Applications of Principal Component Analysis in Fixed Income Portfolios

Methods in Statistical Finance STAT W4400 Project

Group 4

In order of Presentation

Linan Qiu <lq2137@columbia.edu>

Daniel O'Shaughnessy <djo2128@columbia.edu>

Akshat Sinha <as4724@columbia.edu>

Christophe Sabourin <christophe.sabourin1@gmail.com>

Max Mattioli <m.max@columbia.edu>

Project Outline

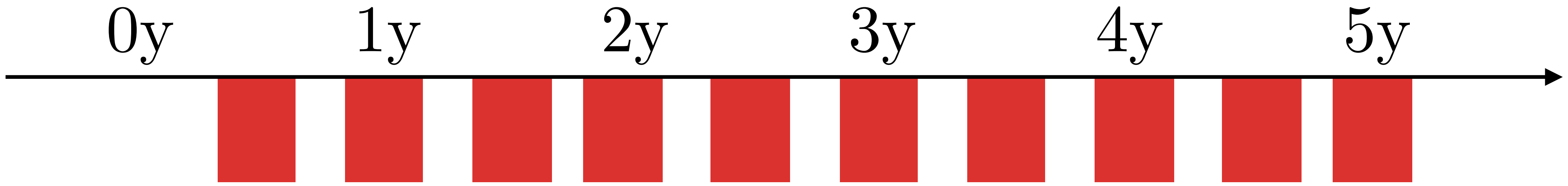
- Overview of applications of PCA in fixed income analysis
- Interest Rate Swap analysis
  - Overview of Interest Rate Swaps
  - PCA on Interest Rate Swaps as an alternative to duration analysis
- Fixed income portfolio risk modeling
  - Descriptive statistics for data
  - Fixed income portfolio risk model
  - Regression and results

Principal Component Analysis on Interest Rate Swaps

What are Interest Rate Swaps

- Over-the-counter agreement between two parties to exchange a fixed cash flow for a floating cashflow

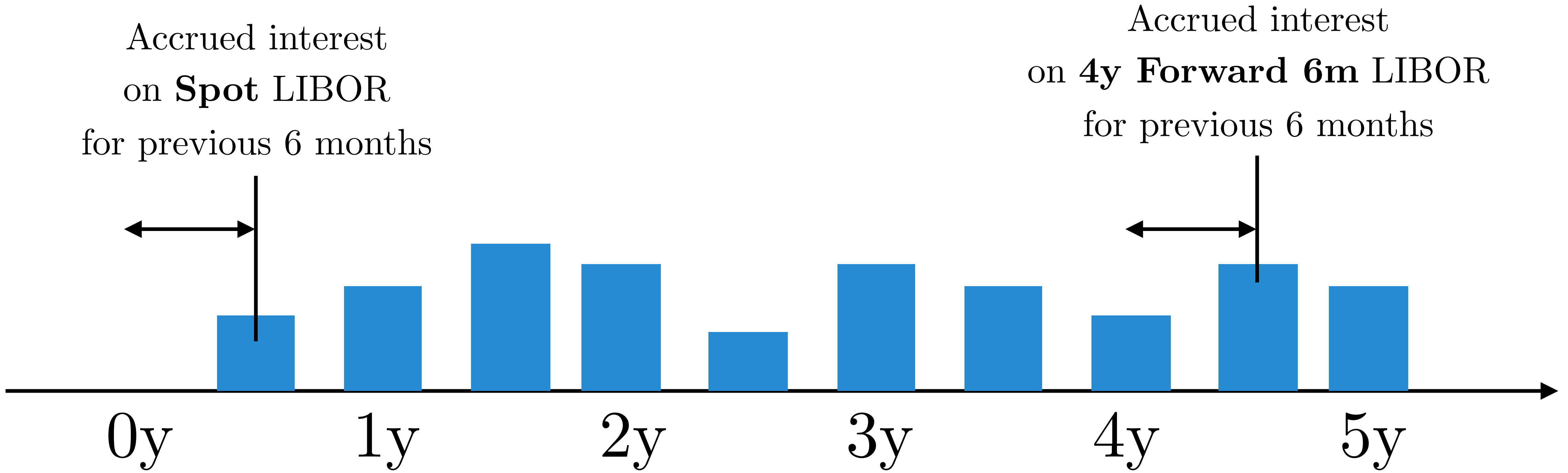
Pay Fixed



What are Interest Rate Swaps

- Over-the-counter agreement between two parties to exchange a fixed cash flow for a floating cashflow

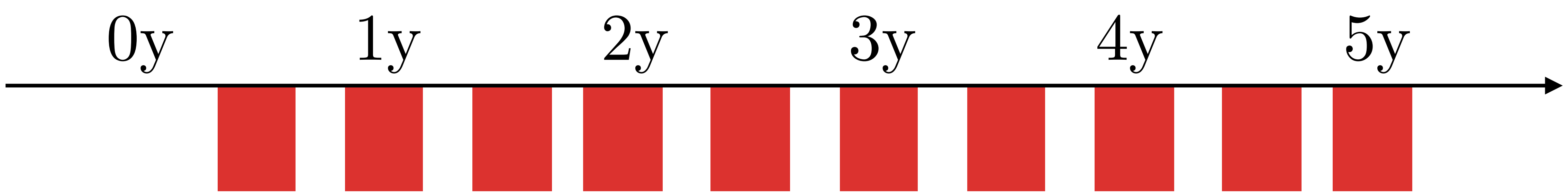
Receive Float



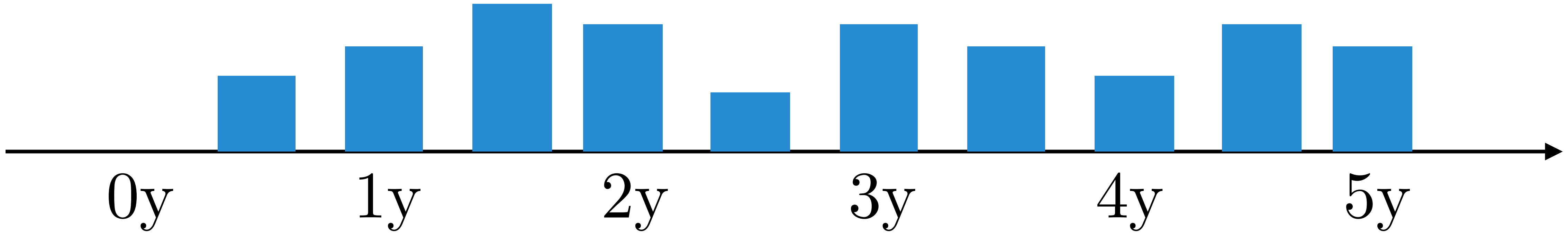
What are Interest Rate Swaps

- Over-the-counter agreement between two parties to exchange a fixed cash flow for a floating cashflow

Pay Fixed



Receive Float



What are Interest Rate Swaps

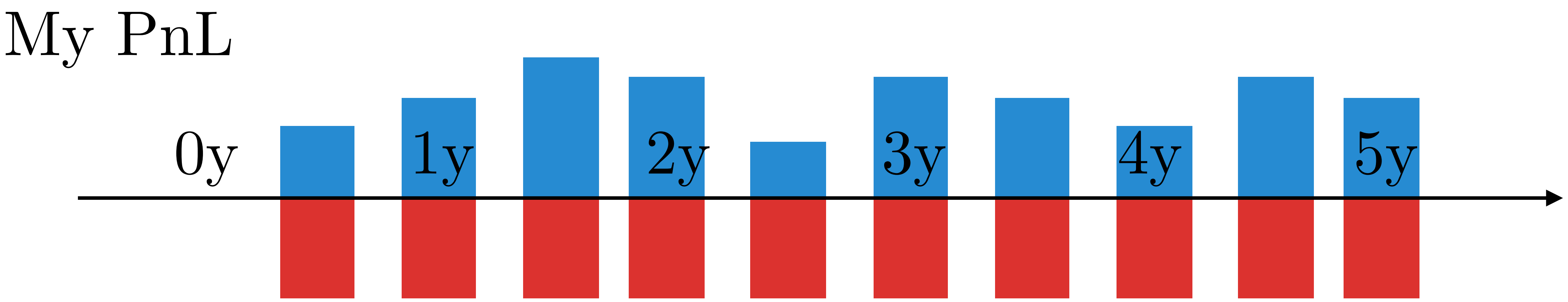
- Over-the-counter agreement between two parties to exchange a fixed cash flow for a floating cashflow

Time	Six Month LIBOR rate (%)	Floating Cash Flow Received	Fixed Cash Flow Paid	Net Cash Flow
0y	4.2%			
0.5y	4.8%	2.1	-2.5	-0.4
1y	5.3%	2.4	-2.5	-0.1
1.5y	5.5%	2.65	-2.5	0.15
2y	5.6%	2.75	-2.5	0.25
2.5y	5.9%	2.8	-2.5	0.3
3y	6%	2.95	-2.5	0.45
3.5y	6.1%	3	-2.5	0.5
4y	6.5%	3.05	-2.5	0.55
4.5y	7.1%	3.25	-2.5	0.75
5y	7.5%	3.55	-2.5	1.05

Citations: Hull, Options, Futures and Other Derivatives 7e

What are Interest Rate Swaps

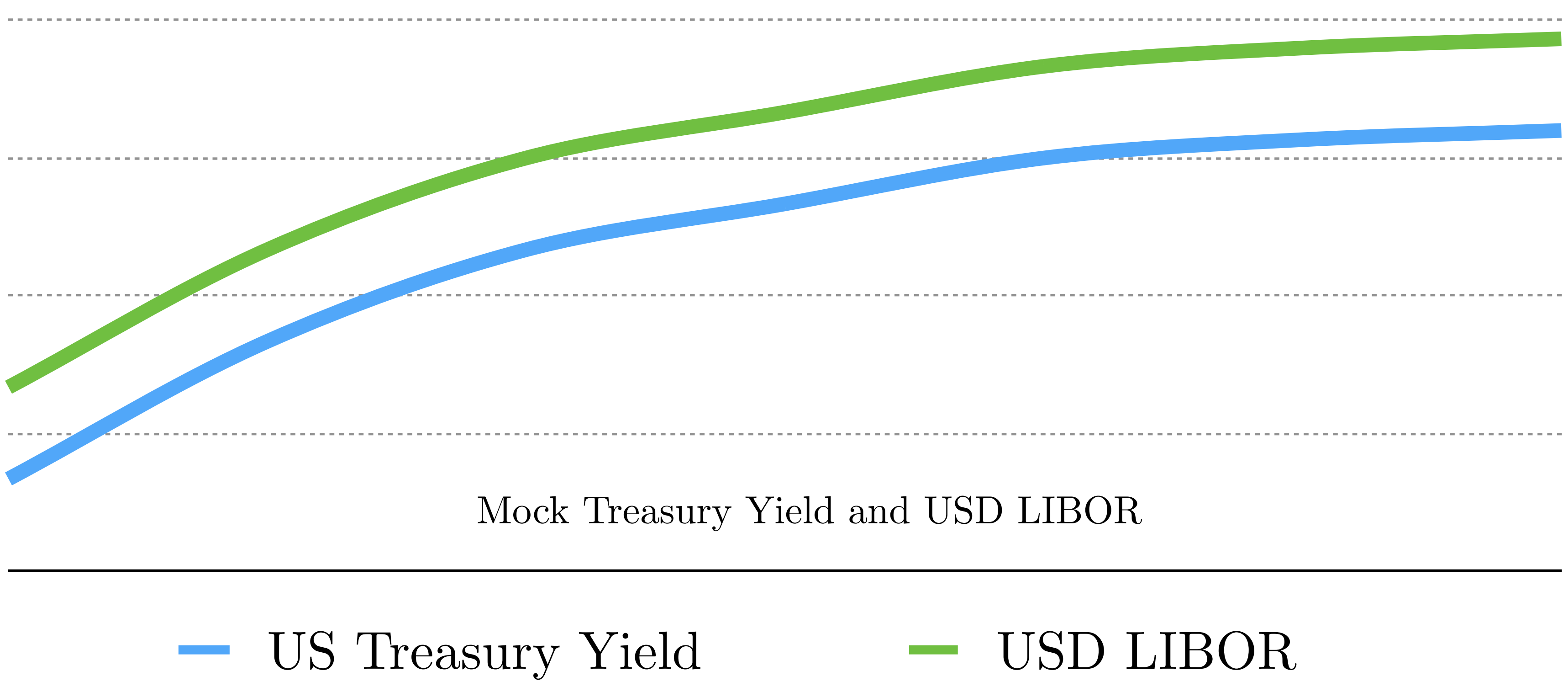
- Over-the-counter agreement between two parties to exchange a fixed cash flow for a floating cashflow



- The discount rate used for calculating the fixed leg is called the **swap rate**
- The swap rate makes the present value of the fixed leg equal to the present value of the floating leg
- This makes the entire contract value 0 (as swaps always are)

LIBOR and USD Interest Rate

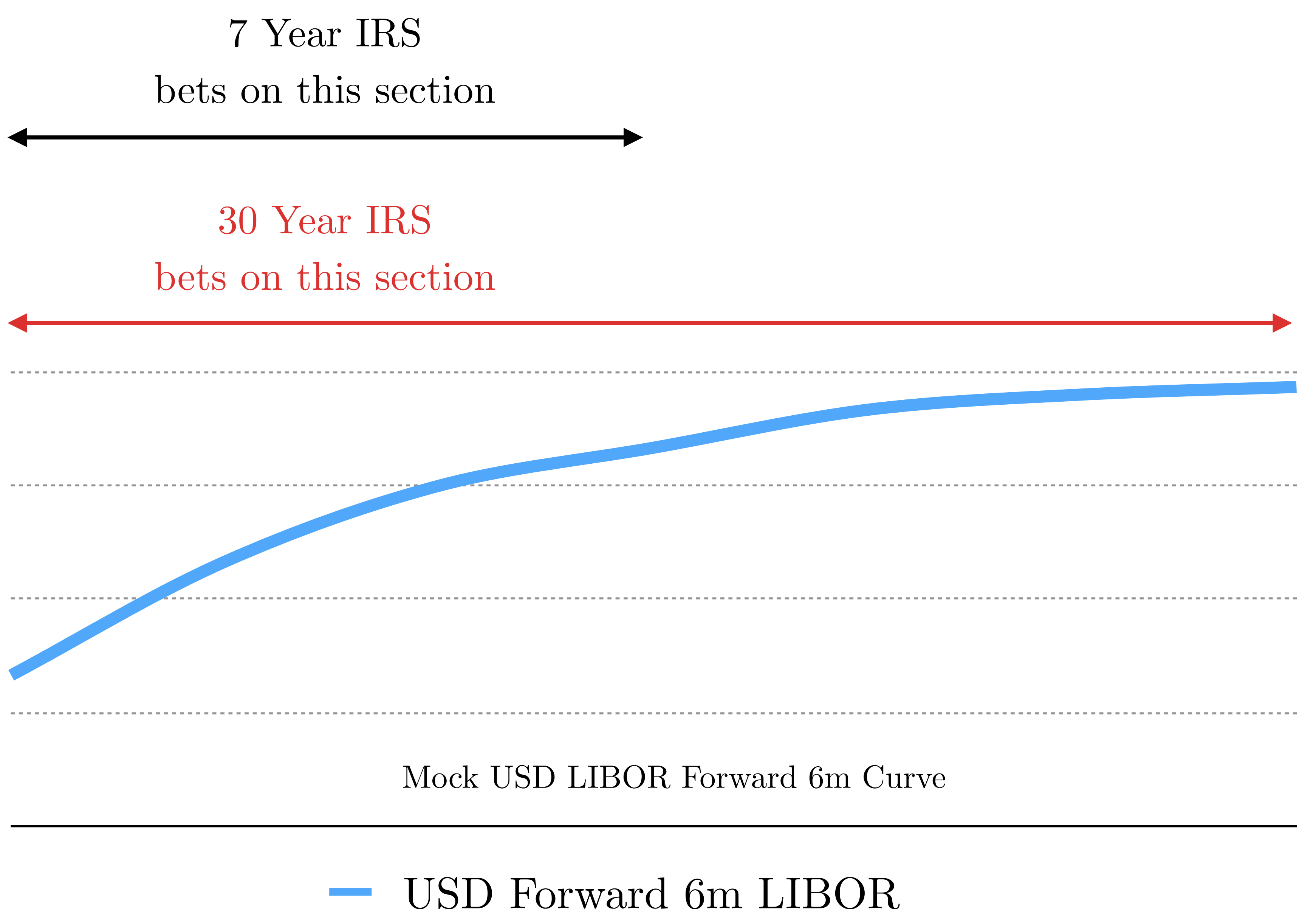
- The LIBOR is used as the reference rate for the floating legs in interest rate swaps.
- LIBOR is the rate that AA rated banks borrow from each other.
- Over different periods (ranging from say spot to 12 months) the USD LIBOR forward curve is usually above the Treasury Yield curve with mostly the same shape.
- Hence, the LIBOR forward curve is often used by speculators to speculate on the underlying treasury yield curve (or the ECB rate curve if EUR denominated IRS are used instead).



Citations: Hull, Options, Futures and Other Derivatives 7e

IRS and Speculation

- Entering into an IRS is equivalent to making a bet on a section of the forward curve.



Citations: Hull, Options, Futures and Other Derivatives 7e

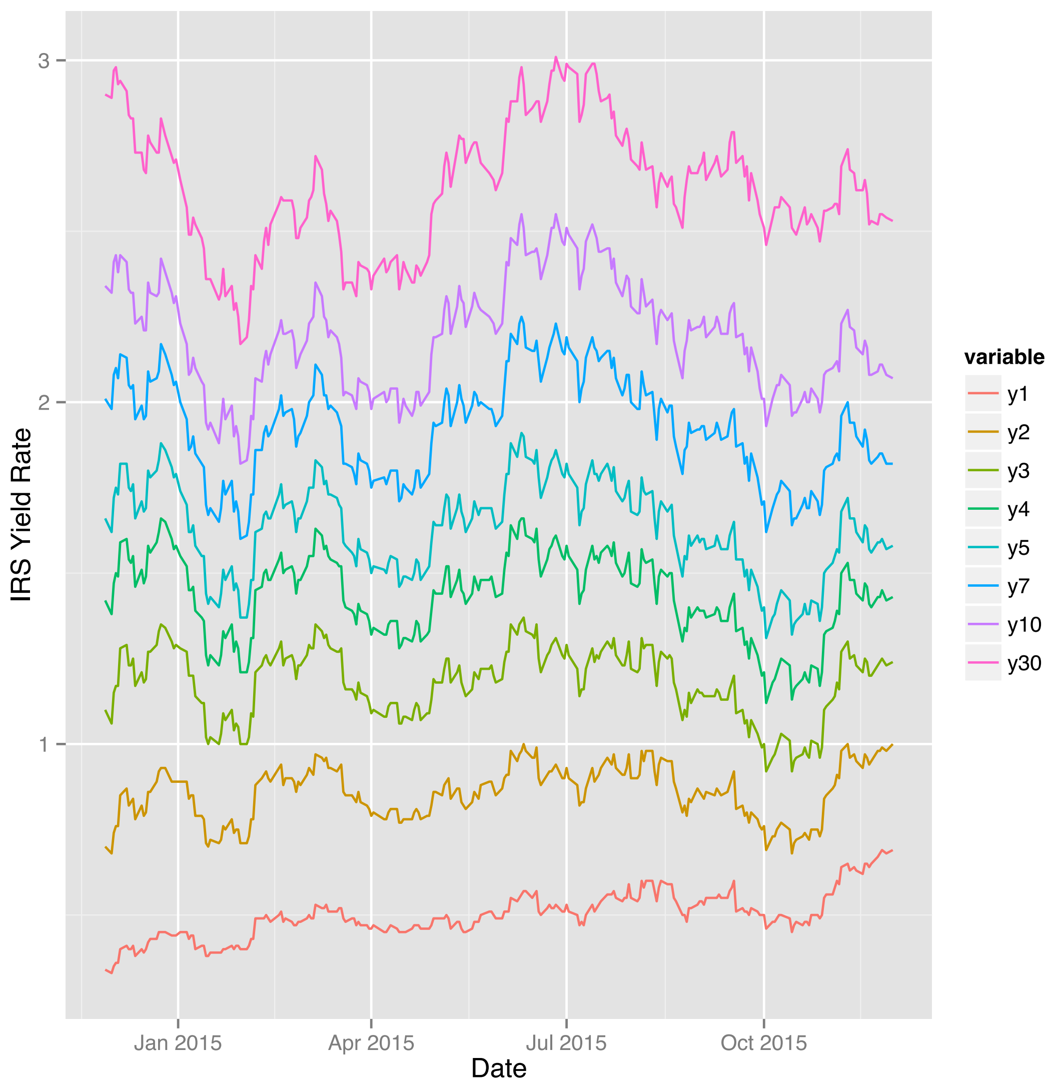
IRS and Speculation

- Entering into an IRS is equivalent to making a bet on a section of the forward curve.
- In terms of directions:
  - If I **pay fixed** (receive float), when the yield curve goes up, I profit (since I'm paying less than I would have).
  - If I **receive fixed** (pay float), when the yield curve goes up, I lose (since I'm paying more).

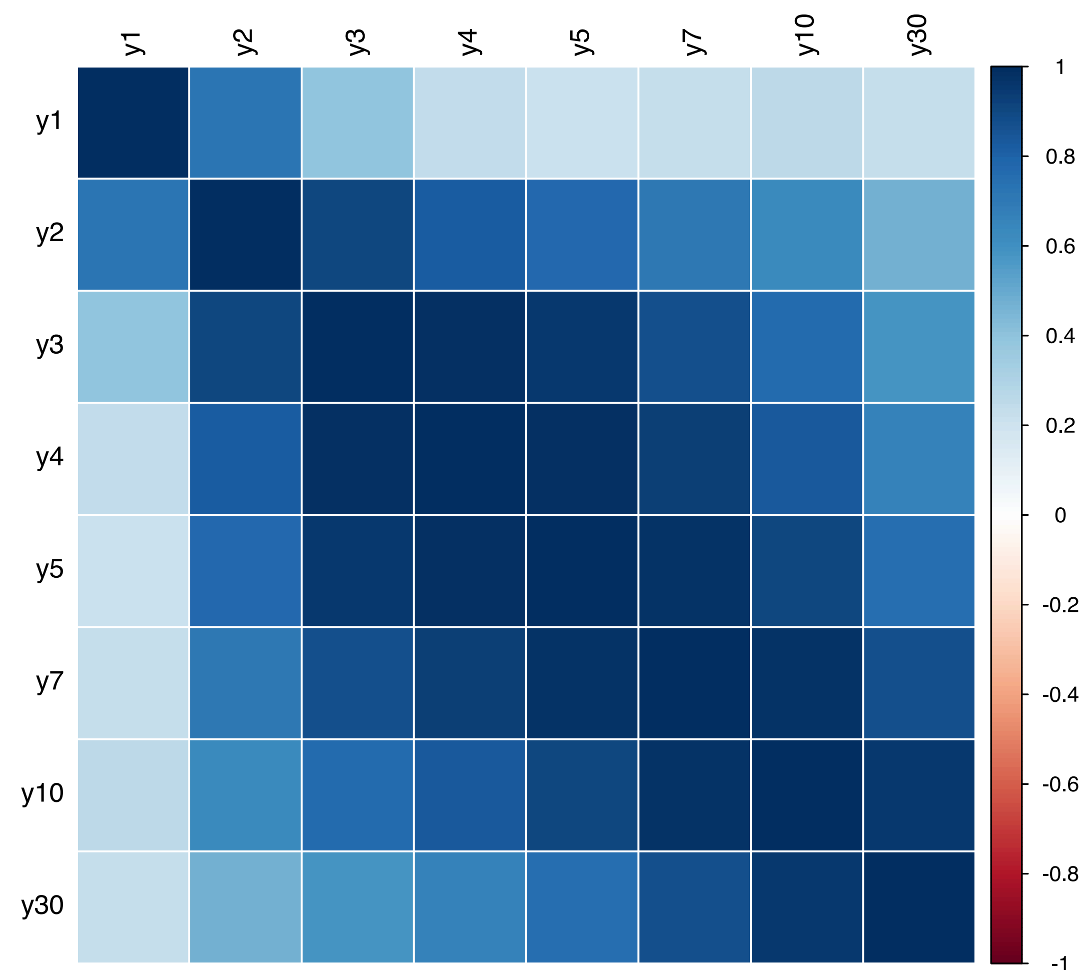
Citations: Hull, Options, Futures and Other Derivatives 7e

A. PCA on Vanilla IRS

Descriptive Data for IRS Rates Used



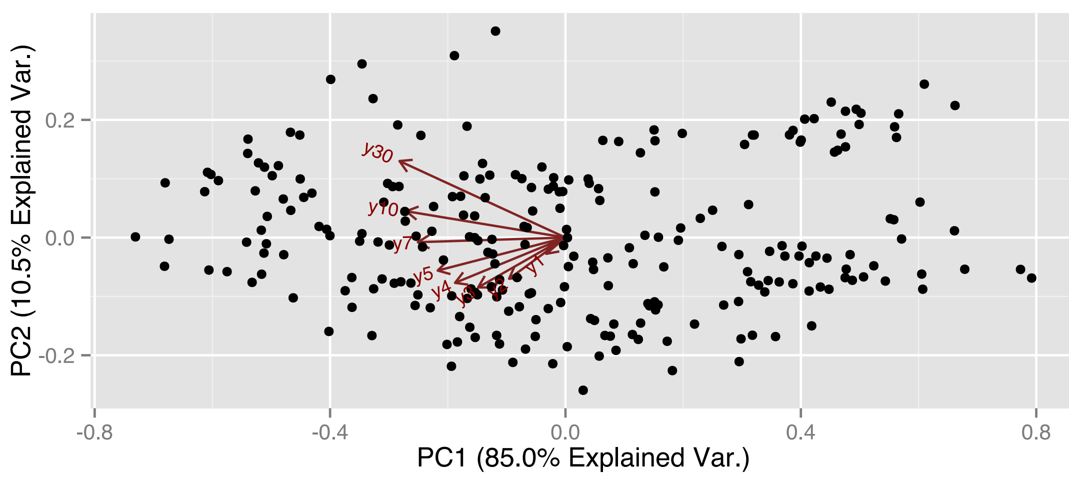
Time Series of Swap Rates for Year to Date



Covariance Matrix of Swap Rates

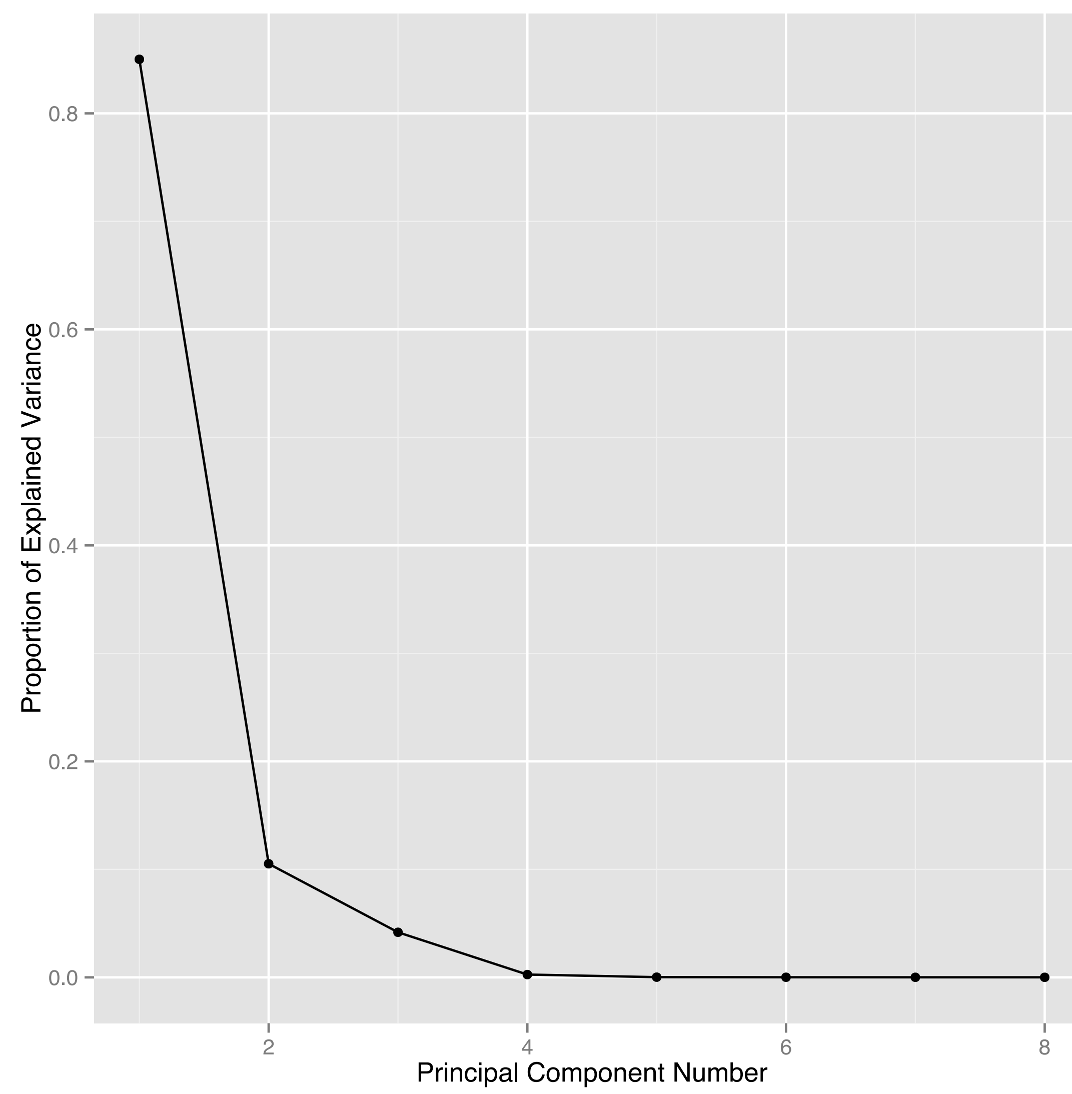
PCA Results

Biplot of First 2 PCs



- PC1: 85.0% of variance
- PC2: 10.5% of variance
- PC3: 4.1% of variance
- First 3 PCs are highly explanatory and accounts for 99.7% of variance

Screeplot of PCs



Interpretation of Loadings

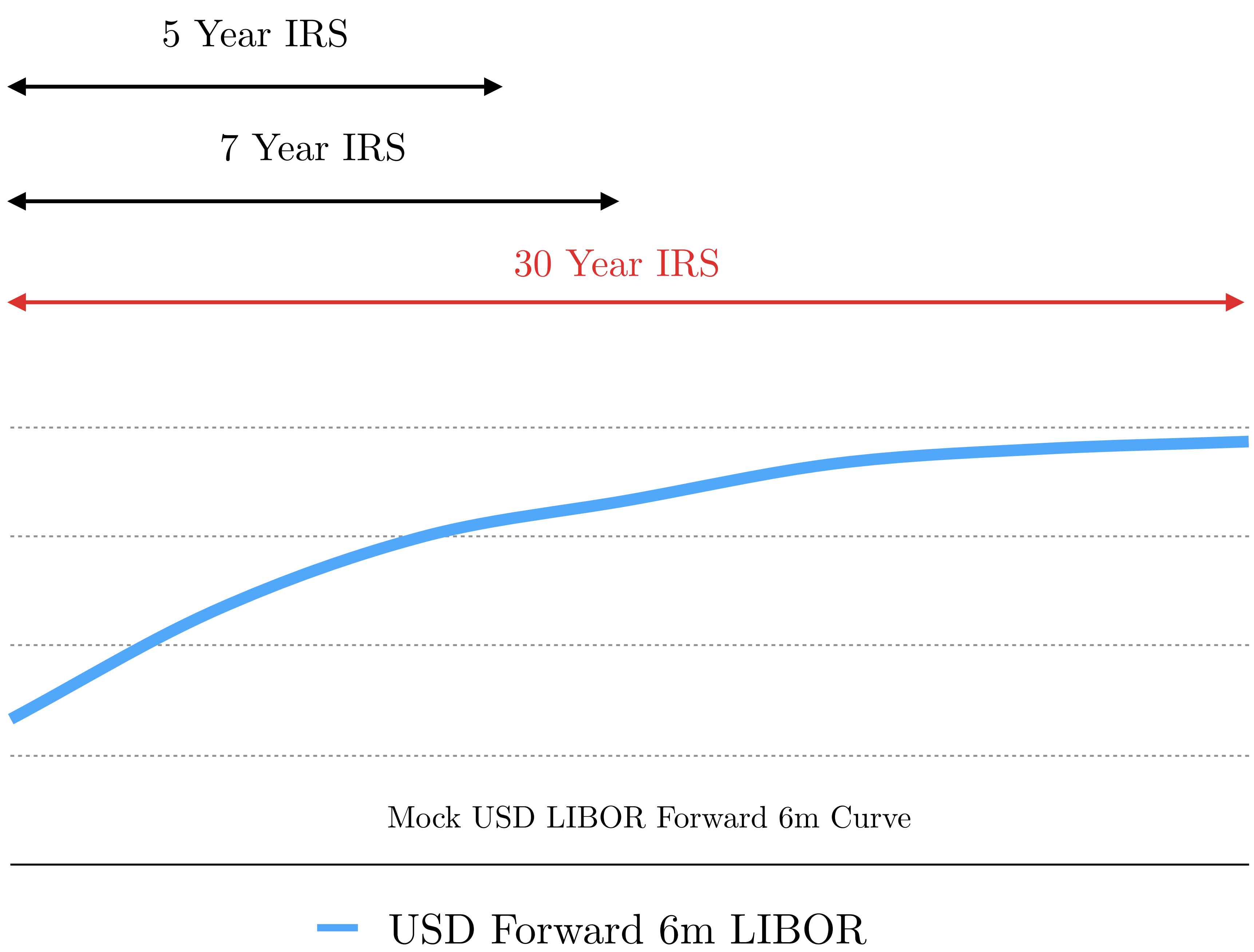
	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
y1	-0.06	-0.14	0.81	-0.17	-0.52	-0.11	-0.10	-0.04
y2	-0.17	-0.35	0.43	0.04	0.63	0.43	0.20	0.22
y3	-0.26	-0.42	0.03	0.32	0.27	-0.51	-0.24	-0.51
y4	-0.33	-0.38	-0.19	0.30	-0.30	-0.25	0.20	0.65
y5	-0.38	-0.28	-0.24	0.00	-0.39	0.56	0.23	-0.46
y7	-0.44	-0.04	-0.15	-0.42	0.05	0.14	-0.73	0.21
y10	-0.47	0.22	0.00	-0.55	0.15	-0.37	0.51	-0.08
y30	-0.49	0.64	0.19	0.54	0.01	0.11	-0.07	0.00

Loadings of Principal Components

- PC1: Directional movements in the yield curve
- PC2: Slope movements in the yield curve
- PC3: Curvature movements in the yield curve

Shortcoming

- We are always over-counting the short end of the curve

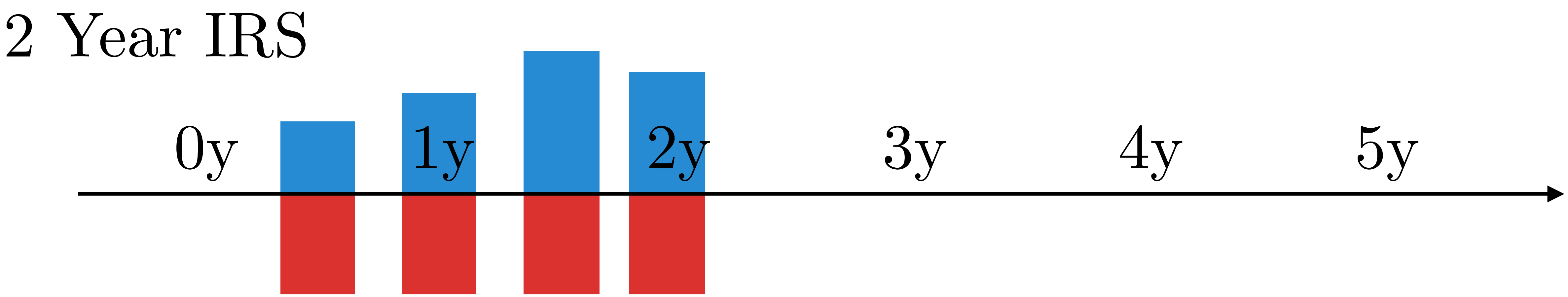


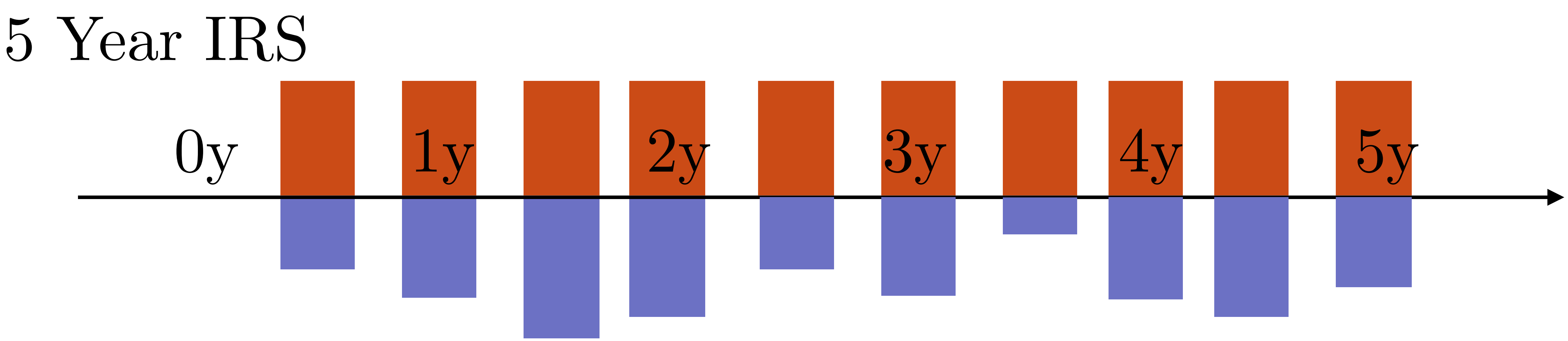
Citations: Hull, Options, Futures and Other Derivatives 7e

B. PCA on Curve Rates

Curve Trades

- Imagine a trade like this
  - I pay fixed (receive float) on one 2 year IRS: I profit from the yield curve going up at the short end
  - I pay float (receive fixed) on one 5 year IRS: I profit from the yield curve going down at the long end

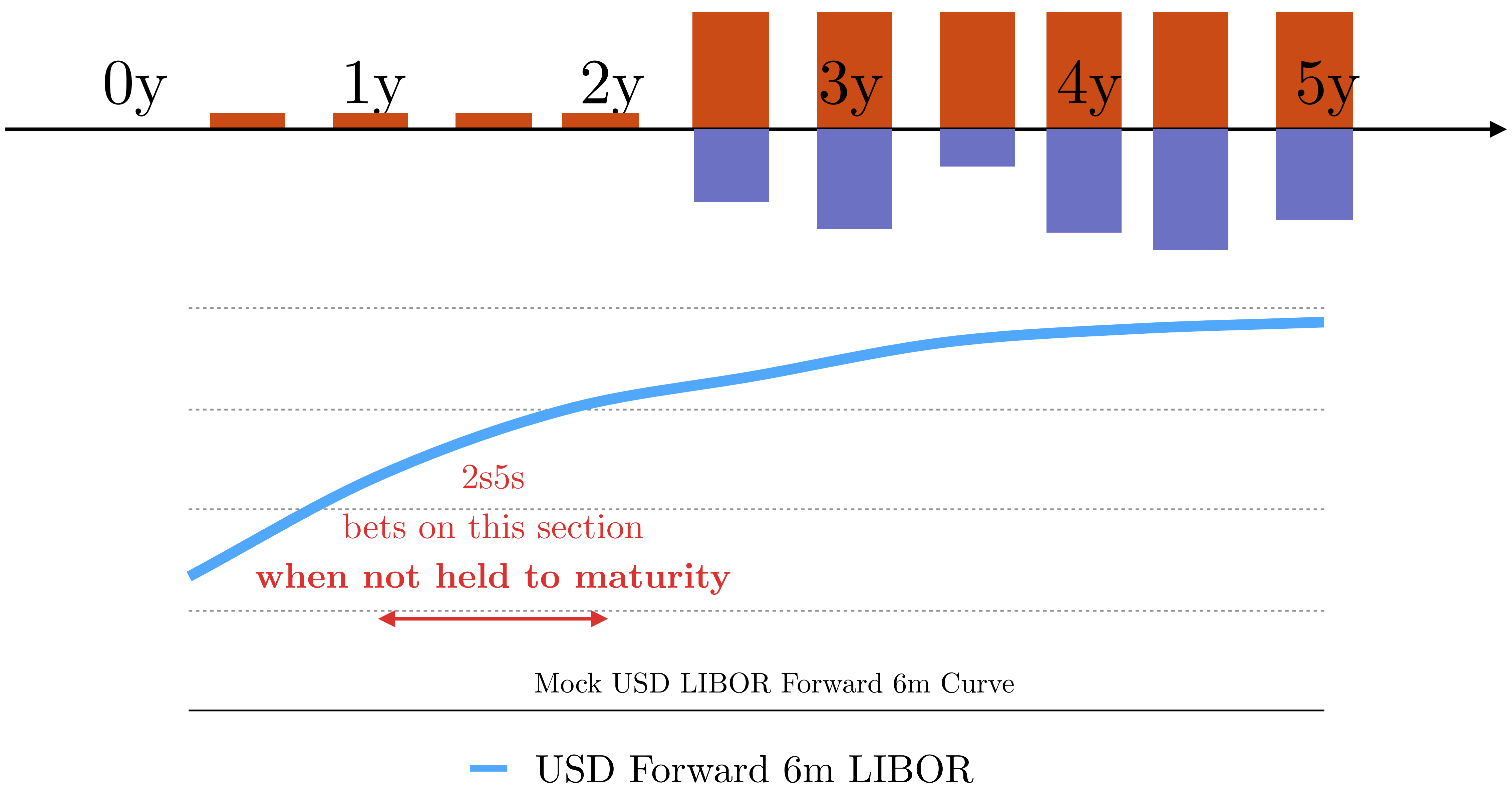




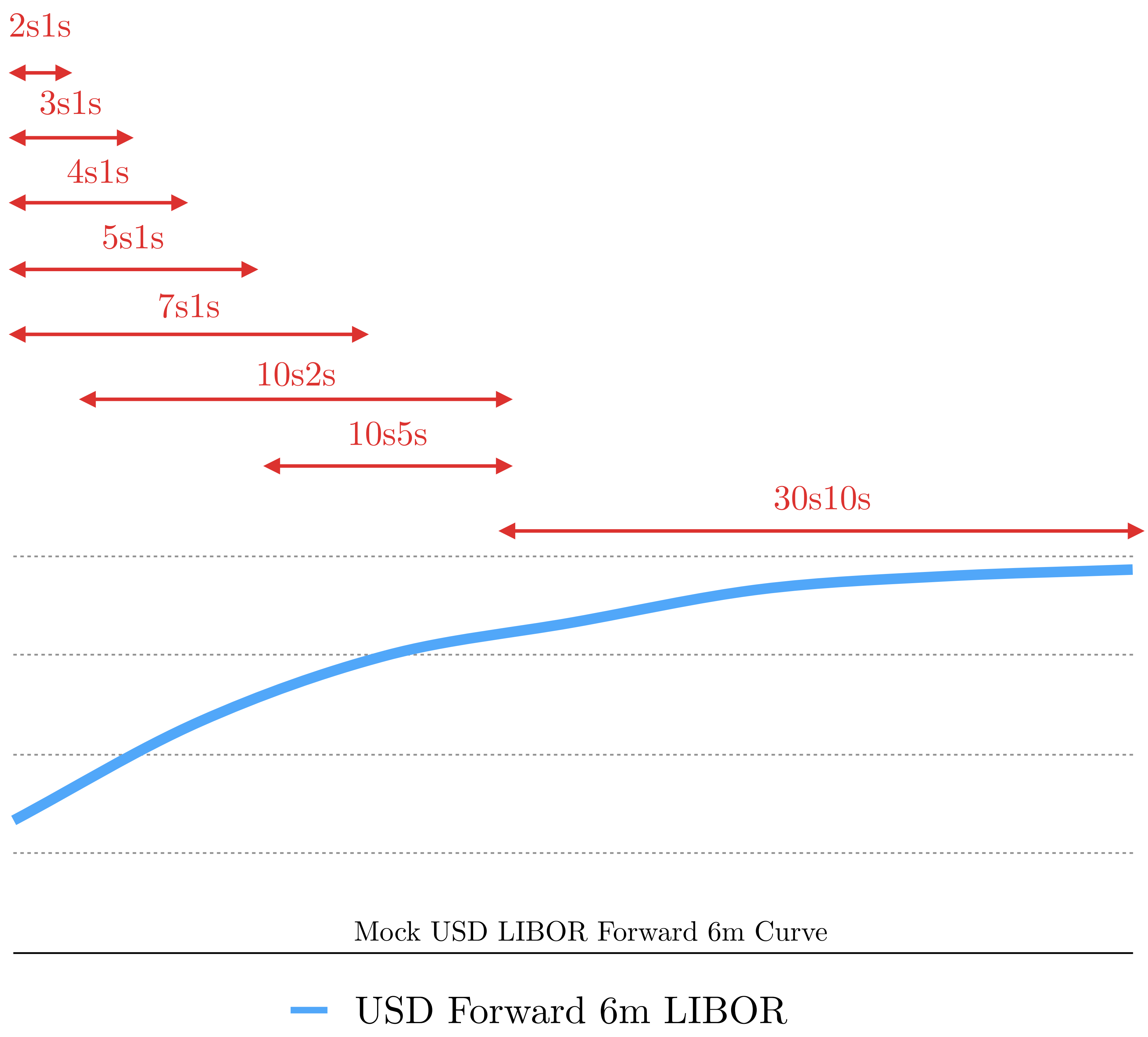
Curve Trades

- Floating legs for the period of the shorter maturity IRS cancel out.
- Fixed legs result in a fixed difference.

Combined Cashflows

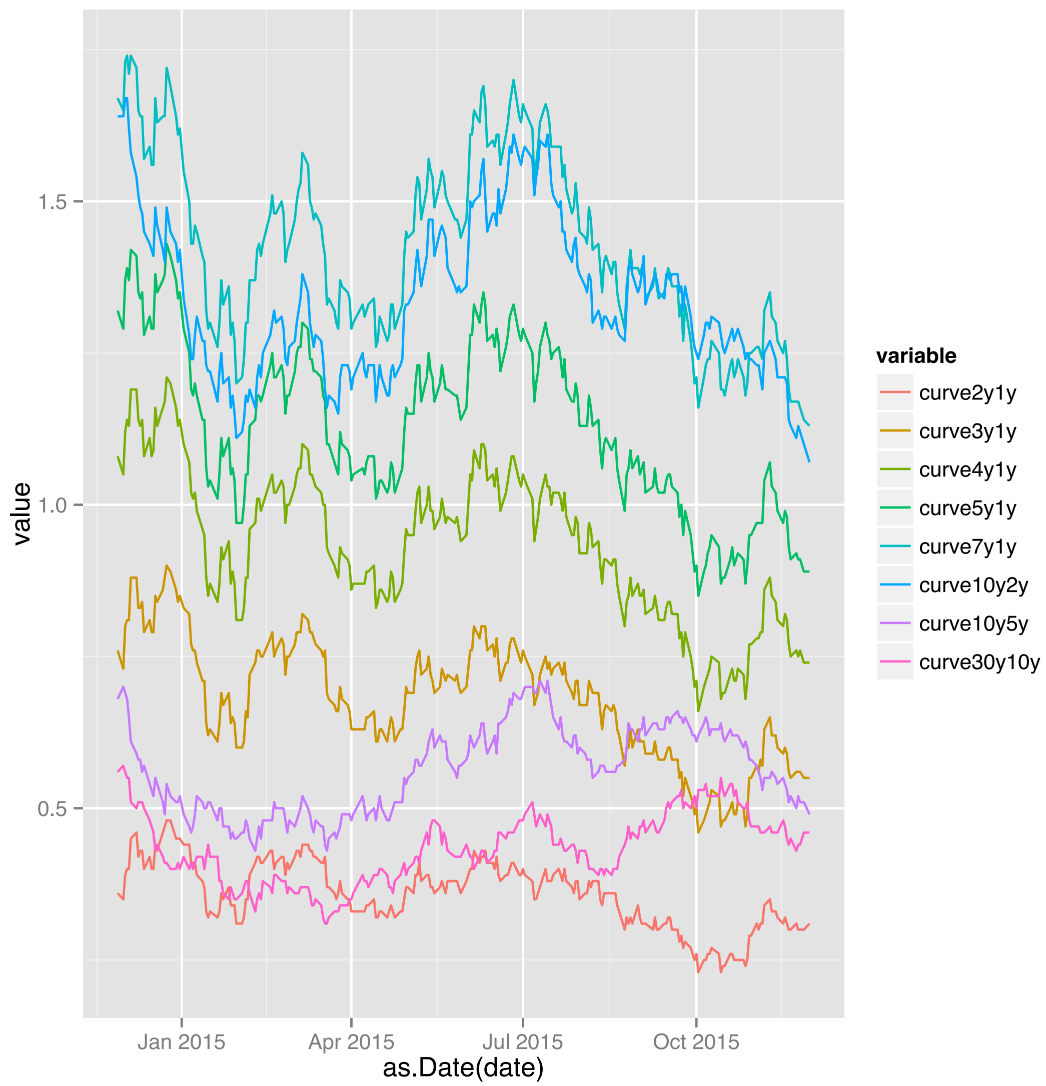


Curve Pairs Used

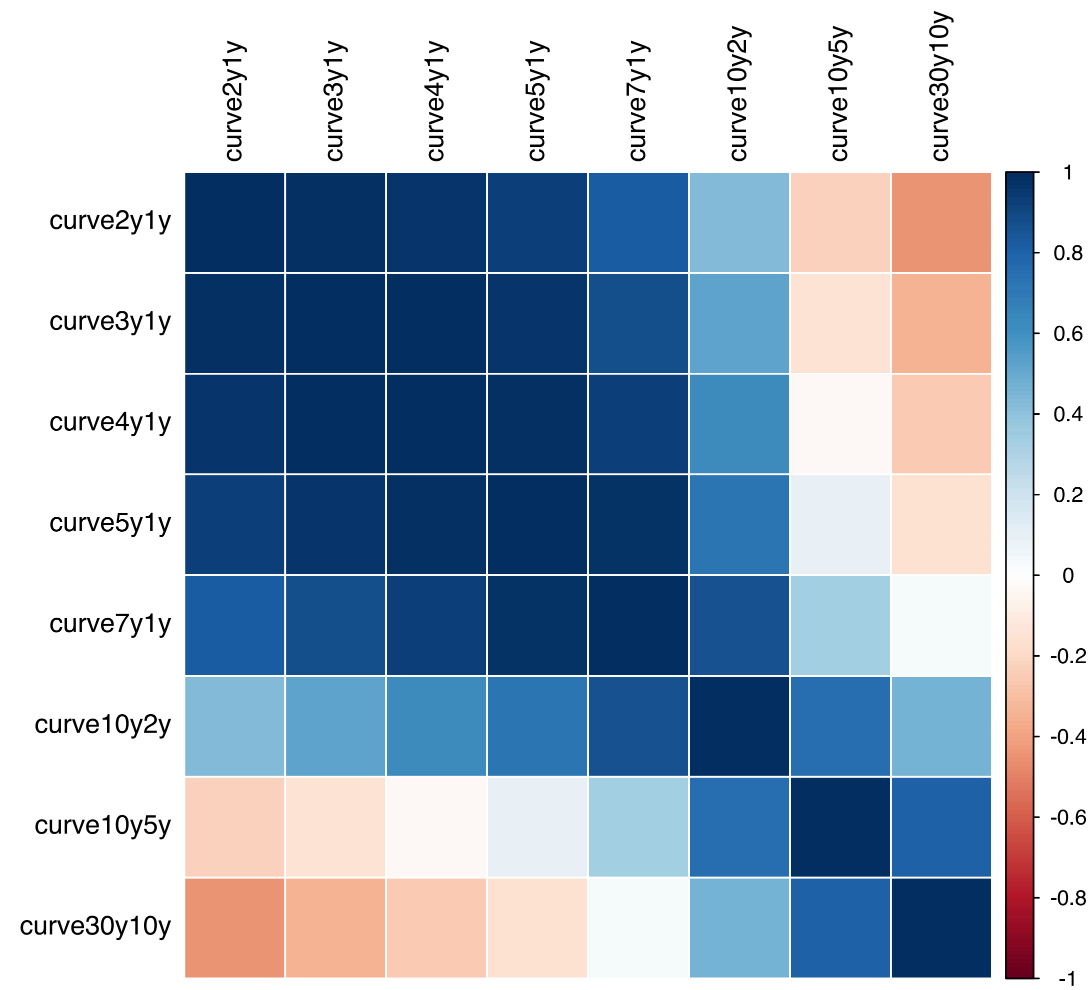


Descriptive Data for Curve Rates Used

- Curve rates calculated as: $C = S_{0,t_2} - S_{0,t_1}$



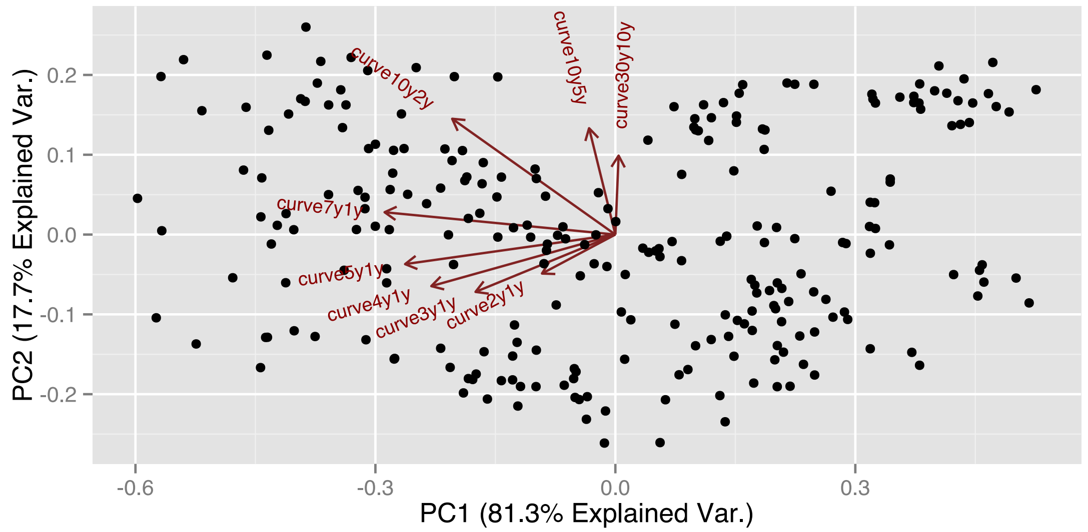
Time Series of Curve Rates for Year to Date



Covariance Matrix of Curve Rates

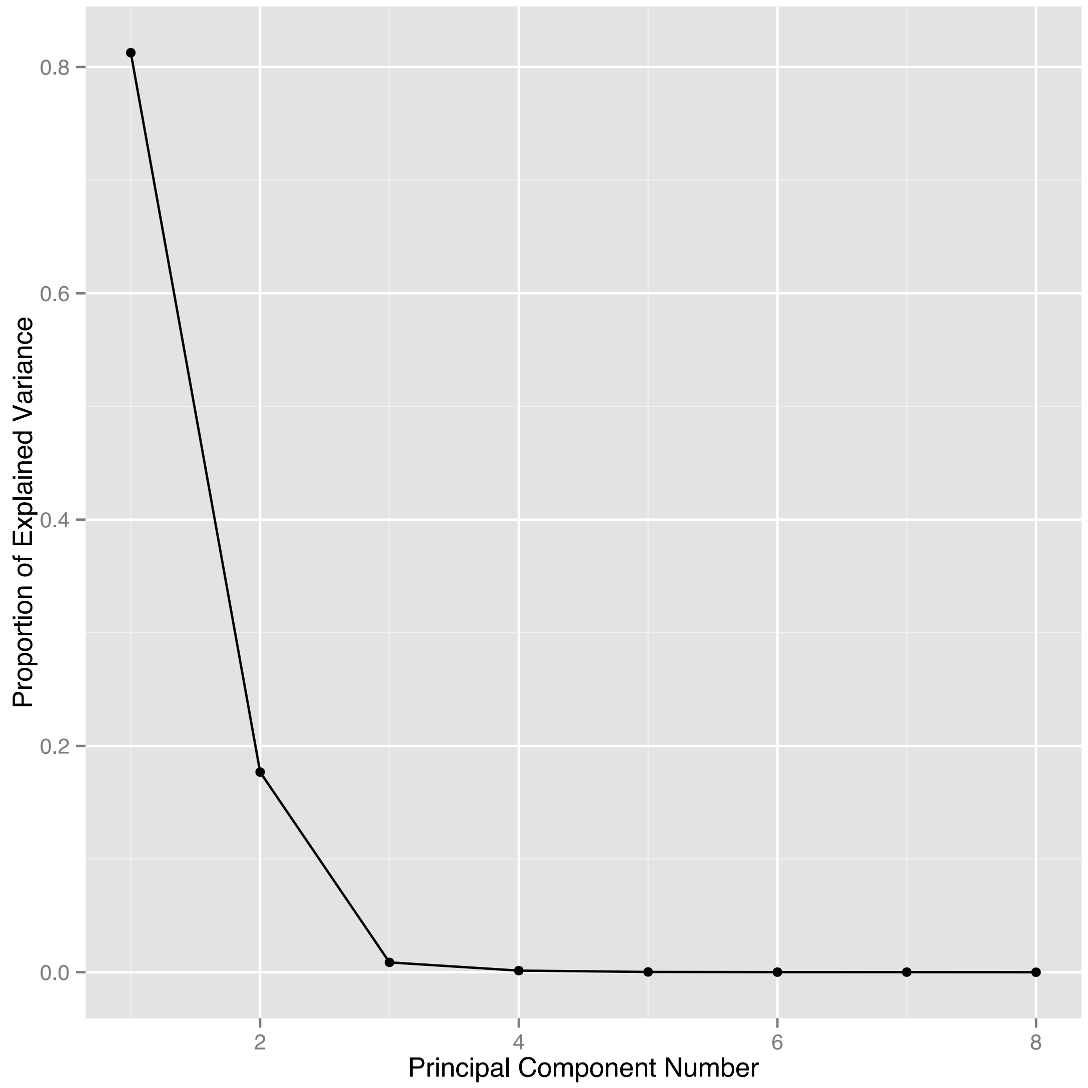
PCA Results

Biplot of First 2 PCs



- PC1: 81.3% of variance
- PC2: 17.7% of variance
- PC3: 0.87% of variance
- First 3 PCs are highly explanatory and accounts for 99.8% of variance

Screeplot of PCs



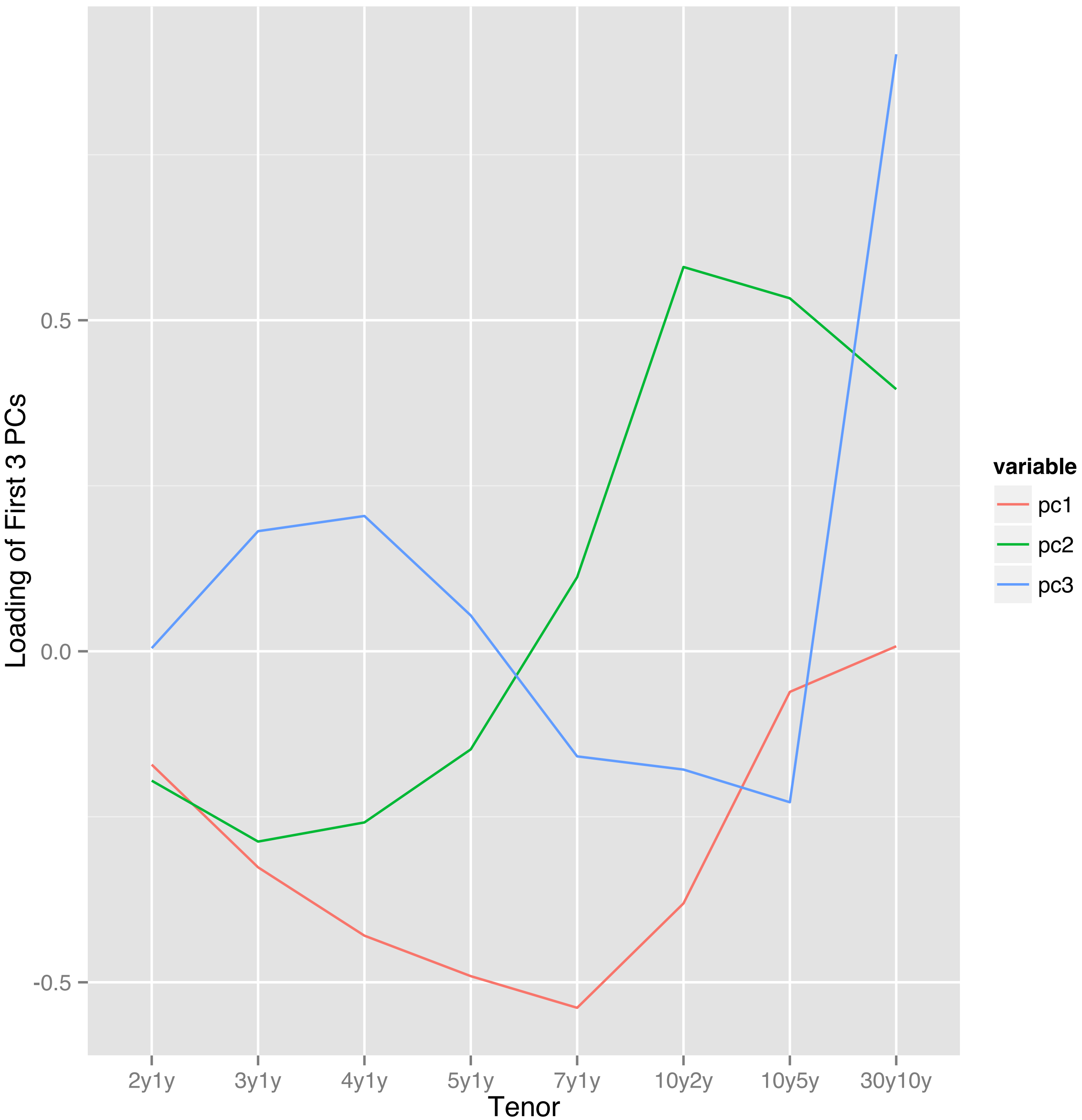
Interpretation of Loadings

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
curve2y1y	-0.171	-0.195	0.005	0.668	0.256	-0.362	-0.201	-0.500
curve3y1y	-0.326	-0.287	0.181	0.249	-0.531	0.575	-0.322	0.000
curve4y1y	-0.430	-0.259	0.204	-0.130	-0.429	-0.486	0.520	-0.000
curve5y1y	-0.491	-0.148	0.054	-0.214	0.338	-0.251	-0.511	0.500
curve7y1y	-0.539	0.112	-0.159	0.115	0.455	0.451	0.499	-0.000
curve10y2y	-0.381	0.580	-0.179	-0.339	-0.206	-0.066	-0.272	-0.500
curve10y5y	-0.061	0.533	-0.228	0.543	-0.288	-0.176	0.038	0.500
curve30y10y	0.008	0.396	0.902	0.079	0.149	0.033	0.022	0.000

Loadings of Principal Components

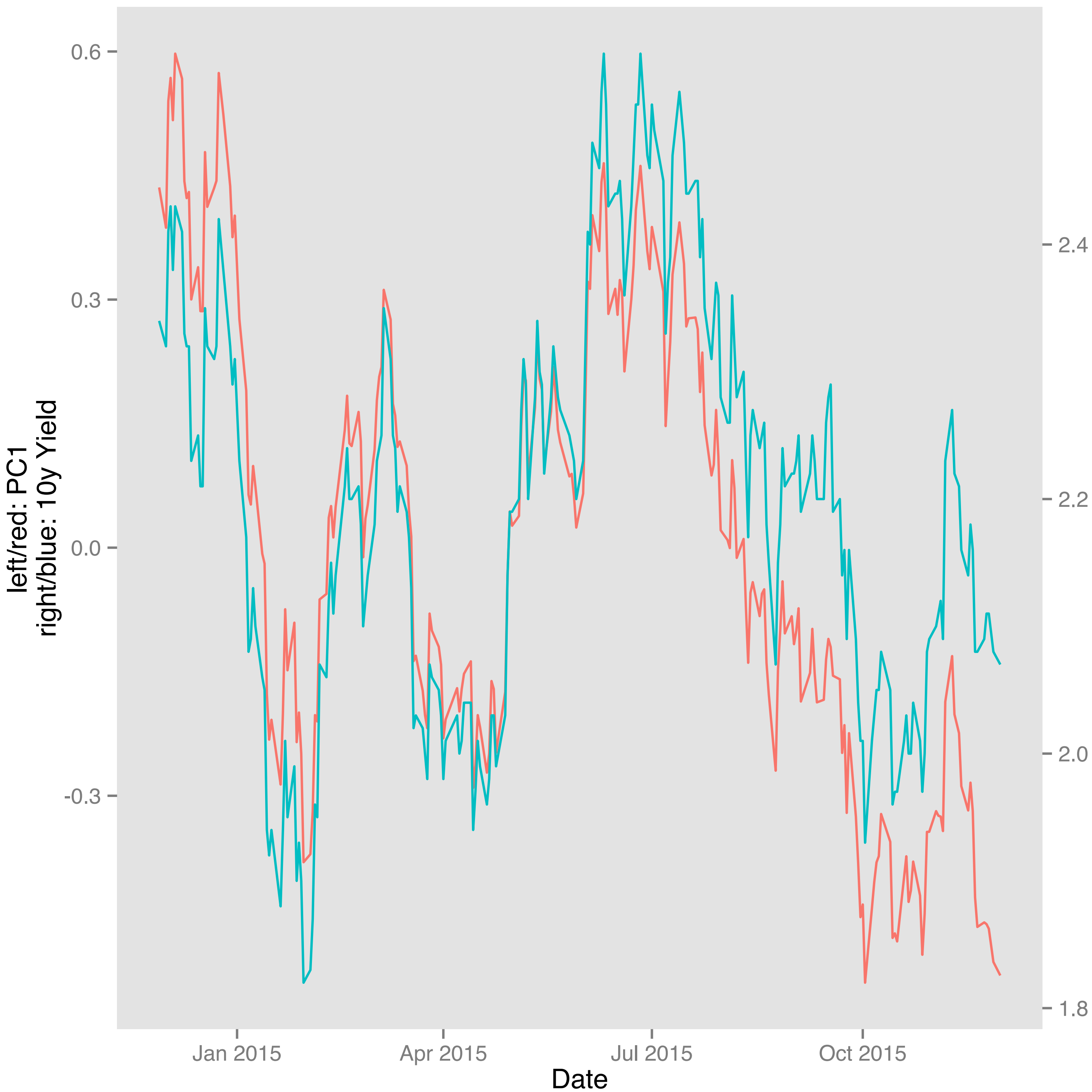
- PC1: Directional movements in the yield curve
- PC2: Slope movements in the yield curve
- PC3: Curvature movements in the yield curve

Interpretation of Loadings



Loadings of Principal Components

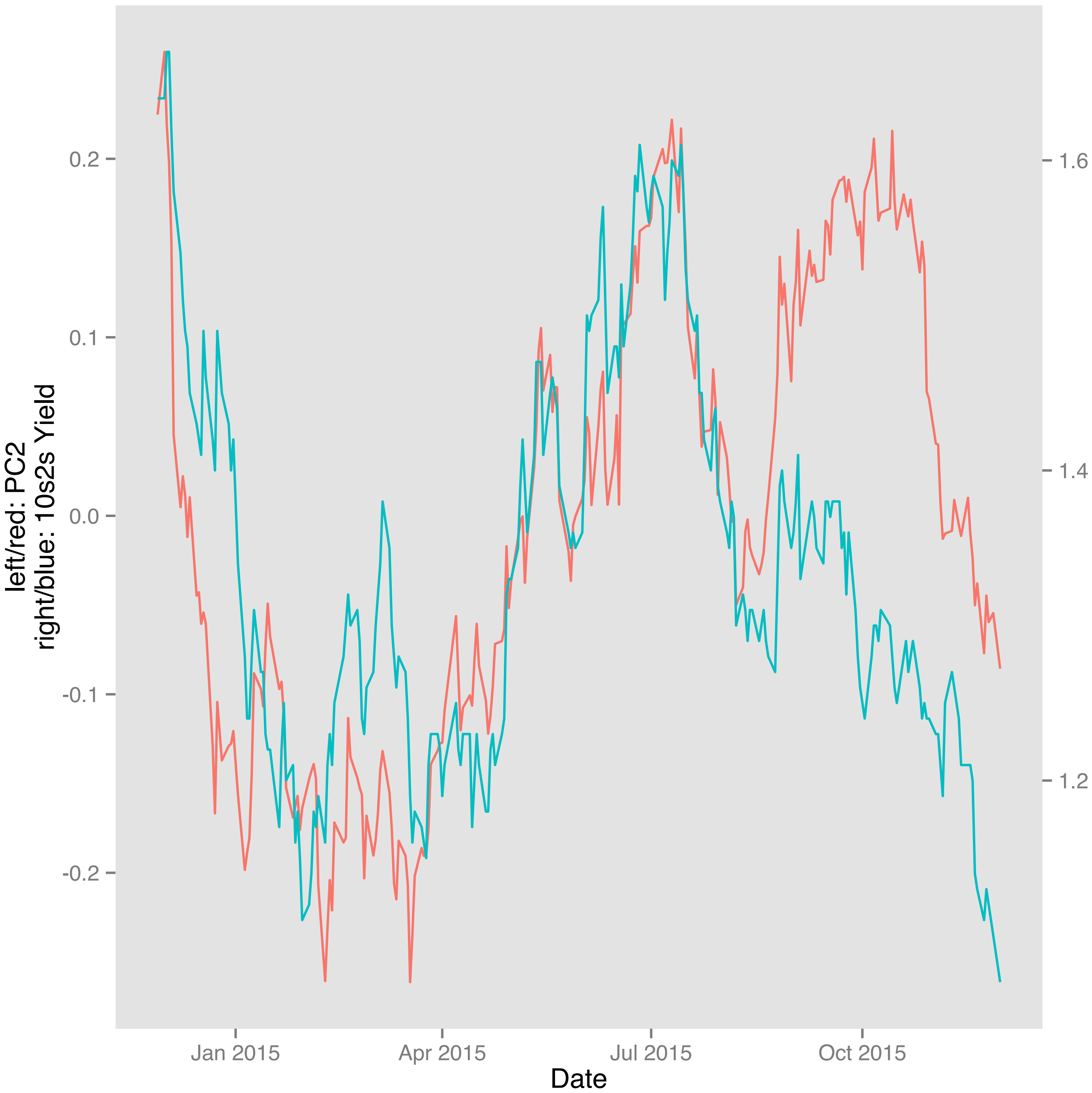
Correlating PC1 and 10 Year IRS Swap Rate



Left/Red: PC1

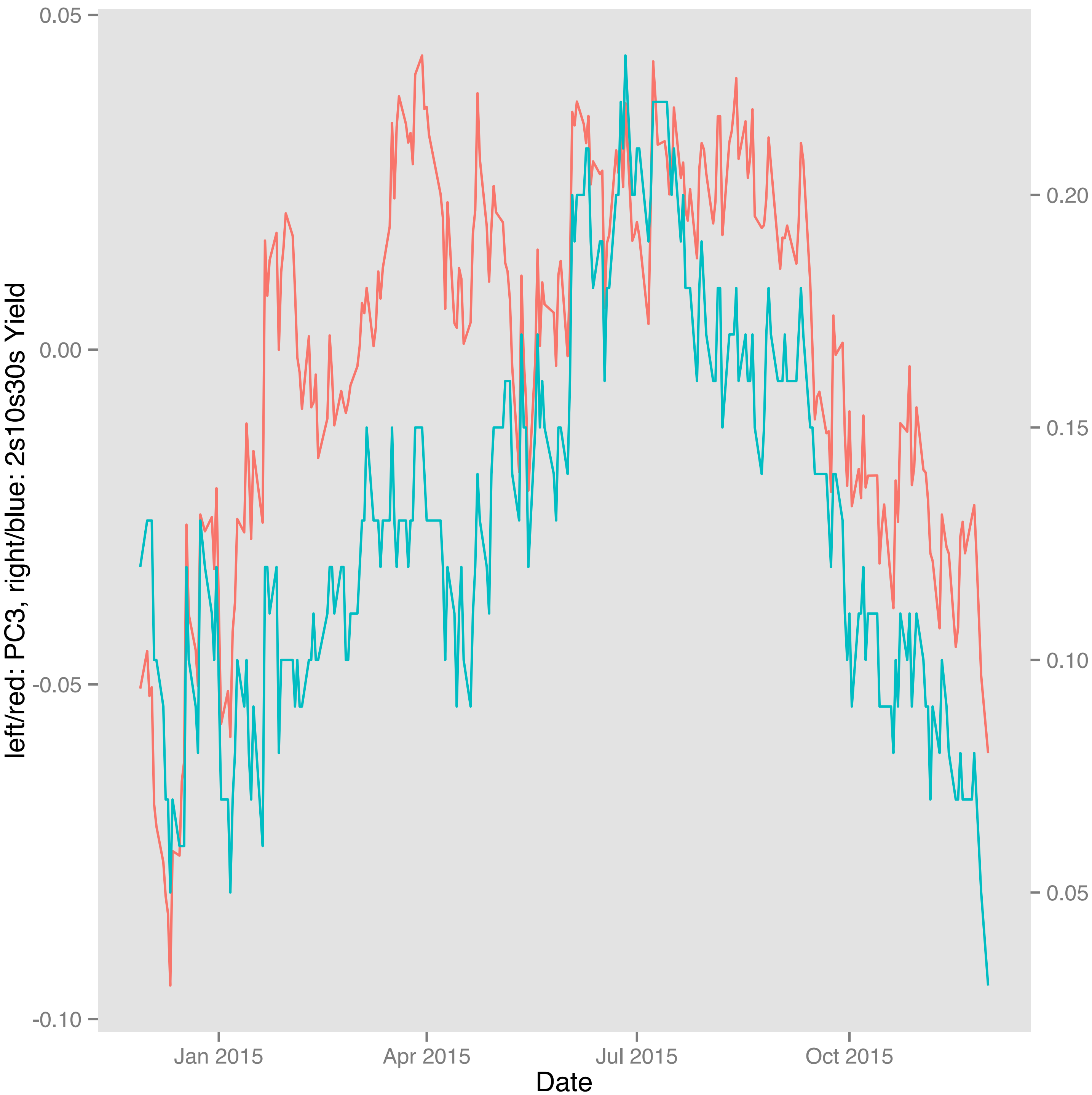
Right/Blue: 10 Year IRS Swap Rate

Correlating PC2 and 10s2s Curve Rate



Left/Red: PC2
Right/Blue: 10s2s Curve Rate

Correlating PC3 and 2s10s30s Butterfly Rate



Left/Red: PC3
Right/Blue: 2s10s30s Butterfly Rate

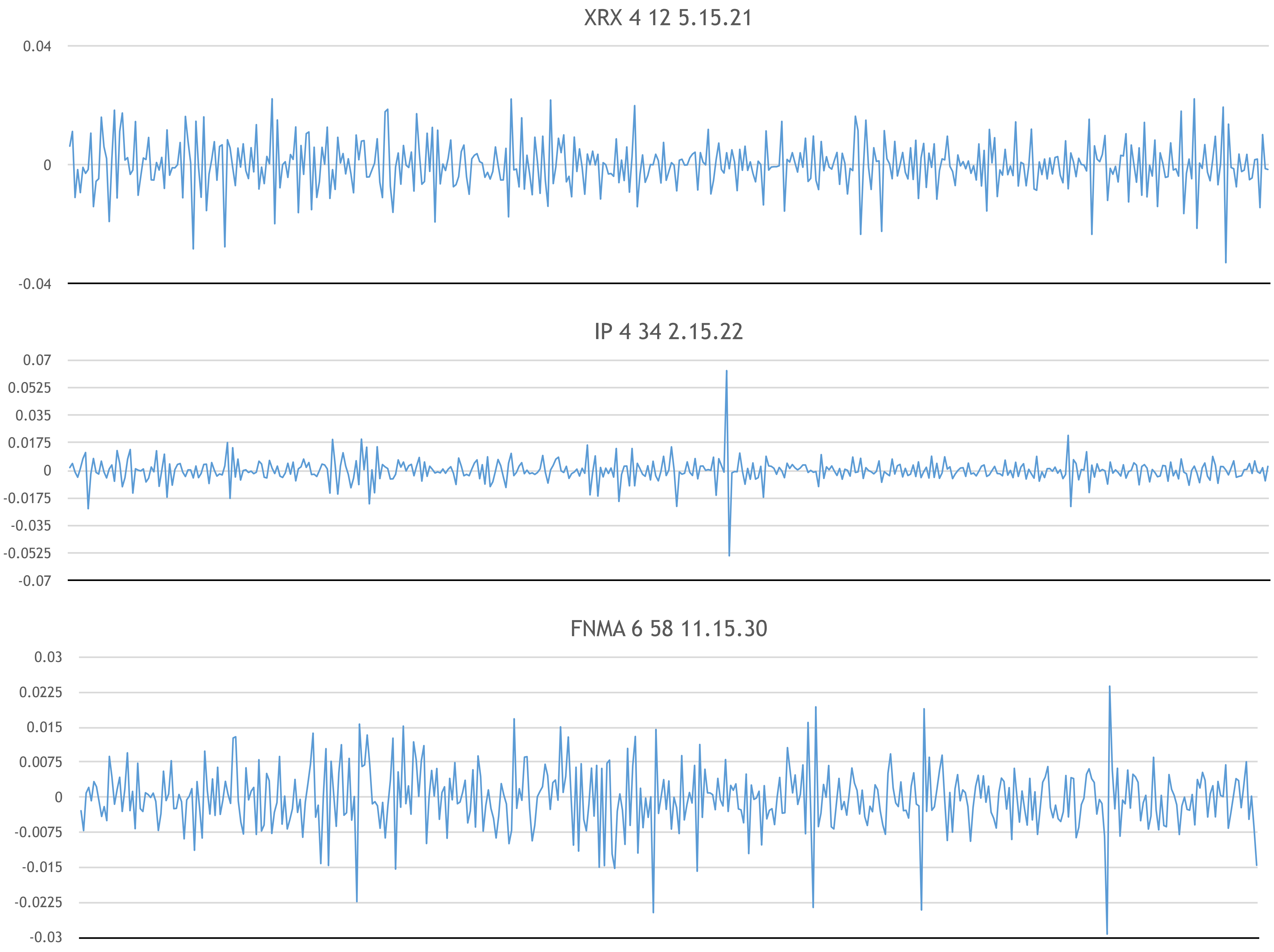
Conclusion

- More robust than simple duration analysis
- Can be used for
  - Hedge ratio generation between two IRS securities
  - Neutralizing PC1, PC2, and/or PC3 risks in a large existing IRS portfolio

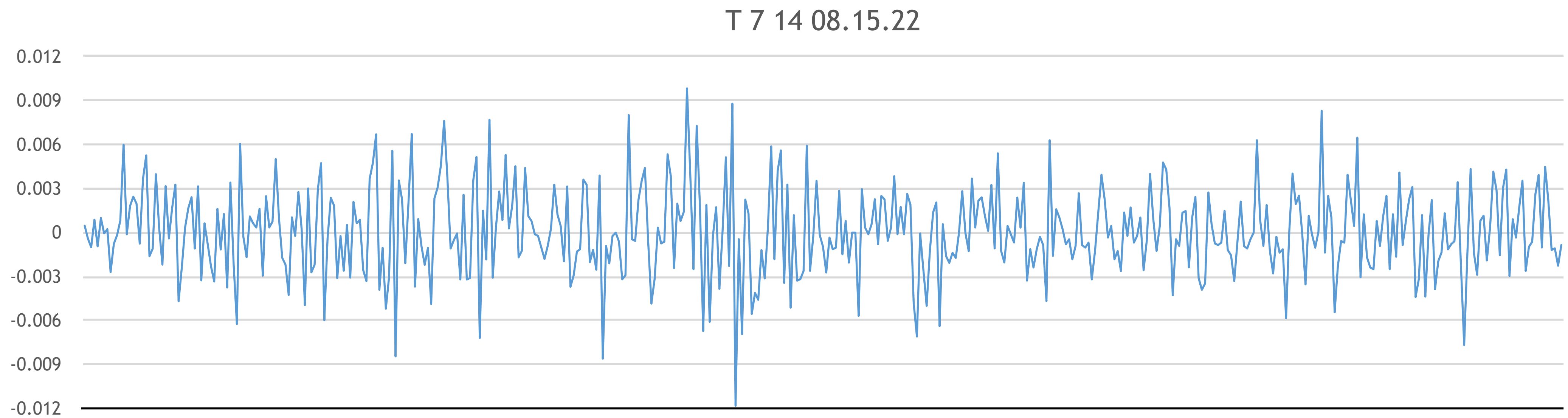
Fixed Income Portfolio Risk Modeling

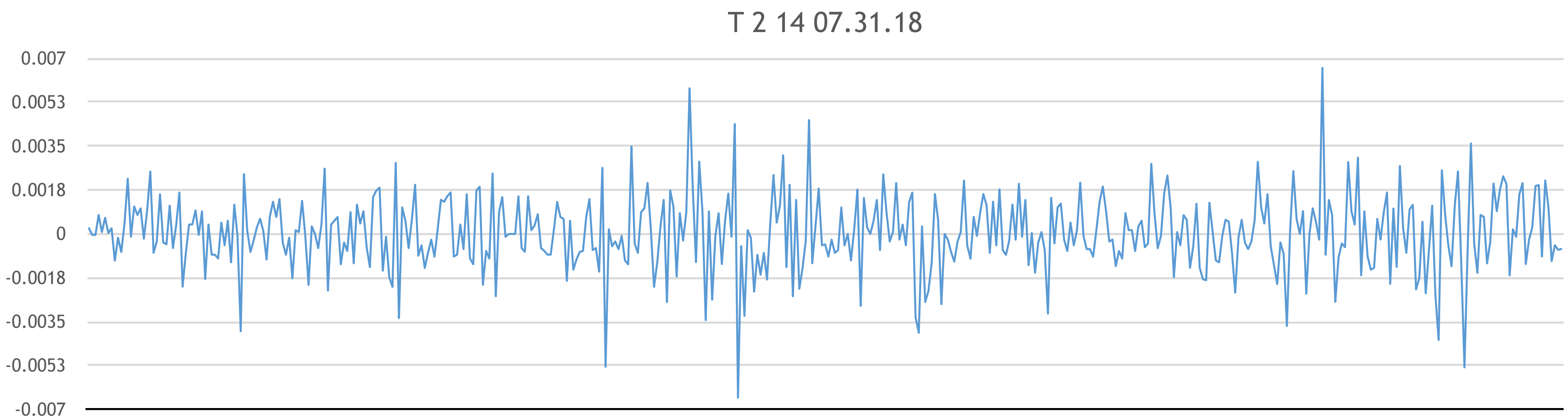
Descriptive Statistics of Data

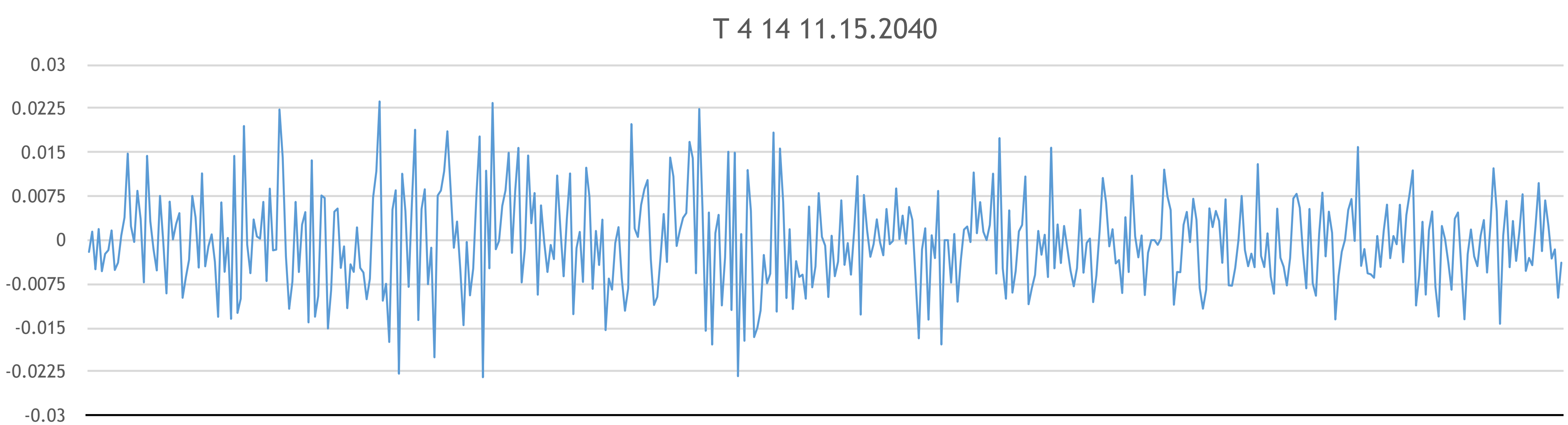
Descriptive Statistics: Return Series



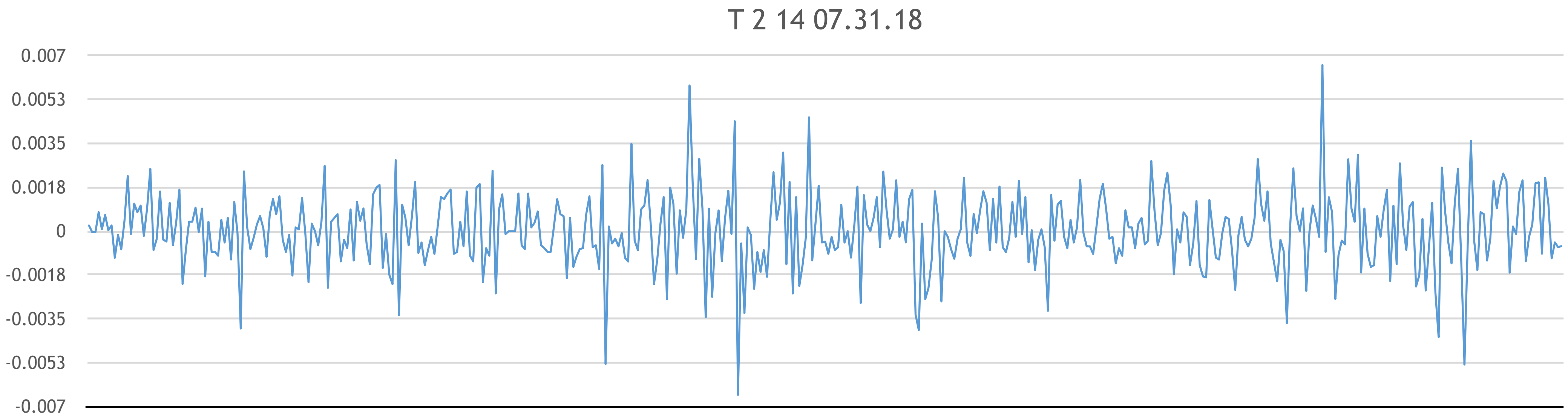
Descriptive Statistics: Return Series

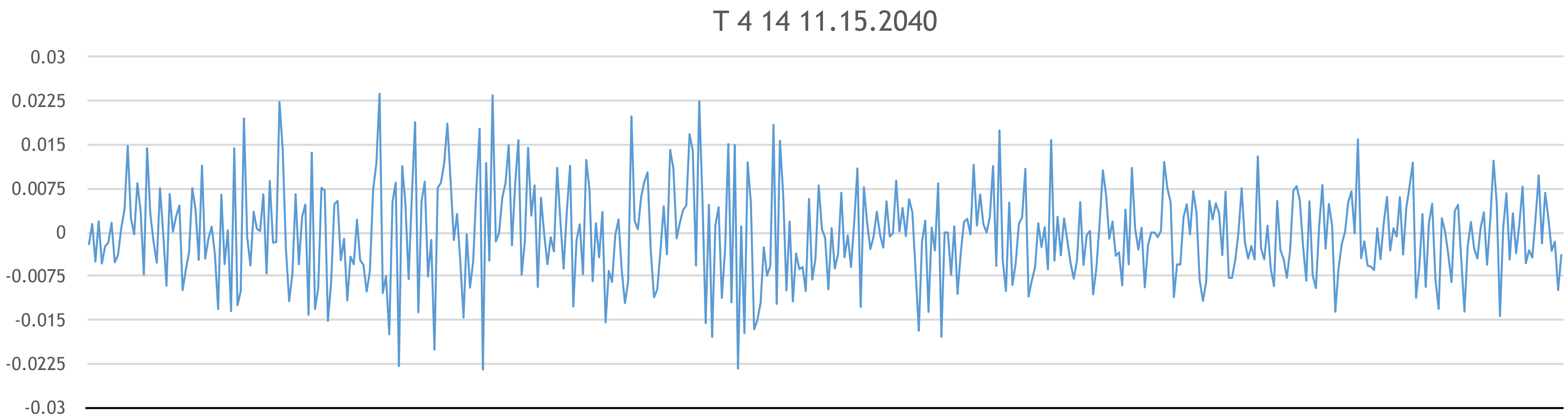






Descriptive Statistics: Return Series

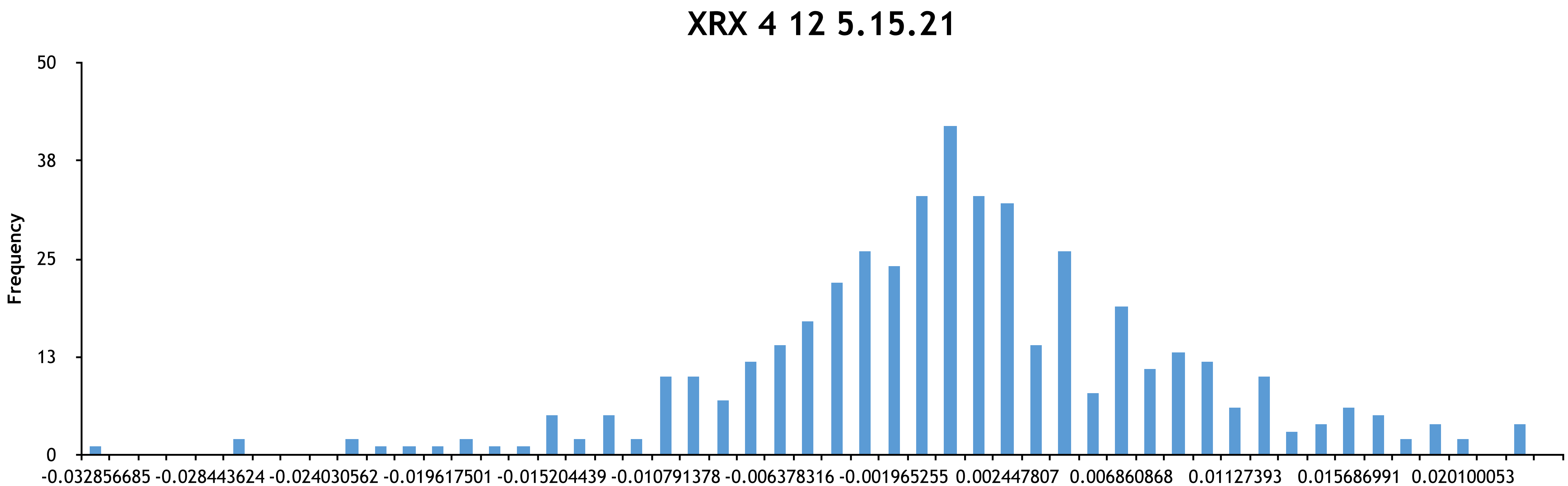


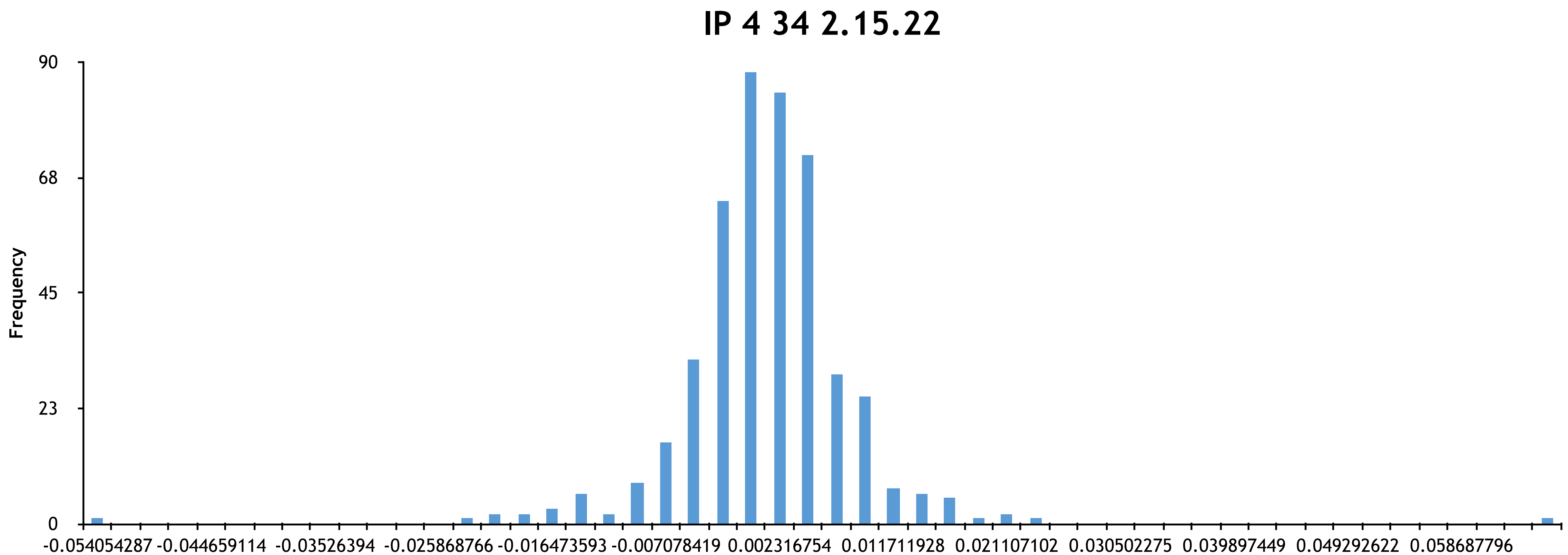


Descriptive Statistics: Summary

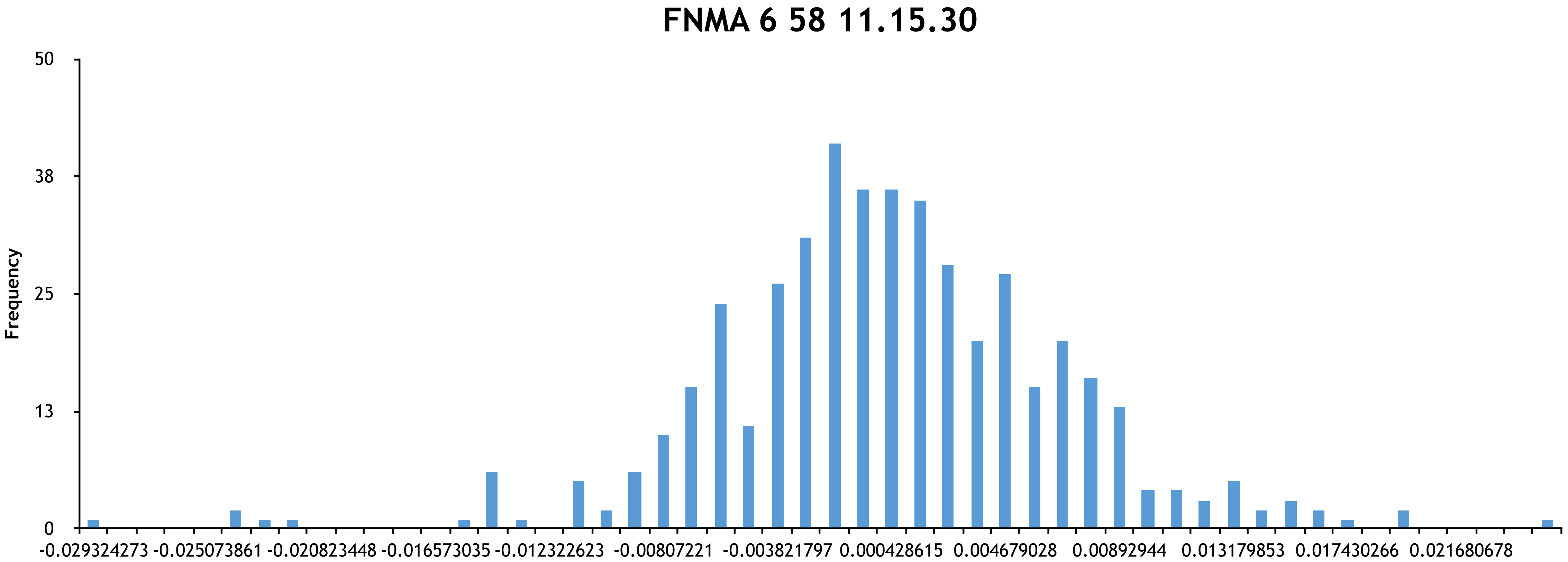
	XRX 4 12 5.15.21	IP 4 34 2.15.22	FNMA 6 58 11.15.30	T 7 14 08.15.22	T 2 14 07.31.18	T 4 14 11.15.2040	FHLM 2 38 01.22	HUD 2.56 08.01.21
MEAN	0.0000486	-0.0000184	-0.0002016	0.0000663	0.0000290	-0.0002844	-0.0000930	-0.0000811
STDEV	0.0081360	0.0072687	0.0064942	0.0029797	0.0014866	0.0080912	0.0041003	0.0024886
SKEW	-0.1735524	0.3937087	-0.2596869	0.0365424	-0.0814206	0.1779466	-0.4374094	0.1688861
KURT	1.2067322	19.2731195	2.1772646	0.7495469	2.3186874	0.1997959	7.7614271	0.6117801

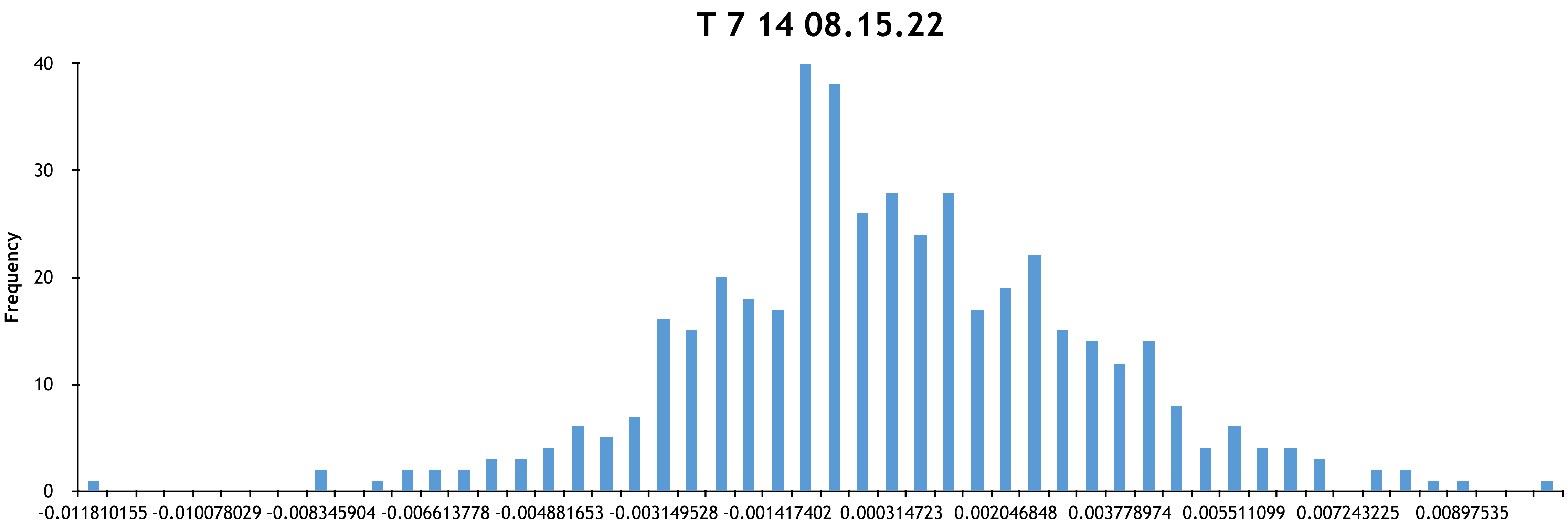
Descriptive Statistics: Histograms



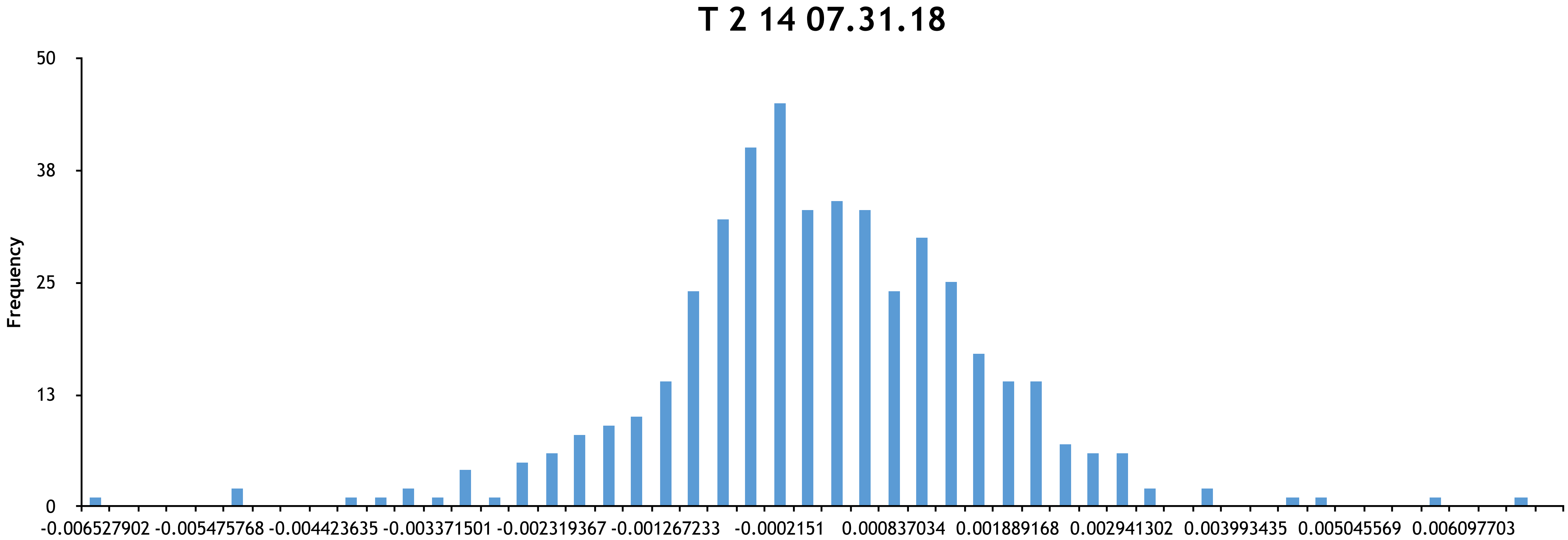


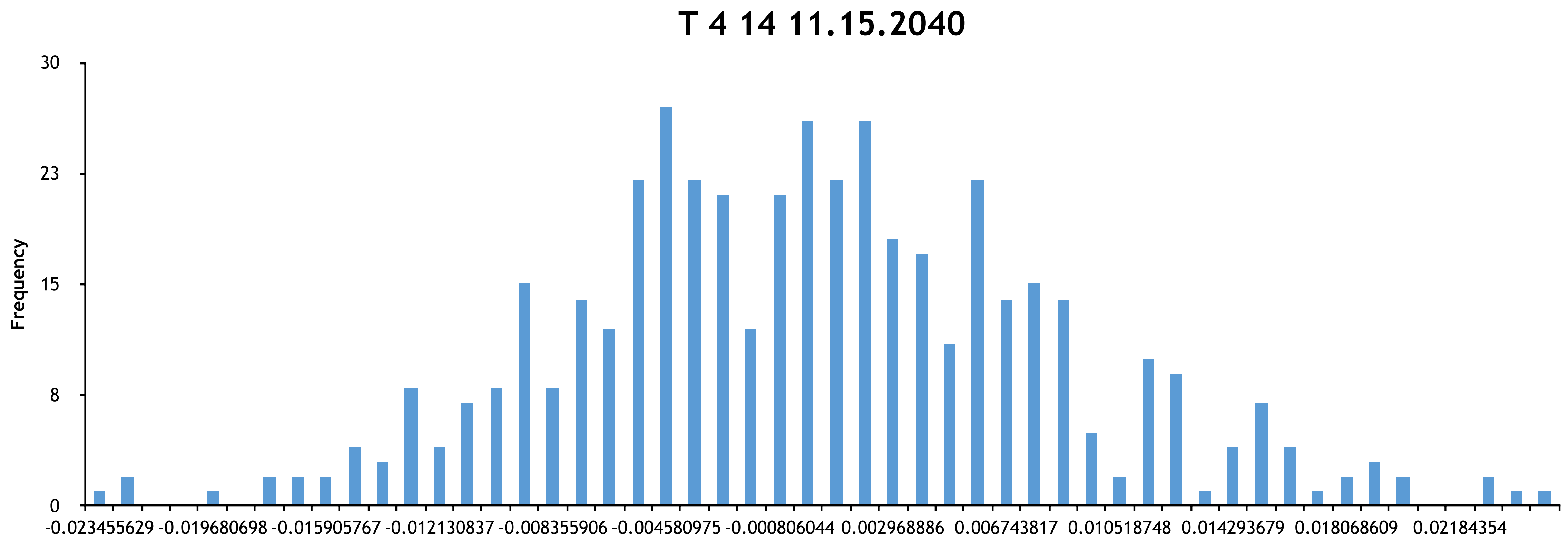
Descriptive Statistics: Histograms



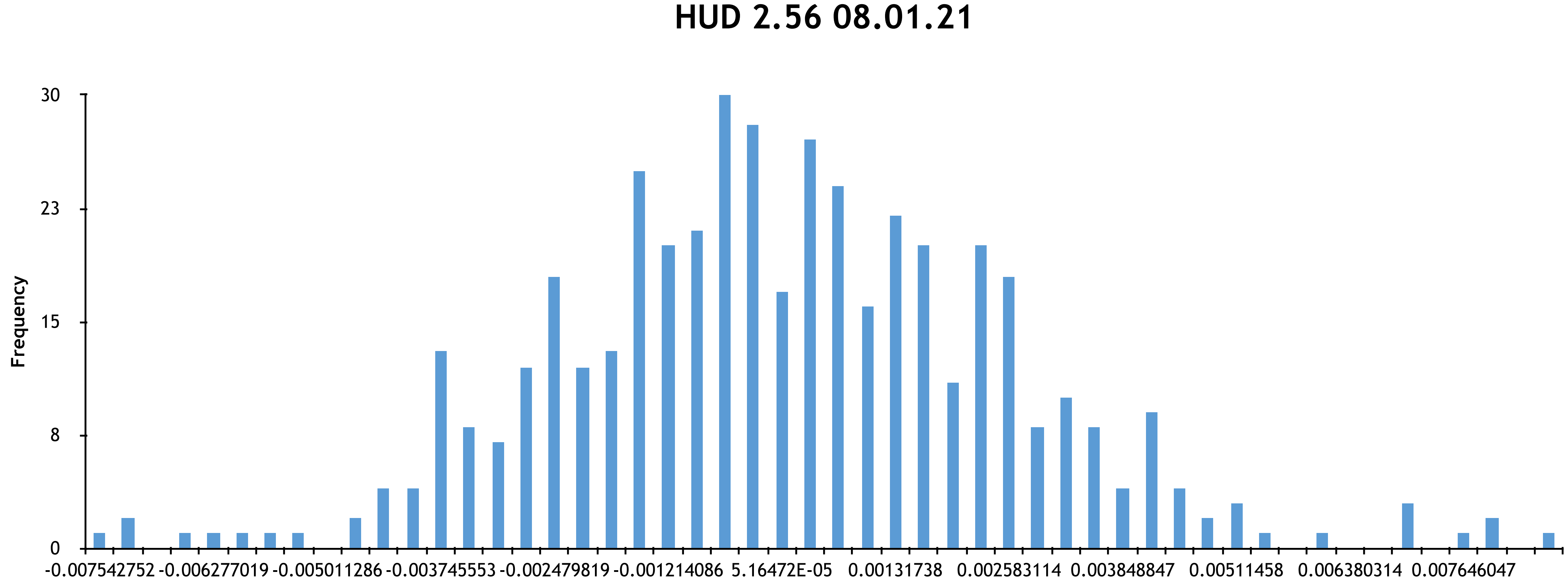


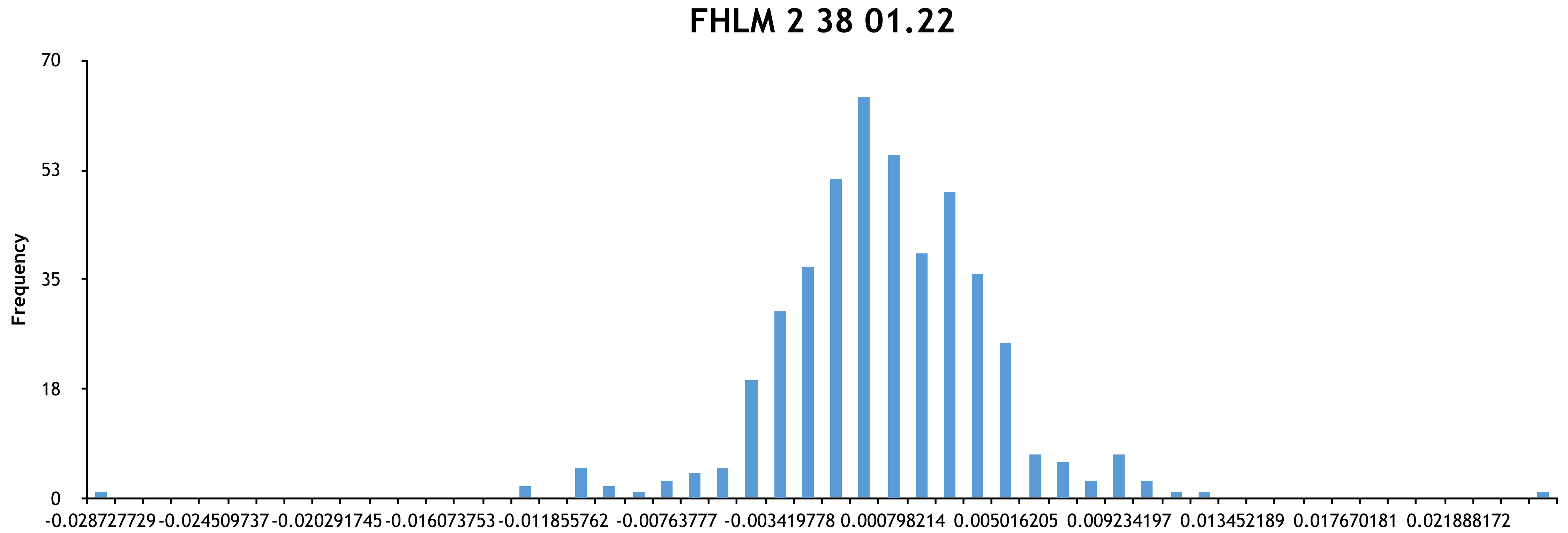
Descriptive Statistics: Histograms



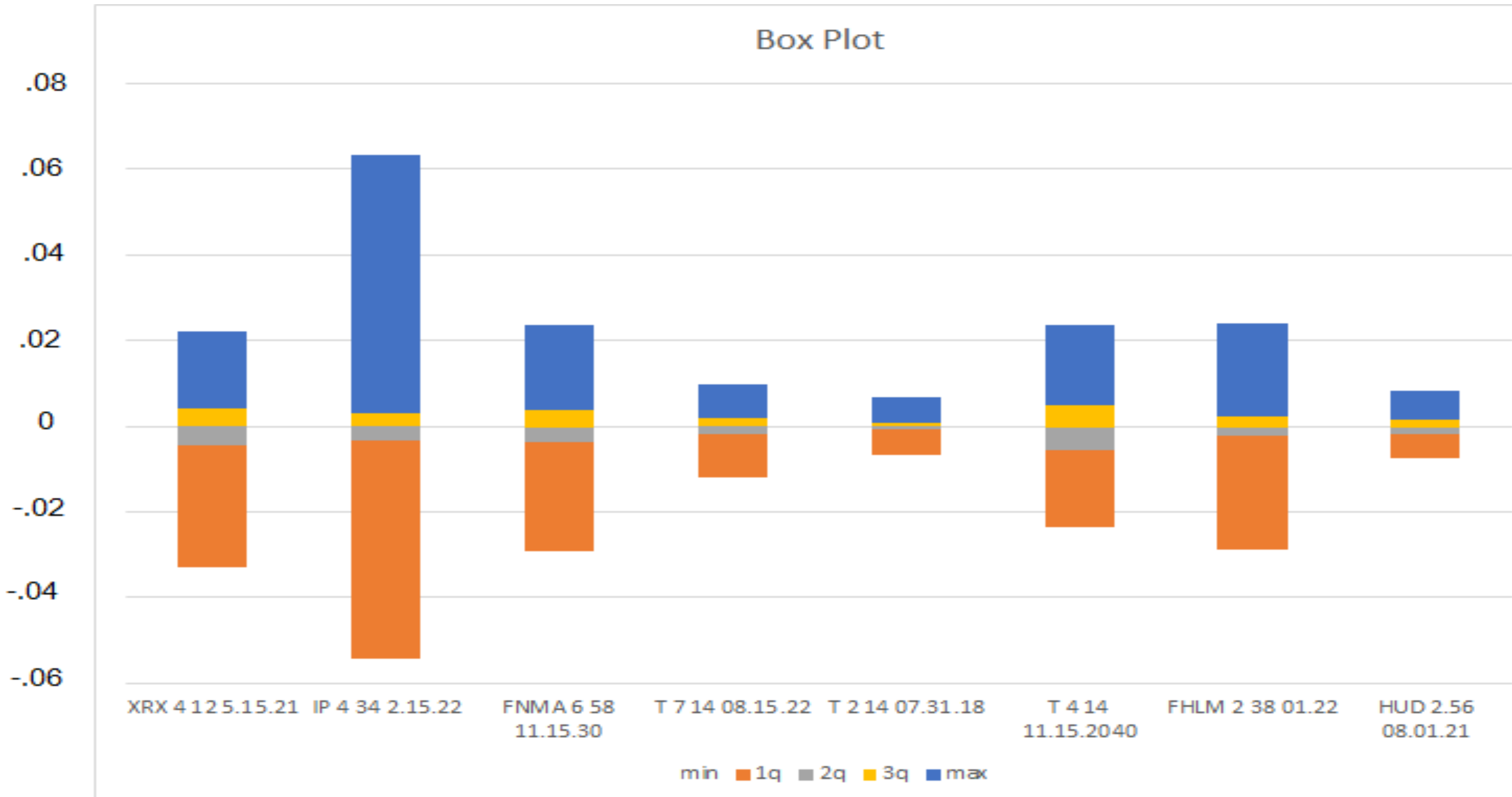


Descriptive Statistics: Histograms

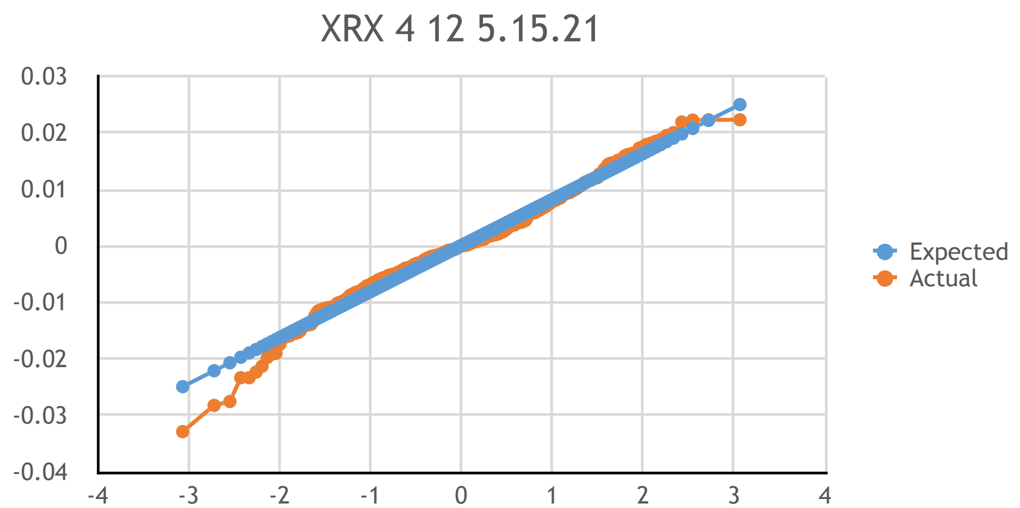


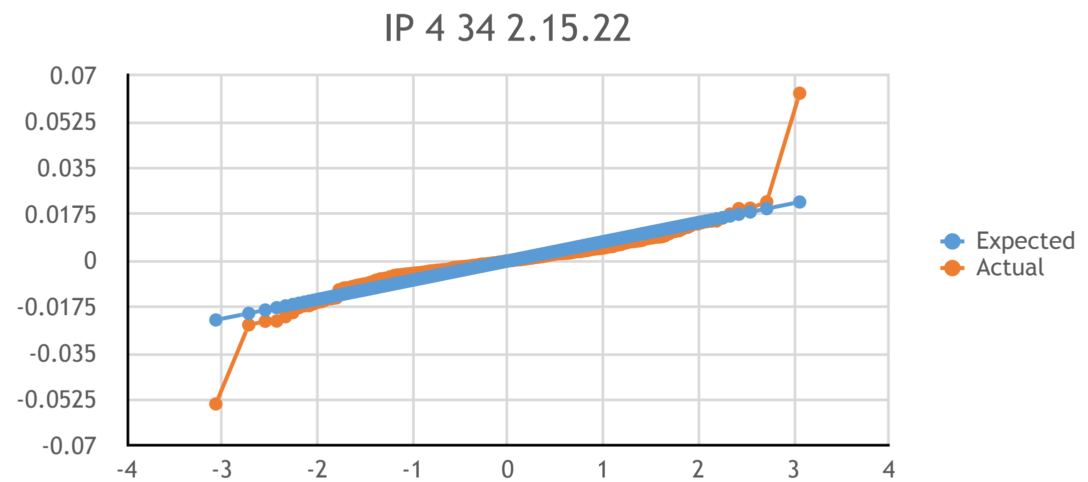


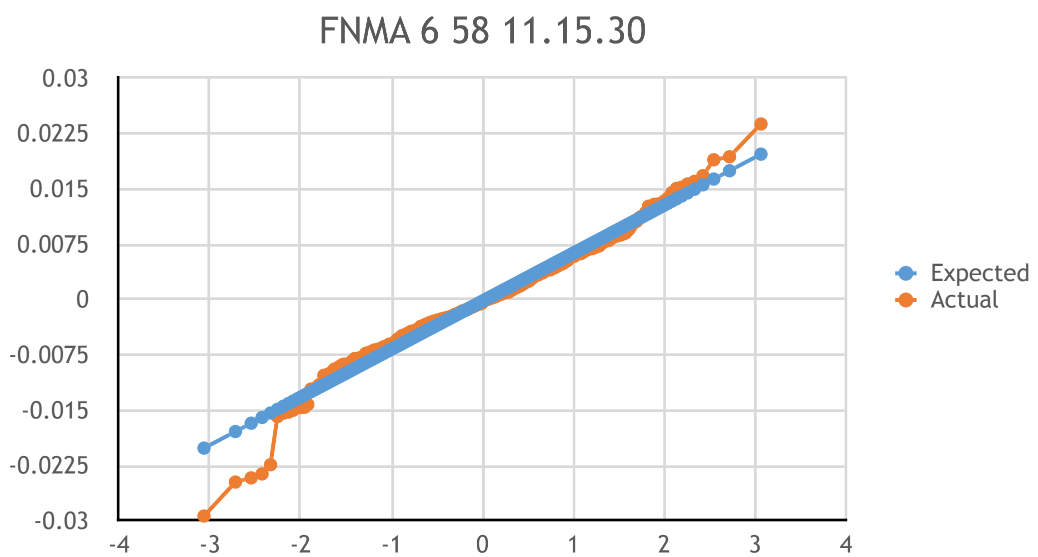
Descriptive Statistics: Box Plot

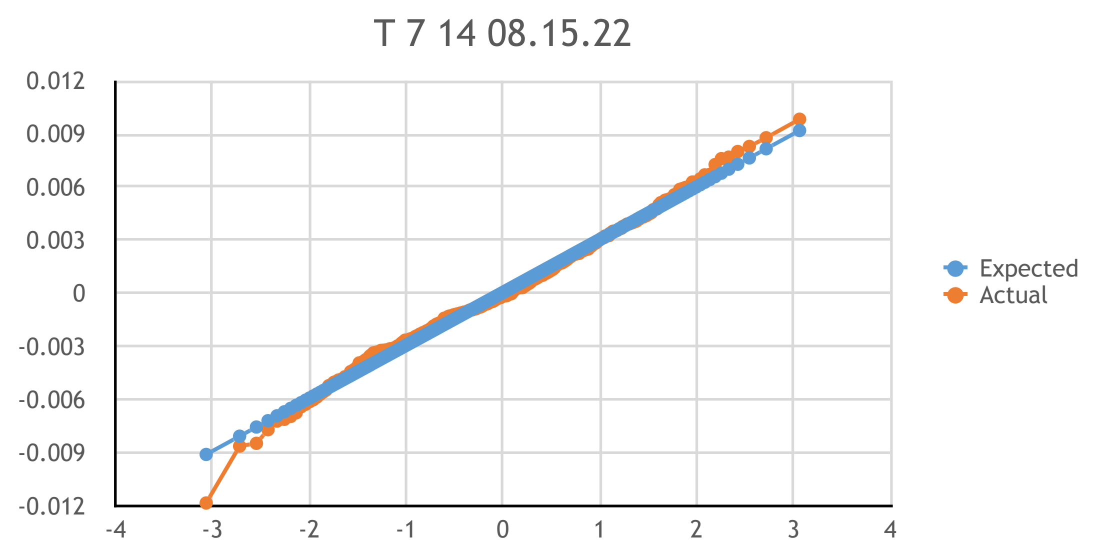


Descriptive Statistics: Q-Q plots

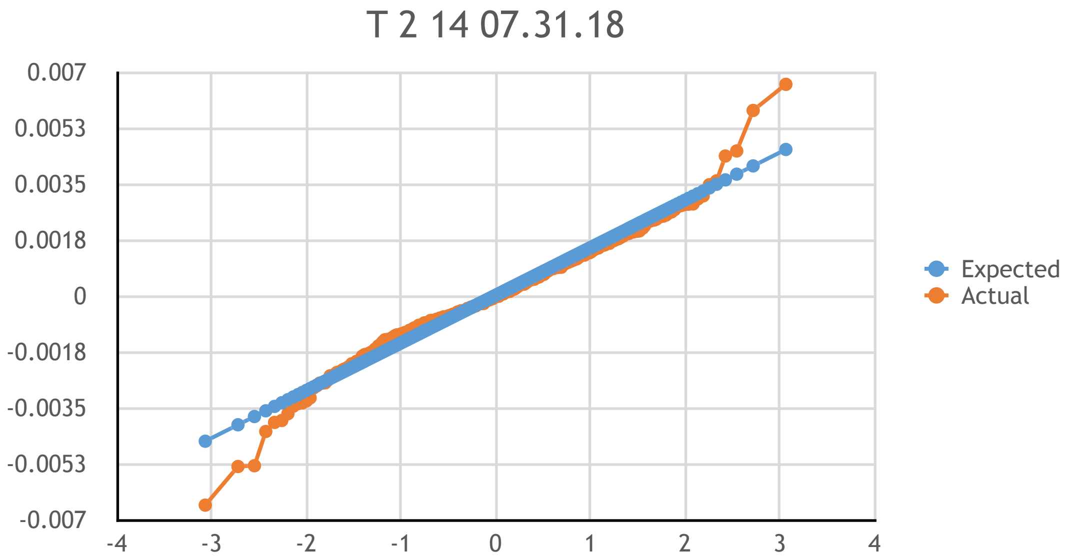


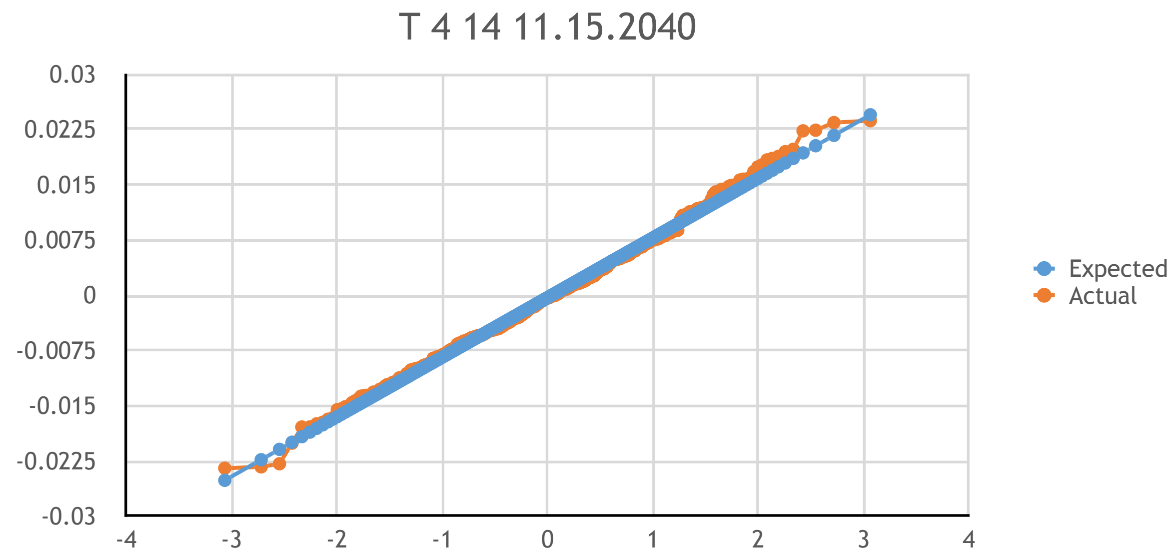


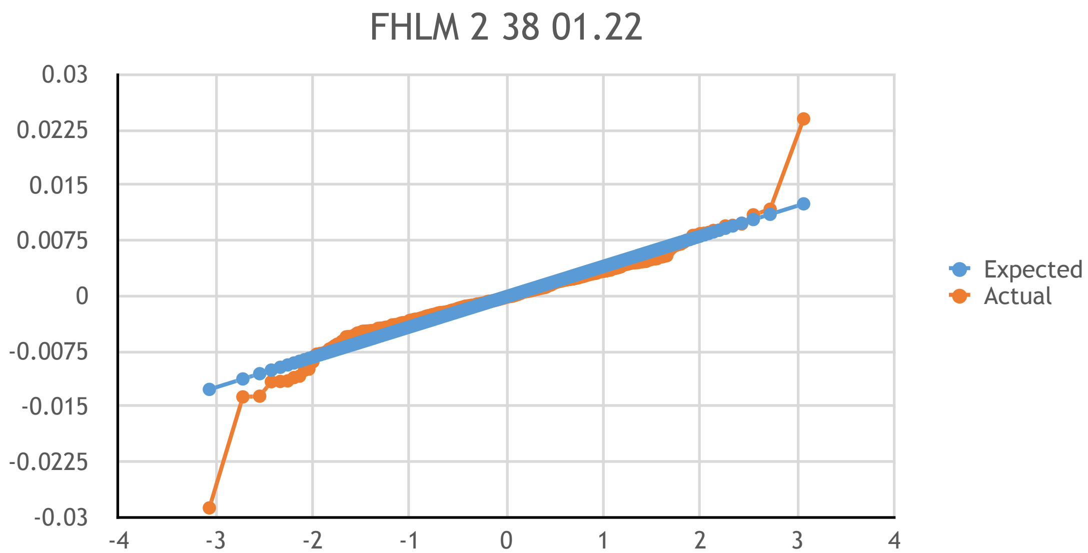


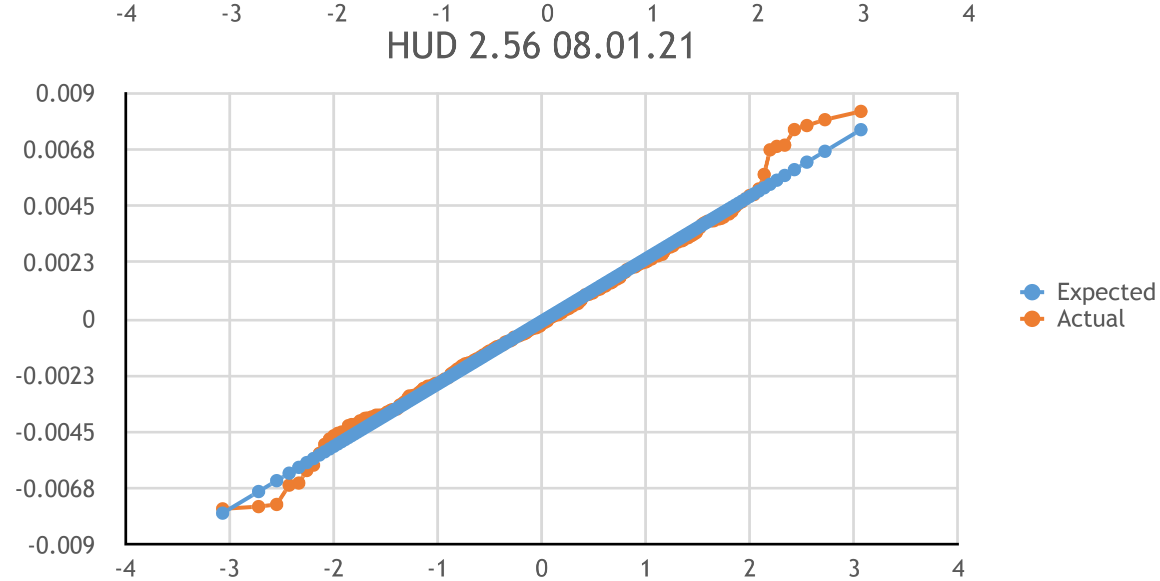


Descriptive Statistics: Q-Q Plots









Descriptive Statistics: Return Correlation

	XRX 4 12 5.15.21	IP 4 34 2.15.22	FNMA 6 58 11.15.30	T 7 14 08.15.22	T 2 14 07.31.18	T 4 14 11.15.204	FHLM 2 38 01.22	HUD 2.56 08.01.21
XRX 4 12 5.15.21	1.00000	0.08833	0.19668	0.22428	0.24595	0.17098	0.16918	0.19987
IP 4 34 2.15.22	0.08833	1.00000	0.21969	0.30606	0.26435	0.30683	0.21094	0.29086
FNMA 6 58 11.15.30	0.19668	0.21969	1.00000	0.68775	0.54706	0.74105	0.45615	0.50698
T 7 14 08.15.22	0.22428	0.30606	0.68775	1.00000	0.90048	0.88580	0.66710	0.78573
T 2 14 07.31.18	0.24595	0.26435	0.54706	0.90048	1.00000	0.68045	0.62219	0.73847
T 4 14 11.15.2040	0.17098	0.30683	0.74105	0.88580	0.68045	1.00000	0.59240	0.66521
FHLM 2 38 01.22	0.16918	0.21094	0.45615	0.66710	0.62219	0.59240	1.00000	0.54862
HUD 2.56 08.01.21	0.19987	0.29086	0.50698	0.78573	0.73847	0.66521	0.54862	1.00000

PCA Analysis

PCA Analysis

- Analyzing the principal components of the model through summary statistics
- Building histograms to gain a feel for the shape of the distribution
- Determining the autocorrelation function for each variable
- Fitting a model for the principal components

PCA Analysis

	A	B	C	D	E
1	Date	Portfolio	pca1	pca2	pca3
2	2013-11-21	0.270858	-0.15598	0.122195	-0.02235
3	2013-11-22	0.369259	0.004133	-0.05189	0.005935
4	2013-11-25	-0.00392	0.028928	-0.00496	0.009251
5	2013-11-26	0.339977	0.024036	-0.02346	-0.00698
6	2013-11-27	-0.25647	-0.04506	0.040418	-0.01479
7	2013-11-29	-0.03877	-0.0505	0.00962	0.000287
8	2013-12-02	-0.37129	-0.08154	7.19E-05	-0.02498
9	2013-12-03	0.176263	0.040785	0.010677	0.018889
10	2013-12-04	-0.31575	-0.10464	0.011308	-0.02254
11	2013-12-05	-0.2591	-0.03659	9.29E-05	-0.01024
12	2013-12-06	0.091499	0.03372	-0.0134	0.010611
13	2013-12-09	0.179391	0.001368	-0.02158	-0.00619
14	2013-12-10	0.292906	0.032707	-0.01172	0.004973
15	2013-12-11	-0.21927	-0.02166	0.007423	-0.00032
16	2013-12-12	-0.17907	-0.15487	-0.02867	-0.01514
17	2013-12-13	0.060009	0.020317	-0.00667	0.002667
18	2013-12-16	-0.24269	0.049222	0.004726	-0.0008
19	2013-12-17	0.646895	0.009123	0.055653	0.017533
20	2013-12-18	-0.52923	-0.02365	0.031396	-0.01656
21	2013-12-19	-0.22542	-0.1511	-0.06293	-0.01859

- Data collected for two years
- Portfolio Value is second variable
- Last three variables are principal components
- We want to have the portfolio value as a function of the 3 principal components

Summary Statistics

summary(pca.s$V2)

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-1.21300	-0.19810	0.02253	0.01506	0.25230	0.99490

summary(pca.s$V3)

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-0.326800	-0.057860	0.004868	0.000000	0.054200	0.381000

summary(pca.s$V4)

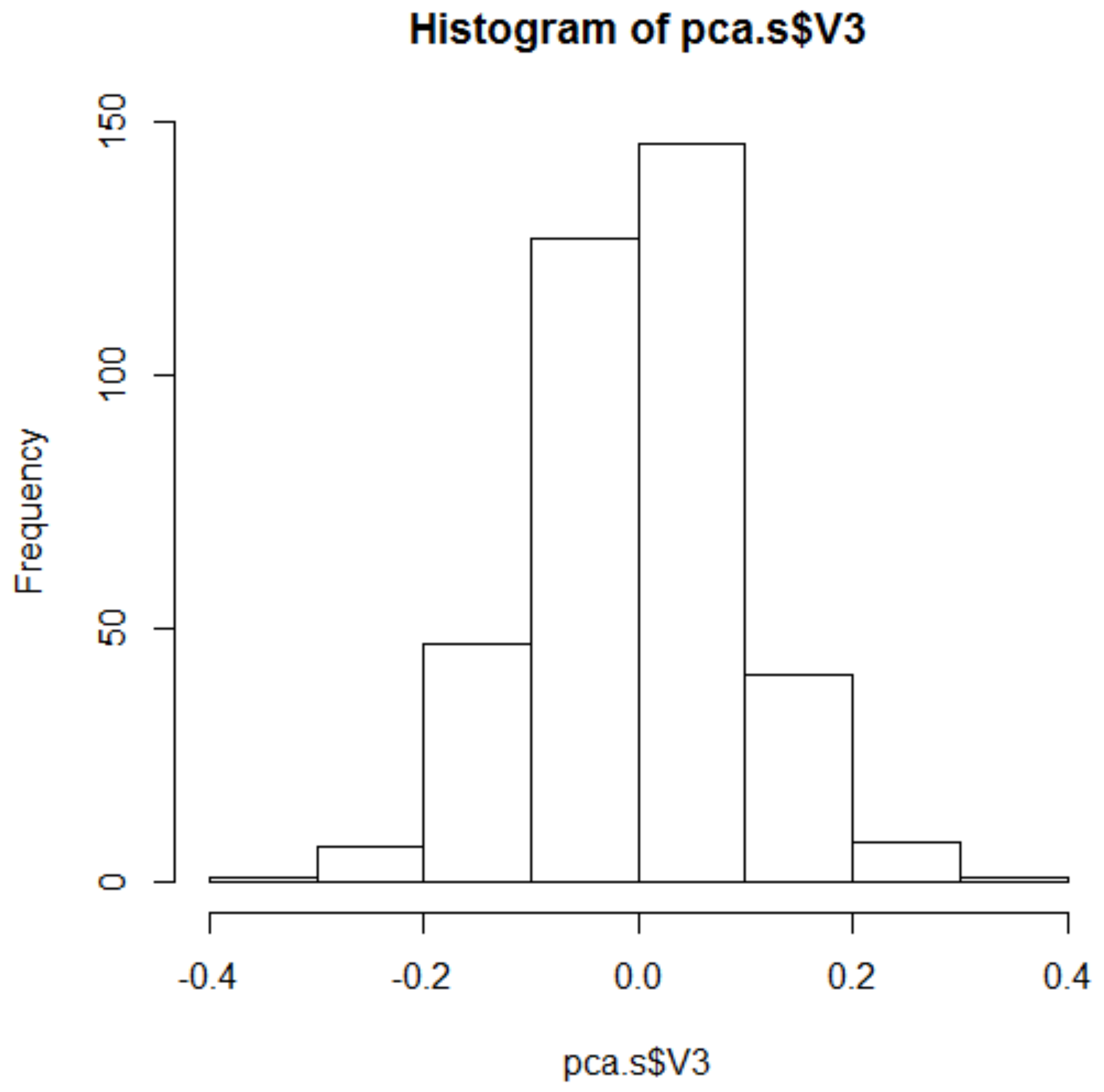
Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-0.110300	-0.016700	-0.001056	0.000000	0.018290	0.122200

summary(pca.s$V5)

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-0.0376300	-0.0054530	0.0006101	0.0000000	0.0056060	0.0351700

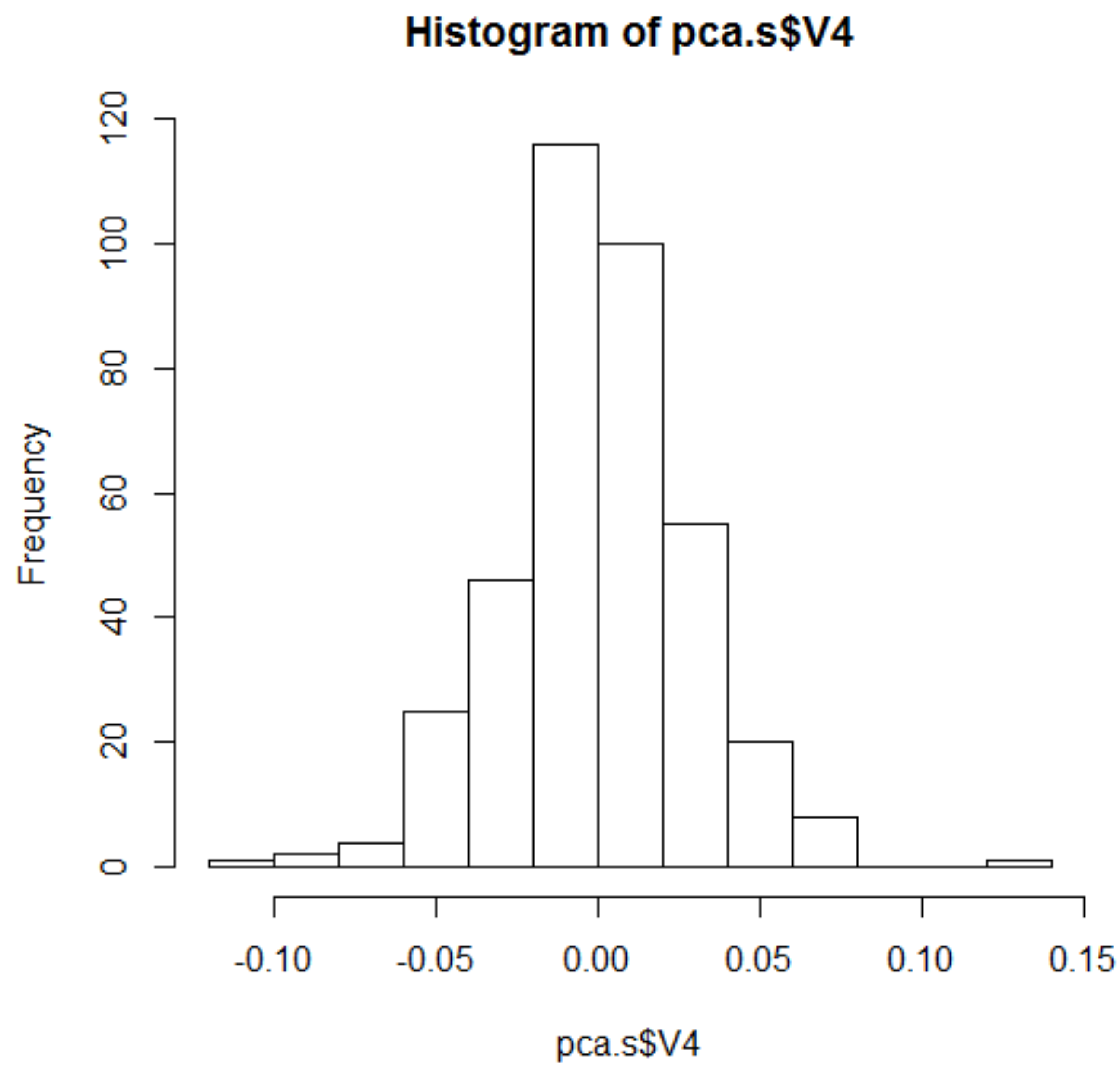
First Principal Component-Directional

- hist(pca.s$V3)
- We can describe this component as being heavily left skewed, with the bulk of the data being negative, implying negative direction.



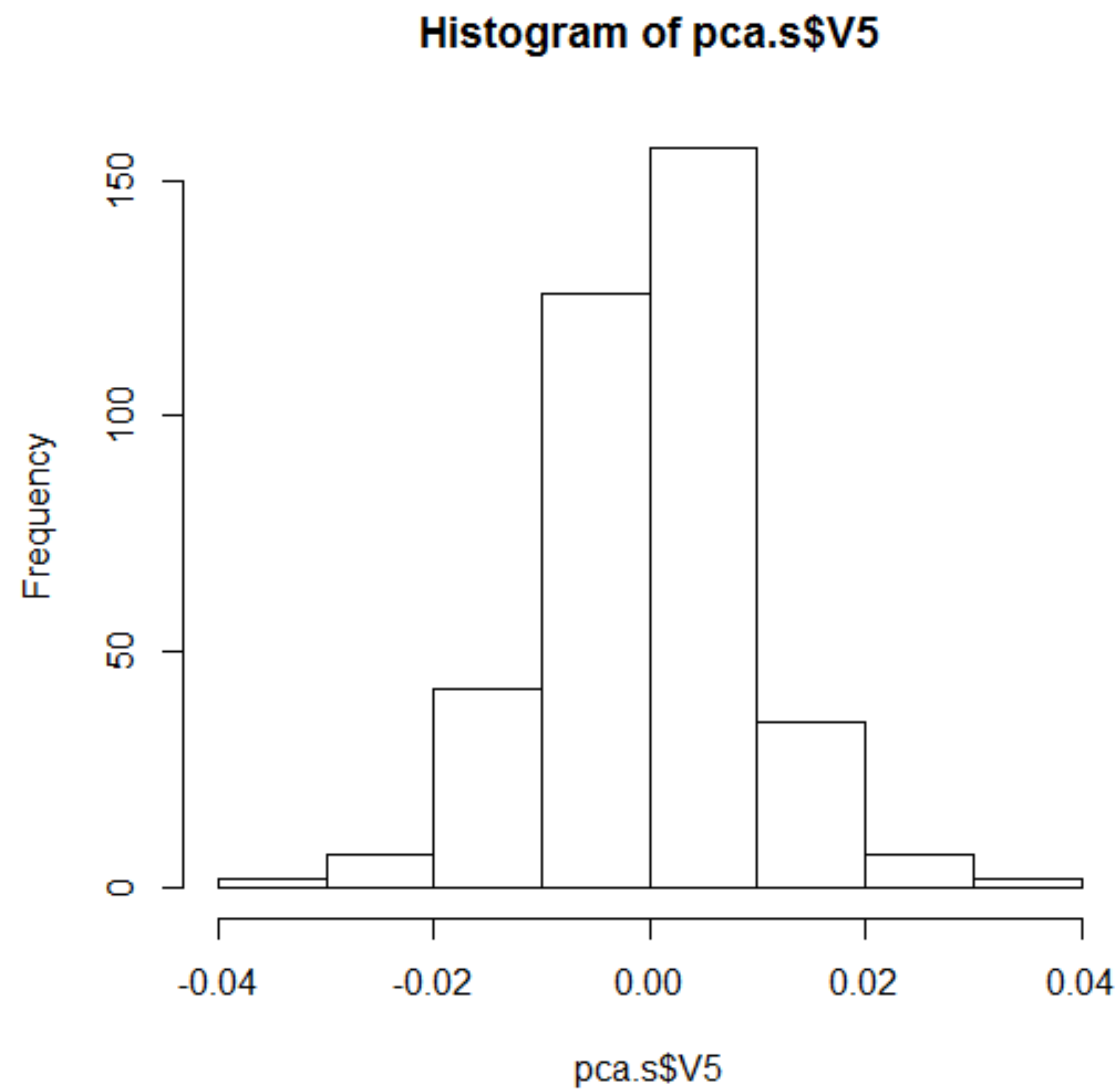
Second Principal Component-Slope

- hist(pca.s$V4)
- This component defines slope, which tends to be closer to a normal distribution, but still slightly skewed to the left.

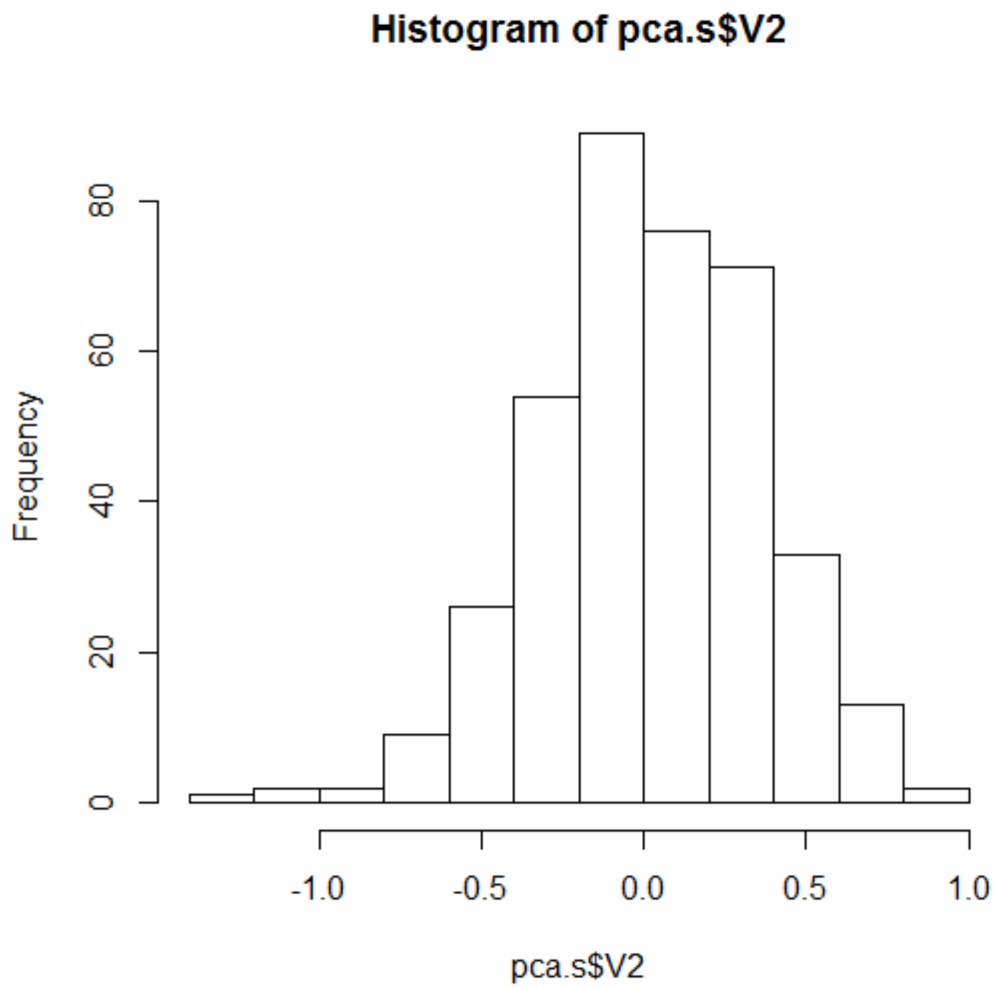


Third Principal Component- Curvature

- hist(pca.s$V5)
- The third component, curvature, appears to be skewed to the right, which implies an overall positive curvature.

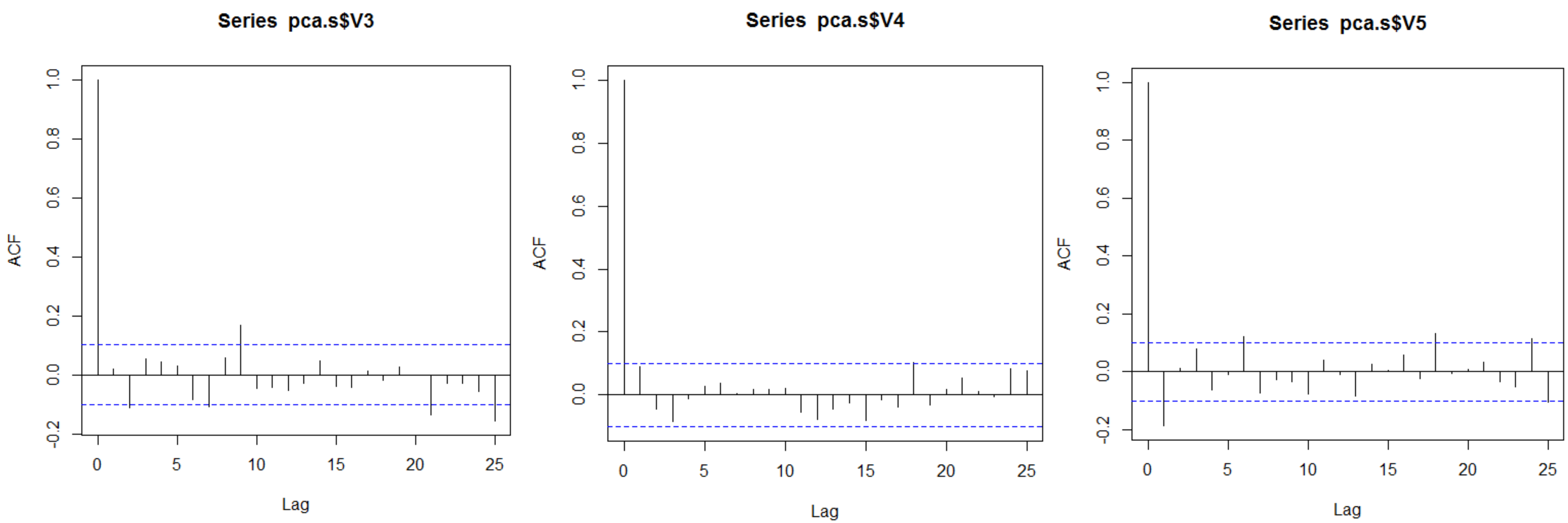


Portfolio Value

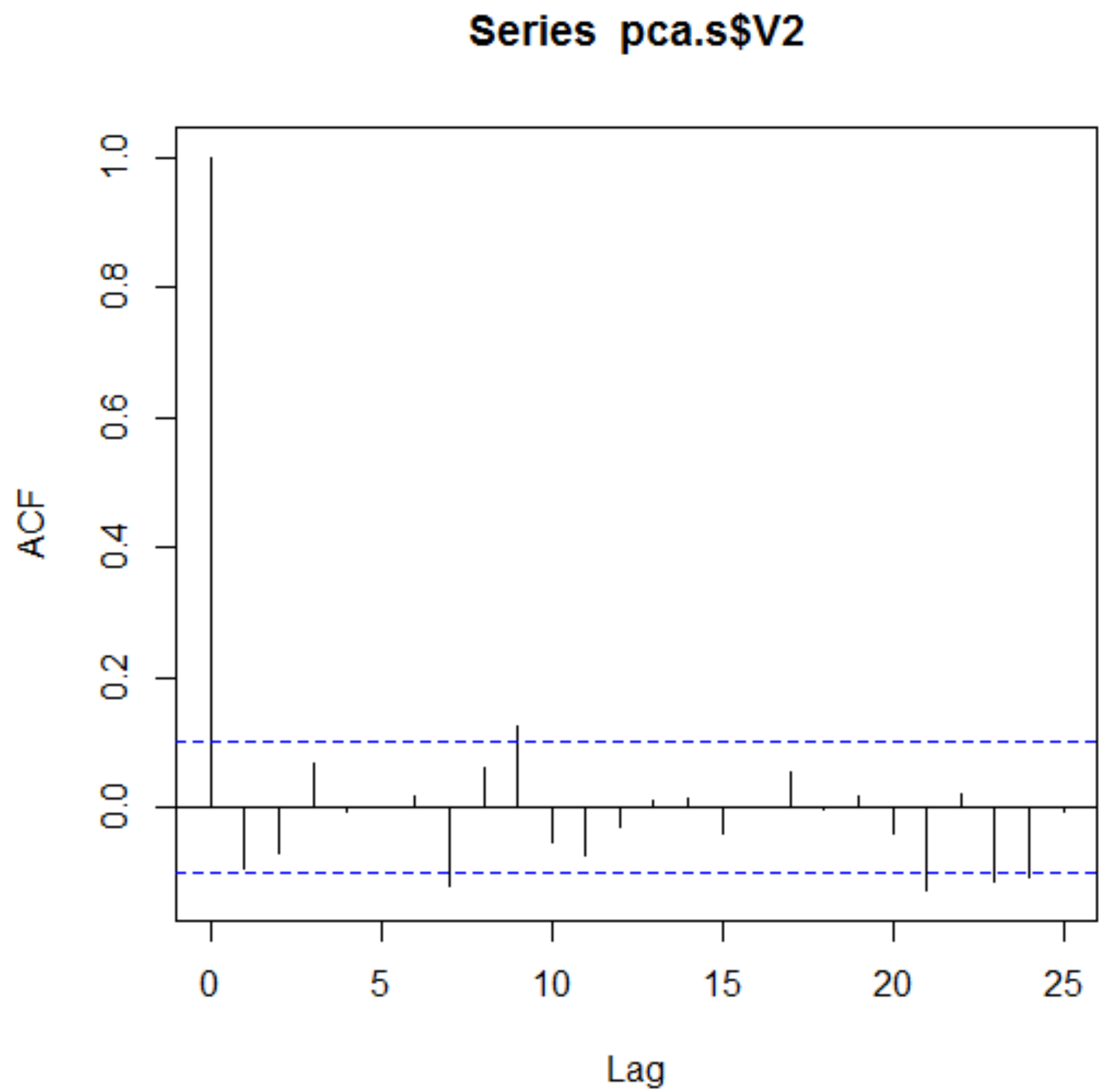


- hist(pca.s$V2)
- Given the general proportion of variation for the first two principal components, it is not surprising that the histogram shows a left skew for portfolio value.

Autocorrelation Function - Principal Components



Autocorrelation Function- Portfolio Value



Multivariate GARCH (lag=1)

```
> print(Bcoeff)
                portfolio_value.l1      pc1.l1       pc2.l1     pc3.l1         const
portfolio_value      -0.1566293269  0.34642604 -0.903310137 -0.2149591  1.682479e-02
pc1                   0.1083367859 -0.21632299 -0.062209644 -0.4587356 -1.268405e-03
pc2                  -0.0148398079  0.03993689  0.069953660 -0.1797497 -8.704620e-05
pc3                  -0.0002552395  0.01116540 -0.002625736 -0.1885876  6.835248e-05
```

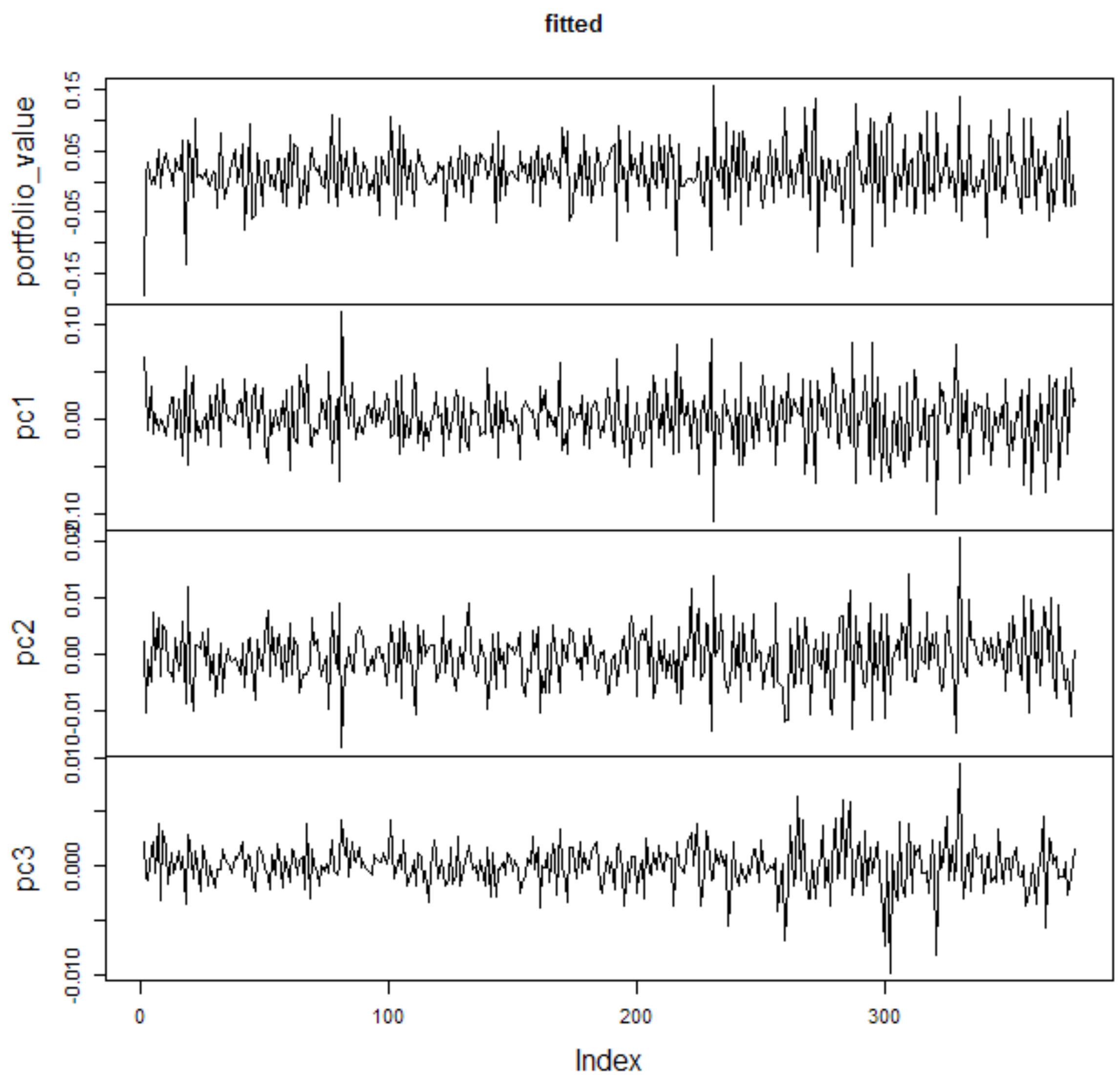
```
> print(Bcov)
$portfolio_value
                            [,1]          [,2]          [,3]          [,4]          [,5]
portfolio_value.l1  4.107433e-03 -0.0088820977  5.215514e-03  0.0066520643 -6.421149e-05
pc1.l1             -8.882098e-03  0.0545176914 -1.124155e-02 -0.0148372276  1.456608e-04
pc2.l1              5.215514e-03 -0.0112415507  3.977967e-01  0.0081433355 -7.697183e-05
pc3.l1              6.652064e-03 -0.0148372276  8.143335e-03  3.4480913504 -1.602266e-04
const              -6.421149e-05  0.0001456608 -7.697183e-05 -0.0001602266  3.199049e-04

$pc1
                            [,1]          [,2]          [,3]          [,4]          [,5]
portfolio_value.l1  2.761291e-04 -5.971140e-04  3.506217e-04  0.0004471963 -4.316726e-06
pc1.l1             -5.971140e-04  3.665044e-03 -7.557322e-04 -0.0009974577  9.792291e-06
pc2.l1              3.506217e-04 -7.557322e-04  2.674255e-02  0.0005474495 -5.174562e-06
pc3.l1              4.471963e-04 -9.974577e-04  5.474495e-04  0.2318037690 -1.077150e-05
const              -4.316726e-06  9.792291e-06 -5.174562e-06 -0.0000107715  2.150615e-05

$pc2
                            [,1]          [,2]          [,3]          [,4]          [,5]
portfolio_value.l1  2.587231e-05 -5.594745e-05  3.285200e-05  4.190069e-05 -4.044618e-07
pc1.l1             -5.594745e-05  3.434015e-04 -7.080940e-05 -9.345821e-05  9.175025e-07
pc2.l1              3.285200e-05 -7.080940e-05  2.505682e-03  5.129406e-05 -4.848379e-07
pc3.l1              4.190069e-05 -9.345821e-05  5.129406e-05  2.171918e-02 -1.009251e-06
const              -4.044618e-07  9.175025e-07 -4.848379e-07 -1.009251e-06  2.015049e-06

$pc3
                            [,1]          [,2]          [,3]          [,4]          [,5]
portfolio_value.l1  3.012527e-06 -6.514423e-06  3.825230e-06  4.878843e-06 -4.709482e-08
pc1.l1             -6.514423e-06  3.998507e-05 -8.244923e-06 -1.088211e-05  1.068324e-07
pc2.l1              3.825230e-06 -8.244923e-06  2.917572e-04  5.972590e-06 -5.645368e-08
pc3.l1              4.878843e-06 -1.088211e-05  5.972590e-06  2.528944e-03 -1.175154e-07
const              -4.709482e-08  1.068324e-07 -5.645368e-08 -1.175154e-07  2.346288e-07
```

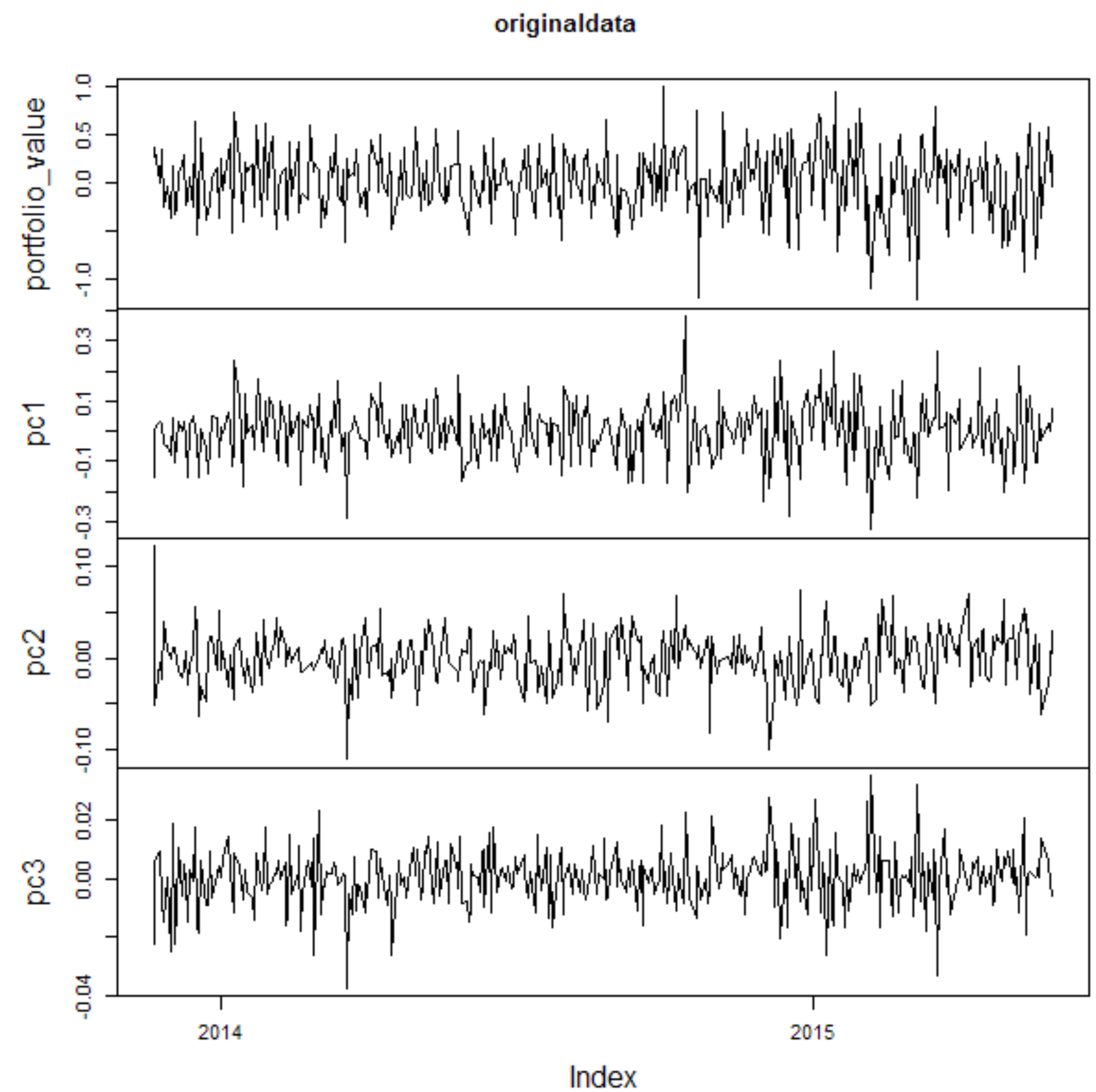
Multivariate GARCH (lag=1)

```
> print(tstat)
  portfolio_value        pc1         pc2        pc3
1      -2.4439248  6.5195833 -2.91749875 -0.1470559
2       1.4836853 -3.5732474  2.15512848  1.7657347
3      -1.4322087 -0.3804138  1.39748612 -0.1537235
4      -0.1157621 -0.9528011 -1.21968148 -3.7501060
5       0.9406743 -0.2735121 -0.06132068  0.1411119
```

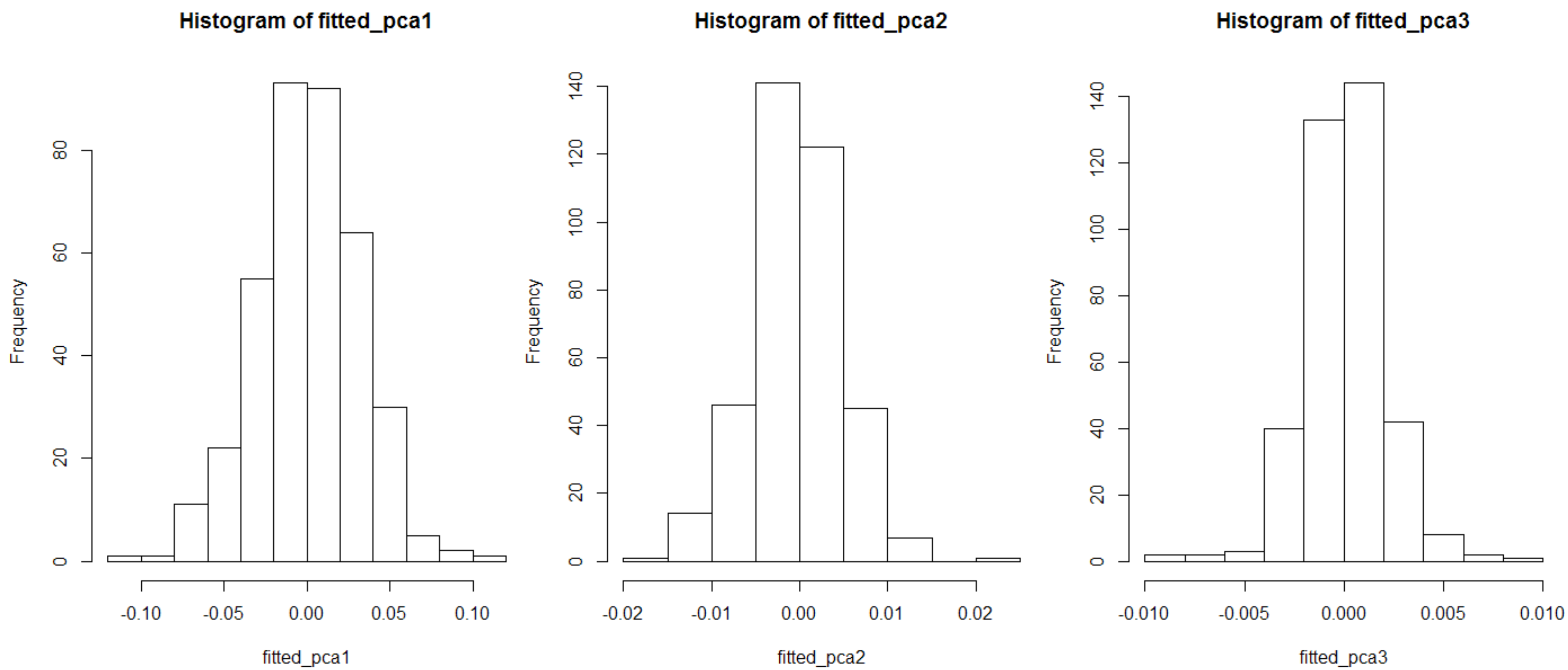
```
> print(pstat)
  portfolio_value          pc1         pc2          pc3
1      0.01452845 7.050294e-11 0.003528511 0.8830879125
2      0.13789242 3.525814e-04 0.031151782 0.0774403673
3      0.15208410 7.036383e-01 0.162267437 0.8778277611
4      0.90784107 3.406909e-01 0.222585643 0.0001767599
5      0.34687177 7.844596e-01 0.951103819 0.8877815560
```

Multivariate GARCH (lag=1)





Fitted Principal Components using MGarch Model



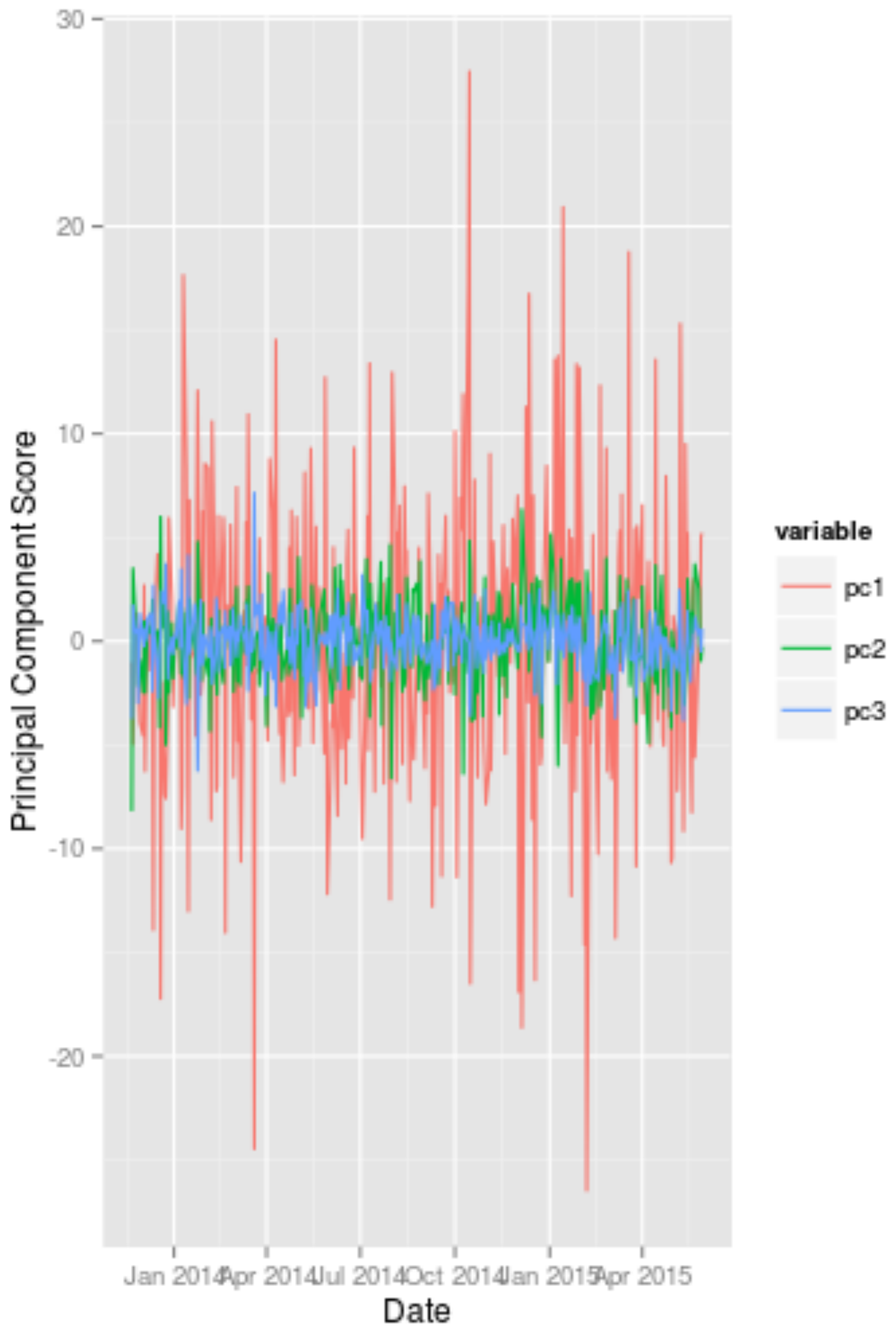
Monte Carlo Simulation of Value-at-Risk Calculation from Principal Components

Outline

- Obtain Principal Components
- Regress Portfolio Returns on Principal Components
- Fit distribution on principal components
- Take 100,000 samples from this distribution and use the regression to obtain simulated portfolio values
- Calculate VAR and Expected Shortfall on this

Regression

- Principal Components are extracted for change in US Swap yield.
- This is regressed with log returns of the portfolio.
- US Swap yields price in the expected LIBOR rate and the credit risk of the banks. It is therefore, a good proxy for interest rate risk.
- In-sample period is from 11/20/2013 to 05/29/2015 while out-sample is taken upto 6/1/2015.

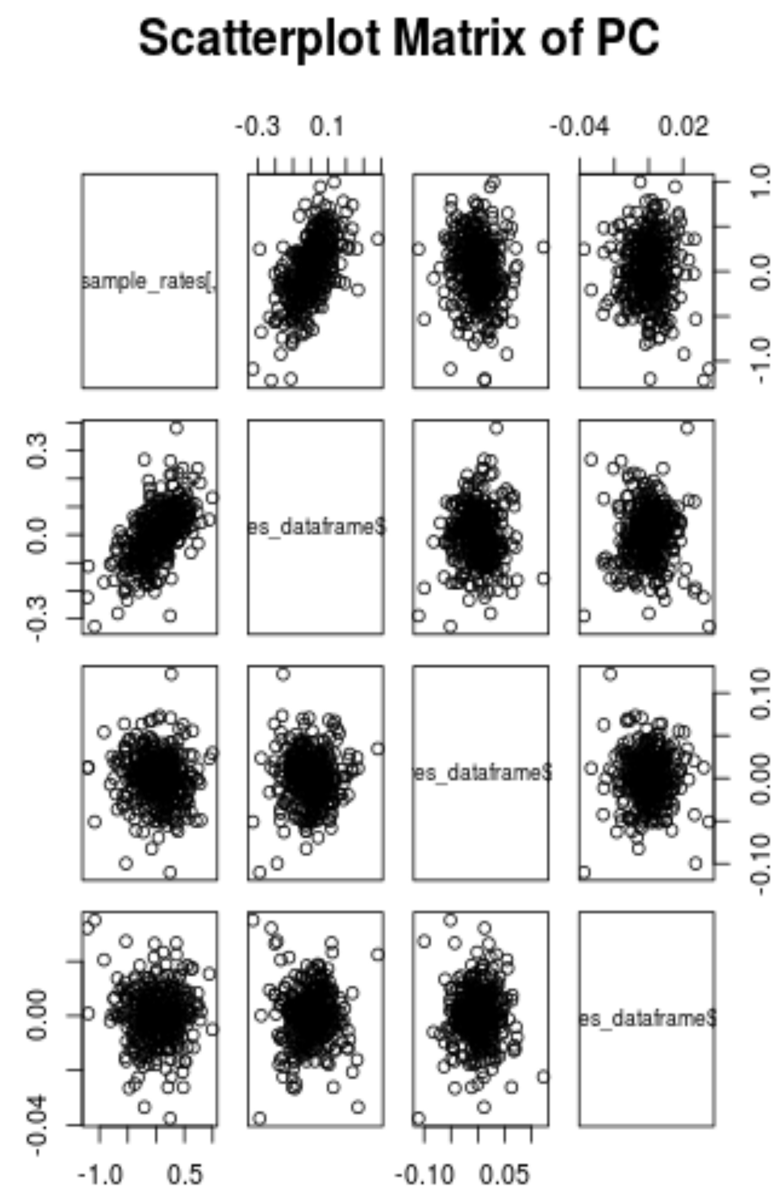


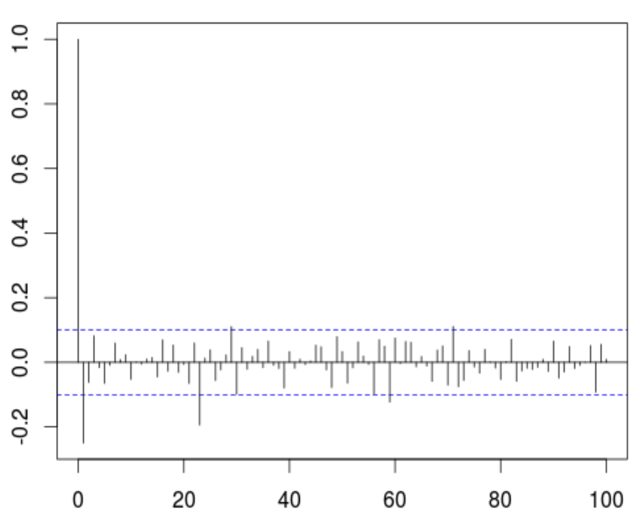
Regression

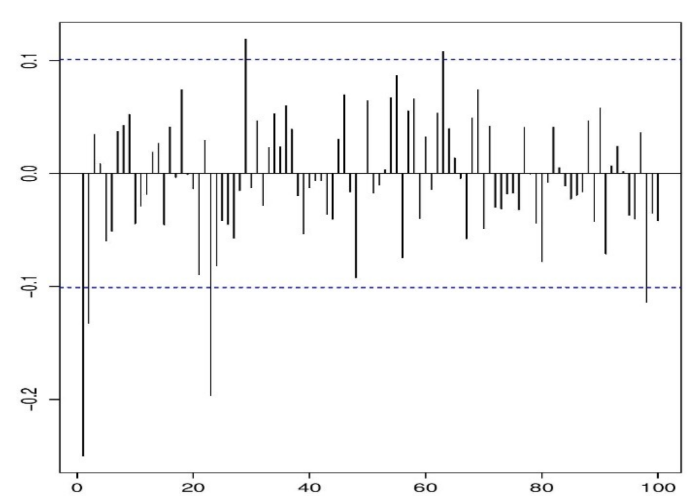
- Portfolio Value ~ PC1 + PC2

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	0.01296	0.01430	0.907	0.36518	
pc1[-1,]	2.19418	0.15087	14.544	< 2e-16	***
pc2[-1,]	-1.58862	0.51337	-3.094	0.00212	**

Residual standard error: 0.2775 on 374 degrees of freedom
Multiple R-squared: 0.3698, Adjusted R-squared: 0.3664
F-statistic: 109.7 on 2 and 374 DF, p-value: < 2.2e-16



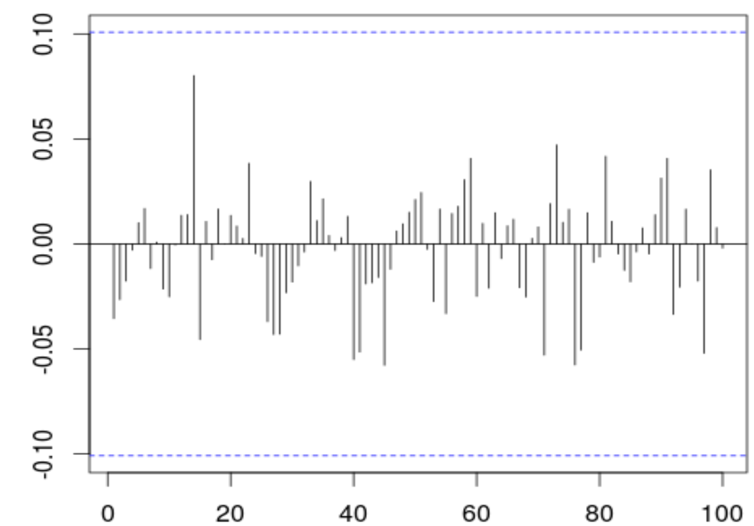


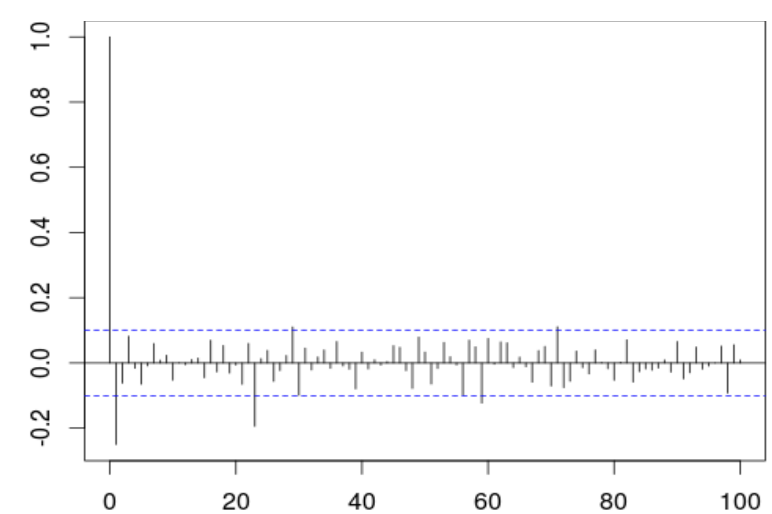


Regression

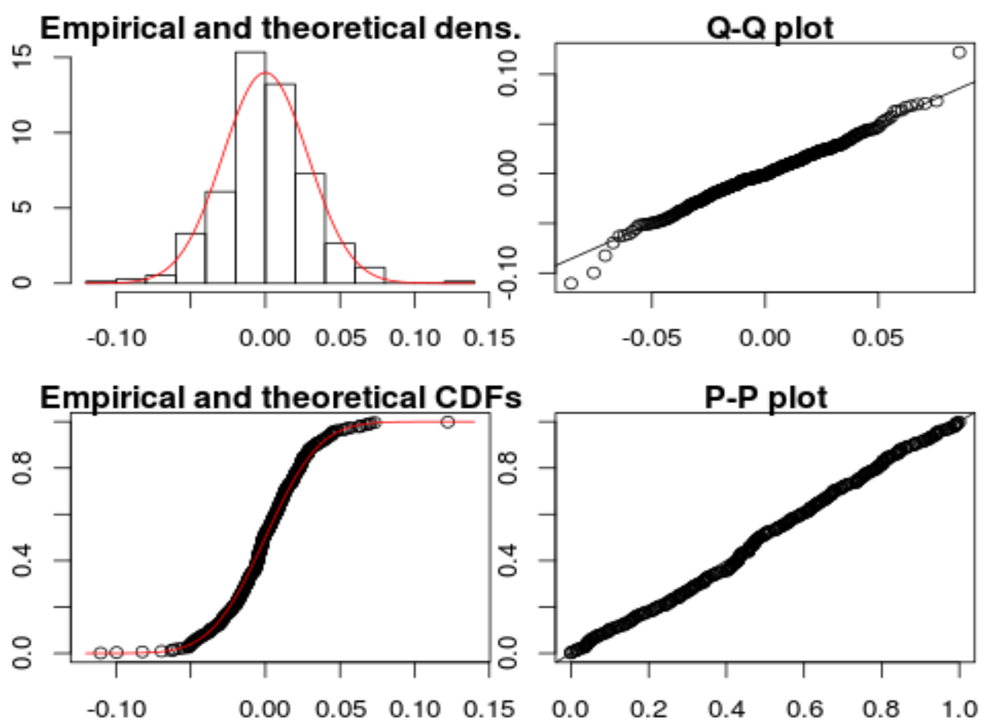
- Portfolio Value ~ PC1 + PC2 + Portfolio Value(-1)

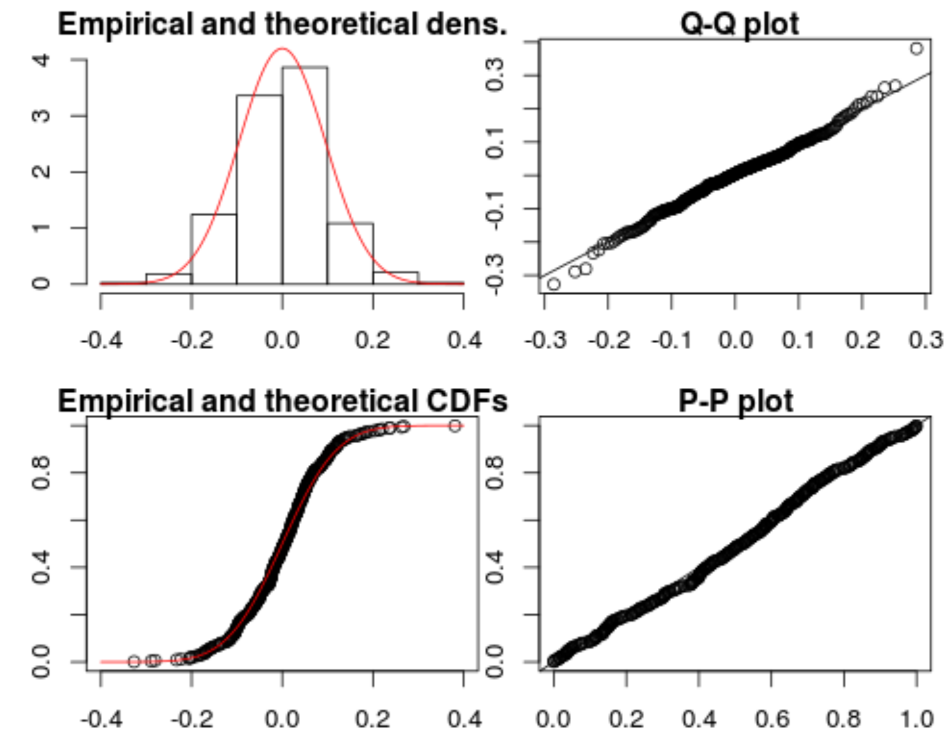
```
Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)          0.01721    0.01338   1.286    0.199
pc1[-1, ]            2.49631    0.14689  16.994  < 2e-16 ***
pc2[-1, ]           -2.01734    0.48360  -4.172 3.77e-05 ***
lag(portfolio, -1) -0.29699    0.04019  -7.391 9.69e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2596 on 373 degrees of freedom
Multiple R-squared:  0.4503,    Adjusted R-squared:  0.4459
F-statistic: 101.8 on 3 and 373 DF,  p-value: < 2.2e-16
```





Distribution of PC - Parametric





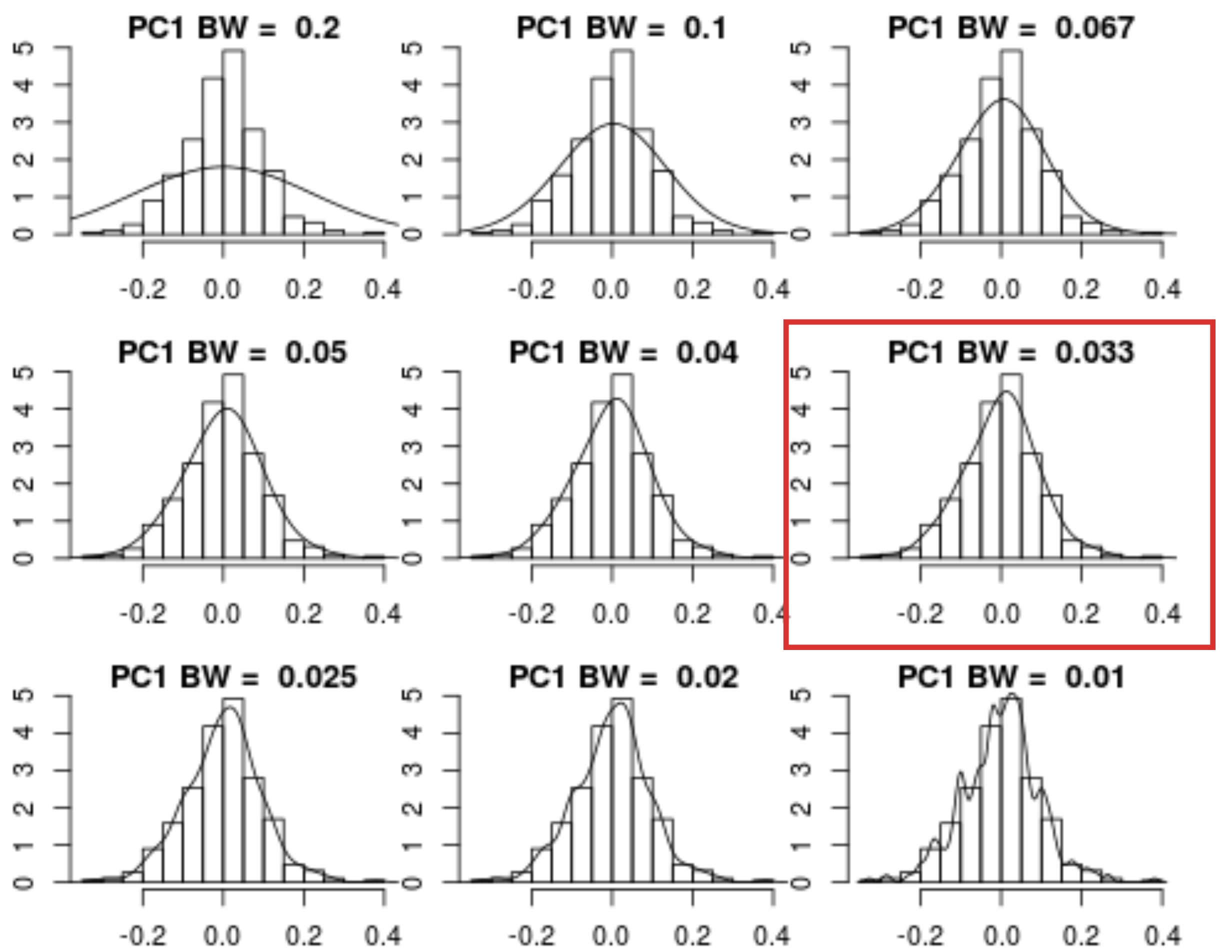
- PC1 Shapiro-Wilk normality testdata: W = 0.99136, p-value = 0.02646 **(FAIL)**
- PC2 Shapiro-Wilk normality testdata: W = 0.98851, p-value = 0.004493 **(FAIL)**
- Due to presence of fat tails, we go for t-distribution.

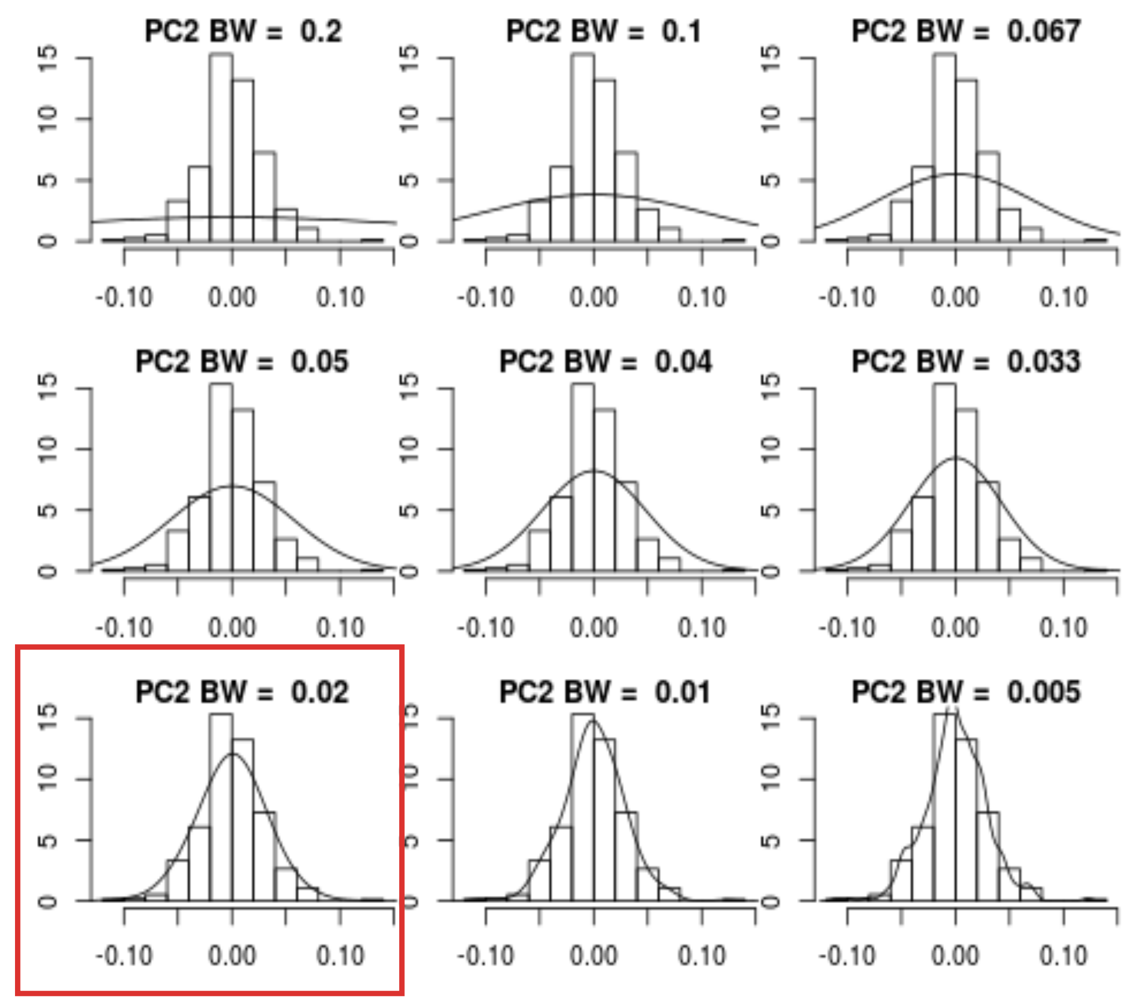
Distribution of PC - Parametric

	PC1	PC2
Mean	0.000654	0.000159
SD	0.095224	0.028545
DF	8.521494	7.830668

Citations:

Distribution of PC - Kernel





BW = 0.02296478

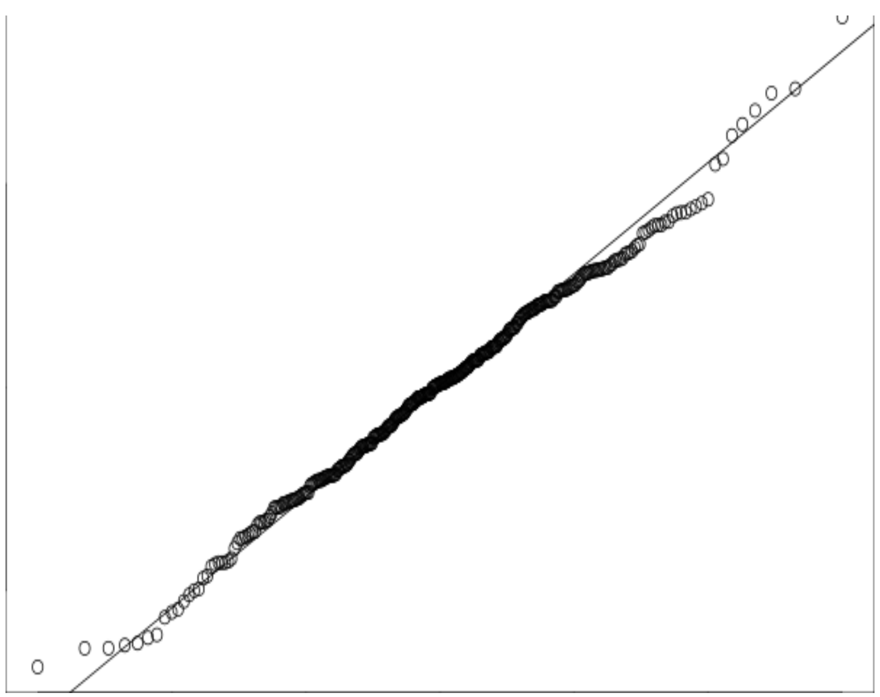
BW = 0.007170068

Sampling a Kernel

- Repeat N times:
  - Sample a point from data with replacement.
  - Pick a random number from the kernel centered at the sampled point, with a standard deviation equal to bandwidth.

Simulating Portfolio Returns

- Combine sampled data from the fitted distribution of PCs, with same coefficients as that of the regression.
- Portfolio Return = αPC1 + βPC2 + γ
- Introduce AR component to simulated values.
  - Portfolio Return = Portfolio Return + Portfolio Return(-1)
- Add normal i.i.d noise with same parameters as residual of regression



QQ plot for residuals

Kolmogrov–Smirnov Test

- The Kolmogrov-Smirnov test is a non-parametric test to find if two samples are from the same distribution.
- Parametric:
  - Data: Portfolio Return and Simulated Portfolio Return
  - Result: D = 0.096151, p-value = 0.001893
  - Alternative hypothesis: two-sided
- Non-parametric:
  - data: Portfolio Return and Simulated Portfolio Return
  - D = 0.096071, p-value = 0.001915
  - alternative hypothesis: two-sided
- Clearly, the simulated portfolio value has identical distribution with the original portfolio value.

Risk Calculation

- VAR and Expected Shortfall is calculated on the simulated portfolio returns.

	Parametric	Non-parametric	In-sample	Out-sample
		Without IDD		
Value at risk (95%)	4260.37	4295.92	5499.71	5765.29
Expected shortfall	5526.12	5565.13	7518.70	6992.21
		With IDD		
Value at risk (95%)	6128.78	6079.08	5499.71	5765.29
Expected shortfall	7743.63	7678.92	7518.70	6992.21

Conclusion

- VAR and ES calculated from factor model is computationally less intensive.
- The accuracy of risk numbers is dependent on variance of portfolio returns captured by the model.
- Principal Components allows us to use univariate distribution directly, as they are orthogonal. This keeps the model simple.
- This approach is well suited for large institutional funds and investment banks, with portfolios of thousands of securities.

Citations:

Future work

- Compare PCA based approach to Copular approach for measuring risk.
- Apply semi-parametric distributions to measure risk.

Citations: