Lecture 1
Overview of Nonstandard Models

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Mexican Numerical Simulations School (03-06 Oct 2016)

This Lecture

- In this lecture series, I'll give a brief overview of some most widely used tools to test cosmological models against observations
- To put things into context, the first lecture will be an overview of theoretical possibilities. The idea is to give sufficient information for people to appreciate the diversity of models, the basic logic of how such a huge number of models should be studied, and the need for a range of tools/methods/algorithms to study them
- It is not our aim to give a comprehensive review of all models
- The next few lectures will be about the tools/methods/algorithms themselves, and how these can be applied to test models with observations

Challenge from the Cosmic Acceleration

- It is well established that our Universe has been experience a phase of accelerated Hubble expansion
- Although a cosmological constant can explain most data to date, it is theoretical difficult to contrive an extremely tiny value that is required to match observations
- Many alternatives have been proposed, but 20 years on there is still no commonly agreed preferred model
- With observational data to improve vastly in the coming years, it is time to reconsider the whole picture and our position in it, and equip ourselves with essential tools to tackle this challenge

The Previous Cosmological Model

energy momentum tensor
(dark and normal matter)

curvatures	$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$	Einstein's equation
	$\Downarrow$	
expansion rate	$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G \rho_{\mathrm{m}}$	Friedmann equations
acceleration rate	$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho_{\mathrm{m}} + 3P_{\mathrm{m}})$	

ä > 0: accelerated expansion
ä < 0: decelerated expansion

Why does the Previous Model Fail?

energy momentum tensor
(dark and normal matter)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

$$\Downarrow$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G \rho_{\rm m}$$

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho_{\rm m} + 3P_{\rm m})$$

Observations (SNe, BAO, CMB, LSS, etc.):
ä > 0

Cannot be explained by the Friedmann eqn. with normal matter

Where could the Previous Model Fail?

modified gravity
gravity theory incorrect on large scales?

dark energy
missing matter species in our current model?

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

$$\Downarrow$$

mistreatment in applying GR?
backreaction

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G \rho_{\rm m}$$

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho_{\rm m} + 3P_{\rm m})$$

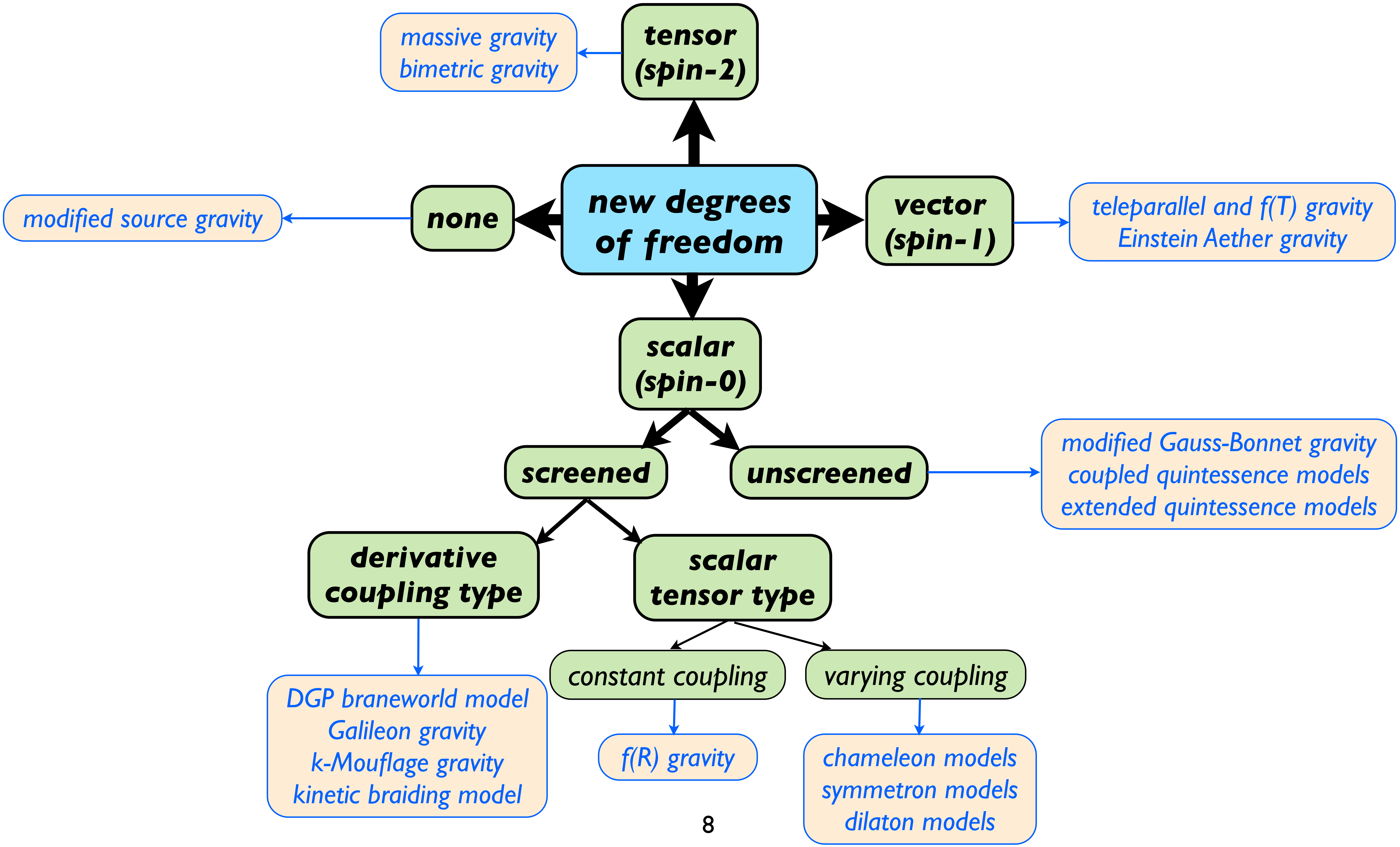
Observations (SNe, BAO, CMB, LSS, etc.):
ä > 0

misinterpretation of data?

Cosmological Constant

- ΛCDM is the current standard cosmological model
- The simplest possibility: a mysterious term with constant (in time) and homogeneous (in space) energy density as the Universe expands
- But not so simple: quantum field theory predicts quantum fluctuations in vacuum, which behave like a cosmological constant macroscopically, but the value is $\sim 10^{120}$ times too large!
- Commonly assumed that exact cancellation due to some symmetry makes this vacuum energy exactly zero, and some other mechanism generates a small value that we observe today
- Multiverse & anthropic principle?

Alternatives?



What should We Expect to Learn?

- As the previous slide shows, there is a vast number of models, most of which are phenomenological, despite people's claims that they are motivated by theories of fundamental physics
- Cosmological constant (single) vs. alternatives (many)
- Observations do not (yet) have sufficient precisions to reveal the fundamental nature of the true model
- As such, these models are better to be used as what they really are: *toy models the constraints on which can tell us how much deviation from ΛCDM (in every known direction) is allowed*
- Confirmation of no deviation from ΛCDM will still be useful, as it will rule out many possibilities and precisely test GR on cosmic scales

Should We Study All Models in Detail?

- Unlike simple ΛCDM, many of the non-standard models are very complicated: impractical to study them all in great detail
- Many of them share similarities in their cosmological behaviours: unnecessary to study them all in great detail
- A more efficient way is to *abstract* the most representative models and their properties, *categorise* them and *parameterise* them, in the hope that studies of such models/parameterisations could lead to generic conclusions?

Examples of Parameterisations (1)

- The simplest and most well-known parameterisation is one that works for the dark energy equation of state $w = P_{DE}/\rho_{DE}$
- For example: $w(a) = w_0 + w_1(1\text{-}a)$
- The parameterisation can be used to study cosmology (e.g., the background expansion history) without knowing a specific model
- In appropriate limit, e.g., $w_0 \to -1$, $w_1 \to 0$, the parameterisation gives the ΛCDM model as a special case. Therefore, constraints on the parameters w_0 and w_1 can tell how much deviation from ΛCDM is allowed by observations.

Examples of Parameterisations (2)

- Another often used example is the parameterisation of the Newton constant, which often gets modified and becomes a function of space and time in non-standard models
- For example: G(k, a) in Fourier space

$$G(k,a) = \frac{4a^2k^2 + 3m^2}{3a^2k^2 + 3m^2} G_{\mathrm{N}}$$

- Again, this goes back to $G(k,a)=G_N$ (the standard value of Newton's constant) at early times ($a \rightarrow 0$) and/or on large scales ($k \rightarrow 0$). The parameterisation is controlled by a parameter *m*, which can be constrained by observations

Examples of Parameterisations (3)

- At perturbative level, there exists a theoretical framework according to which, if certain (reasonable) assumptions are made, then the equations governing the evolution of large-scale structure have unambiguously fixed forms. Such assumptions include:
- *the model respects general covariance (is a basic property of GR)*;
- *the field equations are coupled differential equations not containing higher than second-order derivatives*;
- *the basic content of the new physics is known, e.g., it is scalar field*;
- *the background evolution in the model is fixed.*
- Will not discuss further here: interested people check literature

Drawbacks of Parameterisations

- It is a tradeoff between being *sufficiently generic* and being *reasonably faithful*. For example, if a parameterisation only works for one specific model, there is no advantage at all; but if it is designed to capture one or two properties of many models, it will fail to capture all properties of individual models given the rich diversity of models.
- There is still a degree of guesswork as to what the parameterisation should look like: it cannot be arbitrary, because otherwise it may not represent any known model.

Categorisation + Parameterisation

model A
model B → parameterisation I
model C

model D
model E → parameterisation II
model F

model G
model H → parameterisation III
model J

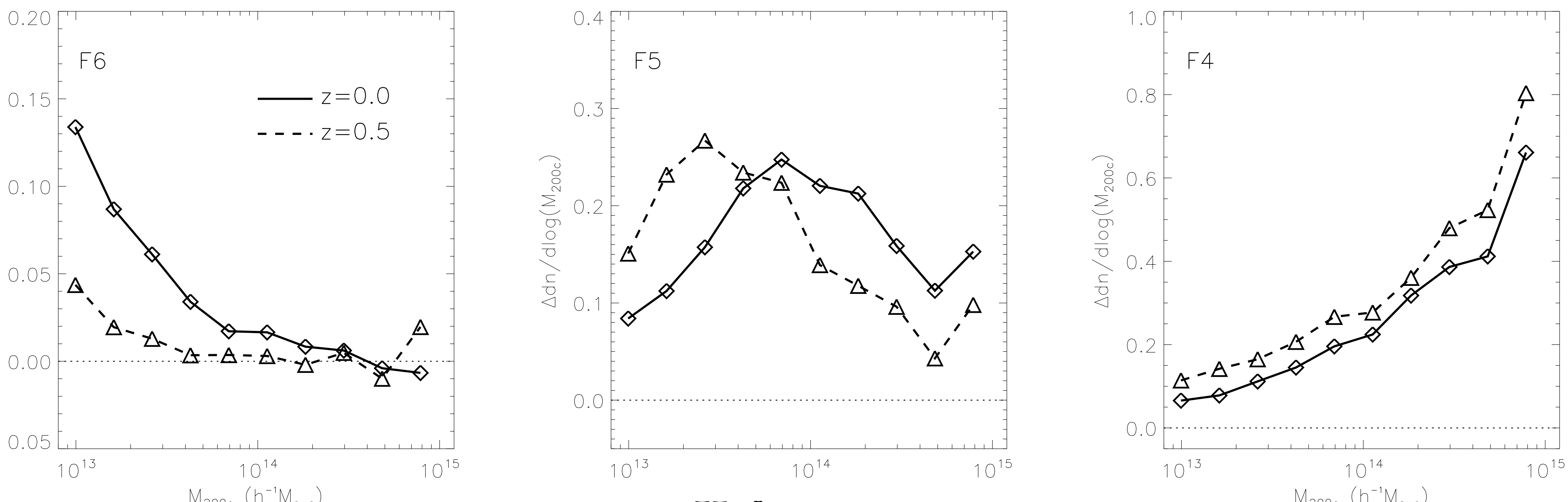
… …

Group similar models together before parameterisation

f(R) Gravity - an Example

varying model parameters

Halo mass function diff from GR



Halo mass

By varying the parameter used in the parameterisation, one can get a range of qualitatively different behaviours seen in many other models. A representative model that can be a good testbed.

Scalar Fields

- Scalar fields are the most popular candidates of nonstandard models to explain the cosmic acceleration, and will be focus of this lecture series.
- So what is a scalar field?
- It's a field that permeates space and exists everywhere, like the familiar electromagnetic (EM) field. The difference is that an EM field is a vector field (e.g., the electric field as 3 components), while a scalar field is a scalar (i.e., it has one value at each space point)
- The field quanta of the EM field is photon (spin-1); the field quanta of a scalar field is a spin-0 particle, e.g., Higgs boson.
- But in cosmology we are primarily interested in classical scalar fields (not quantised particles)

Scalar Fields: Analogy to EM Fields

	vector field	scalar field
examples	the EM field	the Higgs field
symbol	**E**, **B**	φ
energy & pressure	yes	yes
self interaction	E with M	usually yes
other interaction	charged particles	‘charged’ particles
dynamical eqn.	Maxwell eqn.	Klein-Gordon eqn.

Quintessence Scalar Field

kinetic potential

EM analogy

energy density
$$\rho(\varphi) = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$

pressure
$$P(\varphi) = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

$$\boxed{\rho_{\mathrm{EM}} = \frac{1}{2}\left(\mathbf{E}^2 + \mathbf{B}^2\right)}$$

dynamical eqn.
$$\Box\varphi + \frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi} = 0$$

self interaction

interaction with other matter

$$\boxed{\begin{aligned} \nabla \cdot \mathbf{B} &= 0 \\ \nabla \cdot \mathbf{E} &= \rho \\ \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} &= 0 \\ \nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} &= \mathbf{J} \end{aligned}}$$

Scalar Field Dynamics

scalar field in a Minkowski space

$$\Box\varphi + \frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi} = 0$$

$$\Downarrow$$

$$\ddot{\varphi} + \frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi} = 0$$

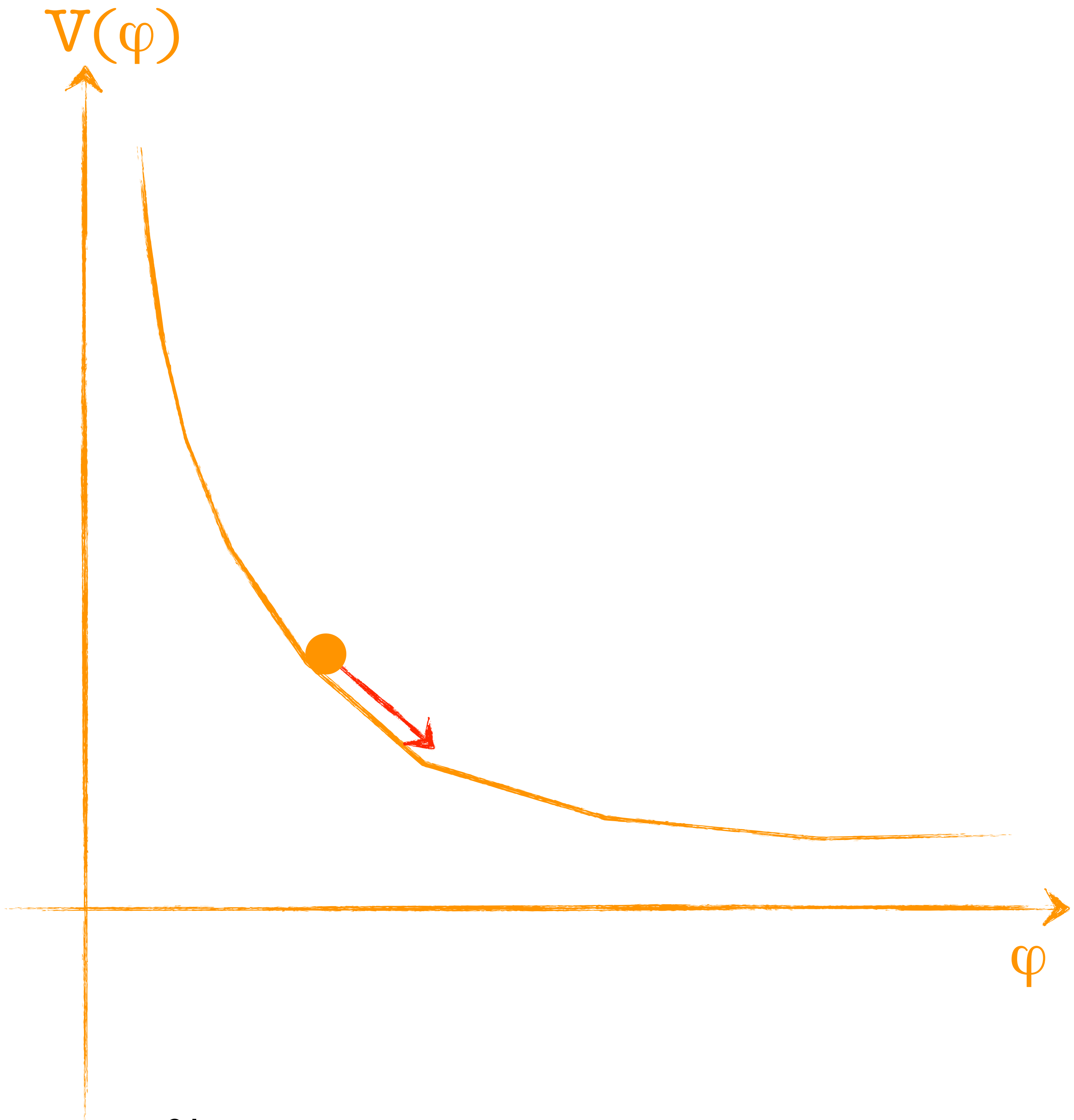
$$\longleftrightarrow$$

point mass in a conservative potential

$$\ddot{x} + \frac{\mathrm{d}V(x)}{\mathrm{d}x} = 0$$

Scalar Field Dynamics

$$\begin{aligned} \ddot{x} + \frac{\mathrm{d}V(x)}{\mathrm{d}x} &= 0 \\ \ddot{\varphi} + \frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi} &= 0 \end{aligned}$$



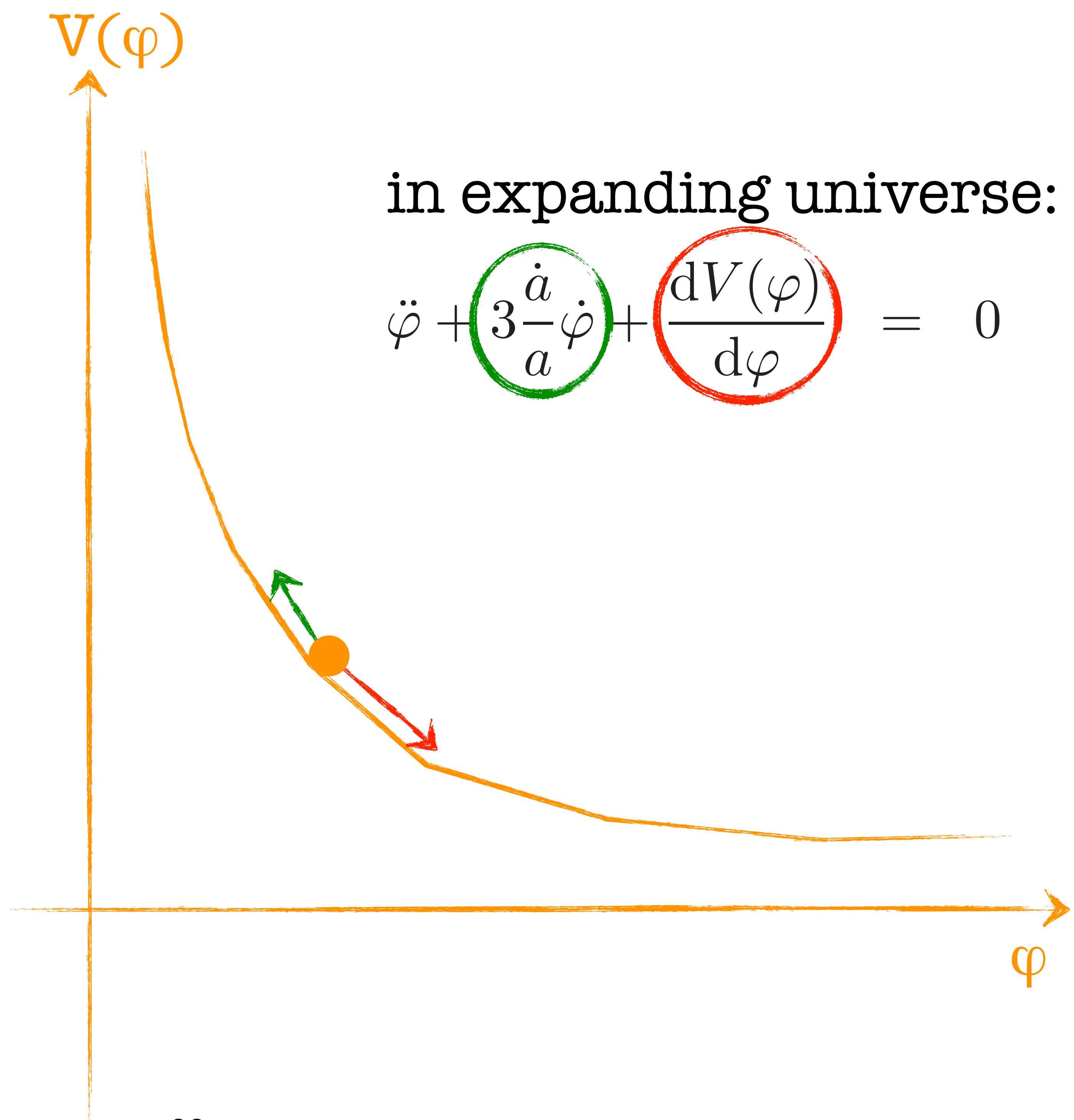
For understanding its dynamics, a scalar field can often be thought of as a ball rolling down a hill with a slope

Scalar Field Dynamics

$$\ddot{x} + \frac{dV(x)}{dx} = 0$$

$$\ddot{\varphi} + \frac{dV(\varphi)}{d\varphi} = 0$$

For understanding its dynamics, a scalar field can often be thought of as a ball rolling down a hill with a slope



in expanding universe:

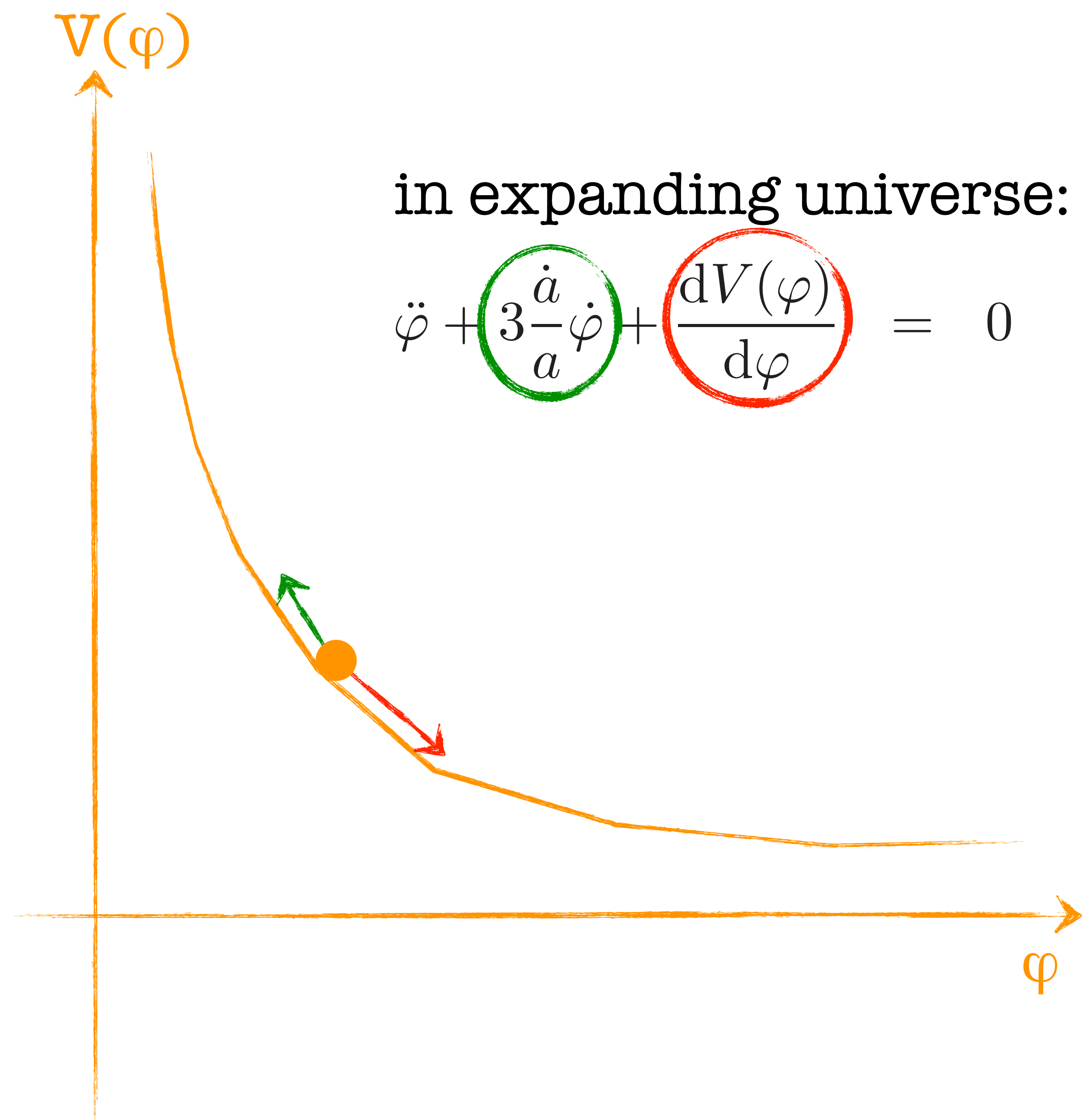
$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} + \frac{dV(\varphi)}{d\varphi} = 0$$

Dynamics of a Quintessence Field

If the potential is sufficiently 'flat', the scalar field would 'slowly' roll down the potential hill.

$$\rho(\varphi) = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \sim \text{const.}$$
$$P(\varphi) = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \sim \text{const.}$$

Similar to cosmological constant!



in expanding universe:

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} + \frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi} = 0$$

Quick Exercises

- Convince yourself that the above analogy of the evolution of a scalar field to the rolling of a ball in a potential works (we will use it below)
- Assuming that $dV(\varphi)/d\varphi$ is small and therefore can be neglected, solve the remaining part of the scalar field dynamical equation and explain why the first time derivative of φ will be small

Dynamics of a Quintessence Field

- Small spatial perturbations of the scalar field, written as $\varphi(\mathbf{x},t) = \varphi(t) + \delta\varphi(\mathbf{x},t)$, satisfy the following linearised equation:

$$\ddot{\delta\varphi} + 3\frac{\dot{a}}{a}\dot{\delta\varphi} + \frac{1}{a^2}\nabla^2\delta\varphi + \boxed{V_{,\varphi\varphi}(\bar{\varphi})\delta\varphi - 2\Phi\ddot{\bar{\varphi}} - 4\dot{\bar{\varphi}}\left(\dot{\Phi} + \frac{\dot{a}}{a}\Phi\right)} = 0$$

green: standard wave equation
blue: damping/friction term
red: external driving force

Dynamics of a Quintessence Field

- Small spatial perturbations of the scalar field, written as $\varphi(\mathbf{x},t) = \varphi(t) + \delta\varphi(\mathbf{x},t)$, satisfy the following linearised equation:

$$\delta\ddot{\varphi} + 3\frac{\dot{a}}{a}\delta\dot{\varphi} + \frac{1}{a^2}\nabla^2\delta\varphi \boxed{} = 0$$

If the potential is sufficiently 'flat', $V_{,\varphi\varphi} = d^2V(\varphi)/d\varphi^2$ is negligible, and so as the other terms in the red box. The wave is damped with no driving force; and $\delta\varphi$ decreases in time. This means that any small perturbation in the scalar field dies off, and the field becomes increasingly homogeneous.

Rich Phenomenology of Scalar Fields

- The quintessence model described above is perhaps the simplest scalar field model that is of interest in cosmology
- In it, a single scalar field has a self-interaction described by a potential $V(\varphi)$, and has no interaction with anything else
- In practice, however, the lack of clue about the fundamental origin of the cosmic acceleration means that we cannot simply rule out other, more complicated possibilities
- These include: multiple scalar fields, scalar field self-interaction described by a non-standard kinetic energy, scalar field interacting with normal and/or dark matter, scalar field interacting with space-time curvature, etc.

Coupled Scalar Field Models

- Most scalar field models involve some sort of interaction with the rest of the Universe: normal matter, dark matter, curvature, etc.
- Here we consider the most widely studied form of such interaction:

$$\Box\varphi + \boxed{\frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi}} = \boxed{0}$$

$$\Box\varphi + \boxed{\frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi}} = \boxed{C(\varphi)\rho_{\mathrm{m}}}$$

interaction with other matter

self interaction

Coupled Scalar Field Models

- The interaction between the scalar field and matter will naturally:
- change the dynamics of the scalar field, which can be seen from the dynamical equation directly

$$\Box\varphi + \frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi} = 0$$

$$\Box\varphi + \underbrace{\boxed{\frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi}}}_{\text{self interaction}} = \boxed{C(\varphi)\rho_{\mathrm{m}}}$$

interaction with other matter

Coupled Scalar Field Models

- The interaction between the scalar field and matter will naturally:
- change the dynamics of matter particles. This change can be written either in the form of a new force on particle acceleration equations, or as an additional term in the Poisson equation (we use the latter here)

$$\boxed{\nabla^2\Phi \quad = \quad 4\pi G\rho_{\rm m}} + \boxed{\beta(\varphi)\nabla^2\varphi}$$

standard Poisson equation

additional term arising from the interaction with scalar field. This actually is a nuisance, as such an extra term is likely to violate local gravity constraints

Screening Mechanisms

- A great effort in recently studies of scalar field cosmology has been devoted to models with ‘screening mechanisms’
- These are inherent dynamical properties of the theories, which ensures that the extra term in the modified Poisson eqn becomes negligible in environments resembling those where local gravity tests have been carried out [this will be called new force later]

$$\begin{aligned} \nabla^2\Phi &= 4\pi G\rho_{\rm m} + \beta(\varphi)\nabla^2\varphi \quad ??? \\ \Box\varphi + \frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi} &= C(\varphi)\rho_{\rm m} \end{aligned}$$

Screening Mechanisms

$$
\begin{aligned}
\nabla^2\Phi &= 4\pi G\rho_{\rm m} + \beta(\varphi)\nabla^2\varphi \\
\Box\varphi + \frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi} &= C(\varphi)\rho_{\rm m}
\end{aligned}
$$

$$
\begin{aligned}
\varphi &\to 0 && \text{chameleon mechanism} \\
\beta(\varphi) &\to 0 && \text{symmetron mechanism} \\
\frac{\nabla^2\varphi}{4\pi G\rho_{\rm m}} &\to 0 && \text{Vainshtein mechanism}
\end{aligned}
$$

Screening Mechanisms

$$\nabla^2\Phi = 4\pi G\rho_{\rm m} + \beta(\varphi)\nabla^2\varphi$$

$$\Box\varphi + \boxed{\frac{{\rm d}V(\varphi)}{{\rm d}\varphi} = C(\varphi)\rho_{\rm m}}$$

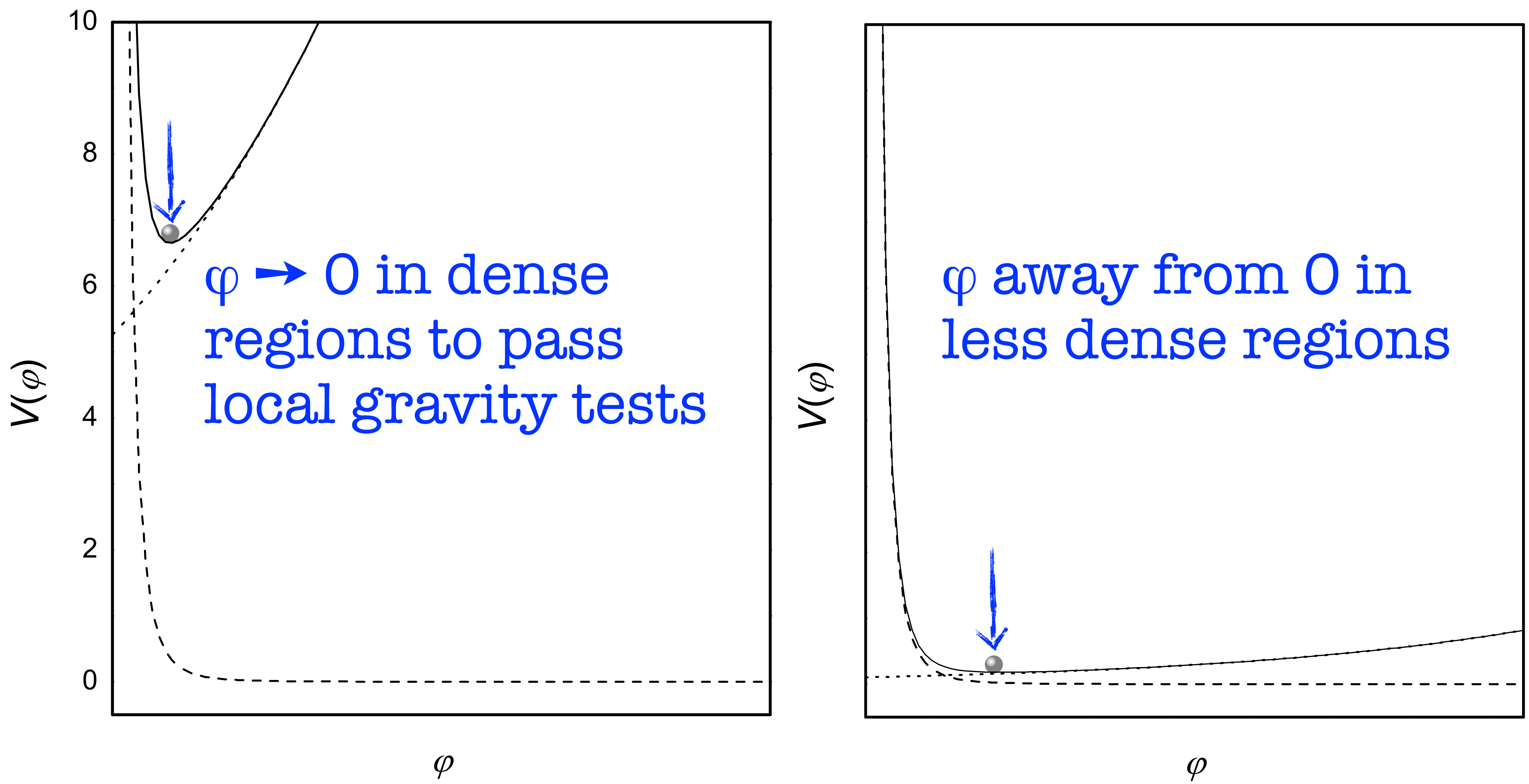
$$\varphi \to 0$$ chameleon mechanism

$$\beta(\varphi) \to 0$$ symmetron mechanism

$$\frac{\nabla^2\varphi}{4\pi G\rho_{\rm m}} \to 0$$ Vainshtein mechanism

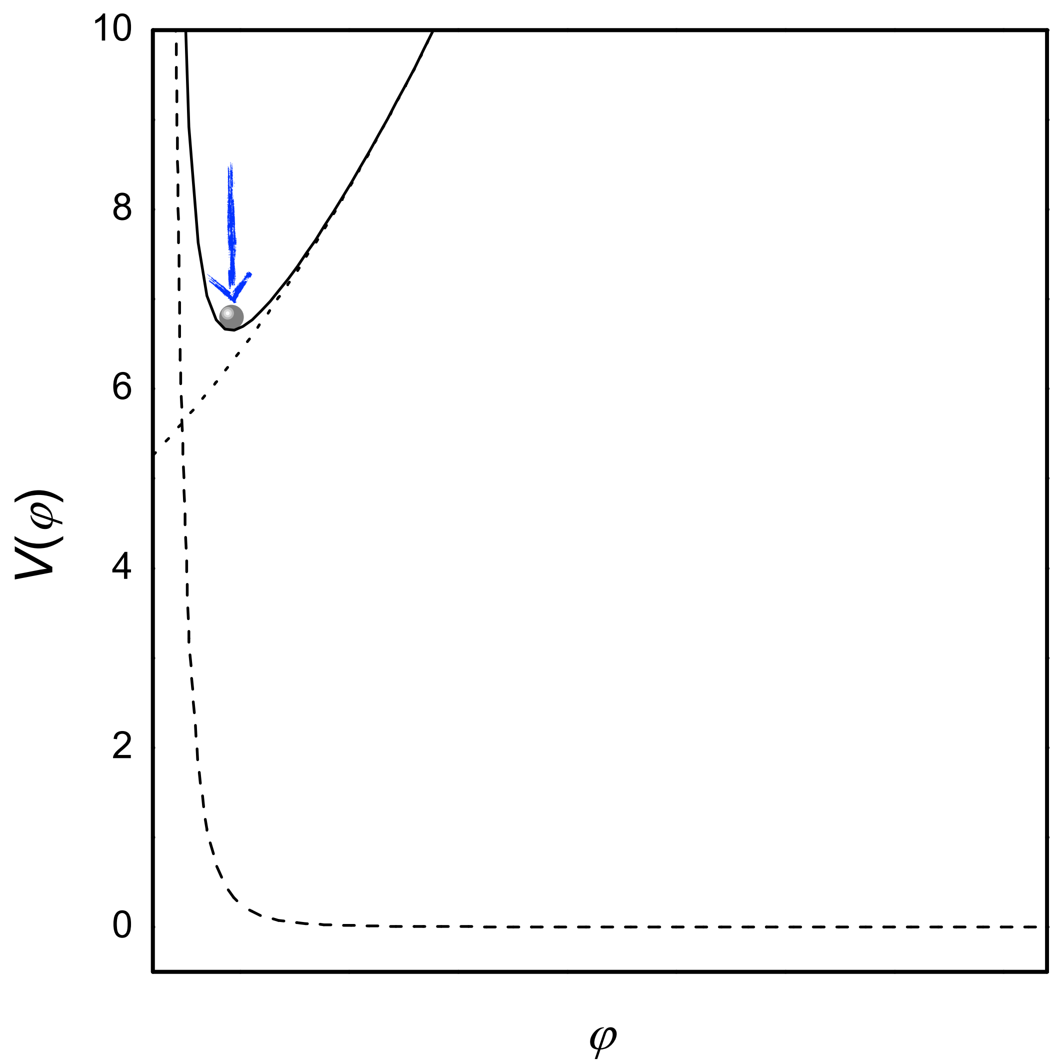
To achieve these in dense regions require intelligent design of the scalar field's self interaction and interaction with matter

Chameleon Mechanism & Models



$$
\begin{aligned}
\nabla^2\Phi &= 4\pi G\rho_{\mathrm{m}} + \beta(\varphi)\nabla^2\varphi \\
\Box\varphi + \frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi} &= C(\varphi)\rho_{\mathrm{m}}
\end{aligned}
$$

Chameleon Mechanism & Models



$$\nabla^2\varphi + m^2\varphi = 0$$

$$m^2(\varphi) \equiv \frac{\mathrm{d}^2V(\varphi)}{\mathrm{d}\varphi^2} + \rho_{\mathrm{m}}\frac{\mathrm{d}C(\varphi)}{\mathrm{d}\varphi}$$

simplified eqn near minimum

$$\varphi(r) \propto \frac{1}{r}\exp(-mr)$$

an extra force of Yukawa type!

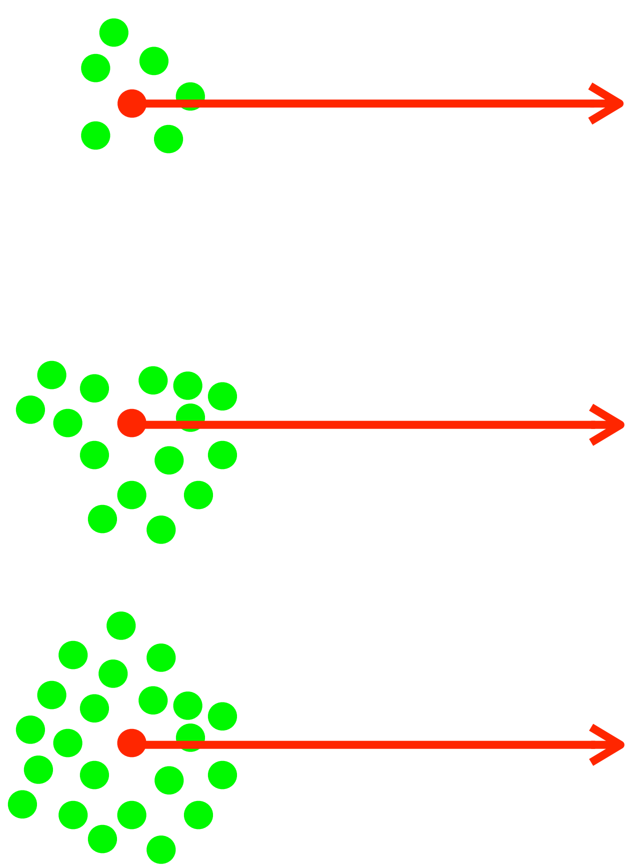
$$\nabla^2\Phi = 4\pi G\rho_{\mathrm{m}} + \beta(\varphi)\nabla^2\varphi$$

$$\Box\varphi + \frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi} = C(\varphi)\rho_{\mathrm{m}}$$

Chameleon Mechanism & Models

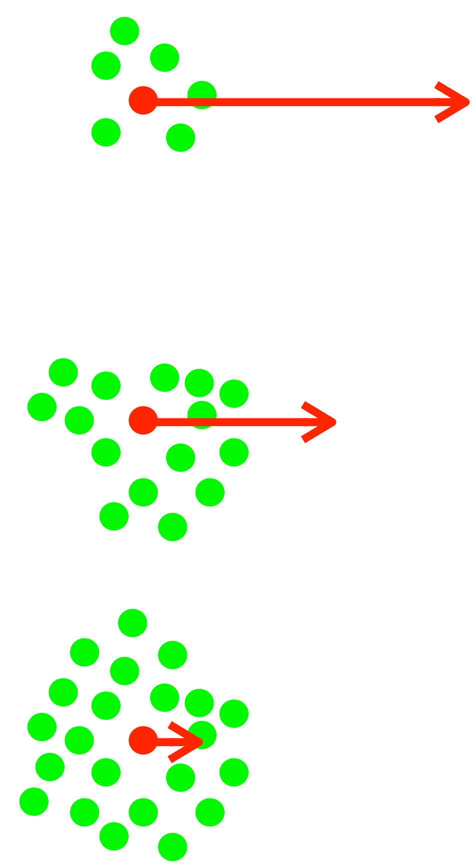
GR force (1 particle)

many particles



the new force (1 particle)

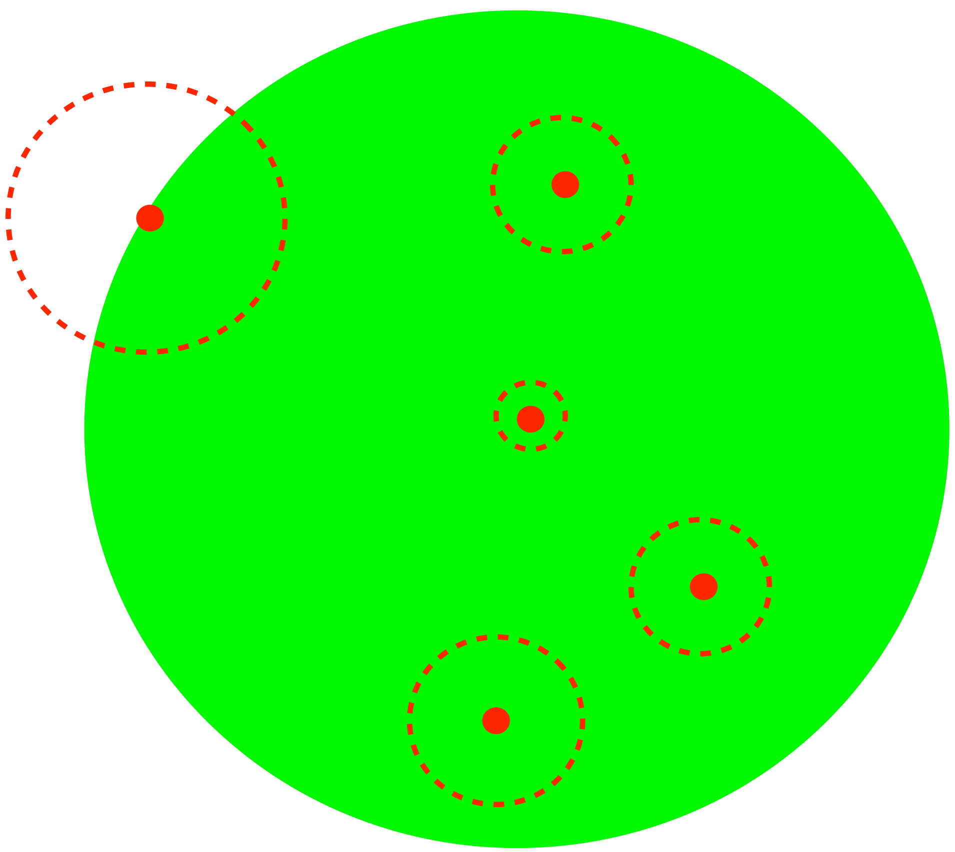
many particles

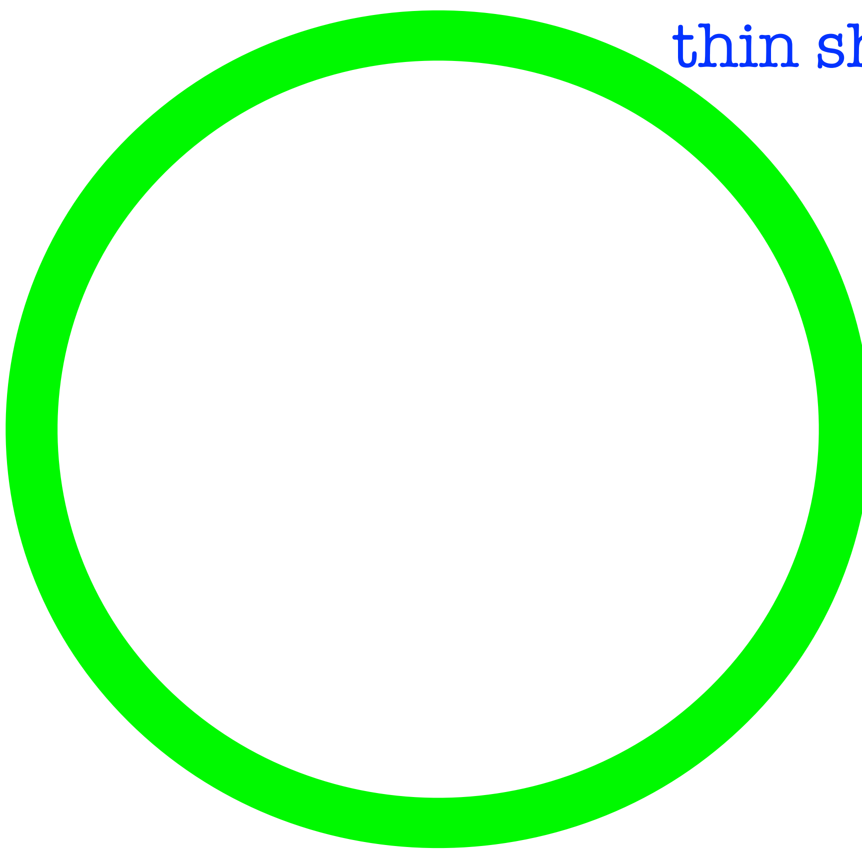


Yukawa force:
larger m, shorter range

$$\boxed{\varphi(r) \quad \propto \quad \frac{1}{r}\exp(-mr)}$$

Chameleon Mechanism & Models



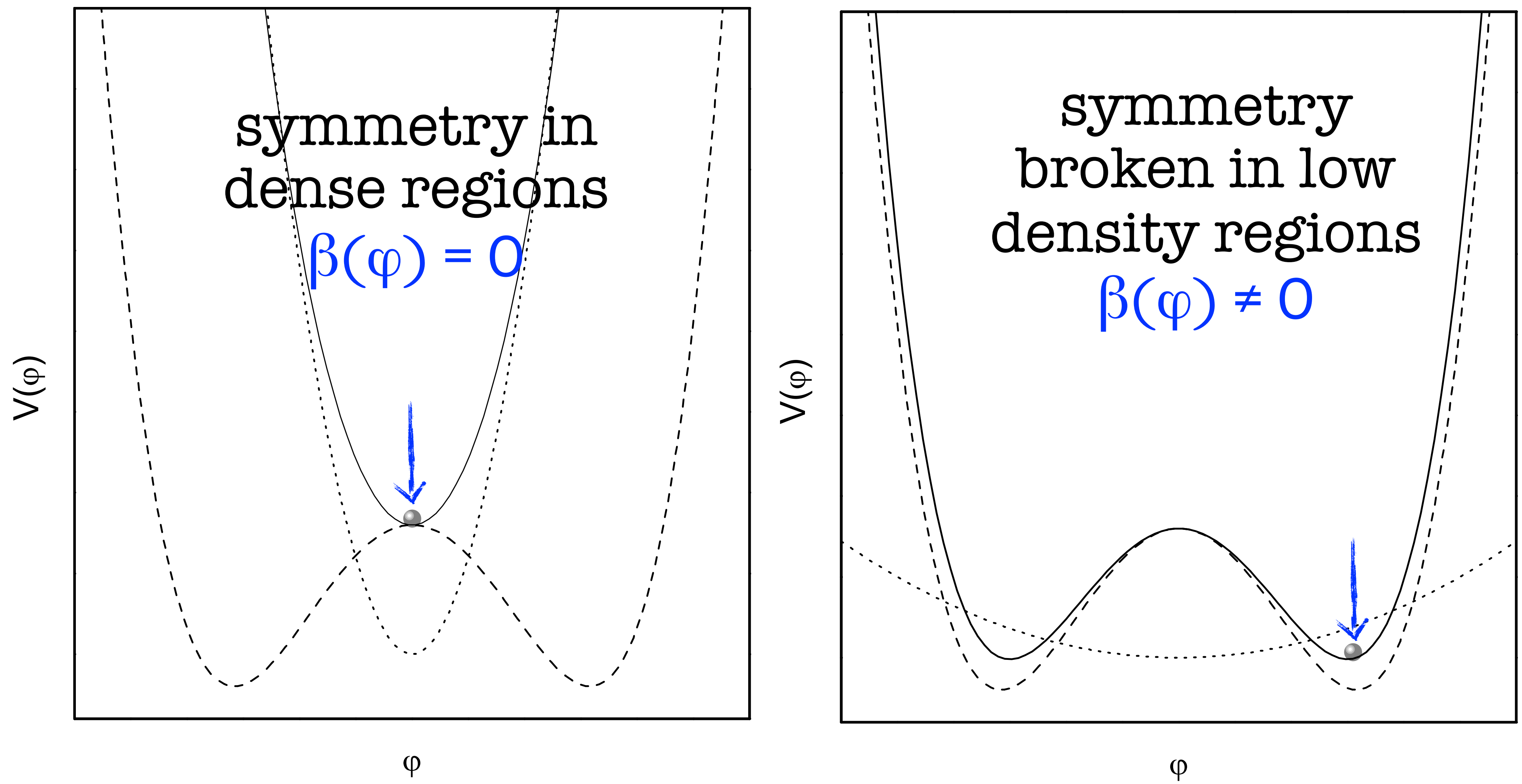


dashed circles: regions affected by the deviation from GR.

The new force produced by inner particles too short ranged

only mass in this thin shell contributes to new force felt by external particles

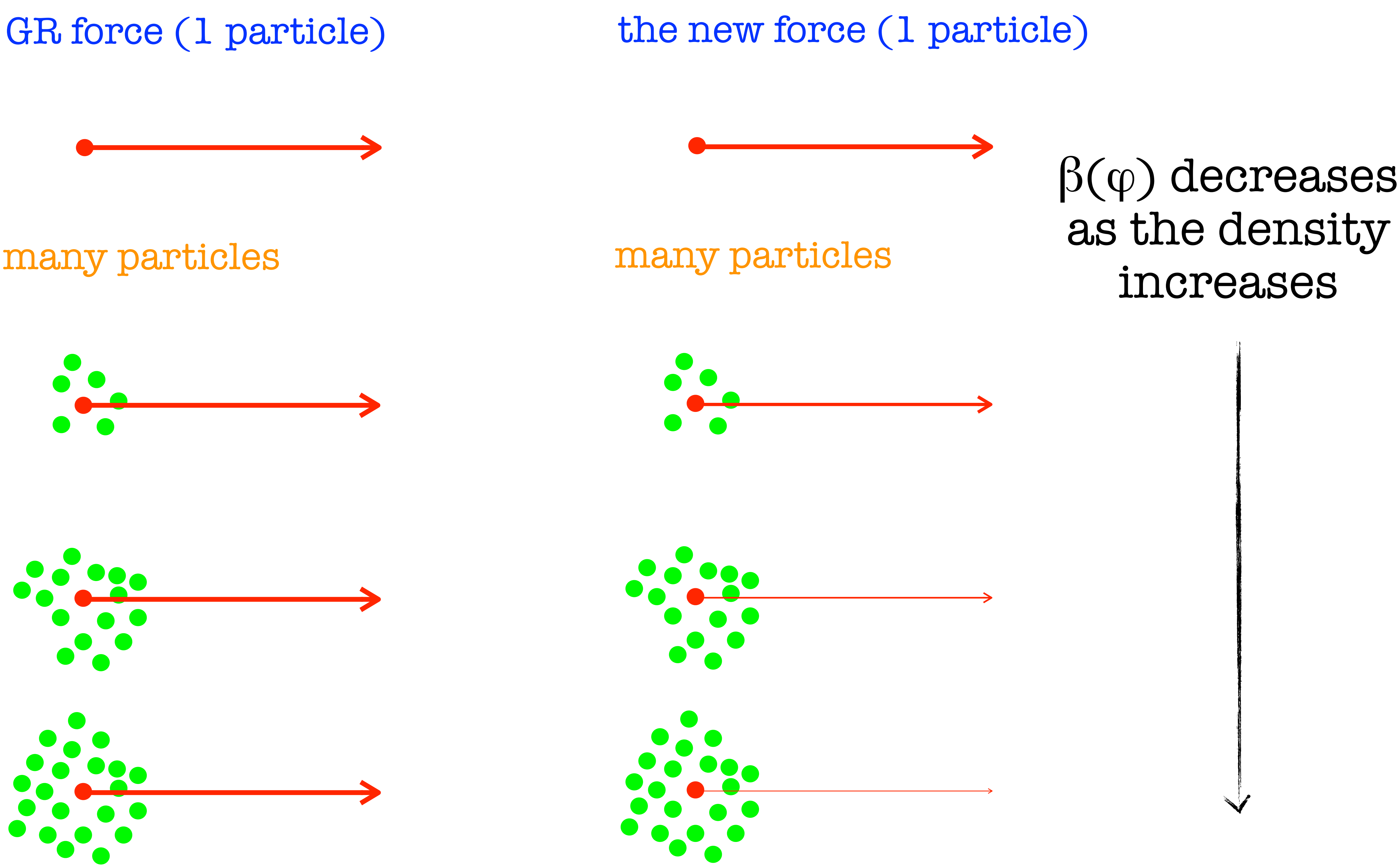
Symmetron Mechanism & Models



$$\nabla^2\Phi = 4\pi G\rho_{\rm m} + \beta(\varphi)\nabla^2\varphi$$

$$\Box\varphi + \frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi} = C(\varphi)\rho_{\rm m}$$

Symmetron Mechanism & Models



Vainshtein Mechanism & Models

- Both chameleon and symmetron (as well as dilaton models, which are similar to symmetron in spirit and so will not be discussed here) types of models *rely on some sort of nonlinear self-interaction potential $V(\varphi)$ to dynamically drive the scalar field to a state that minimises the new force in high-density regions*.

- Though the new force can be as strong as standard gravity in low density regions, so there is still a cosmological effect.

$$
\begin{aligned}
\nabla^2\Phi &= 4\pi G\rho_{\rm m} + \beta(\varphi)\nabla^2\varphi \\
\Box\varphi + \frac{\mathrm{d}V(\varphi)}{\mathrm{d}\varphi} &= C(\varphi)\rho_{\rm m}
\end{aligned}
$$

vanishes in dense regions;
large in low density regions

Vainshtein Mechanism & Models

- But a self-interaction potential $V(\varphi)$, which is a nonlinear algebraic function of the scalar field φ, is not the only way a scalar field could interact with itself.

- There can also be what people call derivative interactions, which involve nonlinear powers of the scalar field derivatives in the equation

$$
\nabla^2\varphi + \begin{matrix} \left(\nabla^2\varphi\right)^2 \\ \nabla^i\nabla^j\varphi\nabla_i\nabla_j\varphi \\ \left(\nabla^2\varphi\right)^3 \\ \nabla^i\nabla^j\varphi\nabla_j\nabla^k\varphi\nabla_k\nabla_i\varphi \\ \nabla^2\varphi\nabla^i\nabla^j\varphi\nabla_i\nabla_j\varphi \\ |\nabla\varphi|^2 \\ \cdots \end{matrix} = C(\varphi)\rho_{\rm m}
$$

$$
\nabla^2\Phi = 4\pi G\rho_{\rm m} + \beta(\varphi)\nabla^2\varphi
$$

???

Vainshtein Mechanism & Models

$$\nabla^2\varphi + \begin{matrix} (\nabla^2\varphi)^2 \\ \nabla^i\nabla^j\varphi\nabla_i\nabla_j\varphi \\ (\nabla^2\varphi)^3 \\ \nabla^i\nabla^j\varphi\nabla_j\nabla^k\varphi\nabla_k\nabla_i\varphi \\ \nabla^2\varphi\nabla^i\nabla^j\varphi\nabla_i\nabla_j\varphi \\ |\nabla\varphi|^2 \\ \ldots \end{matrix} = C(\varphi)\rho_{\mathrm{m}}$$

$$\nabla^2\Phi = 4\pi G\rho_{\mathrm{m}} + \beta(\varphi)\nabla^2\varphi$$

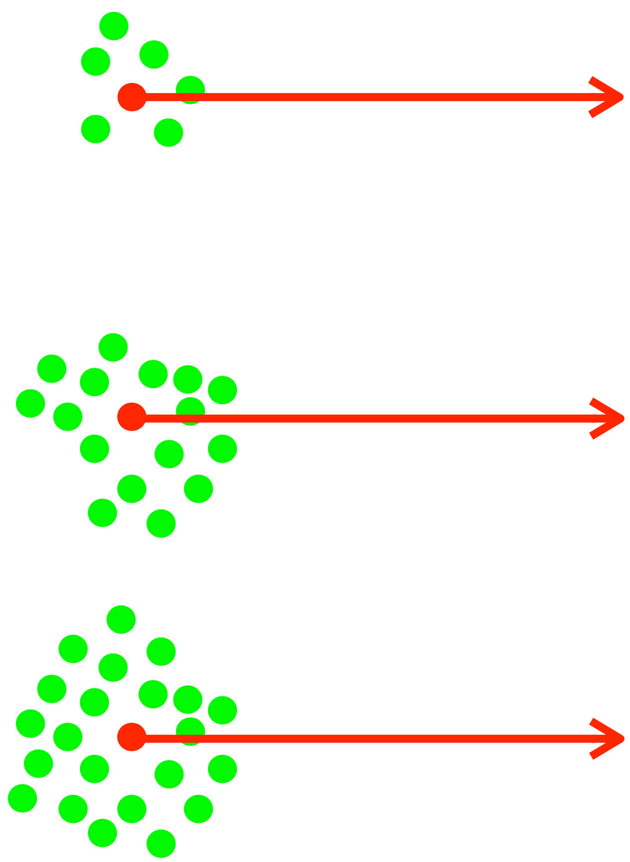
dense regions: $\nabla^2\varphi$ small, higher order terms even smaller, $\nabla^2\varphi \sim C(\varphi)\rho_m$. New force comparable to standard Newtonian force.

underdense regions: $\nabla^2\varphi$ big, higher order terms even bigger, $\nabla^2\varphi \ll C(\varphi)\rho_m$. New force much smaller than standard Newtonian force and therefore negligible. Screening works.

Vainshtein Mechanism & Models

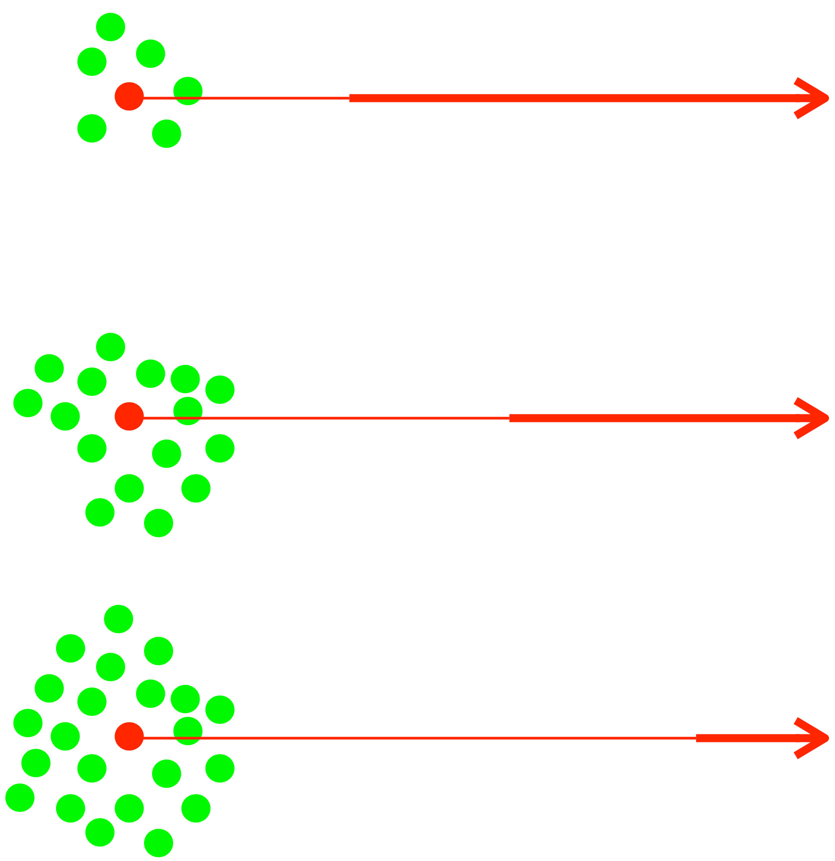
GR force (1 particle)

many particles

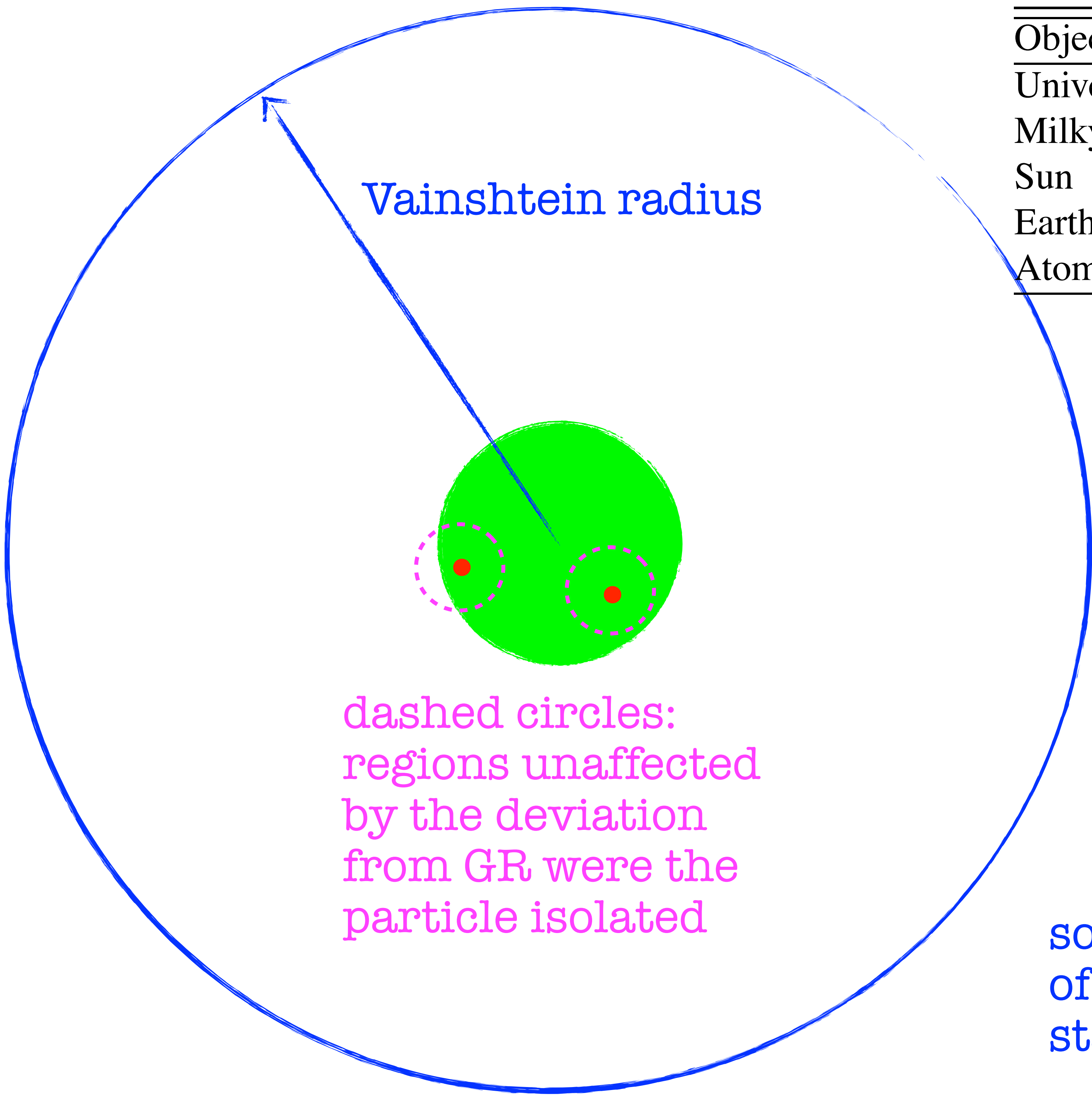


the new force (1 particle)

many particles

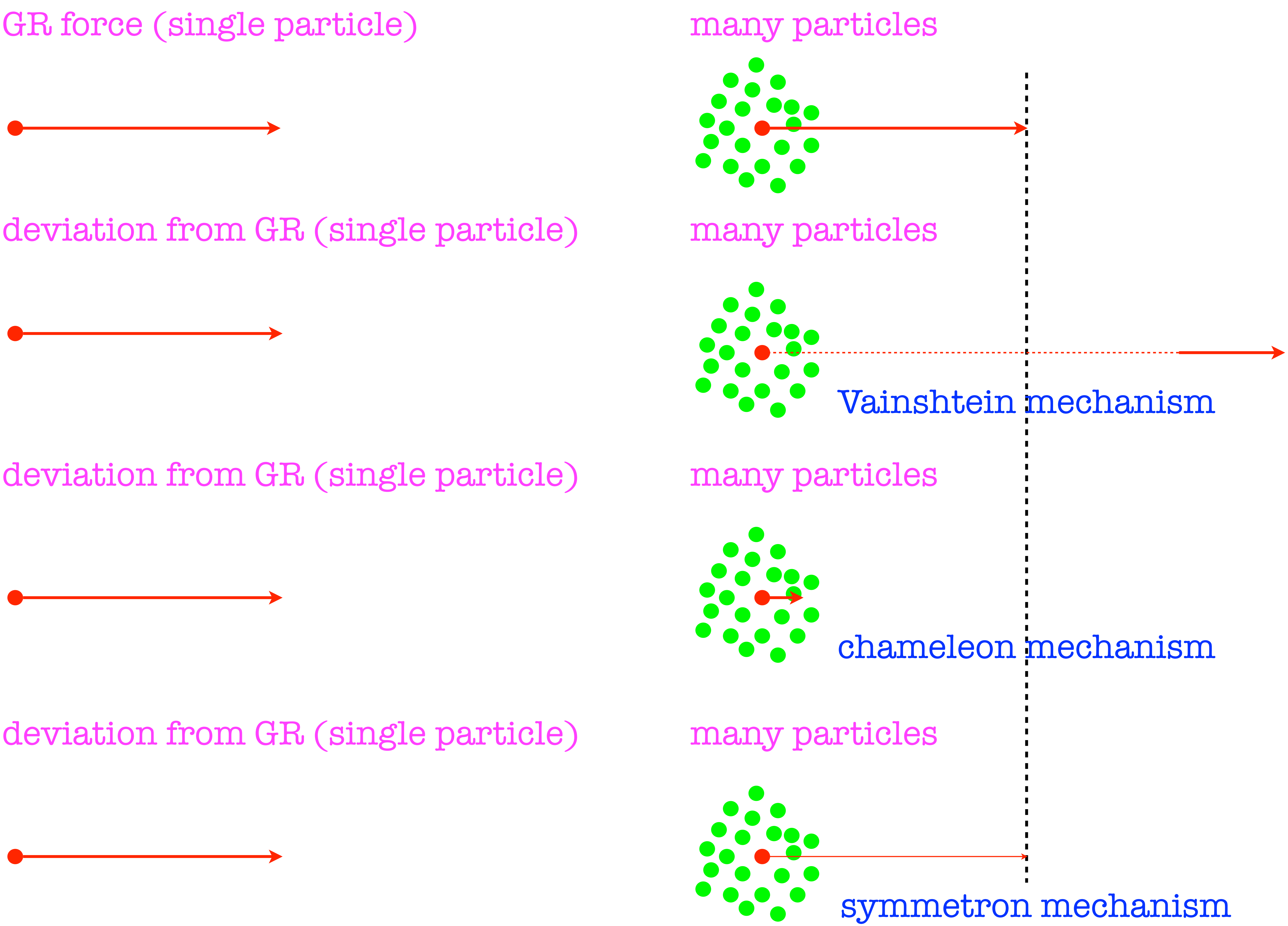


Vainshtein Mechanism & Models

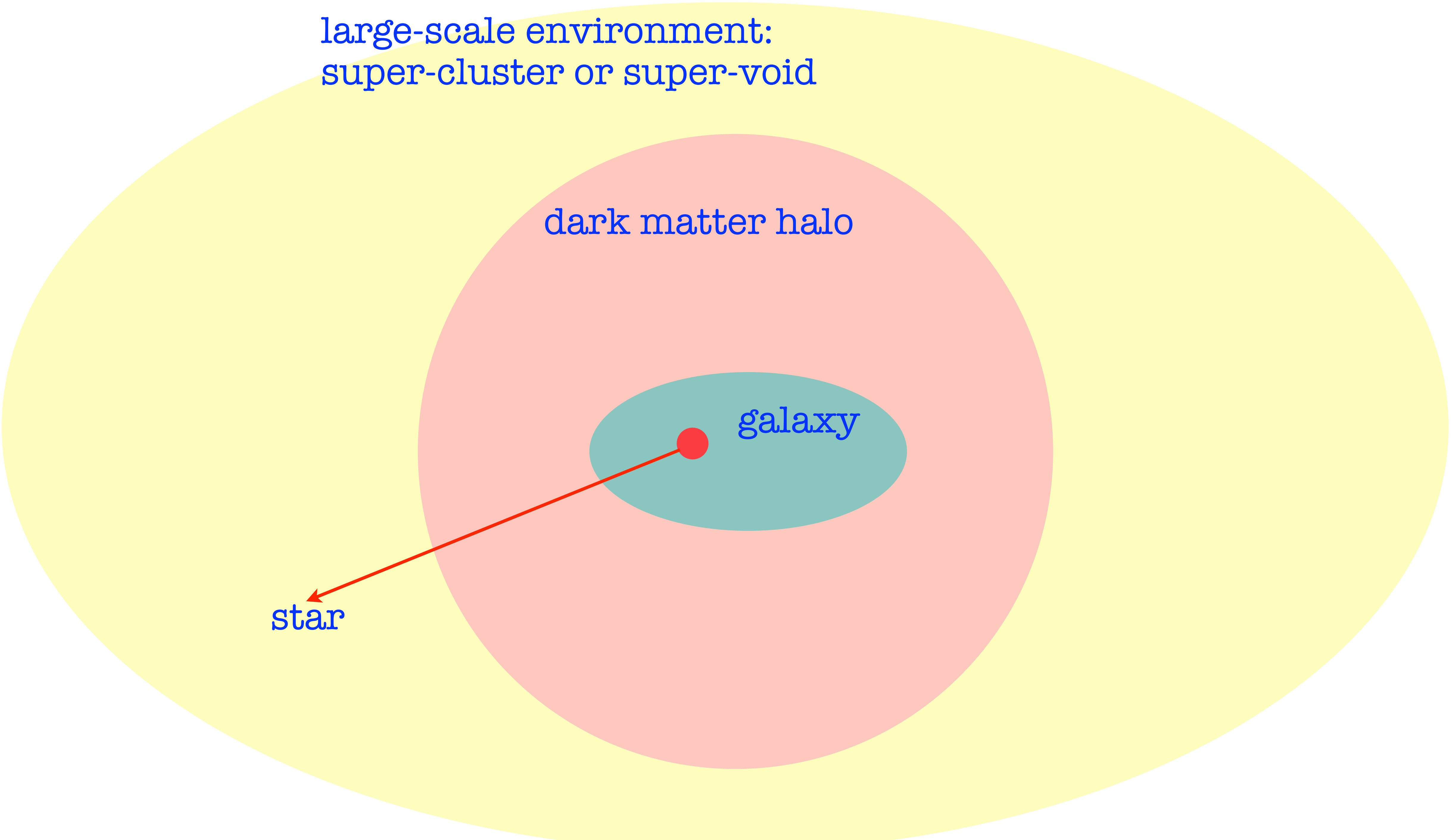


Object	R	r_s	r_*
Universe	$\sim 1.2 \times 10^{26}$	$\sim 4.5 \times 10^{25}$	$\sim 8.6 \times 10^{25}$
Milky Way	$\sim 0.9 \times 10^{21}$	$\sim 2 \times 10^{15}$	$\sim 3 \times 10^{22}$
Sun	$\sim 0.7 \times 10^{9}$	$\sim 3 \times 10^{3}$	$\sim 3.5 \times 10^{18}$
Earth	$\sim 6 \times 10^{6}$	$\sim 9 \times 10^{-3}$	$\sim 5 \times 10^{16}$
Atom	$\sim 5 \times 10^{-11}$	$\sim 1.8 \times 10^{-54}$	$\sim 3 \times 10^{-1}$

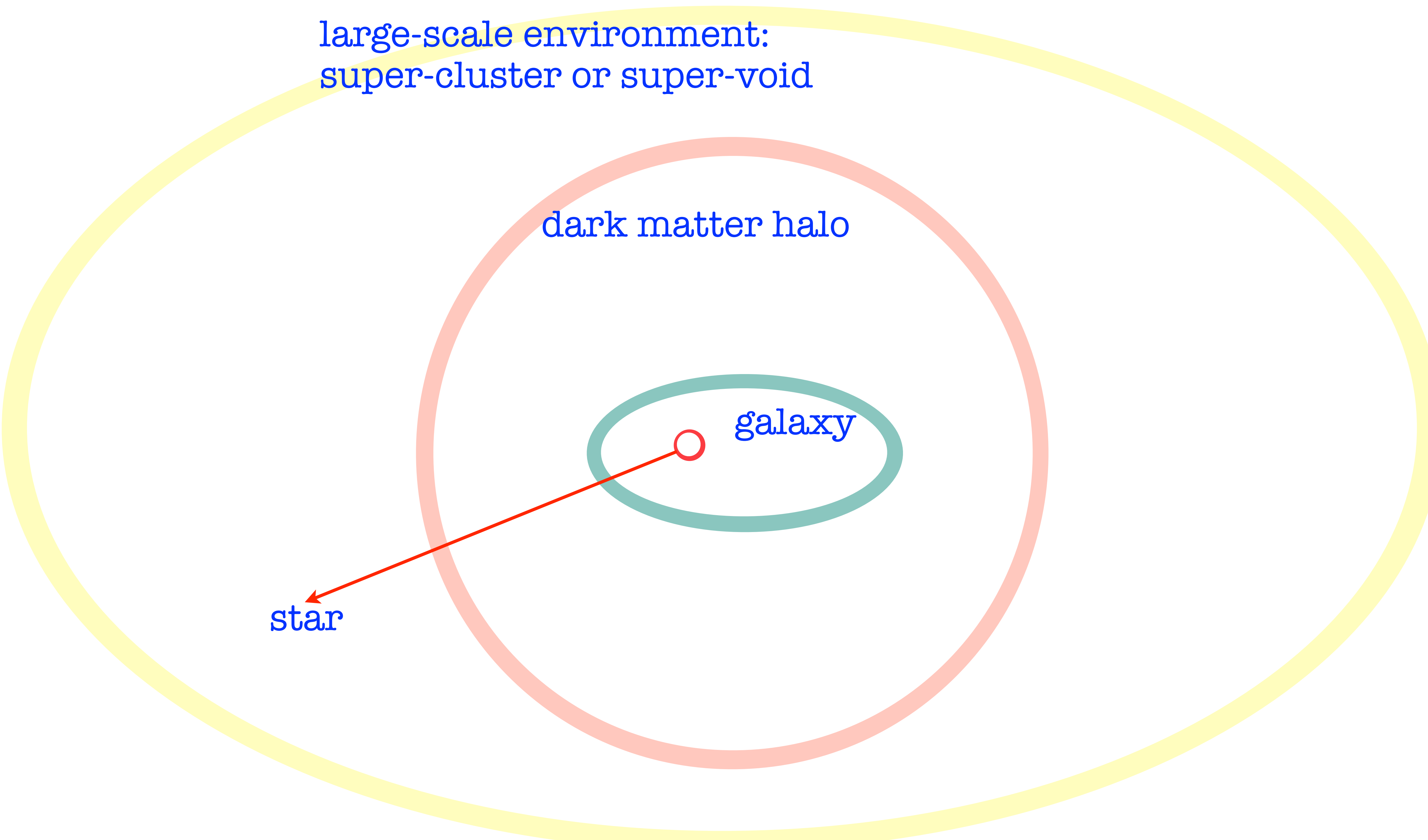
solid circle: Vainshtein radius
of the spherical body below which
standard gravity is recovered


GR force (single particle)
many particles
deviation from GR (single particle)
many particles
Vainshtein mechanism
deviation from GR (single particle)
many particles
chameleon mechanism
deviation from GR (single particle)
many particles
symmetron mechanism

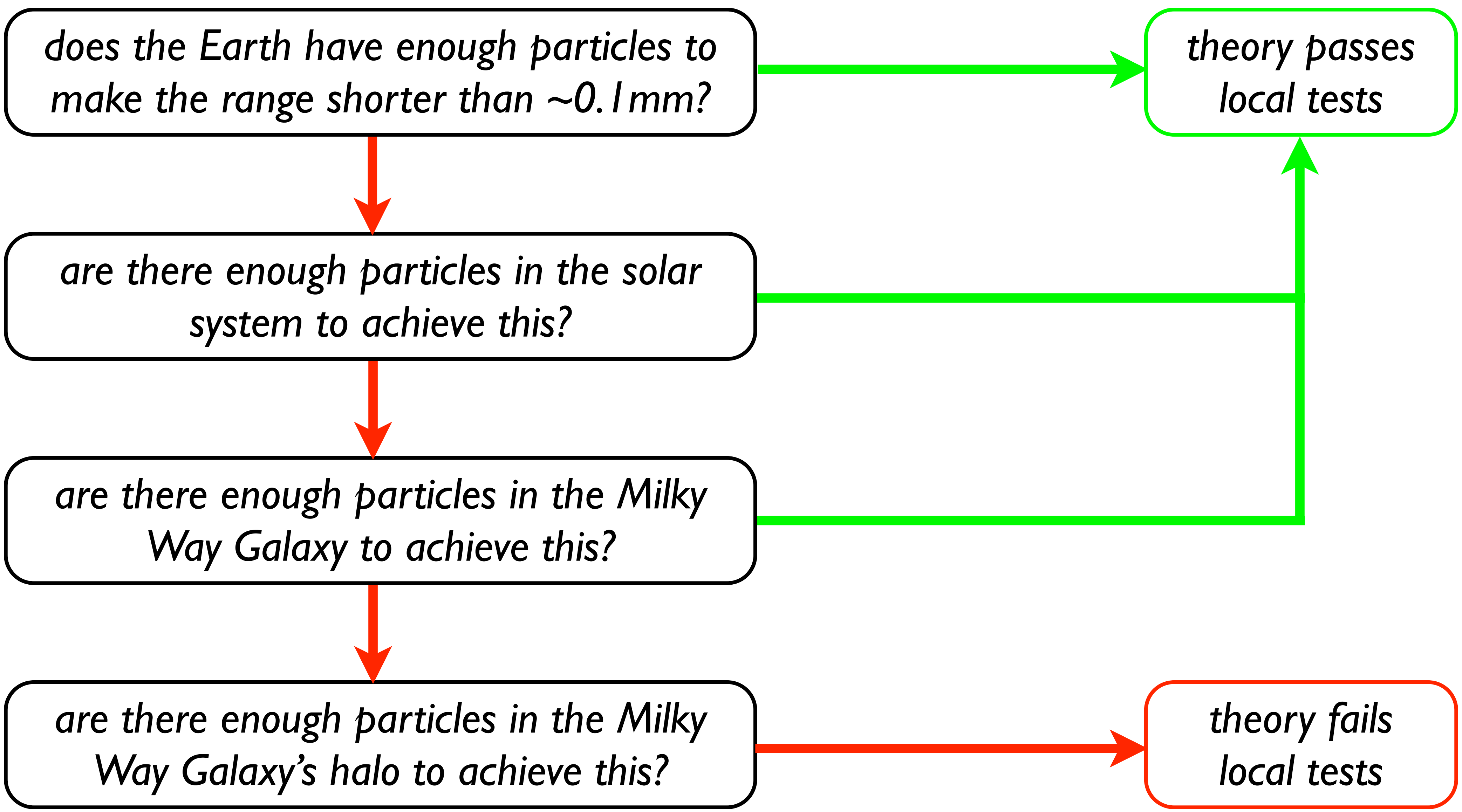
Environmental Dependence (Chameleon)



Environmental Dependence (Chameleon)



Environmental Dependence (Chameleon)



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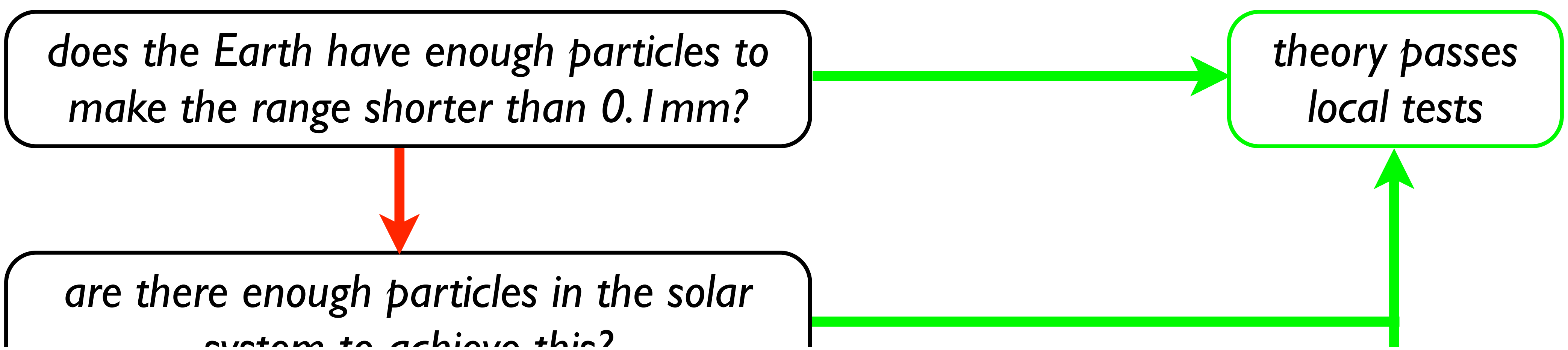

Behaviour of the scalar field on small length scales can be affected by its behaviour on very large length scales.

This environmental dependence means that one has to solve the behaviour of the scalar field in the **whole** system, even if the aim is to study it on small scales.

The complexity of matter distribution means that numerical simulations is the only way to make accurate predictions.