Numerical Methods

Exercises week 4

1 Non Matlab exercises

1. In the proof of the convergence of the Euler scheme we needed the following statement: if U_n satisfies the difference equation

$$U_n = (1 + K_1\Delta t)U_{n-1} + M\Delta t^2,\ U_0 = 0$$

then

$$U_n = ((1 + K_1\Delta t)^n - 1)\,\frac{M}{K_1}\Delta t.$$

Prove this.

2. Check that if the function f is independent of x, then the forth order Runge-Kutta scheme for the equation $\dot{x} = f(t)$ is reduced to Simpsons quadrature rule.

2 Matlab exercises

1. Consider a system of ODE describing a pendulum:

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= -x_1 \end{aligned}$$

   (a) Solve the pendulum equation.
   (b) Define function `function v=fun(x)`, where v is the vector $[x(2), -x(1)]$ as in the pendulum equation.
   (c) Implement the Euler method.
   (d) Implement the Midpoint method.
   (e) Implement the Trapezoidal method.

(f) Fix some initial value x_0 and the time interval $[0, t_f]$. Take several values of Δt decreasing to zero and compute the error for each method. Plot the error as a function of Δt.

(g) Change m-file "fun.m" so it describes the Lorenz equation:

$$\begin{aligned} \frac{dy_1}{dt} &= \sigma(y_2 - y_1) \\ \frac{dy_2}{dt} &= (\tau - y_3)y_1 - y_2 \\ \frac{dy_3}{dt} &= y_1 y_2 - \beta y_3 \end{aligned} \tag{1}$$

where $\sigma = 10, \tau = 28, \beta = 8/3$.

Make a figure with a few solutions of the Lorentz equation in the phase space.