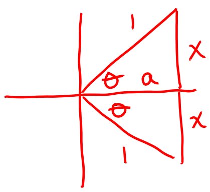
$c^2 = 3^2 + 4^2$

$c^2 = 25 \quad c = \pm 5 \to c = 5$

$\cos\theta = \frac{A}{H} = \boxed{\frac{4}{5}}$

(b) Find an equivalent algebraic expression (so no trig functions allowed!) for $\cos(\sin^{-1} x)$.

$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$ (b/c of restriction on range of $\sin^{-1} x$)



Let $\theta = \sin^{-1} x$

θ could be in QI or QIV

Since $\sin\theta = x = \frac{x}{1}$,

we can label triangle

x = opposite, 1 = hypotenuse

$\cos(\sin^{-1} x) = \cos\theta$

$= \frac{\sqrt{1-x^2}}{1}$

$\boxed{= \sqrt{1-x^2}}$

Need to find a:

$a^2 + x^2 = 1$

$a^2 = 1 - x^2$

$a = \pm\sqrt{1-x^2}$

But a is in right half-plane, so

$a > 0$

$a = \sqrt{1-x^2}$

C Basic graphing

Question 10. **Sketch** the graph of each of the functions below, indicating x- and y-intercepts and asymptotes, and **write the domain and range** of each function. No calculators!

(a) $f(x) = 3x - 2$

(b) $f(x) = x^2$

(c) $f(x) = x^3$

(d) $f(x) = |x|$

(e) $f(x) = \frac{1}{x}$

(f) $f(x) = \sqrt{x}$

*most are in resource in D2L folder

(g) $f(x) = e^x$

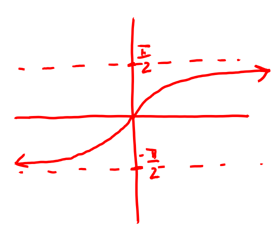
(h) $f(x) = \ln x$

(i) $f(x) = \sin x$

(j) $f(x) = \cos x$

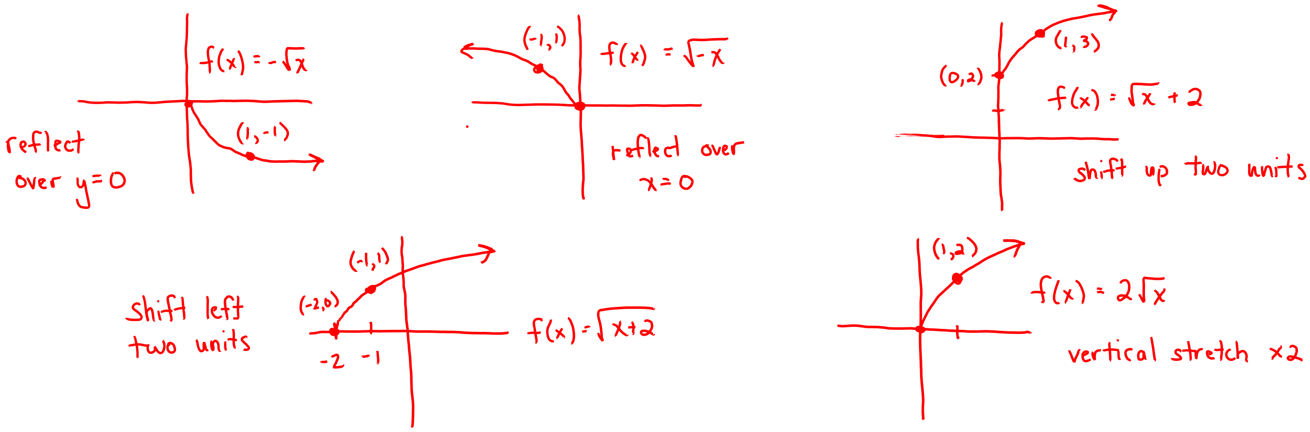
(k) $f(x) = \tan x$

(l) $f(x) = \tan^{-1} x$

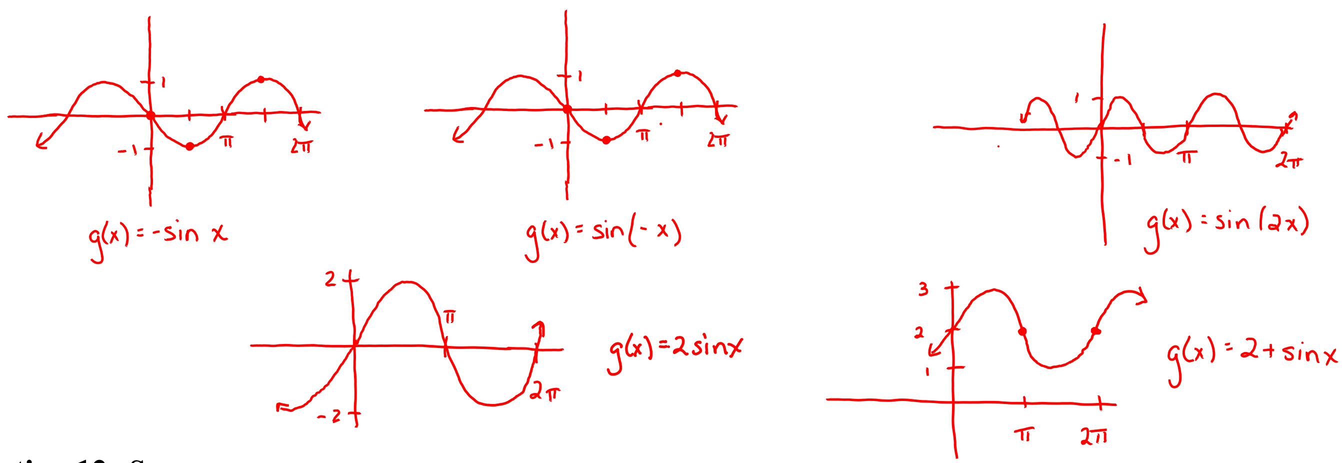


Question 11. Use transformations to sketch the graphs of each of the following:

(a) $f(x) = -\sqrt{x},\ f(x) = \sqrt{-x},\ f(x) = \sqrt{x} + 2,\ f(x) = \sqrt{x+2},\ f(x) = 2\sqrt{x}$



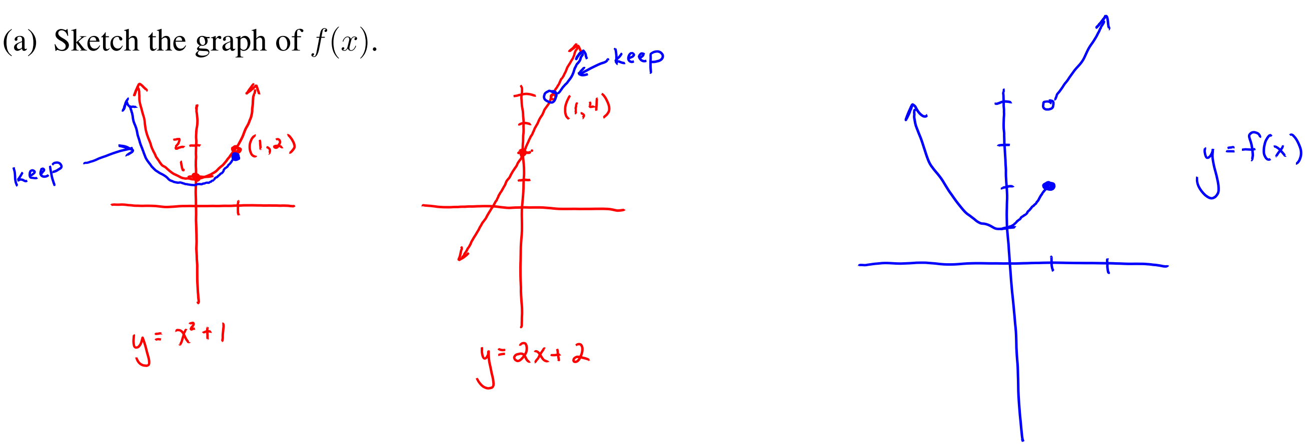
(b) $g(x) = -\sin x,\ g(x) = \sin(-x),\ g(x) = \sin(2x),\ g(x) = 2\sin x,\ g(x) = 2 + \sin x$



Question 12. Suppose

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 1 \\ 2x + 2 & \text{if } x > 1. \end{cases}$$

(a) Sketch the graph of $f(x)$.



(b) Find $f(-2)$, $f(1)$, and $f(5)$.

$f(-2) = (-2)^2 + 1 = 5 \qquad f(1) = 1^2 + 1 = 2 \qquad f(5) = 2 \cdot 5 + 2 = 12$

D Exponential and Logarithmic functions

Question 13. First a review of basic exponent rules: Simplify each expression *without using a calculator*.

(a) $(-2)^4 = (-2)(-2)(-2)(-2)$
$= \boxed{16}$

(b) $-2^4 = -(2)(2)(2)(2)$
$= \boxed{-16}$

(c) $2^{-4} = \frac{1}{2^4} = \boxed{\frac{1}{16}}$

(d) $\frac{3^{15}}{3^{17}} = \frac{1}{3^{17-15}} = \frac{1}{3^2} = \boxed{\frac{1}{9}}$

(e) $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^3 = \frac{5^3}{2^3} = \boxed{\frac{125}{8}}$

(f) $81^{-3/4} = \frac{1}{81^{\frac{3}{4}}} = \frac{1}{(81^{\frac{1}{4}})^3} = \frac{1}{\sqrt[4]{81}^{\,3}}$
$= \frac{1}{3^3} = \boxed{\frac{1}{27}}$

Question 14. Simplify the following:

(a) $\ln e^{\sin x} = \boxed{\sin x}$
b/c $\ln e^A = A$

(b) $\log_2 \frac{1}{8} = -3$

2 to what power is $\frac{1}{8}$?

(c) $\ln 8 - \ln 2 = \ln \frac{8}{2} = \ln 4$

(d) $e^{x+\ln(5)} = e^x \cdot e^{\ln 5} = e^x \cdot 5 = 5e^x$
$(x^{a+b} = x^a \cdot x^b)$ $(e^{\ln A} = A)$

Question 15. If $f(x) = e^{2x} - 3e^x + 1$, find $f(0)$ and $f(\ln 2)$.

$f(0) = e^0 - 3e^0 + 1 = 1 - 3(1) + 1 = \boxed{-1}$

$f(\ln 2) = e^{2\ln 2} - 3e^{\ln 2} + 1 =$
$= (e^{\ln 2})^2 - 3e^{\ln 2} + 1 = 2^2 - 3 \cdot 2 + 1 = \boxed{-1}$

Note: $x^{ab} = (x^a)^b$, so $e^{2\ln 2} = (e^{\ln 2})^2$

Question 16. Use properties of logarithms to expand the following into a sum, difference, and/or multiple of logarithms.

(a) $\ln\left(\frac{x \sin x}{\sqrt{x+2}}\right) = \ln(x \sin x) - \ln(x+2)^{\frac{1}{2}}$

$\boxed{= \ln x + \ln(\sin x) - \frac{1}{2}\ln(x+2)}$

(b) $\ln(e^x \sqrt{x} \cos x)^3 = 3\ln(e^x \cdot x^{\frac{1}{2}} \cdot \cos x)$

$\sqrt{x} = x^{\frac{1}{2}}$

$= 3(\ln e^x + \ln x^{\frac{1}{2}} + \ln(\cos x))$ $\quad * \ln e^x = x$

$= \boxed{3(\underline{x + \frac{1}{2}\ln x + \ln(\cos x)})}$

parentheses around this needed!

E Functions

Question 17. If $f(x) = x^2 + 4$, evaluate and simplify $\frac{f(2+h) - f(2)}{h}$.

$$f(2+h) = (2+h)^2 + 4 = (2+h)(2+h) + 4$$
$$= 4 + 4h + h^2 + 4$$
$$= 8 + 4h + h^2$$

$$\frac{f(2+h) - f(2)}{h} = \frac{(8 + 4h + h^2) - (2^2 + 4)}{h}$$
$$= \frac{4h + h^2}{h}$$
$$= \frac{\cancel{h}(4+h)}{\cancel{h}}$$
$$= 4 + h \qquad (h \neq 0)$$

Question 18. Find the domain:

(a) $g(x) = \dfrac{\sqrt[3]{x}}{x^2 + 1}$.

$\sqrt[3]{x}$: all reals

$x^2 + 1$: all reals

$x^2 + 1 = 0$
no solutions

$\boxed{\text{Domain: } (-\infty, \infty)}$

(b) $f(x) = \dfrac{\ln(x+2)}{x-1}$

$\ln(x+2)$ needs $x + 2 > 0$
$x > -2$

$x - 1 \neq 0$, so $x \neq 1$

$\boxed{\text{Domain: } (-2, 1) \cup (1, \infty)}$

(c) $h(x) = \sqrt{e^x + 1}$

$e^x + 1$: all reals

$\sqrt{e^x+1}$ needs $e^x + 1 \geq 0$
$e^x \geq -1$
always true!

$\boxed{\text{Domain: } (-\infty, \infty)}$

Question 19. Function composition

(a) If $f(x) = x^2 + 2x$ and $g(x) = \sin x$, find $g \circ f$ and $f \circ g$.

$(g \circ f)(x) = g(f(x)) = \sin(x^2 + 2x)$

$(f \circ g)(x) = f(g(x)) = (\sin x)^2 + 2(\sin x) = \sin^2 x + 2 \sin x$

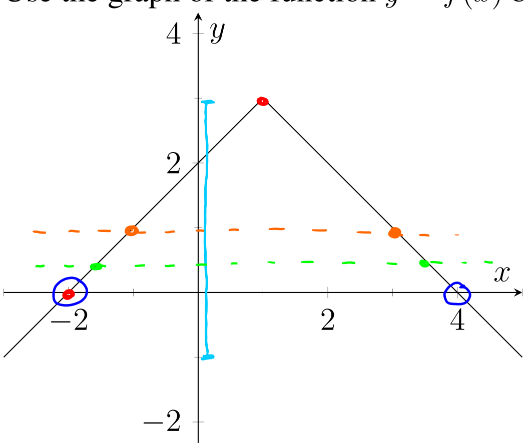
(b) Let $h(x) = \sqrt{x^3 + 2}$. Find two functions $f(x)$ and $g(x)$ so that $h(x) = g(f(x))$. (Don't use $f(x) = x$ or $g(x) = x$.)

$f(x) = x^3 + 2, \quad g(x) = \sqrt{x}$

or: $f(x) = x^3, \quad g(x) = \sqrt{x+2}$

F Interpreting graphs of functions

Use the graph of the function $y = f(x)$ below to answer the questions in this section.



Question 20. Find $f(1)$ and $f(-2)$.

$f(1) = 3, \quad f(-2) = 0$

Question 21. How many zeros does $f(x)$ have on the domain $[-3, 5]$?

Two zeros $(x = -2, \; x = 4)$

Question 22. Find all solutions to the equation $f(x) = 1$ on the interval $[-3, 5]$.

$x = -1, \quad x = 3$

Question 23. Find the range of $f(x)$ on the domain $[-3, 5]$.

$-1 \le y \le 3$

Question 24. Find all values of x for which $(x, 1/2)$ is a point on the graph of $y = f(x)$.

$x = -1.5, \quad x = 3.5$

G Algebra skills

Question 25. Subtract:

(a) $\frac{1}{x} - \frac{1}{\sin x} = \frac{1}{x} \cdot \frac{\sin x}{\sin x} - \frac{1}{\sin x} \cdot \frac{x}{x}$

common denominator $= \frac{\sin x}{x \sin x} - \frac{x}{x \sin x}$

$= \frac{\sin x - x}{x \cdot \sin x}$ * can't be simplified!

(b) $\frac{1}{(x+h)+1} - \frac{1}{x+1} = \frac{1}{x+h+1} \cdot \frac{x+1}{x+1} - \frac{1}{x+1} \cdot \frac{x+h+1}{x+h+1}$

common denominator $= \frac{x+1}{(x+1)(x+h+1)} - \frac{x+h+1}{(x+1)(x+h+1)}$

distribute negative $= \frac{x+1-x-h-1}{(x+1)(x+h+1)} = -\frac{h}{(x+1)(x+h+1)}$

Question 26. Simplify the following:

(a) $\frac{x^2+x-6}{x+3} = \frac{\cancel{(x+3)}(x-2)}{\cancel{x+3}}$

$= \boxed{x-2}$ $(x \neq -3)$

(b) $\frac{x^2 \cdot \frac{1}{x} - 2x \ln x}{(x^2)^2} = \frac{x - 2x \ln x}{x^4}$

$x^2 \cdot \frac{1}{x} = x$

$(x^a)^b = x^{ab}$ $= \frac{\cancel{x}(1-2\ln x)}{x^{\cancel{4}\,3}} = \boxed{\frac{1-2\ln x}{x^3}}$

(c) $\frac{\frac{3}{x}}{\frac{y}{x^2}} = \frac{3}{\cancel{x}} \cdot \frac{x^{\cancel{2}\,1}}{y}$ * mult. by reciprocal

$= \boxed{\frac{3x}{y}}$

Question 27. Rewrite as a product, sum, or difference using negative exponents:

(a) $\frac{\sin x}{x^2} = \boxed{(x^{-2})(\sin x)}$

(b) $\frac{1-\sqrt{x}}{x^3} = x^{-3}(1 - x^{\frac{1}{2}})$ distribute

$= \boxed{x^{-3} - x^{-\frac{5}{2}}}$

note: $x^{-3} \cdot x^{\frac{1}{2}} = x^{-3+\frac{1}{2}} = x^{-\frac{5}{2}}$

Question 28. Factor the following completely:

Help on some of these can be found in the video solution to Questions 31 and 33 (starred problems)

(a) $9x^2 - 16$ $= (3x+4)(3x-4)$

Difference of squares

(b) $x^5 + 8x^2$ $= x^2(\underbrace{x^3+8}_{\text{sum of cubes}})$

$= x^2(x+2)(x^2-2x+4)$

(c) $x^2 - x - 12$ $= (x-4)(x+3)$

* (d) $x - x^{1/3}$ $= x^{\frac{1}{3}}(x^{\frac{2}{3}}-1)$

* Factor out the smallest exponent

$= x^{\frac{1}{3}}(x^{\frac{2}{9}}-1)(x^{\frac{4}{9}}+x^{\frac{2}{9}}+1)$

Difference of cubes (although this factoring step may not make the expression easier to handle)

* (e) $(x+1)^{-4}(3)(x+5)^2 + (-4)(x+1)^{-5}(x+5)^3$

* Factor out the smallest exponent

$= (x+1)^{-5}(x+5)^2\left[(x+1)(3) + (-4)(x+5)\right]$

$= (x+1)^{-5}(x+5)^2(3x+3-4x-20)$

$= (x+1)^{-5}(x+5)^2(-x-17)$

(f) $\sin^2 x - 2\sin x + 1$

* Like $u^2 - 2u + 1 = (u-1)(u-1)$

$= (\sin x - 1)(\sin x - 1)$

$= (\sin x - 1)^2$

(g) $xe^x - e^x$

$= e^x(x-1)$

* (h) $x\ln x + x$

$= x(\ln x + 1)$

Video solution in D2L

Question 29. Write the following expression so that there is no root in the numerator, and then simplify.

$$\frac{\sqrt{x+h}-1-(\sqrt{x}-1)}{h}$$ *First distribute negative

$$=\frac{\sqrt{x+h}-1-\sqrt{x}+1}{h}$$ *add −1 and +1

$$=\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})}$$ multiply top and bottom by conjugate

$$=\frac{\sqrt{x+h}^{2}+\cancel{\sqrt{x}\sqrt{x+h}}-\cancel{\sqrt{x}\sqrt{x+h}}-\sqrt{x}^{2}}{h(\sqrt{x+h}+\sqrt{x})}$$ offset offset distribute (conjugate magic!)

$$=\frac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})}$$

$$=\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}=\boxed{\frac{1}{\sqrt{x+h}+\sqrt{x}}}$$

Video solution in D2L

Question 30. Solve for z in the following and simplify:

(a) $2x+2yz=\dfrac{x^2\cdot 4yz-2y^2\cdot 2x}{(x^2)^2}$ multiply both sides by $(x^2)^2=x^4$

$x^4(2x+2yz)=4x^2yz-4xy^2$ distribute

$2x^5+2x^4yz=4x^2yz-4xy^2$ sort z terms to left, others to right

$2x^4yz-4x^2yz=-4xy^2-2x^5$ factor out z

$z(2x^4y-4x^2y)=-4xy^2-2x^5$ divide

$z=\dfrac{-4xy^2-2x^5}{2x^4y-4x^2y}=\boxed{\dfrac{-2y^2-x^4}{x^3y-2yx}}$ (simplify $2x$ out of top and bottom)

(b) $2xy+x^2z=e^{xy}(y+xz)$ distribute

$2xy+x^2z=ye^{xy}+xze^{xy}$ sort terms

$x^2z-xze^{xy}=ye^{xy}-2xy$ factor out z

$z(x^2-xe^{xy})=ye^{xy}-2xy$ divide

$$\boxed{z=\frac{ye^{xy}-2xy}{x^2-xe^{xy}}}$$

H Solving Equations

Video solution to starred questions in D2L

Question 31. Solve the following equations (Hint: Many involve using the factoring you already did in an earlier question.)

(a) $9x^2 - 16 = 0$

$(3x+4)(3x-4)=0$

$3x+4=0 \quad 3x-4=0$

$\boxed{x=-\frac{4}{3} \qquad x=\frac{4}{3}}$

(b) $x^2 - x - 12 = 0$

$(x-4)(x+3)=0$

$x-4=0 \quad x+3=0$

$\boxed{x=4 \qquad x=-3}$

(c) $x^2 - x = 13$

$x^2-x-13=0$ doesn't factor, use quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{1\pm\sqrt{1-4(1)(-13)}}{2(1)}$

$\boxed{x=\frac{1\pm\sqrt{53}}{2}}$

(d) $x^2 - x + 12 = 0$

Doesn't factor, use quadratic formula

$x=\frac{1\pm\sqrt{1-4(1)(12)}}{2(1)}$ ← Discriminant is negative, $\boxed{\text{no real solutions}}$

* (e) $x - x^{1/3} = 0$

$$x^{\frac{1}{3}}\left(x^{\frac{2}{3}} - 1\right) = 0$$

$$x^{\frac{1}{3}} = 0 \qquad\qquad x^{\frac{2}{3}} - 1 = 0$$

$$\sqrt[3]{x} = 0 \qquad\qquad \sqrt[3]{x^2} = 1$$

$$x^2 = 1$$

$$\boxed{x = 0 \qquad\qquad x = 1,\ x = -1}$$

* (f) $(x+1)^{-4}(3)(x+5)^2 + (-4)(x+1)^{-5}(x+5)^3 = 0$

$$(x+1)^{-5}(x+5)^2(-x-17) = 0$$

$$\frac{(x+5)^2(-x-17)}{(x+1)^5} = 0$$

$$(x+5)^2(-x-17) = 0$$

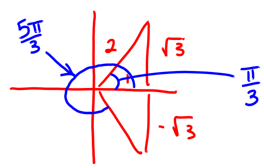
$$x+5 = 0 \qquad -x-17 = 0$$

$$\boxed{x = -5 \qquad x = -17}$$

Question 32. Solve the following trigonometric equations:

(a) $\cos x = \frac{1}{2}$ on $[0, 2\pi]$

$$\boxed{x = \frac{\pi}{3}, \quad x = \frac{5\pi}{3}}$$



(b) $\sin^2 x + 1 = 2\sin x$ on $[-2\pi, 2\pi]$ (Use your factoring above)

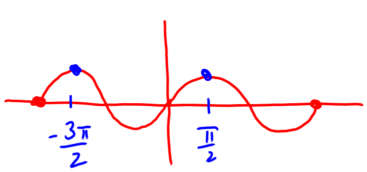
$$\sin^2 x - 2\sin x + 1 = 0$$

$$(\sin x - 1)^2 = 0$$

$$\sin x - 1 = 0$$

$$\sin x = 1$$

$$\boxed{x = -\frac{3\pi}{2}, \quad x = \frac{\pi}{2}}$$



Question 33. Solve the following equations using the factoring you already completed in an earlier question.

(a) $xe^x - e^x = 0$

$e^x(x-1) = 0$

$e^x = 0$ no solutions

$x - 1 = 0$

$\boxed{x = 1}$

(b) $x \ln x + x = 0$

$x(\ln x + 1) = 0$

$\cancel{x = 0}$ not in domain $\ln 0$ DNE

$\ln x + 1 = 0$

$\ln x = -1$

$e^{-1} = x$

$\boxed{x = \frac{1}{e}}$

I Solving Inequalities

Question 34. Solve the following inequalities and express the answer using interval notation.

(a) $3 + 7x < 5$

$7x < 2$

$x < \frac{2}{7}$

$\boxed{\left(-\infty, \frac{2}{7}\right)}$

(b) $-3 < 7 - 2x \leq 15$

$-7 \quad -7 \quad -7$

Subtract -7 from all 3 "sides"

$-10 < -2x \leq 8$

Divide by -2, reverse inequalities

$5 > x \geq -4$

$\boxed{[-4, 5)}$

Question 35. Solve the following inequalities and express the answer using interval notation.

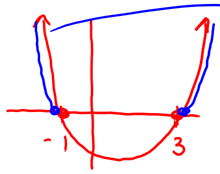
(a) $x^2 \geq 2x + 3$

$x^2 - 2x - 3 \geq 0$

$(x-3)(x+1) \geq 0$

$\boxed{(-\infty, -1] \cup [3, \infty)}$

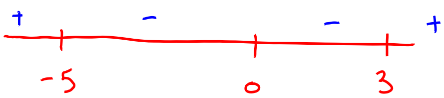
Blue region at left



Where $y \geq 0$

Graph of $y = x^2 - 2x - 3$

(b) $x^2(x-3)(x+5) \leq 0$



zeros of function, sign must be the same in each interval

test -10, -1, 1, and 10

	x^2	$x-3$	$x+5$	overall
-10	+	−	−	+
-1	+	−	+	−
1	+	−	+	−
10	+	+	+	+

here the product of the terms is < 0

$\boxed{(-5, 0) \cup (0, 3)}$

J Detecting errors

For each of the following equations, determine whether it is True (i.e. true for *all* values of x and y) or False (meaning at least sometimes false).

(a) $(x + y)^2 = x^2 + y^2$ False

test $x = 1$, $y = 2$

(b) $\sqrt{x^2 + 4} = x + 2$ False

test $x = 1$

(c) $\sqrt{4x^2} = 2x$ $(x \geq 0)$ True!

(d) $\sin(x + y) = \sin x + \sin y$ False

test $x = \frac{\pi}{4}$, $y = \frac{\pi}{4}$

(e) $\dfrac{1}{x + y} = \dfrac{1}{x} + \dfrac{1}{y}$ False

test $x = 1$, $y = 2$