

## Chapter 6 Section 4

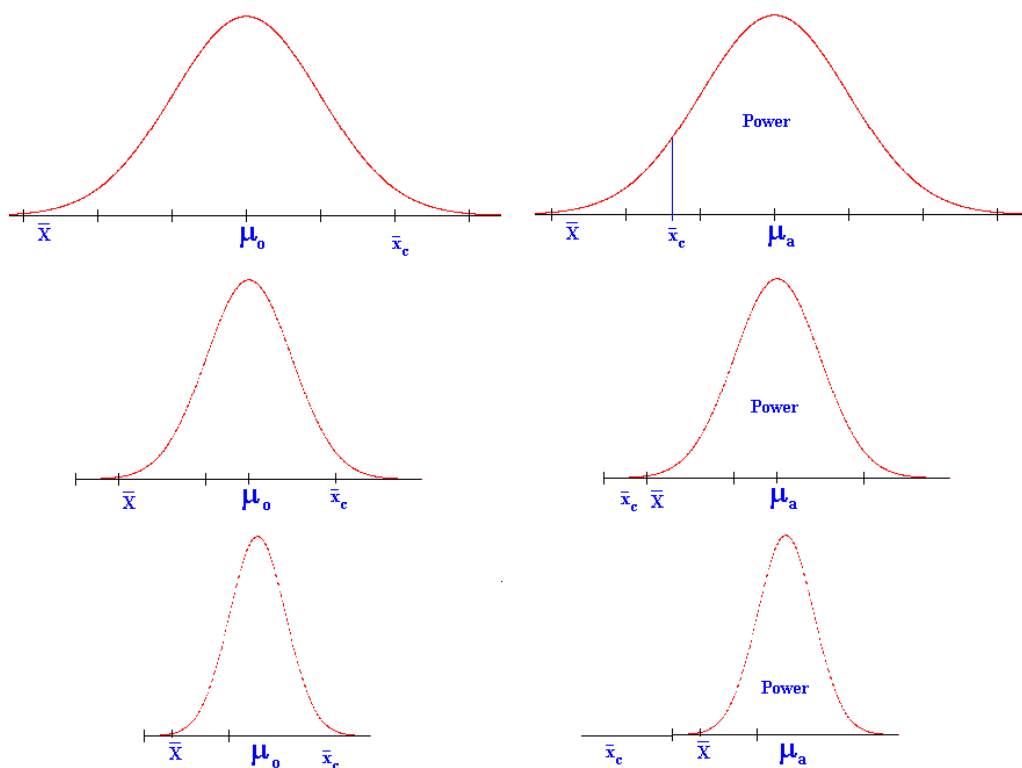
### Homework A

**6.106 Make a recommendation.** Your manager has asked you to review a research proposal that includes a section on sample size justification. A careful reading of this section indicates that the power is 20% for detecting an effect that you would consider important. Write a short report for your manager explaining what this means and make a recommendation on whether or not this study should be run.

**You would not want to run this test with only 20% power. Since this means that if the effect that you are looking for is really present, then the test you are about to run will only have a 20% chance of detecting it.**

**6.107 Explain power and sample size.** Two studies are identical in all respects except for the sample sizes. Consider the power versus a particular sample size- Will the study with the larger sample size have more power or less power than the one with the smaller sample size? Explain your answer in terms that could be understood by someone with very little knowledge of statistics.

**The larger the sample size the more power a study will have. Why? Power is about the ability to reject the null hypothesis when it is false. As you increase the sample size the variability of  $\bar{x}$  reduces  $\frac{\sigma}{\sqrt{n}}$ . Now the critical value is tied to the null mean, thus as  $n$  increase, the critical value gets closer to the null mean and away from the alternative mean increasing power.**



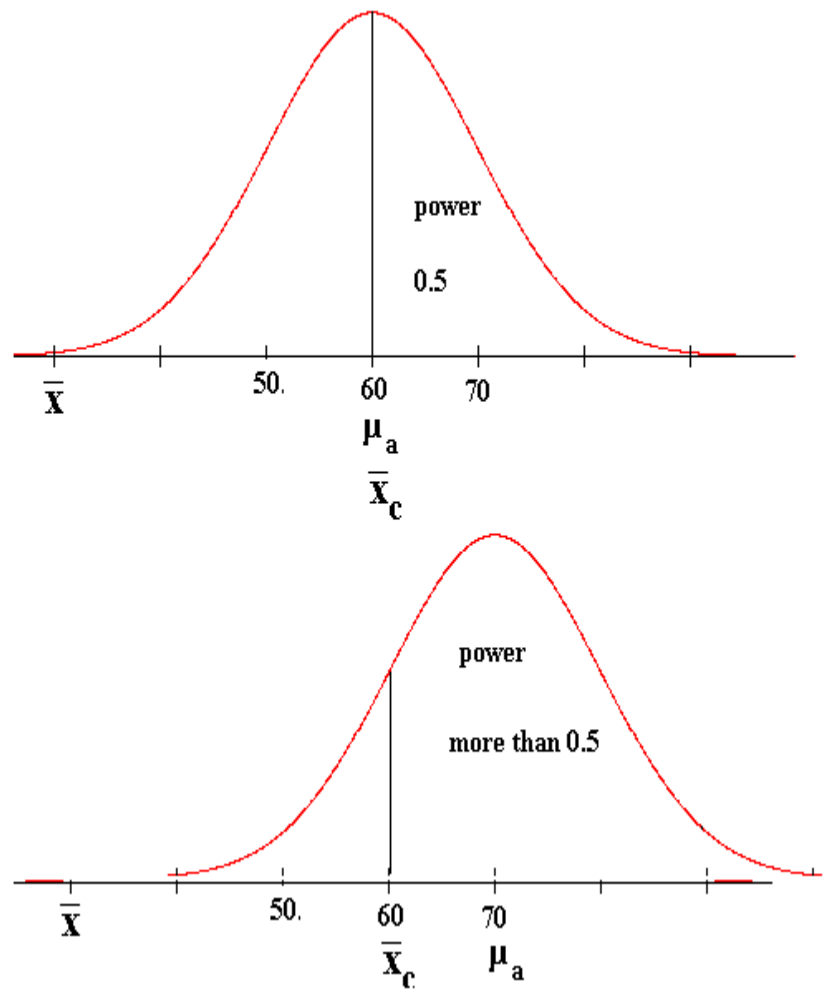
**6.109 More on the power for a different alternative.** A one-sided test of the null hypothesis  $\mu_0 = 50$  versus the alternative  $\mu_{a1} = 60$  has power equal to 0.5. Will the power for the alternative  $\mu_{a2} = 70$  be higher or lower than 0.5? Draw a picture and use this to explain your answer.

If at  $\mu_a = 60$  the power is 0.5 that means the critical value is also at 60 (see picture).

Now if we change the alternative mean to 70,  $\mu_a = 70$ , the critical value stays the same at 60. However, the graph itself shifts so that the new center is 70.

You can see that the power increases.

This makes sense since the further away the null and alternative mean are, if the alternative mean is correct, a random sample should produce a sample average close to  $\mu_a$  and not  $\mu_0$ . The likely hood of rejecting the null increases.



**6.110 Power of the random north-south distribution of trees test.** In Exercise 6.66 (page 392) you performed a two-sided significance test of the null hypothesis that the average north-south location of the longleaf pine trees sampled in the Wade Tract was  $\mu = 100$ . There were 584 trees in the sample and the standard deviation was assumed to be 58. The sample mean in that analysis was  $\bar{x} = 99.74$ . Use the *Power* applet to compute the power for the alternative  $\mu = 99$  using a two-sided test at the 5% level of significance.

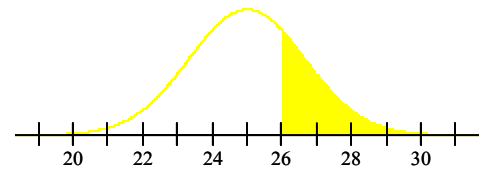
**6.112 Mail-order catalog sales.** You want to see if a redesign of the cover of a mail-order catalog will increase sales. A very large number of customers will receive the original catalog, and a random sample of customers will receive the one with the new cover. For planning purposes, you are willing to assume that the sales from the new catalog will be approximately Normal with  $\sigma = 50$  dollars and that the mean for the original catalog will be,  $\mu = 25$  dollars. You decide to use a sample size of  $n = 900$ . You wish to test

$$H_0: \mu = 25 \quad H_a: \mu > 25$$

You decide to reject  $H_0$  if  $\bar{x} > 26$ .

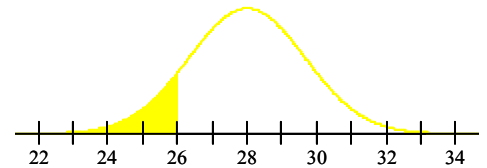
(a) Find the probability of a Type I error, that is, the probability that your test rejects  $H_0$  when in fact  $\mu = 25$  dollars.

$$Z = \frac{26 - 25}{\frac{50}{\sqrt{900}}} = 0.6 \quad P(\bar{x} > 26) = P(Z > 0.6) = 0.2743$$



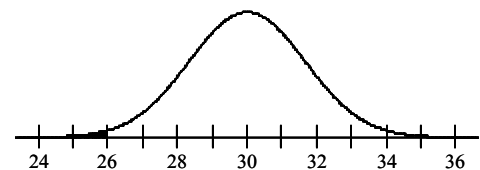
(b) Find the probability of a Type II error when  $\mu = 28$  dollars. This is the probability that your test accepts  $H_0$  when in fact  $\mu = 28$ .

$$Z = \frac{26 - 28}{\frac{50}{\sqrt{900}}} = -1.2 \quad P(\bar{x} < 26) = P(Z < -1.2) = 0.1151$$



(c) Find the probability of a Type II error when  $\mu = 30$ .

$$Z = \frac{26 - 30}{\frac{50}{\sqrt{900}}} = -2.4 \quad P(\bar{x} < 26) = P(Z < -2.4) = 0.00820$$



(d) What did you learn from exercise (b) and (c)?

**The probability of committing a Type II error decreases the further  $\mu_a$  is away from  $\mu_0$ .**

(e) “You decide to reject  $H_0$  if  $\bar{x} > 26$ .” What is the value, 26, called?

**The critical value.**

(d) The distribution of sales is not Normal, because many customers buy nothing. Why is it nonetheless reasonable in this circumstance to assume that the mean will be approximately Normal? This is a very important question. Make sure you truly understand the answer.

**The sample size is 900, thus a sample size of 900 should be large enough (unless there is some severe skewness) to call upon the Central Limit Theorem, which says that the distribution of the sample mean should be approximately normal.**

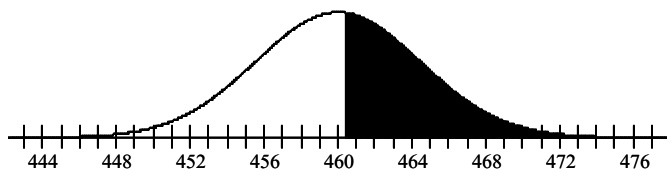
**6.113 Power of the mean SAT score test.** Example 6.16 (page 385) gives a test of a hypothesis about the SAT scores of California high school students based on an **SRS** of 500 students. The hypotheses are

$$H_0: \mu = 450 \quad H_a: \mu > 450$$

Assume that the population standard deviation is  $\sigma = 100$ . The test rejects  $H_0$ , at the 1% level of significance when  $z \geq 2.326$ .

Is this test sufficiently sensitive to usually detect an increase of 10 points in the population mean SAT score? Answer this question by calculating the power of the test against the alternative  $\mu = 460$

$$\bar{x}_c = 2.326 \left( \frac{100}{\sqrt{500}} \right) + 450 = 460.40$$



$$Z = \frac{460.40 - 460}{\frac{100}{\sqrt{500}}} = 0.08944 \quad P(\bar{x}_c > 460.40) = P(Z > 0.08944) = 0.4644$$

**This test would require a larger sample size in order to increase the power to an appropriate level (i.e. the test is not sensitive enough).**

**6.114 Choose the appropriate distribution.** You must decide which of two discrete distributions a random variable  $X$  has. We will call the distributions  $p_0$  and  $p_1$ . Here are the probabilities they assign to the values  $x$  of  $X$ :

You have a single observation on  $X$  and wish to test  $H_0: p_0$  is correct  $H_a: p_1$  is correct

$X$	0	1	2	3	4	5	6
$P_0$	0.1	0.1	0.1	0.2	0.1	0.1	0.3

$X$	0	1	2	3	4	5	6
$P_1$	0.3	0.1	0.1	0.2	0.1	0.1	0.1

One possible decision procedure is to reject  $H_0$  only if  $X = 0$  or  $X = 1$ .

(a) Find the probability of a Type I error, that is, the probability that you reject  $H_0$  when  $p_0$  is the correct distribution.

$X$	<b>0</b>	<b>1</b>	2	3	4	5	6
$P_0$	0.1	0.1	0.1	0.2	0.1	0.1	0.3

$$P(\text{Type I error}) = P(X = 0 \text{ or } X = 1)$$

$$= 0.1 + 0.1$$

$$= 0.2$$

**Notice that I had to use the table with  $P_0$  since this is the null value.**

(b) Find the probability of a Type II error.

$X$	0	1	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
$P_1$	0.3	0.1	0.1	0.2	0.1	0.1	0.1

$$P(\text{Type II error}) = P(X = 2 \text{ or } X = 3 \text{ or } X = 4 \text{ or } X = 5 \text{ or } X = 6)$$

$$= 1 - P(X = 0 \text{ or } X = 1)$$

$$= 1 - (0.3 + 0.1)$$

$$= 0.6$$

**Notice that I had to use the table with  $P_1$ , since I am assuming that the null value is not correct.**

(c) What is the value  $X = 1$  called? **The critical value.**

**6.115 A Web-based business.** You are in charge of marketing for a Web site that offers automated medical diagnoses. The program will scan the results of routine medical tests (pulse rate, blood pressure, urinalysis, etc.) and either clear the patient or refer the case to a doctor. You are marketing the program for use as part of a preventive-medicine system to screen many thousands of persons who do not have specific medical complaints. The program makes a decision about each patient.

- (a) What are the two hypotheses and the two types of error that the program can make? Describe the two types of error in terms of "false-positive" and "false-negative" test results.
- (b) The program can be adjusted to decrease one error probability at the cost of an increase in the other error probability. Which error probability would you choose to make smaller, and why? (This is a matter of judgment. There is no single correct answer.)

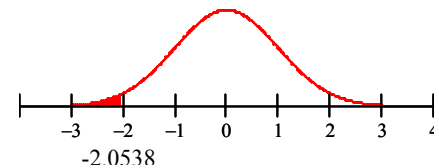
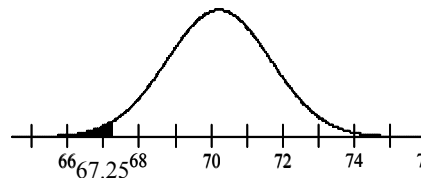
1. You want to run the following test  $H_0: \mu = 70.22$   $H_a: \mu < 70.22$ , where  $\sigma = 4.34$ , with  $n = 9$ . And you will reject the null hypothesis at the 2% significance level. The effect you want to measure is 5 units.

a. What is the probability of a Type I error?

$$P(\text{Type I error}) = 0.02$$

b. What is the power of this test?

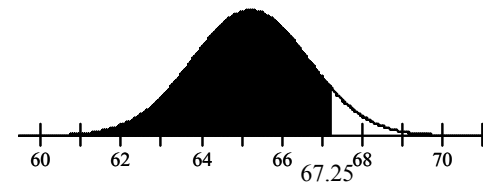
$$\sigma_{\bar{x}} = \frac{4.34}{\sqrt{9}} = 1.4467$$



We need to calculate the critical value.

$$\bar{x}_c = -2.0538(1.4467) + 70.22 = 67.25$$

The effect you want to measure is 5 units. Thus,  $\mu_a = 65.22$



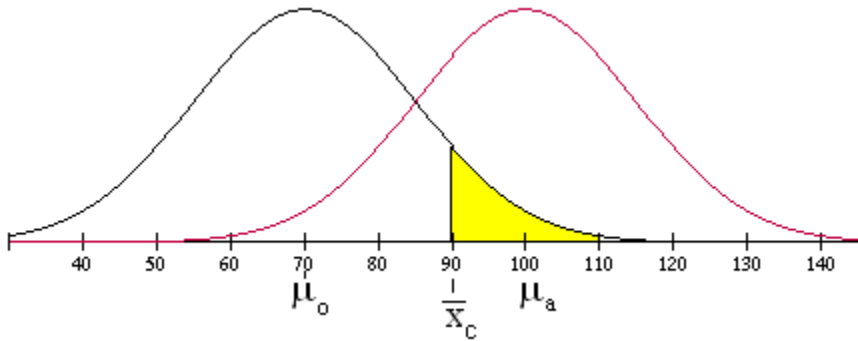
$$Z = \frac{67.25 - 65.22}{\frac{4.34}{\sqrt{9}}} = 1.4043 \quad P(\bar{x}_c < 67.25) = P(Z < 1.4043) = 0.9197$$

This test has at a minimum 91.97% power which means we have a 91.97% chance to detect an effect as large as 5 units or larger if the effect is really present.

c. What is the probability of a Type II error?

$$\begin{aligned} \text{P(Type II error)} &= 1 - \text{P(power of test)} \\ &= 1 - 0.9197 \\ &= 0.0803 \end{aligned}$$

2. Shade the correct region that corresponds to a type I error.  $\mu_o = 70$  (null mean),  $\mu_a = 100$  (particular alternative mean), and  $\bar{x}_c = 90$  (critical value). Keep in mind that only one of these graphs can exist at a time depending on the question asked.



3. Shade the correct region that corresponds to the power.  $\mu_o = 70$  (null mean),  $\mu_a = 100$  (particular alternative mean), and  $\bar{x}_c = 90$  (critical value). Keep in mind that only one of these graphs can exist at a time depending on the question asked.

