EXERCISES FOR CHAPTER I

I.1. The position of the maxima and minima in special Fourier approximations. Show that the extrema of the Fourier approximation

$$S_{2n-1} = \sin x + \frac{1}{3} \sin 3x + \cdots + \frac{1}{2n-1} \sin (2n-1) x$$

lie at equal distances, and that except for the extremum $x = \pi/2$ which is common to all S, they lie between the extrema of S_{2n+1}.

I.2. Summation of certain arithmetic series. Compute the higher analogues to the Leibniz series (2.8) and to the series Σ_2, Σ_4 in (2.18).

I.3. Expansion of sin x in a cosine series. Expand $\sin x$ between 0 and π in a cosine series,

a) by considering $\sin x$ continued as an even function in the interval $-\pi < x < 0$, or

b) by substituting $b = 0$ and $c = \pi$ in (4.5).

I.4. Spectral resolutions of certain simple time processes. Compute the spectra of the time processes which are indicated in Fig. 33a and 33b from their Fourier integral and represent them graphically. In the same manner compute the spectrum of a sine wave $\sin 2\pi t/\tau$, which is bounded on both sides and which ranges from $t = -T$ to $t = +T$ where $T = n\tau$ (Fig. 33c), and deduce from this the fact that the width of a spectral line varies inversely with its life span. An absolutely sharp, completely monochromatic spectral line would therefore need a completely unperturbed sine wave that extends to infinity in both directions.

I.5. Examples of the method of complex integration. Give the reasons for the result of exercise I.4a (*Dirichlet discontinuous factor*) by the method of complex integration; also, resolve the spectrum of the sine curve bounded on both sides into the spectra of two waves that are bounded on one side.

I.6. Compute the first Hermite and Laguerre polynomials from their orthogonality condition by the method applied to spherical harmonics, normalizing so that the leading term of $H_n(x)$ is $(2x)^n$ and the constant term of $L_n(x)$ is $n!$.

For the definition of these polynomials see the table on p. 27.