Cosmo-Statistics with Mathematica Wolfram
Mini-Taller 2017: Métodos Numéricos y Estadísticos en Cosmología

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All the initial codes to perform the following exercises can be found in `https://github.com/celia-escamilla-rivera?tab=repositories`
Due that we will working with Mathematica Wolfram[1] files (.nb or .m), the codes will not be displayed correctly in the above url, therefore is strongly recommended to "Clone or download" each repository in your own computer.

I. $H(z)$ DATA ON DARK ENERGY PARAMETERISATIONS.

Background theory: Different methods and data sets are being used to reconstruct the dark energy (DE) equation of state $w = p_{DE}/\rho_{DE}$ and thereby also to test the concordance model (which has $w = -1$). The results vary significantly according to the methods and data sets used, and the error bars and uncertainties are large. One of these data sets are the $H(z)$ data obtained from differential ages of galaxies. On one hand, these data can be used together with other cosmological test in order to get useful consistency checks or tighter constrains on models. On the other hand, these data comes from a Hubble function which is not integrated over, allowing the reduce of systematic errors and to get cleaner constraints on the cosmological parameters.

Task 1: Using the $H(z)$ data reported in [JCAP 0707 (2007) 015], and the Low Correlation parameterisation (Wang parameterisation) [Y. Wang, Phys. Rev. D 77 (2008) 123525]

$$E^2(z) \; = \; \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1-2w_0+3w_{0.5})} \times \exp\left[\frac{9(w_0 - w_{0.5})z}{1+z}\right], \qquad (1.1)$$

compute the χ^2-statistics, best fit values $(w_0, w_{0.5})$ and report the confidence contours. Check the differences.

To perform the task go to:
`https://github.com/celia-escamilla-rivera/H-data-on-dark-energy-parameterisations.`
and work with the file:

- Hcode_Task1.nb

II. COMPARISON OF STANDARD RULER AND STANDARD CANDLE ON DARK ENERGY PARAMETERISATIONS.

Background theory: There are two classes of probes that may be used to observe the expansion rate $H(z)$ or equivalently $w(z)$.

[1] Since Mathematica Wolfram is a software with license, there is an option of 15-days trial in `https://www.wolfram.com/mathematica/trial/`

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- *Standard Candles* are luminous sources of known intrinsic luminosity which may be used to measure the luminosity distance which, assuming flatness, is connected to $H(z)$ as

$$d_L(z) = c(1+z)\int_0^z dz' \frac{1}{H(z')} \tag{2.1}$$

Useful standard candles in cosmology are Type Ia supernovae (SnIa) and the less accurate but more luminous Gamma Ray Bursts.

- *Standard Rulers* are objects of known comoving size which may be used to measure the angular diameter distance which, in a flat universe, is related to $H(z)$ as

$$d_A(z) = \frac{c}{1+z}\int_0^z dz' \frac{1}{H(z')}. \tag{2.2}$$

The most useful standard ruler in cosmology is the last scattering horizon, the scale of which can be measured either directly at $z \simeq 1089$ through the CMB temperature power spectrum or indirectly through Baryon Acoustic Oscillations (BAO) on the matter power spectrum at low redshifts. Clusters of galaxies and radio galaxiesmay also be used as standard rulers under certain assumptions but they are less accurate than CMB+BAO.

A. Standard rulers data

We use the datapoints $(R, l_a, \Omega_b h^2)$ where R, l_a are two shift parameters:

- The scaled distance to recombination

$$R = \sqrt{\Omega_m \frac{H_0^2}{c^2}}\, r(z_{CMB}) \tag{2.3}$$

where $r(z_{CMB})$ is the comoving distance from the observer to redshift z and is given by

$$r(z) = \frac{c}{H_0}\int_0^z \frac{dz}{E(z)} \tag{2.4}$$

with $E(z) = H(z)/H_0$.

- The angular scale of the sound horizon at recombination

$$l_a = \pi \frac{r_(a_{CMB})}{r_s(a_{CMB})} \tag{2.5}$$

where $r_s(a_{CMB})$ is the comoving sound horizon at recombination given by

$$r_s(a_{CMB}) = \frac{c}{H_0}\int_0^{a_{CMB}} \frac{c_s(a)}{a^2 E(a)} da \tag{2.6}$$

with the sound speed being $c_s(a) = 1/\sqrt{3(1+\bar{R}_b a)}$ and $a_{CMB} = \frac{1}{1+z_{CMB}}$, where $z_{CMB} = 1089$. The quantity $\bar{R}_b$, is actually the photon-baryon energy-density ratio, and its value can be calculated using $\bar{R}_b = \frac{3}{4}\frac{\Omega_b h^2}{\Omega_\gamma h^2} = 31500\Omega_b h^2 (T_{CMB}/2.7K)^{-4}$.

For a flat prior, the 3-year WMAP data (WMAP3) measured best fit values are

$$\mathbf{\bar{V}_{CMB}} = \begin{pmatrix} \bar{R} \\ \bar{l}_a \\ \bar{\Omega}_b h^2 \end{pmatrix} = \begin{pmatrix} 1.70 \pm 0.03 \\ 302.2 \pm 1.2 \\ 0.022 \pm 0.00082 \end{pmatrix} \tag{2.7}$$

The corresponding normalised covariance matrix is

$$\mathbf{C_{CMB}^{norm}} = \begin{pmatrix} 1 & -0.09047 & -0.01970 \\ -0.09047 & 1 & -0.6283 \\ -0.01970 & -0.6283 & 1 \end{pmatrix} \tag{2.8}$$

from which the covariance matrix can be found to be:

$$(C_{CMB})_{ij} = (C_{CMB}^{norm})_{ij}\ \sigma_{\bar{V}^i_{CMB}} \sigma_{\bar{V}^j_{CMB}} \tag{2.9}$$

where $\sigma_{\bar{V}^i_{CMB}}$ are the 1σ errors of the measured best fit values of eq. (2.7).

We thus use equations (2.7), (2.3) and (2.5) to define

$$\mathbf{X_{CMB}} = \begin{pmatrix} R - 1.70 \\ l_a - 302.2 \\ \Omega_b h^2 - 0.022 \end{pmatrix}, \tag{2.10}$$

and construct the contribution of CMB to the χ^2 as

$$\chi^2_{CMB} = \mathbf{X_{CMB}}^{\mathbf{T}} \mathbf{C_{CMB}}^{\mathbf{-1}} \mathbf{X_{CMB}} \tag{2.11}$$

with

$$\mathbf{C_{CMB}}^{-1} = \begin{pmatrix} 1131.32 & 4.8061 & 5234.42 \\ 4.8061 & 1.1678 & 1077.22 \\ 5234.42 & 1077.22 & 2.48145 \times 10^6 \end{pmatrix}, \tag{2.12}$$

Notice that χ^2_{CMB} depends on four parameters (Ω_m, Ω_b, w_0 and w_1).

B. BAO

As in the case of the CMB, we apply the maximum likelihood method using the datapoints

$$\mathbf{\bar{V}_{BAO}} = \begin{pmatrix} \frac{r_s(z_{CMB})}{D_V(0.2)} = 0.1980 \pm 0.0058 \\ \frac{r_s(z_{CMB})}{D_V(0.35)} = 0.1094 \pm 0.0033 \end{pmatrix}, \tag{2.13}$$

where the dilation scale

$$D_V(z_{BAO}) = \left[\left(\int_0^{z_{BAO}} \frac{dz}{H(z)} \right)^2 \frac{z_{BAO}}{H(z_{BAO})} \right]^{1/3} \tag{2.14}$$

encodes the visual distortion of a spherical object due to the non-Euclidianity of a FRW spacetime, and is equivalent to the geometric mean of the distortion along the line of sight and two orthogonal directions. We thus construct

$$\mathbf{X_{BAO}} = \begin{pmatrix} \frac{r_s(z_{\rm dec})}{D_V(0.2)} - 0.1980 \\ \frac{r_s(z_{\rm dec})}{D_V(0.35)} - 0.1094 \end{pmatrix}, \tag{2.15}$$

and using the inverse covariance matrix

$$\mathbf{C_{BAO}}^{-1} = \begin{pmatrix} 35059 & -24031 \\ -24031 & 108300 \end{pmatrix}, \tag{2.16}$$

we find the contribution of BAO to χ^2 as

$$\chi^2_{BAO} = \mathbf{X_{BAO}}^{\mathbf{T}} \mathbf{C_{BAO}}^{-\mathbf{1}} \mathbf{X_{BAO}} \tag{2.17}$$

Task 2: Using the information given above, construct the contours for a CPL parametrisation using BAO+CMB data. You need to:

- Find the minimum of the χ^2-statistics with the priors $\Omega_m = 0.24$ and $\Omega_b = 0.042$
- Find the χ^2-statistics between the priors given for (Ω_m, Ω_b) and ΛCDM
- Compute the σ-distance between BAO+CMB and ΛCDM. Tip: Use the above results.
- Compute the contour plot at 1-2σ of confidence.

To perform the task go to:
`https://github.com/celia-escamilla-rivera/Standard-ruler-candle-DE`
and work with the files:

- SN_BAO_CMB_Task2.nb
- datacosmo.dat

III. TENSION BETWEEN SN AND BAO DATA SETS

Background theory: Giving the heterogeneous origin of the Union2 supernova dataset, and the procedures to reduce data described in [Astrophys. J. 686 (2008) 749.], we will work with an alternative version of χ^2, which consists in minimizing the quantity

$$\tilde{\chi}^2_{\rm SN}(\boldsymbol{\theta}) = c_1 - \frac{c_2^2}{c_3} \tag{3.1}$$

with respect to the other parameters. Here

$$c_1 = \sum_{j=1}^{\mathcal{N}_{\rm SN}} \frac{(\mu(z_j, \Omega_m; \mu_0 = 0, \boldsymbol{\theta})\} - \mu_{obs}(z_j))^2}{\sigma^2_{\mu,j}}\,, \tag{3.2}$$

$$c_2 = \sum_{j=1}^{\mathcal{N}_{\rm SN}} \frac{(\mu(z_j, \Omega_m; \mu_0 = 0, \boldsymbol{\theta})\} - \mu_{obs}(z_j))}{\sigma^2_{\mu,j}}\,, \tag{3.3}$$

$$c_3 = \sum_{j=1}^{\mathcal{N}_{\rm SN}} \frac{1}{\sigma^2_{\mu,j}}\,. \tag{3.4}$$

It is easy to see that $\tilde{\chi}^2_{SN}$ is just a version of χ^2_{SN}, minimized with respect to μ_0. To that end, it suffices to notice that

$$\chi^2_{\mathrm{SN}}(\mu_0, \boldsymbol{\theta}) = c_1 - 2c_2\mu_0 + c_3\mu_0^2 \,, \tag{3.5}$$

which clearly becomes minimum for $\mu_0 = c_2/c_3$, and so we can see $\tilde{\chi}^2_{\mathrm{SN}} \equiv \chi^2_{\mathrm{SN}}(\mu_0 = 0, \boldsymbol{\theta})$.

Task 3: Using the Levenberg-Marquardt algorithm, compute the confidence contour for the CPL parameterisation using standard ruler data (Baryon Acoustic Oscillations -BAO-) and SNe Ia standard candle data from a heterogeneous sample (UNION2)

To perform the task go to:
`https://github.com/celia-escamilla-rivera/Tension-between-SN-and-BAO`
and work with the files:

- Tension_Task3.nb
- union2.txt
- the BAO data explained above and your own Levenberg-Marquardt algorithm.

IV. CONFRONTING CPL PARAMETERISATION WITH ΛCDM USING JLA DATA SET

Task 4: Using the Levenberg-Marquardt algorithm, compute the confidence contour for the CPL parameterisation using JLA data set and compare with ΛCDM model.

To perform the task go to:
`https://github.com/celia-escamilla-rivera/Confronting-CPL-with-LCDM-using-JLA`
and work with the files:

- Contour_construction_Task4.nb
- jla_mub_complete.txt
- and your own Levenberg-Marquardt algorithm (or MCMC).