

CyberSecurity: Principle and Practice

BSc Degree in Computer Science
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Lesson 4: Cryptographic Tools pt.1

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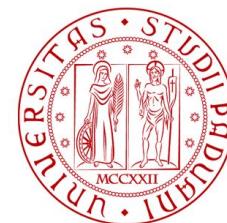
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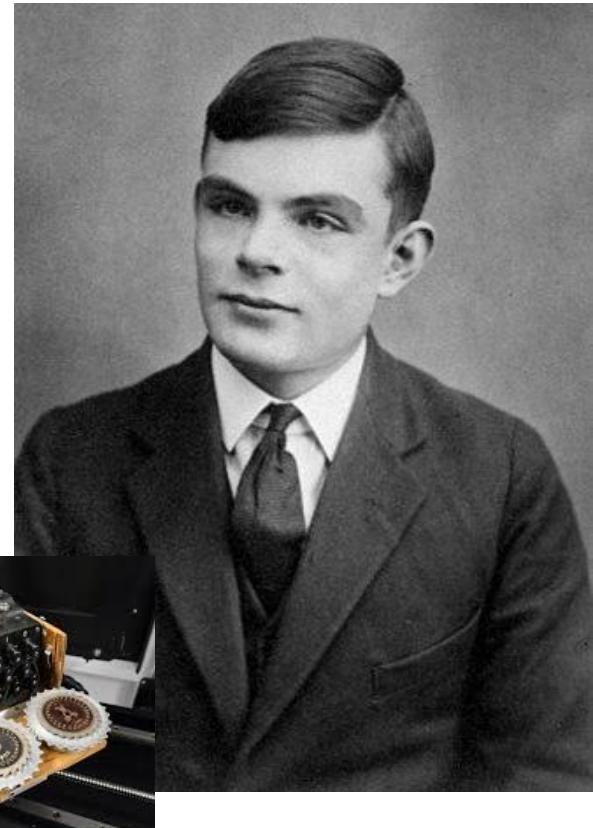
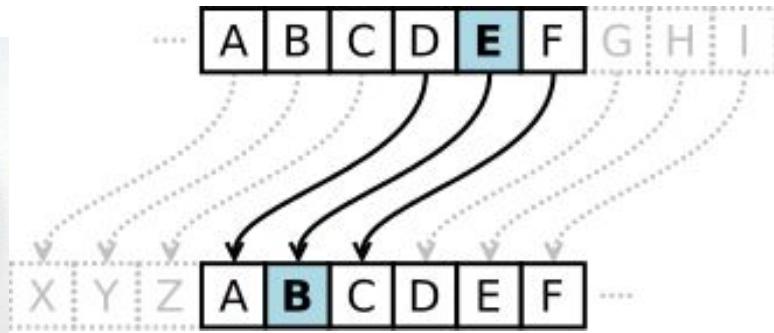


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DIPARTIMENTO 1
MATEMATICA

Historical Facts

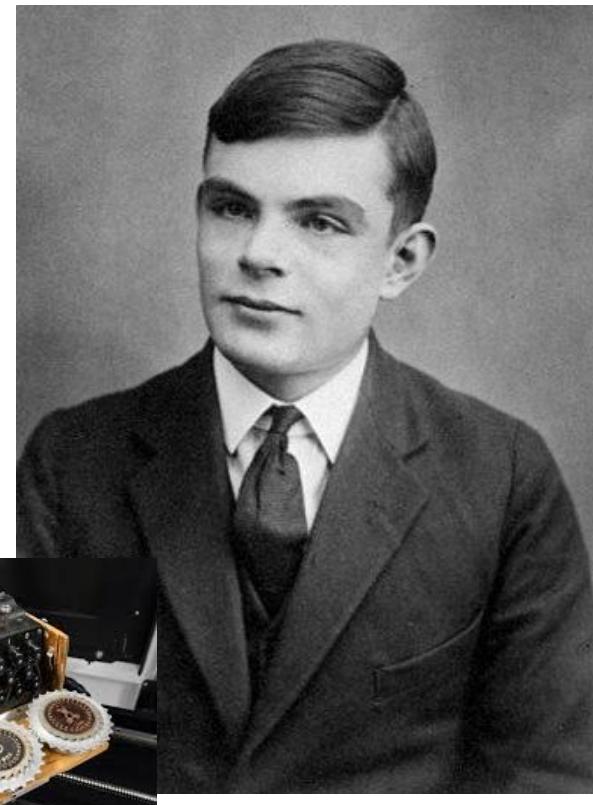
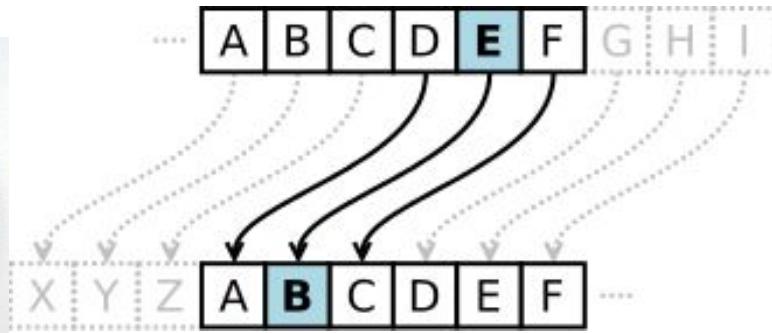


Ceasar Cipher: private correspondence (~50BC)



Alan Turing: decryption of German's ciphers during WWII (1940s)

Historical Facts



Ceasar Cipher: private correspondence (~50BC)



Alan Turing: decryption of German's ciphers during WWII (1940s)

Introduction



- Cryptographic algorithms important element in security services
- Review various types of elements
 - symmetric encryption
 - public-key (asymmetric) encryption
 - secure hash functions
- Example of encryption



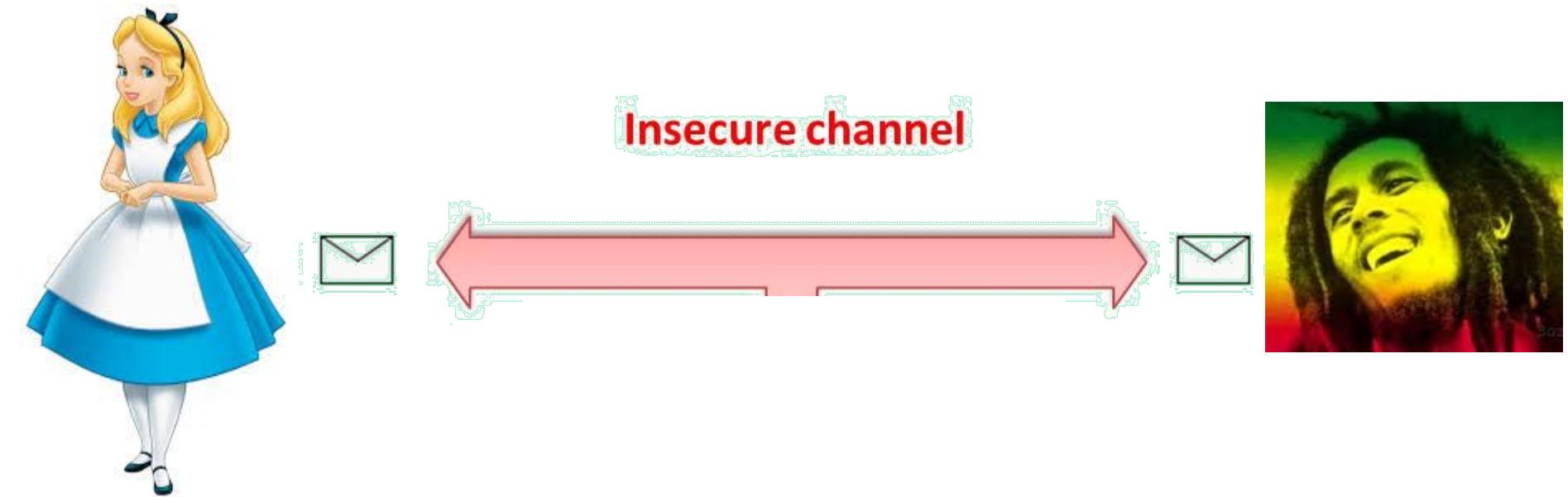
Encryption



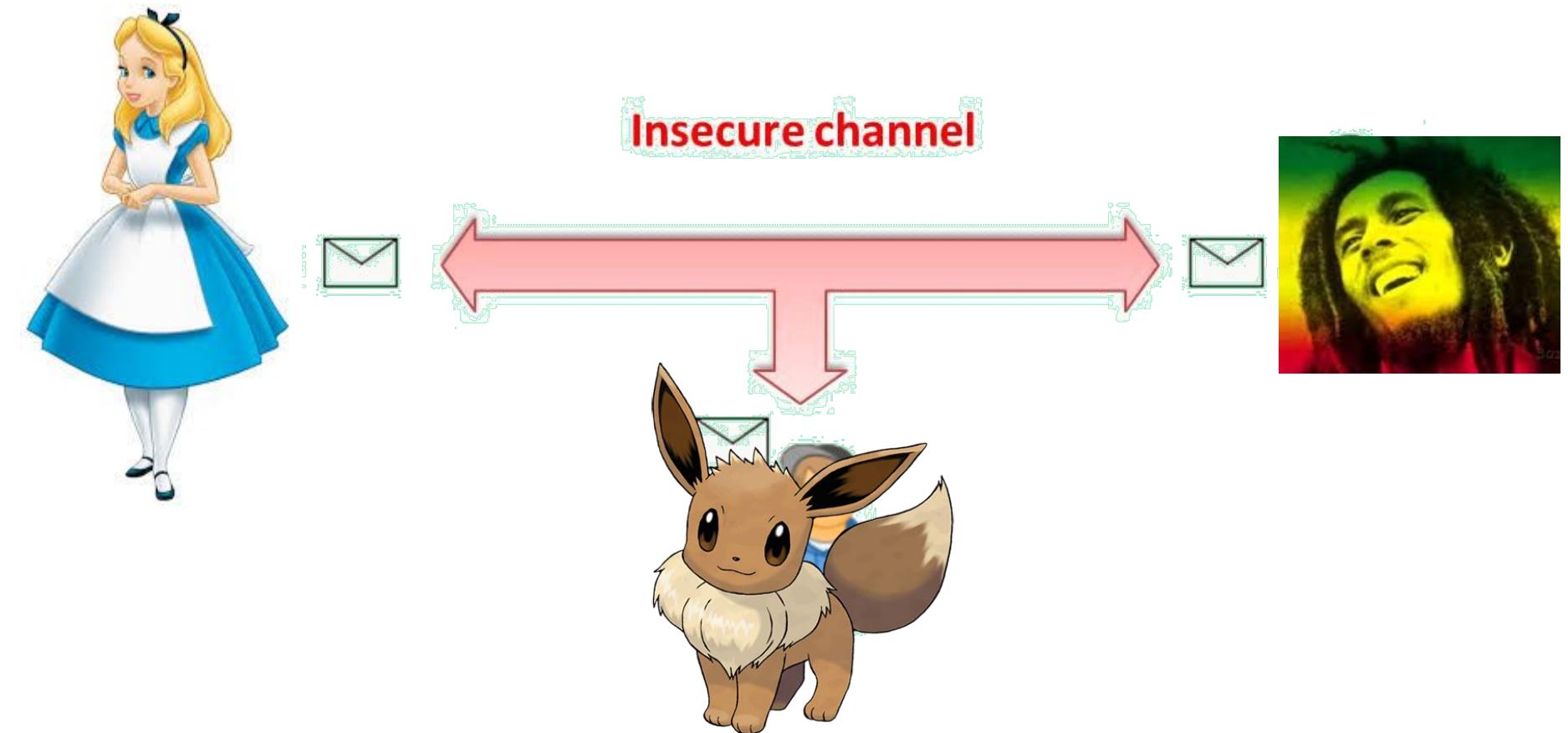
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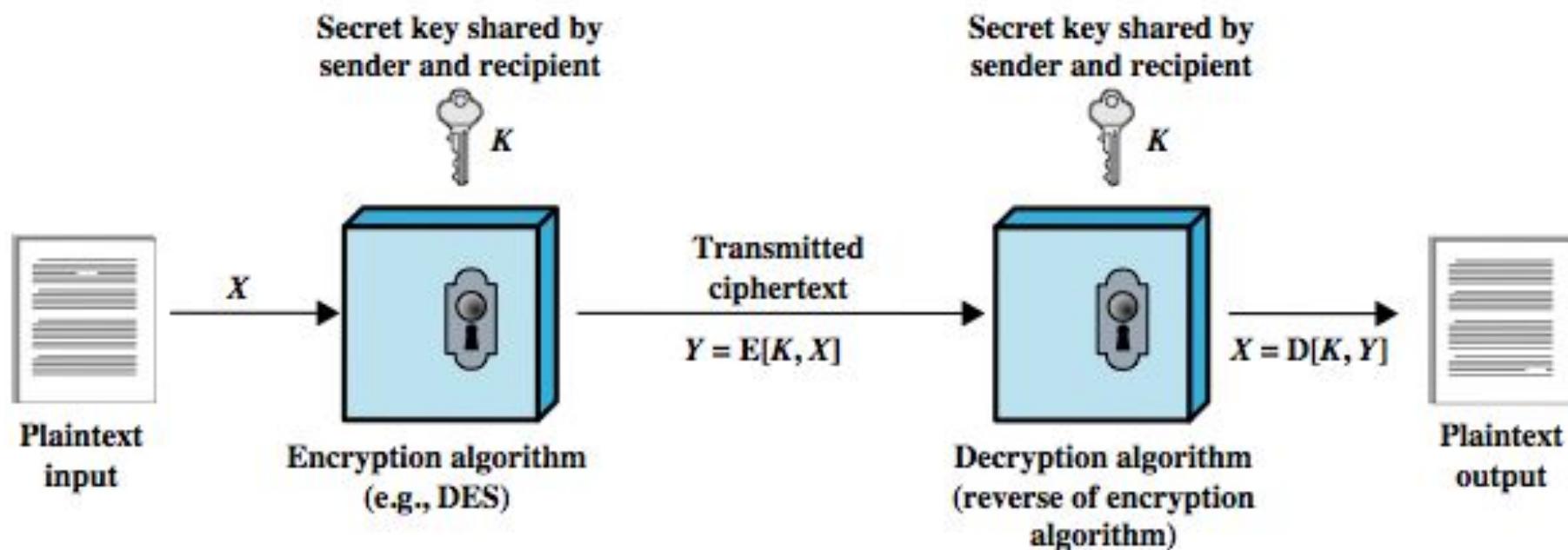
Encryption



Encryption



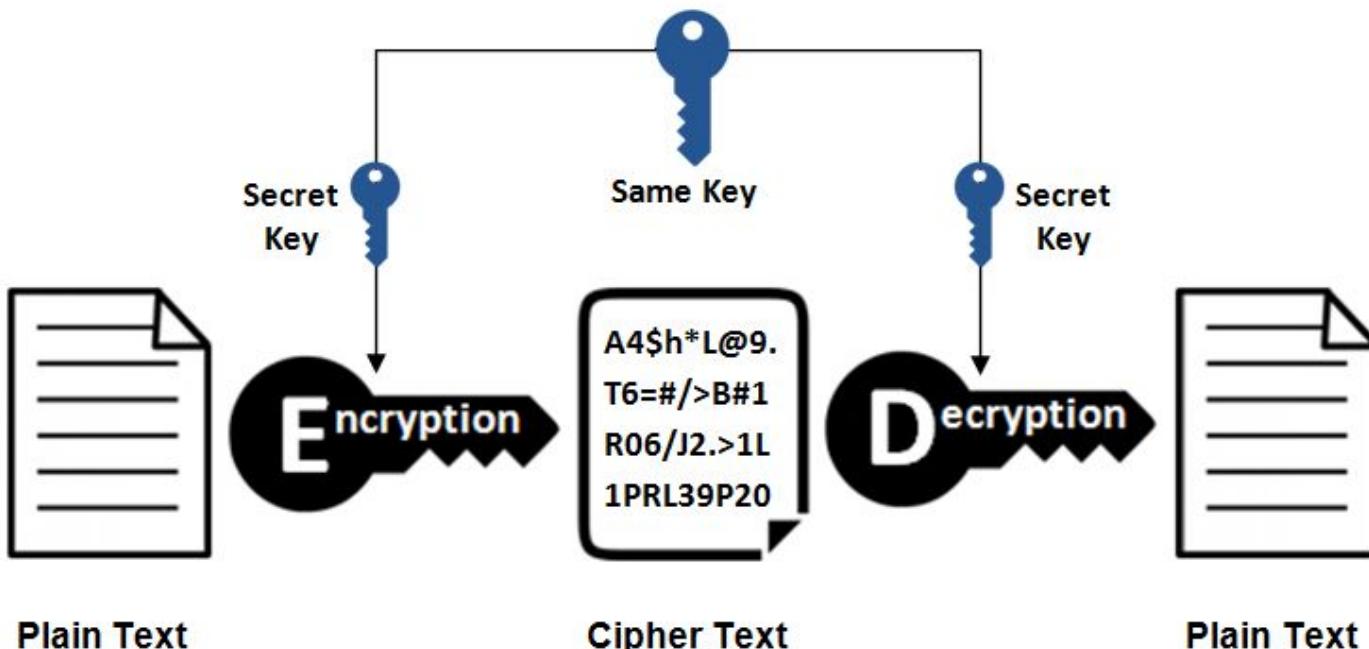
Symmetric Encryption



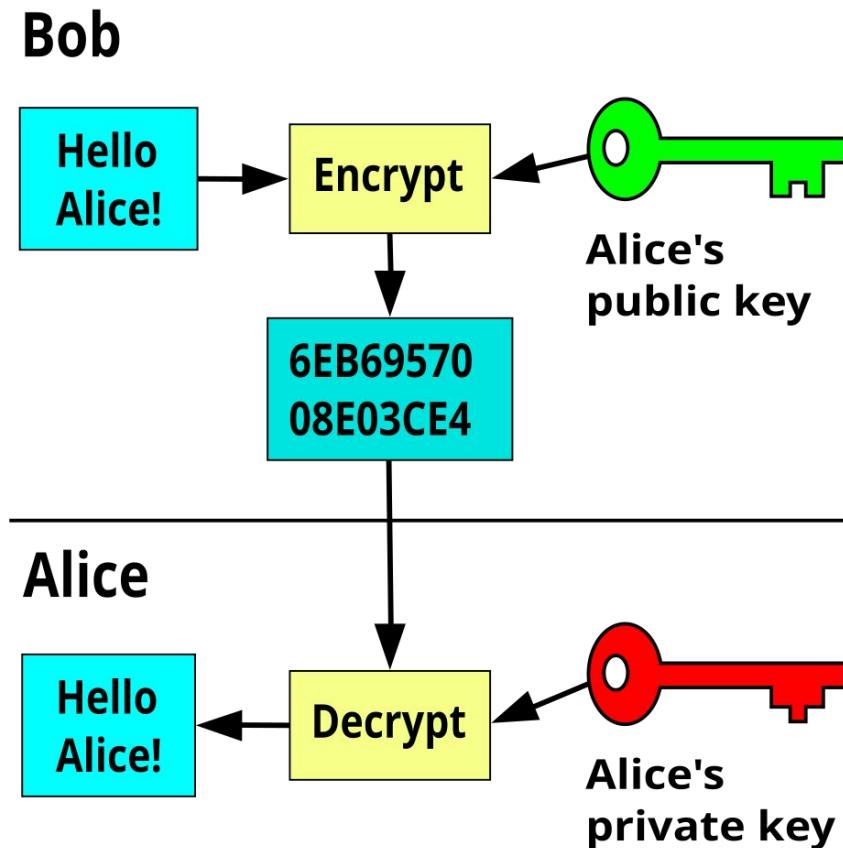
Symmetric Encryption



Symmetric Encryption



Asymmetric Encryption



Symmetric Encryption - Threats

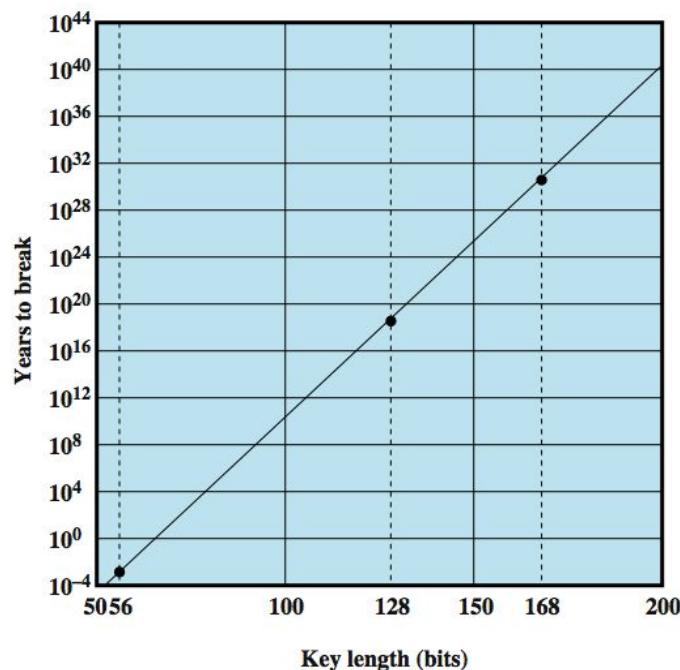


- Cryptanalysis- breaking cryptographic systems or algorithms (*focused on identifying weaknesses*)
 - Rely on nature of the algorithm
 - Plus some knowledge of plaintext characteristics
 - Even some sample plaintext-ciphertext pairs
 - Exploits characteristics of algorithm to deduce specific plaintext or key
- Brute-force attack
 - Try all possible keys on some ciphertext until get an intelligible translation into plaintext

Exhaustive Key Search



Key Size (bits)	Number of Alternative Keys	Time Required at 1 Decryption/ μ s	Time Required at 10^6 Decryptions/ μ s
32	$2^{32} = 4.3 \times 10^9$	$2^{31} \mu\text{s} = 35.8$ minutes	2.15 milliseconds
56	$2^{56} = 7.2 \times 10^{16}$	$2^{55} \mu\text{s} = 1142$ years	10.01 hours
128	$2^{128} = 3.4 \times 10^{38}$	$2^{127} \mu\text{s} = 5.4 \times 10^{24}$ years	5.4×10^{18} years
168	$2^{168} = 3.7 \times 10^{50}$	$2^{167} \mu\text{s} = 5.9 \times 10^{36}$ years	5.9×10^{30} years
26 characters (permutation)	$26! = 4 \times 10^{26}$	$2 \times 10^{26} \mu\text{s} = 6.4 \times 10^{12}$ years	6.4×10^6 years



Symmetric Encryption



	DES	Triple DES	AES
Plaintext block size (bits)	64	64	128
Ciphertext block size (bits)	64	64	128
Key size (bits)	56	112 or 168	128, 192, or 256

DES = Data Encryption Standard

AES = Advanced Encryption Standard

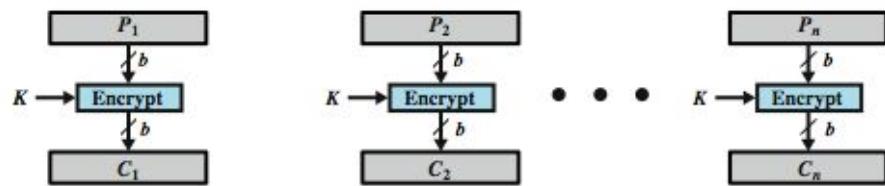
DES and Triple-DES



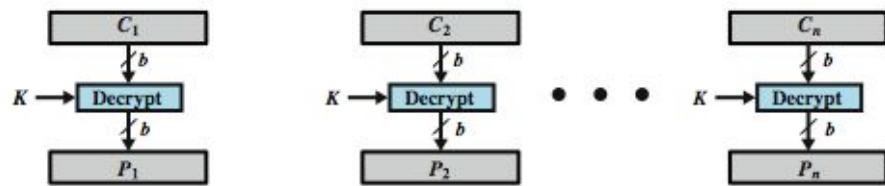
- Data Encryption Standard (DES) is the most widely used encryption scheme
 - Uses 64 bit plaintext block and 56 bit key to produce a 64 bit ciphertext block
 - Concerns about algorithm & use of 56-bit key
- Triple-DES
 - Repeats basic DES algorithm three times
 - Using either two or three unique keys
 - Much more secure but also much slower

Block vs. Stream Ciphers

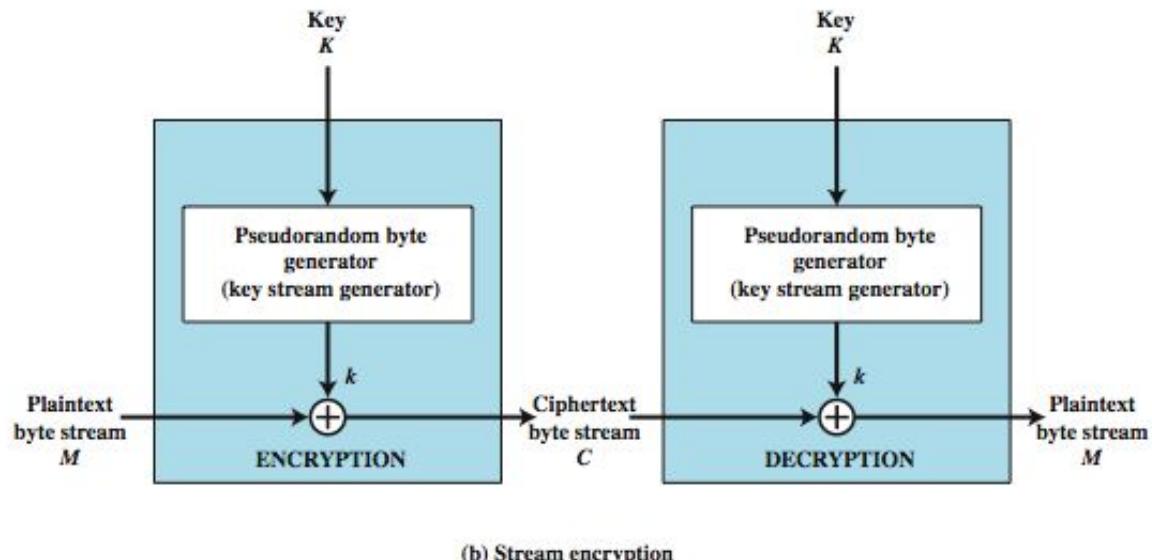
Encryption



Decryption



(a) Block cipher encryption (electronic codebook mod



(b) Stream encryption

Stream Ciphers



$$P = P_1 P_2 P_3, \dots$$

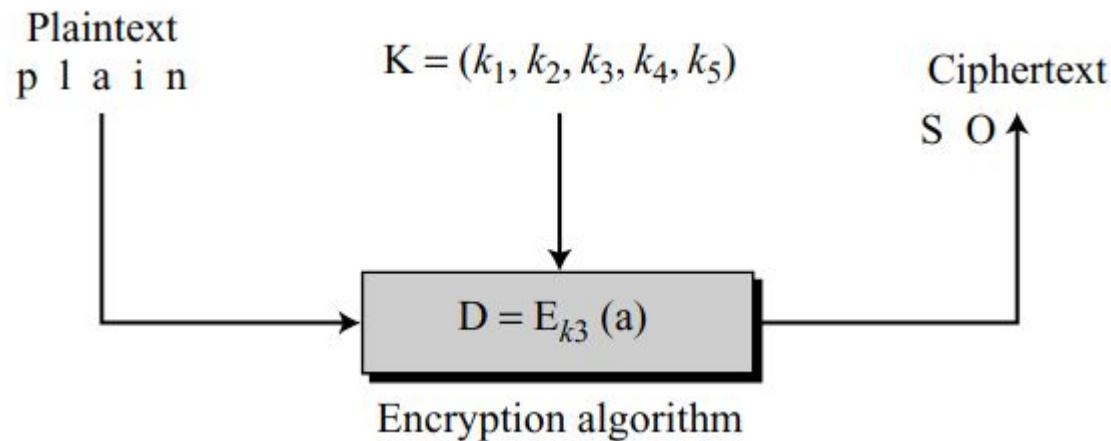
$$C = C_1 C_2 C_3, \dots$$

$$K = (k_1, k_2, k_3, \dots)$$

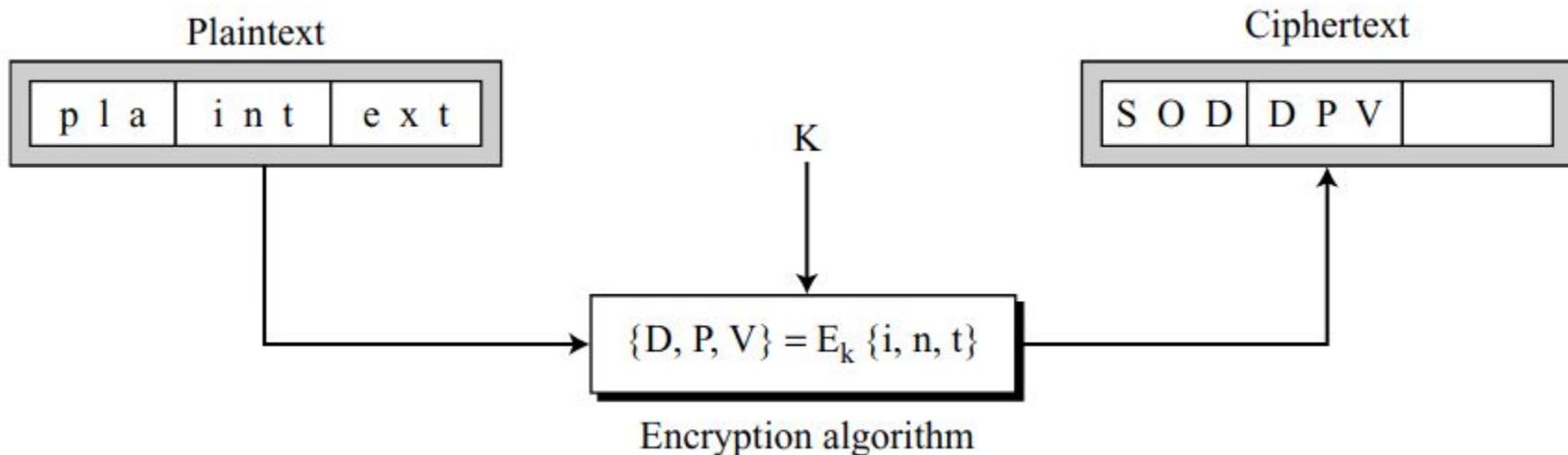
$$C_1 = E_{k1}(P_1)$$

$$C_2 = E_{k2}(P_2)$$

$$C_3 = E_{k3}(P_3) \dots$$



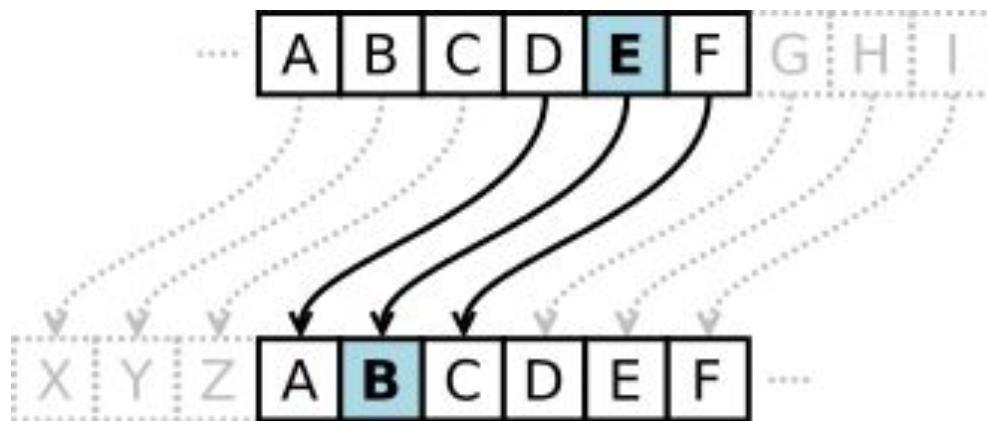
Block Ciphers



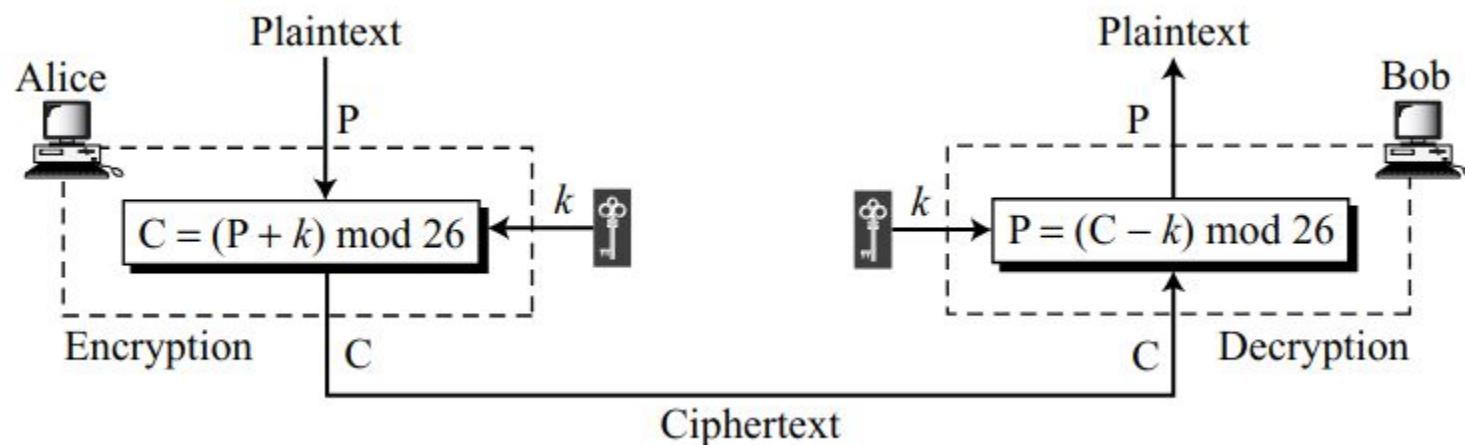
- a group of plaintext symbols of size m ($m > 1$) are encrypted together creating a group of ciphertext of the same size.
- **a single key is used** to encrypt the whole block.

Example 1 - Caesar Cipher

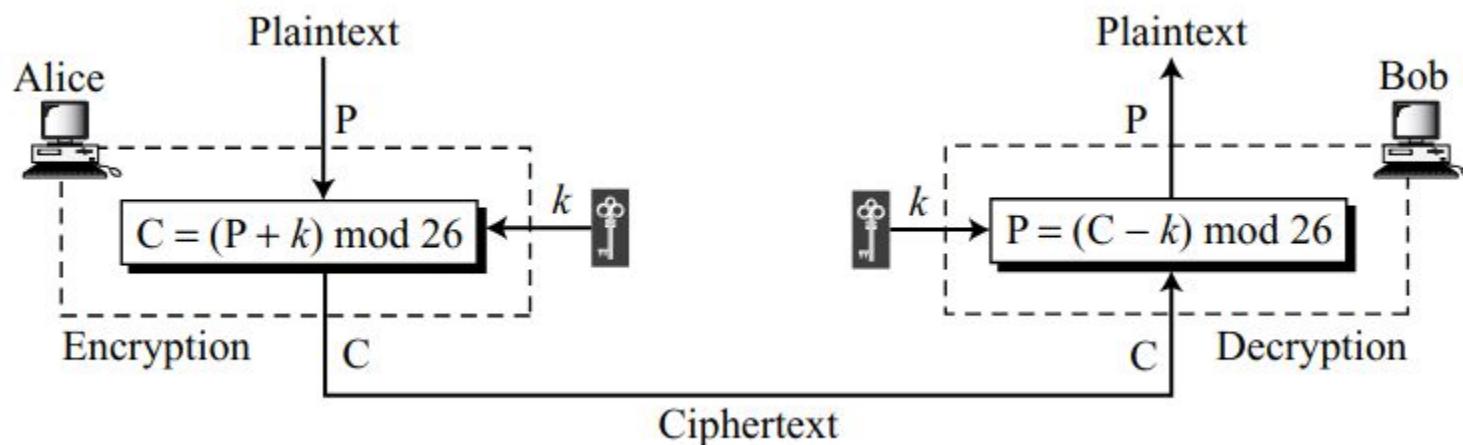
- Substitution cipher
 - the alphabet is shifted
 - one of the easiest ciphers (and not really secure)



Example 1 - Additive Cipher



Example 1 - Additive Cipher



When the cipher is additive, the plaintext, ciphertext, and key are integers in \mathbb{Z}_{26} .

Example 1 - Additive Cipher

Plaintext →	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext →	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Value →	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Use the additive cipher with key = 15 to encrypt the message “hello”.

Plaintext: h → 07

Encryption: $(07 + 15) \bmod 26$

Ciphertext: 22 → W

Plaintext: e → 04

Encryption: $(04 + 15) \bmod 26$

Ciphertext: 19 → T

Plaintext: l → 11

Encryption: $(11 + 15) \bmod 26$

Ciphertext: 00 → A

Plaintext: l → 11

Encryption: $(11 + 15) \bmod 26$

Ciphertext: 00 → A

Plaintext: o → 14

Encryption: $(14 + 15) \bmod 26$

Ciphertext: 03 → D

Example 1 - Additive Cipher

Plaintext →	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext →	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Value →	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Use the additive cipher with key = 15 to encrypt the message “WTAAD”.

Ciphertext: W → 22

Decryption: $(22 - 15) \bmod 26$

Plaintext: 07 → h

Ciphertext: T → 19

Decryption: $(19 - 15) \bmod 26$

Plaintext: 04 → e

Ciphertext: A → 00

Decryption: $(00 - 15) \bmod 26$

Plaintext: 11 → l

Ciphertext: A → 00

Decryption: $(00 - 15) \bmod 26$

Plaintext: 11 → l

Ciphertext: D → 03

Decryption: $(03 - 15) \bmod 26$

Plaintext: 14 → o

Example 1 - Caesar Cipher



- Cyphertext:
“QEB NRFZH YOLTK CLU GRJMP LSBO QEB IXWV ALD”

Any ideas?

Example 1 - Caesar Cipher



- Cyphertext:
 - “QEB NRFZH YOLTK CLU GRJMP LSBO QEB IXWV ALD”
- Solution: try all the possible combinations of alphabets (shifts)
- Cryptanalysis + brute force in this case is easier than cryptanalysis
- Plaintext: “THE QUICK BROWN FOX JUMPS OVER THE LAZY DOG”

Cryptographic Tools: XOR



- XOR is it widely adopted in crypto algorithms
 - Boolean operation
 - $0 \text{ xor } 0 = 0$
 - $0 \text{ xor } 1 = 1$
 - $1 \text{ xor } 0 = 1$
 - $1 \text{ xor } 1 = 0$
 - Represented with the symbol “ \wedge ”
- `enc_message = clear_message ^ key`

Cryptographic Tools: XOR



Properties:

- XOR is **commutative**

$$a \wedge b = b \wedge a$$

- XOR is **associative**

$$a \wedge (b \wedge c) = (a \wedge b) \wedge c$$

- Anything XORed with **itself is zero**

$$a \wedge a = 0$$

- Anything **XORed with zero is anything**

$$a \wedge 0 = a$$

Cryptographic Tools: XOR



```
enc_message = clear_message ^ key
```

```
clear_message = enc_message ^ key
```

```
key = clear_message ^ enc_message
```

Cryptographic Tools: XOR



- **XOR** is used between a **key** and a **message**
 - Often $\text{len}(\text{key}) << \text{len}(\text{message})$
 - We “repeat the key” on the message
- Example
 - `clear_message = "THIS IS A MESSAGE"`
 - `key = "YOU"`

T	H	I	S		I	S		A		M	E	S	S	A	G	E
Y	O	U	Y	O	U	Y	O	U	Y	O	U	Y	O	U	Y	O

Cryptographic Tools: XOR



T	H	I	S		I	S		A		M	E	S	S	A	G	E
84	72	73	83	32	73	83	32	65	32	77	69	83	83	65	71	69

Y	O	U	Y	O	U	Y	O	U	Y	O	U	Y	O	U	Y	O
89	79	85	89	79	85	89	79	85	89	79	85	89	79	85	89	79

Cryptographic Tools: XOR



msg	84	72	73	83	32	73	83	32	65	32	77	69	83	83	65	71	69
key	89	79	85	89	79	85	89	79	85	89	79	85	89	79	85	89	79
enc	13	7	28	10	111	28	10	111	20	121	2	16	10	28	20	30	10

The XOR between two integer it is the result of the xor of their binary representations.

- $84 = 1010100$
- $89 = 1011001$
- $13 = 0001101$

XOR - Kasiski Elimination



Kasiski elimination:

- Technique to **attack substitution ciphers**
 - E.g., **Vigenère** cipher
(Polyalphabetic cipher, based on initial idea of **Bellaso**)
- Involve the **inspection of character sequences** inside a ciphertext
 - We look for anomaly amount of **repetitions**
 - At least sequences with more than 3 characters
 - An anomaly might be **derived by a repetition on the plaintext**
- Useful to identify the key length
 - ... and cryptanalysis

XOR - Kasiski Elimination



13	7	28	10	111	28	10	111	20	121	2	16	10	28	20	30	10

T	H	I	S		I	S		A		M	E	S	S	A	G	E
Y	O	U	Y	O	U	Y	O	U	Y	O	U	Y	O	U	Y	O

Questions? Feedback? Suggestions?



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