these regions (spatially speaking they are spherical crescents) by five repeated reflections.

IV.7. Mapping a) *of the plane parallel plate into two tangent spheres,* b) *of two concentric spheres into a plane and a sphere.* Investigate the three-dimensional figure into which the plane parallel plate of p. 74 together with its mirror images are transformed upon inversion. The sphere of inversion is best situated so that it is tangent to the boundary planes of the plate. Show that the plate is thus mapped into the exterior of two tangent spheres; and its mirror images are mapped into the space between two interior tangent spheres. b) Show that two concentric spheres can be inverted into a plane and a sphere. Hence, conversely, we can transform the potential of a sphere towards a plane into the much simpler boundary value problem for two concentric spheres. The same holds for the potential of two arbitrary non-intersecting spheres.

IV.8. Evaluation of two expressions involving Bessel functions. In equation (5) of Appendix I to Chapter IV determine

$$H_n(\varrho)\, I_n'(\varrho) - H_n'(\varrho)\, I_n(\varrho) \tag{I}$$

and in equation (20b) of the same appendix determine

$$\zeta_n(\varrho)\, \psi_n'(\varrho) - \zeta_n'(\varrho)\, \psi_n(\varrho). \tag{II}$$

EXERCISES FOR CHAPTER V

V.1. Normalization questions. Normalize the functions $I_n(\lambda r)$ and $\psi_n(k r)$ of (26.3) and (26.2) to 1 for the basic interval $0 < r < a$ with the boundary conditions $I_n'(\lambda a) = 0$ and $\psi_n'(k a) = 0$ in analogy to equation (20.19).

V.2. The Gauss theorem of arithmetic mean in potential theory. Prove the theorem: The value of a potential function $\overline{u}$ at any point P of its domain of regularity S is equal to the arithmetic mean u of its values on an arbitrary sphere K_a, which has radius a and center P and which lies entirely in S.

V.3. Summation formulas over the roots of Bessel functions. Verify that the coefficients A_{nm} in the expansion (27.13) equal those of (27.14), and derive interesting identities for the Ψ_n from the equality of the coefficients of $\Pi_n^m(\cos\vartheta_0)\, e^{-im\varphi_0}$ in these two expansions. These identities can be rewritten as identities for the ψ_n.