Fizica III – Electromagnetism

Aplicatii # 5 - Linii electrice.
Regim variabil in timp. Adaptare.

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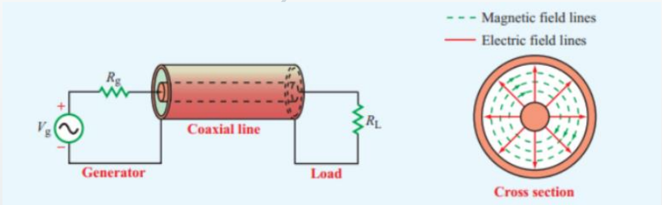
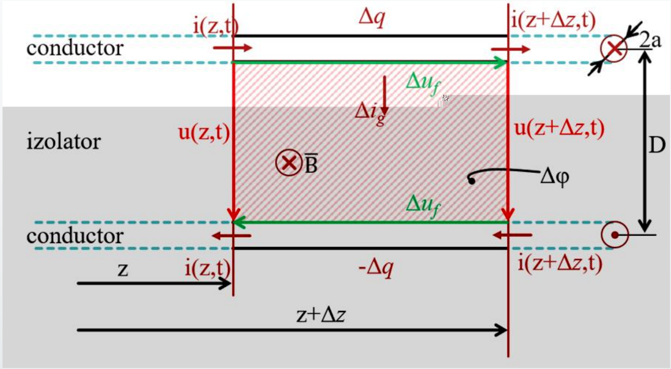
Linii electrice (circuite electrice cu parametri distribuiti)

1. Concepte utile*
   - 1.1. Modelul liniei. Parametri lineici. Ecuatiile liniilor (telegrafistilor).
   - 1.2. Linia fara pierderi. Regim tranzitoriu. Impedanta liniei.
   - 1.3. Reflexia pe linii fara pierderi. Adaptare.
   - 1.4. Coeficientul de reflexie
   - 1.5. Linii cu pierderi.
2. Experimente virtuale simple
3. Experiment real

*Acesta este un rezumat. Pentru detalii vedeti cursul, cap 4.

1. Concepte utile
1.1. Modelul liniei. Parametrii lineici. Ecuatiile liniilor (telegrafistilor).



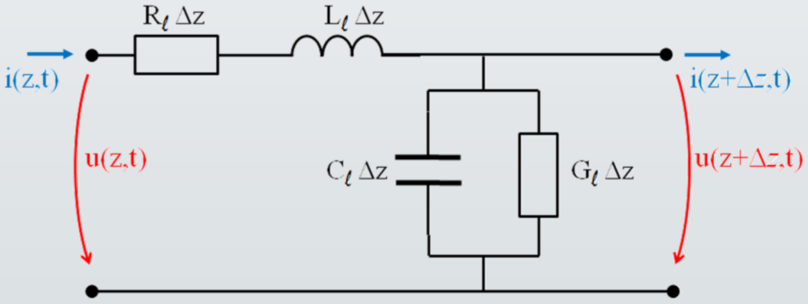


$$2\Delta u_f \approx R_l\, i(z,t)\, \Delta z \qquad R_l\left[\frac{\Omega}{\mathrm{m}}\right]$$

$$\Delta\varphi \approx L_l\, i(z,t)\Delta z \qquad L_l\left[\frac{\mathrm{H}}{\mathrm{m}}\right]$$

$$\Delta i_g \approx G_l\, u(z,t)\Delta z \qquad G_l\left[\frac{\mathrm{S}}{m}\right]$$

$$\Delta q \approx C_l\, u(z,t)\Delta z \qquad C_l\left[\frac{\mathrm{F}}{m}\right]$$



$$-\frac{\partial u(z,t)}{\partial z} = R_l\, i(z,t) + L_l \frac{\partial i(z,t)}{\partial t} \qquad \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

Pierderile de tensiune pe unitatea de lungime a liniei = caderile de tensiune lineice pe conductoarele liniei + viteza de variatie a fluxului lineic ($L_l i$)

$$-\frac{\partial i(z,t)}{\partial z} = G_l\, u(z,t) + C_l \frac{\partial u(z,t)}{\partial t} \qquad \left[\frac{\mathrm{A}}{\mathrm{m}}\right]$$

Pierderile de curent pe unitatea de lungime a liniei = curentul prin dielectric, pe unitatea de lungime a liniei + viteza de variatie a sarcinii lineice ($C_l u$)

1. Concepte utile
Linia fara pierderi. Regim tranzitoriu. Impedanta liniei

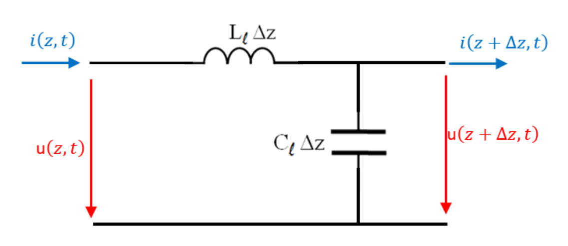
- Linia fara pierderi este linia in care nu exista pierderi Joule.
- Formal $R_l = 0,\ G_l = 0$, dar in realitate o comportare ca a liniei fara pierderi este obtinuta in conditiile in care componentele resistive din ecuatiile de ordin 1 sunt neglijabile in raport cu cele reactive:

$$|R_l i| \ll \left|L_l \frac{\partial i}{\partial t}\right|$$

$$|G_l u| \ll \left|C_l \frac{\partial u}{\partial t}\right|$$

$$-\frac{\partial u}{\partial z} = L_l \frac{\partial i}{\partial t}$$

$$-\frac{\partial i}{\partial z} = C_l \frac{\partial u}{\partial t}$$



$$u_d(z,t) = f(\theta_-)$$

$$u_i(z,t) = g(\theta_-)$$

$Z_0[\Omega]$ = impedanța caracteristică a liniei fara pierderi
$Y_0[S]$ = admitanța caracteristică a liniei fara pierderi

$$-\frac{\partial^2 u(z,t)}{\partial z^2} = L_l C_l \frac{\partial^2 u(z,t)}{\partial t^2}$$

$$\theta_- = t - z/v$$

$$\theta_+ = t + z/v$$

$$L_l v = L_l \frac{1}{\sqrt{L_l C_l}} = \boxed{\sqrt{\frac{L_l}{C_l}} \overset{\text{not}}{=} Z_0} = \frac{1}{Y_0};\quad Y_0 = \sqrt{\frac{C_l}{L_l}}$$

$$\boxed{v = \frac{1}{\sqrt{L_l C_l}}}$$

$$\boxed{\begin{aligned} u(z,t) &= u_d(z,t) + u_i(z,t) + U_0 \\ i(z,t) &= \frac{1}{Z_0} u_d(z,t) - \frac{1}{Z_0} u_i(z,t) + I_0 \end{aligned}}$$

$$i_d(z,t) = \frac{1}{Z_0} u_d(z,t) \qquad i_i(z,t) = -\frac{1}{Z_0} u_i(z,t)$$

Daca linia este plasata in aer (vid), atunci

$$v = \frac{1}{\sqrt{L_l C_l}} = \frac{1}{\sqrt{\frac{\mu_0}{P}\varepsilon_0 P}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c \approx 3 \cdot 10^8 \frac{\text{m}}{\text{s}}$$

1. Concepte utile
1.2. Linia fara pierderi. Regim tranzitoriu. Impedanta liniei

1)Solutia ecuatiilor liniei **fara** pierderi in regim variabil in timp:

$$\begin{cases} u(z,t) = u_d(z,t) + u_i(z,t) + U_0 \\ i(z,t) = \underbrace{\frac{u_d(z,t)}{Z_0}}_{i_d(z,t)} - \underbrace{\frac{u_i(z,t)}{Z_0}}_{i_i(z,t)} + I_0 \end{cases}$$

Atentie! $I_0 \neq \frac{U_0}{Z_0}$

2)Viteza de propagare, impedanța / admitanța carcateristică

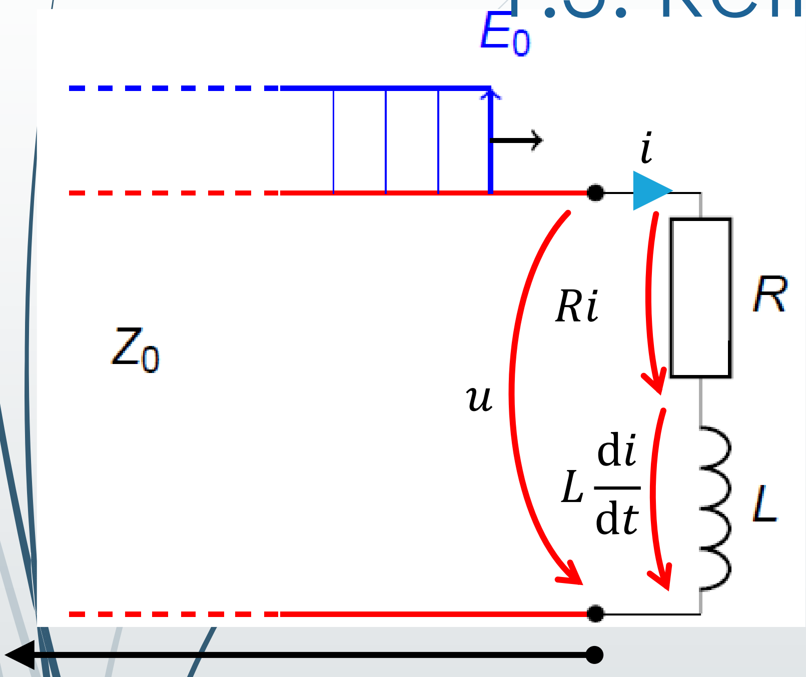
$$v = \frac{1}{\sqrt{L_l C_l}}; \qquad Z_0 = \sqrt{\frac{L_l}{C_l}}; \qquad Y_0 = \sqrt{\frac{C_l}{L_l}}; \qquad Z_0 = \frac{1}{Y_0}$$

3) $u_d(z,t), u_i(z,t)$ → unde elementare de tensiune

$i_d(z,t) = \frac{u_d(z,t)}{Z_0}$, $i_i(z,t) = -\frac{u_d(z,t)}{Z_0}$ → unde elementare de current

4)Expresile undelor elementare de tensiune și curent se determină din condițiile concrete în care funcționează linia (condiții inițiale și la capete – conform T. de unicitate, sec. 4.5.).
5)Constantele U_0 si I_0 sunt componente continui arbitrare, aceleasi in toate punctele liniei, care se determina din conditiile initiale.

1. Concepte utile
1.3. Reflexia pe linia fara pierderi. Adaptare.



In scrierea expresiilor generale, se inlocuieste t cu t-x/v.

$$u_d(x,t) = E_0$$

$$u_i(x,t) = \left(\frac{2\,Z_0 E_0}{Z_0+R} e^{-\frac{\left(t-\frac{x}{v}\right)}{\tau}} - \frac{E_0(Z_0-R)}{Z_0+R}\right) 1\left(t-\frac{x}{v}\right)$$

$$i_d(x,t) = \frac{u_d}{Z_0} = \frac{E_0}{Z_0}$$

$$i_i(x,t) = -\frac{u_i(x,t)}{Z_0}$$

$$\tau = \frac{L}{Z_0+R}$$

Se demonstreaza ca (vedeti curs)

$$u_i(t) = \left(\frac{2\,Z_0 E_0}{Z_0+R} e^{-\frac{t}{\tau}} - \frac{E_0(Z_0-R)}{Z_0+R}\right) 1(t)$$

$$i_i(t) = -\frac{u_i(t)}{Z_0}$$

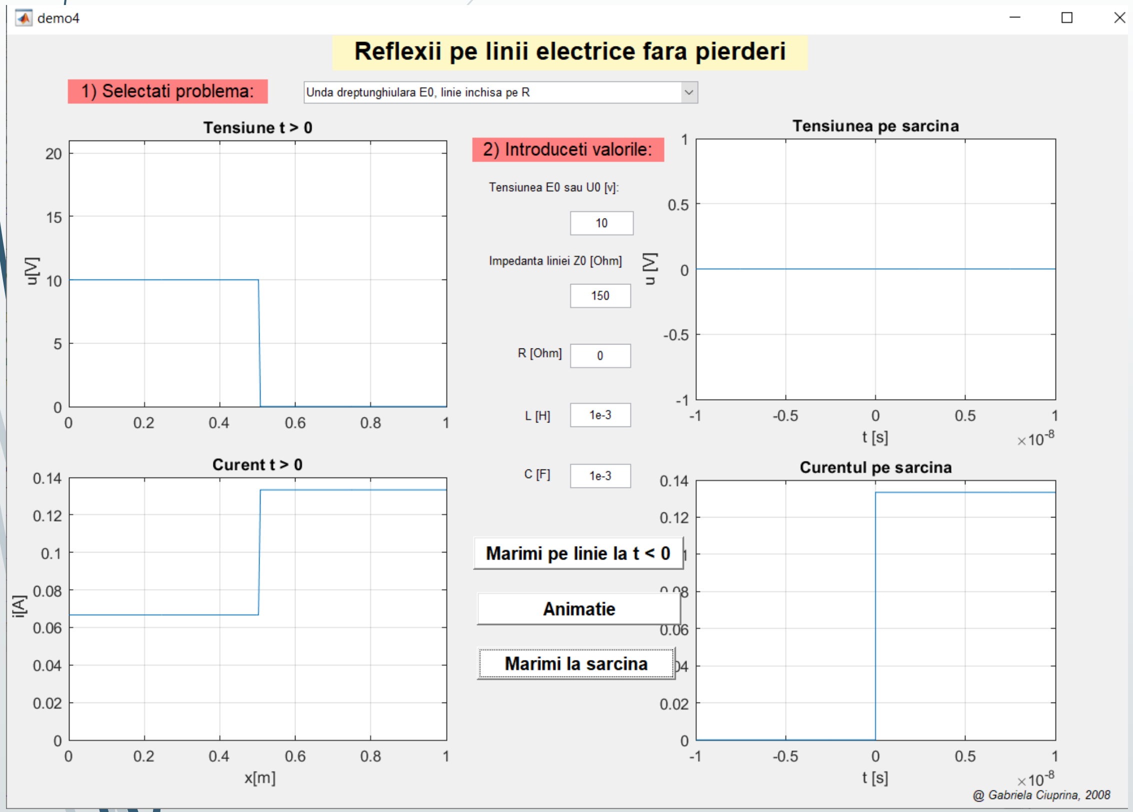
In cazul $L = 0$

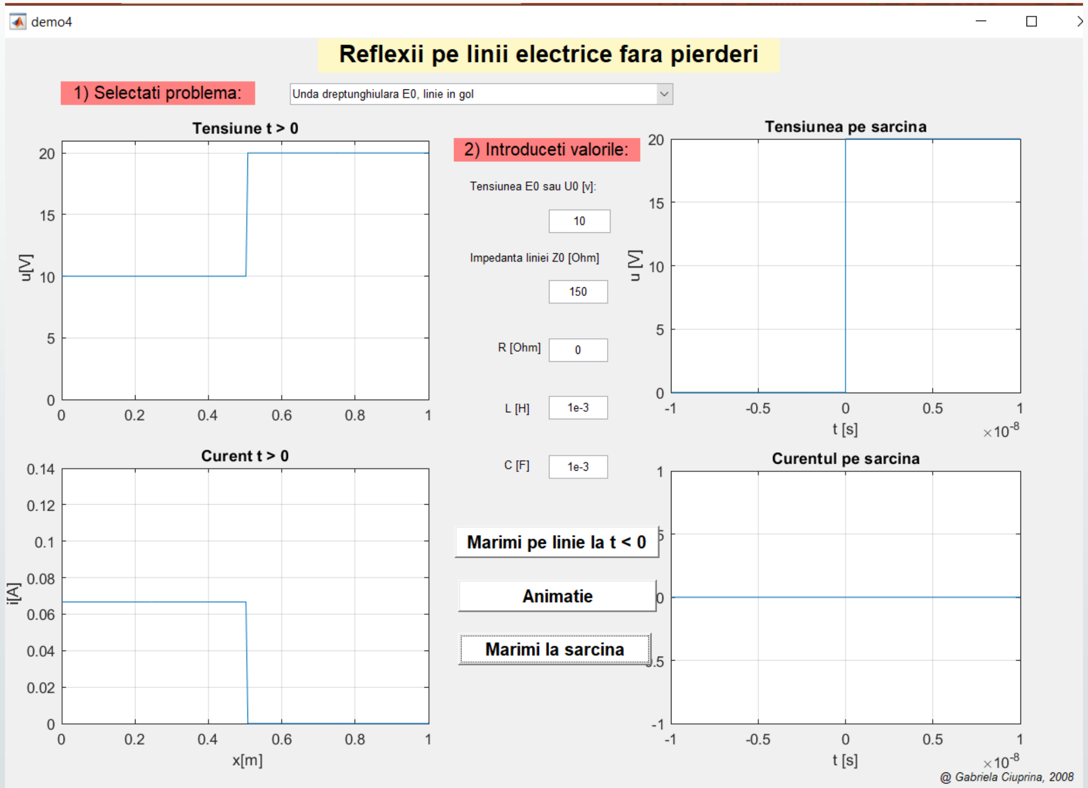
$$u_i(x,t) = \left(-\frac{E_0(Z_0-R)}{Z_0+R}\right) 1\left(t-\frac{x}{v}\right)$$

$$i_i(x,t) = -\frac{u_i(x,t)}{Z_0}$$

Daca $Z_0 = R$ atunci nu exista unda inversa (nu exista reflexii). Se spune ca sarcina este **adaptata** liniei.

1. Concepte utile
1.3. Reflexia pe linia fara pierderi. Adaptare.



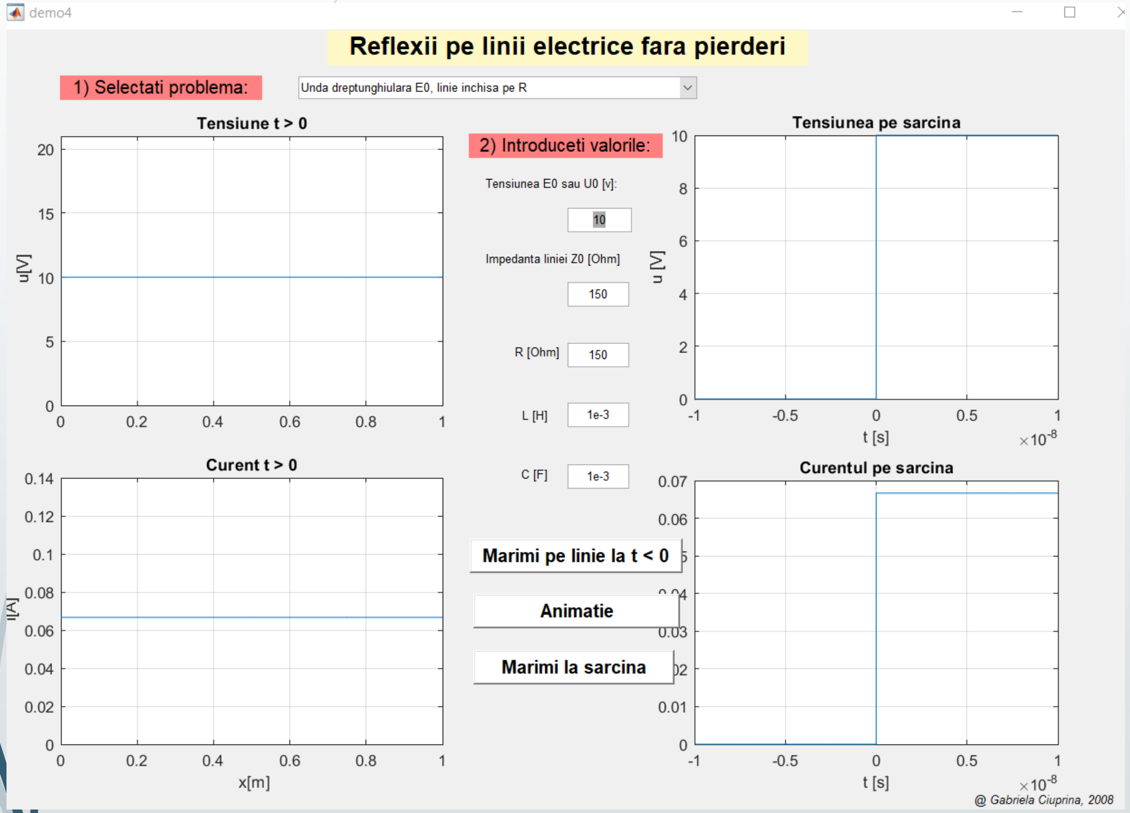


Linie in scurt, tensiunea devine zero, curentul se dubleaza.

Linie in gol, curentul devine zero, tensiunea se dubleaza.

Vedeti demo matlab – demo4.m

1. Concepte utile
1.3. Reflexia pe linia fara pierderi. Adaptare.



Linie inchisa pe un R adaptat.

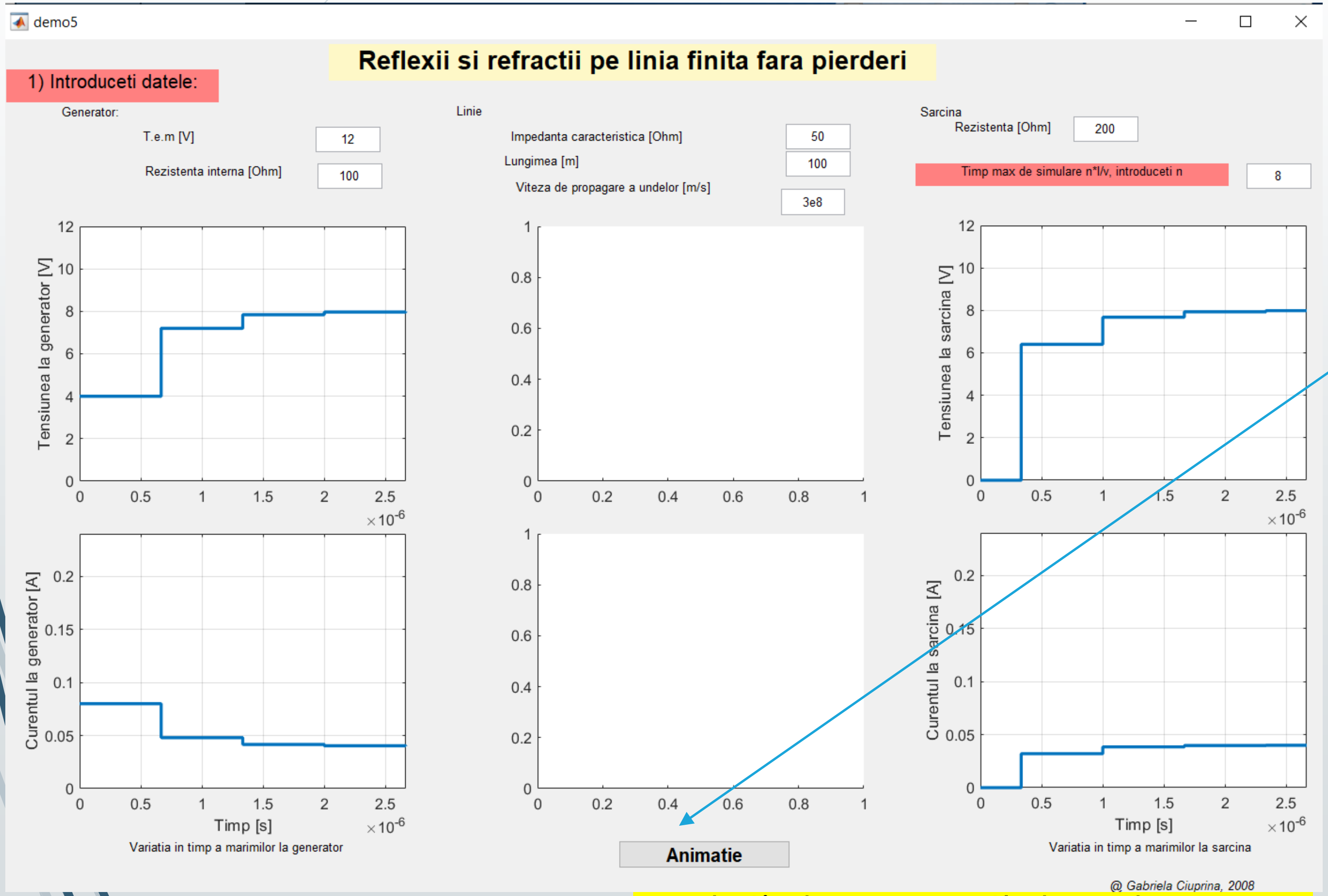
Nu exista reflexii.

Reflexiile perturba puternic semnalele digitale transmise pe linii, deci acestea trebuie adaptate.

Se spune ca linia adaptata este "transparenta", indiferent de lungimea ei.

Vedeti demo matlab – demo4.m

1. Concepte utile
1.3. Reflexia pe linia fara pierderi. Adaptare.

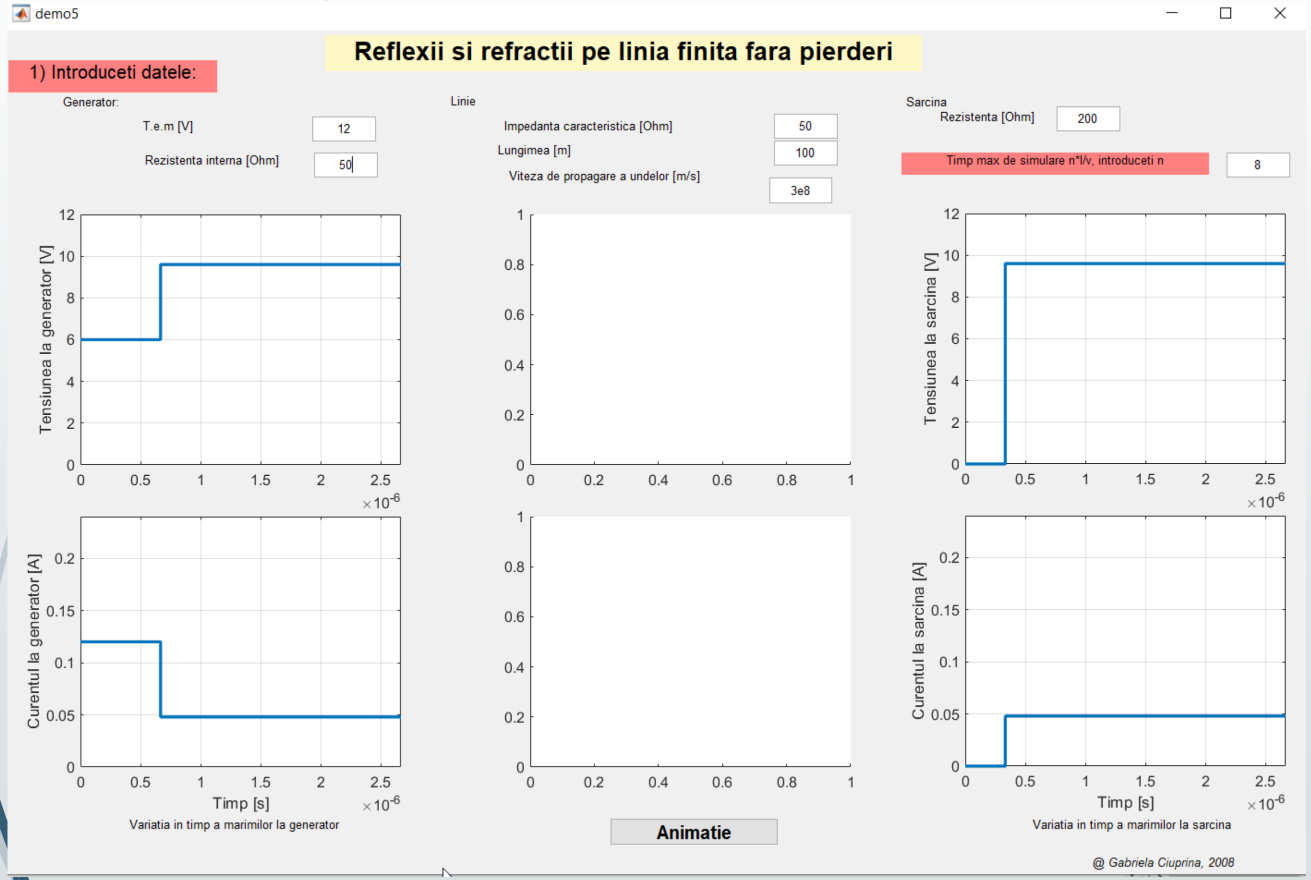


Puteti vedea animatia, si reflectarea semnalului la ambele capete.

Aici nici sarcina nici generatorul nu sunt adaptate

Vedeti demo matlab – demo5.m

1. Concepte utile
1.3. Reflexia pe linia fara pierderi. Adaptare.

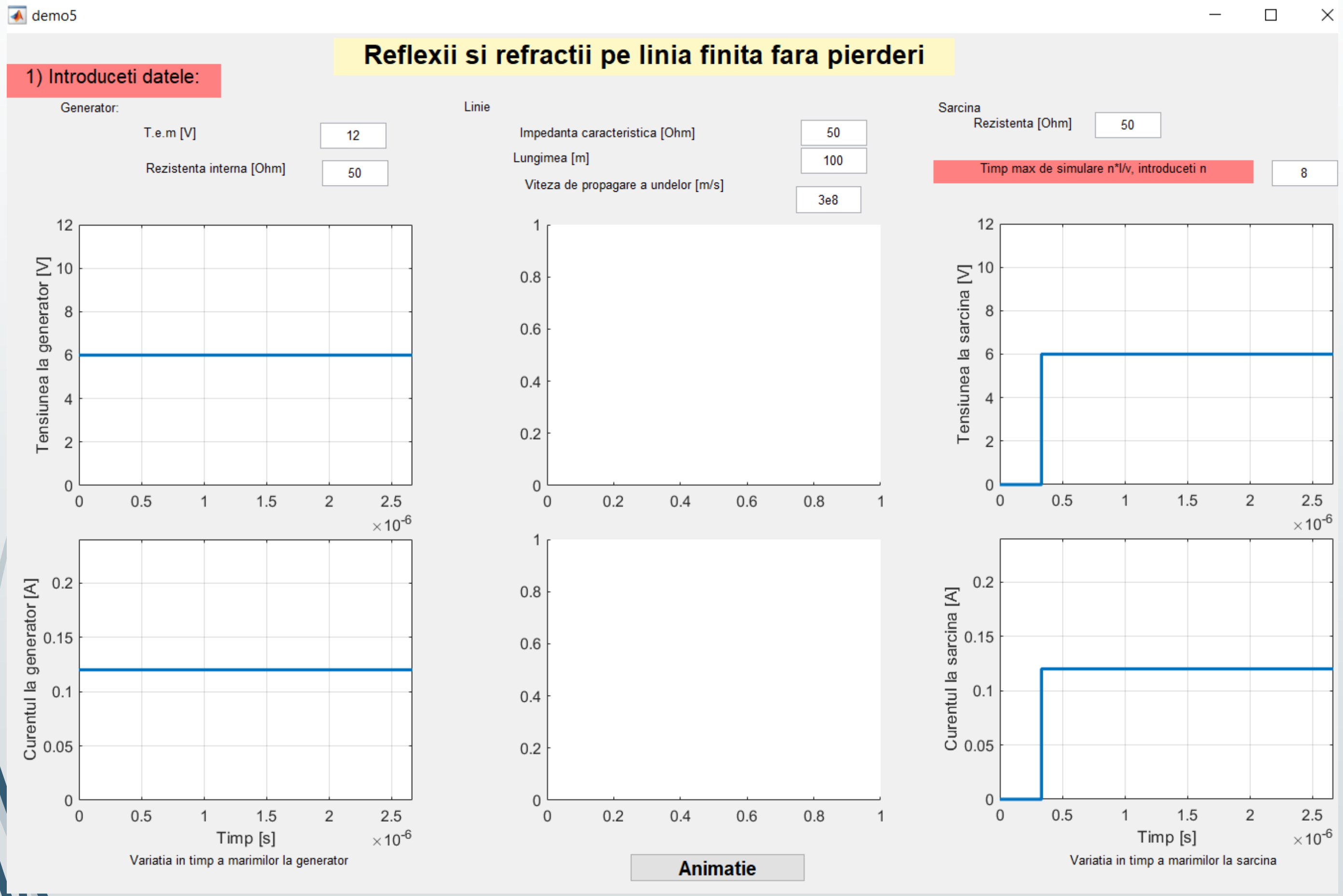


Aici generatorul este adaptat, dar sarcina nu.

Are loc o singura reflexie la sarcina.
Unda inversa generate de aceasta reflexie nu mai este reflectata cand ajunge la generatorl

Vedeti demo matlab – demo5.m

1. Concepte utile
1.3. Reflexia pe linia fara pierderi. Adaptare.



Aici atat generatorul este adaptat cat si sarcina

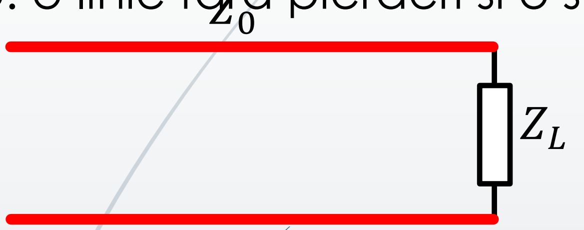
Nu are loc nicio reflexie.

Se observa doar intarzierea semnaluui.

Vedeti demo matlab – demo5.m

1. Concepte utile
1.4. Coeficientul de reflexie

Pp. o linie fara pierderi si o sarcina rezistiva



$$\boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

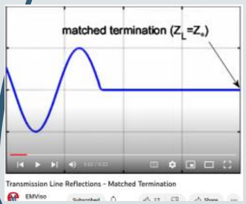
Coeficient de reflexie $u_{i1}(0,t) = \Gamma u_{d1}(0,t)$

Linia adaptata : $Z_L = Z_0 \Rightarrow \Gamma = 0$

nu exista reflexii

Vedeti animatie de la

https://www.youtube.com/watch?v=C0VFNrkBfFI&list=PL2fRCJxWQiS8B-CrI1R7fE5_ZlX7iFRF3&index=3



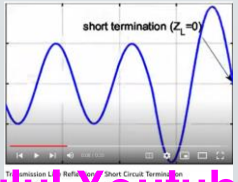
Linia in **scurt** : $Z_L = 0 \Rightarrow \Gamma = -1$

totul se reflecta, unda reflectata este cu semn schimbat fata de una incidenta

Daca unda incidenta este sinusoidala, atunci apar unde stationare.

$$u_1(x,t) = E_0 \sin(\omega t - \beta x) - E_0 \sin(\omega t + \beta x) = E_0 \left(\sin \frac{\omega t - \beta x + \omega t + \beta x}{2} \sin \frac{\omega t - \beta x - \omega t - \beta x}{2} \right)$$

$$u_1(x,t) = E_0 \sin(\omega t) \sin(\beta x)$$

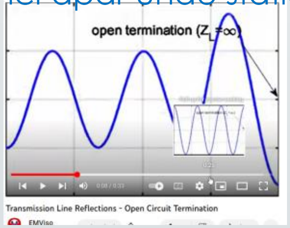


https://www.youtube.com/watch?v=Rs3dfV4Hi_w&list=PL2fRCJxWQiS8B-CrI1R7fE5_ZlX7iFRF3&index=1

Linia in **gol** : $Z_L \to \infty \Rightarrow \Gamma = 1$

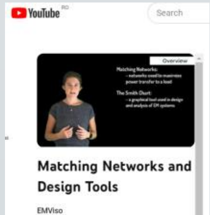
totul se reflecta, unda reflectata este identica cu unda incidenta a de una incidenta

Daca unda incidenta este sinusoidala, atunci apar unde stationare.



https://www.youtube.com/watch?v=qxG30qRUAI8&list=PL2fRCJxWQiS8B-CrI1R7fE5_ZlX7iFRF3&index=3

Va recomandam cu caldura canalul Youtube EMViso si playlist-ul "Matching networks and design tools" https://www.youtube.com/playlist?list=PL2fRCJxWQiS_eJhVoYMeFRk1Ew1OqYz0g



by Kathryn Leigh Smith, Asst. Prof. in the Department of Electrical and Computer Engineering at the University of North Carolina-Charlotte in partnership with Ansys.

1. Concepte utile
1.5. Linia cu pierderi

Lina cu pierderi este caracterizata de toti cei 4 parametri lineici: $R_l\left[\frac{\Omega}{m}\right], L_l\left[\frac{H}{m}\right], G_l\left[\frac{S}{m}\right], C_l\left[\frac{F}{m}\right]$.

Pentru o propagare a semnalelor fara distorsiuni trebuie indeplinita **conditia Heaviside***:

https://en.wikipedia.org/wiki/Oliver_Heaviside

$$\frac{R_l}{L_l} = \frac{G_l}{C_l} \overset{\text{not}}{=} \frac{1}{\tau}$$

ConditiaHeaviside poate fi indeplinitaprin pupinizare (inserierea periodica de bobine pe linie – se numesc bobine Pupin)
https://en.wikipedia.org/wiki/Loading_coil

https://en.wikipedia.org/wiki/Mihajlo_Pupin

In acest caz $\alpha = \sqrt{R_l G_l}$

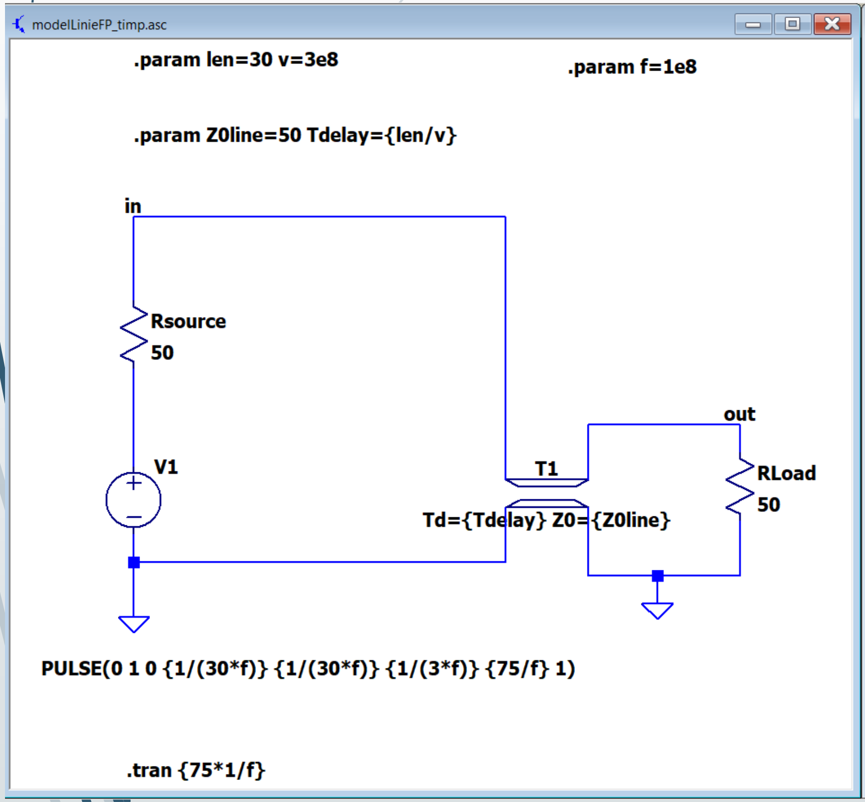
Semnalele se propaga atenuat dar nedistorsionat.

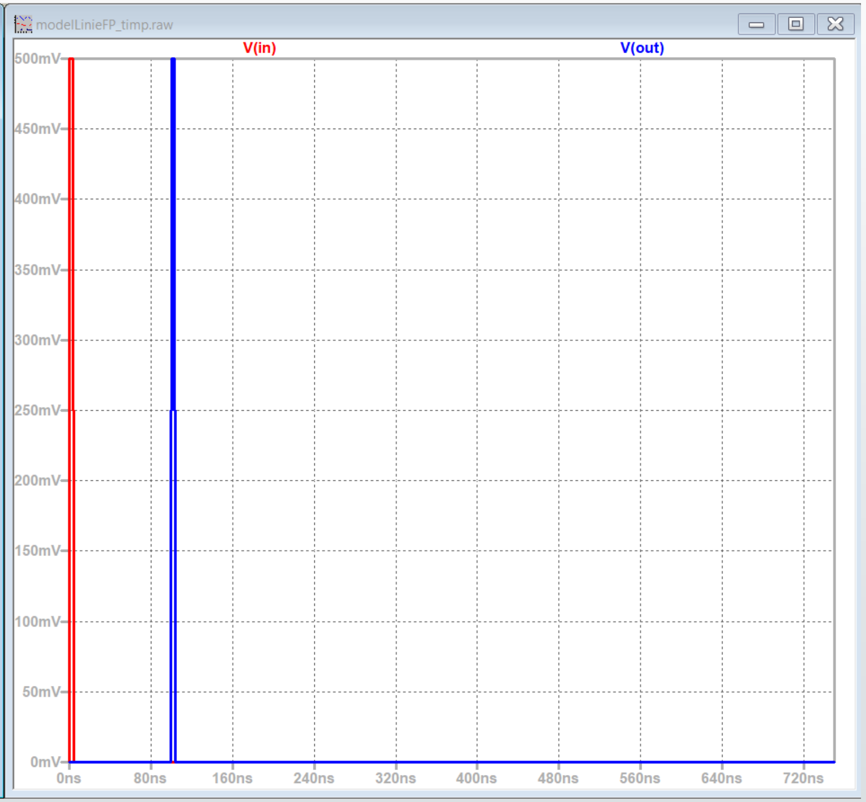
$$v = \frac{1}{\sqrt{L_l C_l}} \qquad Z_c = \sqrt{\frac{L_l}{C_l}} \overset{\text{not}}{=} Z_0$$

*Linia cu pierderi se analizeaza mai simplu in regim armonic permanent, vedeti cap 4 din curs pentru detalii suplimentare.

2. Experimente virtuale #1 a)

Intarzierea semnalelor transmise pe linie





Linie fara pierderi, adaptata si la sarcina si la generator.

$R_S = 50\ \Omega, Z_0 = 50\ \Omega, Z_L = 50\ \Omega$

Sursa genereaza un impuls cu valoarea maxima de 1 V.

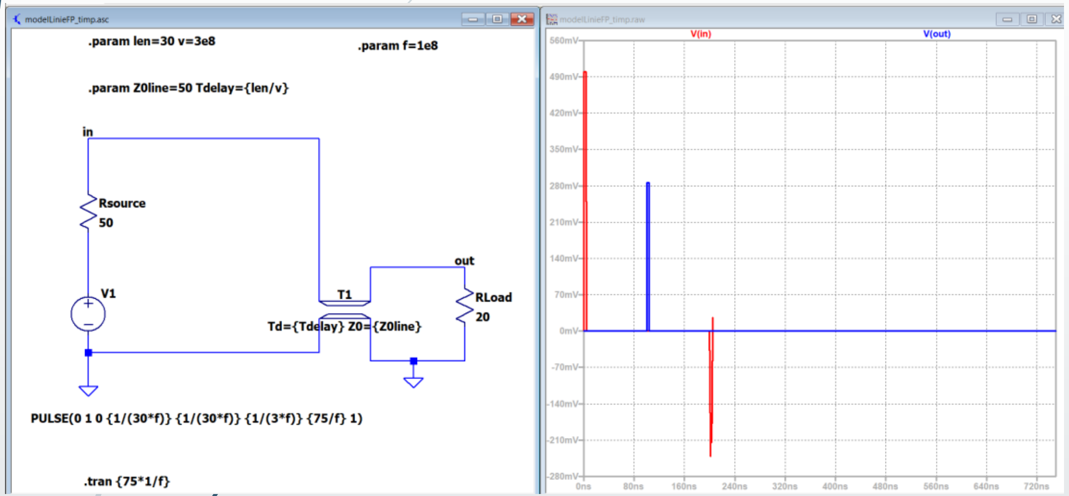
La borna in amplitudinea este de 500 mV pentru ca la bornele de intrare de la linie, impedanta echivalenta este 50 Ohm.

Intarzierea semnalului la sarcina este corelata cu lungimea liniei.
30 m/3e8 m/s = 100 nsec

2. Experimente virtuale #1b)

Linie fara pierderi, adaptata numai la generator.

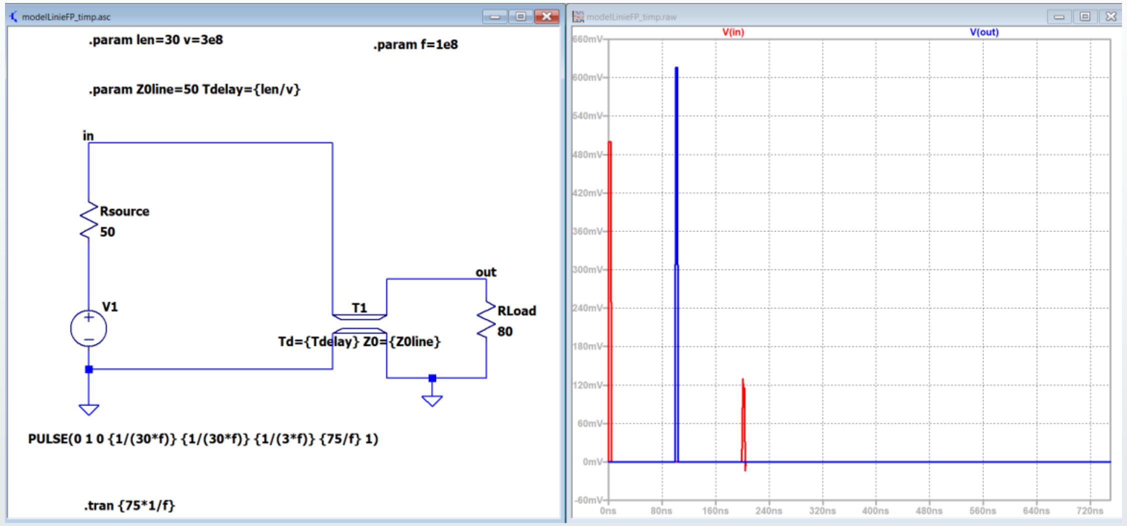
Intarzierea semnalelor transmise pe linie



$$\boldsymbol{R_S = 50\ \Omega, Z_0 = 50\ \Omega, Z_L = 20\ \Omega}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{20 - 50}{20 + 50} = -0.43$$

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{50 - 50}{50 + 50} = 0$$



$$\boldsymbol{R_S = 50\ \Omega, Z_0 = 50\ \Omega, Z_L = 80\ \Omega}$$

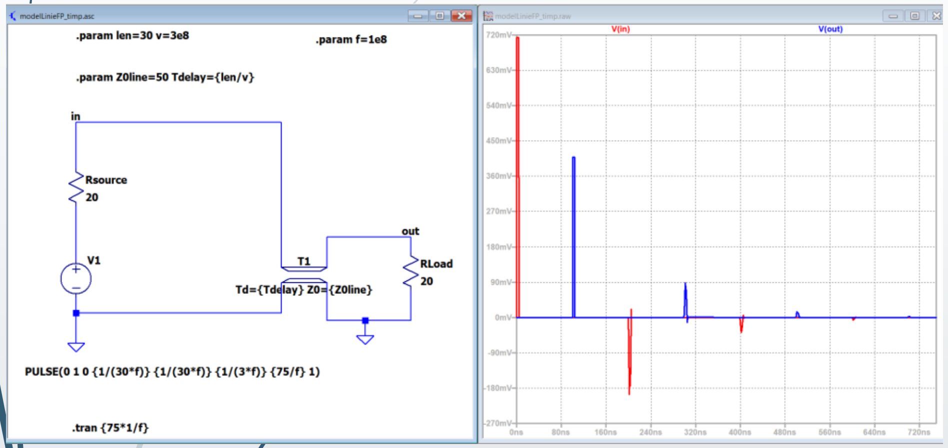
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{80 - 50}{80 + 50} = 0.23$$

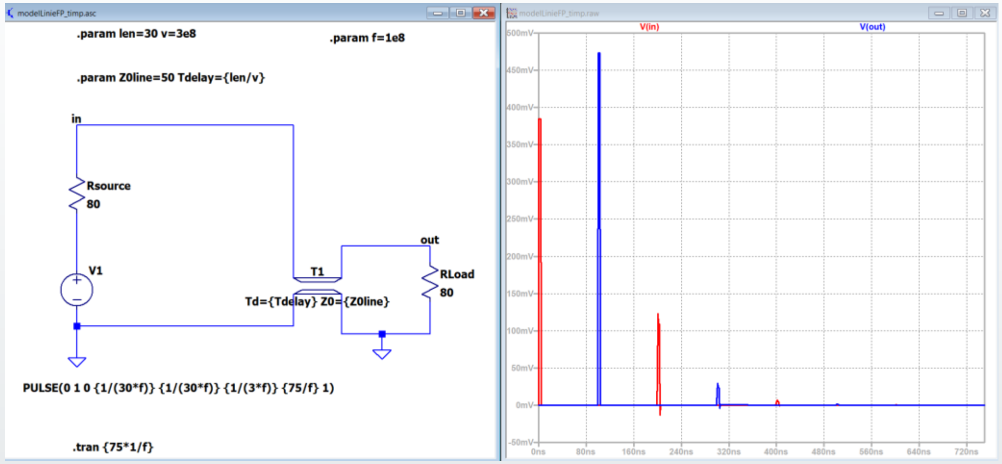
$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{50 - 50}{50 + 50} = 0$$

2. Experimente virtuale #1c)

Linie fara pierderi, neadaptata la ambele capete.

Intarzierea semnalelor transmise pe linie





$$\boldsymbol{R_S = 20\ \Omega, Z_0 = 50\ \Omega, Z_L = 20\ \Omega}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{20 - 50}{20 + 50} = -0.43$$

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{20 - 50}{20 + 50} = -0.43$$

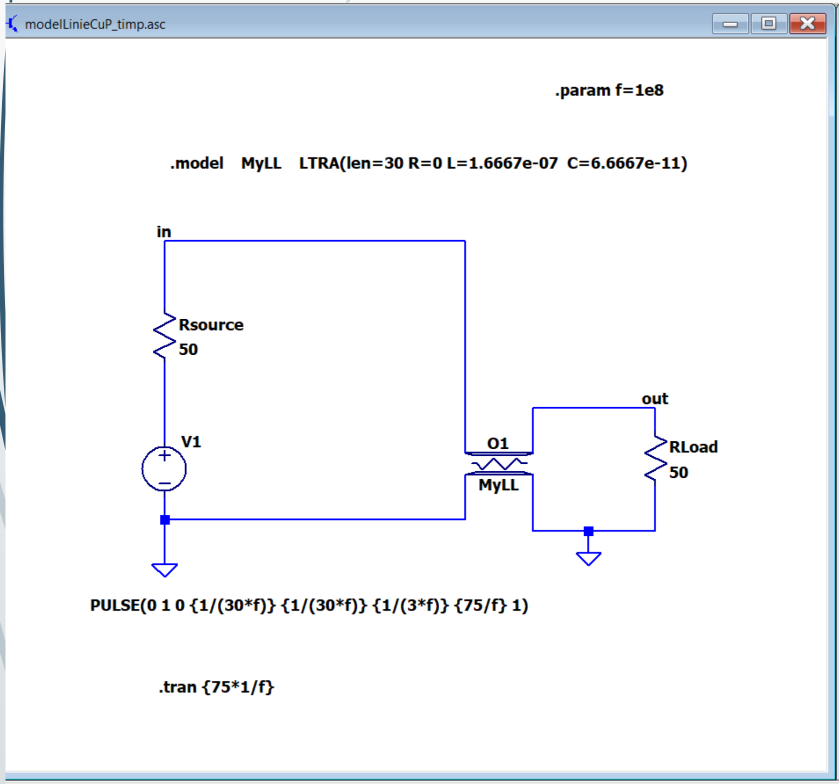
$$\boldsymbol{R_S = 80\ \Omega, Z_0 = 50\ \Omega, Z_L = 80\ \Omega}$$

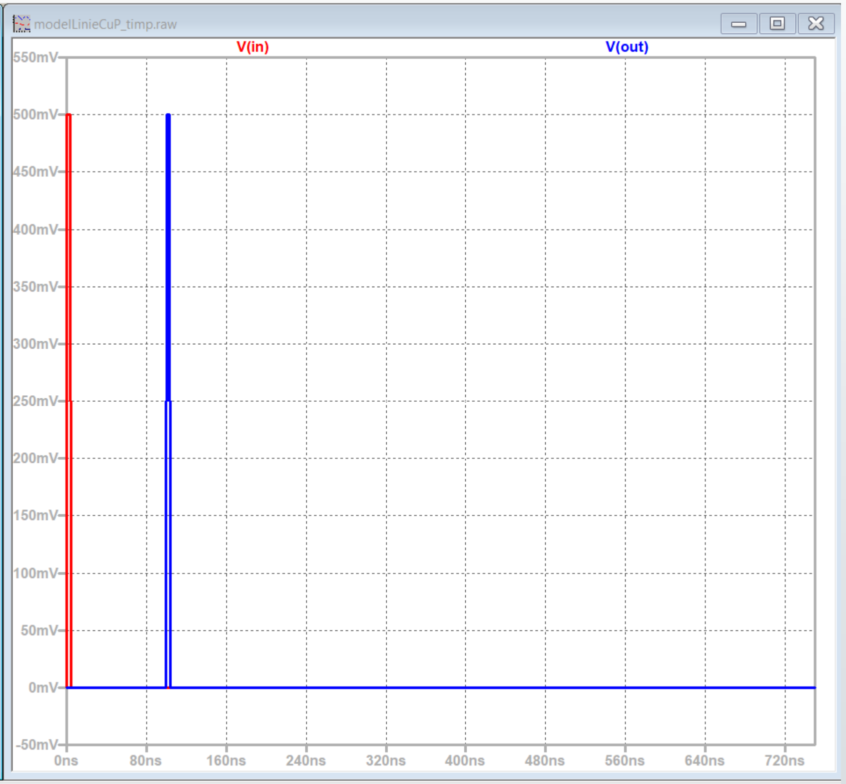
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{80 - 50}{80 + 50} = 0.23$$

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{80 - 50}{80 + 50} = 0.23$$

2. Experimente virtuale #2 a)

Folosirea unui model de linie cu pierderi in LTSPICE. Nu este posibil decat cazul $G_l = 0$





Se dau: lungimea liniei si parametrii lineici R_l, L_l, C_l.

Acest exemplu reprezinta de fapt linia far pierderi din exemplele anterioare:

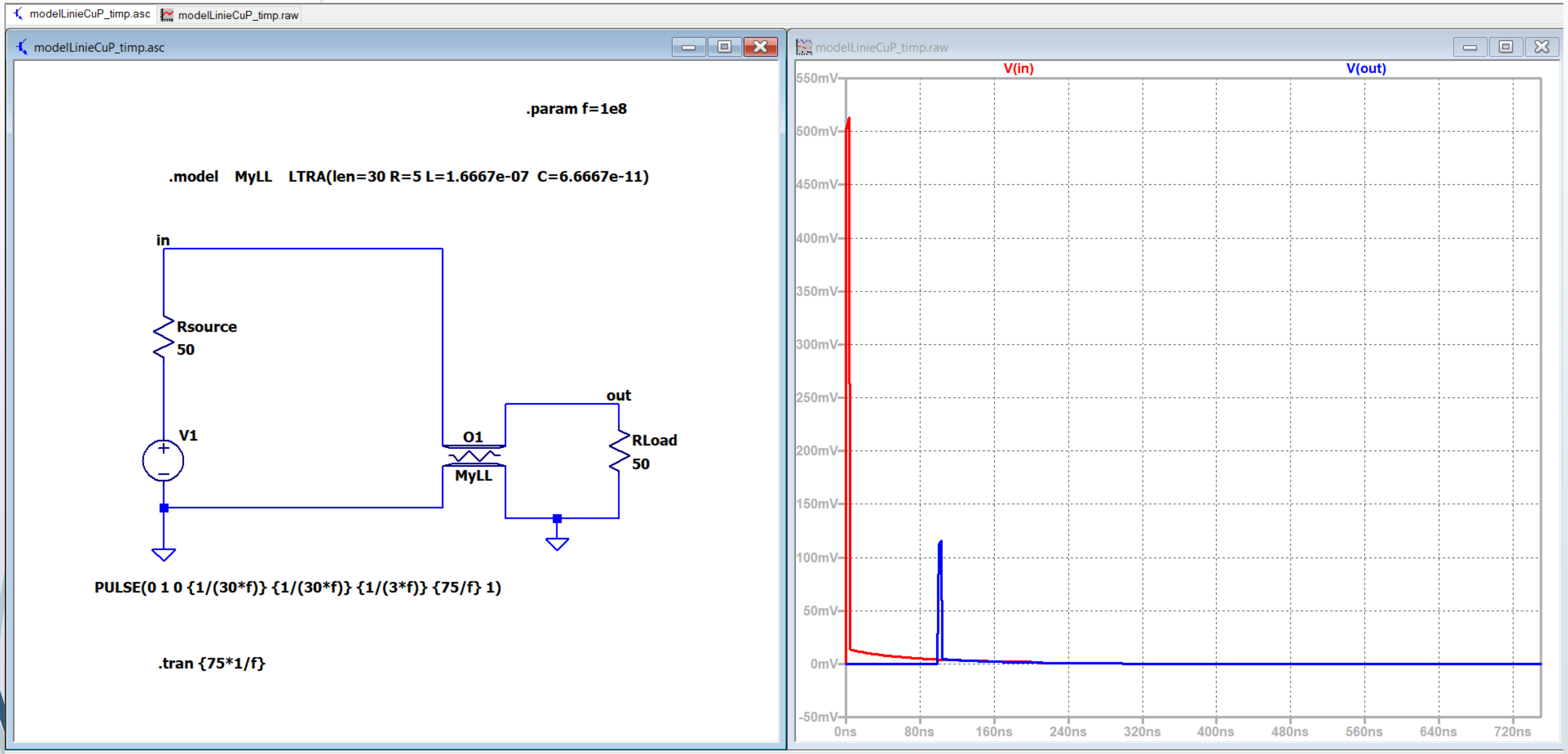
$$R_l = 0$$

$$L_l = \frac{Z_0}{c} = \frac{50}{3 \cdot 10^8} = 1.6667 \cdot 10^{-7} \frac{\mathrm{H}}{\mathrm{m}}$$

$$C_l = \frac{1}{Z_0\, c} = \frac{1}{50 \cdot 3 \cdot 10^8} = 6.6667 \cdot 10^{-11} \frac{\mathrm{F}}{\mathrm{m}}$$

Linie adaptata la ambele capete

2. Experimente virtuale #2 b)



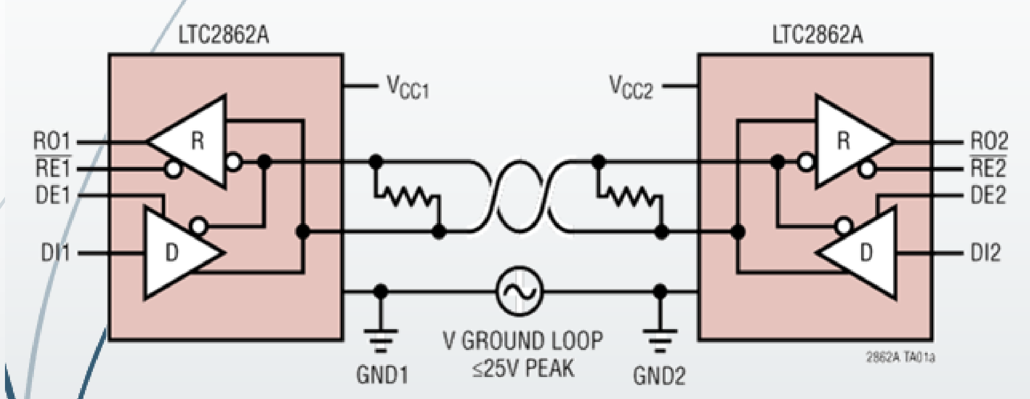
Linia anterioara dar

$$R_l = 5\frac{\Omega}{m}$$

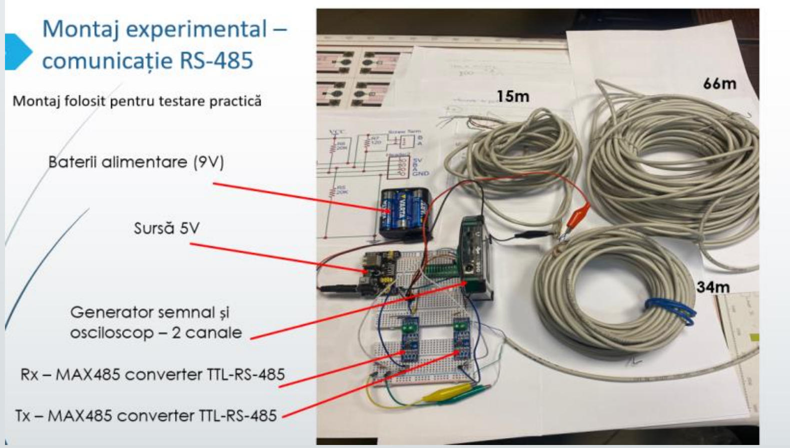
Semnalul se atenueaza.

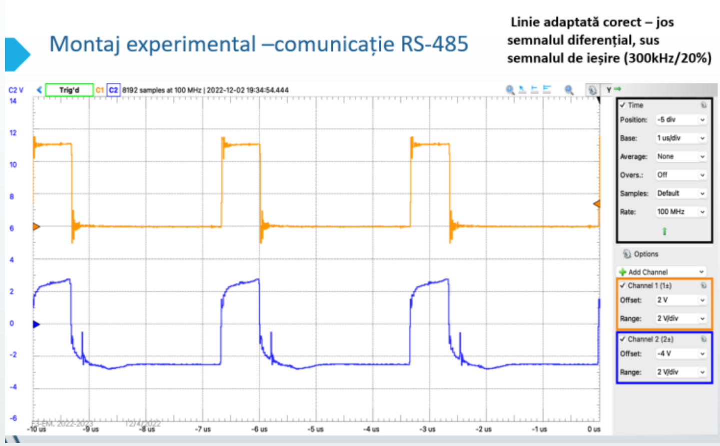
3. Experiment real

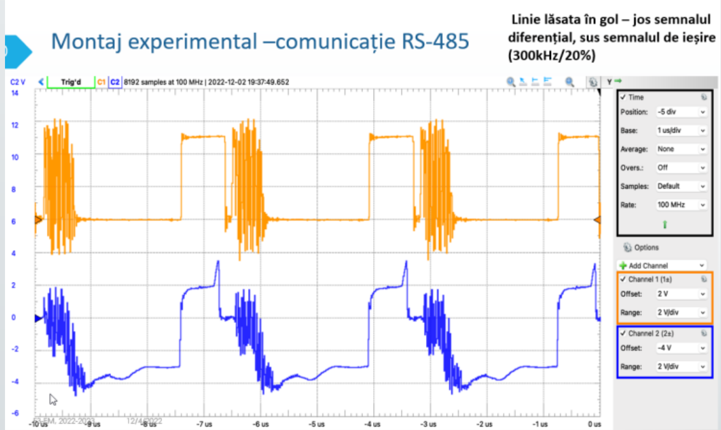
Vedeti detalii in aplicatii5_F3_partea2.pdf



https://www.analog.com/media/en/technical-documentation/data-sheets/ltc2862a.pdf







CONCLUZIE

Parametrii lineici determina

- ***intarzierea si perturbarea semnalului** transmis pe linie, deci si*
- *frecventa maxima la care poate fi folosit un cablu pentru transmiterea datelor digitale.*

*Adaptarea inseamna asigurarea **egalitatii dintre impedanta sarcinii si impedanta liniei**. Ea trebuie realizata la **ambele capete** ale unei linii pe care se transmit semnale.*

Notare

- Rezolvati quiz-ul P5.

- Pentru bonus (pana in saptamana 14)
  - – crearea unor figuri/animatii proprii illustrative pentru cursul de EM, folosind coduri proprii si instrumente software mai performante, de exemplu https://vtk.org/, https://www.paraview.org/
  - - realizarea unor experimente virtuale/reale care sa ilustreze conceptele discutate.