L'Hôpital's Rule

- Used to evaluate limits that result in indeterminate form $\left(\frac{0}{0}\right)$

"If you have two functions that are differentiable near a point a, and both approach 0 or $\pm\infty$ as x approaches a, then the limit of their quotient can be found by taking the derivative of the numerator and the derivative of the denominator."

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Evaluate the limit using L'Hôpital's Rule

$$\lim_{x \to -\infty} \frac{\ln(x^4+5)}{11x+2}$$

f' $$= \frac{4x^3}{x^4+5} \cdot \frac{1}{11}$$

$$= \frac{4x^3}{11(x^4+5)}$$

f'' $$= \frac{12x^2}{44x^3} = \frac{4x^2(3)}{4x^2(11x)}$$

$$= \frac{3}{11x}$$

$$= 0$$

Rule: $K/\infty = 0$

$K/-\infty = 0$

$f(x) = \ln(x^4+5)$ Chain Rule $f'(u) \cdot u'$

$f = \ln(u) \quad f' = 1/u$

$u = x^4+5 \quad u' = 4x^3$

$f'(x) = \frac{1}{x^4+5} \cdot 4x^3 = \frac{4x^3}{x^4+5}$

$\frac{d}{dx} \ln(x) = \frac{1}{x}$

Does L'Hôpital's Rule apply? If yes, use it to evaluate the limit.

$$\lim_{x \to -29} \frac{x^2 - 841}{725 - 4x - x^2}$$

$$= 2x / (-4 - 2x)$$

$$\lim_{x \to -29} \frac{2(-29)}{-4 - 2(-29)} = \frac{-58}{54}$$

L'Hôpital's Rule applies & the limit evaluates to $\frac{-58}{54}$.

Evaluate the limit using L'Hôpital's Rule.

$$\lim_{x \to 0} \frac{3^x - 7^x}{x}$$

3^x : $f'(x) = a^x \cdot \ln(a)$

$= 3^x \cdot \ln(3)$

$$= \frac{3^x \ln(3) - 7^x \ln(7)}{1}$$

$g(x) = x$

$g'(x) = 1$

$$\lim_{x \to 0} 3^0 \ln(3) - 7^0 \ln(7)$$

$$= 1(\ln(3)) - 1(\ln(7))$$

$$= \ln(3) - \ln(7) = \ln(3/7)$$

Evaluate the limit.

$$\lim_{x \to \infty} \frac{x^2}{e^{9x}}$$

f' $= \frac{2x}{9e^{9x}}$

f'' $= \frac{2}{81e^{9x}}$

$\lim_{x \to \infty} = 0$

$\frac{d}{dx} e^{9x}$

$f = e^u \quad f' = e^u$

$u = 9x \quad u' = 9$

$e^{9x} \cdot 9 = 9e^{9x}$

Product Rule

$\frac{d}{dx} 9e^{9x}$ $\quad f'g + g'f$

$f = 9 \quad f' = 0$

$g = e^{9x} \quad g' = 9e^{9x}$

$0(e^{9x}) + 9e^{9x}(9)$

$= 81e^{9x}$

Evaluate the limit.

$$\lim_{x \to 0} \frac{\sin(7x)}{x^2 + 12x}$$

$f' = \frac{7\cos(7x)}{2x + 12}$

$\lim_{x \to 0} \frac{7\cos(0)}{12} = \frac{7}{12}$

$\cos(0) = 1$

$f = \sin(u) \quad f' = \cos(u)$

$u = 7x \quad u' = 7$

$\cos(7x) \cdot 7 = 7\cos(7x)$