Patch-based learning of parameter maps for total variational image restoration

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Total Variation image reconstruction

For $m, n \in \mathbb{N}, m \leq n$ and given $\mathbf{y} \in \mathbb{R}^m$, seek $\mathbf{x} \in \mathbb{R}^n$, such that

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e} \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{m \times n}$ models blur and possible under-sampling, $\mathbf{e} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{Id})$.

Due to *ill-posedness*, problem (1) can be solved by minimizing the sum of a data fidelity term and a regularization term encoding *a priori* assumptions on the solution.

A popular choice in imaging is the Total Variation (TV) semi-norm due to its:

- Edge-preserving behaviour
- Noise removal properties
- Convexity which provides efficient algorithmic solvers.

For $0 < \epsilon \ll 1$, the smoothed problem formulation is:

$$\hat{\mathbf{x}} \in \arg\min_{\mathbf{x} \in \mathbb{R}^n} \frac{\mu}{2} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \mathrm{TV}_\epsilon(\mathbf{x}), \tag{2}$$

where $\mathrm{TV}_\epsilon(\mathbf{x}) := \sum_{i=1}^n \|(\nabla \mathbf{x})_i\|_2 = \sum_{i=1}^n \sqrt{(\nabla_h \mathbf{x})_i^2 + (\nabla_v \mathbf{x})_i^2 + \epsilon^2}$.

The hyperparameter $\mu > 0$ balances the effect of the regularization against the data fit. Its choice is crucial for good reconstructions.

(a) $\mathbf{x}$ (b) $\mathbf{y}$ (c) $\hat{\mathbf{x}}$, $\mu = 20$ (d) $\hat{\mathbf{x}}$, $\mu = 100$

Space-variant reconstruction: interpretation

Statistical viewpoint

Maximize posterior:

$$\mathbf{x}^* \in \arg\max_{\mathbf{x}} \; P(\mathbf{x}|\mathbf{y}, \mathbf{A}).$$

Apply *Bayes' formula*:

$$\mathbf{x}^* \in \arg\max_{\mathbf{x}} \frac{P(\mathbf{x})P(\mathbf{y}|\mathbf{x}, \mathbf{A})}{P(\mathbf{y})}$$

Optimization viewpoint

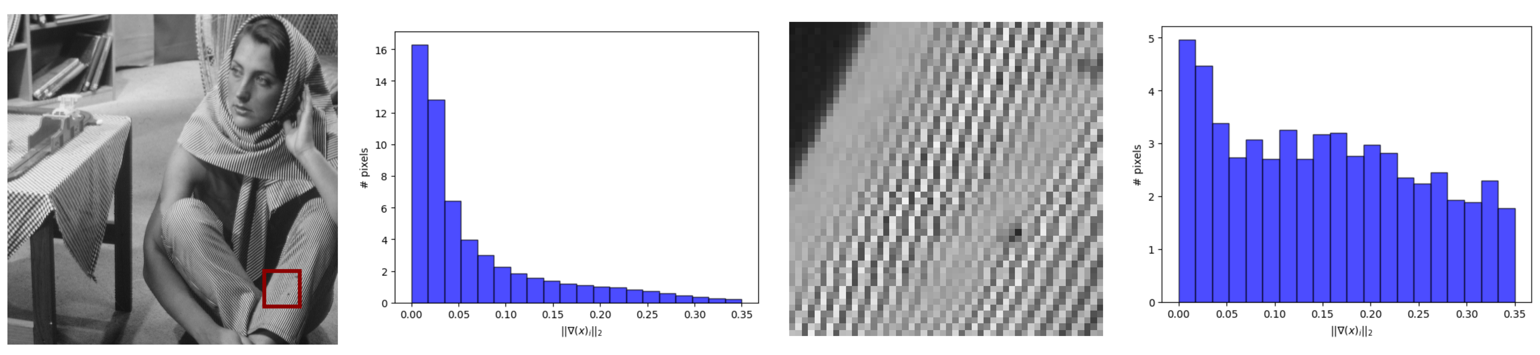
Take the negative log-likelihood

$$\mathbf{x}^* \in \arg\min_{\mathbf{x}} \; -\ln(P(\mathbf{x})P(\mathbf{y}|\mathbf{x}, \mathbf{A}))$$

$$P(\mathbf{x}) \propto e^{-R(\mathbf{x})}, \; P(\mathbf{y}|\mathbf{x}, \mathbf{A}) \propto e^{\frac{-\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2}{2\sigma^2}}$$

$$\Rightarrow \; \mathbf{x}^* \in \arg\min_{\mathbf{x}} \; R(\mathbf{x}) + \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2$$

A TV prior assumes that the gradient magnitudes $q_i := \|(\nabla \mathbf{x})_i\|_2$ follow a space-invariant one-parameter half-Laplacian Distribution (hLD) with **global scale parameter** λ, which does not adapt to local image structures.



(a) $\mathbf{x}$ (b) hist of $\|(\nabla \mathbf{x})_i\|_2$ (c) Detail of $\mathbf{x}$ (d) hist of $\|(\nabla \mathbf{x})_i\|_2$

We allow the gradient scale to **locally adapt** to the pixel-dependent structure, which means considering the adaptive prior:

$$P(q_i; \lambda_i) = \begin{cases} \lambda_i e^{-\lambda_i q_i} & q_i \geq 0 \\ 0 & q_i < 0 \end{cases},$$

or, equivalently, a pixel-dependent fidelity resulting in an adaptive version of (2):

$$\hat{\mathbf{x}} \in \arg\min_{\mathbf{x}} \sum_{i=1}^n \frac{\mu_i}{2} |(\mathbf{A}\mathbf{x})_i - y_i|^2 + \mathrm{TV}_\epsilon(\mathbf{x}), \quad \boldsymbol{\mu} = (\mu_i) \in \mathbb{R}^n. \tag{3}$$

Outlook

- Train on multiple Gaussian noise levels for more flexibility.
- Test on different noise distributions (Poisson, Poisson-Gaussian...).
- Test on different patch sizes. Balancing network capacity to learn and spatial adaptation.
- Comparison with algorithmic unrolling where $\boldsymbol{\mu}$-maps are not computed patch-wise
- Test on a biological imaging datasets, where fewer data are avaiable.

Learning parameter maps

Idea: Training a neural network $f_{\boldsymbol{\theta}}$ on natural image patches so that $f_\theta(\mathbf{y}) = \boldsymbol{\mu}$.

Dataset: *Berkeley Dataset*, $J = 300$ natural images. For each image j, $N = 126$ overlapping patches $\tilde{\mathbf{x}}_n^j$ of size 64x64 are taken. For each patch a global parameter $\mu_{j,n}^{GT}$ is estimated by solving (2) by a golden-section search maximizing SSIM and using FISTA. Training is performed so that:

$$\hat{\boldsymbol{\theta}} \in \arg\min_{\boldsymbol{\theta}} \sum_{j,n} |f_{\boldsymbol{\theta}}(\mathbf{y}_n^j) - \mu_{j,n}^{GT}|^2$$

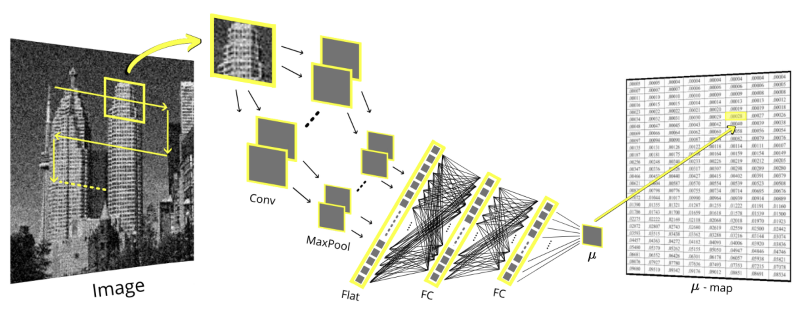


Figure 3: At inference, the network computes a $\boldsymbol{\mu}$-map by sliding over the image, i.e. $f_{\hat{\boldsymbol{\theta}}}(\mathbf{y}) = \mu_{i,j}$, the center of the patch.

Denoising

We start considering $\mathbf{A} = \mathbf{Id}$ in (3) and fixed noise of $\sigma^2 = 0.01$.

$\texttt{net\_error} = \frac{1}{N \max_{j,n} \; \mu_{j,n}^{GT}} \sum_{j,n} |\hat{\mu}_{j,n} - \mu_{j,n}^{GT}| = \mathbf{0.06}$, $\frac{1}{JN} \sum_{j,n} \mu_{j,n}^{GT} = \mathbf{48.25}$.

Note that the trained network $f_{\hat{\boldsymbol{\theta}}}$ estimates optimal adaptive parameter maps <u>without</u> the need of ground truth.

(a) $\mathbf{y}$, SSIM=0.38. (b) $\hat{\mathbf{x}}$ (SSIM: 0.75) - $\hat{\mathbf{x}}^{SV}$ (SSIM=**0.78**) (c) $\boldsymbol{\mu}$ map.

We tested the network on 30 images and the average SSIM improvement is: **0.02**, with a much better visual improvement especially in textured regions.

Deblurring

We consdier $\mathbf{A}$ being a 2D convolution operator modelling Gaussian blur with $\sigma_{\mathrm{PSF}} = 1$ and noise of variance $\sigma^2 = 0.01$.

$\texttt{net\_error} = \frac{1}{N \max_{j,n} \; \mu_{j,n}^{GT}} \sum_{j,n} |\hat{\mu}_{j,n} - \mu_{j,n}^{GT}| = \mathbf{0.07}$, $\frac{1}{JN} \sum_{j,n} \mu_{n,j}^{GT} = \mathbf{70.64}$.

Deblurring seems to require higher μ_i (i.e. less regularization).

(a) $\mathbf{y}$, SSIM=0.29 (b) $\hat{\mathbf{x}}$ (SSIM: 0.70) - $\hat{\mathbf{x}}^{SV}$ (SSIM=**0.71**) (c) $\boldsymbol{\mu}$ map.

We tested the network on 30 images and the average SSIM improvement is: **0.01**

References

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