

Name: Starlan DavisNet ID: cdavis10

Exam 2

MATH 1190 - Calculus 1 (Fall 2025) - section SSS

Q1) (3 points each) Find the derivative of y with respect to x.

$$\bullet y = e^{\cos x} + \pi^3$$

$$e^{\cos x} \cdot (-\sin(x))$$

$$(-\sin(x)) e^{\cos(x)}$$

$$\bullet y = \frac{3 \tan x}{x^{\frac{1}{5}} - 2x^3 + 5}$$

$$f = 3 \tan x \quad f' = 3 \sec^2(x)$$

$$g = x^{\frac{1}{5}} - 2x^3 + 5 \quad g' = \frac{1}{5} x^{-\frac{4}{5}} - 6x^2$$

$$g' = \frac{1}{5} x^{-\frac{4}{5}} - 6x^2$$

$$\frac{\frac{1}{5} x^{-\frac{4}{5}} - 6x^2}{(x^{\frac{1}{5}} - 2x^3 + 5)^2}$$

$$\frac{3 \sec^2(x) (x^{\frac{1}{5}} - 2x^3 + 5) - (\frac{1}{5} x^{-\frac{4}{5}} - 6x^2) 3 \tan x}{(x^{\frac{1}{5}} - 2x^3 + 5)^2}$$

$$\bullet y = \ln(\sin(2 - 2x^{-3}))$$

$$\frac{1}{\cos(2 - 2x^{-3}) \cdot 6x^{-4}}$$

$$f = \sin(u) \quad f' = \cos(u)$$

$$u = 2 - 2x^{-3} \quad u' = 6x^{-4}$$

$$\cos(2 - 2x^{-3}) \cdot 6x^{-4}$$

$$\bullet y = \tan^{-1} 3x$$

$$\frac{3}{1 + 3x^2}$$

$$\frac{d}{dx} \tan^{-1} = \frac{u'}{1 + u^2}$$

Q2) (3 points each) Find the limit:

$$a) \lim_{x \rightarrow 0} \frac{e^x - x - 1}{\cos x - 1}$$

$$\frac{d}{dx} \left(\frac{e^x - x - 1}{\cos x - 1} \right)$$

$$\frac{1 - \cos(0) + 1}{(\cos(0) - 1)^2}$$

$$f = e^x - x - 1 \quad f' = e^x - 1$$

$$g = \cos x - 1 \quad g' = -\sin x$$

$$\frac{e^x - 1 (\cos x - 1) - (e^x - x - 1) (-\sin x)}{(\cos x - 1)^2}$$

$$\frac{1 - \cos(0) + 1}{(-1 + 0 + 1)(-1)}$$

$$b) \lim_{x \rightarrow 0} \frac{\sin(5x)}{x^2 + 3x}$$

$$\frac{1}{x} = \frac{\cos(5x) \cdot 5}{2x + 3}$$

$$\frac{5 \cos(x)}{2x + 3}$$

Q 3) (10 points) Consider the function $f(x) = x^3 - 6x^2 + 1$ answer the following questions:

a) Determine critical point (s).

$$\frac{d}{dx} x^3 - 6x^2 + 1$$
$$3x^2 - 12x = 0$$

$$S =$$

b) Interval of increase and decrease.

Increasing: -----

Decreasing: -----

c) Determine, local maximum and minimum.

Local Max: (... , ...)

Local Min: (... , ...)

d) Interval of concave up and concave down.

Concave up: -----

Concave down: -----

e) Coordinate of the inflection point if it does exist.

Q 5) (4 points) A raindrop is a perfect sphere. When the radius is 0.2 mm, the radius is increasing at rate of 0.001 mm per second. At what rate is the volume of the raindrop changing at that moment? Give an exact answer. Include correct units.

$$r = 0.2$$

$$\text{rad. rate} = 0.001$$

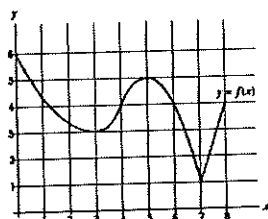
$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi (0.2)^3$$

$$0.034$$

$$V = 0.034 \text{ mm/s}^3$$

Q6) (3 points) Use the graph on $[0, 8]$ to answer the following questions:



$$f(c) = \frac{f(b) - f(a)}{b - a} = \frac{4 - 6}{8 - 0} = \frac{-2}{8}$$

a) What are the critical points of F (only x values)?

$$0, 3, 5, 7, 8$$

b) For each critical points $X=c$ that you identified in part (a), does $f'(c) = 0$ or is $f'(c)$ undefined?

c) What is the absolute maximum and absolute minimum of, $f(x)$ on $[0, 8]$?

$$\text{max} = (0, 6)$$

$$\text{min} = (7, 1)$$

Q 7) (5 points) Differentiate the following function:

$$f(x) = (2x^2 + 1)e^x$$

$$f'g + g'f$$

$$4x(e^x) + e^x(2x^2 + 1)$$

$$\frac{d}{dx}(2x^2 + 1)$$

$$f = 2x^2 + 1 \quad f' = 4x$$

$$e^x(4x + 2x^2 + 1)$$

$$= 2 \cdot 2x^{2-1} + 0$$

$$g = e^x$$

$$g' = e^x$$

$$e^x(2x^2 + 4x + 1)$$

$$= 4x$$

Q8) (5 points) find equation of the tangent line to the function $y = \frac{6}{x^4+2}$ at $x=1$.

$$f = 6 \quad f' = 0$$

$$g = x^4 + 2 \quad g' = 4x^3$$

$$y - f(a) = f'(a)(x - a)$$

$$f(a) = \frac{6}{1+2} = 2$$

$$\frac{0(x^4+2) - 6(4x^3)}{(x^4+2)^2}$$

$$\frac{0 - 24x^3}{(x^4+2)^2}$$

$$f'(a) = \frac{-24}{(1+2)^2} = \frac{-24}{9}$$

$$y = \frac{-24}{9}(x - a) + 2$$

Q9) (4 points) A ball is thrown vertically upward from ground level at a velocity of 15 m/sec. The equation of motion for the ball thrown vertically upward from ground level is, $s(t) = 15t - 0.3t^2$.

$$\text{pos: } s(t)$$

$$v: s'(t)$$

$$a: s''(t)$$

a) Write the expression for the velocity.

$$\frac{d}{dt}(15t - 0.3t^2) = 15 - 0.6t$$

b) Write the expression for the acceleration.

$$\frac{d}{dt}(15 - 0.6t) = -0.6$$

c) The maximum height reached by the ball.