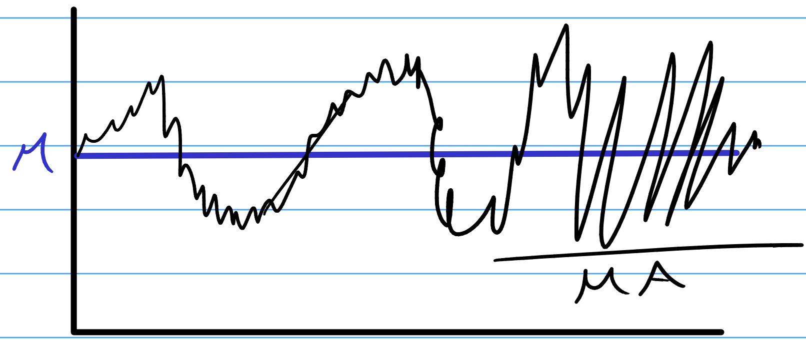
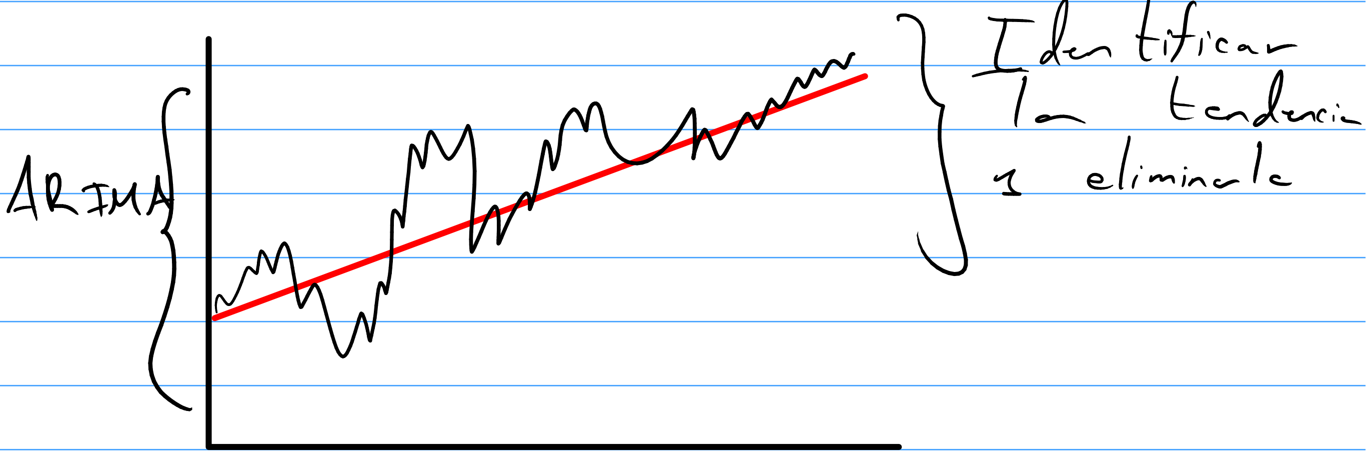
Modelos ARIMA

- A R } Autoregresive
- I } Integrated
- M A } Moving Average





Podemos expresar a los modelos ARIMA de la siguiente forma:

$$\phi_p(B)(1-B)^d X_t = \Theta_q(B) Z_t$$

↳ ARIMA(p, d, q)

$(1-B)X_t = X_t - X_{t-1}$

Si $d=0$, tenemos un proceso ARMA(p,q)

↳ De hecho, si $Y = (1-B)^d X_t \Rightarrow Y$ es un proceso ARMA(p,q) $(d=1)$

En finanzas, es común trabajar con series de tiempo transformándolas de la siguiente forma:

$$Y_t = \frac{\overbrace{X_t - X_{t-1}}^{\text{tendencia}}}{\underbrace{X_{t-1}}_{\text{normaliza los datos}}}$$

$\rightarrow$ Cambio Porcentual de una serie de tiempo.

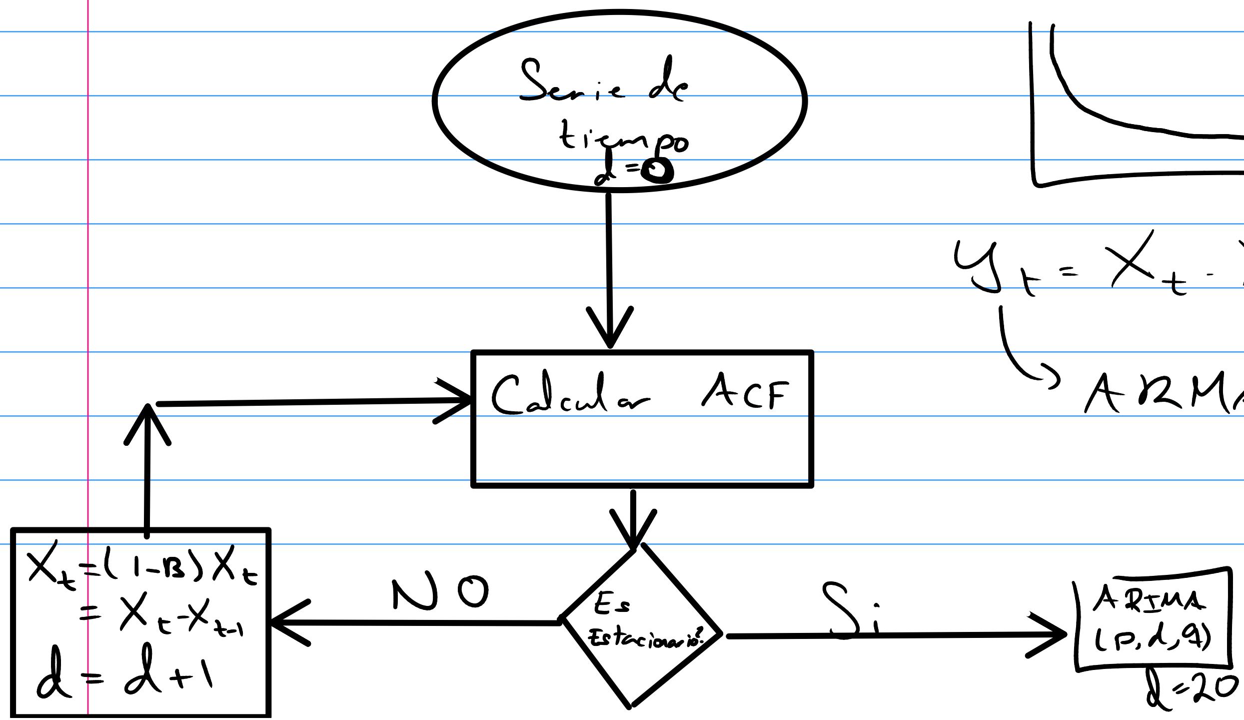
$$\left(Y_t = X_t - X_{t-1} \right)$$

$$X\ (USD) \rightarrow \frac{X_t - X_{t-1}}{X_{t-1}} \frac{(\cancel{USD})}{(\cancel{USD})} \quad \%$$

$$(1-B)^2 X_t = (1-B)(1-B)X_t$$

$$= (1-B)(\underbrace{X_t - X_{t-1}}_{Y}) = (1-B)Y$$

¿Cómo determinar el valor de d?



$Y_t = X_t - X_{t-1}$

$\rightarrow$ ARMA

Ejemplo:

* Caminata aleatoria:

$$X_t = X_{t-1} + Z_t, \qquad Z_t \sim WN(0, \sigma^2)$$

$$X_t - X_{t-1} = Z_t \iff (1-B)X_t = Z_t$$

$\hookrightarrow$ ARIMA$(0,1,0)$

$$Y_t = X_t - X_{t-1} \Rightarrow Y_t = Z_t$$

Qué pasa si ahora, nuestro modelo es de la siguiente forma:

$$(1-B)X_t = \Theta_0 + Z_t \quad \text{ó}$$

$$X_t = X_{t-1} + \Theta_0 + Z_t$$

$$X_t = (X_{t-2} + \Theta_0 + Z_{t-1}) + \Theta_0 + Z_t$$

$$X_t = X_{t-2} + 2\Theta_0 + Z_{t-1} + Z_{t-2}$$

$$\vdots$$

$$X_t = X_K + \underbrace{(t-K)\Theta_0}_{\text{Drift}} + \sum_{j=K+1}^{t} Z_j$$

$$Y_t = (1-B)X_t$$

$$Y_t = \Theta_0 + Z_t \rightarrow Y_t - Y_{t-1} = (\cancel{\Theta_0} + Z_t) - (\cancel{\Theta_0} + Z_{t-1})$$

$$(1-B)Y_t = (1-B)Z_t$$

$$\text{ARIMA}(0,2,1) \Rightarrow (1-B)^2 X_t = (1-B)Z_t$$

Modelo ARIMA$(0,1,1)$ ó IMA$(1,1)$

$$(1-B)X_t = (1-\theta B)Z_t$$

$$X_t = X_{t-1} + Z_t - \theta Z_{t-1}$$

$$\boxed{Z_t = \frac{(1-B)}{1-\theta B} X_t} \qquad \sum_{i=1}^{\infty} x^i = \frac{1}{1-x}$$

$$\frac{1-B}{1-\theta B} = (1-B)(1+\theta B+\theta^2 B^2+\dots)$$

$$= (1+\theta B+\theta^2 B^2+\dots) - (\cancel{B}+\theta B^2+\theta^2 B^3+\dots)$$

$$= 1-(1-\theta)B - (1-\theta)\theta B^2 - (1-\theta)\theta^2 B^3 - \dots$$

Definir $\alpha = 1-\theta$

$$= 1 - \alpha B - \alpha(1-\alpha)B^2 - \alpha(1-\alpha)^2 B^3 - \dots$$

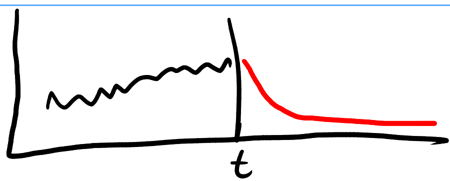
$$\Rightarrow \quad X_t = \alpha \sum_{j=1}^{\infty} (1-\alpha)^{j-1} X_{t-j} + Z_t$$

Supongamos que el mejor estimador para pronosticar el valor futuro de X_t es

$$\hat{X}_t = \mathbb{E}[X_t]$$

$$\hat{X}_t = \mathbb{E}\Big[\alpha \sum_{j=1}^{\infty} (1-\alpha)^{j-1} \overbrace{X_{t-j}}^{\text{observados}} + \overbrace{\cancel{Z_t}}^{0}\Big]$$

$$\hat{X}_t = \alpha \sum_{j=1}^{\infty} (1-\alpha)^{j-1} X_{t-j}$$



$$\hat{X}_{t+1} = \alpha \sum_{j=1}^{\infty} (1-\alpha)^{j-1} X_{t+1-j}$$

$$= \alpha (1-\alpha)^{1-1} X_{t+1-1} + \alpha \sum_{j=2}^{\infty} (1-\alpha)^{j-1} X_{t+1-j}$$

$$= \alpha X_t + \alpha(1-\alpha) \sum_{j=2}^{\infty} (1-\alpha)^{\underbrace{(j-1)-1}_{i}} X_{\underbrace{t+1-j}_{}} \quad \underbrace{t-(j-1)}_{t-i}$$

$i = j-1$

$$= \alpha X_t + (1-\alpha) \underbrace{\alpha \sum_{i=1}^{\infty} (1-\alpha)^{i-1} X_{t-i}}_{\hat{X}_t}$$

$$\hat{X}_{t+1} = \alpha X_t + (1-\alpha) \hat{X}_t$$

→ El pronóstico de $\hat{X}_{t+1}$ podemos verlo como un promedio ponderado entre la observación X_t y el pronóstico $\hat{X}_t$

→ Caso particular de un Filtro de Kalman.