Z_t un proceso estacionario

$\gamma_t = \mathbb{E}[Z_t Z_{t+k}] = \mathbb{E}[Z_t Z_{t+k}] = f(k) \Rightarrow$ Estacionario

$\hookrightarrow \mathbb{E}[Z_t] = 0$

$\mathbb{E}[Z_t^2] < \infty$

$Y_t = X \; ; \quad X \sim \text{Cauchy}(\theta) \; ; \quad \mathbb{E}[X^2] = \infty \Rightarrow$

$K = 0 \; ; \quad \gamma_t^{(k)} = \mathbb{E}[Z_t Z_{t+k}] = \mathbb{E}[Z_t \cdot Z_t] = \mathbb{E}[Z_t^2] = Var(Z_t)$

$\hookrightarrow \gamma_t^{(0)} = \gamma_0 = Var(Z_t) = \sigma^2$

Propiedades de la función de autocovarianza y la función de autocorrelación; para procesos estacionarios

1) $\gamma_0 = Var(Z_t) \; ; \quad \rho_0 = 1 = \rho_t^{(0)} \hookrightarrow \rho(Z_t, Z_t) = 1$

2) $|\gamma_k| \leq \gamma_0 \; ; \quad |\rho_k| \leq 1 \quad |\rho(X,Y)| \leq 1$

$X, Y \; ; \quad \underbrace{Cov(X,Y) \leq \sqrt{Var(X)}\sqrt{Var(Y)}}_{\text{Desigualdad de Cauchy-Schwartz}}$

$\underbrace{Cov(Z_t, Z_{t+k})}_{\text{Cauchy Schwartz}} \leq \underbrace{\sqrt{Var(Z_t)}}_{\sigma = \sqrt{\gamma_0}} \cdot \underbrace{\sqrt{Var(Z_{t+k})}}_{\sigma = \sqrt{\gamma_0}}$

Para un proceso estacionario, $Var(Z_t) = \gamma_0$

$\to f(K)$

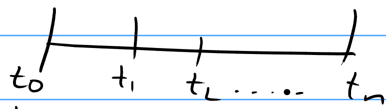
• $\gamma_t^{(K)} = \gamma_t^{(-K)}$ $\forall\, t \in \mathbb{R}$, me voy a tomar $t' = t-K$

$\hookrightarrow \mathbb{E}[Z_t \cdot Z_{t+K}] = \mathbb{E}[Z_{t'} \cdot Z_{t'+K}] = \mathbb{E}[Z_{t-K} \cdot Z_{t-\cancel{K}+\cancel{K}}]$

$= \mathbb{E}[Z_t \cdot Z_{t-K}] = \gamma_t^{(-K)}$

$\mathbb{E}[Z_t \cdot Z_{t+K}] = \gamma_t^{(K)}$; Como Z_t es un proceso estacionario

$\Rightarrow \tilde{\gamma}_t^{(K)} = \gamma_T^{(K)} = \gamma_{t_2}^{(K)}$



$t_1 \le t_2 \le t_3 \le \dots \le t_n$; $t_i \in \mathbb{R}$

$\gamma_{t_1}^{(K)} = \gamma_{t_2}^{(K)} \dots = \gamma_{t_n}^{(K)}$

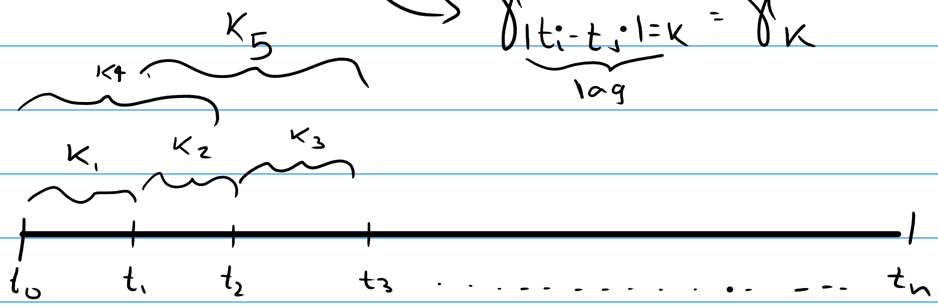
$\to \rho_K = \rho_{-K}$; $\rho_K = \dfrac{\gamma_K}{\underset{\gamma_0}{\sqrt{\sigma_t^2}} \cdot \underset{\gamma_0}{\sqrt{\sigma_t^2}}} = \boxed{\dfrac{\gamma_K}{\gamma_0}} = \dfrac{\gamma_{-K}}{\gamma_0} = \rho_{-K}$

$\rho_K = \dfrac{\gamma_K}{\gamma_0}$; $\gamma_0 = Var(Z_t) = \sigma^2 \in \mathbb{R}$ $\quad \sigma^2 > 0$

→ γ_k, ρ_k ; Son semidefinidas positivas

$$\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \alpha_j \gamma_{|t_i - t_j|} \geq 0$$

$$\gamma_{\underbrace{|t_i - t_j|}_{lag} = k} = \gamma_k$$



$$X = \sum_{i=1}^{n} \alpha_i Z_{t_i}$$

$$Var(X) = \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \alpha_j Cov(Z_{t_i}, Z_{t_j})$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \alpha_j \gamma_{|t_i - t_j|}$$

$$\gamma_{t_i - t_j} = \gamma_{t_j - t_i}$$

$$\gamma_k = \gamma_{-k}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \alpha_j \rho_{|t_i - t_j|} \geq 0$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \alpha_j \rho_{|t_i - t_j|} = \sum_{i}\sum_{j} \alpha_i \alpha_j \frac{\gamma_{|t_i - t_j|}}{\underbrace{\gamma_0}_{\geq 0}} \geq 0$$

Ruido Blanco (WN)

-> Sea Z_t un ruido blanco, el cual definimos como $Z_t = WN$; $\mathbb{E}[WN] = 0$, $Var(WN) = \sigma^2$

$$Z_t \perp Z_s \quad \forall\, t \neq s \in \mathbb{R} \rightarrow Z_t \perp Z_{t+k} \quad \forall\, k \in \mathbb{R}$$

$$\gamma_t^{(k)} = \begin{cases} 0 & k \neq 0 \\ 1 & k = 0 \end{cases}$$

Normalmente, $WN \sim N(0, 1)$ ✓

$$\begin{cases} WN \sim t_{(\nu)} \\ WN \sim Err(\alpha, \beta) \end{cases}$$

-> PACF; AR, MA

AR → autorregresivos

MA → medias móviles

-> Time Series Analysis
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