Applied Optimization

A jewelry box with a square base is to be built w/ silver plated sides, nickel plated top and bottom, and a volume of $44 cm^3$. If nickel plating costs \$1 per cm^2 and silver plating costs \$8 per cm^2, find the dimensions of the box to ~~mini~~ minimize the cost of materials.

$b^2h = 44$

$h = 44/b^2$

nickel: $2 \cdot b^2$ at \$1/cm^2 $= 2b^2$

silver: $4 \cdot b \cdot h$ at \$8/cm^2 $= 32bh$

Total Cost: $2b^2 + 32bh$

$f(b) = 2b^2 + 32b(44/b^2)$

$f(b) = 2b^2 + \frac{1408b}{b^2}$

$= 2b^2 + 1408/b$

$f'(b) = 4b - \frac{1408}{b^2}$

$4b - \frac{1408}{b^2} = 0$

$4b = \frac{1408}{b^2}$

$4b^3 = 1408$

$b^3 = 352$

$b \approx 7.0607$

$\frac{0(b) - 1(1408)}{b^2} = \frac{-1408}{b^2}$

length: 7.0607 cm

height: 0.8826 cm

$h = 44/7.0607^2 \approx 0.8826$

Verify Minimum

$f''(b) = 4 - \frac{2816}{b^3}$

$4 - \frac{2816}{7.0607^3} = 4 - \frac{2816}{352} \approx 3.2017$

$3.2017 > 0$

$\frac{0(b^2) - 2b(1408)}{b^4} = \frac{-2816b}{b^4}$

$= \frac{-2816}{b^3}$

A rectangular bird sanctuary with one side along a straight river is to be constructed so that it contains $32 km^2$ of area. Find the dimensions of the rectangle to minimize the amount of fence necessary to enclose the remaining 3 sides.

Length of fence perpendicular to riverbank- 4 km
length of fence parallel to riverbank- 8 km

area of rect. = $l \cdot h$

$lh = 32$

$h = 32/l$

Fence needed: $f(l) = l + 2(32/l)$

$= l + 64/l$

$f'(l) = 1 - \frac{64}{l^2}$

$1 - \frac{64}{l^2} = 0$

$1 = 64/l^2$

$l^2 = 64$

$l = 8$

$\frac{0(l) - 1(64)}{l^2} = \frac{-64}{l^2}$

Verify Minimum

$f''(l) = 128/l^3$

$128/8^3 = 0.25$

$0.25 > 0$

$\frac{0(l^2) - 2l(-64)}{l^4}$

$= \frac{128l}{l^4} = \frac{128}{l^3}$

$h = 32/l$

$32/8 = 4$

$h = 4$