CSE 1322 Module 4 – Part 2

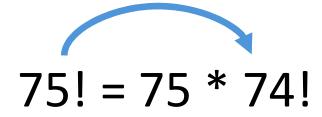
Recursion











Defined in terms of itself



75! = 75 * 74!

74! = 74 * 73!



75! = 75 * 74!

74! = 74 * 73!

73! = 73 * 72!





In general...

$$n! = n * (n - 1)!$$



Recursion

- When a method calls itself
 - Easy to identify
 - Going to create "clones" of the function
 - Usually, the clone has a smaller problem to work
- Requirements
 - Must have the recursive call
 - Must have a terminating condition (Base case)
 - Must make progress towards terminating.



Recursion

- A recursive method works like this:
 - If it is asked the simplest problem, it directly solves it. This is called the base condition/base case.
 - If it is asked a more complex question, it breaks that question into a slightly simpler problem and makes a recursive call to solve the simpler problem.
- Eventually, you get to the simplest problem, and when you plug in the answer from each layer, you can solve the next layer.



Recursion - Technique Design

- 1. Determine the base case, i.e. the stopping point for the recursion. It should normally be the simplest case.
- 2. What is the case that is just one step above it? Can it be generalized enough to fit?



Recursion - Technique Design

- The base case will contain some form of resolution to the method/recursion call.
- This will be usually in the form of a return statement, either by itself (return a void type) or by returning some value.
- We can also resolve a function by naturally reaching the end of the function without encountering any further recursive calls.
 - In Java, the function will just return void by default.



• A recursive declaration of the factorial method is arrived at by observing the following relationship:

$$n! = n * (n - 1)!$$

What is the simplest case / terminating state?

$$0! = 1 \text{ or } 1! = 1$$



• Let's start by defining the method:

```
public static int factorial(int number){
}
```



Factorial Example - Base Case

• As shown previously, we have to ways to handle the base case:

```
if(number == 0){
    return 1;
}
```

• Or

```
if(number == 1){
    return 1;
}
```



Remember:



- When designing the recursive method process and recursive call, always have in mind that we are going to stack up the function stack.
- We can either "work" while we stack up, or "work" when we stack down.
- Or both!



$$1! = 1 + 0!$$

$$2! = 2 + 1!$$

• • •



Factorial Example - Recursive Call

• We are going to use this expression to calculate the factorial:

$$n! = n * (n - 1)!$$

• This is how it will look like in our method:

```
return number * factorial(number - 1);
```



```
public class FactorialExample {
   public static int factorial(int number){
      if(number == 0){
        return 1;
      }
      return number * factorial(number - 1);
   }

   public static void main(String[] args) {
      System.out.println(factorial(5));
   }
}
```



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }

    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

1. ls 5 == 0?

factorial(5)

main()

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
   public static int factorial(int number){
      if(number == 0){
        return 1;
      }
      return number * factorial(number - 1);
   }
   public static void main(String[] args) {
        System.out.println(factorial(5));
   }
}
```

1. Is 5 == 0?

factorial(5)

main()

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(5)

- 1. Is 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

factorial(4)

1. Is 4 == 0?

factorial(5)

1. Is 5 == 0?

2. Return 5 * factorial(4)

main()

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

factorial(4)

1. Is 4 == 0?

factorial(5)

1. Is 5 == 0?

2. Return 5 * factorial(4)

main()

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
   public static int factorial(int number){
      if(number == 0){
        return 1;
      }
      return number * factorial(number - 1);
   }
   public static void main(String[] args) {
      System.out.println(factorial(5));
   }
}
```

factorial(4)

1. Is 4 == 0?

2. Return 4 * factorial(3)

factorial(5)

1. ls 5 == 0?

2. Return 5 * factorial(4)

main()

1. Call factorial(5)

2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(3)

1. Is 3 == 0?

factorial(4)

1. Is 4 == 0?

2. Return 4 * factorial(3)

factorial(5)

1. Is 5 == 0?

2. Return 5 * factorial(4)

main()

1. Call factorial(5)

2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(3)

1. Is 3 == 0?

factorial(4)

1. Is 4 == 0?

2. Return 4 * factorial(3)

factorial(5)

1. Is 5 == 0?

2. Return 5 * factorial(4)

main()

1. Call factorial(5)

2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2)

factorial(4)

- 1. ls 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. Is 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(2)

1. Is 2 == 0?

factorial(3)

1. Is 3 == 0?

2. Return 3 * factorial(2)

factorial(4)

1. ls 4 == 0?

2. Return 4 * factorial(3)

factorial(5)

1. Is 5 == 0?

2. Return 5 * factorial(4)

main()

1. Call factorial(5)

2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
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    }
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        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(2)

1. Is 2 == 0?

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2)

factorial(4)

- 1. ls 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. ls 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
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    }
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        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(2)

- 1. Is 2 == 0?
- 2. Return 2 * factorial(1)

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2)

factorial(4)

- 1. ls 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. Is 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
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    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

```
factorial(1)
```

1. Is 1 == 0?

factorial(2)

- 1. Is 2 == 0?
- 2. Return 2 * factorial(1)

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2)

factorial(4)

- 1. ls 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. Is 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(1)

1. Is 1 == 0?

factorial(2)

1. Is 2 == 0?

2. Return 2 * factorial(1)

factorial(3)

1. Is 3 == 0?

2. Return 3 * factorial(2)

factorial(4)

1. ls 4 == 0?

2. Return 4 * factorial(3)

factorial(5)

1. ls 5 == 0?

2. Return 5 * factorial(4)

main()

1. Call factorial(5)

2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(1)

- 1. Is 1 == 0?
- 2. Return 1 * factorial(0)

factorial(2)

- 1. Is 2 == 0?
- 2. Return 2 * factorial(1)

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2)

factorial(4)

- 1. ls 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. ls 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
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    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(0)

1. Is 0 == 0?

factorial(1)

- 1. Is 1 == 0?
- 2. Return 1 * factorial(0)

factorial(2)

- 1. Is 2 == 0?
- 2. Return 2 * factorial(1)

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2)

factorial(4)

- 1. Is 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. Is 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



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public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(0)

1. Is 1 == 0?

factorial(1)

1. Is 1 == 0?

2. Return 1 * factorial(0)

factorial(2)

1. Is 2 == 0?

2. Return 2 * factorial(1)

factorial(3)

1. Is 3 == 0?

2. Return 3 * factorial(2)

factorial(4)

1. Is 4 == 0?

2. Return 4 * factorial(3)

factorial(5)

1. Is 5 == 0?

2. Return 5 * factorial(4)

main()

1. Call factorial(5)

2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(0)

- 1. Is 1 == 0?
- 2. Return 1

factorial(1)

- 1. Is 1 == 0?
- 2. Return 1 * factorial(0)

factorial(2)

- 1. Is 2 == 0?
- 2. Return 2 * factorial(1)

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2)

factorial(4)

- 1. Is 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. ls 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
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        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(1)

- 1. Is 1 == 0?
- 2. Return 1 * factorial(0) 1

factorial(2)

- 1. Is 2 == 0?
- 2. Return 2 * factorial(1)

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2)

factorial(4)

- 1. Is 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. Is 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(1)

- 1. Is 1 == 0?
- 2. Return 1 * 1

factorial(2)

- 1. Is 2 == 0?
- 2. Return 2 * factorial(1)

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2)

factorial(4)

- 1. Is 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. ls 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

```
factorial(1)
```

- 1. Is 1 == 0?
- 2. Return 1

factorial(2)

- 1. Is 2 == 0?
- 2. Return 2 * factorial(1)

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2)

factorial(4)

- 1. ls 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. Is 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(2)

- 1. Is 2 == 0?
- 2. Return 2 * factorial(1) 1

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2)

factorial(4)

- 1. ls 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. ls 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(2)

- 1. Is 2 == 0?
- 2. Return 2 * 1

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2)

factorial(4)

- 1. Is 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. ls 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(2)

- 1. Is 2 == 0?
- 2. Return 2

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2)

factorial(4)

- 1. ls 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. Is 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * factorial(2) 2

factorial(4)

- 1. Is 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. Is 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(3)

- 1. Is 3 == 0?
- 2. Return 3 * 2

factorial(4)

- 1. Is 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. Is 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(3)

- 1. Is 3 == 0?
- 2. Return 6

factorial(4)

- 1. Is 4 == 0?
- 2. Return 4 * factorial(3)

factorial(5)

- 1. Is 5 == 0?
- 2. Return 5 * factorial(4)

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
   public static int factorial(int number){
      if(number == 0){
        return 1;
      }
      return number * factorial(number - 1);
   }
   public static void main(String[] args) {
      System.out.println(factorial(5));
   }
}
```

factorial(4)

1. Is 4 == 0?

2. Return 4 * factorial(3) 6

factorial(5)

1. Is 5 == 0?

2. Return 5 * factorial(4)

main()

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

main()

1. Call factorial(5)

factorial(4)
1. Is 4 == 0?
2. Return 4 * 6

factorial(5)

1. Is 5 == 0?

2. Print returned value

2. Return 5 * factorial(4)



```
public class FactorialExample {
        if(number == 0){
            return 1;
        return number * factorial(number - 1);
   public static void main(String[] args) {
        System.out.println(factorial(5));
```

factorial(4) 1. Is 4 == 0?

2. Return 24

factorial(5)

1. Is 5 == 0?

2. Return 5 * factorial(4)

main()

1. Call factorial(5)

2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(5)

1. ls 5 == 0?

2. Return 5 * factorial(4) 24

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(5)

1. ls 5 == 0?

2. Return 5 * 24

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
    public static int factorial(int number){
        if(number == 0){
            return 1;
        }
        return number * factorial(number - 1);
    }
    public static void main(String[] args) {
        System.out.println(factorial(5));
    }
}
```

Week-8/Recursion/FactorialExample.java

factorial(5)

1. Is 5 == 0?

2. Return 120

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
   public static int factorial(int number){
      if(number == 0){
        return 1;
      }
      return number * factorial(number - 1);
   }

   public static void main(String[] args) {
      System.out.println(factorial(5));
   }
}
```

main()

- 1. Call factorial(5)
- 2. Print returned value



```
public class FactorialExample {
   public static int factorial(int number){
      if(number == 0){
        return 1;
      }
      return number * factorial(number - 1);
   }

   public static void main(String[] args) {
      System.out.println(factorial(5));
   }
}
```



Common Programming Error

- If a recursive method does not have a base care or if the recursive call is structured so it does not converge to the base case, the recursion will never terminate.
- Since it does not terminate, this leads to an infinite recursion where the function keeps calling itself indefinitely until the system runs out of memory.
- This error is analogous to the problem of an infinite loop in an iterative (non-recursive) solution.



Stack Overflow

- The problem mentioned before is referred to a Stack Overflow. This occurs when you recurse too many times and run out of memory.
- Often, it is more efficient to perform calculations via iteration (looping), but it can also be easier to express an algorithm via recursion.
- Recursion is especially useful for non-linear situations.



In-Class problem

- Write a method that calculates exponential function using recursion.
- The method should take in two positive integers: the base and the exponent (in this order).
- The method should return the result of the exponential function.
- Solution will be posted to Github:
 - Week-8/Recursion/ExponentialExample.java



What is the output?

```
public static void printNumbers(int n){
   if(n > 0){
       System.out.print(n + " ");
       printNumbers(n - 1);
       System.out.print(n + " ");
   }
}

public static void main(String[] args) {
   printNumbers(10);
}
```

Week-8/Recursion/Example.java

