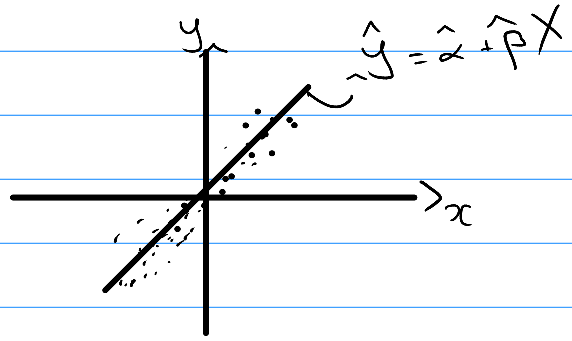
Ruido Blanco $\{w_t\}_{t=1}^{\infty}$ $\mathbb{E}[w_t]=0$, $Var(w_t)=\sigma_w^2$

$Cov(w_t, w_{t+h})=0 \iff h\neq 0$

Modelo de regresión lineal:

$$y=\alpha+\beta X+w_t \quad ; \quad w_t\sim N(0,\sigma^2)$$

$$\hookrightarrow \hat{y}=\hat{\alpha}+\hat{\beta}X$$



- → Proceso estacionario de orden 1
- ⋮
- → Proceso estacionario de orden n

Proceso fuertemente estacionarios

· Orden 1: Sea $\{X_t\}_{t=1}^{\infty}$ un proceso estocástico, decimos que X_t es estacionario de orden 1 si $\mathbb{P}[X_{t_1}\leq x]=\mathbb{P}[X_{t_2}\leq x]=\dots=\mathbb{P}[X_{t_n}\leq x]$ para una colección de v.a's $\{X_{t_1}, X_{t_2},\dots, X_{t_n}\}$

· Orden 2: si $\mathbb{P}[X_{t_i}\leq x_1, X_{t_j}\leq x_2]=\mathbb{P}[X_{t_k}\leq x_1, X_{t_l}\leq x_2]$

⋮

Orden n: → → → → → → →

→ Fuertemente estacionario $\iff$ Estacionario de orden n $\forall n\geq 1$

Ejemplo: Función de autocovarianza de una media móvil.

$\mathbb{E}[w_t]=0$ / $Cov(w_t, w_{t+h})=0$

$Var(w_t)=\sigma_w^2$

$\rightarrow$ Sea $\{w_t\}_{t=1}^{\infty}$ un ruido blanco,

Definamos a $v_t = \frac{1}{3}(w_{t-1}+w_t+w_{t+1})$

$\gamma_v(s,t) = Cov(v_s, v_t) = Cov\left(\frac{1}{3}[w_{t-1}+w_t+w_{t+1}], \frac{1}{3}[w_{s-1}+w_s+w_{s+1}]\right)$

$= \frac{1}{9} Cov(w_{t-1}+w_t+w_{t+1}, w_{s-1}+w_s+w_{s+1})$

$= \frac{1}{9}\Big[\underbrace{\mathbb{E}[(w_{t-1}+w_t+w_{t+1})(w_{s-1}+w_s+w_{s+1})]}_{*} - \cancel{\mathbb{E}[w_{t-1}+w_t+w_{t+1}]}^{0}\,\mathbb{E}[w_{s-1}+w_s+w_{s+1}]\Big]$

$s=t$

$Var(w_{t-1}+w_t+w_{t+1})$

$* = \frac{1}{9}\mathbb{E}[(w_{t-1}+w_t+w_{t+1})(w_{t-1}+w_t+w_{t+1})] = \frac{1}{9}\mathbb{E}[(w_{t-1}+w_t+w_{t+1})^2]$

$= \frac{3}{9}\sigma_w^2$

$\rightarrow w_t$; $\mathbb{E}[w_t]=0$ $Var(w_t)=\sigma_w^2$ $\forall t \in R$

$Var(w_{t-1}+w_t+w_{t+1}) = \mathbb{E}[(w_{t-1}+w_t+w_{t+1})^2] - \cancel{\mathbb{E}^2[w_{t-1}+w_t+w_{t+1}]}^{0}$

$= \mathbb{E}\Big[w_{t-1}(\underbrace{w_{t-1}}_{\sigma_w^2}+\underbrace{\cancel{w_t}}_{0}+\underbrace{\cancel{w_{t+1}}}_{0}) + w_t(\underbrace{\cancel{w_{t-1}}}_{0}+\underbrace{w_t}_{\sigma_w^2}+\underbrace{\cancel{w_{t+1}}}_{0})$

$+ w_{t+1}(\underbrace{\cancel{w_{t-1}}}_{0}+\underbrace{\cancel{w_t}}_{0}+\underbrace{w_{t+1}}_{\sigma_w^2})\Big] = \sigma_w^2+\sigma_w^2+\sigma_w^2 = 3\sigma_w^2$

$\gamma(t,s) = \gamma(s,t)$; $\underbrace{\gamma(t,t+h)}_{\gamma_h} = \underbrace{\gamma(t,t-h)}_{\gamma_{-h}}$

$s = t+1$; $s = t = -1$; $|s-t| = \underline{1}$

$\gamma_{\upsilon}(t,t+1) = Cov(\upsilon_t, \upsilon_{t+1})$

$= \frac{1}{9} Cov\left(\frac{1}{3}(w_{t-1} + w_t + w_{t+1}), \frac{1}{3}(w_t + w_{t+1} + \cancel{w_{t+2}})\right)$ σ_w^2 σ_w^2

$= \frac{1}{9} \cdot 2\sigma_w^2 = \frac{2}{9}\sigma_w^2$

$|s-t| = 2$; $s = t+2$, $s = t-2$

$\gamma_{\upsilon}(t,t+2) = \frac{1}{9} Cov(\cancel{w_{t-1}} + \cancel{w_t} + \underbrace{w_{t+1}, w_{t+1}}_{\sigma_w^2} + \cancel{w_{t+2}} + \cancel{w_{t+3}}) = \frac{1}{9}\sigma_w^2$

$|s-t| > 2$, $s = t+h$, $h > 2$

$\gamma_{\upsilon}(t,t+h) = \frac{1}{9}(Cov(\cancel{w_{t-1}} + \cancel{w_t} + \cancel{w_{t+1}}, w_{t+h-1} + w_{t+h} + w_{t+h+1}) = 0$

$$\gamma_{\upsilon}(t,s) = \begin{cases} \frac{3}{9}\sigma_w^2 & s = t \rightsquigarrow h = 0 \\ \frac{2}{9}\sigma_w^2 & |s-t| = 1 \rightarrow |h| = 1 \\ \frac{1}{9}\sigma_w^2 & |s-t| = 2 \rightarrow |h| = 2 \\ 0 & e.o.c. \rightsquigarrow |h| > 2 \end{cases}$$

Debilmente estacionario
$E[\upsilon_t] = 0$
$E[|\upsilon_t|] < \infty$
$\gamma_h = f(h)$

$s = t+h$; $h \in R$

↳ ¿Es débilmente estacionario $\upsilon_t \overset{?}{=} \frac{1}{3}(w_{t-1} + w_t + w_{t+1})$

$E[\upsilon_t] = 0$ $E[|\upsilon_t|] < E[\upsilon_t^2] < \infty$

↳ $Var(\upsilon_t) < \infty \rightsquigarrow Var(\upsilon_t) = \frac{3}{9}\sigma_w^2 < \infty$

$\Rightarrow$ El proceso υ_t es débilmente estacionario

$s = t+h \quad ; \quad h \in \mathbb{R}$

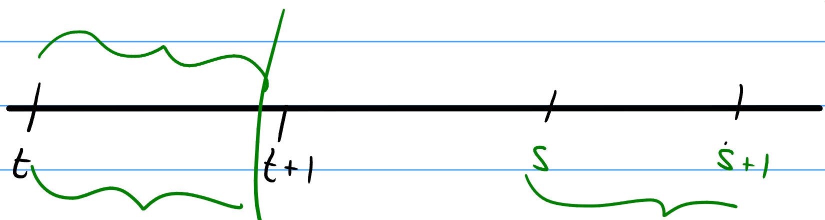
$$\gamma_h = \begin{cases} \frac{3}{9}\sigma_w^2 & |h| = 0; \\ \frac{2}{9}\sigma_w^2 & |h| = 1; \\ \frac{1}{9}\sigma_w^2 & |h| = 2; \\ 0 & |h| > 2; \end{cases}$$

En cualquier caso,

$\gamma_h = \gamma(t, t+h)$ sólo depende de h

$\Rightarrow$ El proceso v_t es débilmente estacionario.

$$Cov(X_t, X_{t+1}) = Cov(X_s, X_{s+1})$$



$$\neq Cov(X_s, X_{s+2})$$

-> Función de autocovarianza para una caminata aleatoria

$$X_t = \sum_{i=1}^{t} w_i \; ; \quad w_i \text{ es un ruido blanco}$$

$$w_i \sim WN(0, \sigma_w^2)$$

$$\gamma_x(t,s) = Cov(X_t, X_s) = Cov\left(\sum_{i=1}^{t} w_i, \sum_{j=1}^{s} w_j\right)$$

Si $s > t \Rightarrow s = t+h \; ; \; h > 0 \quad , \quad t, h, s \in \mathbb{N}$

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