Calculus: Velocity ~~& Tangent Line~~

VELOCITY

S(t) is a position function. It describes where an object is positioned at time t.

For average velocity, find the total change in position over a time interval. [a,b] represents a ~~hold~~ time interval.

$$V_{avg} = \frac{S(b) - S(a)}{b - a}$$

- The numerator is the displacement (Change in position)
- The denominator is the amount of time that has ~~tu~~ elapsed.
- The Slope of the secant line between a and b is between the ~~pants~~ points (a, s(a)) and (b, s(b)).

For instantaneous velocity, find the velocity of an object at a single instance in time.

$$V(t) = S'(t) = \lim_{\Delta t \to 0} = \cancel{S(t)} \ \frac{S(t+\Delta t) - S(t)}{\Delta t}$$

- Instantaneous velocity is the derivative of the position function.
- V(t) = Slope of Tangent Line

1. A particle's position is given by $S(t) = 2t^2 + 3t$, t as seconds. What is the average velocity on the interval [1,4]?

$$V_{avg} = \frac{S(b) - S(a)}{b - a}$$

$S(b) = S(4) = 2(4)^2 + 3(4) = 32 + 12 = 44$

$S(a) = S(1) = 2(1)^2 + 3(1) = 2 + 3 = 5$

$b = 4 \quad a = 1$

$$\frac{44 - 5}{4 - 1} = \frac{39}{3} = 13$$

2. A particle moves according to $S(t) = \sqrt{t+1}$.
Find the average velocity on $[3, 7]$.

$V_{avg} = \dfrac{S(b) - S(a)}{b - a}$

$S(b) = S(7) = \sqrt{(7)+1} = \sqrt{8}$
$S(a) = S(3) = \sqrt{(3)+1} = \sqrt{4} = 2$
$b = 7$, $a = 3$

$\dfrac{\sqrt{8} - 2}{7 - 3} = \dfrac{\sqrt{8} - 2}{4}$ or $\dfrac{2\sqrt{2} - 2}{4}$

3. The position of an object is $S(t) = \ln(t+2)$. Find the average velocity between $t = 1$ and $t = 4$.

$V_{avg} = \dfrac{S(b) - S(a)}{b - a}$

$S(b) = S(4) = \ln(4 + 2) = \ln(6)$
$S(a) = S(1) = \ln(1 + 2) = \ln(3)$

$\dfrac{\ln(6) - \ln(3)}{4 - 1} = \dfrac{\ln(6) - \ln(3)}{3}$ or $\dfrac{\ln(6/3)}{3}$

quotient rule of natural log

4. A particle's position is $S(t) = 1/t$, $t > 0$. Find the instantaneous velocity at $t = 2$.

$V(t) = S'(t) = \cancel{A}$

quotient rule for derivatives

$\dfrac{d}{dx} \dfrac{1}{t} = \dfrac{f'g - g'f}{g^2}$, $f = 1$ $f' = 0$, $g = t$ $g' = 1$ $\approx \dfrac{0(t) - 1(1)}{t^2} = \dfrac{-1}{t^2} \approx -1/2^2 = -1/4$

5. For $S(t) = \sin(t)$, find the instantaneous velocity at $t = \pi$.

$V(t) = S'(t)$ $\quad \dfrac{d}{dx} \sin(t) = \cos(t)$ $\quad \cos(\pi) = -1$

Trig. Function ~~Rules of~~ Derivatives ~~Rul~~

6. For $S(t) = t^2 e^t$, find the Inst. velocity at $t = 1$.

$V(t) = S'(t)$ $\quad \dfrac{d}{dx} t^2 e^t = f'g + g'f$, $f = t^2$ $f' = 2t$, $g = e^t$ $g' = e^t$

product rule of derivatives

$2t(e^t) + e^t(t^2) = e^t(2t + t^2)$
$e^1(2(1) + (1)^2)$
$= 3e$

7. Find the Inst. velocity for #3 at $t = 3$.

$V(t) = S'(t)$ $\quad \dfrac{d}{dx} \ln(t+2) = \dfrac{1}{t+2}$ $\quad \dfrac{1}{(3)+2} = \dfrac{1}{5}$

Chain rule & Derivative rule for natural log