week 6

# FOPDT: First Order Plus Dead Time

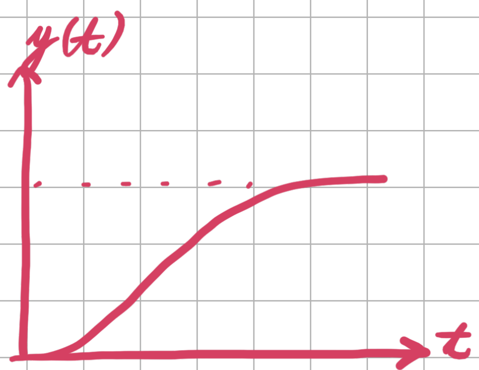
Real-Time experiment First Order + Dead Time

$$mc_p \frac{dT}{dt} = UA(T_\infty - T) + \alpha Q$$

$$T_p \frac{dT}{dt} = -T + K_p u(t)$$

$$\tau \frac{dy}{dt} = -y + K_p u(t)$$ variable of interest is $y(t)$

$$\tau_p \frac{dy}{dt} = -y + K_p u(t - \theta_p)$$


$$G(s) = \frac{K_p e^{-\theta_p s}}{T_p s + 1}$$

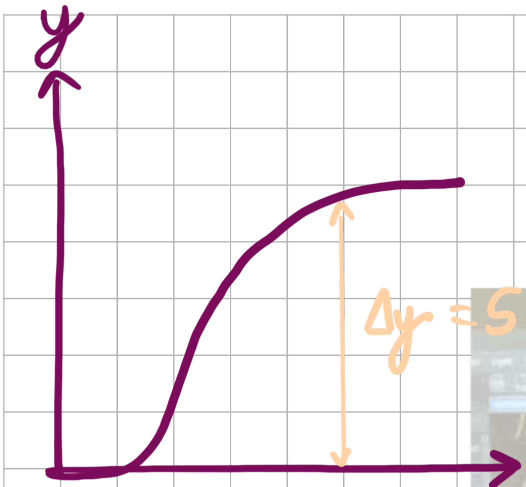
$u(t)$: input

$y(t)$: output

$k_p$: st-st constant

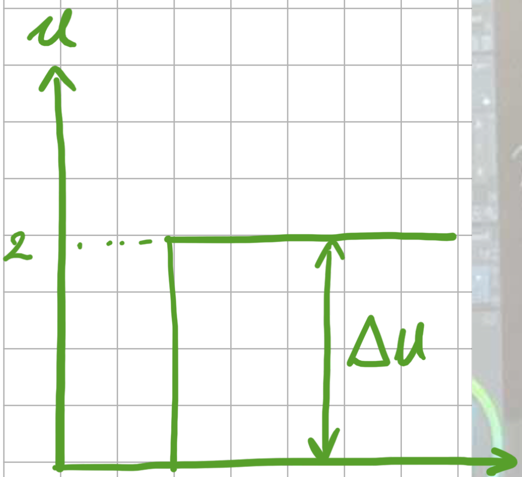
$T_p$: Time constant

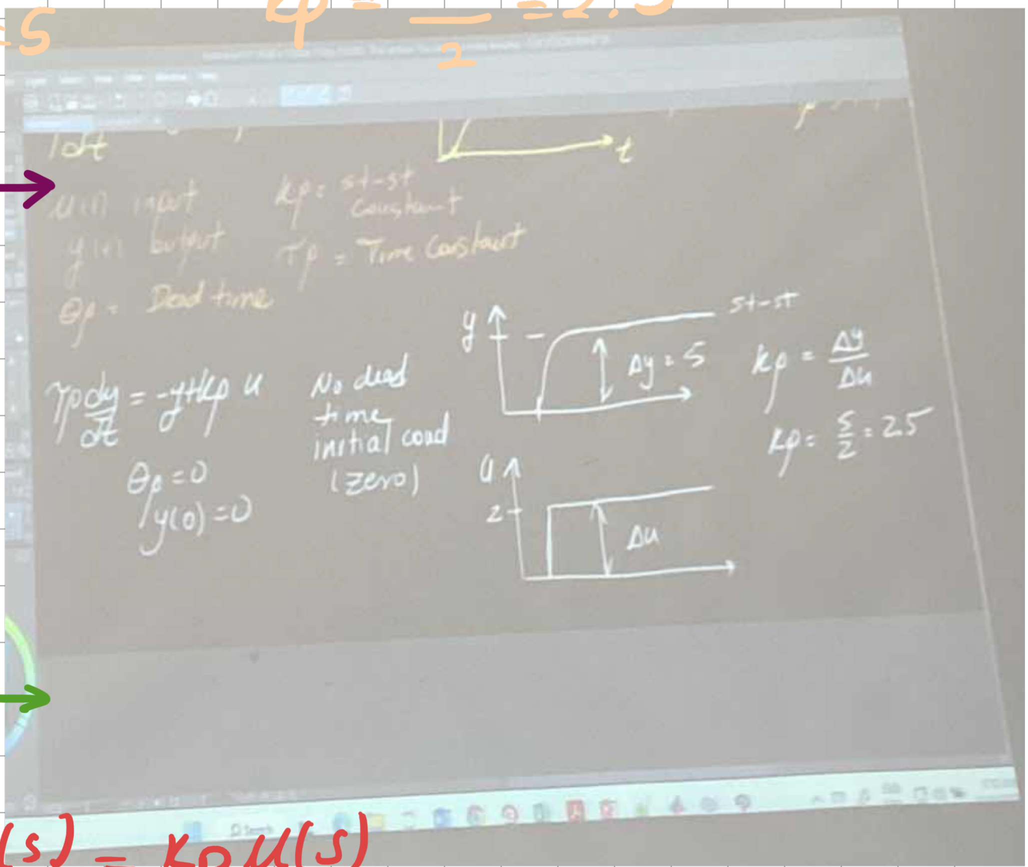
$\theta_p$: Dead Time




$$k_p = \frac{\Delta y}{\Delta u}$$

$$k_p = \frac{5}{2} = 2.5$$




$$\tau_p s\, Y(s) + Y(s) = k_p U(s)$$

$$(\tau_p s + 1)\, Y(s) = K_p U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{K_p}{\tau_p s + 1} \longrightarrow \text{step } U(s) = \frac{1}{s}$$

$$Y(s) = \frac{K_p}{s(\tau_p s + 1)} = \frac{K_p/\tau_p}{s\left(s + \frac{1}{\tau_p}\right)} = \underset{s=0}{\frac{A}{s}} + \underset{s=-1/\tau_p}{\frac{B}{s + \frac{1}{\tau_p}}}$$

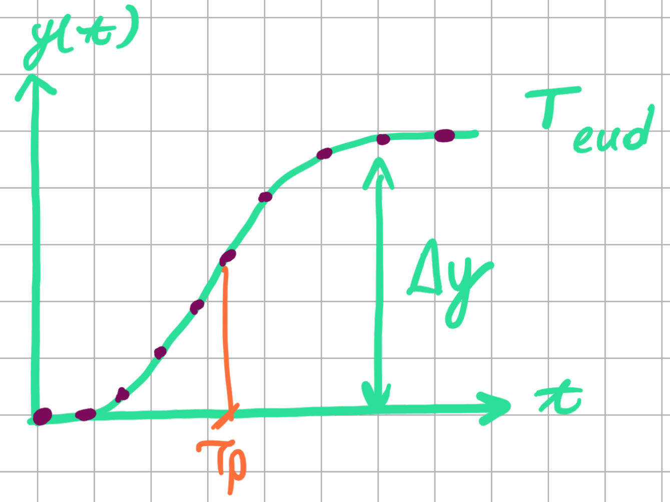
$$\left.\frac{K_p/\tau_p}{s + \frac{1}{\tau_p}}\right|_{s=0} \qquad A = K_p \; , \qquad \left.\frac{K_p/\tau_p}{s}\right|_{s=\frac{1}{\tau_p}} \qquad B = -K_p$$

$$Y(s) = \frac{K_p}{s} + \frac{-K_p}{s+\frac{1}{\tau_p}} \; ; \quad y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

$$y(t) = K_p\, u(t) - K_p\, e^{-\frac{1}{\tau_p}t}\, u(t)$$

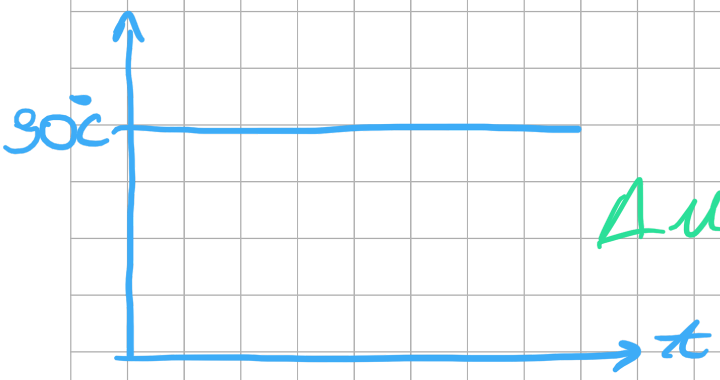
$$= K_p\left(1 - e^{-\frac{1}{\tau_p}t}\right) u(t)$$

$$y(t) = K_p\left(1 - e^{-\frac{1}{\tau_p}t}\right) u(t) \cdot s(t-\theta_p)$$




$$\Delta y = T_{end} - T(1)$$

$$K_p = \frac{\Delta y}{\Delta u}$$




$$\Delta u = 90^\circ C$$

$$y(\tau_p) = T(1) + 0.632\,\Delta y = 51.92\ ^\circ C$$

$$\tau_p = 145 - 22 = 123$$

$\tau_p$ was 122.5 (week 3)

Regression $2^{nd}$ order MIMO systems

(Ignorign Radiotion Energy)

Energy Balance of the heaters:

$$① \quad mc_p \frac{dT_1}{dt} = \underbrace{\mu A(T_\infty - T_1)}_{\text{Convective}} + \underbrace{\mu A_s \overbrace{(T_2 - T_1)}^{DT}}_{\text{heat transfer}} + \underbrace{Q_1}_{\text{Heater}}$$

$$② \quad mc_p \frac{dT_2}{dt} = \mu A(T_\infty - T_2) + \mu A_s(T_1 - T_2) + Q_2$$

$$DT = T_2 - T_1$$

$$① \quad mc_p \frac{dT_1}{dt} + \mu A(T_1 - T_\infty) = \mu A_s DT + Q_1$$

$$② \quad mc_p \frac{dT_2}{dt} + \mu A(T_2 - T_\infty) - \mu A_s DT + Q_2$$

$$① \quad \frac{mc_p}{\mu A}\frac{dT_1}{dt} + (T_1 - T_\infty) = \overbrace{\frac{\cancel{\mu}A_s}{\cancel{\mu}A}}^{k_3} DT + \overbrace{\frac{1}{\mu A}}^{k_1} Q_1$$

$$② \quad \underbrace{\frac{mc_p}{\mu A}}_{T_{12}}\frac{dT_2}{dt} + (T_2 - T_\infty) = \underbrace{\frac{-\cancel{\mu}A_s}{\cancel{\mu}A}}_{k_4} DT + \underbrace{\frac{1}{\mu A}}_{k_2} Q_2$$

$$T_{12}\frac{dT_1}{dt} + (T_1 - T_\infty) = k_3 DT + k_1 Q_1$$

$$T_{12}\frac{dT_2}{dt} + (T_2 - T_\infty) = -k_4 DT + k_2 Q_2$$

# Energy Balance Sensors

$$\tau \frac{dT_c}{dt} + T_c = T_i$$

Sensor's model

$$T_c \frac{dT_{c1}}{dt} = T_1 - T_{c1}$$

Only convective energy!

$$T_c \frac{dT_{c2}}{dt} = T_2 - T_{c2}$$