Numerical Methods

Weeks 9, 10 code

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function [V, variance] = mc_euro_call_imp(S, E, T, r, sigma, N)
% MC_EURO_CALL_IMP - Pricing of European call option by the Monte Carlo method
%                    with importance sampling
% S         - The underlying price
% E         - Strike price
% T         - Time to expiration
% r         - The interest rate
% sigma     - The volatility
% N         - Number of MC samples
% V         - The corresponding option price

eps0 = (log(E/S) - (r-0.5*sigma^2)*T)/(sigma * sqrt(T));
Ihat = 1.0 - normcdf(eps0);
epshat = norminv(1.0 - rand(N,1) * Ihat);

ST = S*exp((r-0.5*sigma^2)*T+sigma*sqrt(T)*epshat);
payoff = Ihat*exp(-r*T)*max(0.,ST-E);
V = mean(payoff);
variance = var(payoff);
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function [V, variance] = mc_euro_call_imp_str(S, E, T, r, sigma, N, NB)
% MC_EURO_CALL_IMP - Pricing of European call option by the Monte Carlo method
%                    with importance sampling + stratified sampling
% S         - The underlying price
% E         - Strike price
% T         - Time to expiration
% r         - The interest rate
% sigma     - The volatility
% N         - Number of MC samples
% NB        - Number of partition elements
% V         - The corresponding option price

eps0 = (log(E/S) - (r-0.5*sigma^2)*T)/(sigma * sqrt(T));
Ihat = 1.0 - normcdf(eps0);
du = Ihat / NB;
Y  = zeros(NB,1);
VY = zeros(NB,1);

for k=1:NB
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    epshat = norminv(1.-(k-1)*du - rand(N/NB,1) * du);
    ST = S*exp((r-0.5*sigma^2)*T+sigma*sqrt(T)*epshat);
    payoff = Ihat*exp(-r*T)*max(0.,ST-E);
    Y(k) = mean(payoff);
    VY(k) = var(payoff);
end

V = mean(Y);
variance = sum(VY)/NB;
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function [V, variance] = mc_euro_call_con(S, E, T, r, sigma, N)
% MC_EURO_CALL_CON - Pricing of European call option by the Monte Carlo method
%                    with control variate
% S         - The underlying price
% E         - Strike price
% T         - Time to expiration
% r         - The interest rate
% sigma     - The volatility
% N         - Number of MC samples
% V         - The corresponding option price

EST = S*exp(r*T);

x = randn(N,1);
ST = S*exp((r-0.5*sigma^2)*T+sigma*sqrt(T)*x);
payoff = exp(-r*T)*max(0.,ST-E);
c = -(payoff'*ST/N - mean(payoff)*EST) / (EST^2*(exp(sigma^2*T)-1.));
payoff_con = payoff + c*(ST - EST);
V = mean(payoff_con);
variance = var(payoff_con);
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function [V, variance] = mc_euro_call_delta_fd(S, E, T, r, sigma, N, dS)
% MC_EURO_CALL_DELTA_FD - Delta of European call option by the Monte Carlo
% method
% S         - The underlying price
% E         - Strike price
% T         - Time to expiration
% r         - The interest rate
% sigma     - The volatility
% N         - Number of MC samples
% V         - The corresponding option price

x = randn(N,1);
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ST1 = (S+dS)*exp((r-0.5*sigma^2)*T+sigma*sqrt(T)*x);
payoff1 = exp(-r*T)*max(0.,ST1-E);
ST2 = (S-dS)*exp((r-0.5*sigma^2)*T+sigma*sqrt(T)*x);
payoff2 = exp(-r*T)*max(0.,ST2-E);
delta = (payoff1-payoff2)/(2.*dS);
V = mean(delta);
variance = var(delta);
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function [V, variance] = mc_euro_call_delta_pd(S, E, T, r, sigma, N)
% MC_EURO_CALL_DELTA_PD - Delta of European call option by the Monte Carlo
% method
% S         - The underlying price
% E         - Strike price
% T         - Time to expiration
% r         - The interest rate
% sigma     - The volatility
% N         - Number of MC samples
% V         - The corresponding option price

x = randn(N,1);
ST = S*exp((r-0.5*sigma^2)*T+sigma*sqrt(T)*x);
delta = exp(-r*T)*heaviside(ST-E).*ST/S;
V = mean(delta);
variance = var(delta);
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function [V, variance] = mc_euro_call_delta_lr(S, E, T, r, sigma, N)
% MC_EURO_CALL_DELTA_LR - Delta of European call option by the Monte Carlo
% method
% S         - The underlying price
% E         - Strike price
% T         - Time to expiration
% r         - The interest rate
% sigma     - The volatility
% N         - Number of MC samples
% V         - The corresponding option price

x = randn(N,1);
ST = S*exp((r-0.5*sigma^2)*T+sigma*sqrt(T)*x);
delta = exp(-r*T)*max(0.,ST-E).*(log(ST/S)-(r-0.5*sigma^2)*T)/(S*T*sigma^2);
V = mean(delta);
variance = var(delta);
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