Ch. 6.2 – HOMEWORK SOLUTIONS

6.50 What's wrong? Here are several situations where there is an incorrect application of the ideas presented in this section. Write a short paragraph explaining what is wrong in each situation and why it is wrong.

- **(a)** A researcher tests the following null hypothesis: H_0: $\bar{x}$= 23.
- **(b)** A random sample of size 30 is taken from a population that is assumed to have a standard deviation of 5. The standard deviation of the sample mean is 5/30.
- **(c)** A study with $\bar{x}$= 45 reports statistical significance for $H_a : \mu > 50$.
- **(d)** A researcher tests the hypothesis H_0: $\mu = 350$ and concludes that the population mean is equal to 350.

Solution

6.50. (a) The null hypothesis should be a statement about μ, not the sample mean $\bar{x}$.
(b) The standard deviation of the sample mean is 5/sqrt(30). **(c)** $\bar{x}$ = 45 would not make us inclined to believe that $\mu > 50$ over the (presumed) null hypothesis $\mu = 50$. **(d)** Even if we fail to reject H_0, we are not sure that it is true.

Note: *That is, "not rejecting H_0" is different from "knowing that H_0 is true." This is the same distinction we make about a jury's verdict in a criminal trial: If the jury finds the defendant "not guilty," that does* not *necessarily mean that they are sure he/she is innocent. It simply means that they were not sufficiently convinced of his/her guilt.*

6.51 What's wrong? Here are several situations where there is an incorrect application of the ideas presented in this section. Write a short paragraph explaining what is wrong in each situation and why it is wrong.

- **(a)** A significance test rejected the null hypothesis that the sample mean is equal to 500.
- **(b)** A test preparation company wants to test that the average score of their students on the ACT is better than the national average score of 21.2. They state their null hypothesis to be $H_0 : \mu > 21.2$.
- **(c)** A study summary says that the results are statistically significant and the P-value is 0.98.
- **(d)** The z statistic is equal to 0.018. Because this is less than α= 0.05, the null hypothesis was rejected.

Solution

6.51. (a) Hypotheses should be stated in terms of the population mean, not the sample mean.
(b) The null hypothesis H_0 should be that there is no change (μ = 21.2). **(c)** A small P-value is needed for significance; P = 0.98 gives no reason to reject H_0. **(d)** We compare the P-value, not the z-statistic, to α. (In this case, such a small value of z would have a very large P-value—close to 0.5 for a one-sided alternative, or close to 1 for a two-sided alternative.)

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6.52 Determining hypotheses. State the appropriate null hypothesis H_0 and alternative hypothesis H_a in each of the following cases.

(a) A 2008 study reported that 88% of students owned a cell phone. You plan to take an SRS of students to see if the percentage has increased.
(b) The examinations in a large freshman chemistry class are scaled after grading so that the mean score is 75. The professor thinks that students who attend early morning recitation sections will have a higher mean score than the class as a whole. Her students this semester can be considered a sample from the population of all students she might teach, so she compares their mean score with 75.
(c) The student newspaper at your college recently changed the format of their opinion page. You take a random sample of students and select those who regularly read the newspaper. They are asked to indicate their opinions on the changes using a five-point scale: −2 if the new format is much worse than the old, −1 if the new format is somewhat worse than the old, 0 if the new format is the same as the old, +1 if the new format is somewhat better than the old, and +2 if the new format is much better than the old.

Solution

6.52. (a) We are checking to see if the proportion p increased, so we test H_0: $p = 0.88$ versus H_a: $p > 0.88$. **(b)** The professor believes that the mean μ for the morning class will be higher, so we test H_0: $\mu = 75$ versus H_a: $\mu > 75$. **(c)** Let μ be the mean response (for the population of all students who read the newspaper). We are trying to determine if students are neutral about the change, or if they have an opinion about it, with no preconceived idea about the direction of that opinion, so we test H_0: $\mu = 0$ versus H_a: $\mu \neq 0$.

6.53 More on determining hypotheses. State the null hypothesis H_0 and the alternative hypothesis H_a in each case. Be sure to identify the parameters that you use to state the hypotheses.

(a) A university gives credit in first-year calculus to students who pass a placement test. The mathematics department wants to know if students who get credit in this way differ in their success with second-year calculus. Scores in second-year calculus are scaled so the average each year is equivalent to a 77. This year 21 students who took second-year calculus passed the placement test.
(b) Experiments on learning in animals sometimes measure how long it takes a mouse to find its way through a maze. The mean time is 20 seconds for one particular maze. A researcher thinks that playing rap music will cause the mice to complete the maze slower. She measures how long each of 12 mice takes with the rap music as a stimulus.
(c) The average square footage of one-bedroom apartments in a new student-housing development is advertised to be 880 square feet. A student group thinks that the apartments are smaller than advertised. They hire an engineer to measure a sample of apartments to test their suspicion.

Solution

6.53. (a) If μ is the mean score for the population of placement-test students, then we test H_0: $\mu = 77$ versus H_a: $\mu \neq 77$ because we have no prior belief about whether placement-test students will do better or worse. **(b)** If μ is the mean time to complete the maze with rap music playing, then we test H_0: $\mu = 20$ seconds versus H_a: $\mu > 20$ seconds because we believe rap music will make the mice finish more slowly. **(c)** If μ is the mean area of the apartments, we test H_0: $\mu = 880$ ft^2 versus H_a: $\mu < 880$ ft^2, because we suspect the apartments are smaller than advertised.

6.55 Translating research questions into hypotheses.

Translate each of the following research questions into appropriate H_0 and H_a.

(a) Census Bureau data show that the mean household income in the area served by a shopping mall is $42,800 per year. A market research firm questions shoppers at the mall to find out whether the mean household income of mall shoppers is higher than that of the general population.

(b) Last year, your online registration technicians took an average of 0.4 hours to respond to trouble calls from students trying to register. Do this year's data show a different average response time?

Solution

6.55. (a) H_0: $\mu = \$42,800$ versus H_a: $\mu > \$42,800$, where μ is the mean household income of mall shoppers. **(b)** H_0: $\mu = 0.4$ hr versus H_a: $\mu \neq 0.4$ hr, where μ is this year's mean response time.

6.56 Computing the *P*-value.

A test of the null hypothesis H_0: $\mu = \mu_0$ gives test statistic $z = 1.63$.

(a) What is the P-value if the alternative is H_a: $\mu > \mu_0$?

(b) What is the P-value if the alternative is H_a: $\mu < \mu_0$?

(c) What is the P-value if the alternative is H_a: $\mu \neq \mu_0$?

Solution

6.56. (a) For H_a: $\mu > \mu_0$, the P-value is $P(Z > 1.63) = 0.0516$.

(b) For H_a: $\mu < \mu_0$, the P-value is $P(Z < 1.63) = 0.9484$.

(c) For H_a: $\mu \neq \mu_0$, the P-value is $2P(Z > 1.63) = 2(0.0516) = 0.1032$.

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6.57 More on computing the *P*-value.
A test of the null hypothesis H_0: $\mu = \mu_0$ gives test statistic $z = -1.82$.

- **(a)** What is the P-value if the alternative is H_a: $\mu > \mu_0$?
- **(b)** What is the P-value if the alternative is H_a: $\mu < \mu_0$?
- **(c)** What is the P-value if the alternative is H_a: $\mu \neq \mu_0$?

Solution

6.57. (a) For H_a: $\mu > \mu_0$, the P-value is $P(Z > -1.82) = 0.9656$.

(b) For H_a: $\mu < \mu_0$, the P-value is $P(Z < -1.82) = 0.0344$.

(c) For H_a: $\mu \neq \mu_0$, the P-value is $2P(Z < -1.82) = 2(0.0344) = 0.0688$.

6.58 A two-sided test and the confidence interval.
The P-value for a two-sided test of the null hypothesis H_0: $\mu = 30$ is 0.032.

- **(a)** Does the 95% confidence interval include the value 30? Why?
- **(b)** Does the 90% confidence interval include the value 30? Why?

Solution

6.58. Recall the statement from the text: "A level α two-sided significance test rejects . . . H_0: $\mu = \mu_0$ exactly when the value μ_0 falls outside a level $1 - \alpha$ confidence interval for μ."

(a) No, 30 is not in the 95% confidence interval because $P = 0.032$ means that we would reject H_0 at $\alpha = 0.05$. **(b)** No, 30 is not in the 90% confidence interval because we would also reject H_0 at $\alpha = 0.10$.

6.59 More on a two-sided test and the confidence interval. A 90% confidence interval for a population mean is (25, 32).

- **(a)** Can you reject the null hypothesis that $\mu = 24$ against the two-sided alternative at the 10% significance level? Why?
- **(b)** Can you reject the null hypothesis that $\mu = 30$ against the two-sided alternative at the 10% significance level? Why?

Solution

6.59. See the quote from the text in the previous solution. **(b)** Yes, we would reject H_0: $\mu = 24$; the fact that 24 falls outside the 90% confidence interval means that $P < 0.10$.

(a) No, we would not reject H_0: $\mu = 30$ because 30 falls inside the confidence interval, so $P > 0.10$.

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6.61 Peer pressure and choice of major. A recent study followed a cohort of students entering a business/economics program. All students followed a common track during the first three semesters and then chose to specialize in either business or economics. Through a series of surveys, the researchers were able to classify roughly 50% of the students as either peer driven (ignored abilities and chose major to follow peers) or ability driven (ignored peers and chose major based on ability). When looking at entry wages after graduation, the researchers conclude that a peer-driven student can expect an average wage that is 13% less than that of an ability-driven student. The report states that the significance level is $P = 0.09$. Can you be confident of this statement regarding the wage decrease? Discuss.

Solution

6.61. $P = 0.09$ means there is some evidence for the wage decrease, but it is not significant at the $\alpha = 0.05$ level. Specifically, the researchers observed that average wages for peer-driven students were 13% lower than average wages for ability-driven students, but (when considering overall variation in wages) such a difference might arise by chance 9% of the time, even if student motivation had no effect on wages.

6.62 Symbol of wealth in ancient China? Every society has its own symbols of wealth and prestige. In ancient China, it appears that owning pigs was such a symbol. Evidence comes from examining burial sites. The skulls of sacrificed pigs tend to appear along with expensive ornaments. This suggests that the pigs, like the ornaments, signal the wealth and prestige of the person buried. A study of burials from around 3500 B.C. concluded that, "there are striking differences in grave goods between burials with pig skulls and burials without them…. A test indicates that the two samples of total artifacts are significantly different at the 0.01 level." Explain clearly why "significantly different at the 0.01 level" gives good reason to think that there really is a systematic difference between burials that contain pig skulls and those that lack them.

Solution

6.62. If the presence of pig skulls were not an indication of wealth, then differences similar to those observed in this study would occur less than 1% of the time by chance.

6.65 Sleep quality and elevated blood pressure. A recent study looked at $n = 238$ adolescents, all free of severe illness. Subjects wore a wrist actigraph, which allowed the researchers to estimate sleep patterns. For those subjects classified as having low sleep efficiency, they are reported as having an average systolic blood pressure that is 4 mm Hg higher than other children with a standard deviation of the mean equal to 1.2 mm Hg. Based on these results, test whether this difference is significant at the 0.01 level.

Solution

6.65. If μ is the mean difference between the two groups of children, we test H_0: $\mu = 0$ versus H_a: $\mu \neq 0$. The test statistic is $z = \frac{4-0}{1.2} \doteq 3.33$, for which software reports $P \doteq 0.0009$—very strong evidence against the null hypothesis.

Note: *The exercise reports the standard deviation of the mean, rather than the sample standard deviation; that is, the reported value has already been divided by* $\sqrt{238}$.

6.68 Who is the author? Statistics can help decide the authorship of literary works. Sonnets by a certain Elizabethan poet are known to contain an average of $\mu = 8.9$ new words (words not used in the poet's other works). The standard deviation of the number of new words is $\sigma = 2.5$. Now a manuscript with six new sonnets has come to light, and scholars are debating whether it is the poet's work. The new sonnets contain an average of $\bar{x} = 10.2$ words not used in the poet's known works. We expect poems by another author to contain more new words, so to see if we have evidence that the new sonnets are not by our poet we test

$$H_0: \mu = 8.9$$
$$H_a: \mu > 8.9$$

Give the z test statistic and its P-value. What do you conclude about the authorship of the new poems?

Solution

6.68. For testing these hypotheses, we find $z = \frac{10.2 - 8.9}{2.5/\sqrt{6}} \doteq 1.27$. This is not significant ($P = 0.1020$); there is not enough evidence to conclude that these sonnets were not written by our poet. (That is, we cannot reject H_0.)

6.69 Attitudes toward school. The Survey of Study Habits and Attitudes (SSHA) is a psychological test that measures the motivation, attitude toward school, and study habits of students. Scores range from 0 to 200. The mean score for U.S. college students is about 115, and the standard deviation is about 30. A teacher who suspects that older students have better attitudes toward school gives the SSHA to 25 students who are at least 30 years of age. Their mean score is $\bar{x} = 127.8$.

(a) Assuming that $\sigma = 30$ for the population of older students, carry out a test of

$$H_0: \mu = 115$$
$$H_a: \mu > 115$$

Report the P-value of your test, and state your conclusion clearly.

(b) Your test in part (a) required two important assumptions in addition to the assumption that the value of σ is known. What are they? Which of these assumptions is most important to the validity of your conclusion in part (a)?

Solution

6.69. (a) $z = \frac{127.8 - 115}{30/\sqrt{25}} \doteq 2.13$, so the P-value is $P = P(Z > 2.13) = 0.0166$. This is strong evidence that the older students have a higher SSHA mean. **(b)** The important assumption is that this is an SRS from the population of older students. We also assume a Normal distribution, but this is not crucial provided there are no outliers and little skewness.

6.70 Nutritional intake in Canadian high performance athletes. Since previous studies have reported that elite athletes are often deficient in their nutritional intake (e.g., total calories, carbohydrates, protein), a group of researchers decided to evaluate Canadian high performance athletes. A total of $n = 324$ athletes from eight Canadian sports centers participated in the study. One reported finding was that the average caloric intake among the $n = 201$ women was 2403.7 kcal/day. The recommended amount is 2811.5 kcal/day. Is there evidence that female Canadian athletes are deficient in the caloric intake?

(a) State the appropriate H_0 and H_a to test this.
(b) Assuming a standard deviation of 880 kcal/day, carry out the test. Give the P-value, and then interpret the result in plain language.

Solution

6.70. (a) Because we suspect that athletes might be deficient, we use a one-sided alternative: H_0: $\mu = 2811.5$ kcal/day versus H_a: $\mu < 2811.5$ kcal/day. **(b)** The test statistic is $z = \frac{2403.7 - 2811.5}{880/\sqrt{201}} \doteq -6.57$, for which $P < 0.0001$. There is strong evidence of below-recommended caloric consumption among female Canadian high-performance athletes.

6.73 Level of nicotine in cigarettes. According to data from the Tobacco Institute Testing Laboratory, Camel Lights king size cigarettes contain an average of 0.9 milligrams of nicotine. An advocacy group commissions an independent test to see if the mean nicotine content is higher than the industry laboratory claims.

(a) What are H_0 and H_a?
(b) Suppose that the test statistic is $z = 1.83$. Is this result significant at the 5% level?
(c) Is the result significant at the 1% level?

Solution

6.73. For (b) and (c), either compare with the critical values in Table D or determine the *P*-value (0.0336). **(a)** H_0: $\mu = 0.9$ mg versus H_a: $\mu > 0.9$ mg. **(b)** Yes, because $z > 1.645$ (or because $P < 0.05$). **(c)** No, because $z < 2.326$ (or because $P > 0.01$).

6.77 Understanding levels of significance. Explain in plain language why a significance test that is significant at the 5% level must always be significant at the 10% level.

Solution

6.77. When a test is significant at the 5% level, it means that if the null hypothesis were true, outcomes similar to those seen are expected to occur fewer than 5 times in 100 repetitions of the experiment or sampling. "Significant at the 10% level" means we have observed something that occurs in fewer than 10 out of 100 repetitions (when H_0 is true). Something that occurs "fewer than 5 times in 100 repetitions" also occurs "fewer than 10 times in 100 repetitions," so significance at the 5% level implies significance at the 10% level (or any higher level).

6.78 More on understanding levels of significance. You are told that a significance test is significant at the 5% level. From this information can you determine whether or not it is significant at the 1% level? Explain your answer.

Solution

6.78. Something that occurs "fewer than 5 times in 100 repetitions" is not necessarily as rare as something that occurs "less than once in 100 repetitions," so a test that is significant at 5% is not necessarily significant at 1%.