independent not only of z, but also of y, so that the quadratic convection terms ($\mathbf{v}$ grad) $\mathbf{v}$ vanish. The Navier-Stokes equation for v is then according to v.II, equation (16.1)

$$\frac{\partial v}{\partial t} - k\frac{\partial^2 v}{\partial x^2} = -\frac{1}{\varrho}\frac{\partial p}{\partial y}; \tag{1}$$

where k is the kinematic viscosity; the right side is *independent of* x due to the corresponding equation for the vanishing x-component of velocity, hence it is a function of t only, say $a(t)$.

The flow is to be bounded at $x = 0$ by a fixed plate, which is at rest up to the time $t = 0$ and thereafter is in motion with the velocity $v_0(t)$. Due to the adhesion of the fluid to the plate we have for $x = 0$:

$$v = \begin{cases} 0 \ldots t \leq 0, \\ v_0(t) \ldots t > 0. \end{cases} \tag{2}$$

For the linear Couette flow (see v.II, Fig. 19b) we would have to add further boundary conditions on a plane that is at rest at a finite distance from $x = 0$. However, for the sake of simplicity, we shall consider this plate situated at infinity. The limiting case obtained in this manner is known in fluid dynamics as *plate flow*. For this flow we have, in addition to (2), the condition for $x = \infty$:

$$v = 0 \quad \text{and} \quad p = p_0 \text{ (independent of } y\text{)}. \tag{3}$$

From this it follows that $a(t) = 0$, so that (1) goes over into the equation of heat conduction.

We are thus led to a boundary value problem, which is a specialization of the problem illustrated by Fig. 13 only in that we now have $x_1 = \infty$ and $x_0 = 0$, and is a simplification of that problem because in the initial state in which the plate and hence the fluid are at rest, we have:

$$v = 0 \quad \text{for} \quad t = 0 \quad \text{and} \quad \text{all} \quad x > 0. \tag{4}$$

The solution is obtained as in (12.18), if the principal solution V is replaced by a suitable Green's function. Discuss the resulting velocity profile $v(x)$ for increasing values of t.

EXERCISES FOR CHAPTER III

III.1. Linear conductor with external heat conduction according to Fourier. Let the initial temperature for $x > 0$ be $u(x,0) = f(x)$. How must this function be continued for $x < 0$ so that the condition is

$$\frac{\partial u}{\partial n} + h\,u = 0$$

satisfied for $x = 0$?

III.2. Deduce *the normalization condition in anharmonic analysis* by specialization from Green's theorem.

III.3. Experimental determination of the ratio of outer and inner heat conductivity. A rod is to be kept at the fixed temperatures u_1 and u_3 at its ends $x = 0$ and $x = l$ and is to be in a *stationary* state after the effect of an arbitrary initial state dies out. The flow of temperature would then be *linear* if the lateral surface of the rod were adiabatically closed. Hence, at the middle section of the rod $x = l/2$ we would have temperature $u_2 = (u_1 + u_3)/2$. Hence from the measurement of

$$q = \frac{u_1 + u_3}{2\,u_2}$$

we can determine the ratio of outer and inner heat conduction (essentially our constant h). Deduce the relation between q and h needed for the evaluation of the measurement; according to the above $q = 1$ corresponds to $h = 0$.

III.4. Determination of the ratio of heat conductivity $\varkappa$ to electric conductivity σ. A metal rod is to be heated electrically, where the current i per unit of length gives the rod the Joule heat $i^2/q\sigma$ (q = cross-section of the rod); the rod is to be insulated against lateral heat conduction. Write the differential equation of the stationary state and adapt it to the boundary conditions $u = 0$ for $x = 0$ and $x = l$ that are realized in water baths. The potential difference V at the ends of the rod and the maximal temperature U at the mid-section of the rod are to be measured. From them we are to compute the ratio $\varkappa/\sigma$. For pure metals this ratio has a universal value (Wiedemann-Franz law).

EXERCISES FOR CHAPTER IV

IV.1. Power series expansion of $I_n(\varrho)$. Compute this expansion from the integral representation (19.18)

a) for integral n,

b) for arbitrary n

with the help of a general definition of the Γ- function.

IV.2. Deduce *the so-called circuit relations for H_n^1 and H_n^2* for integral n from the integral representations (19.22).