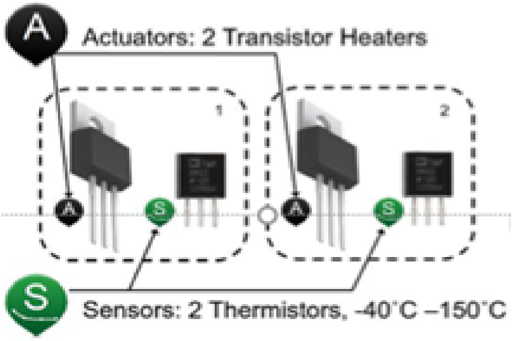
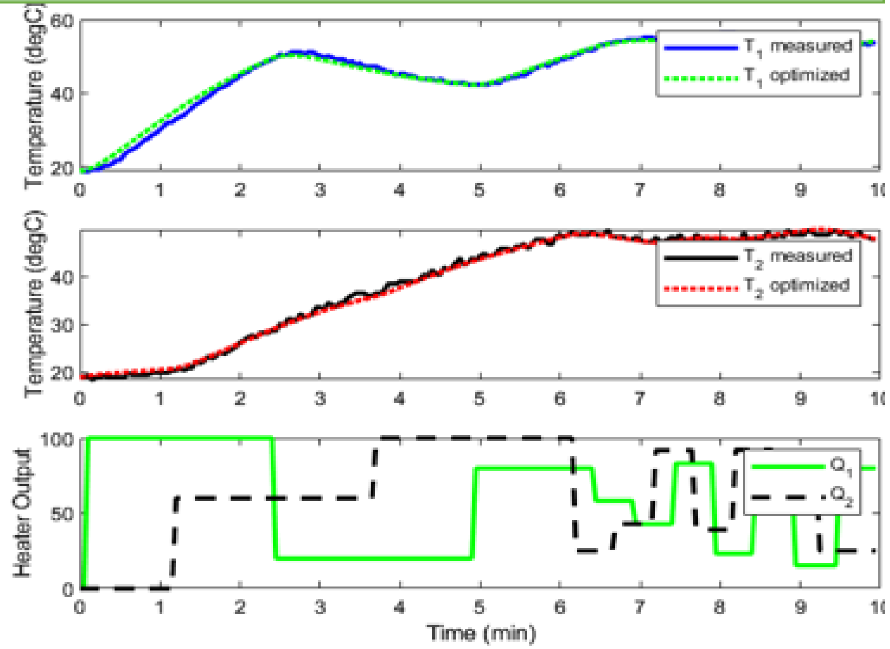
Data-Driven Modelling

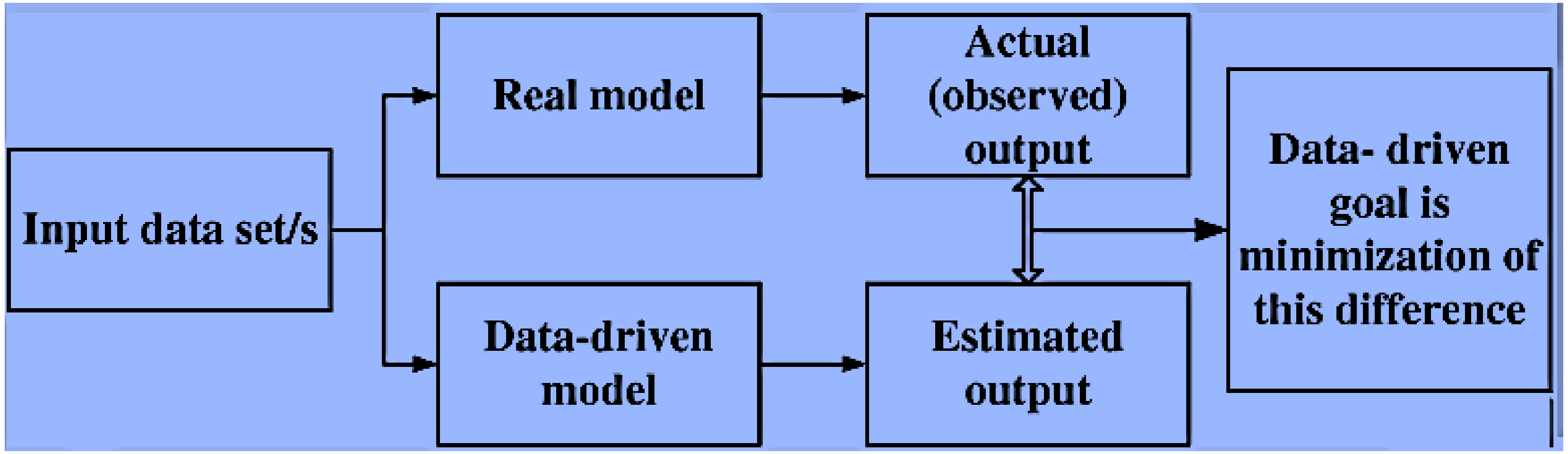
Semi-Empirical Model Estimation: Second-Order Regression

Control Tech LAB





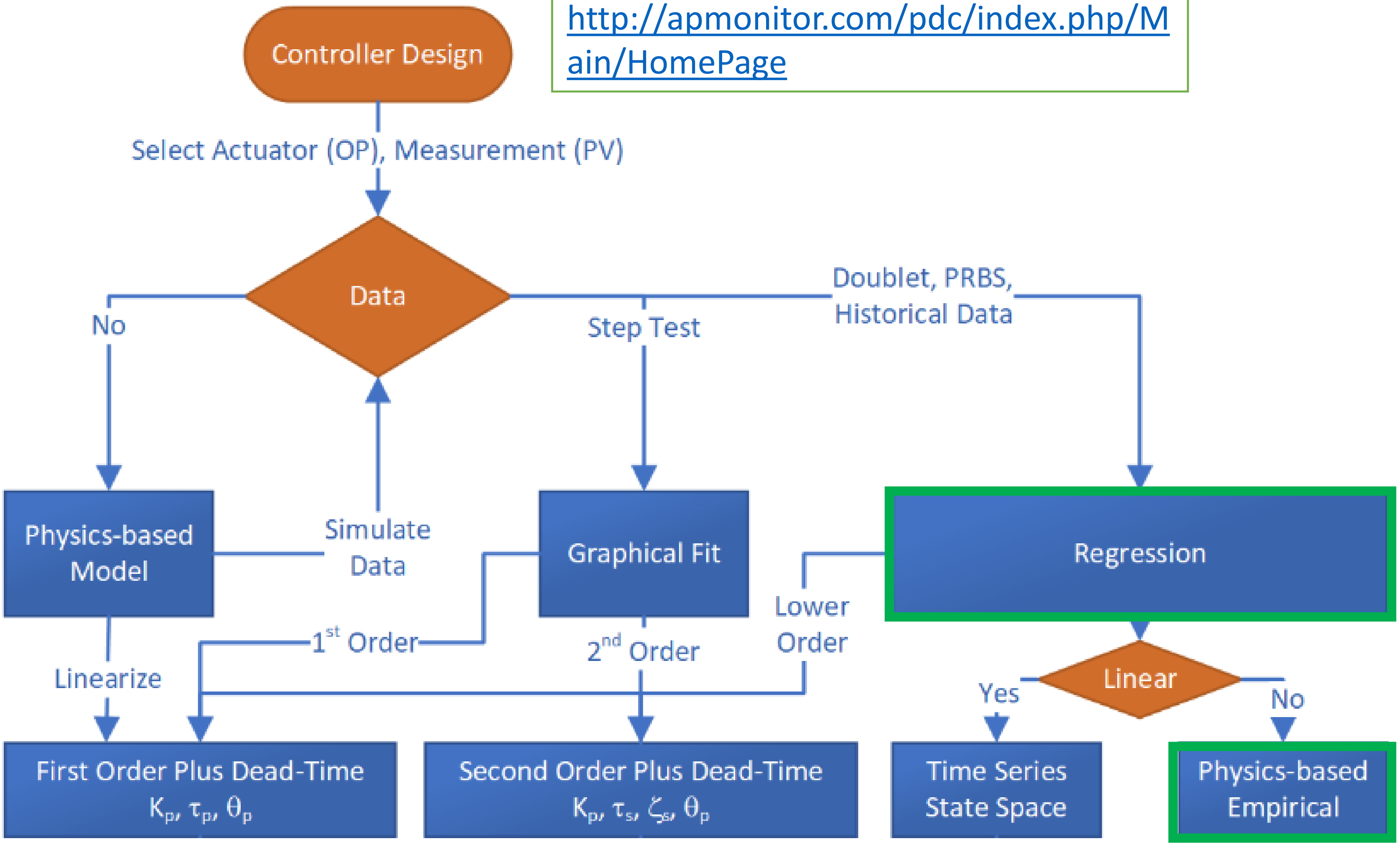
Adjust the parameters and achieve alignment between the model and the measured values.



Working with:

1. Introduction to Semi-Empirical Model Estimation with a Presentation
2. Second Order Regression (A live Script that includes a video)

http://apmonitor.com/pdc/index.php/Main/HomePage



Using the APMonitor Server for Real-time

The APMonitor Modeling Language is optimization software for mixed-integer and differential algebraic equations. It is coupled with large-scale solvers for linear, quadratic, nonlinear, and mixed integer programming. Modes of operation include data reconciliation, real-time optimization, dynamic simulation, and nonlinear predictive control. It is freely available through MATLAB, Python, or from a web browser interface.

Semi-Empirical Model Estimation : Second Order Regression

System identification using empirical data. The predictions are aligned to the measured values through an optimizer that adjusts the empirical parameters to minimize a sum of squared error or sum of absolute values objective.

The objective is to fit **empirical and physics-based predictions** to the data for a two-heater model of the temperature control lab. Parameters are adjusted to minimize the sum of squared errors (SSE) or the integral absolute error (IAE) between the model-predicted values and the measured values.

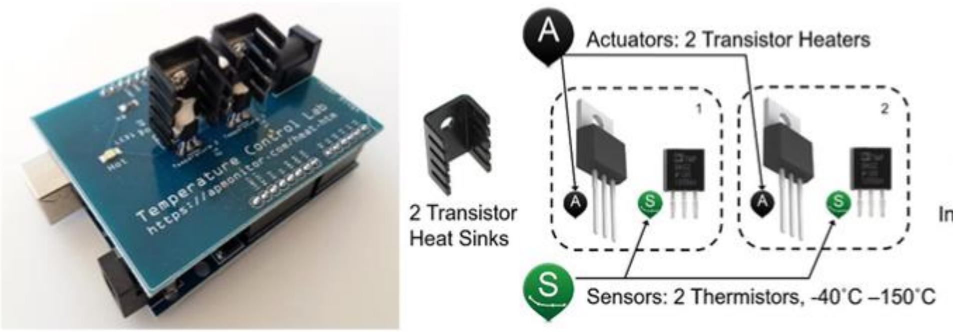
$$IAE_{model} = \sum_{i=0}^{n} |T_{1,meas,i} - T_{1,pred,i}| + |T_{2,meas,i} - T_{2,pred,i}|$$

An optimizer is used to adjust the parameters and achieve alignment between the model and the measured values.

https://apmonitor.com/do/index.php/Main/TCLabD

Regression MIMO System

Transient model between the two heater power outputs and the two temperature sensors. An energy balance describes the transient temperature response of heaters with temperature sensor.



This model represents the energy balance equation with convective heat transfer, radiative heat transfer, and the heater energy inputs. The additional blue terms are heat transfer

convective

radiative

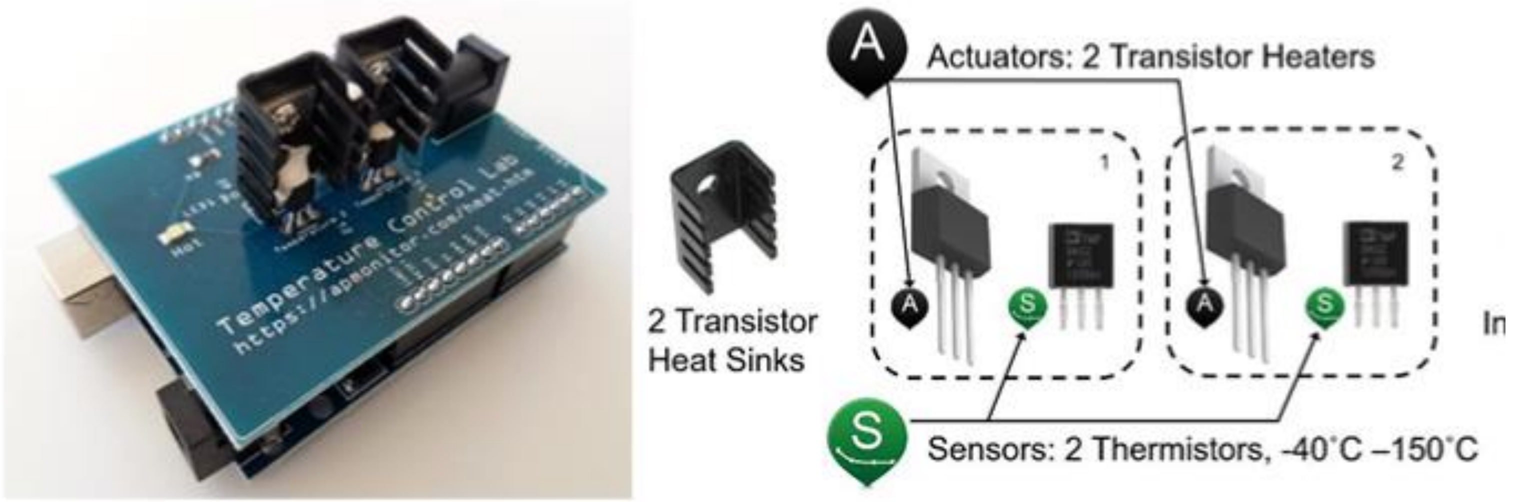
heater inputs

$$mC_p \frac{dT_1}{dt} = UA(T_\infty - T_1) + \epsilon\sigma A_s(T_\infty^4 - T_1^4) + UA_s(T_2 - T_1) + \epsilon\sigma A_s(T_2^4 - T_1^4) + Q_1$$

$$mC_p \frac{dT_2}{dt} = UA(T_\infty - T_2) + \epsilon\sigma A_s(T_\infty^4 - T_2^4) + \underbrace{UA_s(T_1 - T_2) + \epsilon\sigma A_s(T_1^4 - T_2^4)}_{\text{heat transfer}} + Q_2$$

Regression MIMO System

The heater and temperature sensor are assumed to be at the same temperature.



Sensor Model

$$\tau_c \frac{dT_{C1}}{dt} = T_1 - T_{C1}$$

$$\tau_c \frac{dT_{C2}}{dt} = T_2 - T_{C2}$$

You can assume that conduction is negligible and that the only heat transferred is through radiation to the surroundings or convection or radiation to the surrounding air or from the heater nearby. The heaters are initially off and the heaters and sensors are initially at ambient temperature.

The MIMO System Dynamics: 2 heaters and a first order equation per each heater

MIMO System → ignore Radiative energy

Energy Balance ↓

Heaters

$$\text{①}\quad mc_p \frac{dT_1}{dt} = \underbrace{UA(T_\infty - T_1) + UA_s(\underbrace{T_2 - T_1}_{DT})}_{\text{convective heat transfer}} + \underbrace{\alpha_1 Q_1}_{\text{Heater input}}$$

$$\text{②}\quad mc_p \frac{dT_2}{dt} = UA(T_\infty - T_2) + UA_s(T_1 - T_2) + \alpha_2 Q_2$$

$$\boxed{DT = T_2 - T_1}$$

$$\text{①}\quad mc_p \frac{dT_1}{dt} + \underline{UA}(T_1 - T_\infty) = UA_s D_T + \alpha_1 Q_1$$

$$\text{②}\quad mc_p \frac{dT_2}{dt} + UA(T_2 - T_\infty) = -UA_s DT + \alpha_2 Q_2$$

$$\text{①}\quad \overset{\tau_{12}}{\frac{mc_p}{UA}} \frac{dT_1}{dt} + (T_1 - T_\infty) = \overset{k_3}{\frac{UA_s}{UA}} DT + \overset{k_1}{\frac{\alpha_1}{UA}} Q_1$$

$$\text{②}\quad \frac{mc_p}{UA} \frac{dT_2}{dt} + (T_2 - T_\infty) = -\frac{UA_s}{UA} DT + \overset{k_2}{\frac{\alpha_2}{UA}} Q_2$$

τ_{12}

k_1

k_2

k_3

T_C

$$\tau_{12} = \frac{mc_p}{UA}\ ; \quad k_1 = \frac{\alpha_1}{UA}\ ; \quad k_2 = \frac{\alpha_2}{UA}\ ; \quad k_3 \ \frac{\cancel{U}A_s}{\cancel{U}A}$$

$$\text{①}\quad \tau_{12} \frac{dT_1}{dt} + (T_1 - T_\infty) = k_3 DT + k_1 Q_1$$

$$\text{②}\quad \tau_{12} \frac{dT_2}{dt} + (T_2 - T_\infty) = -k_3 DT + k_2 Q_2$$

The MIMO System Dynamics: 2 temperature sensors and a first order equation per each sensor

Sensors

$\downarrow$ input

Output

③ $$C_{T_{c1}} \frac{dT_{c1}}{dt} = T_1 - T_{c1}$$

④ $$C_{T_{c2}} \frac{dT_{c2}}{dt} = T_2 - T_{c2}$$

Model

Initial guess for each of the unknown system parameters

Ambient temperature

Heaters initial input

Variables start with ambient temperature

Build the dynamic equations using the written variables and parameters

```
Constants
   Ta = 23          ! degC

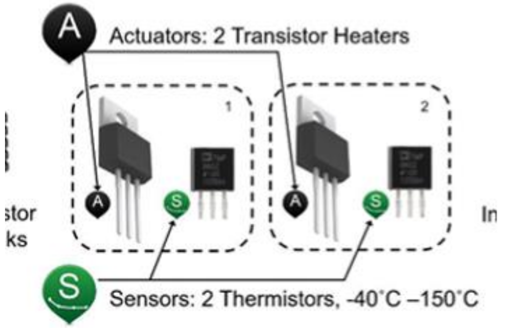
Parameters
   K1 = 0.607    > 0.1      < 1.0
   K2 = 0.293    > 0.1      < 1.0
   K3 = 0.24     > 0.0001   < 1.0
   tau12 = 192   > 50.0     < 250.0   ! sec
   tau3 = 15     > 10.0     < 20.0    ! sec

Parameters
   Q1 = 0
   Q2 = 0

Variables
   TH1 = Ta
   TH2 = Ta
   TC1 = Ta
   TC2 = Ta

Intermediates
   DT = TH2 - TH1

Equations
   tau12 * $TH1 + (TH1-Ta) = K1*Q1 + K3*DT
   tau12 * $TH2 + (TH2-Ta) = K2*Q2 - K3*DT
   tau3 * $TC1 = -TC1 + TH1
   tau3 * $TC2 = -TC2 + TH2
```



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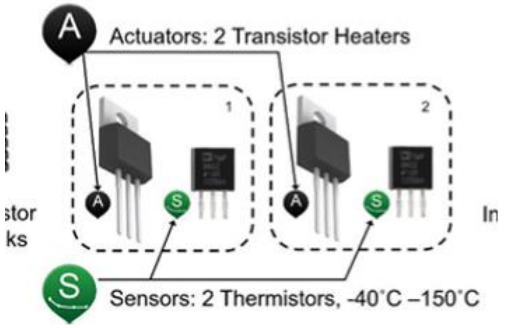
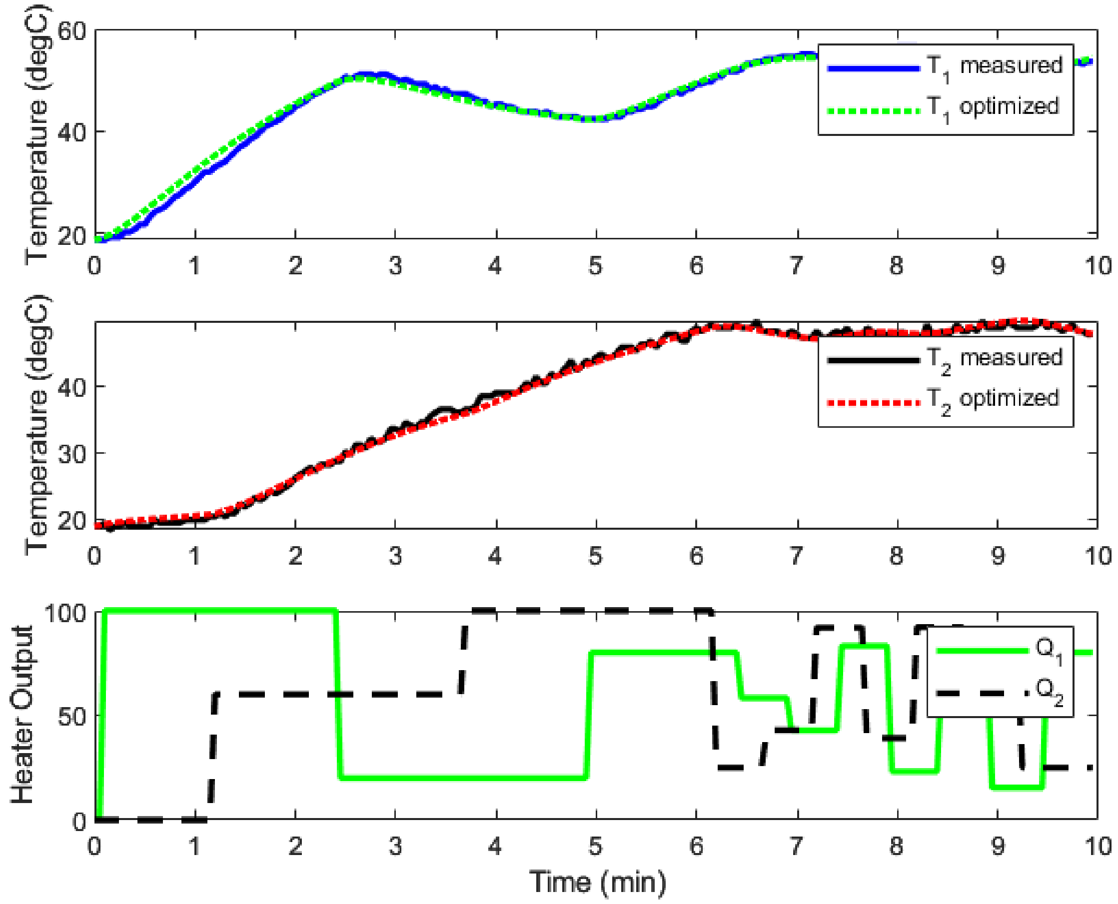
  TC2 = Ta

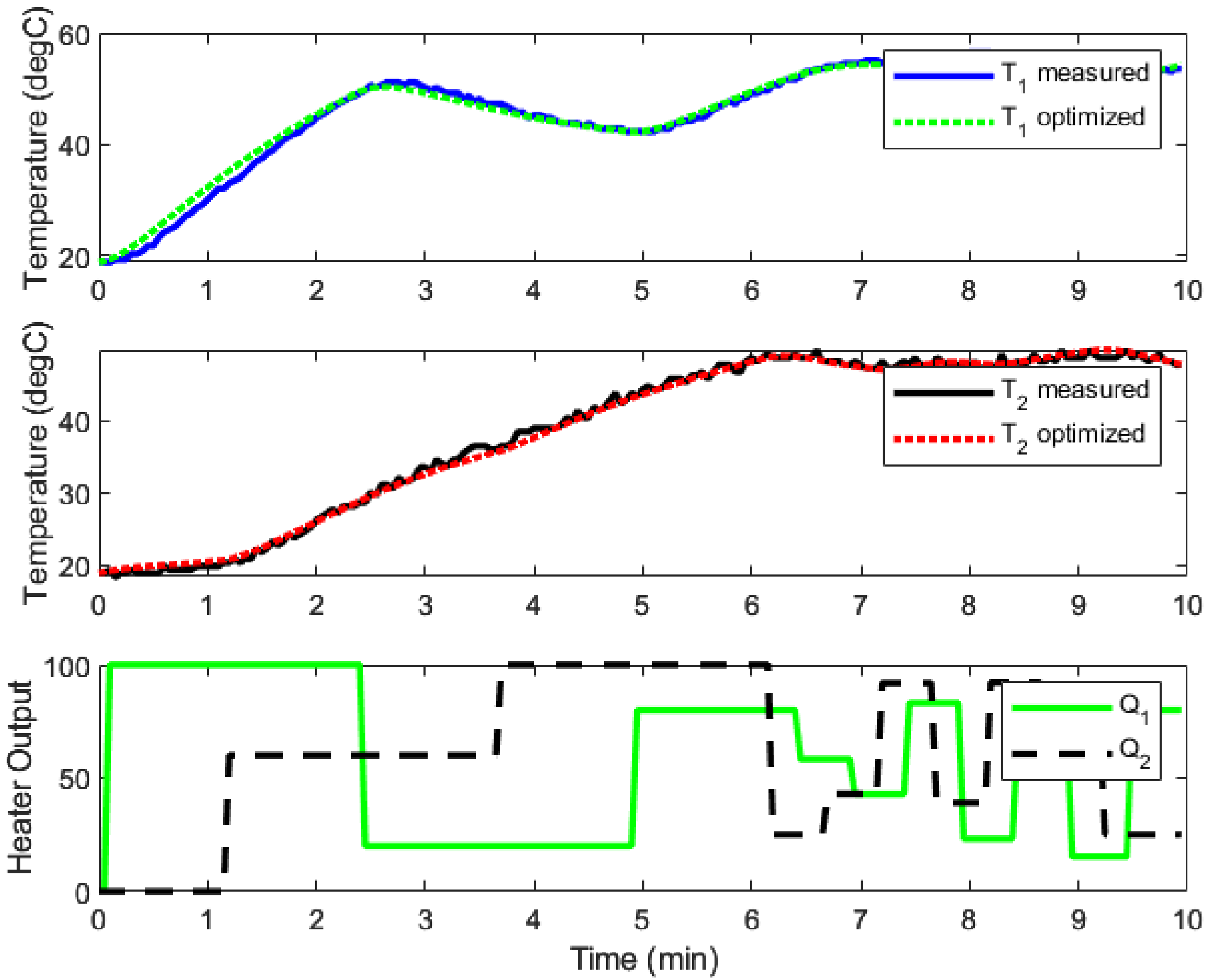
Intermediates
  DT = TH2 - TH1

Equations
  tau12 * $TH1 + (TH1-Ta) = K1*Q1 + K3*DT
  tau12 * $TH2 + (TH2-Ta) = K2*Q2 - K3*DT
  tau3 * $TC1 = -TC1 + TH1
  tau3 * $TC2 = -TC2 + TH2
```



K1: 0.7235
K2: 0.42225
K3: 0.40065
tau12: 211.018
tau3: 15





K1: 0.7235

K2: 0.42225

K3: 0.40065

tau12: 211.018

tau3: 15

We are using experimental data coming from the TC Lab

10-minute data collection period that includes rapid and slow asynchronous (staggered) steps of the heaters with varying magnitude and direction. https://www.youtube.com/watch?v=iTlENSZBxHw

Control Tech LAB

Video Objectives: A data collection period that encompasses step inputs for heaters with varying magnitudes.