Solve for $z$ in each equation and simplify if possible:

(a) $\dfrac{y \cdot 1 - xz}{y^2} + \dfrac{xz - 1 \cdot y}{x^2} = 2(1 \cdot y + xz)$

(b) $(-\sin(x^2 + 2y))(2x + 2z) + e^y + xe^y z = 0$

(c) $e^{\cos x}(-\sin x) + e^{\sin y}(\cos y)(z) = y + xz$

**Solutions on the next page**

(a) $\dfrac{y \cdot 1 - xz}{y^2} + \dfrac{xz - 1 \cdot y}{x^2} = 2(1 \cdot y + xz)$

Multiply both sides by $x^2y^2$ to clear the denominators

$$x^2y^2\left[\frac{y-xz}{y^2} + \frac{xz-y}{x^2}\right] = x^2y^2\left[2(y+xz)\right]$$

Distribute

$$x^2(y-xz) + y^2(xz-y) = 2x^2y^2(y+xz)$$

Distribute more

$$x^2y - x^3z + xy^2z - y^3 = 2x^2y^3 + 2x^3y^2z$$

Subtract/keep terms with $z$ on left, move all others to the right

$$-x^3z + xy^2z - 2x^3y^2z = 2x^2y^3 - x^2y + y^3$$

Factor out $z$

$$z(-x^3 + xy^2 - 2x^3y^2) = 2x^2y^3 - x^2y + y^3$$

Divide to isolate $z$

$$\boxed{z = \frac{2x^2y^3 - x^2y + y^3}{-x^3 + xy^2 - 2x^3y^2}}$$

* Note that the only common factor in the terms in the numerator is a $y$, and in the denominator is $x$, so there is no simplifying possible.

(b) $(-\sin(x^2+2y))(2x+2z)+e^y+xe^yz=0$

No fractions to clear

Distribute

Careful: Do NOT "distribute" the sine function to its argument, because that's not valid.

$$(2x)(-\sin(x^2+2y))+(2z)(-\sin(x^2+2y))+e^y+xe^yz=0$$

$$-2x\sin(x^2+2y)-2z\sin(x^2+2y)+e^y+xe^yz=0$$

Keep terms with z on left, move others to the right

$$-2z\sin(x^2+2y)+xe^yz=2x\sin(x^2+2y)-e^y$$

Factor out a z

$$z(-2\sin(x^2+2y)+xe^y)=2x\sin(x^2+2y)-e^y$$

Divide to isolate z

$$\boxed{z=\frac{2x\sin(x^2+2y)-e^y}{-2\sin(x^2+2y)+xe^y}}$$

* No common factors in all terms, so no simplifying possible

(c) $e^{\cos x}(-\sin x) + e^{\sin y}(\cos y)(z) = y + xz$

No fractions to clear

Everything is already distributed

Move $z$ terms to left, others to right

$$e^{\sin y}(\cos y)(z) - xz = y + e^{\cos x}(\sin x)$$

Factor out $z$

$$z\left(e^{\sin y}(\cos y) - x\right) = y + e^{\cos x}(\sin x)$$

Divide to isolate $z$

$$\boxed{z = \frac{y + e^{\cos x}(\sin x)}{e^{\sin y}\cos y - x}}$$

No common factors, so no simplifying possible