

# L'Hôpital's Rule

Used to evaluate limits that result in indeterminate form ( $\frac{0}{0}$ )

"If you have two functions that are differentiable near a point  $a$ , and

both approach 0 or  $\pm\infty$  as  $x$

approaches  $a$ , then the limit of their

quotient can be found by taking the

derivative of the numerator and the derivative of the denominator."

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

Evaluate the limit using L'Hôpital's Rule

$$\lim_{x \rightarrow -\infty} \frac{\ln(x^4 + 5)}{11x + 2}$$

$$f' = -\frac{4x^3}{x^4 + 5} \cdot \frac{1}{11}$$

$$= \frac{4x^3}{11(x^4 + 5)}$$

$$f(x) = \ln(u) \quad \text{Chain rule}$$

$$f = \ln(u) \quad f' = \frac{1}{u}$$

$$f'' = -\frac{12x^2}{44x^3} = \frac{4x^2(3)}{4x^2(11x)}$$

$$u = x^4 + 5 \quad u' = 4x^3$$

$$= \frac{3}{11x}$$

$$f'(x) = \frac{1}{x^4 + 5} \cdot 4x^3 = \frac{4x^3}{x^4 + 5}$$

$$= \textcircled{0}$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

Rule :  $\frac{0}{0} = 0$

$\frac{0}{-\infty} = 0$

Does L'Hôpital's Rule apply? If yes, use it to evaluate the limit.

$$\lim_{x \rightarrow -29} \frac{x^2 - 841}{729 - 4x - x^2}$$

$$= \frac{2x}{(-4 - 2x)}$$

$$\lim_{x \rightarrow -29} \frac{2(-29)}{-4 - 2(-29)} = \frac{-58}{54}$$

L'Hôpital's Rule applies & the limit evaluates to  $\frac{-58}{54}$ .

Evaluate the limit using L'Hôpital's Rule.

$$\lim_{x \rightarrow 0} \frac{3^x - 7^x}{x}$$

$$3^x : f(x) = a^x \cdot \ln(a)$$

$$= 3^x \ln(3) - 7^x \ln(7)$$

$$= 3^x \cdot \ln(3)$$

1

$$g(x) = x$$

$$g'(x) = 1$$

$$\lim_{x \rightarrow 0} 3^0 \ln(3) - 7^0 \ln(7)$$

$$= 1(\ln(3)) - 1(\ln(7))$$

$$= \ln(3) - \ln(7)$$

$$= \ln\left(\frac{3}{7}\right)$$

Evaluate the limit.

$$\lim_{x \rightarrow \infty} \frac{x^2}{e^{9x}}$$

$$\frac{d}{dx} e^{9x}$$

$$f' = \frac{2x}{9e^{9x}}$$

$$f = e^u \quad f' = e^u$$

$$f'' = \frac{2}{81e^{9x}}$$

$$u = 9x \quad u' = 9$$

$$\lim_{x \rightarrow \infty} = 0$$

$$e^{9x} \cdot 9 = 9e^{9x}$$

Product Rule

$$\frac{d}{dx} 9e^{9x} \quad f'g + g'f$$

$$f = 9 \quad f' = 0$$

$$g = e^{9x} \quad g' = 9e^{9x}$$

$$0(e^{9x}) + 9e^{9x}(9)$$

$$= 81e^{9x}$$

Evaluate the limit.

$$\lim_{x \rightarrow 0} \frac{\sin(7x)}{x^2 + 12x}$$

$$f = \sin(u) \quad f' = \cos(u)$$

$$f' = \frac{7\cos(7x)}{2x + 12} \quad \cos(0) = 1$$

$$u = 7x \quad u' = 7$$

$$\lim_{x \rightarrow 0} \frac{7\cos(0)}{12} = \frac{7}{12}$$

$$\cos(7x) \cdot 7 = 7\cos(7x)$$