Quiz 3 12110644 周昊宇

1° (1) what are the gradients of $\frac{\partial y_k}{\partial w_{kj}}$, $\frac{\partial y_k}{\partial w_{ji}}$ for regression & classification?

regression $\frac{\partial y_k}{\partial w_{kj}} = h'(a_j) \sum_k w_{kj} z_i$

$a_j = \sum_i w_{ji} z_i$, $z_j = h(a_j)$

$a_k = \sum_j w_{kj} z_j$, $y_k = a_k$

$\Rightarrow \frac{\partial y_j}{\partial w_{ji}} = z_i$

classification $\frac{\partial y_k}{\partial a_k} = y_k(1-y_k)$, $y_k = \sigma(a_k)$

$\frac{\partial y_k}{\partial w_{kj}} = y_k(1-y_k) z_i$, $\frac{\partial y_j}{\partial w_{ji}} = y_k(1-y_k) h'(a_j) \sum_k w_{kj} z_i$

(2) what are the gradients of $\frac{\partial E_n}{\partial w_{kj}}$, $\frac{\partial E_n}{\partial w_{ji}}$ for regression & classification?

regression $\frac{\partial E_n}{\partial w_{kj}} = \frac{\partial E_n}{\partial a_k} \frac{\partial a_k}{\partial w_{kj}} = \delta_k z_j$, $\delta_k = \frac{\partial E_n}{\partial a_k} = y_k - t_k$, $y_k = a_k$

$\frac{\partial E_n}{\partial w_{ji}} = \frac{\partial E_n}{\partial a_j} \frac{\partial a_j}{\partial w_{ji}} = \delta_j z_i$, $\delta_j = h'(a_j) \sum_k w_{kj} \delta_k$

classification $y_k = \sigma(a_k)$

(3) what are the gradients of $\frac{\partial y_k}{\partial z_i}$ for regression & classification?

regression $\frac{\partial y_k}{\partial z_i} = \sum_k w_{kj} h'(a_j) \sum_j w_{ji}$

classification $\frac{\partial y_k}{\partial z_i} = y_k(1-y_k) \sum_k w_{kj} h'(a_j) \sum_j w_{ji}$

2° If the prior on w is $p(w) = N(m_0, \Sigma_0^{-1})$, then

(1) what are the MAP solutions of w, $p(w|D)$ for regression & classification where $D = \{[x_1, \dots, x_N], [t_1, \dots, t_N]\}$

regression $w_{MAP} \leftarrow w^{new} = w^{old} - A^{-1} \nabla E(w)$

$\nabla E(w) = \alpha w + \beta \sum_{n=1}^{N} (y_n - t_n) g_n$, $g = \nabla_w y(x, w)$

$p(w|D) \propto p(w) p(t|x, w, \beta)$, $p(w) = N(m_0, \Sigma_0^{-1})$

$p(t|x, w, \beta) = N(t | y(x, w_{MAP}) + g^T_{MAP}(w - w_{MAP}, \beta^{-1})$

$\Rightarrow p(w|D) = N(w | w_{MAP}, A^{-1})$

Classification $w_{MAP} \leftarrow w^{new} = w^{old} - A^{-1} \nabla E(w)$

$$\nabla E(w) = \alpha w + \sum_{n=1}^{N} (y_n - t_n) g_n$$

$$p(w|D) \propto p(w)\, p(D|w), \quad p(w) = N(m_0, S_0^{-1})$$

$$\Rightarrow p(w|D) = N(w | w_{MAP}, A^{-1})$$

2) what are the predictive distribution of $\{x_{N+1}, t_{N+1}\}$ for regression and classification?

regression $p(t|x, D, \alpha, \beta) = N(t | y(x, w_{MAP}), g_{MAP}^T A^{-1} g_{MAP} + \beta^{-1})$

classification $p(t|x, D) = \int p(t|x, w)\, q(w|D)\, dw$

$$= p(t | x, w_{MAP}).$$