# Brownian Motion.

**Q1.** Let $\beta(\cdot)$ be a one dimensional Brownian motion and $\tau_1$ the hitting time of level 1. Show that

$$E_0 \int_0^{\tau_1} 1_{\{0 \le \beta(s) \le 1\}}\, ds = 1.$$

**Q2.** Let $H$ be a hyperplane in $\mathbb{R}^d$ (without loss of generality you may assume that $H = \{x_1 = 0\}$. Why ?) and $\beta(\cdot)$ a $d$-dimensional Brownian motion. For $z \in \mathbb{R}^d$ show that

$$\sup_{t>0} E_z\big[|\beta(t)|\, 1_{\{t<\tau_H\}}\big] < \infty,$$

where $\tau_H$ is the hitting time of the hyperplane $H$.

**Q3.** Do exercise 3.17 from the book of Mörters and Peres.