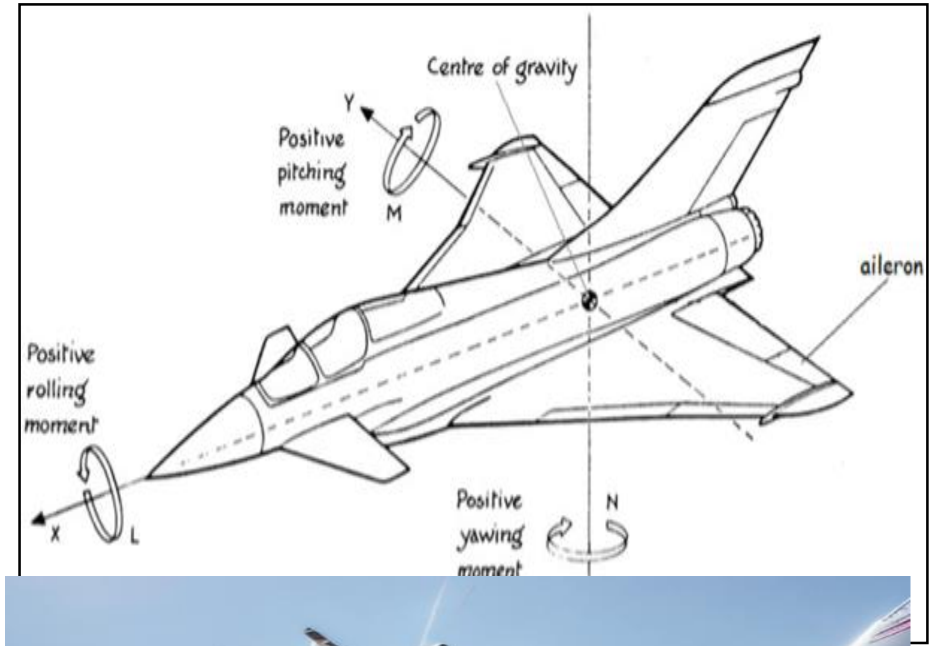
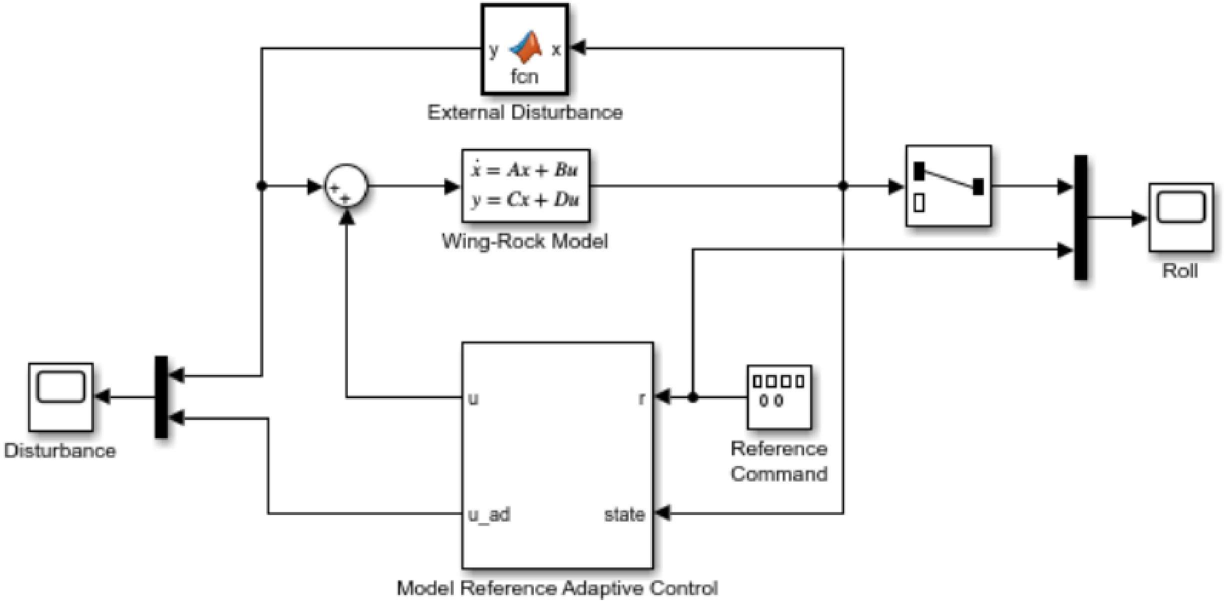
Data-Driven Control

Model Reference Adaptive Control (MRAC)

Control Tech LAB





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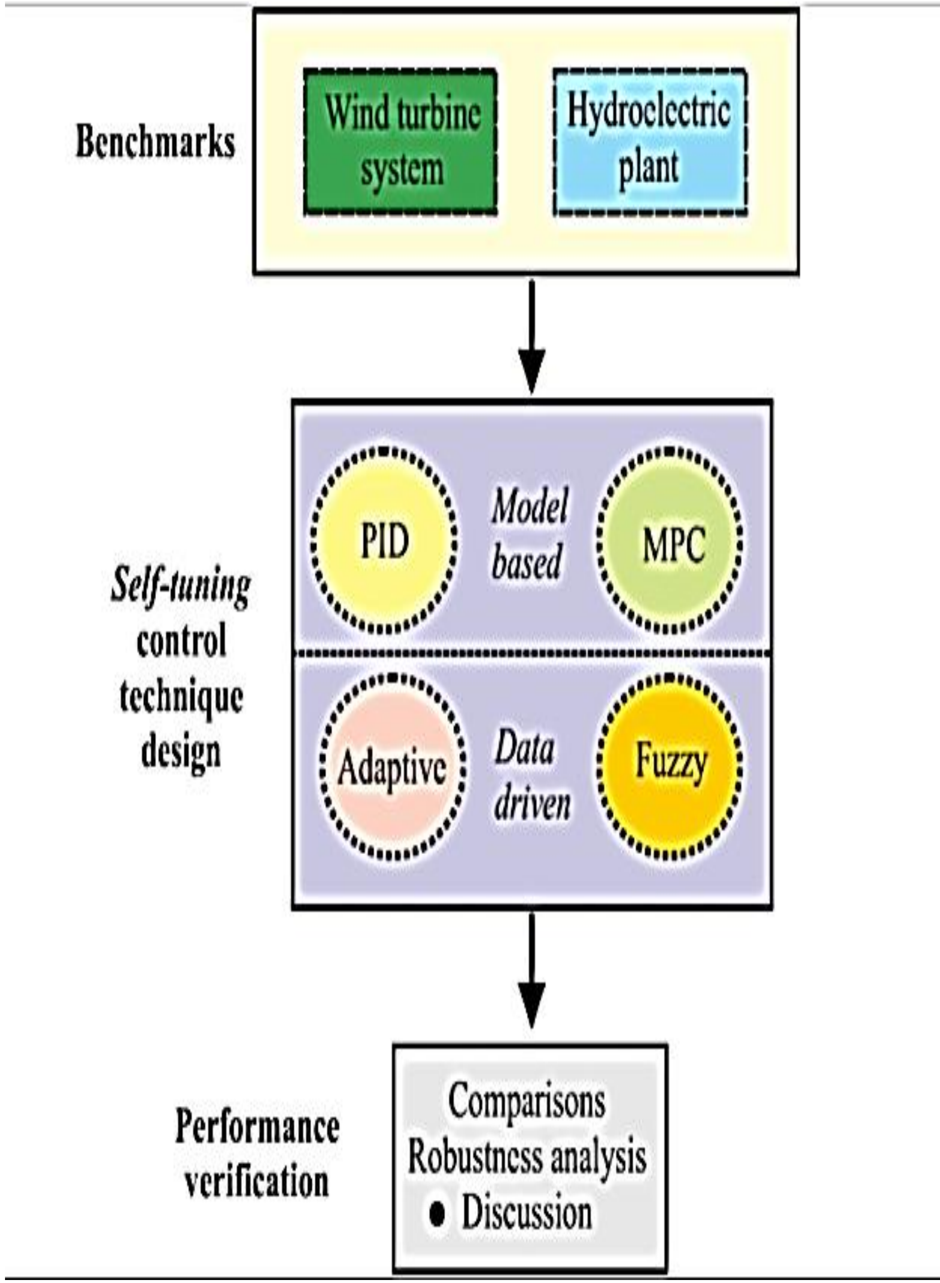
Working with:

1. Introduction Presentation
2. Introduction System Modeling (a Live Script)

Data-driven Control Systems

A control systems in which the identification of the process model and/or the design of the controller are based entirely on experimental data collected from the plant.

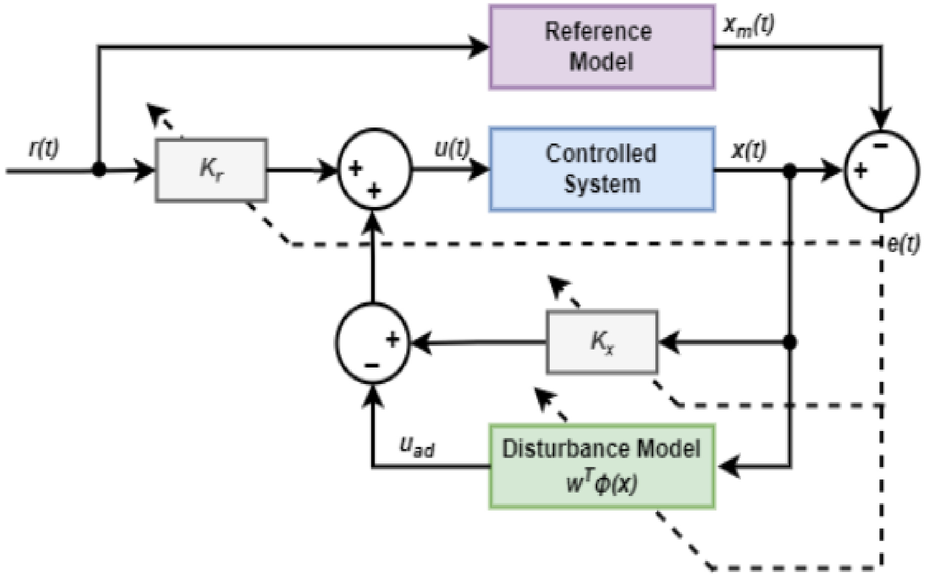
- It is difficult to find a simple reliable model for a physical system and control specifications.
- Direct data-driven methods allow to tune a controller without an identified model of the system.
- It can also simply weight process dynamics of interest inside a cost function, and exclude those dynamics that are out of interest.



Model Reference Adaptive Control (MRAC)

Direct MRAC

A direct MRAC controller has the following control structure.

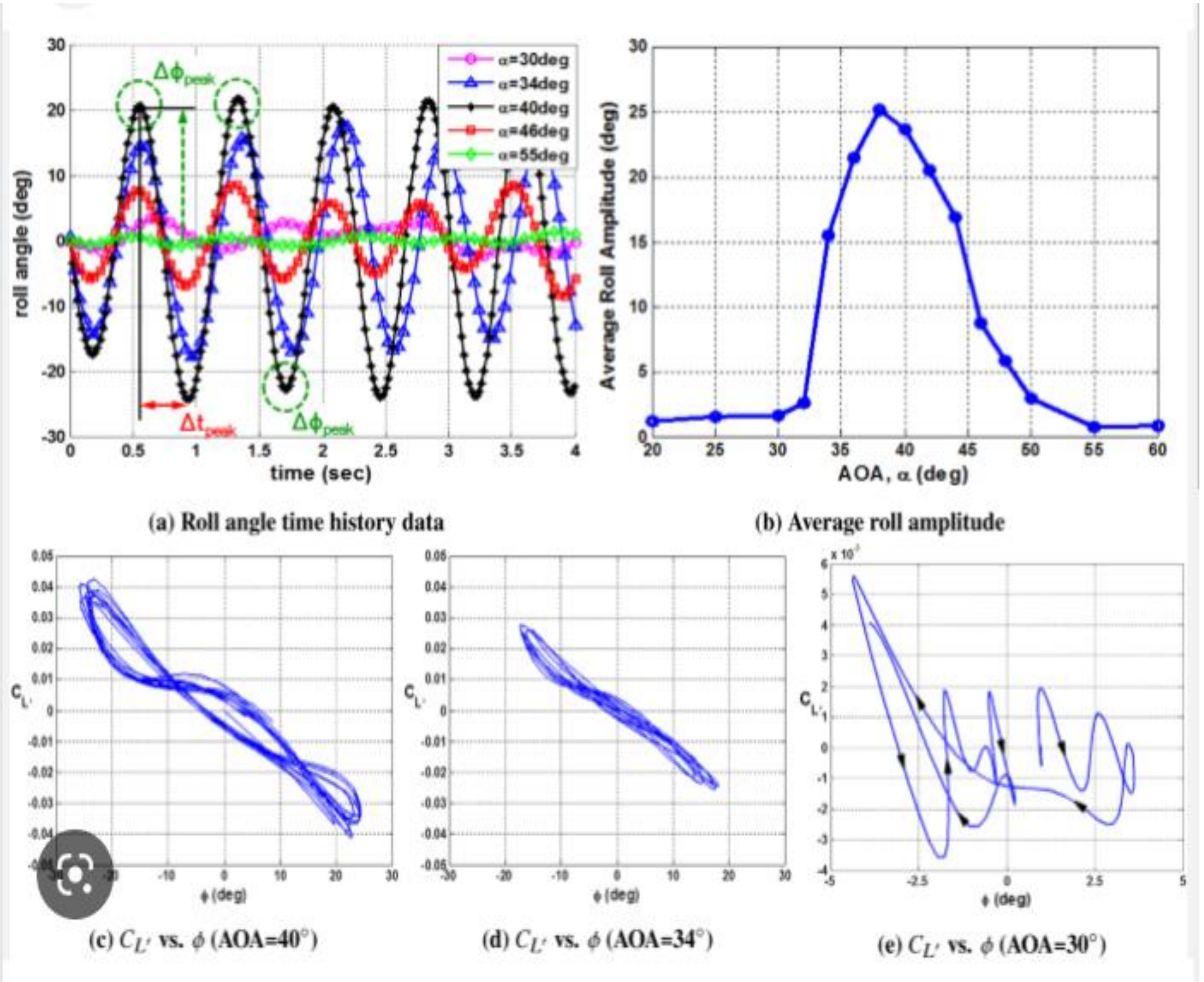


Roll angle

Aircraft Undergoing Wing Rock

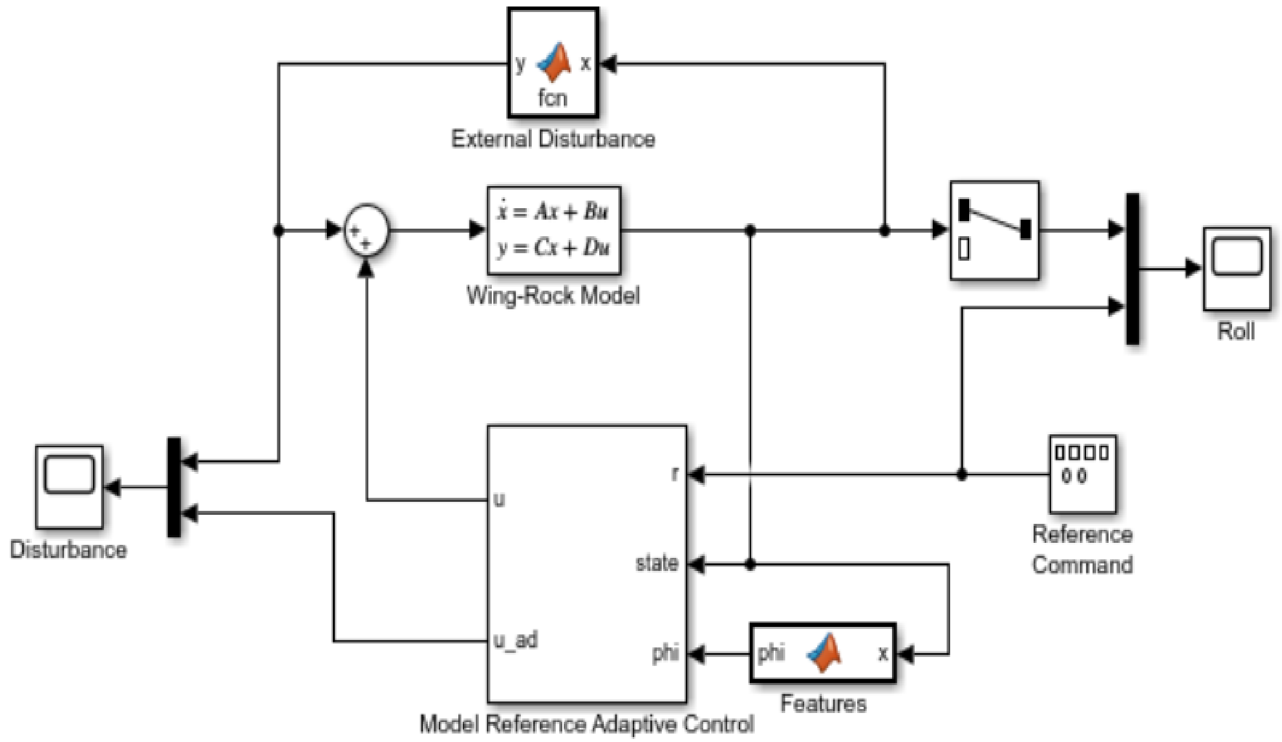
Wing-Rock Control System

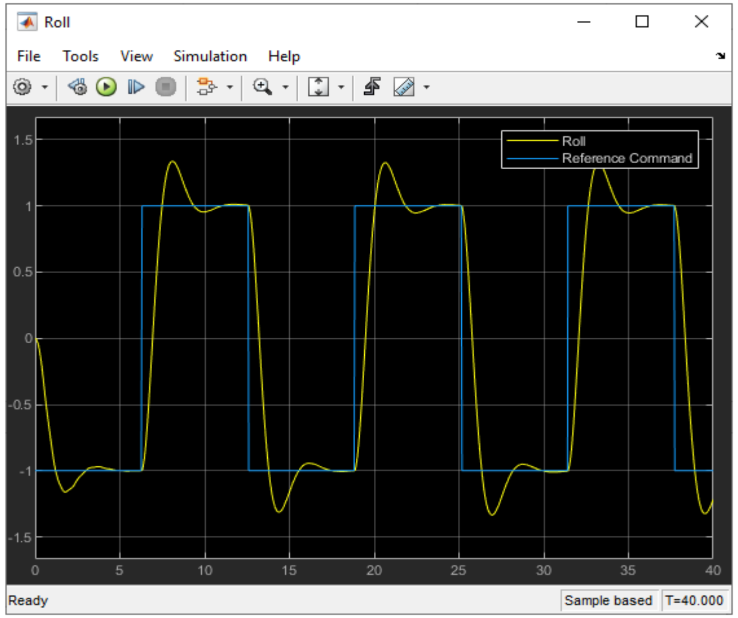
Wing rock is a phenomenon observed in delta wing aircraft flying at low speeds and high angles of attack. The aircraft experiences undesired roll oscillations that make the aircraft more difficult for the pilot to control. The goal of the MRAC controller is to cancel the undesired roll oscillation. You can then design a baseline controller to achieve the desired reference behaviour.

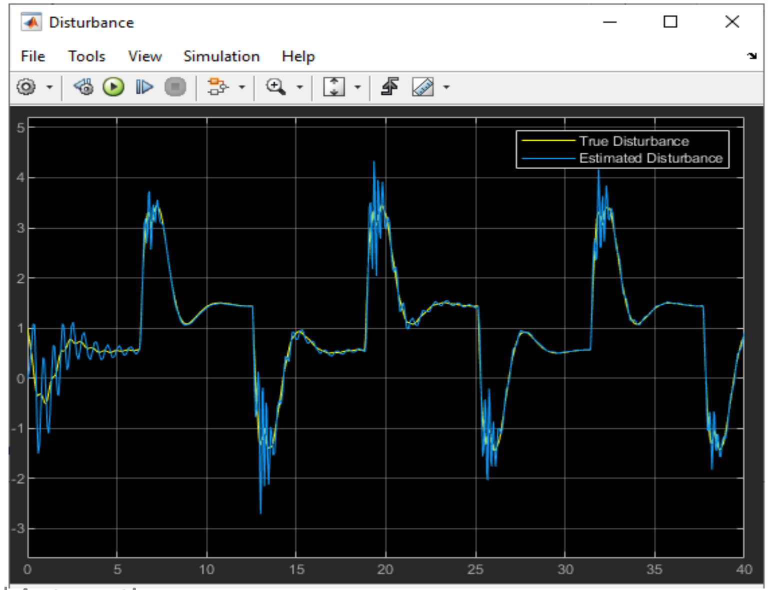


rolling moment coefficients vs Angle Of Attack

Model Reference Adaptive Control (MRAC)





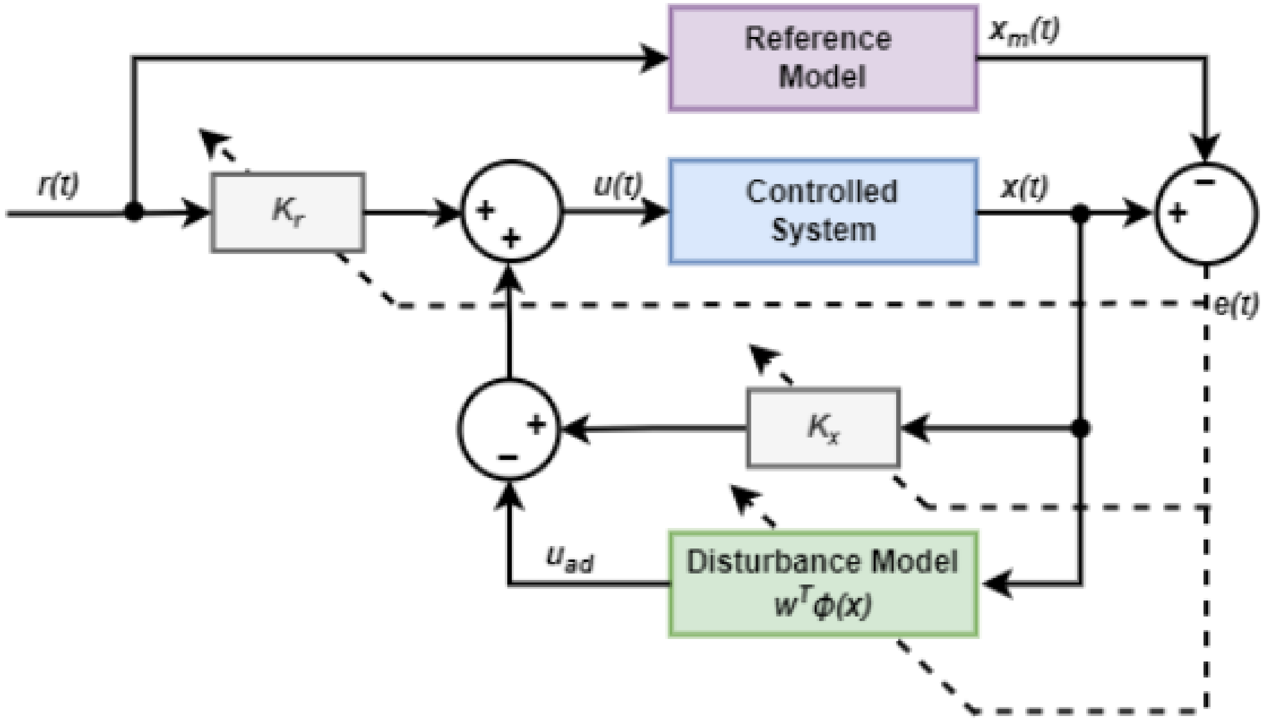


Model Reference Adaptive Control (MRAC) computes control actions to make an uncertain controlled system track the behavior of a given reference plant model.

Direct MRAC — Estimate the feedback and feedforward controller gains based on the real-time tracking error between the states of the reference plant model and the controlled system.

Direct MRAC

A direct MRAC controller has the following control structure.

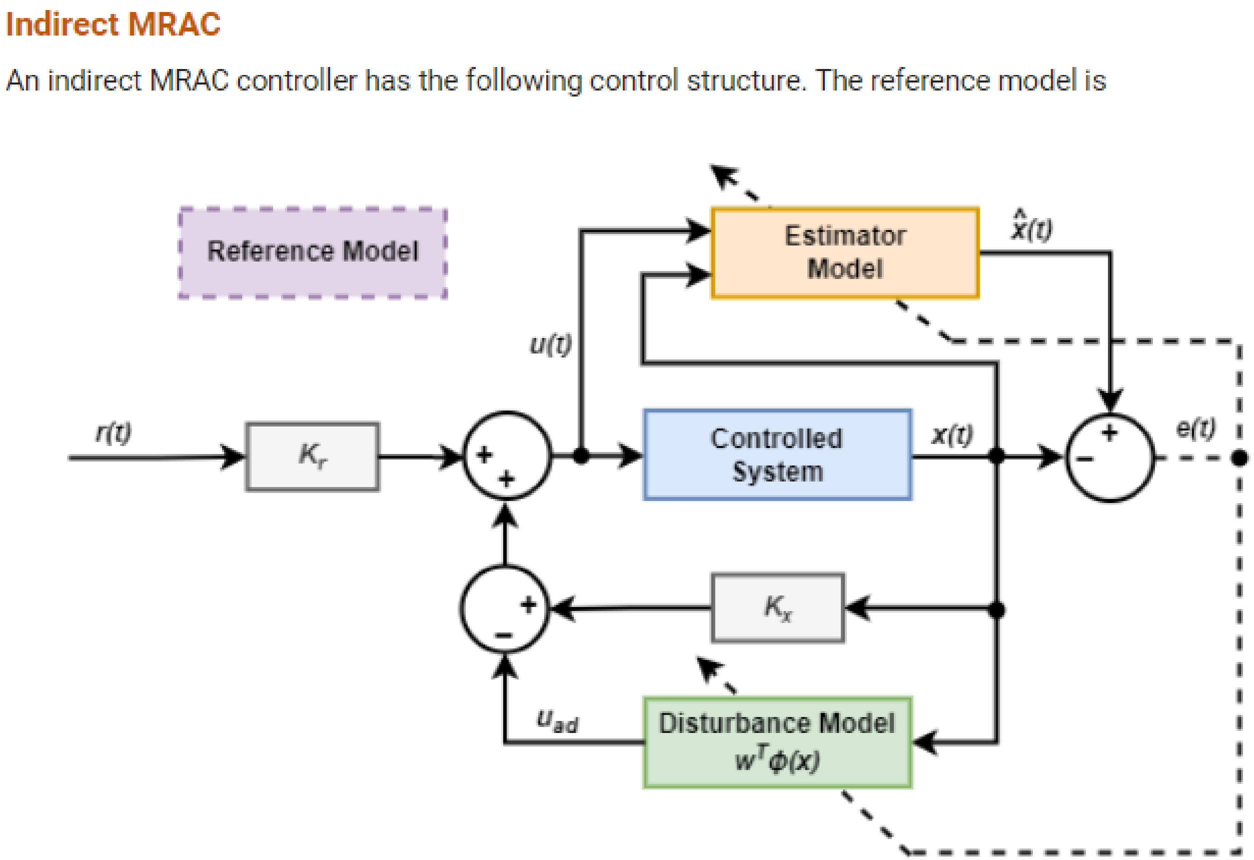


The controller updates the estimated parameters and disturbance model in real-time based on the tracking error.

Indirect MRAC — Estimate the parameters of the controlled system based on the tracking error between the states of the reference plant model and the estimated system. Then, derive the feedback and feedforward controller gains based on the parameters of the estimated system and the reference model.

The controller updates the estimated parameters and disturbance model in real-time based on the tracking error.

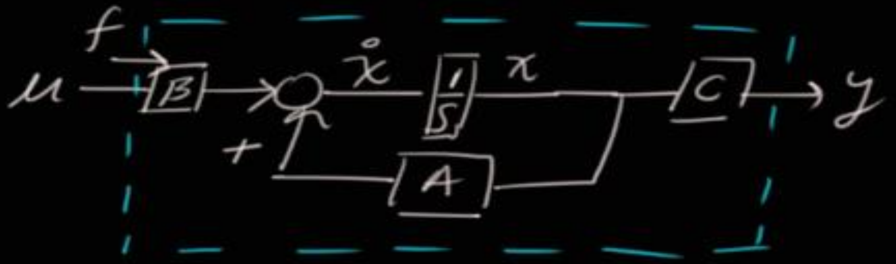
Both direct and indirect MRAC also estimate a model of the external disturbances and uncertainty in the system being controlled. The controller then uses this model to compensate for the disturbances and uncertainty when computing control actions.



System $\dot{x} = Ax + Bu$

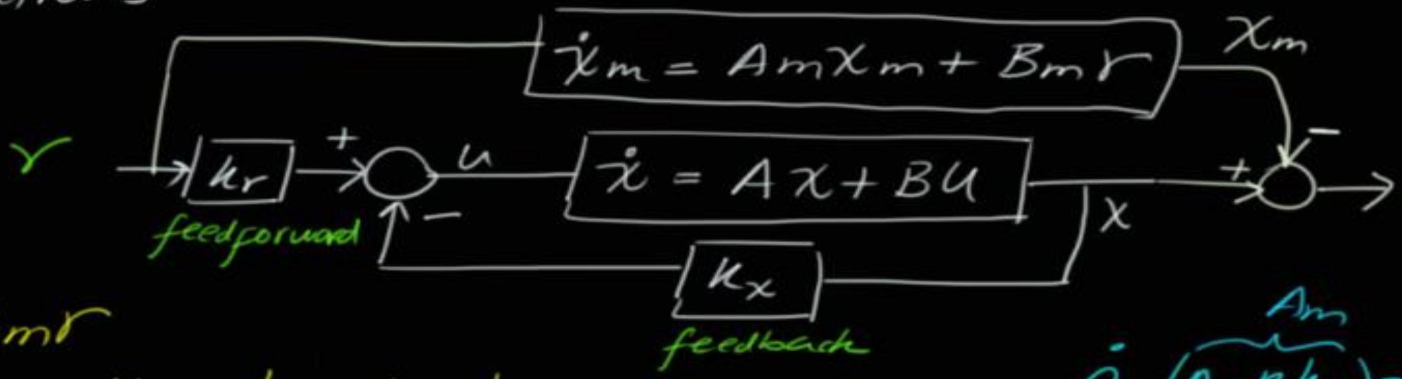
+

- Disturbance $\longrightarrow$ $\dot{x} = Ax + B(u + f)$
- Environmental Changes
- Changes Dynamics
- Changes Parameters
- System Uncertainities
- No modeled dynamics



Control that

Adapts to variations



Error

$e = x - x_m$

$\dot{e} = \dot{x} - \dot{x}_m$

$\dot{x}_m = A_m x_m + B_m r$

$\dot{x} = Ax + Bu \;;\; u = k_r r - k_x x$

$\dot{x} = Ax + B(k_r r - k_x x) = \underbrace{(A - Bk_x)}_{A_m} x + \underbrace{Bk_r}_{B_m} r$

$\dot{e} = \overbrace{(A - Bk_x)}^{A_m} x + \cancel{Bk_r r}$

$\quad - A_m x_m - \cancel{B_m r}$

$\dot{e} = A_m x - A_m x_m$

$\dot{e} = A_m (x - x_m)$

$\quad = A_m e$ stable!

$e = \lim_{t \to \infty} (A_m e) \approx 0$

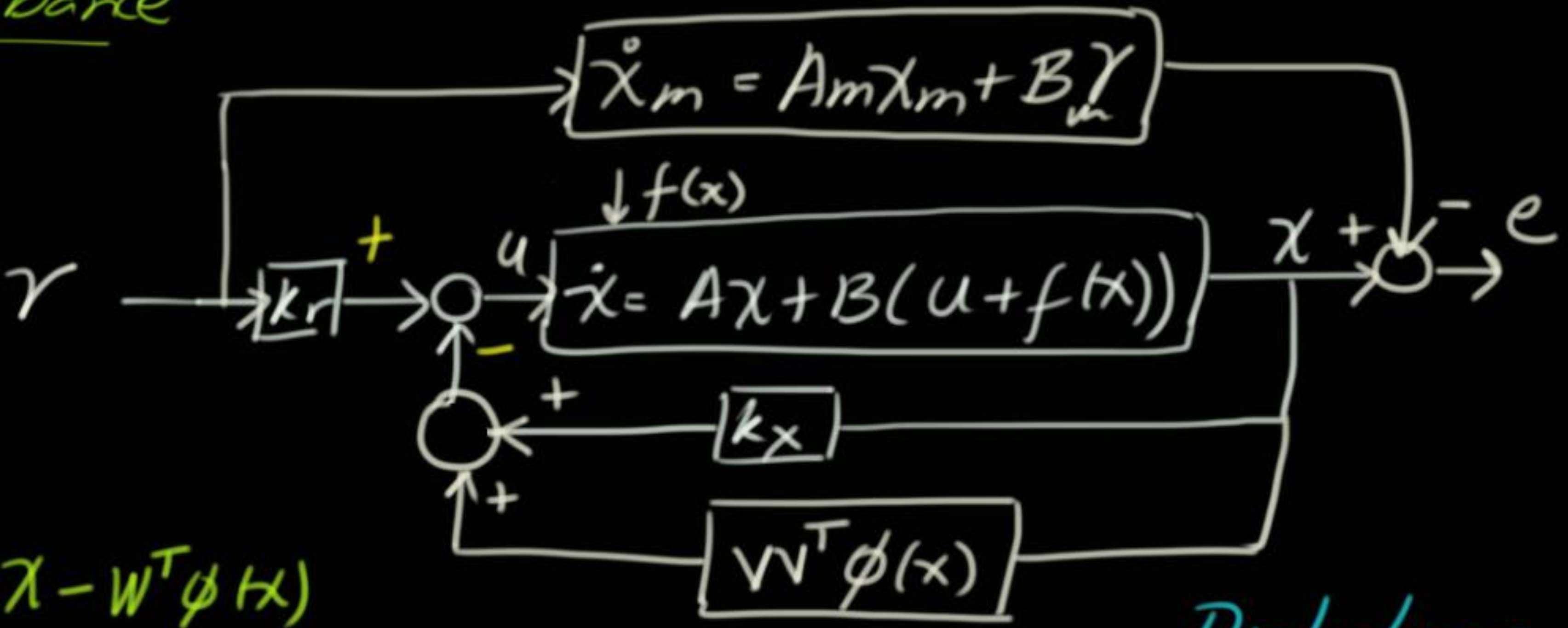
Model Matching

Pole Placement

$A_m = A - Bk_x$

$B_m = Bk_r$

With Disturbance



$$u = k_r r - k_x x - W^T\phi(x)$$

$$\dot{x} = Ax + B(k_r r - k_x x - W^T\phi(x) + f(x))$$

$W^T\phi(x)$ → Set of basic functions

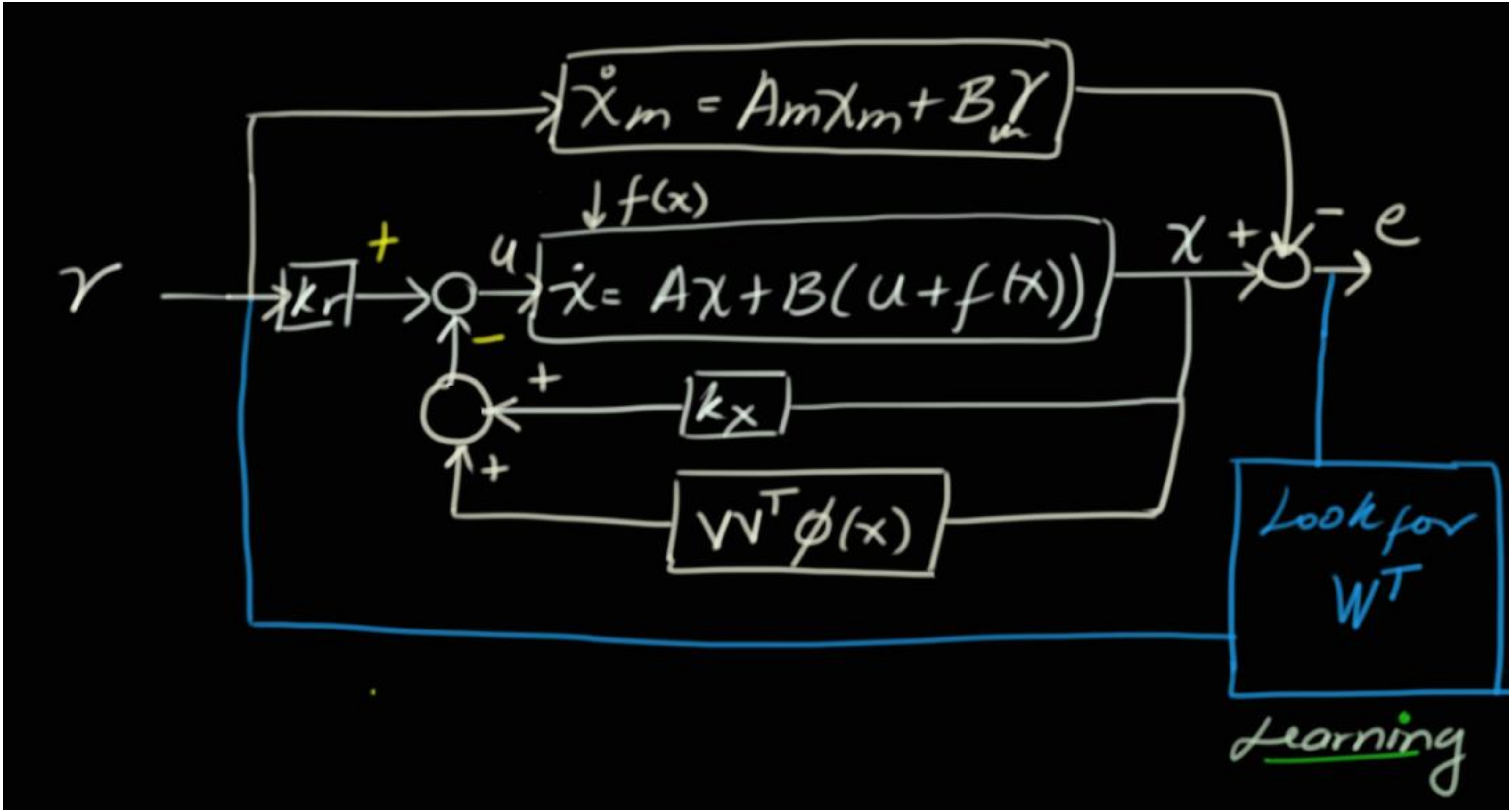
A weighting vector

Disturbance compensation

$$W^T\phi(x) = f(x)$$

$$\downarrow$$

$$\dot{x} = Ax + B(k_r r - k_x x) \checkmark$$



$\dot{x}_m = A_m x_m + B_m r$
$f(x)$
$\dot{x} = Ax + B(u + f(x))$
r
k_r
u
x
e
k_x
$W^T \phi(x)$
Look for W^T
Learning

1) We know the $f(x)$ can be represented using $x \to$ system states

$\underline{\theta}, \underline{\dot{\theta} = P}$ system states $\to f(x) = 1 + 2\theta + 3P + 4|\theta|\theta + 1|P|P + 2\theta^3$

$W_T \, \phi(x)$

$\phi(x) = [1, \theta, P, |\theta|\theta, |P|P, \theta^3]$

$W_T = [1; 2; 3; 4; 1; 2]$

2) Use some of the states as feautures $\longrightarrow \phi(x) = [1 \;\; \theta \;\; P \;\; |\theta|]$

$W_T = [\ldots]$?

3) Use Sum of Gaussians 

Nominal Model → Second Order model $\ddot{x}(t) = u(t) \rightarrow s^2 X(s) = U(s)$

$$\frac{U(s)}{X(s)} = \frac{1}{s^2}$$

state vector

$$x = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \theta \\ p \end{bmatrix}$$

Second order integrator

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u$$ State equation

$$y = \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D} u$$ output equation

Reference Model → stable second order model

$$\ddot{x}_m = -4x_m - 2\dot{x}_m + 4r$$

X_m Reference model state vector

r Roll reference given by the pilot

$$X_m = \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix}; \quad \begin{matrix} \dot{x}_m \\ \ddot{x}_m \end{matrix} = \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} r(t)$$

Pole placement

Model Matching

① $A_m = A - Bk_x$

② $B_m = Bk_r$

① $$\begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_{x1} & k_{x2} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ k_{x1} & k_{x2} \end{bmatrix}$$

feedback gains

$$\left.\begin{aligned} -4 &= -k_{x1} \\ -2 &= -k_{x2} \end{aligned}\right\} \begin{aligned} k_{x1} &= 4 \\ k_{x2} &= 2 \end{aligned}$$

② $$\begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_{r1} \end{bmatrix} \left\{ \begin{array}{l} \text{feedforward gain} \\ 4 = k_{r1} \end{array} \right.$$