Numerical Methods

PDEs II

1 Non Matlab exercises

1. Check that after these coordinate changes

$$\begin{aligned} S &= E\exp(x) \\ t &= T - 2\tau/\sigma^2 \\ V &= Eu(x,\tau) \end{aligned}$$

   the Black–Scholes equation becomes

$$u_\tau = u_{xx} + (k-1)u_x - ku,$$

   where $k = 2r/\sigma^2$ and the initial condition becomes

$$u(x,0) = max(e^x - 1, 0)$$

2. Furthermore, check that this equation can be transformed to the heat equation by the substitution

$$u = \exp(\alpha x + \beta\tau)w(x,\tau),$$

   where $\alpha = -\frac{1}{2}(k-1)$ and $\beta = -\frac{1}{4}(k+1)^2$.

3. Check that the discretization of a parabolic equation with variable coefficients as discussed in Lecture notes ("Partial Differential equations II", page 11,12) produces the same truncation error, i.e. $O(\Delta t) + O(\Delta x^2)$ for the explicit/implicit Euler scheme and $O(\Delta t^2) + O(\Delta x^2)$ for the Crank-Nicolson scheme.

2 Matlab exercises

1. Implement the Crank-Nicolson scheme for the European Call option valuation problem.

2. Implement the Crank-Nicolson scheme for the American Put option valuation problem.