From N-body halo catalogue to HOD catalogue

In this practice project we will do a practice of populating dark matter halo catalogues from N-body simulations with galaxies using the Halo Occupation Distribution (HOD) model (see also Aldo's lectures). Making galaxy-halo connection is an essential first step if we are to use observed galaxy clustering to constrain models like modified gravity, and HOD is one of the most successful models to do this.

This model populates haloes with galaxies in a statistical way. More specifically:

• galaxies are divided into central galaxies and satellite galaxies; the former are assumed to be at the centre of dark matter haloes, while the latter reside in subhaloes. Without a complete subhalo catalogue, an alternative is to assume that the satellites distribute radially following an Navarro-Frenk-White (NFW) density profile (just like dark matter) inside haloes. The NFW profile is given by

$$\rho(r) = \frac{4\rho_s}{\frac{r}{R_s}\left(1+\frac{r}{R_s}\right)^2} = \frac{4\rho_s}{\frac{r}{R_h}\frac{R_h}{R_s}\left(1+\frac{r}{R_h}\frac{R_h}{R_s}\right)^2} = \frac{4\rho_s}{\frac{r}{R_h}c_h\left(1+\frac{r}{R_h}c_h\right)^2} \tag{1}$$

where R_s is the scale radius of the NFW profile, ρ_s is the density at R_s, R_h is the halo radius, and $c \equiv R_h/R_s$ is the 'concentration' parameter of the halo which describes how concentrated the halo density profile is.

• not every dark matter halo hosts a central or a satellite galaxy: massive haloes are more likely to be hosts of centrals, and each halo can host at most one central galaxy; haloes can host more than one satellite galaxies, and the more massive a halo is the more likely it can host more satellites.

• the probability that a given halo with mass M hosts a central galaxy is a uniformly distributed random number, whose mean is given by

$$\langle N_{\rm cen}\rangle = \frac{1}{2}\left[1+{\rm erf}\left(\frac{\log M - \log M_{\rm min}}{\sigma_{\log M}}\right)\right] \tag{2}$$

where ${\rm erf}$ is the error function, $M_{\rm min}$ is a HOD parameter such that haloes more massive than $M_{\rm min}$ are likely to have centrals, $\sigma_{\log M}$ is another HOD parameter.

• the number of satellite galaxies hosted by a given halo of mass M is drawn from a Poisson distribution whose mean is given by

$$\langle N_{\rm sat}\rangle = \begin{cases} \langle N_{\rm cen}\rangle\left(\frac{M-M_0}{M_1}\right)^\alpha & \text{if } M \geq M_0 \\ 0 & \text{if } M < M_0 \end{cases}$$

in which M_1, M_0 and α are also HOD parameters. Use the following convention here: if a halo hosts only one galaxy, namely $N_{\rm cen} = N_{\rm sat} = 1$, then make this galaxy central rather than satellite.

• the mean number of all galaxies inside a halo of mass M is given by $N(M) = \langle N_{\rm cen}(M)\rangle + \langle N_{\rm sat}(M)\rangle$.

Questions

In the links given later there are two halo catalogues, one for standard gravity and the other for a non-standard model (for those who want more information, it is the Hu-Sawicki $f(R)$ model with $|f_{R0}| = 10^{-5}$ and $n = 1$). Both catalogs are generated from the $z = 0$ particle snapshot of a N-body simulation of boxsize $1024h^{-1}$Mpc, particle number $N_p = 1024^3$, and WMAP9 cosmology.

There are 6 columns of each file, and they are respectively: halo mass M_h (in unit of $h^{-1}M_\odot$), x coordinate of halo centre, y coordinate of halo centre, z coordinate of halo centre, halo radius R_h, NFW scale radius R_s. All lengths and coordinates are in unit of h^{-1}Mpc.

0. Plot the differential or cumulative halo mass function of the two halo catalogues, as well as their relative difference. You will see that the $f(R)$ model predicts more haloes than standard gravity.

1. Please generate HOD catalogues for the two halo catalogues, following the specification given above. To do this you can use the following HOD parameters (for both models):

$$\begin{aligned}
\log\left[M_{\rm min}/\left(h^{-1}M_\odot\right)\right] &= 13.09 \\
\log\left[M_1/\left(h^{-1}M_\odot\right)\right] &= 14.00 \\
\log\left[M_0/\left(h^{-1}M_\odot\right)\right] &= 13.077 \\
\sigma_{\log M} &= 0.596 \\
\alpha &= 1.0127.
\end{aligned} \tag{3}$$

Note that haloes are assumed to be spherical; although satellite galaxies are assumed to distribute radially with an NFW profile, their angular distribution should be uniform[1].

Be careful with the subtleties in correctly using random number generators, and in get the NFW profile of satellites correct.

2. Measure the 3D real space 2-point correlation functions of the galaxies in your HOD catalogues. You don't have to write your own code (if you do not already have one to do this). There are codes publicly available for doing this, e.g., the CUTE code (http://members.ift.uam-csic.es/dmonge/CUTE.html) by David Alonso. Plot the correlation functions $\xi(r)$ as a function of galaxy separation r. Are they the same for the two models?

3. Can you vary your HOD parameters for the $f(R)$ model to make its galaxy 2-point correlation function agree with the result of standard gravity?

[1] Note that this does not mean that they distribute uniformly in angular coordinates θ and ϕ. See, for example, the discussion in the following article: http://mathworld.wolfram.com/SpherePointPicking.html