Applied Optimization

Find 2 positive, real numbers such that they sum to 105 and the product of the 1st and the square of the 2nd is the maximum.

$x + y = 105$

• maxmize $x \cdot y^2$

$$x = 105 - y$$
$$f(y) = x \cdot y^2 = (105 - y) \cdot y^2$$

• To find the max., take the derivative & set ~~is~~ equal to zero.

$$\frac{d}{dy}\left[(105-y) \cdot y^2\right] \rightarrow 210y - 3y^2$$

$\rightarrow 105y^2 - y^3$

$$210y - 3y^2 = 0$$
$$3y(70 - y) = 0$$

$3y = 0$

$y = 0$

$70 - y = 0$

$-y = -70$

$y = 70$

$y = 0$ or 70

• Condition for y: positive, real #s

$y = 70$

$$x + y = 105$$
$$x = 105 - y$$
$$x = 105 - 70 = 35$$

x and y are 35 and 70.

2nd Derivative test

Verify $y = 70$ is max.

$f(y) = (105 - y) \cdot y^2$ or $105y^2 - y^3$

$f'(y) = 210y - 3y^2$

$f''(y) = 210 - 6y$

$f''(70) = 210 - 420 = -210$

$f''(70) < 0$

It is the maximum

Find a positive number x such that the sum of x and its reciprocal is as small as possible.

$f(x) = x + \frac{1}{x} \quad , \quad x > 0$

FIRST DERIVATIVE TEST

$\frac{d}{dx}\left(\frac{1}{x}\right) = -1/x^2$

$\frac{f'g - g'f}{g^2} \qquad \frac{0(x) - 1(1)}{x^2} = \frac{-1}{x^2}$

$f'(x) = 1 - \frac{1}{x^2}$

$1 - 1/x^2 = 0$

$-1/x^2 = -1$

$1 = 1x^2$

$\pm 1 = x$

• Condition: $x > 0$

$1 = x$

$x = 1$

Verify $x = 1$ is a minimum

$f''(x) = 0 - 2x/x^4$

$\frac{0(x^2) - 2x(-1)}{x^4} = 2x/x^4$

$2x/x^4 = 2(1)/(1)^4 = 2/1$

$= 2$

$2 > 0$

$x = 1$ is a minimum