✓ 1) Proceso estacionario de orden n

✓ 2) Def. Fn. de autocovarianza

? 3) $\gamma(h) = \gamma(-h)$ } Cuando un proceso es débilmente estacionario

✓ 4) $X_t = \phi X_{t-1} + Z_t$ } $Z_t \sim WN(0, \sigma^2)$

↳ Débilmente estacionario

-> De la pregunta 1

para todo ≠ para alguna n

-> Def. proceso estacionario de orden n:

Sea $\{X_t\}_{t=1}^{\infty}$ un proceso estacionario de orden n, entonces <u>para</u> <u>alguna n</u>,

$$F_{X_{t_1}, X_{t_2}, X_{t_3} \ldots X_{t_n}}(x_{t_1}, x_{t_2}, \ldots, x_{t_n}) =$$

$$F_{X_{t_1+k}, X_{t_2+k}, X_{t_3+k}, \ldots, X_{t_n+k}}(x_{t_1}, x_{t_2}, \ldots, x_{t_n})$$

$K \in \mathbb{R}$

2) $\gamma(t, s) = Cov(X_t, X_s) = \mathbb{E}[(X_t - \mu_t)(X_s - \mu_s)]$

3) $\gamma(h) = \gamma(t, t+h) = Cov(X_t, X_{t+h}) = *$ $\quad \gamma(-h)$

Definimos $s = t+h$ $\quad$ estacionario $\quad j = -h$ $\quad$ //

$* = Cov(X_{s-h}, X_s) = Cov(X_{s+j}, X_s) = \gamma(j)$

$\hookrightarrow \phi[\phi X_{t-2} + Z_t]$

4) Probar que el proceso $X_t = \phi X_{t-1} + Z_t$, $Z \sim WN(0,\sigma^2)$ es débilmente estacionario.

$$\mu_t = \mathbb{E}[\phi X_{t-1} + Z_t] = \phi \mathbb{E}[X_{t-2}]$$

$$X_t = \sum_{t=1}^{\infty} \phi^{t-1} Z_t \Rightarrow \mathbb{E}[X_t] = \sum_{t=1}^{\infty} \phi^{t-1} \overset{0}{\cancel{\mathbb{E}[Z_t]}}$$

$$= 0$$

$$\gamma(h) = Cov(X_t, X_{t+h}) = \mathbb{E}[(X_t - \overset{0}{\cancel{\mu_t}})(X_{t+h} - \overset{0}{\cancel{\mu_{t+h}}})]$$

$$= \mathbb{E}[X_t X_{t+h}]$$

→ Sabemos que $X_t = \phi X_{t-1} + Z_t$

=> Multiplicando por X_{t+h} ambos lados:

$$X_t X_{t+h} = \phi X_{t-1} X_{t+h} + Z_t X_{t+h}$$

→ Si $h = 0$, $h = 1$, $h = -1$; casos importantes.

Si $h = 0 \Rightarrow \mathbb{E}[X_t^2] = \phi \mathbb{E}[X_{t-1} X_t] + \mathbb{E}[Z_t X_t]$

$$= \phi \mathbb{E}[X_{t-1}(\phi X_{t-1} + Z_t)] + \underbrace{\mathbb{E}[Z_t(\overset{0}{\cancel{\phi X_{t-1}}} + \overset{\sigma^2}{\cancel{Z_t}})]}_{\sigma^2}$$

$$= \phi[\phi \mathbb{E}[X_{t-1}^2] + \overset{0}{\cancel{\mathbb{E}[Z_t X_{t-1}]}}] + \sigma^2$$

$$= \phi^2 \mathbb{E}[X_{t-1}^2] + \sigma^2$$

$$\hookrightarrow \mathbb{E}[(\phi X_{t-2}^2 + 2X_{t-2} Z_t + Z_t^2)]$$

Ahora, supongamos una h arbitraria:

$$\gamma(h) = \mathbb{E}[X_t \cdot X_{t+h}] = \mathbb{E}[\phi X_{t-1} X_{t+h}] + \cancel{\mathbb{E}[X_{t+h} Z_t]}^{\,0}$$

$$= \phi\, \mathbb{E}[X_{t-1} X_{t+h}] \rightsquigarrow s = t-1$$

$$= \phi\, \mathbb{E}[X_s X_{s+h+1}]$$

$$= \phi \gamma(h+1)$$

$$= \phi \gamma(h+2)$$

$$= \phi^n \gamma(h+n)$$

$\hookrightarrow |\phi| < 1$