AR(1)

$x_t = \phi x_{t-1} + \omega_t, \quad \omega_t \sim WN(0, \sigma^2)$

$x_t = \phi x_{t-1} + \omega_t$

ACF

$\hookrightarrow \mathbb{E}[X_t X_{t-k}] = \mathbb{E}\left[\left[\phi^k x_{t-k} + \sum_{i=1}^{k} \phi^{k-1} \omega_{t-i}\right][X_{t-k}]\right] = \phi^k \gamma_k$

$\hookrightarrow \rho_k = \phi^k$

PACF

$\phi_{11} = \rho_1$

$$\phi_{22} = \frac{\rho_2 - \sum_{i=1}^{1} \phi_{ij} \rho_{2-j}}{1 - \sum \phi_{ij} \rho_j} = \frac{\rho_2 - \rho_2}{\sim} = 0$$

$\phi_{2,1} = \phi_{11} - \cancelto{0}{\phi_{22}} \phi_{11} = \phi_{11} = \phi^1$

$$\phi_{3,1} = \frac{\rho_3 - \sum_{i=1}^{2} \phi_{2j} \hat{\rho}_{2+1-j}}{1 - \sum_{i=1}^{2} \phi_{2j} \rho_j} = \frac{\rho_3 - \phi_{21}\rho_1 - \phi_{22}\rho_2}{1 - \sum}$$

$$= \frac{\phi^3 - \phi_{21}\rho_2}{\sum} = 0$$

$\S \quad \phi_{kk} = 0 \quad \forall k > 1$

$\phi_{kj} = \phi_{kj}$

$$\phi_{k+1\,k+1} = \frac{\rho_{k+1} - \sum_{j=1}^{k} \phi_{kj} \rho_{k+1-j}}{C} = \rho_{k+1} - \phi_{11}\rho_k = 0$$

$\phi_{11}\rho_k$

$\phi_{k1}\rho_k + \phi_{k2}\rho_{k-1} + \phi_{k3}\rho_{k-2} \dots + \cancelto{0}{\phi_{kk}}\rho_1$

$$x_t = \phi w_{t-1} + w_t$$

$$\rho \to -\frac{\theta_1}{1+\theta_1^2}$$

$$\phi_{11} = \rho_1$$

$$\phi_{22} = \frac{\rho_2 - \phi_{11}\rho_1}{1-\phi_{11}\rho_1} = \frac{\rho_2 - \rho_1^2}{1-\rho_1^2} = = \frac{\left(-\frac{\theta_1}{1+\theta_1}\right)^2}{1-\left(\frac{-\theta_1}{1+\theta_1}\right)^2}$$

$$\theta_{kk} = \frac{-\theta_1^k(1-\theta_1^2)}{1-\theta_1^{2(k+1)}}$$

AR(1)

$x_t = \phi x_{t-1} + w_t$; $\{w_t\}_{t=1}^{\infty}$; secuencia de va's. no correl.

$w_t \sim WN(0, \sigma_w^2)$

$(w_t \sim N(0, \sigma^2)$, x_t $)$

¿μ_t , γ_0 , γ_k , ρ_k → $P_k = \phi_{k,k}$)

$|\phi| < 1$ → $\boxed{x_t = \sum_{j=1}^{\infty} \phi^j w_{t-j} \quad \text{\} Principio de causalidad}}$

$$\mu_t = \mathbb{E}[X_t] = \mathbb{E}\left[\sum_{j=1}^{\infty} \phi^j w_{t-j}\right] \overset{|\phi|<1}{=} \sum_{j=1}^{\infty} \phi^j \mathbb{E}[w_{t-j}] = 0$$

Teorema de convergencia acotada

$$\gamma_k = \mathbb{E}[X_t \cdot X_{t-k}] = \mathbb{E}[(\phi X_{t-1} + w_{t-1})(X_{t-k})]$$

$$= \mathbb{E}[(\phi^2 X_{t-2} + \phi w_{t-2} + w_{t-1}) X_{t-k}]$$

⋮ k-veces

$$= \mathbb{E}\left[\left(\phi^k X_{t-k} + \sum_{j=1}^{k-1} \phi^{j-1} w_{t-j}\right) X_{t-k}\right]$$

$\{\phi^0 w_{t-1}, \dots, \phi^{k-2} w_{t-(k-1)}\}$

$$= \mathbb{E}\left[\phi^k \overbrace{X_{t-k} X_{t-k}}^{\gamma_0} + \cancel{X_{t-k} \sum_{j=1}^{k-1} \phi^{j-1} w_{t-j}}^{\,0}\right]$$

$$= \mathbb{E}[\phi^k X_{t-k} X_{t-k}] = \phi^k \underbrace{\mathbb{E}[X_{t-k}^2]}_{\gamma_0} = \phi^k \gamma_0$$

$|\phi| < 1$

$$\Rightarrow \gamma_k = \phi^k \gamma_0 \rightarrow \rho_k = \frac{\gamma_k}{\gamma_0} = \phi^k \}$$

???

$\{z\}_{i=1}^{n}$ v.a.i.i.d $\rightsquigarrow Var(\sum z_i) = \sum Var(z_i)$

$|\phi| < 1$

$$\gamma_0 = \mathbb{E}[X_{t-k}^2] = \mathbb{E}\left[\left(\sum_{j=1}^{\infty} \phi^j w_{t-j}\right)^2\right] = Var\left(\sum_{j=1}^{\infty} \phi^j w_{t-j}\right)$$

$$= \sum_{j=1}^{\infty} \phi^{2j} Var(w_{t-j})$$

$$= \sum_{j=1}^{\infty} \phi^{2j} \sigma_w^2 = \sigma_w^2 \underbrace{\sum_{j=1}^{\infty} \phi^{2j}}$$

$$\sigma_w^2 \cdot \left(\frac{-\phi^2}{1-\phi^2}\right)$$

$$\boxed{= \frac{-\phi^2 \sigma_w^2}{1-\phi^2}}$$

$$\left(\begin{aligned} S &= a + \cancel{a^2} + \cancel{a^3} + \dots a^n \\ aS &= \cancel{a^2} + \cancel{a^3} + a^4 + \dots + a^{n+1} \\ aS - S &= a - a^{n+1} \\ S(a-1) &= a - a^{n+1} \end{aligned}\right)$$

$$S = \frac{a - a^{n+1}}{a-1} = \frac{a^{n+1} - a}{1-a} = \frac{a(a^n - 1)}{1-a}$$

Si $|a| < 1$ $\lim_{n\to\infty} a^n = 0$

$$\lim_{n\to\infty} \frac{a(\cancel{a^n}^{\to 0} - 1)}{1-a} = \frac{-a}{1-a}$$

$$\rho_k = \phi^k \Rightarrow \rho_1 = \underbrace{\phi}$$

$$Corr(X_t, X_{t-1}) = \hat{\rho} \approx \hat{\phi}$$

$$\hookrightarrow \frac{\sum (X_t - \bar{X})(X_{t-1} - \bar{X})}{\sqrt{\sigma(X_t)} \cdot \sqrt{\sigma(X_{t-1})}}$$

$$\hat{\sigma}_w^2 =$$

Recordemos que

$$\gamma_0 = -\underbrace{\sigma_w^2}\left(\underbrace{\frac{\phi^2}{1-\phi^2}}\right) \qquad \{\hat{\phi}; \hat{\sigma}_w^2\}$$

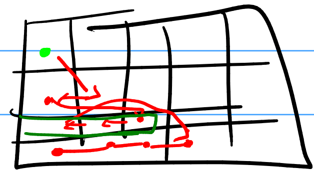
$$\hookrightarrow \frac{1}{n}\sum (X_t - \bar{X})^2$$

Proceso centrado en cero

$$\left(\begin{aligned} &\text{Si } \mu_t = \mu \neq 0 \Rightarrow Y_t = X_t - \mu \\ &\mathbb{E}[X_t] = \mathbb{E}[\cancel{Y_t}^{\to 0} + \mu] = \underline{\mu} \\ &\hat{\mu} = \bar{X} \end{aligned}\right)$$

PACF para AR(1)

$$x_t = \phi x_{t-1} + w_t \quad , \quad w_t \sim WN(0,\sigma_w^2)$$



$$\begin{cases} \hat{\phi}_{k+1,k+1} = \dfrac{\hat{\rho}_{k+1} - \sum_{j=1}^{k} \hat{\phi}_{k,j}\,\hat{\rho}_{k+1-j}}{1 - \sum_{j=1}^{k} \hat{\phi}_{k,j}\,\hat{\rho}_j} \\ \hat{\phi}_{k+1,j} = \hat{\phi}_{k,j} - \hat{\phi}_{k+1,k+1}\,\hat{\phi}_{k,k+1-j} \quad , \; j = 1, \dots, k \end{cases}$$

$$\hat{\phi}_{1,1} = \rho_1$$

$$\rho_k = \phi^k$$

$$\phi_{1,1} = \rho_1$$

$$\phi_{2,2} = \frac{\rho_2 - \sum_{j=1}^{k=1} \hat{\phi}_{1,j}\,\rho_{2-j}}{1 - \sum_{j=1}^{k=1} \hat{\phi}_{1,j}\,\rho_j} = \frac{\hat{\phi}_*^2 - \hat{\phi}_{1,1}\rho_1}{1 - \hat{\phi}_{1,1}\rho_1}$$

$$= \frac{\hat{\phi}_*^2 - \rho_1\rho_1}{1 - \hat{\phi}_{1,1}\rho_1} = \frac{\cancelto{0}{\hat{\phi}^2 - \hat{\phi}\hat{\phi}}}{1 - \hat{\phi}^2} = 0$$

¿Será que $\phi_{k,k} = 0 \;\; \forall\, k > 1, \; k \in \mathbb{N}$? $\rightsquigarrow$ $\underbrace{\text{SI!! :)}}$

$$\underbrace{AR(p) \rightsquigarrow \phi_{k,k} = 0 \;\; \forall\, k > p\,!}$$

$$\hat{\phi}_{2,1} = \hat{\phi}_{1,1} - \cancelto{0}{\hat{\phi}_{2,2}}\,\hat{\phi}_{1,1} \Rightarrow \underbrace{\hat{\phi}_{1,1}}$$

$K = 1$ ✓ $\quad \phi_{1,1} = \rho$

$K = 2 \quad \phi_{2,2} = 0$

Supongamos que $\phi_{K,K} = 0$ para alguna K. (HI)

$\underbrace{\phi_{K+1,K+1}}$ ⇝ P.D. $\phi_{K+1,K+1} = 0$

Sea $R = K - 1$

$$\phi_{R+1,j} = \phi_{R,j} - \phi_{R+1,R+1}\,\hat{\phi}_{R,R+1-j}$$

$$= \phi_{K-1,j} - \cancel{\tilde{\phi}_{K,K}}^{\,\to 0}\,\tilde{\phi}_{K-1,K-j}$$

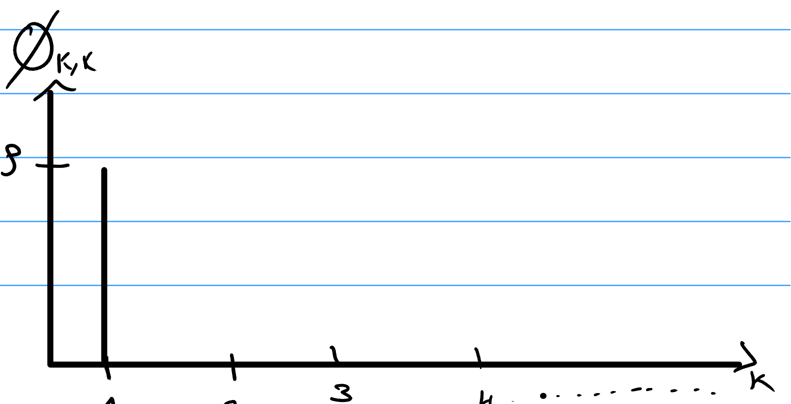
$$\Rightarrow \phi_{K,j} = \phi_{K-1,j} = \phi_{K-2,j} = \phi_{K-3,j} = \dots = \phi_{1,j}$$

$$\phi_{K+1,K+1} = \frac{\hat{\rho}_{K+1} - \sum_{j=1}^{K} \hat{\phi}_{K,j}\,\rho_{K+1-j}}{\underbrace{1 - \sum_{j=1}^{K} \hat{\phi}_{K,j}\,\rho_j}_{A}} = \frac{\hat{\rho}_{K+1} - \hat{\phi}_{K,1}\rho_K - \hat{\phi}_{K,2}\rho_{K-1} - \hat{\phi}_{K,3}\rho_{K-2} \cdots \phi_{K,K}\rho_1}{A}$$

$$= \frac{\hat{\rho}_{K+1} - \hat{\phi}_{1,1}\rho_K - \cancel{\hat{\phi}_{2,2}}^{\,\to 0}\rho_{K-1} - \cancel{\hat{\phi}_{3,3}}^{\,\to 0}\rho_{K-3} \cdots \cancel{\phi_{K,K}}^{\,0}\,\rho_1^{K-1}}{A}$$

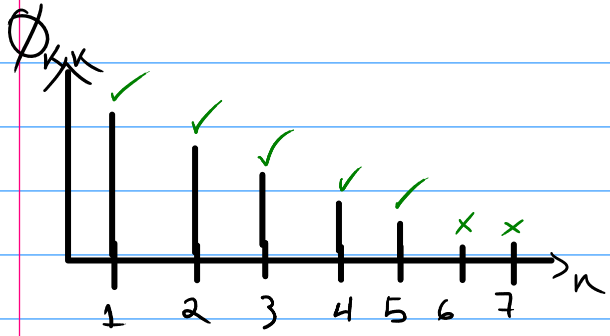
$$= \frac{\phi^{K+1} - \phi\,\phi^{K}}{A} = \frac{\phi^{K+1} - \phi^{K+1}}{A} = 0$$

$$\phi_{K,K} \begin{cases} \phi & K = 1 \\ 0 & \text{e.o.c.} \end{cases}$$



Sea X_t una serie de tiempo tal que $\phi_{K,K} = \hat{\rho}$, para $K=1$, 0 e.o.c. $\longrightarrow$ Un modelo AR(1) es adecuado para X_t.

AR(p)



Un modelo adecuado sería un AR(5)

Modelo de medias móviles de órden 1
~ MA(1)

$$Z_t = W_t - \Theta W_{t-1} = (1-\Theta B) W_t \,; \quad W_t \sim WN(0, \sigma_w^2)$$

¿Z_t es estacionario?

$$\mathbb{E}[Z_t] = \mathbb{E}[W_t] - \mathbb{E}[\Theta W_{t-1}] = 0$$

$$\gamma_k = \mathbb{E}[Z_t Z_{t-k}]$$

- Si $k=0 \Rightarrow \gamma_k = \gamma_0 = \mathbb{E}[Z_t^2] = \mathbb{E}[W_t^2 - \cancel{2\Theta W_t W_{t-1}}^{\,0} + \Theta^2 W_{t-1}^2]$
  $= \sigma_w^2 + \Theta^2 \sigma_w^2 = \boxed{(1+\Theta^2)\sigma_w^2}$

- Si $k=1 \Rightarrow \mathbb{E}[Z_t Z_{t-1}] = \mathbb{E}[(W_t - \Theta W_{t-1})(W_{t-1} - \Theta W_{t-2})]$
  $= \mathbb{E}[\cancel{W_t W_{t-1}}^{\,0} - \Theta W_{t-1}^2 - \Theta \cancel{W_t W_{t-2}}^{\,0} + \Theta^2 \cancel{W_{t-1} W_{t-2}}^{\,0}]$
  $= \boxed{-\Theta \sigma_w^2}$

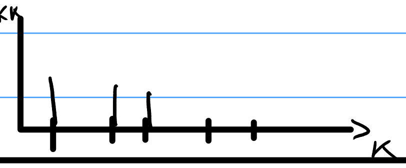
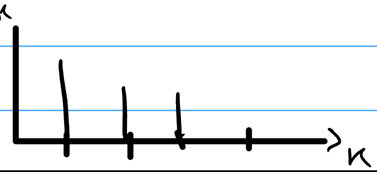
$$\gamma_k = \begin{cases} (1+\Theta^2)\sigma_w^2 & k=0 \\ -\Theta \sigma_w^2 & k=1 \\ 0 & \text{e.o.c.} \end{cases}$$

- $k>1$ $\mathbb{E}[Z_t Z_{t-k}] = \mathbb{E}[\cancel{W_t W_{t-k}}^{\,0} - \Theta \cancel{W_t W_{t-k-1}}^{\,0} - \Theta \cancel{W_{t-1} W_{t-k}} + \Theta^2 \cancel{W_{t-1} W_{t-k-1}}^{\,0}]$
  $= 0$

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{\gamma_k}{(1+\theta^2)\sigma_w^2} = \begin{cases} 1 & k=0 \\ \dfrac{-\theta\cancel{\sigma_w^2}}{(1+\theta^2)\cancel{\sigma_w^2}} & k=1 \\ 0 & e.o.c. \end{cases}$$

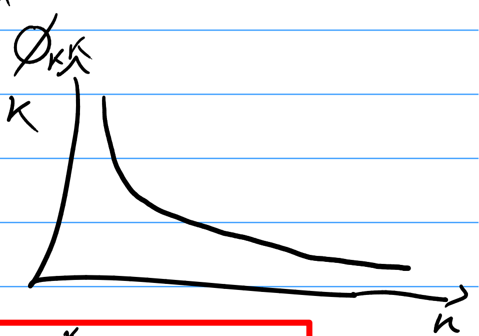
Si Z_t es un proceso MA(1), su función de autocorrelación, solo dependerá de $k=1$, para $k>1$, $\rho_k = 0$

Si X_t una serie de tiempo tal que $\rho_k = 0$ para $k \geq 1$ => el modelo adecuado será un MA(1)

AR(P)	MA(q)
→ PACF	→ ACF
ϕ_{kk} 	ρ_k 

PACF de MA(1)

$$\boxed{\phi_{k,k} = \frac{-\theta^k(1-\theta^2)}{1-\theta^{2(k+1)}}} \underset{\text{Proporcional}}{\propto} -\theta^k$$



$$\boxed{\rho_k = \begin{cases} \dfrac{-\theta}{1+\theta^2} & k=1 \\ 0 & k \geq 1 \end{cases}}$$

$$\phi_{k+1,k+1} = \frac{\rho_{k+1} - \sum_{j=1}^{k} \phi_{kj}\rho_{k+1-j}}{1 - \sum_{j=1}^{k} \phi_{kj}\rho_j}$$

$$\phi_{k+1,j} = \phi_{kj} - \hat{\phi}_{k+1,k+1}\hat{\phi}_{k,k+1-j}$$

Procesos Autoregresivos de orden 2 (AR(2))

$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \omega_t$; $\omega_t \sim WN(0, \sigma_w^2)$

$\to X_t = \phi_1 B X_t + \phi_2 B^2 X_t + \omega_t$

$\to \omega_t = (1 - \phi_1 B - \phi_2 B^2) X_t$

$\to \psi^{(2)}(B) \to$ Brockwell & Davis

ITSA :)

ETSA :(

$\to \psi^{(n)}(B) \to P_2$

→ Propiedades de los procesos AR(2)

· El Proceso AR(2) siempre es <u>invertible</u>.

(AR(1) siempre es invertible, cuando $|\phi| < 1$
$X_t = \sum_{j=1}^{\infty} w_{t-j}$)

· ¿Cuándo se cumple que el proceso AR(2) es estacionario? (Para el AR(1) $\rho_k = \phi^k$)

Para que el proceso AR(2) sea estacionario, las raíces del polinomio $\psi(B) = 1 - \phi_1 B - \phi_2 B^2$ deben de estar afuera del círculo unitario.

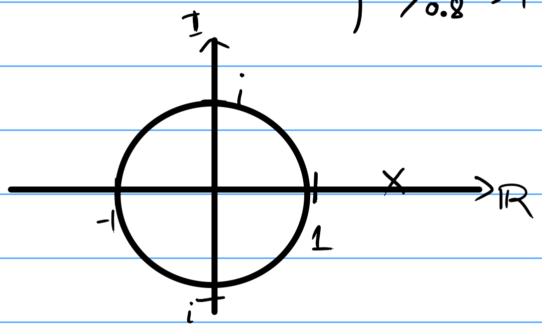
Por ejemplo:

$$\underbrace{(1 - 1.5B + 0.5(B^2)}_{\psi(B)} X_t = w_t$$

$$\psi(B) = (1 - 0.7B)(1 - 0.8B) = 0$$

$$\hookrightarrow B = \begin{cases} 1/0.7 > 1 \\ 1/0.8 > 1 \end{cases}$$

Podemos concluir que el proceso asociado a $\psi(B)$ es estacionario.



$\emptyset$

La condición de estacionariedad del modelo AR(2) puede expresarse en términos de los valores que tome B_1 y B_2.

$$X_t = 3X_{t-2} + w_t \Rightarrow w_t = (1-3B^2)X_t$$

$$\underset{\text{Lag}}{1-3B^2=0 \Leftrightarrow B^2 = \frac{1}{3}} \Leftrightarrow \left\{\begin{array}{l} B_1 = \frac{1}{\sqrt{3}} > 1 \\ B_2 = -\frac{1}{\sqrt{3}} > 1 \end{array}\right\} \text{Estacionario}$$

Ejemplos procesos AR(2)

$$X_t = 2X_{t-1} - X_{t-2} + w_t$$

$$w_t = \underbrace{(1-2B+B^2)}_{\psi(B) = 1-2B+B^2 = (1-B)^2 = 0 \Leftrightarrow \left\{\begin{array}{l} B=1 \\ B=-1 \end{array}\right\}} X_t \Leftrightarrow \quad \text{No es estacionario}$$

Procesos ARMA

$$w_t = \frac{\psi(B)}{\theta(B)} X_t$$

$$X_t = 5X_{t-5} + w_t \to AR(5)$$

$$\boxed{AR(p) \Rightarrow X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \dots + \phi_p X_{t-p} + w_t}$$

$$\hookrightarrow \psi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p$$

$$|B_i| > 1$$

$$X_t = 0.2X_{t-2} + w_t$$

$$\psi(B) = 1 - 0.2B^2 = 0 \Leftrightarrow B \left\{\begin{array}{l} B_1 = \frac{1}{\sqrt{0.2}} > 1 \\ B_2 = \frac{-1}{\sqrt{0.2}} > 1 \end{array}\right.$$

$$(1-\phi_1 B-\phi_2 B^2)=0 \iff \phi_1 B+\phi_2 B^2-1=0$$

$$\iff B_1=\frac{-\phi_1+\sqrt{\phi_1^2+4\phi_2}}{2\phi_2}$$

$$B_2=\frac{-\phi_1-\sqrt{\phi_1^2+4\phi_2}}{2\phi_2}$$

$$1/B_1=\frac{\phi_1+\sqrt{\phi_1^2+4\phi_2}}{2}\ ;\ 1/B_2=\frac{\phi_1-\sqrt{\phi_1^2+4\phi_2}}{2}$$

$$|B_i|>1 \Rightarrow |1/B_i|<1 \quad i=1,2$$

$$|1/B_1\cdot 1/B_2|=|\phi_2|<1 \rightsquigarrow \phi_2\in(-1,1)$$

$$\left(\frac{\theta_1+\sqrt{\phi_1^2+4\phi_2}}{2}\right)\left(\frac{\phi_1-\sqrt{\phi_1^2+4\phi_2}}{2}\right)=\frac{\phi_1^2-\phi_1^2-4\phi_2}{4}=-\phi_2$$

$$B_1=\frac{-\phi_1+\sqrt{\phi_1^2+4\phi_2}}{2\phi_2}\ ;\ \frac{1}{B_1}=\frac{2\phi_2}{-\phi_1+\sqrt{\phi_1^2+4\phi_2}}$$

$$=\frac{2\phi_2}{-\phi_1+\sqrt{\phi_1^2+4\phi_2}}\left(\frac{\phi_1+\sqrt{\phi_1^2+4\phi_2}}{\phi_1+\sqrt{\phi_1^2+4\phi_2}}\right)\rightsquigarrow (a+b)(a-b)=a^2-b^2$$

$$=\frac{2\phi_2\left(\phi_1+\sqrt{\phi_1^2+4\phi_2}\right)}{\cancel{\phi_1^2}+4\phi_2-\cancel{\phi_1^2}}=\frac{\phi_1+\sqrt{\phi_1^2+4\phi_2}}{2}=\frac{1}{B_1}$$

$$\left|\frac{1}{B_1}+\frac{1}{B_2}\right| = \frac{\phi_1+\cancel{\sqrt{\phi_1^2+4\phi_2}}}{2} + \frac{\phi_1-\cancel{\sqrt{\phi_1^2+4\phi_2}}}{2}$$

$$= \frac{2\phi_1+0}{2} = |\phi_1|$$

$$|\phi_1| = \left|\frac{1}{B_1}+\frac{1}{B_2}\right| \leq \left|\frac{1}{B_1}\right| + \left|\frac{1}{B_2}\right| \leq 1+1 = 2$$

$$|\phi_1| < 2$$

Concluyendo: Para que un proceso AR(2) sea estacionario, se tiene que cumplir lo siguiente:

$$\boxed{\begin{array}{c} -2 \leq \phi_1 \leq 2 \\ -1 \leq \phi_2 \leq 1 \end{array}}$$

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + w_t \;; \quad w_t \sim WN(0,\sigma^2)$$

ACF (ρ_K) de un proceso AR(2)

$$\gamma_K = Cov(X_t, X_{t-K}) = \mathbb{E}[X_t X_{t-K}] =$$

$$= \mathbb{E}[X_t X_{t-K}] = \phi_1 \mathbb{E}[X_{t-1} X_{t-K}] + \phi_2 \mathbb{E}[X_{t-2} X_{t-K}] + \mathbb{E}[w_t X_{t-K}]$$

$$\Rightarrow \frac{\gamma_K}{\gamma_0} = \phi_1 \frac{\gamma_{K-1}}{\gamma_0} + \phi_2 \frac{\gamma_{K-2}}{\gamma_0} \rightarrow \underbrace{\rho_K = \phi_1 \rho_{K-1} + \phi_2 \rho_{K-2}}_{\boxed{\text{Ecuaciones en diferencias}}} \; K \geq 1$$

↳ Ecuaciones diferenciales

$$\rho_K = \phi_1 \rho_{K-1} + \phi_2 \rho_{K-2} \;, \quad \rho_0 = \frac{\gamma_0}{\gamma_0} = 1$$

(1)
$$\begin{cases} \rho_1 = \phi_1 + \phi_2 \rho_1 \\ \rho_2 = \phi_1 \rho_1 + \phi_2 \, 1 \end{cases} \rightarrow \boxed{\rho_1 = \frac{\phi_1}{1-\phi_2}}$$

$$\boxed{\rho_2 = \phi_1 \left(\frac{\phi_1}{1-\phi_2}\right) + \phi_2 = \frac{\phi_1^2 + \phi_2(1-\phi_2)}{1-\phi_2}}$$

$$\left.\begin{array}{l} \rho_3 = \phi_1 \rho_2 + \phi_2 \rho_1 \\ \rho_4 = \phi_1 \rho_3 + \phi_1 \rho_2 \end{array}\right\} \rho_K \, ??? \ldots$$

PACF de un proceso AR(2)

$$\rho_K = \phi_1 \rho_{K-1} + \phi_2 \rho_{K-2}$$

$$\phi_{K+1,K+1} = \frac{\rho_{K+1} - \sum_{j=1}^{K} \phi_{Kj} \rho_{K+1-j}}{1 - \sum_{j=1}^{K} \phi_{Kj} \rho_j}$$

$$\phi_{K+1\,j} = \phi_{Kj} - \phi_{K+1\,K+1} \phi_{K,\,K+1-j}$$

$$\underline{\phi_{1,1}} = \rho_1 = \underset{\sim\sim\sim}{\frac{\phi_1}{1-\phi_2}} \Big\}$$

$$\underline{\phi_{2,2}} = \frac{\rho_2 - \phi_{1,1}\rho_1}{1 - \phi_{1,1}\rho_1} = \frac{\rho_2 - \rho_1^2}{1-\rho_1^2} = \frac{\phi_1\rho_1 + \phi_2 - \rho_1^2}{1-\rho_1^2}$$

$$= \frac{\phi_1\left(\frac{\phi_1}{1-\phi_2}\right) + \phi_2 - \left(\frac{\phi_1}{1-\phi_2}\right)^2}{1 - \left(\frac{\phi_1}{1-\phi_2}\right)^2}$$

$$= \frac{\frac{\phi_1^2 + \phi_2 - \phi_2^2}{1-\phi_2} - \left(\frac{\phi_1}{1-\phi_2}\right)^2}{1 - \left(\frac{\phi_1}{1-\phi_2}\right)^2}$$

$$= \frac{\phi_2\left[(1-\phi_2)^2 - \phi_1^2\right]}{(1-\phi_2)^2 - \phi_1^2} = \underset{\sim\sim\sim}{\phi_2} \Big\}$$

$$\phi_{1,2} = \phi_{1,1} - \phi_{2,2}\, \phi_{1,1}$$

$$= \frac{\phi_1}{1-\phi_2} - \phi_2 \cdot \frac{\phi_1}{1-\phi_2} = \frac{\phi_1}{1-\phi_2} \cdot (1-\phi_2) = \phi_1 \}$$

$$\phi_{3,3} = \frac{\rho_3 - \sum_{j=1}^{2} \phi_{2j}\, \rho_{3-j}}{1 - \sum_{j=1}^{2} \phi_{2j}\, \rho_j}$$

$$= \frac{\rho_3 - \phi_{2,1}\rho_2 - \phi_{2,2}\rho_1}{1 - \phi_{2,1}\rho_1 - \phi_{2,2}\rho_2}$$

$$= \frac{\cancel{\phi_1 \rho_2} + \cancel{\phi_2 \rho_1} - \cancel{\phi_1 \rho_2} - \cancel{\phi_2 \rho_1}}{1 - \phi_{2,1}\rho_1 - \phi_{2,2}\rho_2} = 0$$

Proposición

Si nostros tenemos un proceso $AR(p)$

$\Rightarrow \phi_{KK} = 0 \; ; \quad K > P$