$$\boxed{X_t = \psi_1 Z_{t-1} + \psi_2 Z_{t-2} + Z_t \quad ; \quad Z_t \sim WN(0, \sigma^2)}$$

$\boxed{\text{Calcular } \gamma(h)}$ ↳ $X_t \to MA(2)$

Si $\boxed{h=0} \Rightarrow \boxed{Var(X_t)} = \sigma^2(1 + \psi_1 + \psi_2)$

Si $\boxed{h=\pm 1} \Rightarrow \boxed{\mathbb{E}[X_t X_{t-1}]} = Cov(X_t, X_{t-1})$

$$= \mathbb{E}[(\psi_1 Z_{t-1} + \psi_2 Z_{t-2} + Z_t)(\psi_1 Z_{t-2} + \psi_2 Z_{t-3} + Z_{t-1})]$$

$$= \psi_1 \sigma^2 + \psi_1 \psi_2 \sigma^2 = \sigma^2 \psi_1 (1 + \psi_2)$$

Si $\boxed{h = \pm 2} \Rightarrow \mathbb{E}[X_t X_{t-2}] = Cov(X_t, X_{t-2})$

$$= \mathbb{E}[(\psi_1 Z_{t-1} + \psi_2 Z_{t-2} + Z_t)(\psi_1 Z_{t-3} + \psi_2 Z_{t-4} + Z_{t-2})]$$

$$= \psi_2 \sigma^2$$

Si $|h| \geq 3 \Rightarrow \gamma(h) = 0$

$$\boxed{\gamma(h) = \begin{cases} \sigma^2(1 + \psi_1 + \psi_2) & h = 0 \\ \sigma^2 \psi_1 (1 + \psi_2) & |h| = 1 \\ \sigma^2(\psi_2) & |h| = 2 \\ 0 & \text{e.o.c.} \end{cases}}$$

3) Probar que la función de autocorrelación $\rho(h) \in [-1, 1]$

$$\rho(h) = \frac{Cov(X_t, X_{t+h})}{\sqrt{Var(X_t)}\sqrt{Var(X_{t+h})}},$$

ésta es una correlación entre dos variables aleatorias, para cualquier t y h

→ Por la desigualdad de Cauchy-Schwartz

$\left(|\operatorname{Cov}(X,Y)| \leq \sqrt{\operatorname{Var}(X)} \sqrt{\operatorname{Var}(Y)} \right)$

↳ De aquí vemos que $\rho(h) \in [-1, 1]$