Assignment 3 Sample Answers

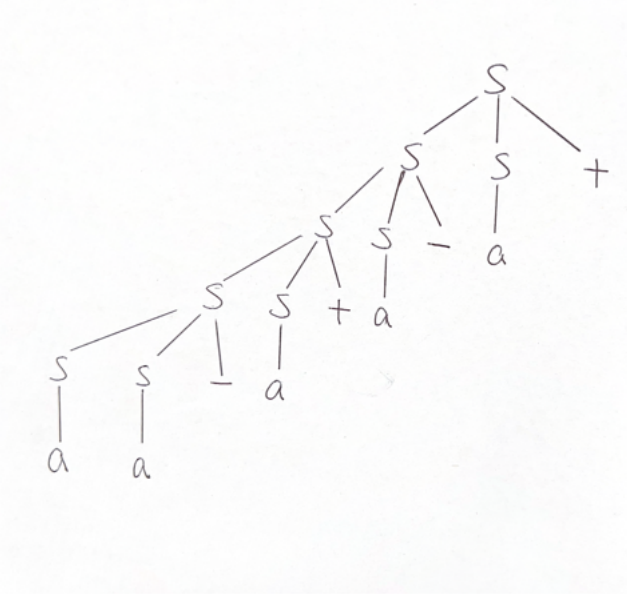
Exercise 1:

1. No. The start symbol S generates either the string “a” or strings ending with “+” or “-”.

2. S => SS+ => SS+ => SS-S+ => SS+S-S+ => SS-S+S-S+ => aS-S+S-S+ => aa-S+S-S+ => aa-a+S-S+ => aa-a+a-S+ => aa-a+a-a+

3. S => SS+ => Sa+ => SS-a+ => Sa-a+ => SS+a-a+ => Sa+a-a+ => SS-a+a-a+ => Sa-a+a-a+ => aa-a+a-a+

4. A possible parse tree is as follows.



Exercise 2:

(1) We first compute the FIRST and FOLLOW sets.

$\text{FIRST}(aB) = \{a\}$

$\text{FIRST}(S * B) = \{a\}$

$\text{FIRST}(\epsilon) = \{\epsilon\}$

$\text{FOLLOW}(S) = \{\$, *\}$

$\text{FOLLOW}(B) = \{\$, *\}$

We can then construct the predictive parsing table based on the above information. We give the resulting table below.

Nonterminals	Input symbol		
	a	$*$	$\$$
S	$s \to aB$	error	error
B	$B \to S * B$	$B \to \epsilon$	$B \to \epsilon$

(2) Yes, the grammar is LL(1).

(3) The parsing steps are listed in the table below:

Matched	Stack	Input	Action
	$S\$$	$aaaa{*}{*}{*}\$$	
	$aB\$$	$aaaa{*}{*}{*}\$$	Output $S \to aB$
a	$B\$$	$aaa{*}{*}{*}\$$	Match a
a	$S * B\$$	$aaa{*}{*}{*}\$$	Output $B \to S * B$
a	$aB * B\$$	$aaa{*}{*}{*}\$$	Output $S \to aB$
aa	$B * B\$$	$aa{*}{*}{*}\$$	Match a
aa	$S * B * B\$$	$aa{*}{*}{*}\$$	Output $B \to S * B$
aa	$aB * B * B\$$	$aa{*}{*}{*}\$$	Output $S \to aB$
aaa	$B * B * B\$$	$a{*}{*}{*}\$$	Match a
aaa	$S * B * B * B\$$	$a{*}{*}{*}\$$	Output $B \to S * B$
aaa	$aB * B * B * B\$$	$a{*}{*}{*}\$$	Output $S \to aB$

$aaaa$	$B*B*B*B\$$	$***\$$	Match a
$aaaa$	$*B*B*B\$$	$***\$$	Output $B \to \epsilon$
$aaaa*$	$B*B*B\$$	$**\$$	Match $*$
$aaaa*$	$*B*B\$$	$**\$$	Output $B \to \epsilon$
$aaaa**$	$B*B\$$	$*\$$	Match $*$
$aaaa**$	$*B\$$	$*\$$	Output $B \to \epsilon$
$aaaa***$	$B\$$	$\$$	Match $*$
$aaaa***$	$\$$	$\$$	Output $B \to \epsilon$

Exercise 3:

(1)

SLR: We compute the canonical LR(0) item sets for the following augmented grammar:

$S' \to S$
$S \to aB$
$B \to S*B \mid \epsilon$

$I_0 = \text{CLOSURE}(\{[S' \to \cdot S]\}) = \{[S' \to \cdot S], [S \to \cdot aB]\}$

$I_1 = GOTO(I_0, S) = CLOSURE(\{[S' \to S \cdot]\}) = \{[S' \to S \cdot]\}$

$I_2 = GOTO(I_0, a) = CLOSURE(\{[S \to a \cdot B]\}) = \{[S \to a \cdot B], [B \to \cdot S*B], [B \to \cdot], [S \to \cdot aB]\}$

$I_3 = GOTO(I_2, S) = CLOSURE(\{[B \to S \cdot * B]\}) = \{[B \to S \cdot * B]\}$

$GOTO(I_2, a) = CLOSURE(\{[S \to a \cdot B]\}) = I_2$

$I_4 = GOTO(I_2, B) = CLOSURE(\{S \to aB \cdot\}) = \{S \to aB \cdot\}$

$I_5 = GOTO(I_3, *)$
$= CLOSURE(\{[B \to S * \cdot B]\})$
$= \{[B \to S * \cdot B], [B \to \cdot S*B], [B \to \cdot], [S \to \cdot aB]\}$

$GOTO(I_5, S) = CLOSURE(\{[B \to S \cdot * B]\}) = \{[B \to S \cdot * B]\} = I_3$
$GOTO(I_5, a) = CLOSURE(\{[S \to a \cdot B]\}) = I_2$

$I_6 = GOTO(I_5, B) = CLOSURE(\{[B \to S*B \cdot]\}) = \{[B \to S*B \cdot]\}$

$FOLLOW(S') = \{\$\}$

$FOLLOW(S) = \{\$,*\}$

$FOLLOW(B) = FOLLOW(S) = \{\$,*\}$

The SLR(1) parsing table is as follows. The empty cells are errors.

State	ACTION			GOTO	
	a	*	$	S	B
0	s2			1	
1			accept		
2	s2	reduce $B \to \epsilon$	reduce $B \to \epsilon$	3	4
3		s5			
4		reduce $S \to aB$	reduce $S \to aB$		
5	s2	reduce $B \to \epsilon$	reduce $B \to \epsilon$	3	6
6		reduce $B \to S * B$	reduce $B \to S * B$		

CLR: Construct the LR(1) item sets

$I_0 = CLOSURE(\{[S' \to \cdot S, \$]\})$
$= \{[S' \to \cdot S, \$], [S \to \cdot aB, \$]\}$

$I_1 = GOTO(I_0, S) = CLOSURE(\{[S' \to S \cdot, \$]\}) = \{[S' \to S \cdot, \$]\}$

$I_2 = GOTO(I_0, a)$
$= CLOSURE(\{[S \to a \cdot B, \$]\})$
$= \{[S \to a \cdot B, \$], [B \to \cdot S * B, \$], [B \to \cdot, \$], [S \to \cdot aB, *]\}$

$I_3 = GOTO(I_2, S) = CLOSURE(\{[B \to S \cdot * B, \$]\}) = \{[B \to S \cdot * B, \$]\}$

$I_4 = GOTO(I_2, a)$
$= CLOSURE(\{[S \to a \cdot B, *]\})$
$= \{[S \to a \cdot B, *], [B \to \cdot S * B, *], [B \to \cdot, *], [S \to \cdot aB, *]\}$

$I_5 = GOTO(I_2, B) = CLOSURE(\{[S \to aB \cdot, \$]\}) = \{[S \to aB \cdot, \$]\}$

$I_6 = GOTO(I_3, *)$
$= CLOSURE(\{[B \to S * \cdot B, \$]\})$
$= \{[B \to S * \cdot B, \$], [B \to \cdot S * B, \$], [B \to \cdot, \$], [S \to \cdot aB, *]\}$

$I_7 = GOTO(I_4, S) = CLOSURE(\{[B \to S \cdot * B, *]\}) = \{[B \to S \cdot * B, *]\}$

$GOTO(I_4, a) = CLOSURE(\{[S \to a \cdot B, *]\}) = I_4$

$I_8 = GOTO(I_4, B) = CLOSURE(\{[S \to aB \cdot, *]\}) = \{[S \to aB \cdot, *]\}$

$GOTO(I_6, S) = CLOSURE(\{[B \to S \cdot * B, \$]\}) = I_3$

$GOTO(I_6, a) = CLOSURE(\{[S \to a \cdot B, *]\}) = I_4$

$I_9 = GOTO(I_6, B) = CLOSURE(\{[B \to S * B \cdot, \$]\}) = \{[B \to S * B \cdot, \$]\}$

$I_{10} = GOTO(I_7, *)$
$= CLOSURE(\{[B \to S * \cdot B, *]\})$
$= \{[B \to S * \cdot B, *], [B \to \cdot S * B, *], [B \to \cdot, *], [S \to \cdot aB, *]\}$

$GOTO(I_{10}, S) = CLOSURE(\{[B \to S \cdot * B, *]\}) = I_7$

$GOTO(I_{10}, a) = CLOSURE(\{[S \to a \cdot B, *]\}) = I_4$

$I_{11} = GOTO(I_{10}, B) = CLOSURE(\{[B \to S * B \cdot, *]\}) = \{[B \to S * B \cdot, *]\}$

The CLR(1) parsing table is as follows. The empty cells are errors.

State	ACTION			GOTO	
	a	*	$	S	B
0	s2			1	
1			accept		
2	s4		reduce by $B \to \epsilon$	3	5
3		s6			
4	s4	reduce by $B \to \epsilon$		7	8
5			reduce by $S \to aB$		
6	s4		reduce by $B \to \epsilon$	3	9
7		s10			
8		reduce by $S \to aB$			

9			reduce by $B \rightarrow S * B$		
10	s4	reduce by $B \rightarrow \epsilon$		7	11
11		reduce by $B \rightarrow S * B$			

LALR: From the LR(1) item sets computed above, we can observe that the following item sets have the same core

I_2 and I_4

I_3 and I_7

I_5 and I_8

I_6 and I_{10}

I_9 and I_{11}

We can merge them to get the new item sets

$I_0 = \{[S' \rightarrow \cdot S, \$], [S \rightarrow \cdot aB, \$]\}$

$I_1 = \{[S' \rightarrow S \cdot, \$]\}$

$I_{24} = \{[S \rightarrow a \cdot B, \$/*], [B \rightarrow \cdot S * B, \$/*], [B \rightarrow \cdot, \$/*], [S \rightarrow \cdot aB, *]\}$

$I_{37} = \{[B \rightarrow S \cdot * B, \$/*]\}$

$I_{58} = \{[S \rightarrow aB \cdot, \$/*]\}$

$I_{610} = \{[B \rightarrow S * \cdot B, \$/*], [B \rightarrow \cdot S * B, \$/*], [B \rightarrow \cdot, \$/*], [S \rightarrow \cdot aB, *]\}$

$I_{911} = \{[B \rightarrow S * B \cdot, \$/*]\}$

$GOTO(I_0, S) = I_1$

$GOTO(I_0, a) = I_{24}$

$GOTO(I_{24}, S) = I_{37}$

$GOTO(I_{24}, a) = I_{24}$

$GOTO(I_{24}, B) = I_{58}$

$GOTO(I_{37},*) = I_{610}$

$GOTO(I_{610},S) = I_{37}$

$GOTO(I_{610},a) = I_{24}$

$GOTO(I_{610},B) = I_{911}$

The LALR(1) parsing table is as follows. The empty cells are errors.

State	ACTION			GOTO	
	a	*	$	S	B
0	s24			1	
1			accept		
24	s24	reduce by $B \to \epsilon$	reduce by $B \to \epsilon$	37	58
37		s610			
58		reduce by $S \to aB$	reduce by $S \to aB$		
610	s24	reduce by $B \to \epsilon$	reduce by $B \to \epsilon$	37	911
911		reduce by $B \to S * B$	reduce by $B \to S * B$		

(2) Yes, the grammar is SLR(1), LR(1), and LALR(1)

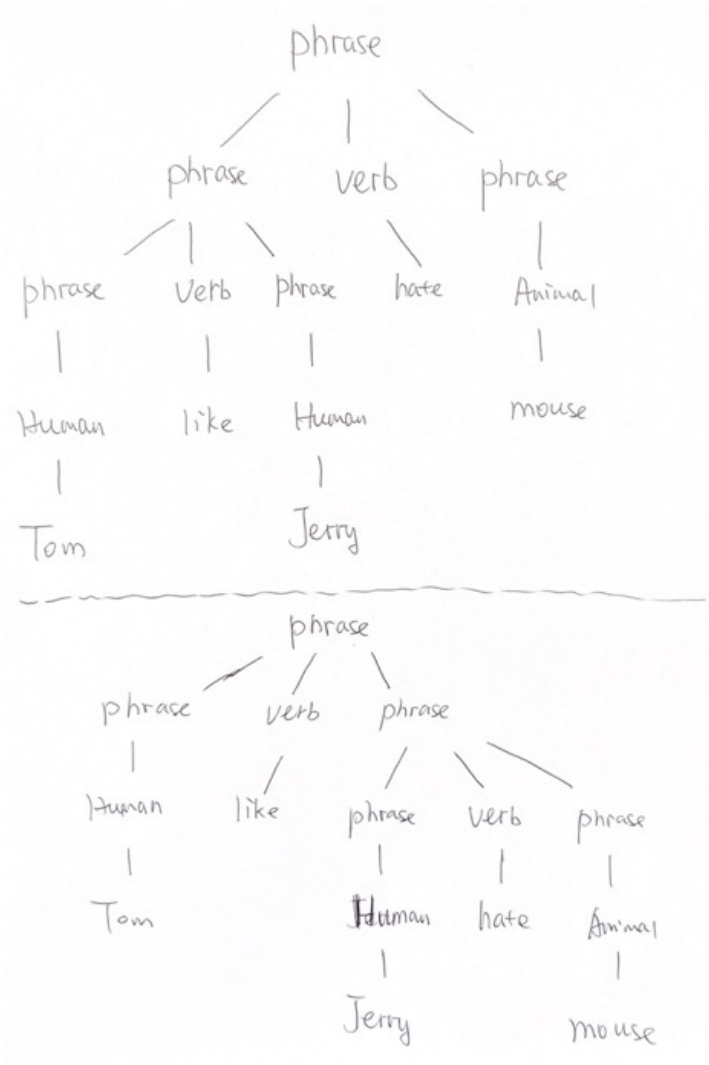
(3) Parsing of aaaa***

Line	Stack	Symbols	Input	Action
(1)	0	$	aaaa***$	shift to 24
(2)	0(24)	$a	aaa***$	shift to 24
(3)	0(24)(24)	$aa	aa***$	shift to 24
(4)	0(24)(24)(24)	$aaa	a***$	shift to 24
(5)	0(24)(24)(24)(24)	$aaaa	***$	reduce by $B \to \epsilon$
(6)	0(24)(24)(24)(24)(58)	$aaaaB	***$	reduce by S→ aB
(7)	0(24)(24)(24)(37)	$aaaS	***$	shift to 610
(8)	0(24)(24)(24)(37)(610)	$aaaS*	**$	reduce by $B \to \epsilon$
(9)	0(24)(24)(24)(37)(610)(911)	$aaaS*B	**$	reduce by $B \to S * B$
(10)	0(24)(24)(24)(58)	$aaaB	**$	reduce by $S \to aB$
(11)	0(24)(24)(37)	$aaS	**$	shift to 610

(12)	0(24)(24)(37)(610)	$aaS*	*$	reduce by $B \rightarrow \epsilon$
(13)	0(24)(24)(37)(610)(911)	$aaS*B	*$	reduce by $B \rightarrow S * B$
(14)	0(24)(24)(58)	$aaB	*$	reduce by $S \rightarrow aB$
(15)	0(24)(37)	$aS	*$	shift to 610
(16)	0(24)(37)(610)	$aS*	$	reduce by $B \rightarrow \epsilon$
(17)	0(24)(37)(610)(911)	$aS*B	$	reduce by $B \rightarrow S * B$
(18)	0(24)(58)	$aB	$	reduce by $S \rightarrow aB$
(19)	01	$S	$	accept

Optional Exercise 1:

The grammar is ambiguous. We can find multiple parse tree for the sentence “Tom like Jerry hate mouse”.



Optional Exercise 2:

There might be multiple answers. One possible grammar is:

S -> aS'
S' -> S+S' | S-S' | ε