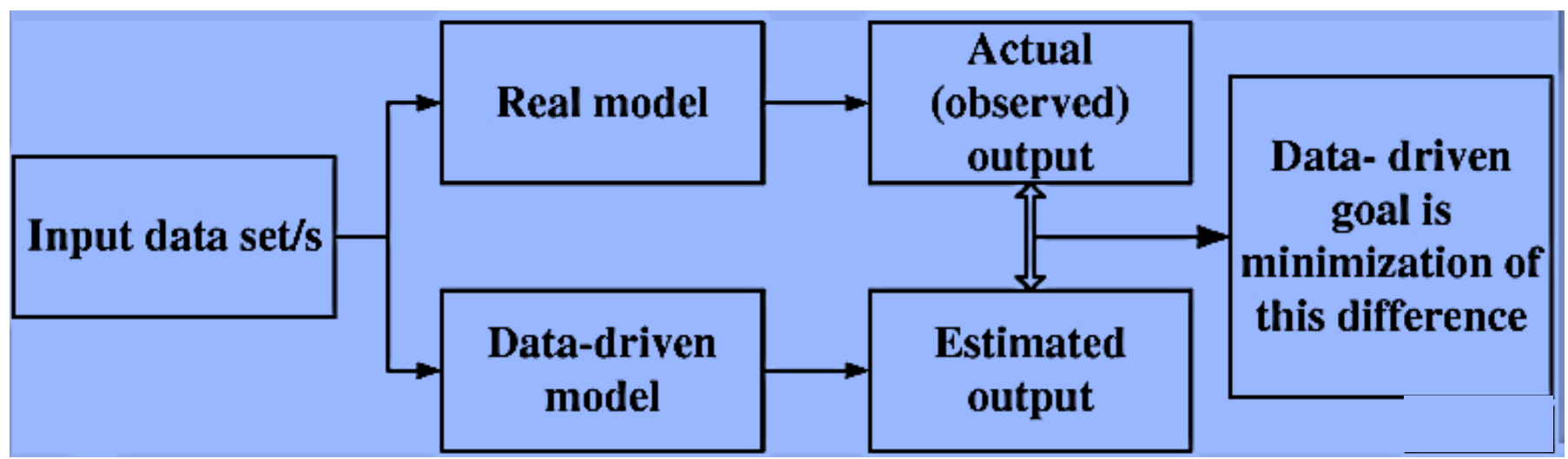
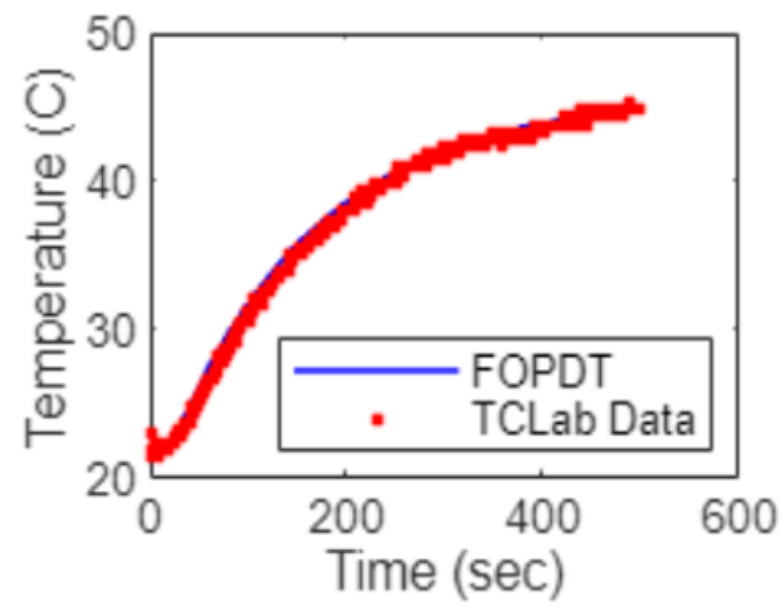
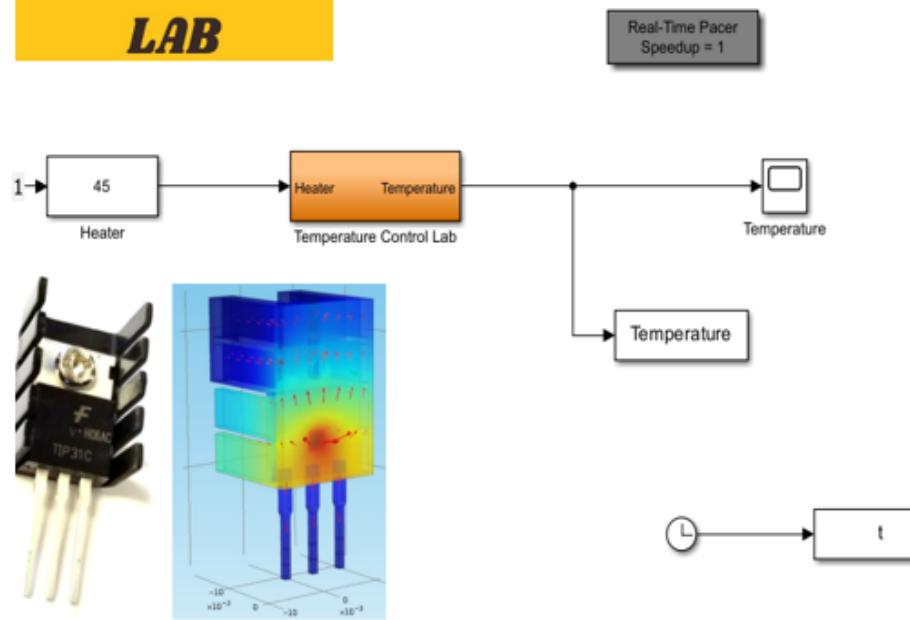


Data-Driven Modelling

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First Order Plus Dead Time Models

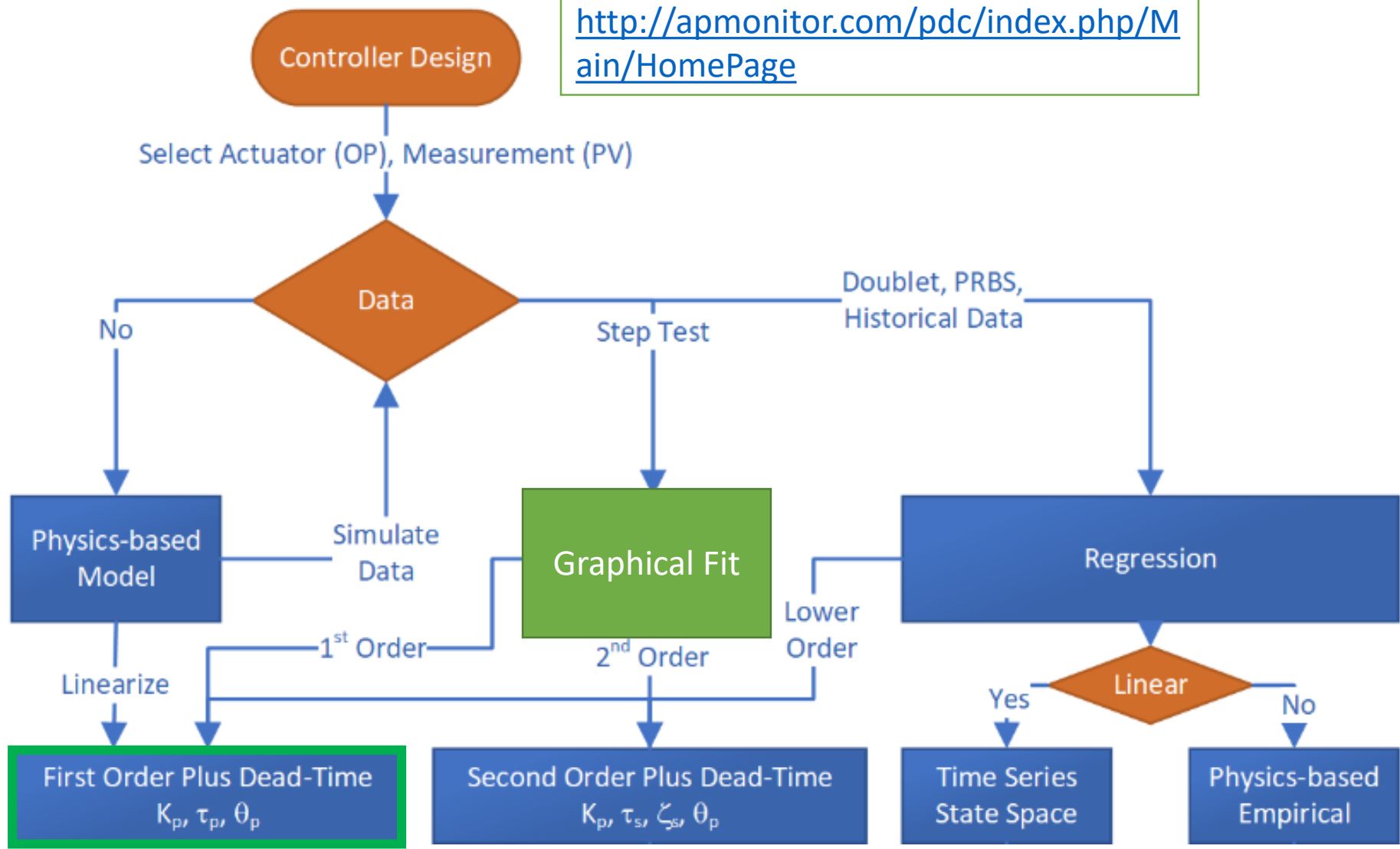
Graphical Fit



Working with:

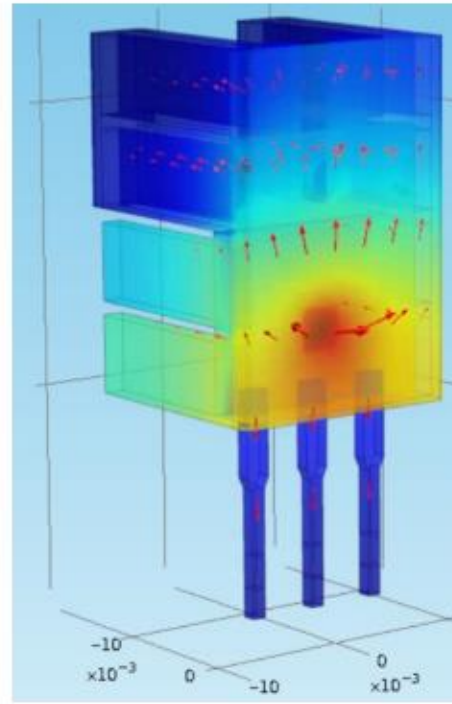
1. Introduction to First Order Plus Dead Time Models with a Presentation
2. FOPD Temperature response to a step (A live Script that includes a video)

<http://apmonitor.com/pdc/index.php/Main/HomePage>



TC Lab Dynamics:

A single heater and the total energy balance.



Quantity
Initial temperature (T_0)
Ambient temperature (T_∞)
Heater output (Q)
Heater factor (α)
Heat capacity (C_p)
Surface Area (A)
Mass (m)
Overall Heat Transfer Coefficient (U)
Emissivity (ϵ)
Stefan Boltzmann Constant (σ)

$$m c_p \frac{dT}{dt} = \sum \dot{h}_{in} - \sum \dot{h}_{out} + Q$$

Q is the rate of heat transfer

$$m c_p \frac{dT}{dt} = U A (T_\infty - T) + \epsilon \sigma A (T_\infty^4 - T^4) + \alpha Q$$

Convection

Radiation

Heater

<https://apmonitor.com/do/index.php/Main/AdvancedTemperatureControl>

REAL TIME EXPERIMENTS - TC Lab – Dynamic model of a single heater

$m = 0.001$; % kg (1 gm) % mass

% heat transfer coefficient

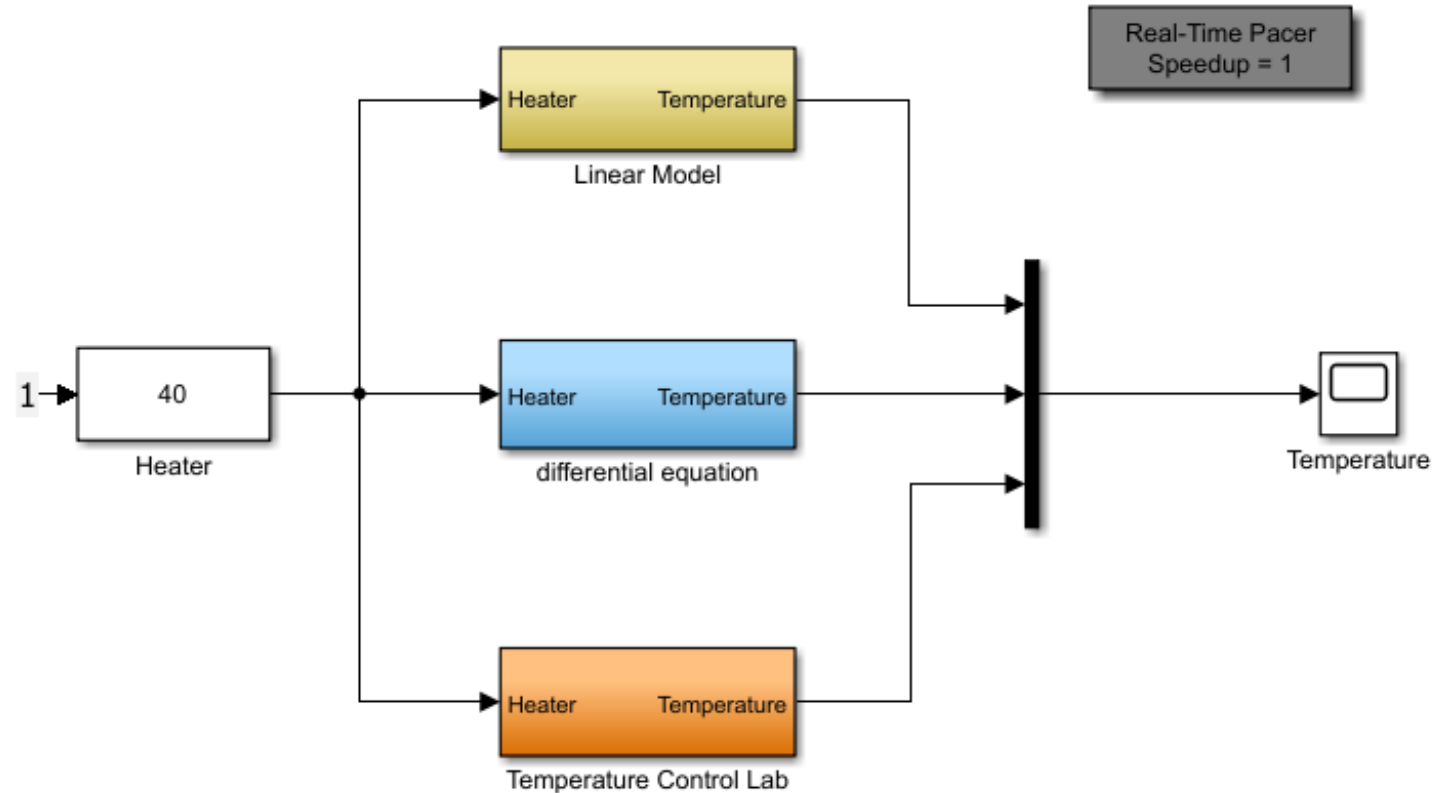
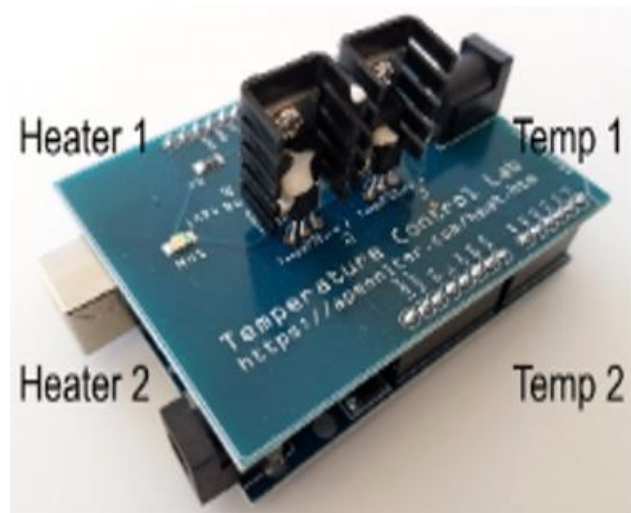
$U = 200$; % W/m²-K

% surface area

$A = 2 / 100^2$; % m²

% heat capacity

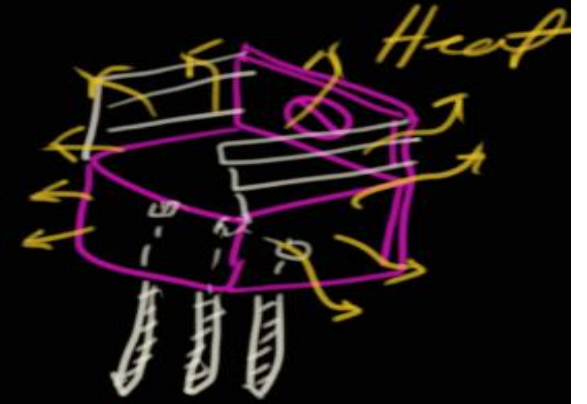
$C_p = 4900.0$; % J/kg-K



REAL-TIME EXPERIMENTS - TC Lab - Dynamic model of a single heater

Total Energy Balance

$$mC_p \frac{dT}{dt} = \underbrace{UA(T_{\infty} - T)}_{\text{Convection}} + \underbrace{\epsilon \sigma (T_{\infty}^4 - T^4)}_{\text{Radiation}} + \underbrace{\alpha Q}_{\text{heater}}$$



If we ignore Radiation components

* $mC_p \frac{dT}{dt} = UA(T_{\infty} - T) + \alpha Q$ First order differential equation

$$\left(\frac{mC_p}{UA} \right) \frac{dT}{dt} + T = T_{\infty} + \left(\frac{1}{UA} \right) \alpha Q$$

$$\tau_p \frac{dT}{dt} + T = T_{\infty} + k_p \alpha Q$$

$$\tau_p = \frac{mC_p}{UA}$$

$$k_p = \frac{1}{UA}$$

Using Laplace and assuming initial conditions 'zero'

$$T(s) = \frac{1}{\tau_p s + 1} T_{\infty}(s) + \frac{k_p}{\tau_p s + 1} \alpha Q(s)$$

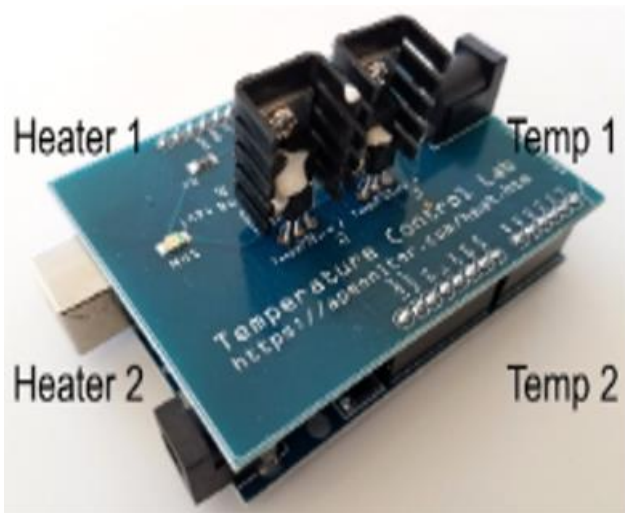
$$T(s) = T_{\infty} + \frac{k_p}{\tau_p s + 1} \alpha Q$$

Using * $\frac{dT}{dt} = \frac{UA(T_{\infty} - T) + \alpha Q}{mC_p}$

REAL-TIME EXPERIMENTS - TC Lab – Dynamic model of a single heater

$m = 0.001$; % kg (1 gm)
% heat transfer coefficient
 $U = 200$; % $W/m^2 \cdot K$
% surface area
 $A = 2 / 100^2$; % m^2
% heat capacity
 $C_p = 4900.0$; % $J/kg \cdot K$

$$\begin{aligned} M &= 0.001 \text{ kg} \\ C_p &= 4900 \frac{J}{kg \cdot K} \\ U &= 200 \frac{W}{m^2 K} \\ A &= \frac{2}{100^2} m^2 \end{aligned} \quad \left. \begin{aligned} \tau_p &= \frac{(0.001 \text{ kg}) / (4900 \frac{J}{kg \cdot K})}{(200 \frac{W}{m^2 K}) (0.0002 m^2)} \\ \tau_p &= 122.5 \frac{J}{W} \\ \tau_p &= 122.5 s \end{aligned} \right\}$$



Temperature response to a step input and obtain the First Order Plus Dead Time FOPDT parameters

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<https://www.youtube.com/watch?v=CJ3OD5WJUTE>

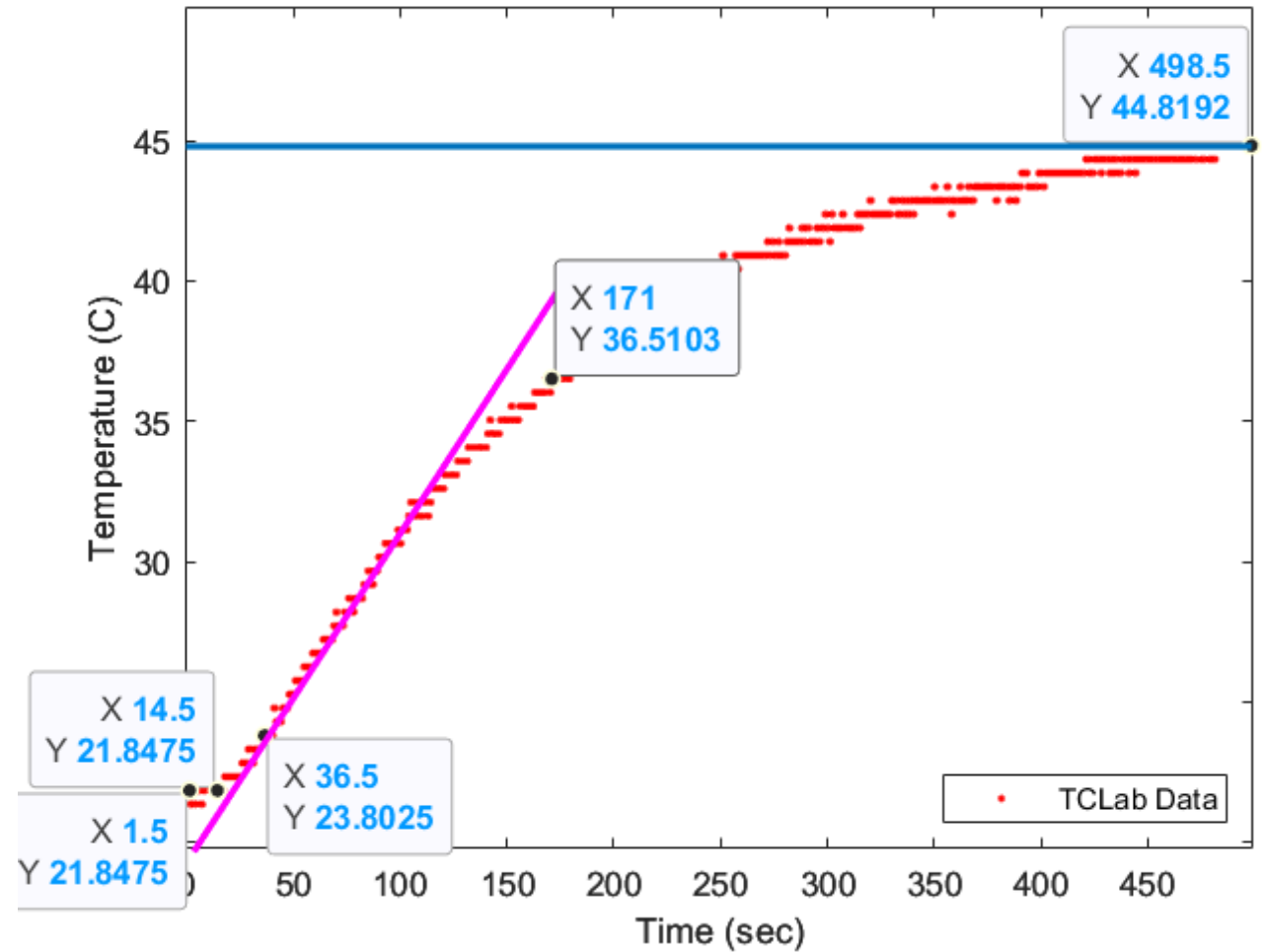
First Order Plus Dead Time Models (FOPDT) with TC lab Real Time Experiments

Temperature response to a step input and obtain the First Order Plus Dead Time FOPDT parameters

Real-Time Experiment and Graphical Fit

dead_time	13
du	90
dy	45.7683
Kp	0.5085
t	1001x1 doub
Tao	123.2000
Temperature	1001x1 doub
tout	1001x1 doub
y_tao	51.9256

```
dy=Temperature(end)-Temperature(1);  
du=45;  
Kp=dy/du;  
dead_time=22-9;  
y_tao=Temperature(1)+0.632*dy;  
Tao=145.2-22;
```



An Introduction to First Order Plus Dead Time Models

A first-order linear system with time delay is a common empirical description of many stable dynamic processes.

Graphical Method: FOPDT to Step Test

After gaining an intuitive understanding of the step response of a single heater model, it is important to understand the mathematical FOPDT equation

$$\tau_p \frac{dy(t)}{dt} = -y(t) + K_p u(t - \theta_p)$$

with variables $y(t)$ and $u(t)$

Where

K_p = Process gain

τ_p = Process time constant

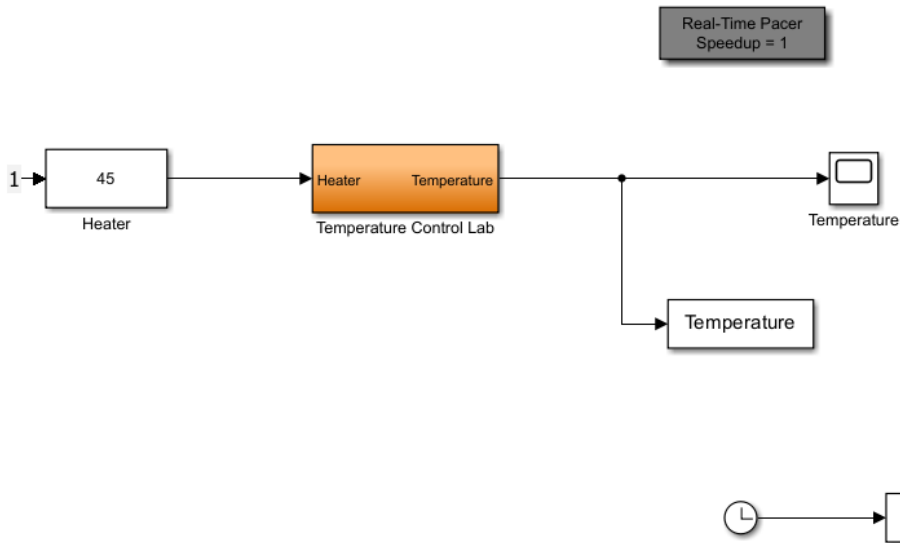
θ_p = Process dead time

An Introduction to First Order Plus Dead Time Models

REAL-TIME EXPERIMENTS - TC Lab -

The Live Script will help us to see the effect of the three adjustable parameters in the FOPDT equation. The gain, time constant, and dead time. Sample data from the TC Lab is used to show how FOPDT models represent measured data.

Use the Live Script: [An Introduction to First Order Plus Dead Time Models](#)



Use the measured temperature data and see how the main dynamic parameters fit the temperature curve.

Obtain the gain, time constant, and dead time