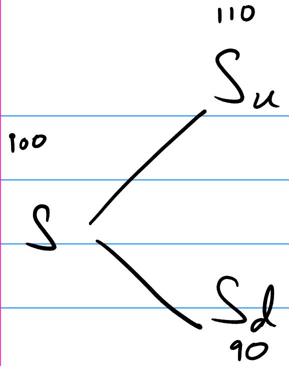
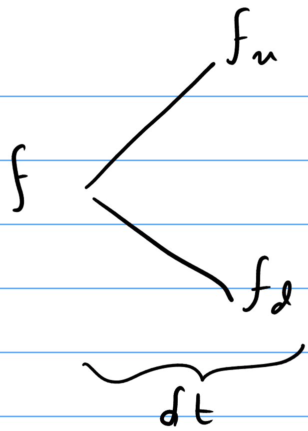
Valoración en tiempo discreto





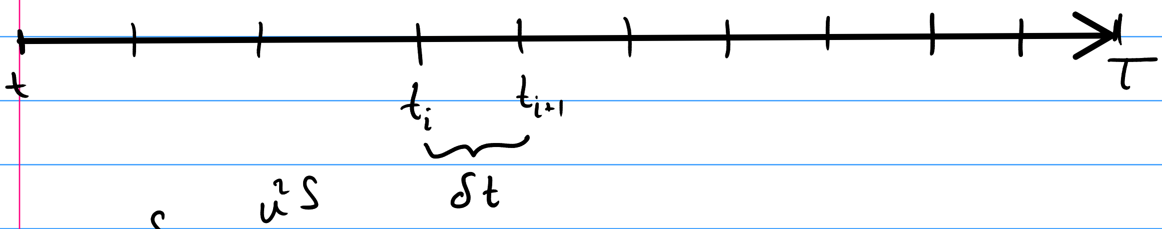
$$B \longrightarrow B e^{r\delta t}$$

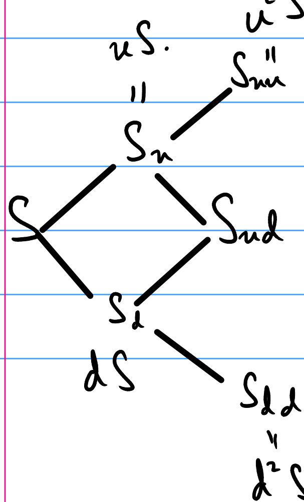
$$\phi S_u + \psi B e^{r\delta t} = f_u$$
$$\phi S_d + \psi B e^{r\delta t} = f_d$$

$$\phi = \frac{f_u - f_d}{S_u - S_d} \qquad \psi = B^{-1} e^{-r\delta t}\left(f_u - \left(\frac{f_u - f_d}{S_u - S_d}\right) S_u\right)$$

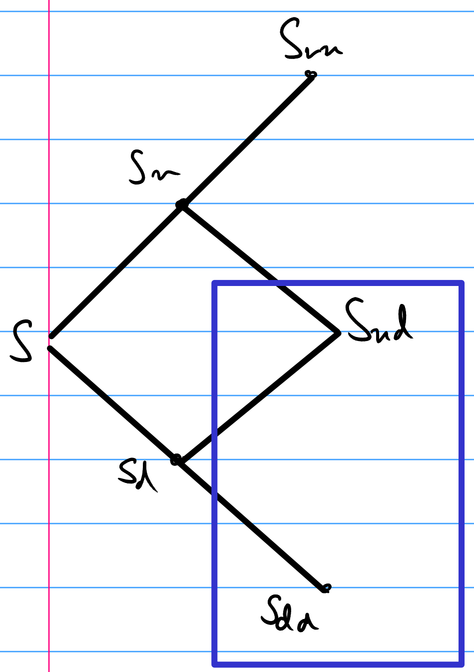
Definiendo $\quad q = \dfrac{S e^{r\delta t} - S_d}{S_u - S_d} \qquad q \in (0,1)$

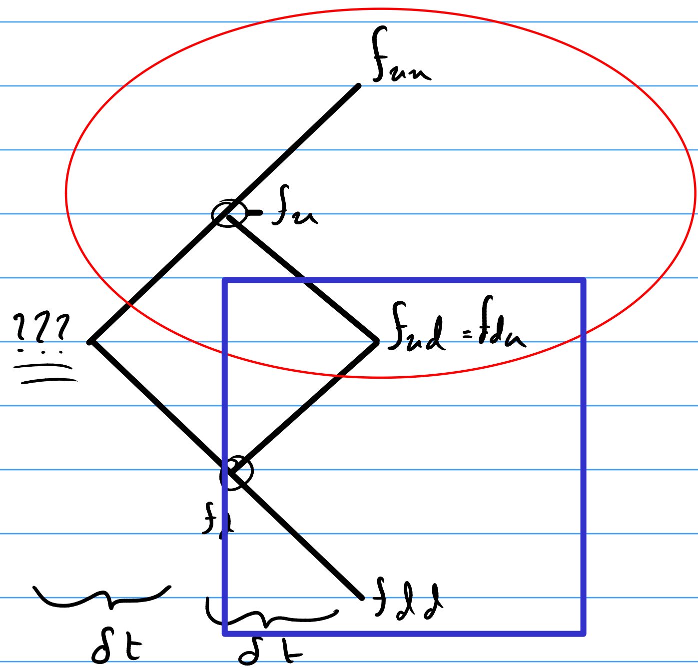
$$V = f = e^{-r\delta t}\left(q f_u + (1-q) f_d\right) \rightsquigarrow e^{-r\delta t}\, \mathbb{E}_t^*[f_T]$$





$S_{ud} = S_{du} = S \iff u = \dfrac{1}{d}$ $\Bigg\}$ árbol multiplicativo que recombine valores





Recordamos que:

$$q = \frac{S e^{r\delta t} - S_d}{S_u - S_d} \quad , \quad q = \frac{S e^{r\delta t} - S_{ud}}{S_{uu} - S_{ud}}$$

$$= \frac{S e^{r\delta t} - S \cdot u d}{S u^2 - S u \cdot d} = \frac{e^{r\delta t} - ud}{u^2 - ud}$$

$$= \frac{e^{r\delta t} - 1}{u^2 - 1}$$

$$f_u = \left[q f_{uu} + (1-q) f_{ud} \right] e^{-r\delta t}$$

$$\phi = \frac{f_{uu} - f_{ud}}{S_{uu} - S_{ud}}$$ # acciones en mi cobertura

$$q = \frac{S e^{r\delta t} - S_{dd}}{S_{ud} - S_{dd}}$$

$$f_d = [\, q f_{ud} + (1-q) f_{dd} \,] e^{-r\delta t}$$

$$\phi = \frac{f_{ud} - f_{dd}}{S_{ud} - S_{dd}}$$

$$f = [\, q f_u + (1-q) f_d \,] e^{-r\delta t}$$

$$q = \frac{S e^{r\delta t} - S_d}{S_u - S_d}$$

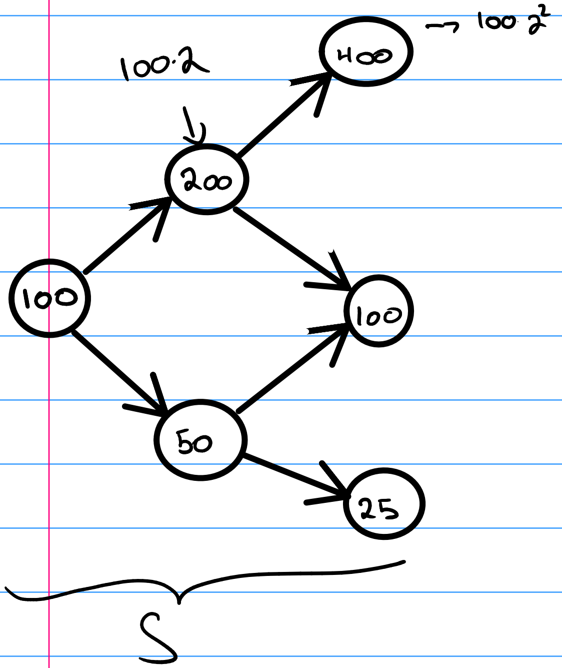
$$\phi = \frac{f_u - f_d}{S_u - S_d}$$

Resumen :

$$q = \frac{S_{now} e^{r\delta t} - S_{down}}{S_{up} - S_{down}} \qquad \phi = \frac{f_{up} - f_{down}}{S_{up} - S_{down}}$$

$$f_{now} = e^{-r\delta t} [\, q f_{up} + (1-q) f_{down} \,]$$

$$\psi = B_{now}^{-1} (f_{up} - \phi S_{now})$$



$$u = e^{\sigma\sqrt{\delta t}} \qquad d = e^{-\sigma\sqrt{\delta t}}$$

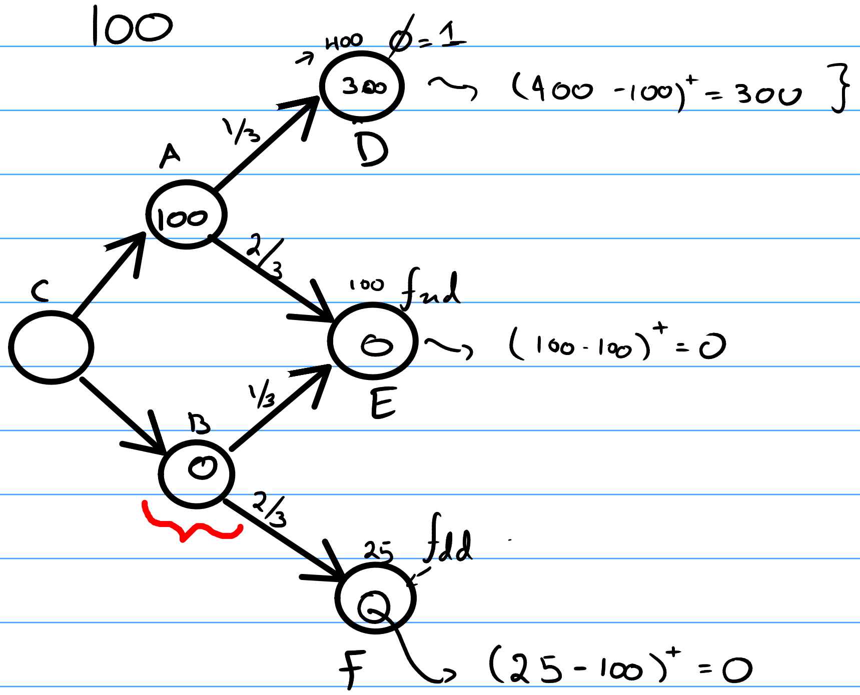
$$u = 2 \qquad d = \frac{1}{2}$$

$$\delta t = \frac{1}{12}$$

$$r = 0.0$$

$(S_T - 100)^+$

Considerar una opción call con strike de 100



Para el nodo A

$$q = \frac{S e^{0\delta t} - 100}{400 - 100} = \frac{200 - 100}{400 - 100} = \frac{100}{300} = \frac{1}{3}$$

$$f_A = e^{0\delta t}\left[\tfrac{1}{3}300 + \tfrac{2}{3}(0)\right] = 100$$

$$\phi = \frac{300 - 0}{400 - 100} = \frac{300}{300} = 1$$

$$\psi = B^{-1}\left[f_D - \phi\, 200\right] = \underbrace{B^{-1}}_{1} \cdot \left[300 - 200\right] = +100$$

$$f_{now} = \phi S_{now} + \psi B = 1 * 200 + (+100) = 300$$

$$\begin{cases} \phi S_u + \psi B \cancel{e^{r\delta t}} = f_u \\ \phi S_d + \psi B \cancel{e^{r\delta t}} = f_d \end{cases}$$

$$\phi(S_u - S_d) = f_u - f_d$$

$$\boxed{\phi = \frac{f_u - f_d}{S_u - S_d}}$$

$$\boxed{\psi = B^{-1}\left[f_u - \phi S_u\right]}$$

Para el nodo B

$$q = \frac{Se^{0\delta t} - S_{dd}}{S_{ud} - S_{dd}} = \frac{50 - 25}{100 - 25} = \frac{25}{75} = \frac{1}{3}$$

$$f_B = \left[\frac{1}{3} \cdot 0 + \frac{2}{3} \cdot 0\right] e^{-\delta t} = 0$$

$$\phi = \frac{f_{ud} - f_{dd}}{S_{ud} - S_{dd}} = \frac{0 - 0}{S_{ud} - S_{dd}} = 0$$

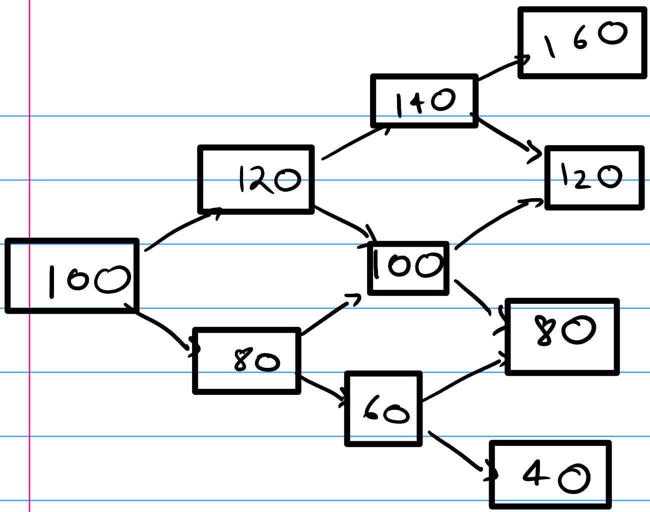
$$\psi = B^{-1}\left[0 - \phi S_{d0}\right] = 0$$

Para el nodo C

$$q = \frac{Se^{r\delta t} - S_d}{S_u - S_d} = \frac{100 - 50}{200 - 50} = \frac{50}{150} = \frac{1}{3}$$

$$f = e^{-0\delta t}\left[100\left(\frac{1}{3}\right) + 0\left(\frac{2}{3}\right)\right] = \frac{100}{3} = 33.\overline{333}$$

$$\phi = \frac{100 - 0}{200 - 50} = \frac{100}{150} = \frac{10}{15} = \frac{2}{3}$$



Consideremos que tenemos un call con strike $K=100$

$dt = \frac{1}{12}$

$r = 0$

→ Determinar el árbol de f