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"Modelo matematico"

1)

- KCL

$I(t) = I_a(t) + I_n(t)$

- Corriente-tensión en C.

$I_n(t) = C_n \dfrac{dv(t)}{dt}$ $\quad I_a(t) = C_a \dfrac{dv_a(t)}{dt}$

$v_a(t)$ → tensión en C_a (entre R_a y C_a)

- Relación $v(t)$, $v_a(t)$ e $I_a(t)$ (por la caída en R_a)

$V(t) = R_a I_a(t) + v_a(t)$

- KVL (serie)

$V_e(t) = L \dfrac{dI(t)}{dt} + R_e I(t) + v(t)$

2)

Relación entre $V_e(t)$ y $V(t)$.

- Por la rama C_n:

$$V(t) = \frac{1}{C_n}\int I_n(t)\,dt = \frac{1}{C_n}\int (I(t) - I_a(t))\,dt$$

$(I_n = I - I_a)$

- Por la rama serie $R_a - C_a$:

$$V(t) = R_a I_a(t) + \frac{1}{C_a}\int I_a(t)\,dt$$

- Igualando ambas expresiones

$$R_a I_a(t) + \frac{1}{C_a}\int I_a(t)\,dt = \frac{1}{C_n}\int (I(t) - I_a(t))\,dt$$

② Pag

Modelo matematico

- aislar I_a (derivar ambos miembros respecto a t)

$$R_a \frac{dI_a}{dt} + \frac{1}{C_a} I_a(t) = \frac{1}{C_n}(I(t) - I_a(t))$$

- Reagrupando terminos en I_a

Modelo integro- → $$R_a \frac{dI_a}{dt} + \left(\frac{1}{C_a} + \frac{1}{C_n}\right) I_a(t) = \frac{1}{C_n} I(t) \qquad (E1)$$

3) Ec. principal (KVL) usando v(t) en terminos de I_a.

- KVL: $V_e(t) = L\frac{dI}{dt} + R_e I(t) + v(t)$

- V(t) en función de I_a (por rama $R_a - C_a$)

$$V(t) = R_a I_a(t) + \frac{1}{C_a}\int I_a(t)\,dt$$

- Sust. en KVL:

Modelo integro- → $$V_e(t) = L\frac{dI}{dt} + R_e I(t) + R_a I_a(t) + \frac{1}{C_a}\int I_a(t)\,dt \qquad (E2)$$

4) Laplace

- De E1:

$$R_a s I_a(s) + \left(\frac{1}{C_a} + \frac{1}{C_n}\right) I_a(s) = \frac{1}{C_n} I(s)$$

L1 → $$\therefore I_a(s) = \frac{1/C_n}{R_a s + \left(\frac{1}{C_a} + \frac{1}{C_n}\right)} I(s) = \frac{1}{C_n} \cdot \frac{1}{R_a s + \left(\frac{1}{C_a} + \frac{1}{C_n}\right)} I(s)$$

- De E2:

$$V_e(s) = LsI(s) + R_e I(s) + R_a I_a(s) + \frac{1}{C_a s} I_a(s)$$

- Agrupando $I_a(s)$ factores:

L2 → $$V_e(s) = (Ls + R_e) I(s) + \left(R_a + \frac{1}{C_a s}\right) I_a(s)$$

- Sust. $I_a(s)$ desde L_1 en L_2. Tero denominador común:

- Denominador común. $A(s) = R_a s + \left(\frac{1}{C_a} + \frac{1}{C_n}\right)$

- Entonces, con L1:

$$I_a(s) = \frac{1/C_n}{A(s)} I(s)$$

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- Sust. en L2

$$Ve(s) = (Ls + Re)I(s) + \left(Ra + \frac{1}{Ca\,s}\right) \cdot \frac{1/Cn}{A(s)} I(s)$$

- Factorizando $I(s)$

$$Ve(s) = I(s)\left[(Ls + Re) + \frac{\frac{1}{Cn}\left(Ra + \frac{1}{Ca\,s}\right)}{A(s)}\right]$$

- Expresar $Vs(s) = v(s)$ (tensión en el nodo).

$$v(s) = \left(Ra + \frac{1}{Ca\,s}\right) I_a(s)$$

- de la relación $v = RaIa + \frac{1}{Ca}\int Ia$ en laplace. Sust. $Ia(s)$:

$$Vs(s) = \left(Ra + \frac{1}{Ca\,s}\right) \cdot \frac{1/Cn}{A(s)} I(s)$$

$\therefore$ La función de transferencia $\frac{Vs(s)}{Ve(s)}$:

$$\frac{Vs(s)}{Ve(s)} = \frac{\left(Ra + \frac{1}{Ca\,s}\right)\frac{1}{Cn\,A(s)}}{(Ls + Re) + \frac{1}{Cn} \cdot \frac{\left(Ra + \frac{1}{Ca\,s}\right)}{A(s)}}$$

- Mult. numerador y denominador por $CnA(s)$:

$$\frac{Vs(s)}{Ve(s)} = \frac{Ra + \frac{1}{Ca\,s}}{CnA(s)(Ls + Re) + \left(Ra + \frac{1}{Ca\,s}\right)}$$

- Sust. $A(s) = Ras + \frac{1}{Ca} + \frac{1}{Cn}$ y simpli.

*Func. Transferencia

$$\boxed{\frac{Vs(s)}{Ve(s)} = \frac{1 + RaCa\,s}{LRaCaCn\,s^3 + (L(Ca + Cn) + ReRaCaCn)s^2 + (Re(Ca + Cn) + RaCa)s + 1}}$$

Error en estado estacionario

$$e = \lim_{s \to 0} \left[1 - \frac{1}{1 + LCn} \right]$$

$$e = \lim_{s \to 0} \left[1 - \frac{1}{1 + (0.04)(0.2)} \right] = 1 - \frac{1}{1 + 0.008} = \frac{1}{1.008}$$

$e = 1 - 0.992 = 0.008 \rightarrow$ Aproximamos a: $\underline{0.01V}$

Estabilidad en lazo abierto de control

$Re = 2\Omega$ $\quad Cn = 0.2$

$Ra = 1\Omega$

$L = 0.04H$

$Ca = 0.20F$

→ Raíces

$a = (0.04)(1)(0.20)(0.20)\, s^3$

$b = ((0.4)(0.2 + 0.2) + (2)(1)(0.2)(0,2))$

$c = (2\,(1 + 0.2) + (1)(0.2))$

$d = 1$

$$\begin{cases} \lambda_1 = -138.2812 \\ \lambda_2 = -0.39929 \\ \lambda_3 = -11.3194 \end{cases}$$

* Estable

Estabilidad en lazo abierto caso

$Re = 2\ \Omega$

$Ra = 20\ \Omega$

$L = 0.04\ H$

$Ca = 0.07\ F$

$Cn = 0.2F$

→ Raíces

$a = (0.04)(20)(0.07)(0.2)$

$b = ((0.04)(0.07 + 0.2) + (2)(20)(0.07)(0.2))$

$c = ((2)(0.07 + 0.2) + (20)(0.07))$

$d = 1$

* Estable

$\lambda_1 = -47.3456 \quad \lambda_2 = -0.6312 \quad \lambda_3 = -2.9874$