CPSC 340 Tutorial 2

September 20, 2021

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  - Other Notation: Pr[We roll a 6]
  - Example: The probability of rolling a 6 on a 6-sided die is $\frac{1}{6}$

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  - Example: $Y \sim \mathcal{N}(0, 1)$
- If $\mathcal{X}$ is the set of all possible outcomes for random variable X, then

$$\sum_{x \in \mathcal{X}} p(x) = 1$$

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  - Example: Probability that we roll 1 or 2

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  - Notation: $A \perp B$ vs $A \not\perp B$

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- Bayes rule:

$$
\begin{aligned}
p(A \mid B) &= \frac{p(A, B)}{p(B)} \\
&= \frac{p(B \mid A)p(A)}{p(B)}
\end{aligned}
$$

Helpful Identities

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- Total Probability: We can partition the probability space and add up joint probabilities
  - Let B, C be a partition of the probability space $\mathcal{X}$
  - Consider the event A
  - Then

$$\begin{aligned} p(A) &= p(A, B) + p(A, C) \\ &= p(A \mid B)p(B) + p(A \mid C)p(C) \end{aligned}$$

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References

https://www.cs.ubc.ca/ schmidtm/Courses/Notes/probabil-
ity.pdf