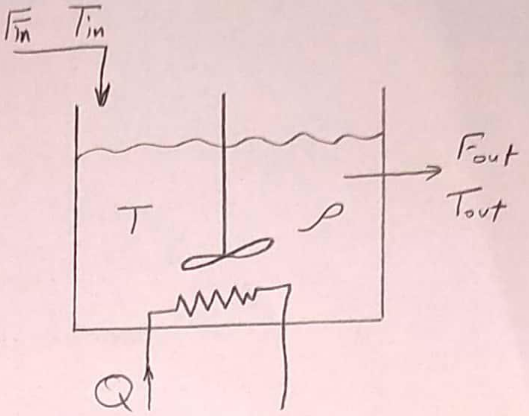
Heating Process

① Total Mass Balance:



$$\frac{dM}{dt} = \dot{m}_{in} - \dot{m}_{out} \quad ; \quad M = \rho . V, \quad \dot{m} = \rho . F$$

$$\frac{d(\rho V)}{dt} = \frac{\rho \, dV}{dt} = \rho F_{in} - \rho F_{out}$$

$$\frac{dV}{dt} = F_{in} - F_{out}$$; Volume is controlled $\Rightarrow$ V is constant

$$0 = F_{in} - F_{out} \Rightarrow F_{in} = F_{out}$$

F_{in} : Input flow rate

F_{out}: Output flow rate

T_{in}: Input Temprature

T_{out}: Output Temprature

C_P: Heat Capacity Factor, const

Q: Manipulated Heat Rate

The system is isolated $\Rightarrow$ No heat losses

Energy Balance: $H = m C_P T = \rho V C_P T$

$$\frac{dH}{dt} = \dot{H}_{in} - \dot{H}_{out} + Q$$

$$\frac{d(mC_PT)}{dt} = \dot{m}_{in} C_P T_{in} - \dot{m}_{out} C_P T_{out} + Q$$

$$m C_P \frac{dT_{out}}{dt} = \rho F_{in} C_P T_{in} - \rho F_{out} C_P T_{out} + Q \quad ; \quad F_{in} = F_{out}$$

$$\rho V C_P \frac{dT_{out}}{dt} = \rho C_P F_{in} (T_{in} - T_{out}) + Q \quad ; \quad m = \rho V$$

$$V \frac{dT_{out}}{dt} = F_{in} (T_{in} - T_{out}) + \frac{Q}{\rho C_P}$$

$$V \frac{dT_{out}}{dt} + F_{in} T_{out} = F_{in} T_{in} + \frac{Q}{\rho C_P}$$

$$\underbrace{\frac{V}{F_{in}}}_{\tau} \frac{dT_{out}}{dt} + T_{out} = \underbrace{1}_{K_{P_1}} . T_{in} + \underbrace{\frac{1}{F_{in} \rho C_P}}_{K_{P_2}} Q$$

$$\tau = \frac{V}{F_{in}} \quad , \quad K_{P_1} = 1$$

$$K_{P_2} = \frac{1}{F_{in} \rho C_P}$$

$$\tau \frac{dT_{out}}{dt} + T_{out} = K_{P_1} T_{in} + K_{P_2} Q$$

Time constant   State constant

$$V \cancel{\frac{dT}{dt}} = F_{in}(T_{in} - T_{out}) + \frac{Q}{\rho C_P}$$

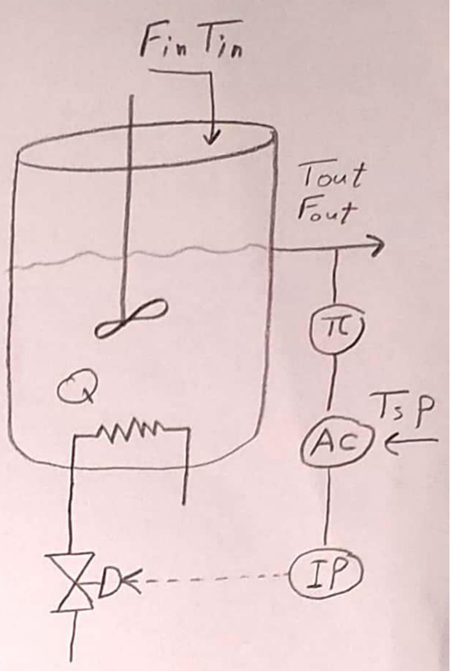
Manipulated

steady state

$$0 = \bar{F}_{in}(\bar{T}_{in} - \bar{T}_{out}) + \frac{\bar{Q}}{\rho C_P}$$

$$\bar{F}_{in}(\bar{T}_{out} - \bar{T}_{in}) = \frac{\bar{Q}}{\rho C_P}$$

$$\bar{T}_{out} = \frac{\bar{Q}}{F_{in}\rho C_P} + \bar{T}_{in}$$



Q manipulated variable

T controlled variable

$$\frac{1}{F_{in}\rho C_P} = 0.05\,^\circ C \frac{min}{Kcal}$$

$$Q = 1200 \frac{Kcal}{min}$$

$$\underbrace{T_{in} = 30\,^\circ C}$$

$$T_{out} = 1200 \frac{Kcal}{min} \cdot 0.05\,^\circ C \frac{min}{Kcal} + 30\,^\circ C$$

$$T_{out} = 90\,^\circ C$$

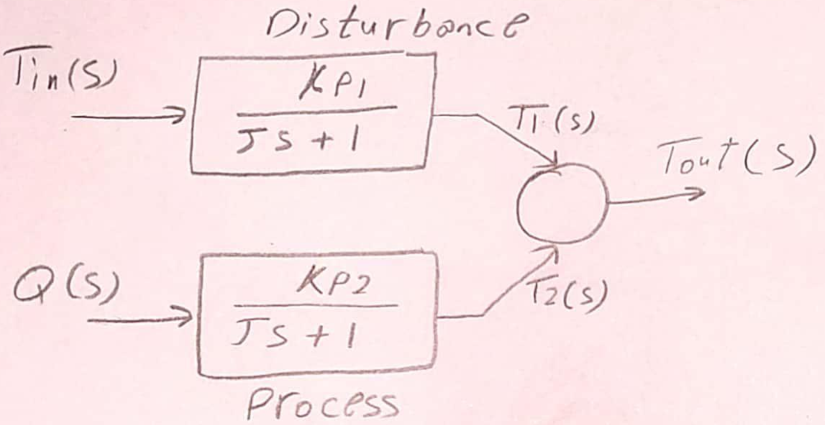
Laplace initial condition 'zero'

$$J s\, T_{out}(s) + T_{out}(s) = K_{P1}\, T_{in}(s) + K_{P2}\, Q(s)$$

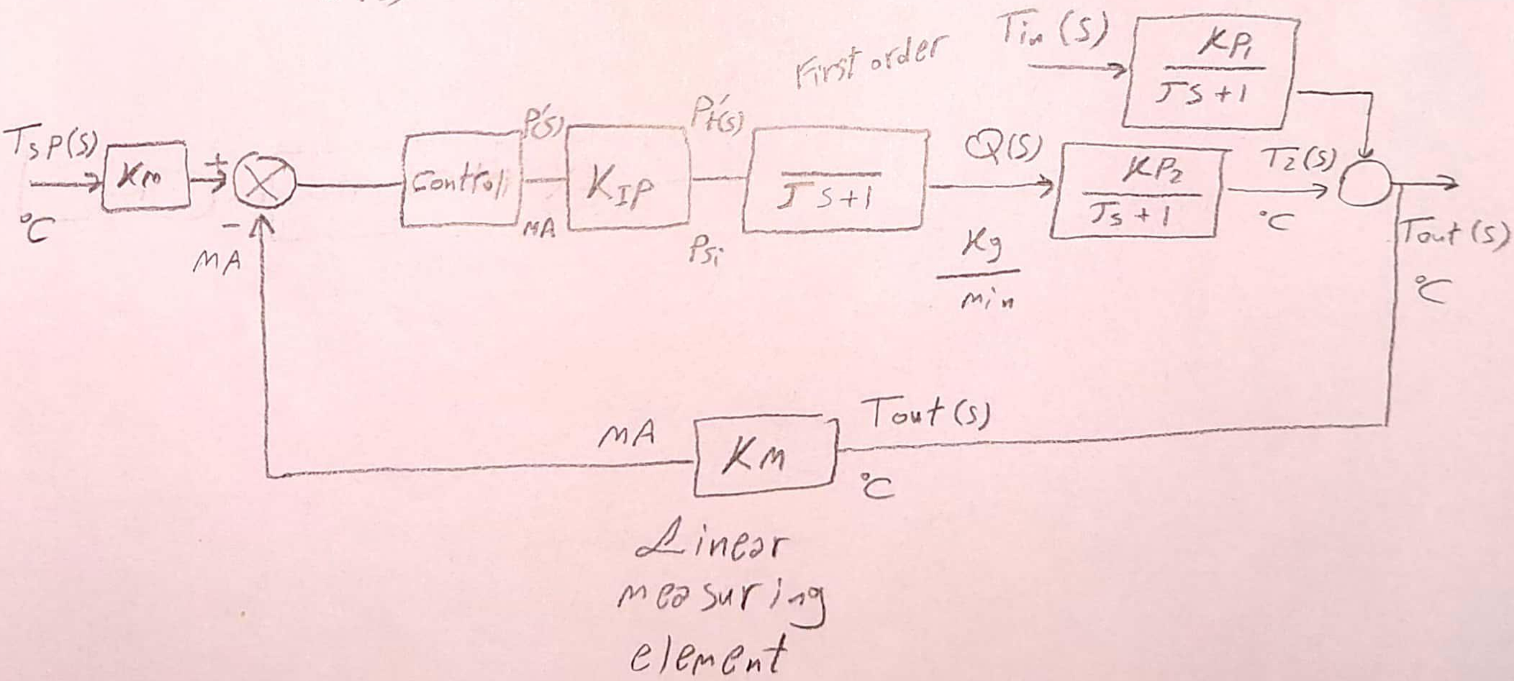
$$(J s + 1)\, T_{out}(s) = K_{P1}\, T_{in}(s) + K_{P2}\, Q(s)$$

$$T_{out}(s) = \frac{K_{P1}}{J s + 1} T_{in}(s) + \frac{K_{P2}}{J s + 1} \underbrace{Q(s)}_{\text{manipulated}}$$

Super Position



$$G_d(s) = \frac{T_1(s)}{T_{in}(s)} = \frac{K_{P1}}{J s + 1}, \quad G_P(s) = \frac{T_2(s)}{Q(s)} = \frac{K_{P2}}{J s + 1}$$



Enthalphy $H = m C_P T$

Total Energy Balance $= \dot{H}_{in} - \dot{H}_{out} + Q$

$$m c_p \frac{dT}{dt} = \underbrace{UA(T_\infty - T)}_{\text{Convection}} + \underbrace{E\sigma(T_\infty^4 - T^4)}_{\text{Radiation}} + \underbrace{\alpha Q}_{\text{Heater}}$$

H: Enthalphy m: mass of transistor

cp: Heat Capacity factor

T: Temprature of transistor

T_∞: aimbient temprature

E: Emissivity

σ: stefan Boltzman constant

A: surface area

U: overall heat transfare coeficient

Q: Heat output

α: factor of the heater

(0 - 100%)

(0 - 1)

if we ignore Radiation:

$$m c_p \frac{dT}{dt} = UA(T_\infty - T) + \alpha Q$$ First order diffirential equation

$$m c_p \frac{dT}{dt} + UAT = UAT_\infty + \alpha Q$$

$$\underbrace{\frac{m c_p}{UA}}_{\tau_p} \frac{dT}{dt} + T = T_\infty + \underbrace{\frac{1}{UA}}_{K_p} \alpha Q \rightarrow \tau_p \frac{dT}{dt} + T = T_\infty + K_p \alpha Q$$

τ_p: Time constant $= \frac{m c_p}{UA}$

K_p: process gain $= \frac{1}{UA}$

Laplace, inatial condition 'zero'

$$\tau_p s T(s) = T_\infty(s) + K_p \alpha Q(s)$$

$$T(s) = \frac{1}{\tau_p s + 1} T_\infty(s) + \frac{K_p}{\tau_p s + 1} \alpha Q(s)$$ Linear model

$$T(s) = T_\infty + \frac{K_p}{\tau_p s + 1} \alpha Q(s)$$

using* $$\frac{dT}{dt} = \frac{UA(T_\infty - T) + \alpha Q}{m c_p}$$ first order diffirential equation

$m = 0{,}001\ Kg$

$C_p = 4900\ \frac{J}{Kg \cdot K}$

$u = 200\ \frac{W}{m^2 K}$

$A = \frac{2}{100^2}\ m^2$

$$\tau_p = \frac{(0{,}001\ Kg)\left(4900\ \frac{J}{Kg \cdot K}\right)}{\left(200\ \frac{W}{m^2 K}\right)\left(0{,}0002\ m^2\right)}$$

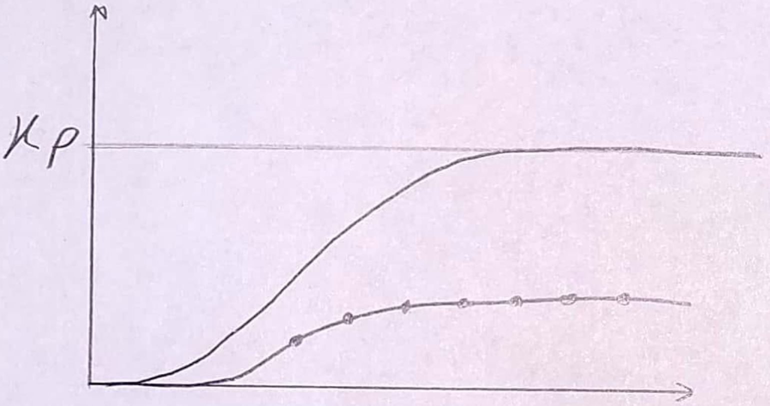
$$\tau_p = 122.5\ \frac{J}{W}$$

$$\tau_p = 122{,}5\ s \quad ; \quad [W] = \left[\frac{J}{s}\right]$$

FOPDT to au sKp input

$$T(s) = \frac{K_p e^{-\theta_p s}}{\tau_p s + 1}$$

θ_p Process dead time



The method approximate high order models to first order model + dead time.

→ Comparing measured response and the interpolated answer.