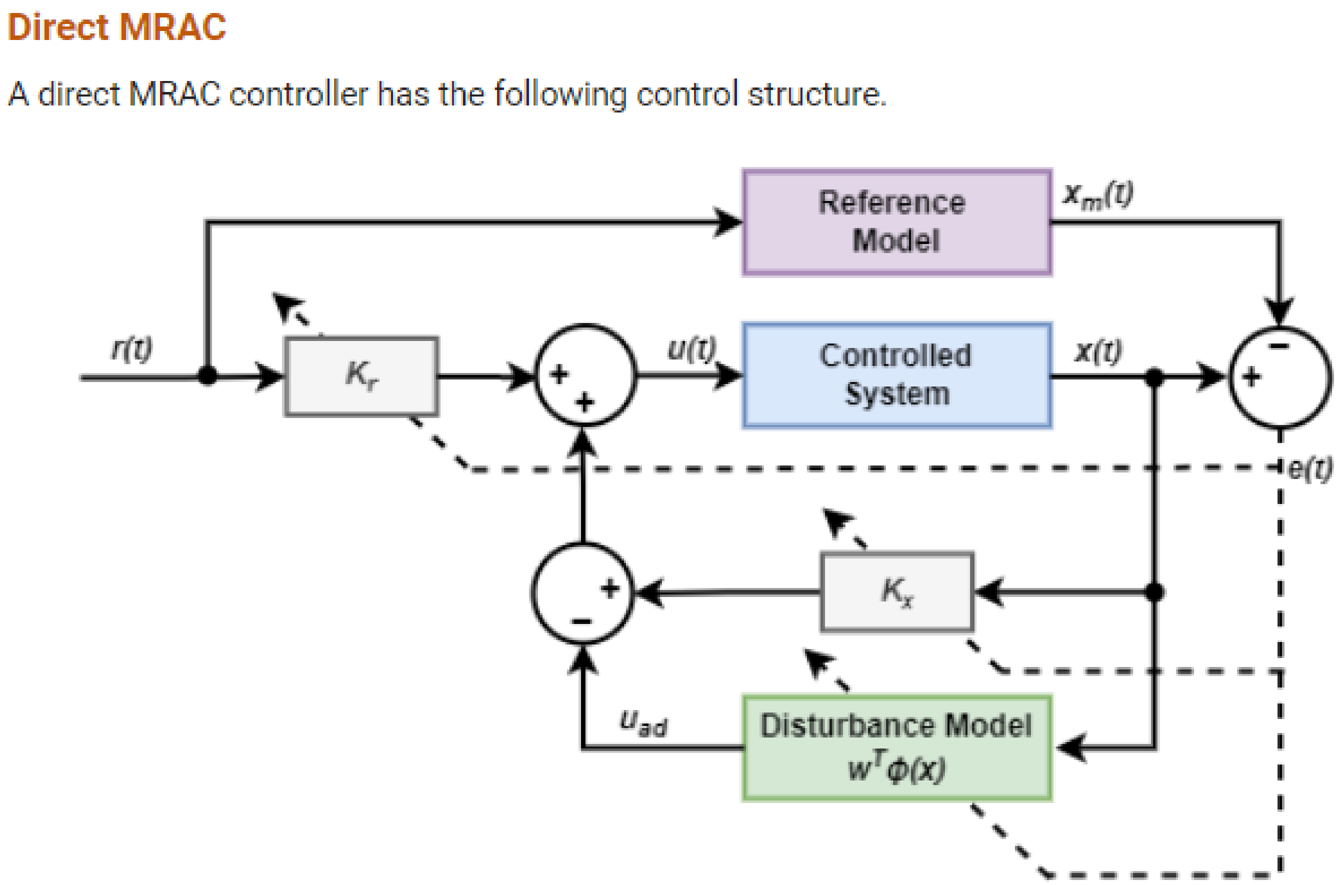
Model Reference Adaptive Control (MRAC) computes control actions to make an uncertain controlled system track the behavior of a given reference plant model.

Direct MRAC — Estimate the feedback and feedforward controller gains based on the real-time tracking error between the states of the reference plant model and the controlled system.

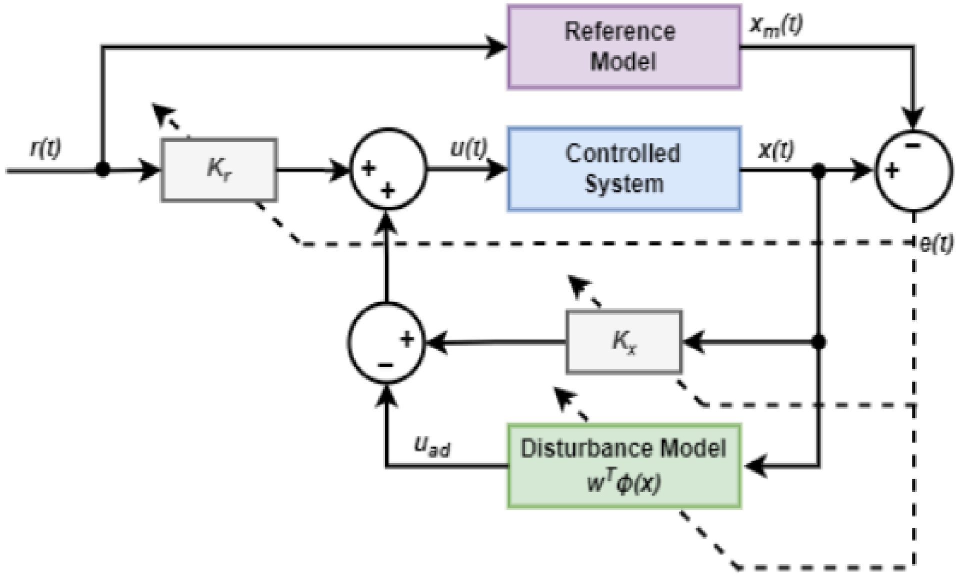
The controller updates the estimated parameters and disturbance model in real-time based on the tracking error.

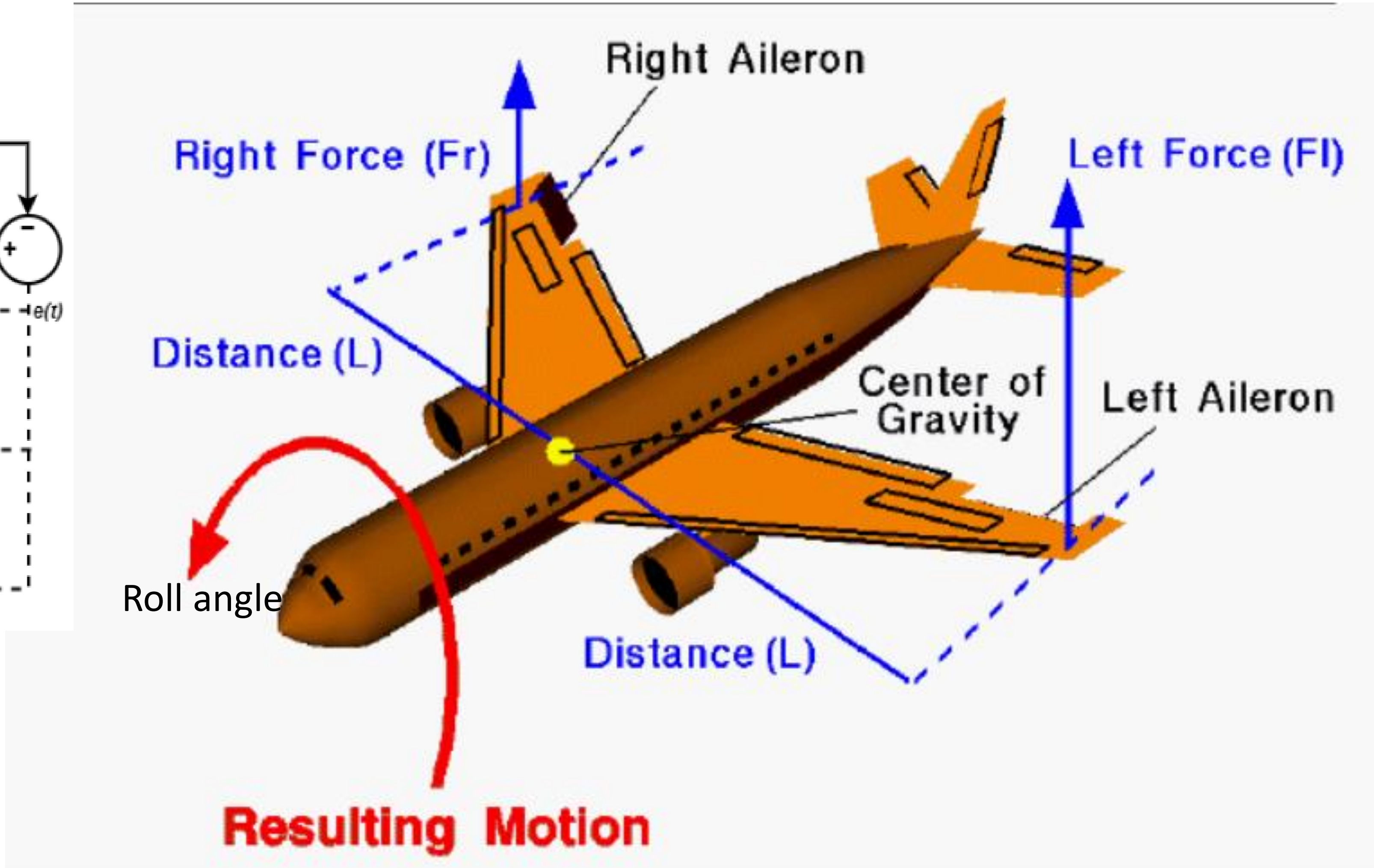


Model Reference Adaptive Control (MRAC)

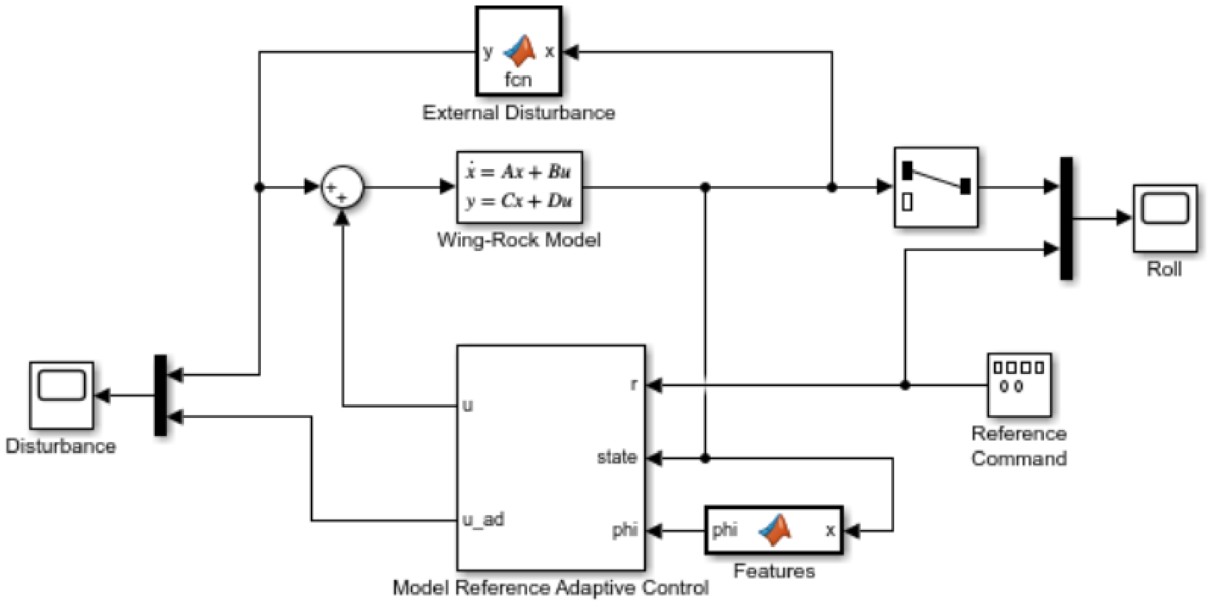
Direct MRAC

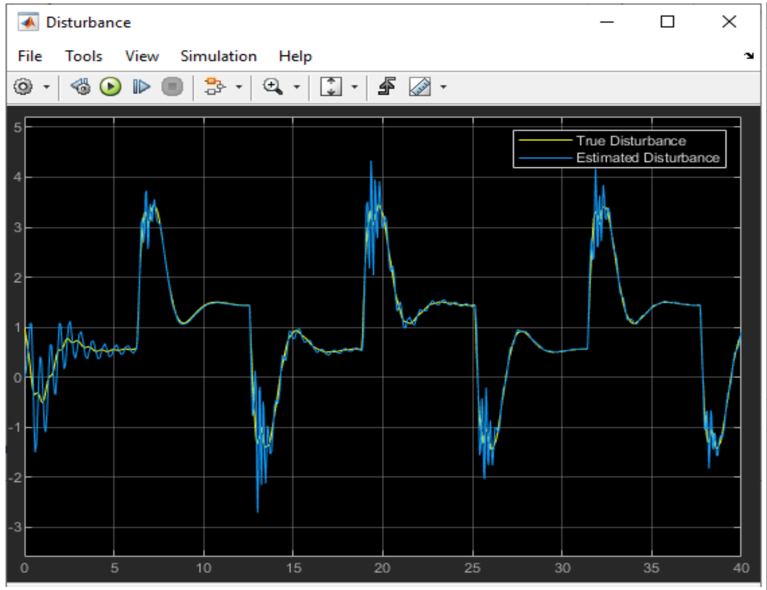
A direct MRAC controller has the following control structure.

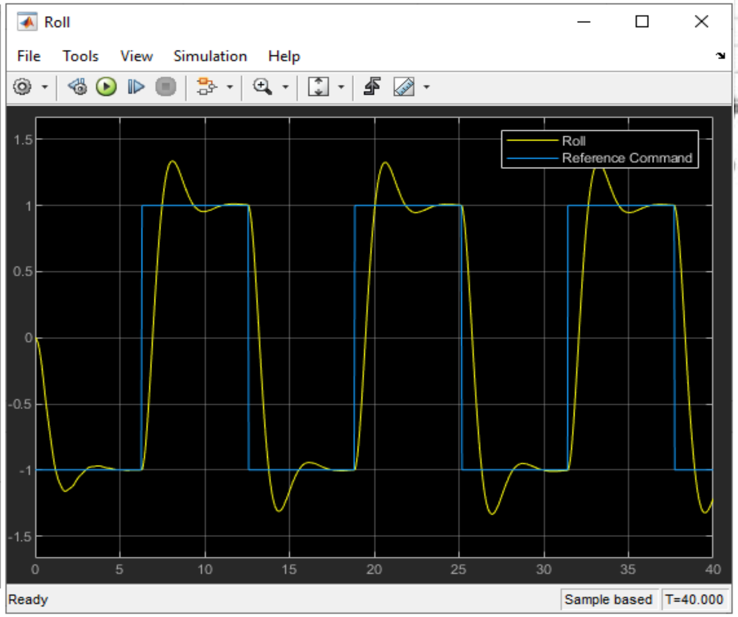




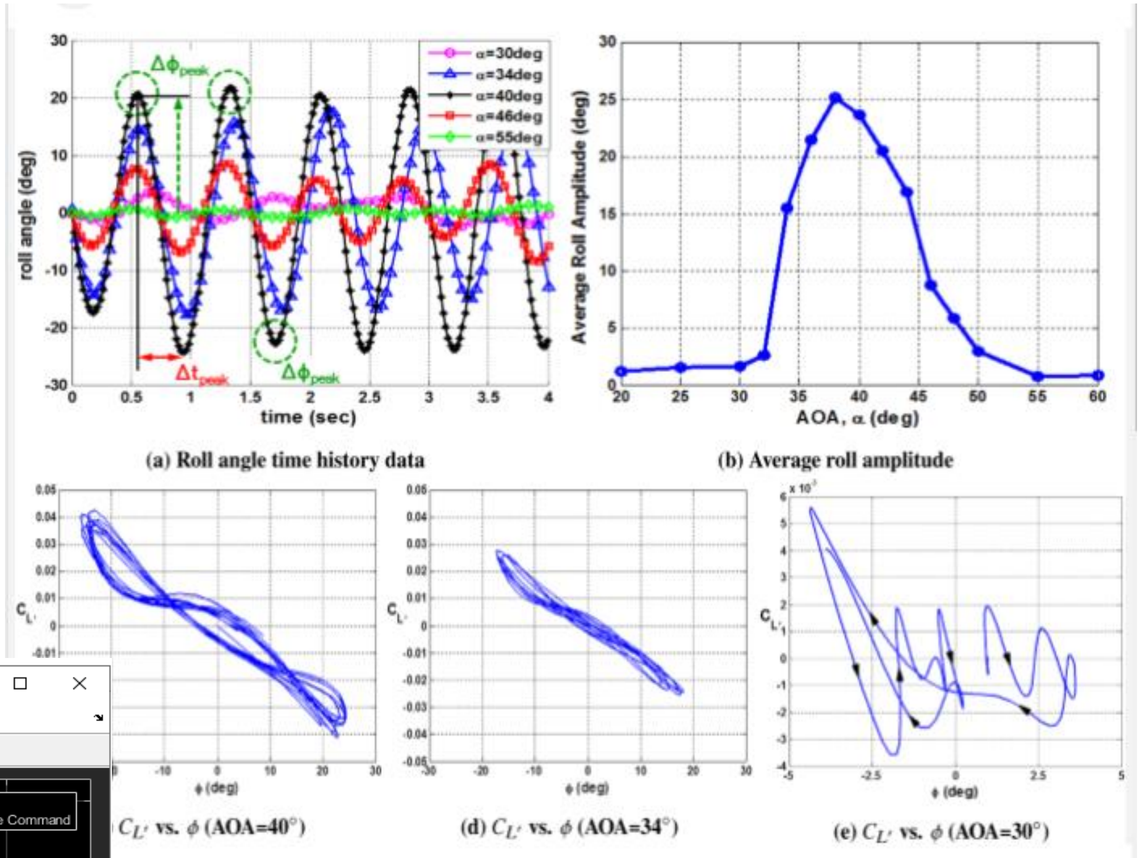
Model Reference Adaptive Control (MRAC)







Aircraft Undergoing Wing Rock



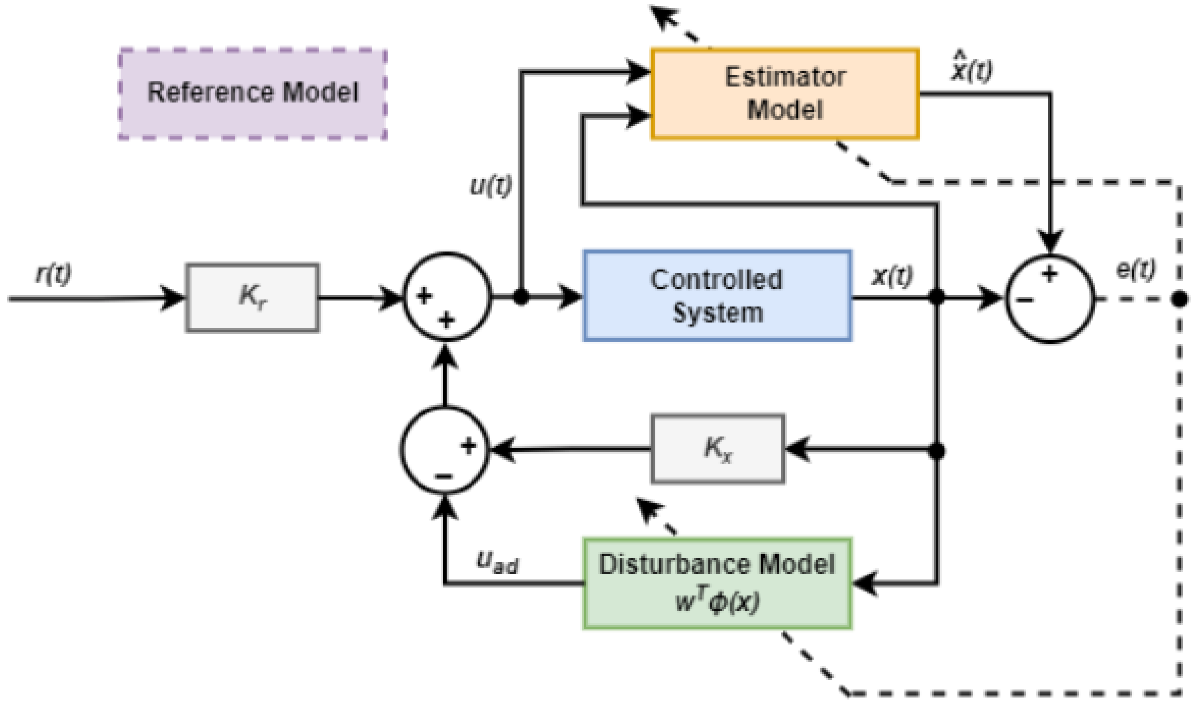
Indirect MRAC — Estimate the parameters of the controlled system based on the tracking error between the states of the reference plant model and the estimated system. Then, derive the feedback and feedforward controller gains based on the parameters of the estimated system and the reference model.

The controller updates the estimated parameters and disturbance model in real-time based on the tracking error.

Both direct and indirect MRAC also estimate a model of the external disturbances and uncertainty in the system being controlled. The controller then uses this model to compensate for the disturbances and uncertainty when computing control actions.

Indirect MRAC

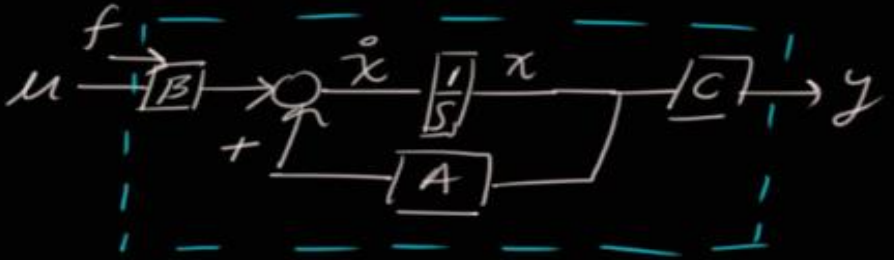
An indirect MRAC controller has the following control structure. The reference model is



System $\dot{x} = Ax + Bu$

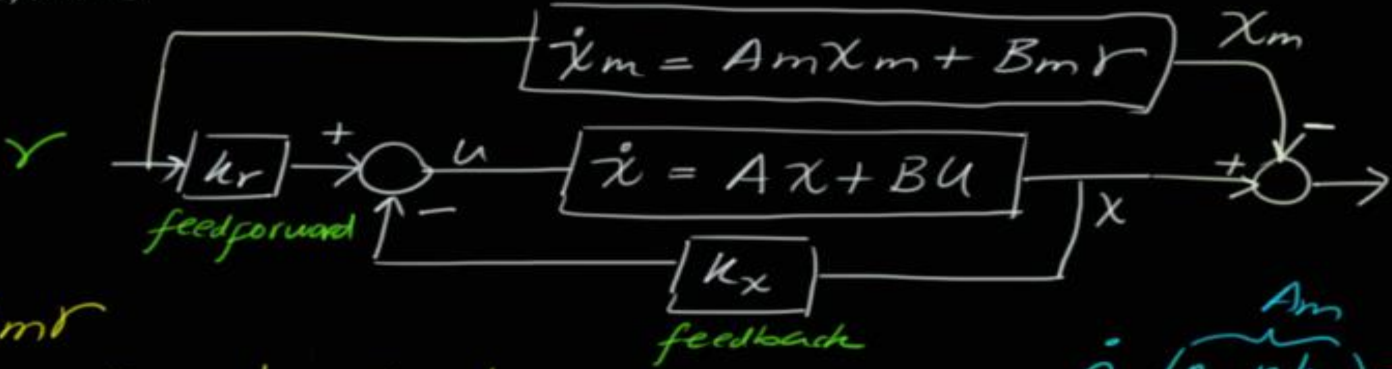
$+$

- Disturbance $\longrightarrow$ $\dot{x} = Ax + B(u+f)$
- Environmental Changes
- changes Dynamics
- Changes Parameters
- system Uncertainities
- No modeled dynamics



Control that

Adapts to variations



Error

$e = x - x_m$

$\dot{e} = \dot{x} - \dot{x}_m$

$\dot{e} = \overbrace{(A - Bk_x)}^{A_m} x + \cancel{Bk_r r}$

$\quad - A_m x_m - \cancel{B_m r}$

$\dot{e} = A_m x - A_m x_m$

$\dot{e} = A_m (x - x_m)$

$\quad = A_m e$ stable!

$e = \lim_{t \to \infty} (A_m e) \approx 0$

$\dot{x}_m = A_m x_m + B_m r$

$\dot{x} = Ax + Bu \; ; \quad u = k_r r - k_x x$

$\dot{x} = Ax + B(k_r r - k_x x) = \underbrace{(A - Bk_x)}_{A_m} x + \underbrace{Bk_r}_{B_m} r$

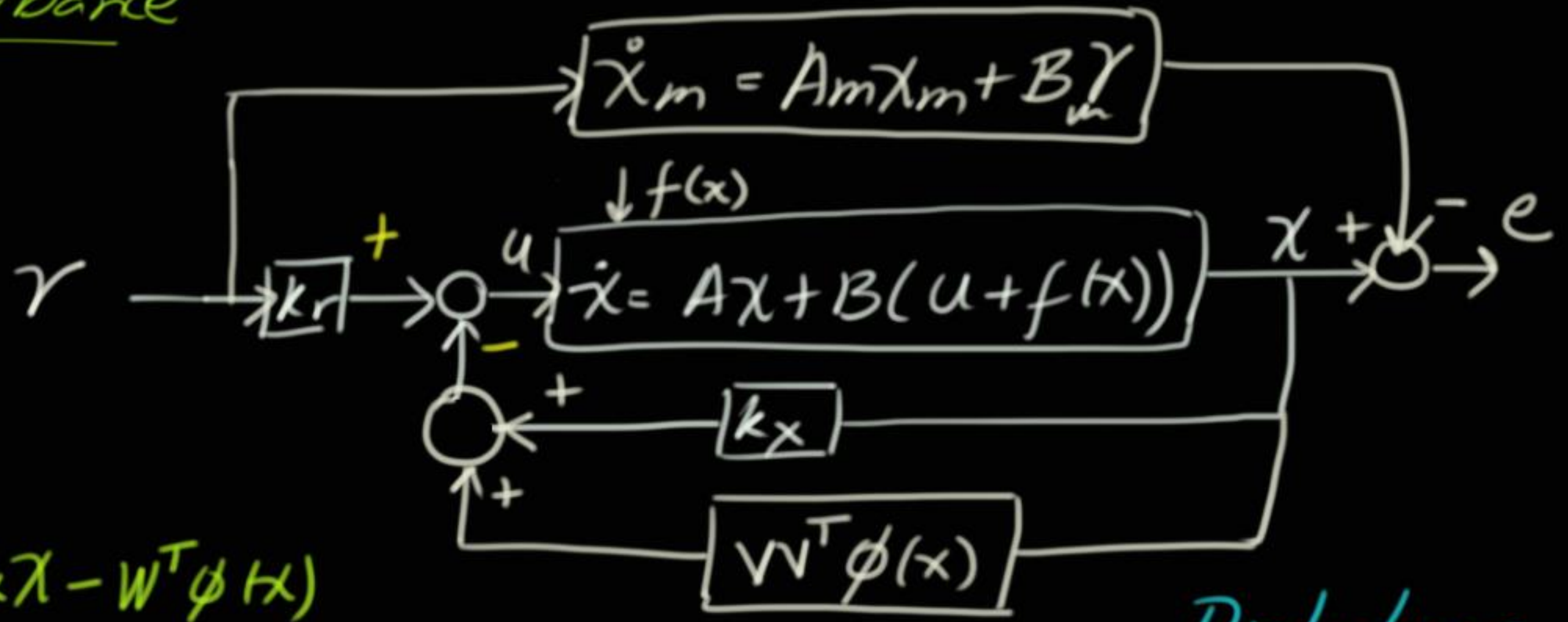
Model Matching

$A_m = A - Bk_x$

$B_m = Bk_r$

Pole Placement

With Disturbance



$$u = k_r r - k_x x - W^T\phi(x)$$

$$\dot{x} = Ax + B(k_r r - k_x x \underbrace{- W^T\phi(x)} + \underbrace{f(x)})$$

$\underline{W^T}\,\underline{\phi(x)}$ → Set of basic functions

A weighting vector

Disturbace compensation

$$W^T\phi(x) = f(x)$$
$$\downarrow$$
$$\dot{x} = Ax + B(k_r r - k_x x) \checkmark$$

1) We know the $f(x)$ can be represented using $x \to$ system states

$\underline{\theta}, \underline{\dot{\theta} = p}$ system states $\to f(x) = 1 + 2\theta + 3p + 4|\theta|\theta + 1|p|p + 2\theta^3$

$W_T \phi(x)$

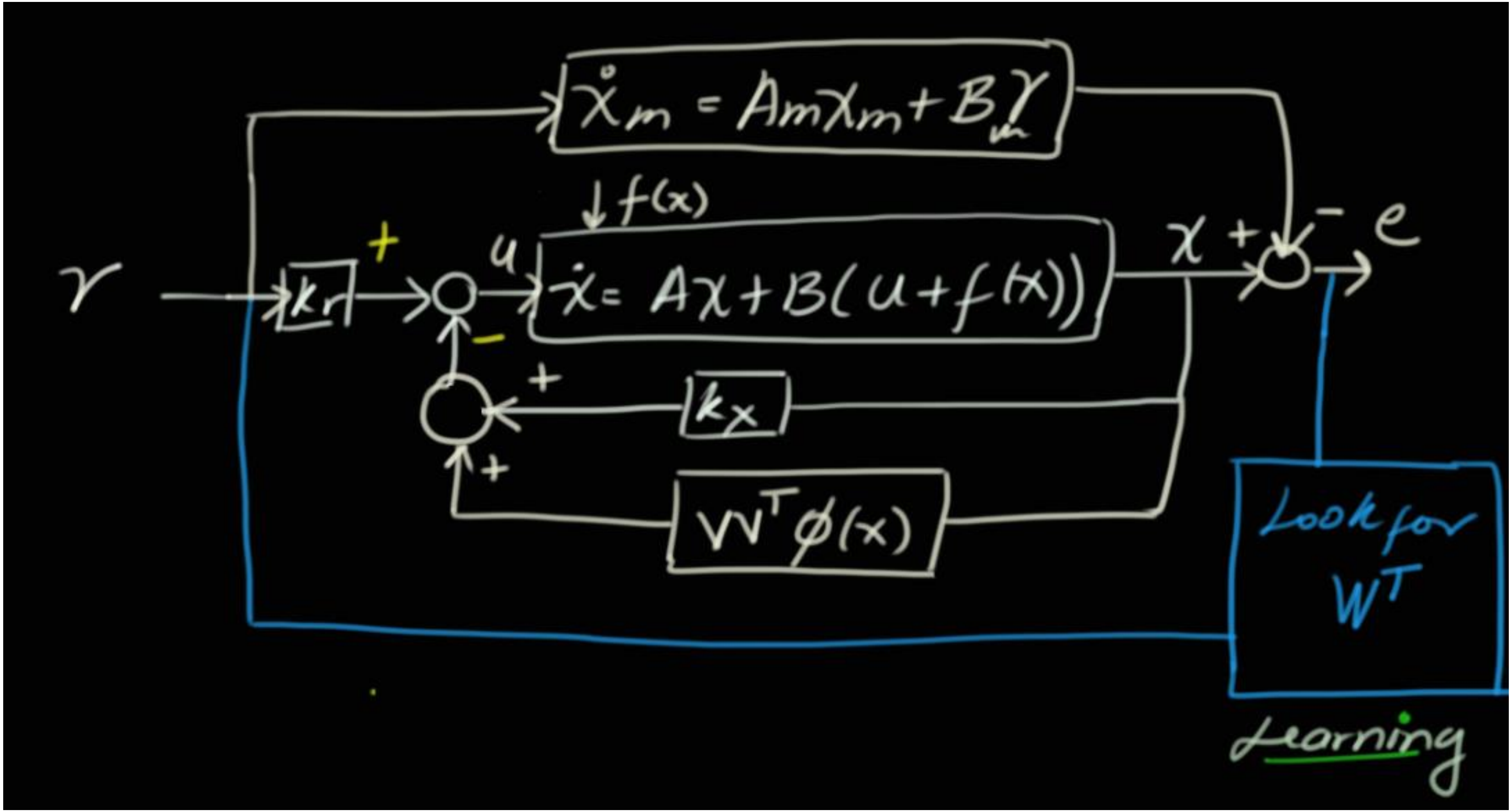
$\phi(x) = [1, \theta, p, |\theta|\theta, |p|p, \theta^3]$

$W_T = [1; 2; 3; 4; 1; 2]$

2) Use some of the states as feautures $\longrightarrow$ $\phi(x) = [1 \ \theta \ p \ |\theta|]$

$W_T = [\ \dots\ ]$?

3) Use Sum of Gaussians 



$\dot{x}_m = A_m x_m + B_m r$
$f(x)$
r
k_r
$+$
u
$\dot{x} = Ax + B(u + f(x))$
x
$+$
$-$
e
$-$
$+$
k_x
$+$
$W^T\phi(x)$
Look for
W^T
Learning

Nominal Model → Second Order model $\ddot{x}(t) = u(t) \rightarrow s^2 X(s) = U(s)$

$$\frac{U(s)}{X(s)} = \frac{1}{s^2}$$

Roll

aileron

state vector

$$x = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \theta \\ p \end{bmatrix}$$

Second order integrator

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u$$

State equation

$$y = \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D} u$$

output equation

Reference Model → stable second order model

$$\ddot{x}_m = -4x_m - 2\dot{x}_m + 4r$$

x_m Reference model state vector

r Roll reference given by the pilot

$$X_m = \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix}; \quad \begin{bmatrix} \dot{x}_m \\ \ddot{x}_m \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} r(t)$$

Pole placement ① $A_m = A - Bk_x$

Model Matching ② $B_m = Bk_r$

① $$\begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_{x1} & k_{x2} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ k_{x1} & k_{x2} \end{bmatrix}$$

feedback gains

$$\left.\begin{aligned} -4 &= -k_{x1} \\ -2 &= -k_{x2} \end{aligned}\right\} \begin{aligned} k_{x1} &= 4 \\ k_{x2} &= 2 \end{aligned}$$

② $$\begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_{r1} \end{bmatrix} \quad \left\{ \begin{array}{l} \text{feedforward gain} \\ 4 = k_{r1} \end{array} \right.$$