Cosmo-Estadística

Mini-Taller 2017: Métodos Numéricos y Estadísticos en Cosmología
Departamento de Física
Cinvestav, México.

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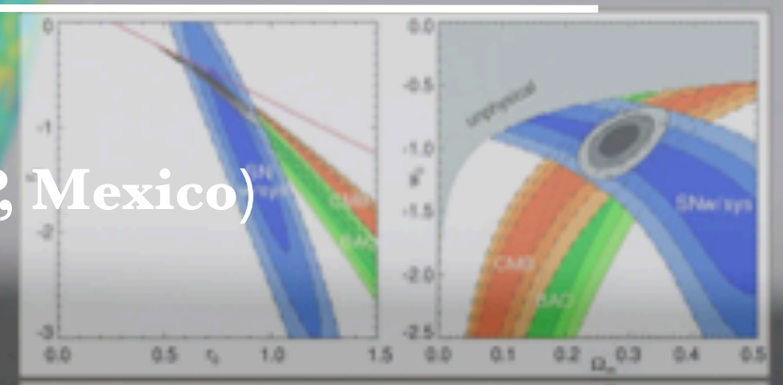
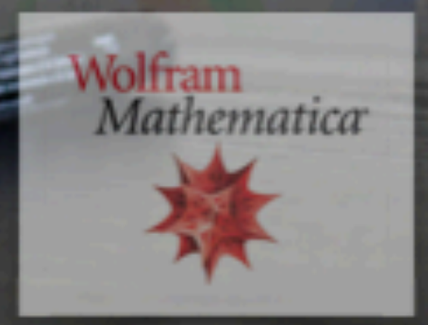




Outline

- Overview
- Cosmology state-of-the-art
- Statistical cosmology
- Modeling recipes: tools and tricks
- Non-parametric methods
- To take away....

- Overview

Curvature determined by content in the spacetime

Metric: way to describe distance/time

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

FRW: $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$

How is metric (distance measure) determinated?

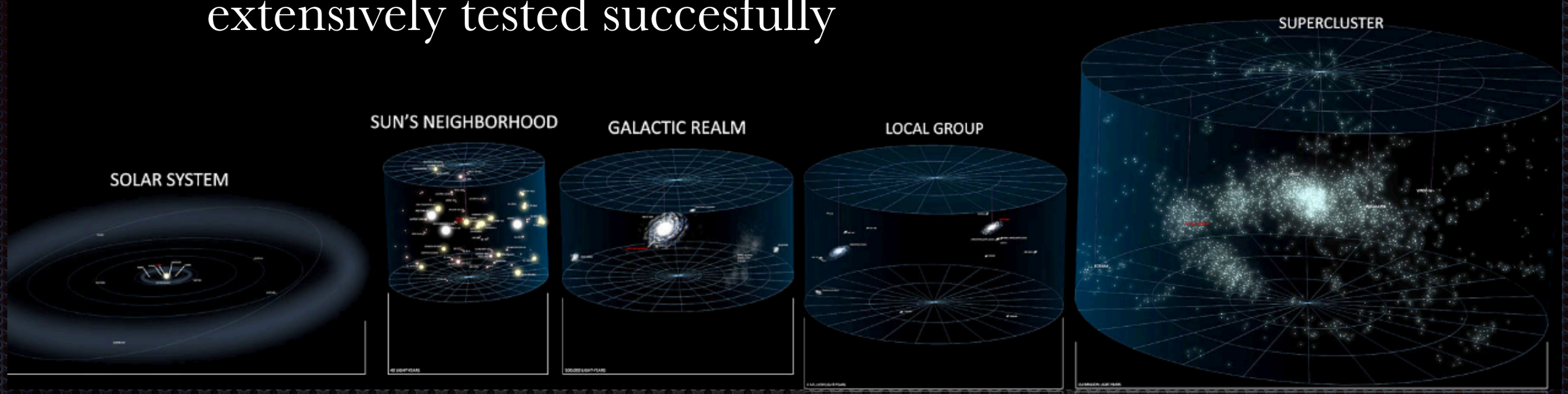
Specify way in which curvature determine/ response to matter

$$\underbrace{G_{\mu\nu}}_{\text{Curvature}} = 8\pi \underbrace{G_0}_{\text{Newton's constant}} \underbrace{T_{\mu\nu}}_{\text{Matter}}$$

$$G_{\mu\nu} \sim g, \partial g, \partial^2 g \qquad T_{\mu\nu} \sim \rho, P$$

Things we know:

- Einstein's General Relativity (GR) has been extensively tested succesfully



- General view → Couple the vacuum theory to matter
- Assume a minimal coupling,
- solve the full Einstein equations and
- compare theoretical models with observations

Re-examine ingredients that went into the cosmological cookbook

Gravity...?
General Relativity

Maybe GR is not the gravitational theory on large scales?
Applying to cosmology is an extrapolation of the validity of GR as the gravitational theory.
Modify Gravity

Geometry...?
Homogeneous and isotropic

Maybe the universe is not homogeneous/ isotropic on larger scale.
Inhomogeneous universes

Content...?
Barions, photons, neutrinos...

Maybe there is more to the content of the universe than we realized.
Dark matter and Dark energy

Structure formation and galaxy rotations...

Cosmic acceleration

- Cosmology state-of-the-art

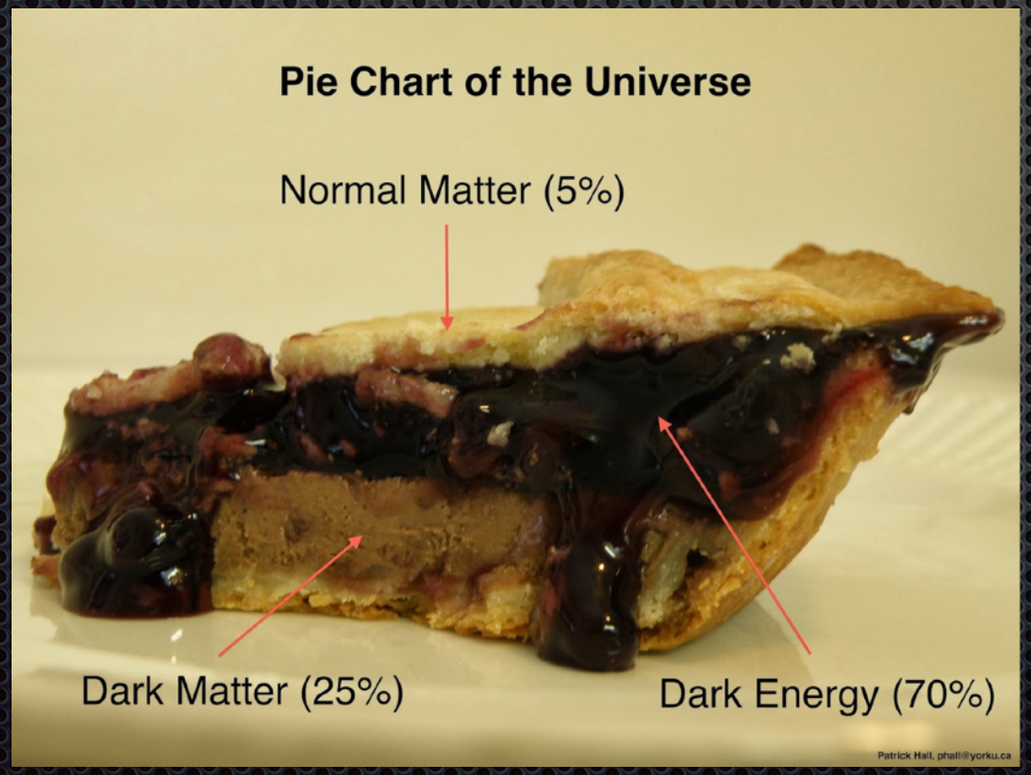
Gravity model
(GR+FRW)

+

Data
(CMB, structure formation, BAO,...)

=

Inconsistency ⟶ Dark matter + Dark energy



What is the Dark energy?

T. Clifton, P. Ferreira, A. Padilla and C. Skordis
Physics Reports 513, 1, 1-189 (2012).

The models zoo

$$\mathcal{L} = R + 2\Lambda$$ Cosmological constant

$$\mathcal{L} = R + F(R)$$

$$\mathcal{L} = R + F(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta})$$

$$\mathcal{L} = R + \partial_\mu\phi\partial^\mu\phi + 2V(\phi)$$ Quintessence

$$\mathcal{L} = R + F(R) + \partial_\mu\partial^\mu\phi + 2V(\phi)$$

$$\mathcal{L} = R - 2f(-\frac{1}{2}\partial_\mu\partial^\mu\phi, \phi)$$ K-essence

$$\mathcal{L} = R - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \lambda(A^\mu A_\mu - 1)$$ Einstein-Aether

$$\vdots$$

Phenomenological construction

Fundamental physics

Theories

Predictions for what such a universe will look like

...distance, abundance of structure, CMB, lensing...

Phenomenological framework

Characteristic observational signatures of broad classes of theories

Constrained

Observations

Use observational probes to uncover properties of dark sector theory

Cosmology over the past 14 years has made the transition to → Precision Cosmology

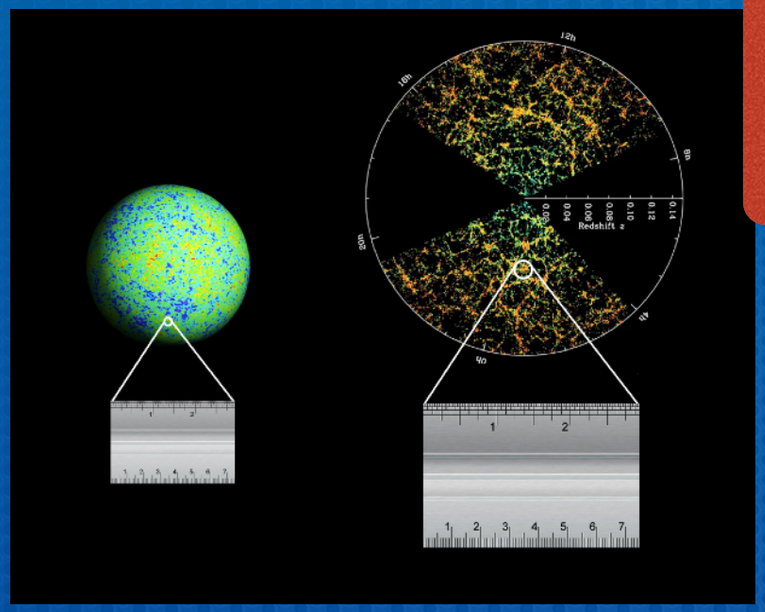
Cosmology has now a standard model → LCDM: only needs few parameters to describe origin composition and evolution

The curse of cosmology

↓

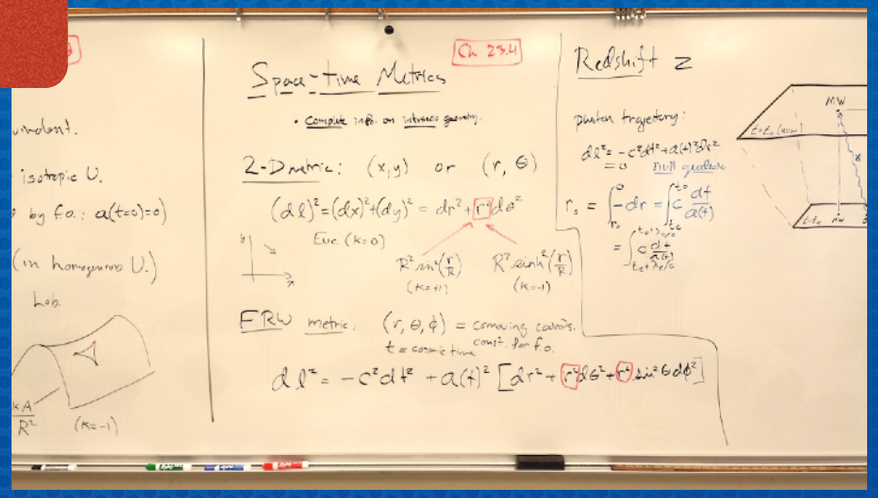
We can only make observations, not experiments… → We fit the models (i.e constrain numerical values of parameters) to observations. → **Results** will depend on the data you consider ("Robutness")

Data
(CMB, structure formation, BAO,...)



Measurement errors:
Cosmic variance

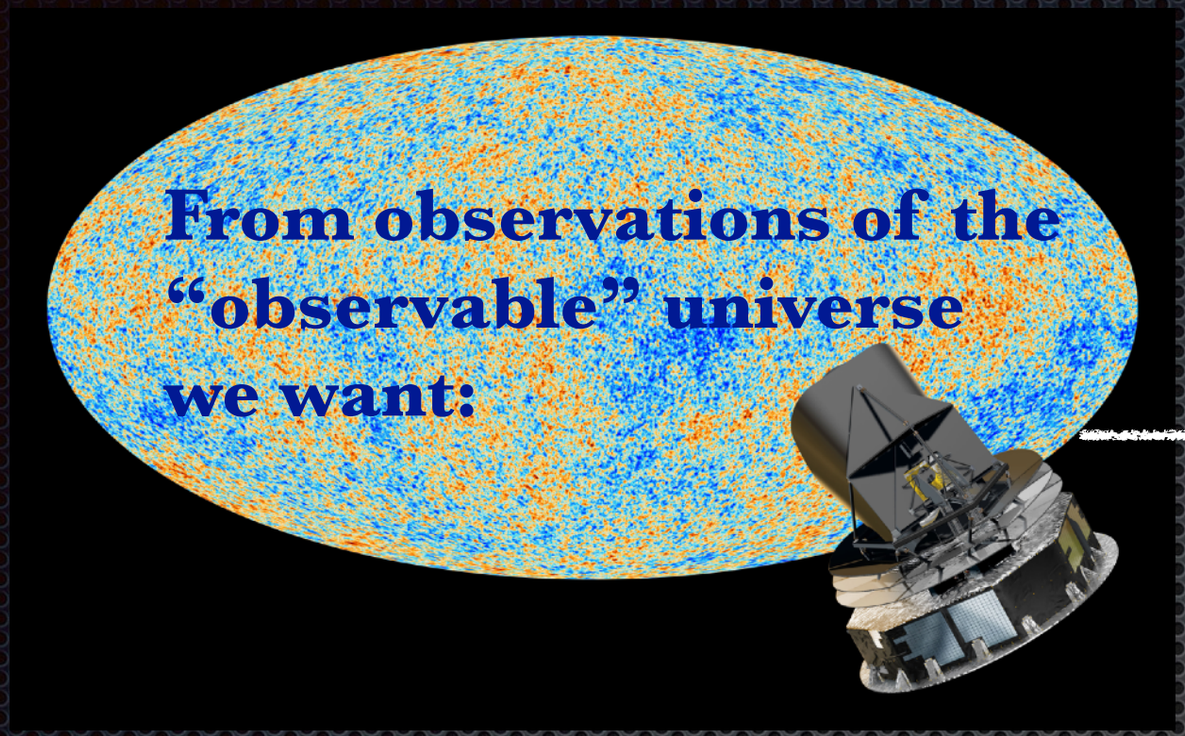
Models
Models parameters
LCDM?
w?
.
.



Most cosmological models are inverse problems, where we have a dataset and want to infer something.

Tasks:
(1) Hypothesis testing
(2) Parameter inference
(3) Model selection

- Statistical cosmology



to make inferences about the properties of the entire universe

Challenges

Important!

(C1) Calculating the errors. → Trust in the error bars…not in its "small" size

(C2) Bayes theorem. → How do we chose the "prior"?

Probability

Random variable: x
Probability distribution: P(x)
Properties:

Non-negative: real number for all real values of x

Normalized $\rightarrow \int dx P(x) = 1.$

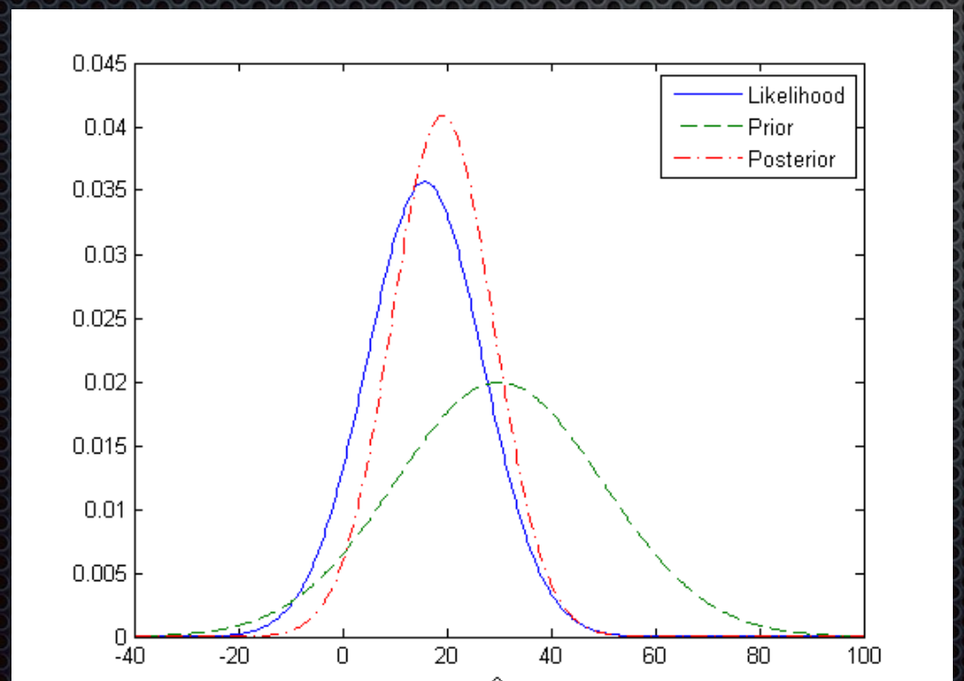
For 2-events: $P(x_1, x_2) = P(x_1) + P(x_2)$

In general: $P(a, b) = P(a)P(b|a)$

$P(b, a) = P(b)P(a|b)$

Bayes theorem

View: what we seem to want from the experiments



Likelihood Prior

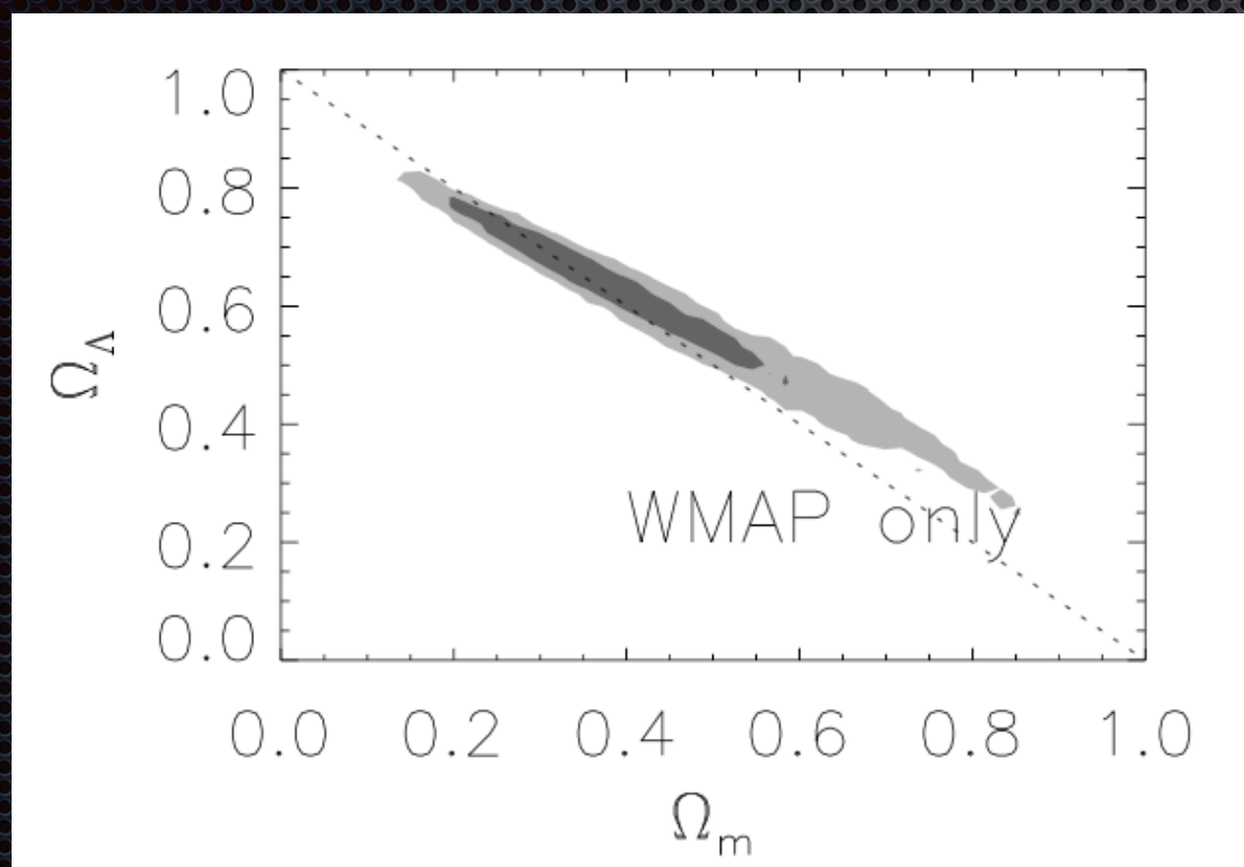
$$P(\theta|x) = \frac{P(x|\theta)P(\theta)}{P(x)}$$

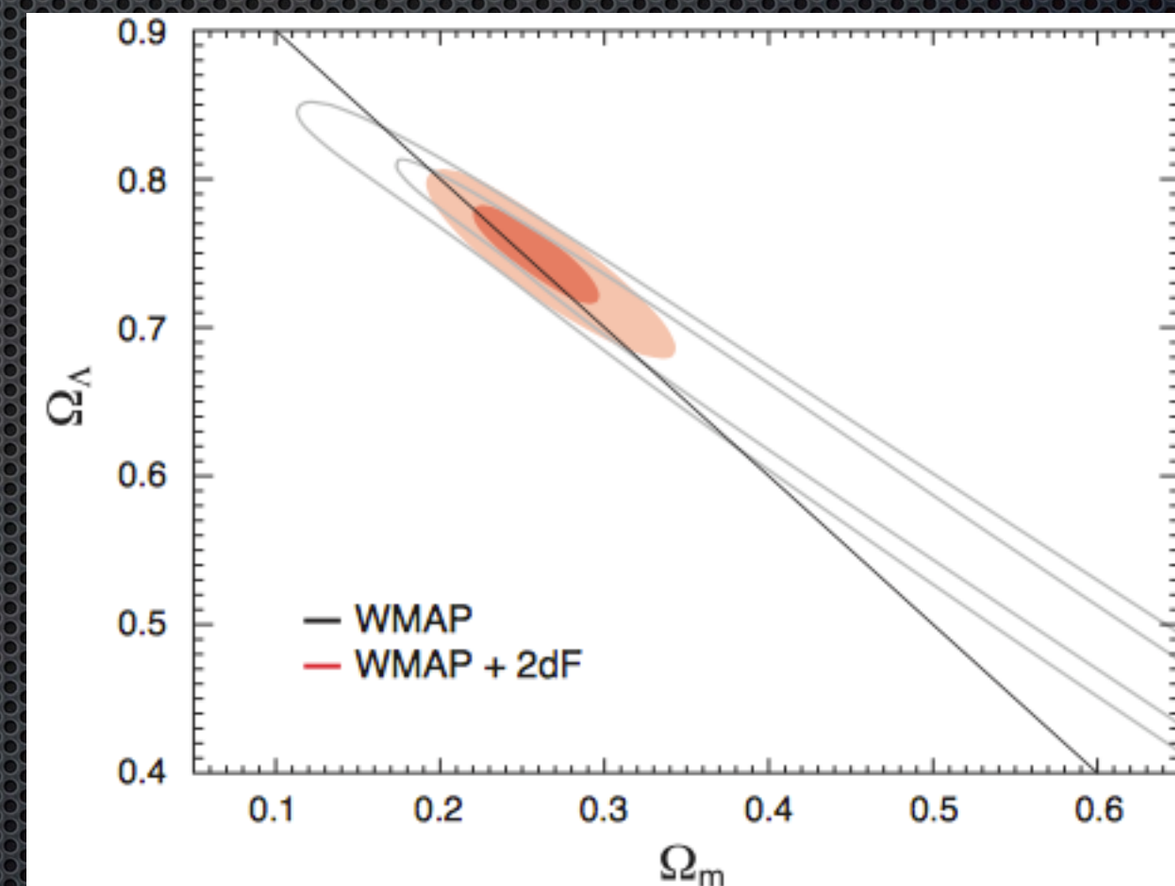
Posterior Evidence

Challenge 2!

Example: Comparison of likelihoods

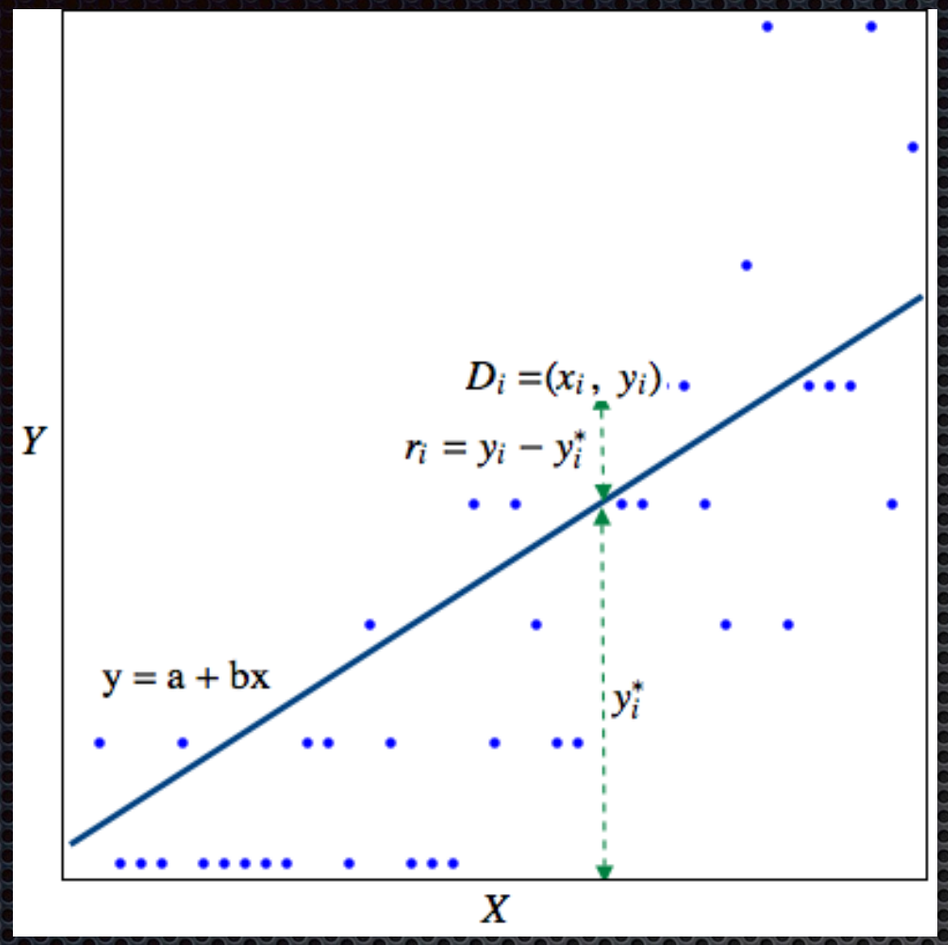
WMAP 1-year results (2003) Vs WMAP 3-year results (2006)





Why the differences? → Challenge 2! : the priors

Modeling the data



Least square fit

$$\chi^2 = \sum_i w_i \left[D_i - y(x_i|\theta)\right]^2$$

Correlation

$$\chi^2 = \sum_{ij} (D_i - y_i)^T F_{ij} (D_i - y_i)$$

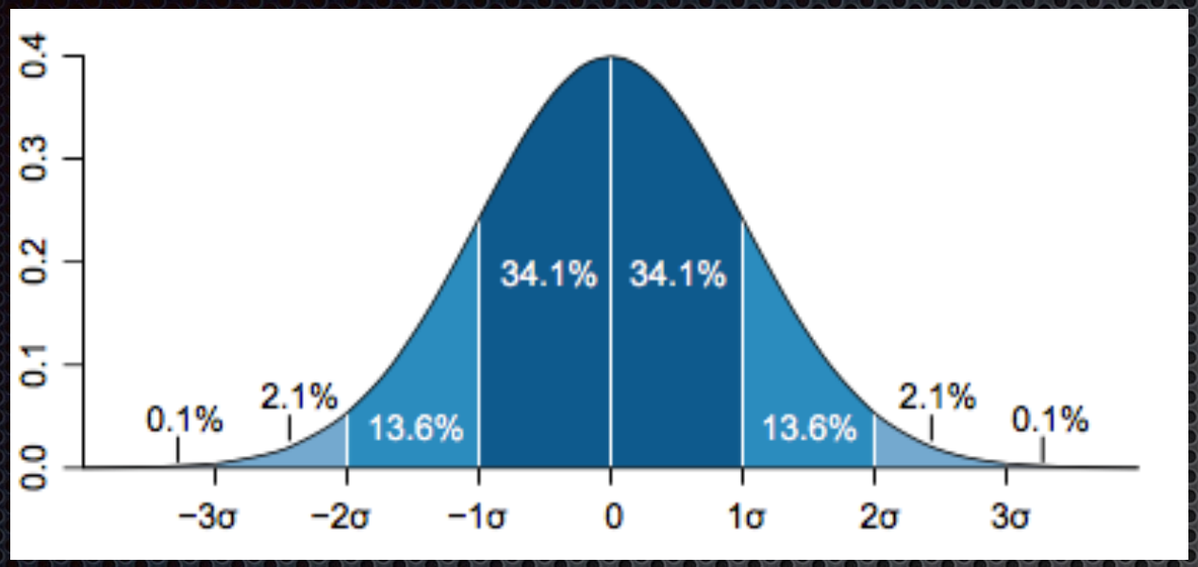
More complicate: use MCMC!

We want:

(1) Best fit parameters
(2) Error estimates on the parameters
(3) A statistical measure of the goodness-of-fit
(4) Bayesian analysis
(5) Figure-of-merit

Confidence regions

Dan Coe. arXiv: 0906.4123v1
Numerical Recipes in C. (1994).

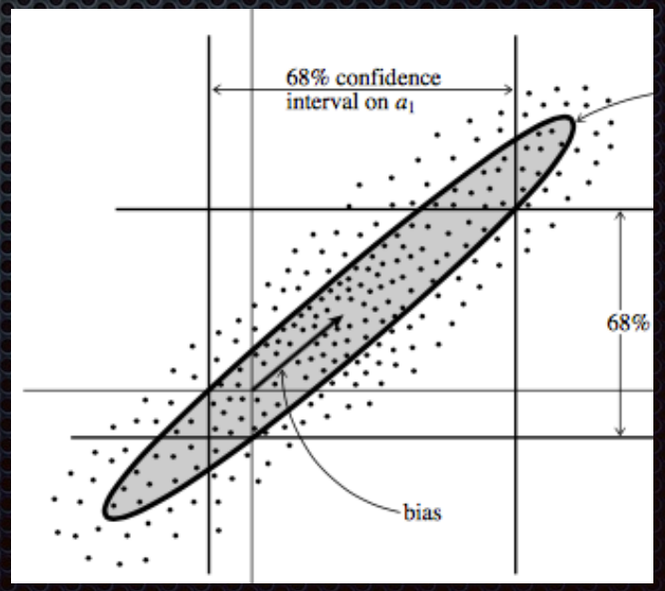


is a region of that M -dimensional space (hopefully a small region) that contains a certain (hopefully large) percentage of the total probability distribution.

$\Delta\chi^2$ as a Function of Confidence Level and Degrees of Freedom						
	ν					
p	1	2	3	4	5	6
68.3%	1.00	2.30	3.53	4.72	5.89	7.04
90%	2.71	4.61	6.25	7.78	9.24	10.6
95.4%	4.00	6.17	8.02	9.70	11.3	12.8
99%	6.63	9.21	11.3	13.3	15.1	16.8
99.73%	9.00	11.8	14.2	16.3	18.2	20.1
99.99%	15.1	18.4	21.1	23.5	25.7	27.8

In 1-dim: line segment centered on the measured value.

In higher dim:



- Modeling recipes: tools and tricks

Ingredients

1. Dark energy models (parameterisations)

2. Astrophysical data sets (SNe Ia, BAO, ...)

3. 2-dim grid algorithm (Computing best fits)

4. Statistical cosmology tools

Studying the nature of dark energy means: studying the equation of state

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0, \qquad \rho(z) = \rho_0(1+z)^{3(1+w(z))}$$

Evolution: Friedmann equation

$$h^2(z) \equiv \frac{H^2(z)}{H_0^2} = \Omega_m(1+z)^3 + (1-\Omega_m)e^{3\int_0^z \frac{1+w(z')}{1+z'}dz'}$$

Bidimensional dark energy models

Linear model

A. Cooray and D. Huterer
Astrophys.J. 513, L95 (1999)

$$w(z) = w_0 + \left(\frac{dw}{dz}\right)_{z=0},$$

$$E(z)^2 = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w_0+w_1)} \times e^{-3w_1 z}.$$

Chevallier-Polarski-Linder model (CPL)

M. Chevallier, D. Polarski and E.V. Linder
Int.J.Mod.Phys. D10, 213 (2001)

$$w(z) = w_0 + \left(\frac{z}{1+z}\right) w_1,$$

$$E(z)^2 = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w_0+w_1)} \times e^{-\left(\frac{3w_1}{1+z}\right)}.$$

Bidimensional dark energy models

E.M.Barboza, Jr and J.S.Alcaniz
Phys.Lett.B. 666, 415 (2008)

Barboza-Alcaniz model (BA)

$$w(z) = w_0 + \frac{z(1+z)}{1+z^2} w_1,$$

$$E(z)^2 = \Omega_m (1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w_0+w_1)} \times e^{-3w_1\left[\frac{z(1+z)}{1+z^2}\right]}.$$

Yun Wang. Phys.Rev.D77:123525
(2008)

Low correlation (Wang's model)

$$w(a) = \left(\frac{a_c - a}{a_c - 1}\right) w_0 + \left(\frac{a-1}{a_c - 1}\right) w_c$$

$$E^2(z) = \Omega_m (1+z)^3 + (1-\Omega_m)(1+z)^{3(1-2w_0+3w_{0.5})} \times e^{\left[\frac{9(w_0 - w_{0.5})z}{1+z}\right]}$$

Recipe 1. Background analysis: testing the dark energy models with observations from SNe Ia and BAO

https://github.com/celia-escamilla-rivera

Astrophysical data: Catalog SNe Ia JLA

$740 \quad events \quad in \quad 0.01 < z < 1.3$

With $N_{\rm JLA} = 31$ binned sample :

$$\mu(z_i, \mu_0) = 5\log_{10}\left[(1+z)\int_0^z d\tilde{z} E^{-1}(\tilde{z}, \Omega_m; w_0, w_1)\right] + \mu_0,$$

$$\chi^2_{\rm SN_{JLA}} = \sum_{i=1}^{N_{\rm JLA}} \frac{\left[\mu(z_i, \Omega_m; \mu_0, w_0, w_1) - \mu_{\rm obs}(z_i)\right]^2}{\sigma^2_{\mu,i}},$$

With $N_{\rm JLA} = 740$ full sample :

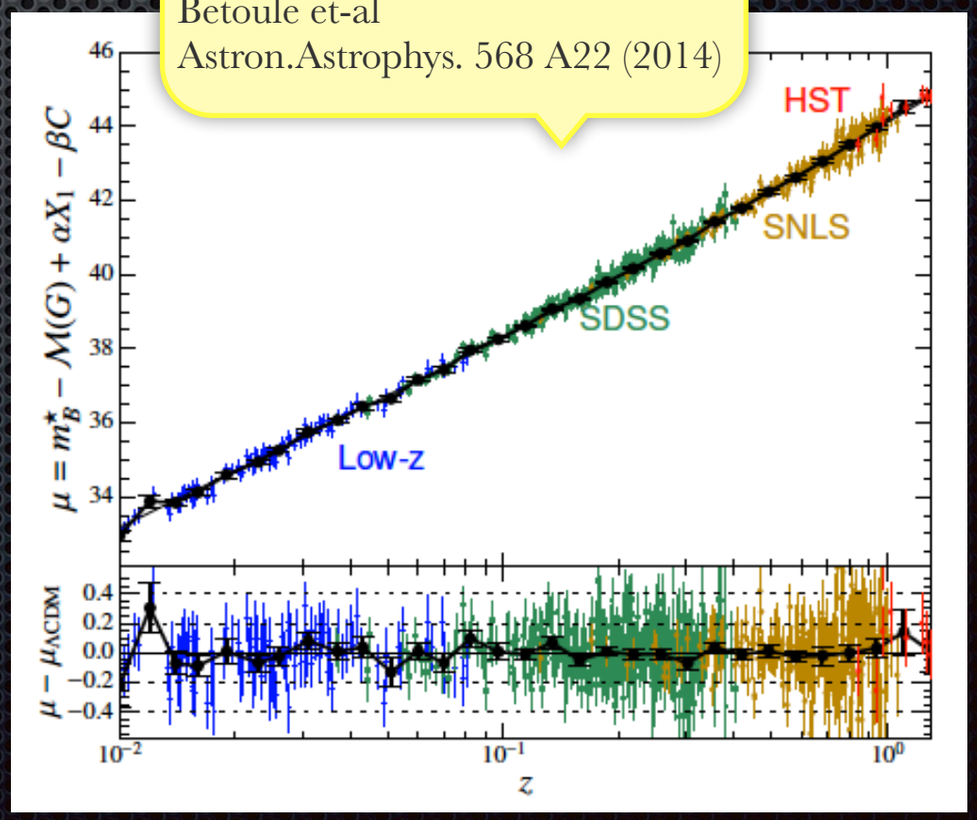
$$\mu_i^{SNIa} = m^\star_{B,i} + \alpha X_{1,i} - \beta C_i - M_B$$

$$\chi^2(\boldsymbol{\theta}, \boldsymbol{\delta}, M_B) = \sum_{i=1}^{740} \frac{\left[\mu_i^{SNIa}(\boldsymbol{\delta}, M_B) - \mu_{\rm th}^{\rm DEmodel}(z_i; \boldsymbol{\theta})\right]^2}{\sigma_i^2 + \sigma^2_{\rm int}}$$

$$\theta := [w_0, w_1]$$

$$\delta := (\alpha, \beta)$$

Betoule et-al
Astron.Astrophys. 568 A22 (2014)



Astrophysical data: Baryon Acoustic Oscillations

$$d_z \equiv \frac{r_s(z_d)}{D_V(z)}, \quad \text{with} \quad r_s(z_d) = \frac{c}{H_0}\int_{z_d}^{\infty} \frac{c_s(z)}{E(z)} \mathrm{d}z, \quad \text{where:} \quad c_s^2 = c^2/3[1 + (3\Omega_{b0}/4\Omega_{\gamma 0})(1+z)^{-1}]$$

$$D_V(z, \Omega_m; w_0, w_1) = \left[(1+z)^2 D_A^2 \frac{cz}{H(z, \Omega_m; w_0, w_1)}\right]^{1/3}$$

$$D_A(z, \Omega_m; w_0, w_1) = \frac{1}{1+z}\int_0^z \frac{c\,\mathrm{d}\tilde{z}}{H(\tilde{z}, \Omega_m; w_0, w_1)}$$

Beutler et al. Mon.Not.Roy.Astron.Soc. 416,3017 (2011)
X.Xu et al. Mon.Not.Roy.Astron.Soc. 427,2146 (2012)
C. Blake et al. Mon.Not.Roy.Astron.Soc.425,405. (2012).
L. Anderson et al. BOSS Collaboration. Mon.Not.Roy.Astron.Soc. 441, 24 (2014)

$d_z(z = 0.106) = 0.336 \pm 0.015$ from 6dFGS
$d_z(z = 0.35) = 0.1126 \pm 0.0022$ from SDSS
$d_z(z = 0.57) = 0.0726 \pm 0.0007$ from BOSS CMASS

+

$d_z(z = 0.44) = 0.073$
$d_z(z = 0.6) = 0.0726$
$d_z(z = 0.73) = 0.0592$
from WiggleZ

$$\mathbf{C}^{-1}_{\mathbf{WiggleZ}} = \begin{pmatrix} 1040.3 & -807.5 & 336.8 \\ -807.5 & 3720.3 & -1551.9 \\ 336.8 & -1551.9 & 2914.9 \end{pmatrix}$$

=

$$\chi^2_{\mathrm{BAO}}(\boldsymbol{\theta}) = \mathbf{X}^T_{\mathbf{BAO}}\mathbf{C}^{-1}_{\mathbf{BAO}}\mathbf{X}_{\mathbf{BAO}}$$

$$\chi^2_{\mathrm{BAO(total)}} = \chi^2_{\mathrm{6dFGS}} + \chi^2_{\mathrm{SDSS}} + \chi^2_{\mathrm{BOSSCMASS}} + \chi^2_{\mathrm{WiggleZ}}$$

Cosmic Chronometers (C-C) data. This kind of sample gives a measurement of the expansion rate without relying on the nature of the metric between the chronometer and us. A full compilation of the latter include 28 measurements of H(z) in the range $0.07 < z < 2.3$.

M. Moresco, et-al. 1201.6658

Data from BAO. Unlike the angular diameter measures given by the transverse BAO scale, the H(z) data can be extracted from the measurements of the line-of-sight of this BAO scale. Because the BAO distance scale is embodied in the CMB, its measurements on DE parameters strongest at low redshift.

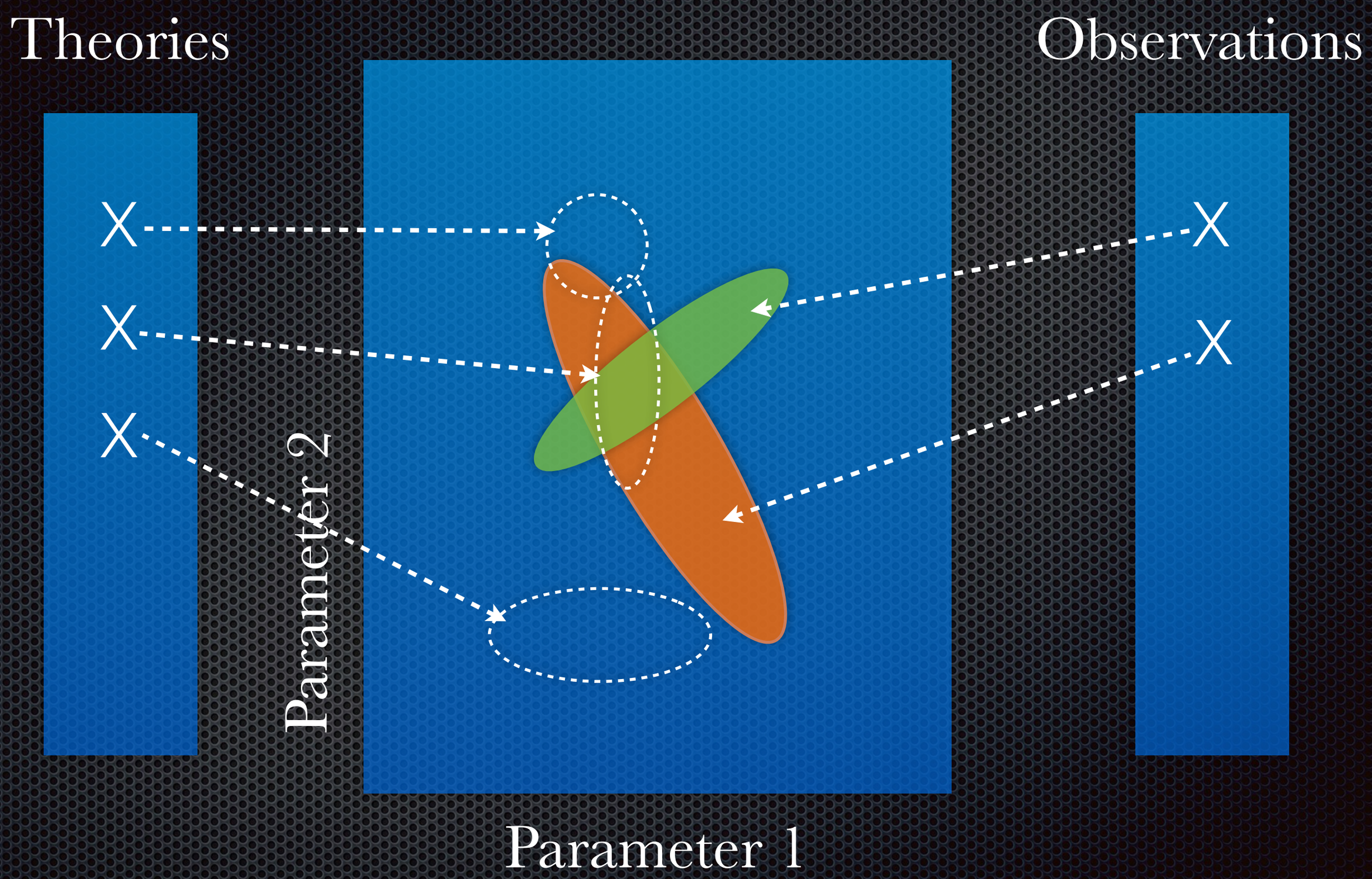
Bao sample data

z	$H(z)$ [kms^{-1}Mpc^{-1}]	$\sigma_H{}^2$
0.24	79.69	2.32
0.34	83.80	2.96
0.43	86.45	3.27
0.44	82.6	7.8
0.6	87.9	6.1
0.73	97.3	7.0

C. Blake, et al. Mon. Not. Roy. Astron. Soc. 425, 405 (2012)
E. Gaztanaga, et al. Mon. Not. Roy. Astron. Soc. 399, 1663 (2009)

Recipe 2. Background analysis: construction of the confidence contours

https://github.com/celia-escamilla-rivera

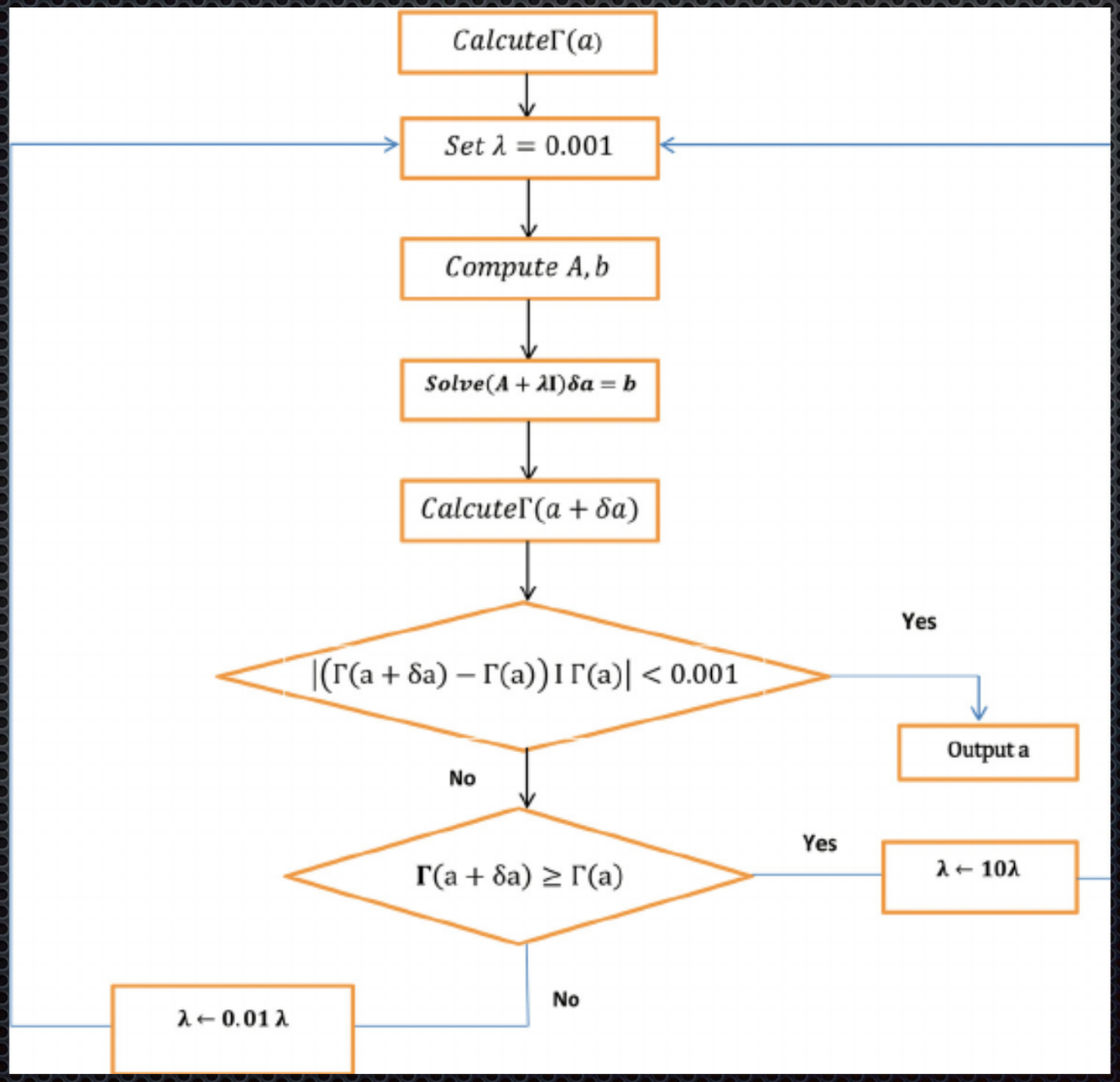

Theories
Observations
X
X
X
X
X
Parameter 2
Parameter 1

2-dim grid: Levenberg-Marquardt algorithm

Two-dimensional grid analysis

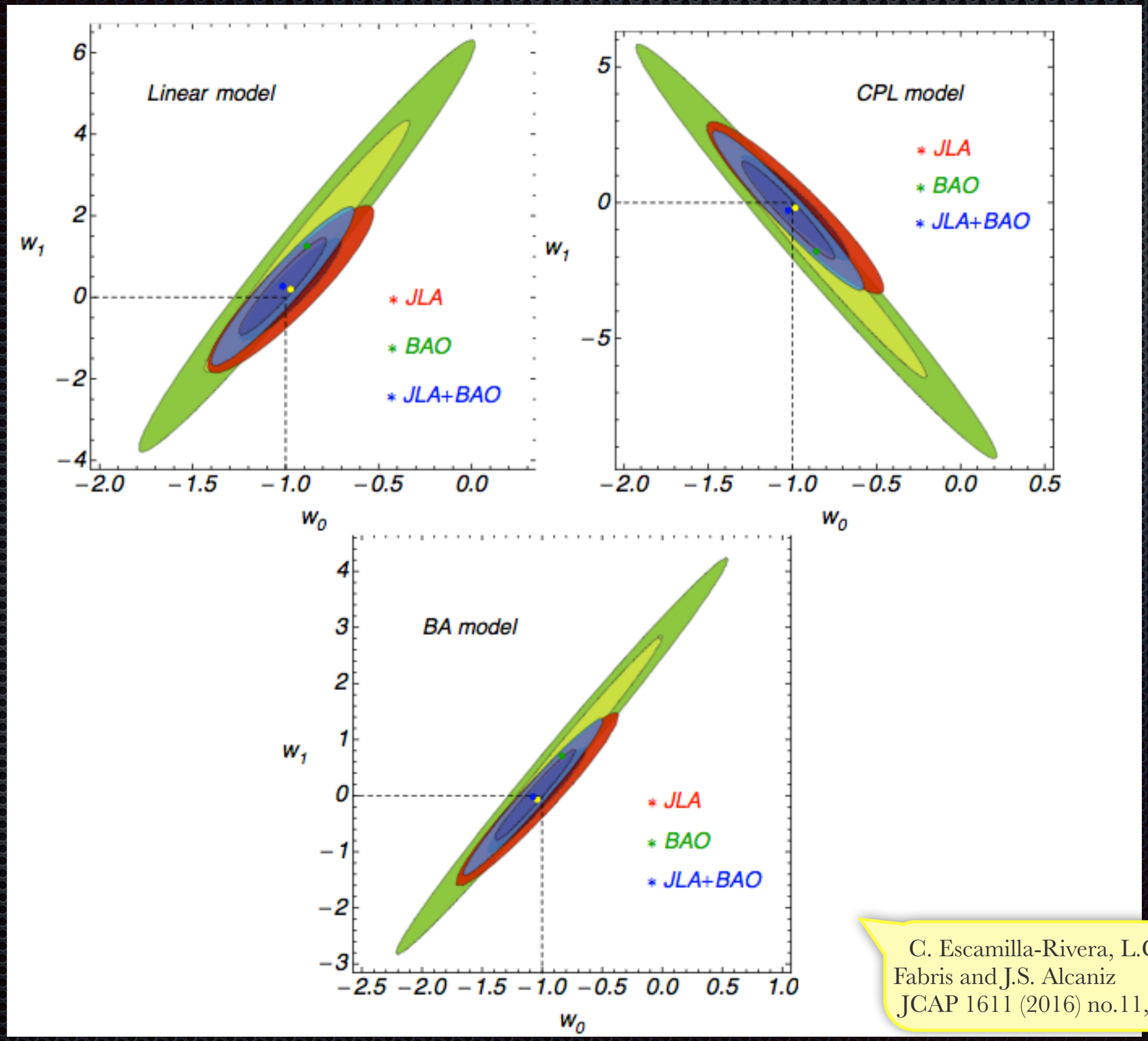
Appropiate $\chi^2(\theta)$ function

Find the values of $\theta : w_0, w_1, \ldots$ that minimize χ^2



Numerical Recipes in C. (1994). Pages 683-685

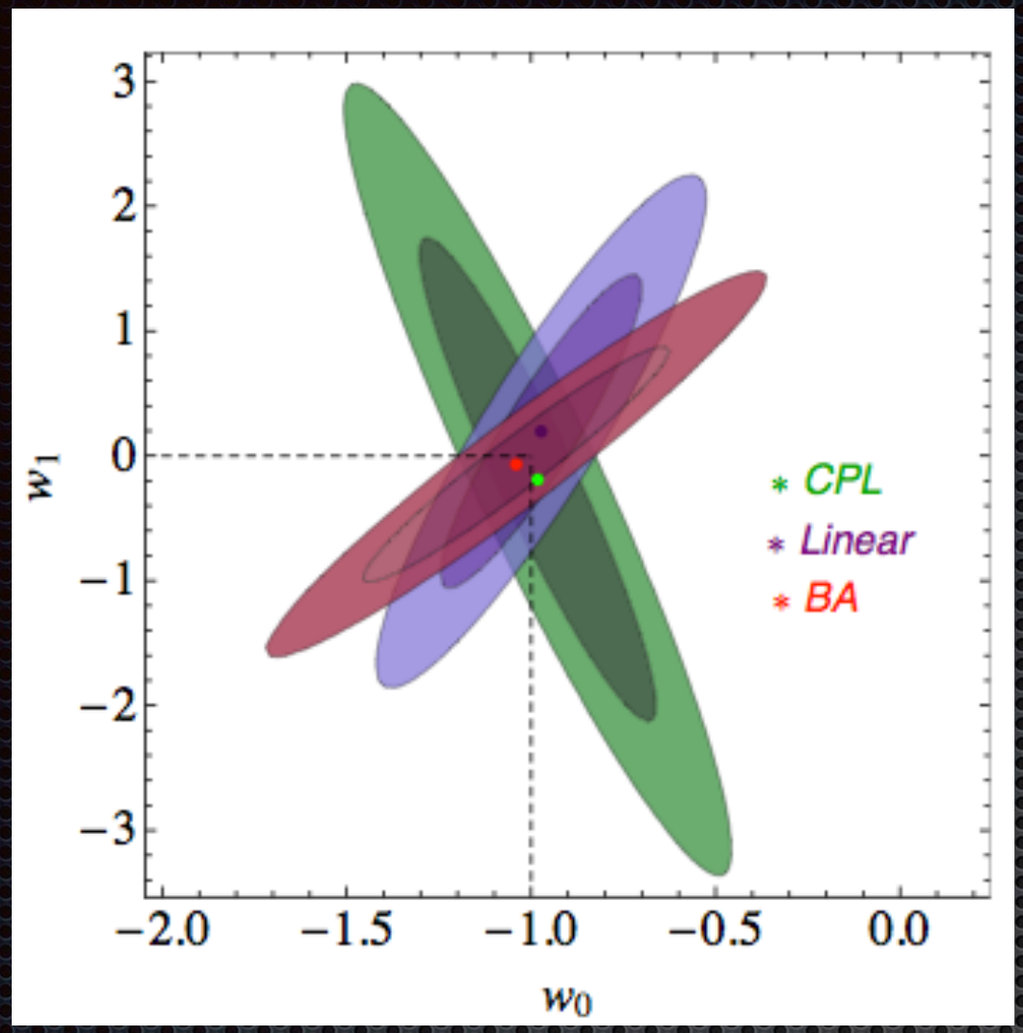
Background analysis



C. Escamilla-Rivera, L.Casarini, J. Fabris and J.S. Alcaniz
JCAP 1611 (2016) no.11, 010

Recipe 3. Background analysis: tension between data sets and dark energy models.

https://github.com/celia-escamilla-rivera



About the likelihood and tension

$$\chi^2_{\rm total} = \chi^2_{\rm SNe\ Ia_{JLA}} + \chi^2_{\rm BAO(total)}$$

Sigma-distance:

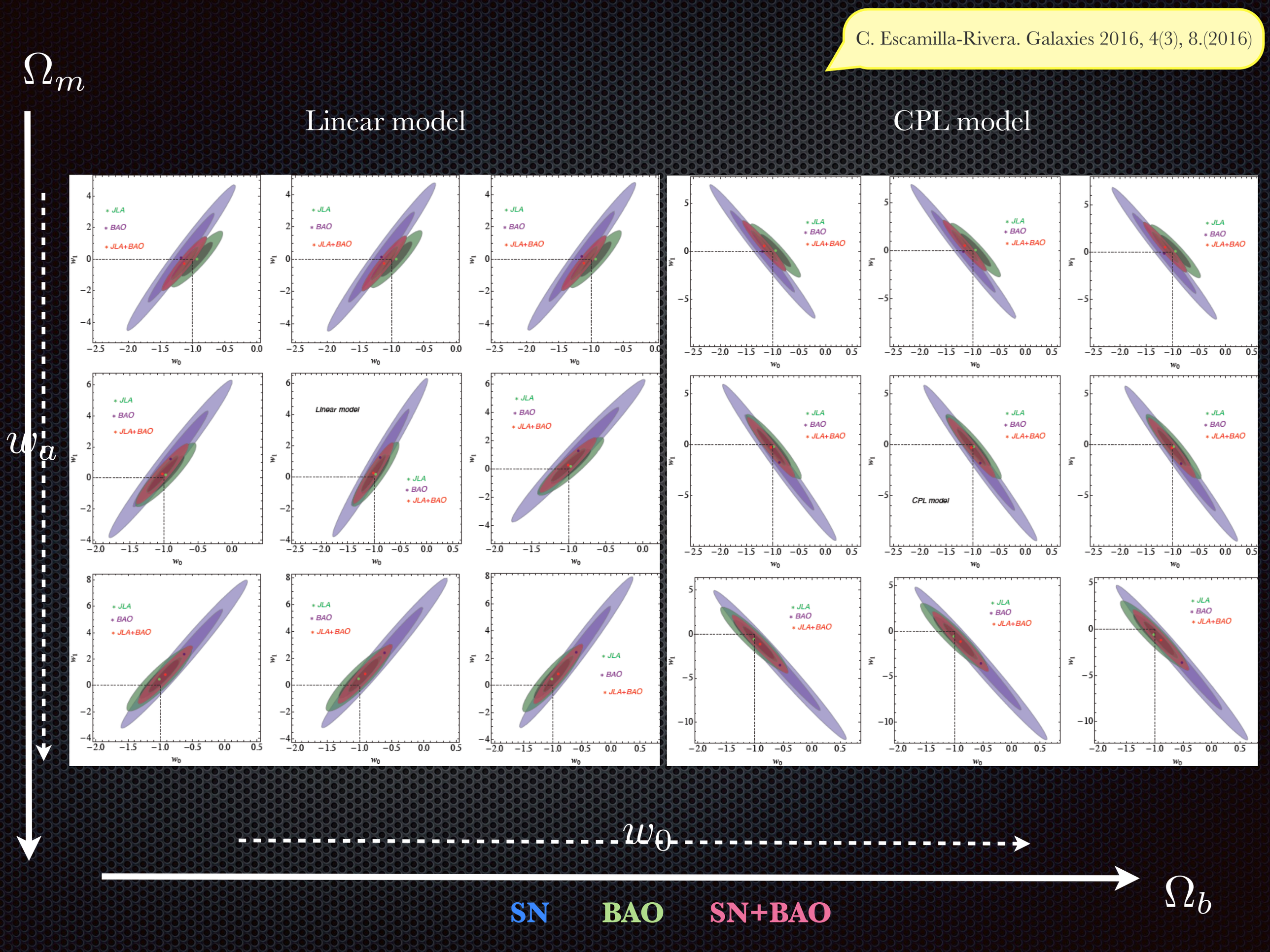
$$1 - \Gamma(1, |\Delta\chi^2_\sigma/2|)/\Gamma(1) = \mathrm{erf}(d_\sigma/\sqrt{2})$$

Recipe 3.1: Tension

$$\chi^2_{tot}([w_0, w_a]_{\mathbf{SNeIa+BAO}}) - \chi^2_{tot}([w_0, w_a]_{\mathbf{SNeIa}})$$

indicates the tension between SNe Ia+BAO and the best fits parameters ($[w_0, w_a]$)

C. Escamilla-Rivera. Galaxies 2016, 4(3), 8.(2016)



Recipe 4. Bayesian statistics

https://github.com/celia-escamilla-rivera

Bayesian model selection

Cosmological model

Astrophysical data

Prior information (free parameters)

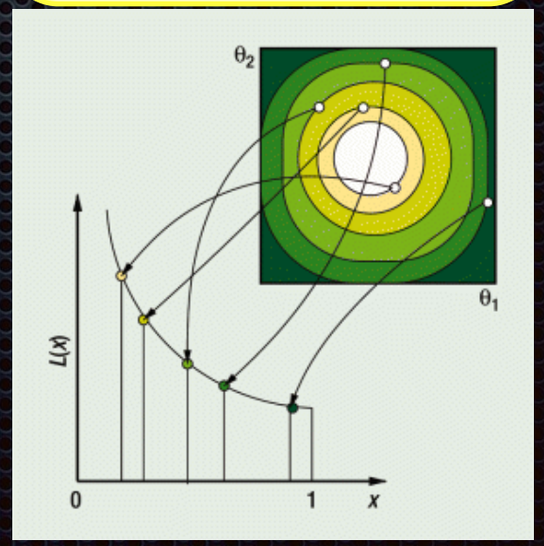
Bayes theorem: update the prior model probability to the posterior model probability $\longrightarrow$ $\mathcal{E} = \int \mathcal{L}(\theta) P(\theta) d\theta.$

Solution: Nested sampling algorithm

A.R.Liddle et al Phys.Rev.D 74, 123506 (2006)

Bayes factor: $\mathcal{B}_{ij} = \mathcal{E}_i / \mathcal{E}_j$

where reference model $(\mathcal{E}_i)$ highest evidence is ΛCDM



Jeffreys's scale:

$\ln B_{ij} < 1$	there is not significant preference for $\mathcal{E}_i$
$1 < \ln B_{ij} < 2.5$	preference substancial
$2.5 < \ln B_{ij} < 5$	strong
$\ln B_{ij} > 5$	decisive

How to report the results…

Table 1. Dark energy parameterizations with best fits and $\sigma-$distances values using SNe Ia JLA data.

Model	Parameterization	$d_\sigma^{\Lambda CDM}$	Best Fit Parameters using SNe Ia JLA data
LCDM	$H^2(z) = H_0^2[\Omega_m(1+z)^3 + (1-\Omega_m)]$	–	$\Omega_m = 0.295 \pm 0.034$
Linear	$H^2(z) = H_0^2[\Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w_0+w_1)}e^{-3w_1 z}]$	0.285	$w_0 = -0.991 \pm 0.036,\ w_1 = 0.297 \pm 0.779$
CPL	$H^2(z) = H_0^2[\Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w_0+w_1)} \times e^{\frac{-3w_1 z}{1+z}}]$	0.258	$w_0 = -0.997 \pm 0.049,\ w_1 = -0.337 \pm 1.822$
BA	$H^2(z) = H_0^2[\Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w_0)} \times (1+z^2)^{3w_1/2}$	0.243	$w_0 = -0.993 \pm 0.034,\ w_1 = -0.245 \pm 0.545$
LC	$H^2(z) = H_0^2[\Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1-2w_0+3w_{0.5})} \times e^{\left[\frac{9(w_0-w_{0.5})z}{1+z}\right]}]$	0.258	$w_0 = -0.997 \pm 0.049,\ w_{0.5} = -1.109 \pm 0.066$
JBP	$H^2(z) = H_0^2[\Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w_0)} \times e^{\frac{3w_1 z^2}{2(1+z)^2}}]$	0.236	$w_0 = -1.013 \pm 0.070,\ w_1 = -0.295 \pm 4.306$
WP	$H^2(z) = H_0^2\left\{\Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3\left[1+\frac{w_0}{1+w_1\ln(1+z)}\right]}\right\}$	0.278	$w_0 = -0.987 \pm 0.040,\ w_1 = -0.169 \pm 0.258$

The cosmological models

Comparison between the prior model (e.g LCDM)

Best fits values

How to report the results…

Table 3. Values of the Figure of Merit for each parameterization.

Model	FoM		
	SNe Ia JLA	BAO	SNe Ia+BAO
Linear	14.203	7.015	23.657
CPL	9.312	4.631	15.681
LC	27.936	13.893	47.043
BA	16.981	8.555	28.437
JBP	6.076	3.024	10.337
WP	26.208	33.996	53.677

Figure of merit: which is generally as the N-dimensional volume enclosed by the confidence contours of the free parameters:

$$FoM_{(w_0,w_1)} = \frac{1}{\sqrt{detCov(w_0, w_1)}}$$

Table 4. Values of Bayes factor for each parameterization.

Model	Bayes Factor $\ln B_{ij}$		
	SNe Ia JLA	BAO	SNe Ia JLA+BAO
Linear	1.904	1.897	1.857
CPL	1.912	1.903	1.875
LC	1.912	1.903	1.875
BA	1.921	1.921	1.921
JBP	1.918	1.912	1.854
WP	1.906	1.891	1.855

Jeffreys's scale:

$$\ln B_{ij} < 1$$

$$1 < \ln B_{ij} < 2.5$$

$$2.5 < \ln B_{ij} < 5$$

$$\ln B_{ij} > 5$$

- Non-parametric methods

A nonparametric method

Methods that can make most of the observational data → Reconstruction of the history of the expansion rate

Proposals

- Principal component analysis
- Nonlinear inverse approach
- Dipole of the luminosity distance method
- Smoothing method
- Gaussian processes

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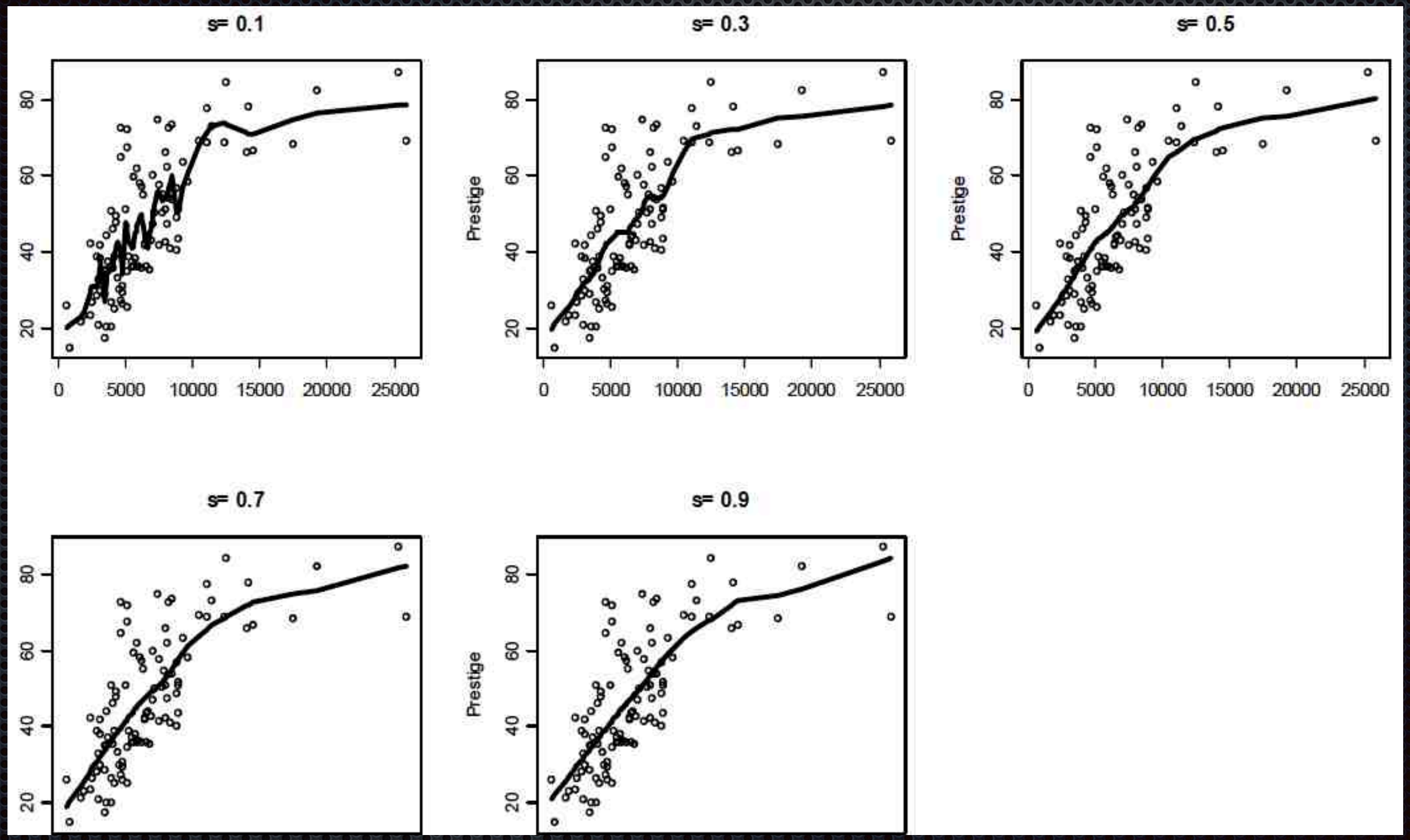
Problems

- Require a fiducial model
- Low efficiency in regions with few data
- Underestimation of the error
- High computacional cost

An example:

J. R. Cook and L. A. Stefanski, J. Am. Stat. Assoc. 89, 1314 (1994).



Gaussian processes (i.e generalisation of a Gaussian distribution) factory

M. Seikel et al. JCAP06(2012)036
http://www.acgc.uct.ac.za/~seikel/GAPP/

$$f(x) \approx \mathcal{GP}(\mu(x), k(x, \tilde{x}))$$

Data entry

Covariance function

$$k(x, \tilde{x}) = \sigma_f^2 e^{\left[-\frac{(x-\tilde{x})^2}{2l^2}\right]}$$

Data entry

Hyperparameters

$$[\sigma_f, l]$$

Reconstruction

$$\mathbf{f}^* \approx \mathcal{N}(\mu^*, K(\mathbf{X}^*, \mathbf{X}^*))$$

$$y \approx \mathcal{N}(\mu, K(\mathbf{X}, \mathbf{X}) + \mathbf{C})$$

Joint distribution

$$\begin{bmatrix} y \\ f^* \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu \\ \mu^* \end{bmatrix}, \begin{bmatrix} K(X,X)+C & K(X,X^*) \\ K(X^*,X) & K(X^*,X^*) \end{bmatrix}\right)$$

Algorithm steps
Tools (Equations)
Remarks

Reconstruction of f

Step 1: Choose the data set

Step 2: Choose a covariance function

Data entry

Step 3: Choose x* where we want the function f* to be estimated

Data entry

Step 4: Decide a prior mean

Data entry

Step 5: Train the hyperparameters (choose initial values for them)

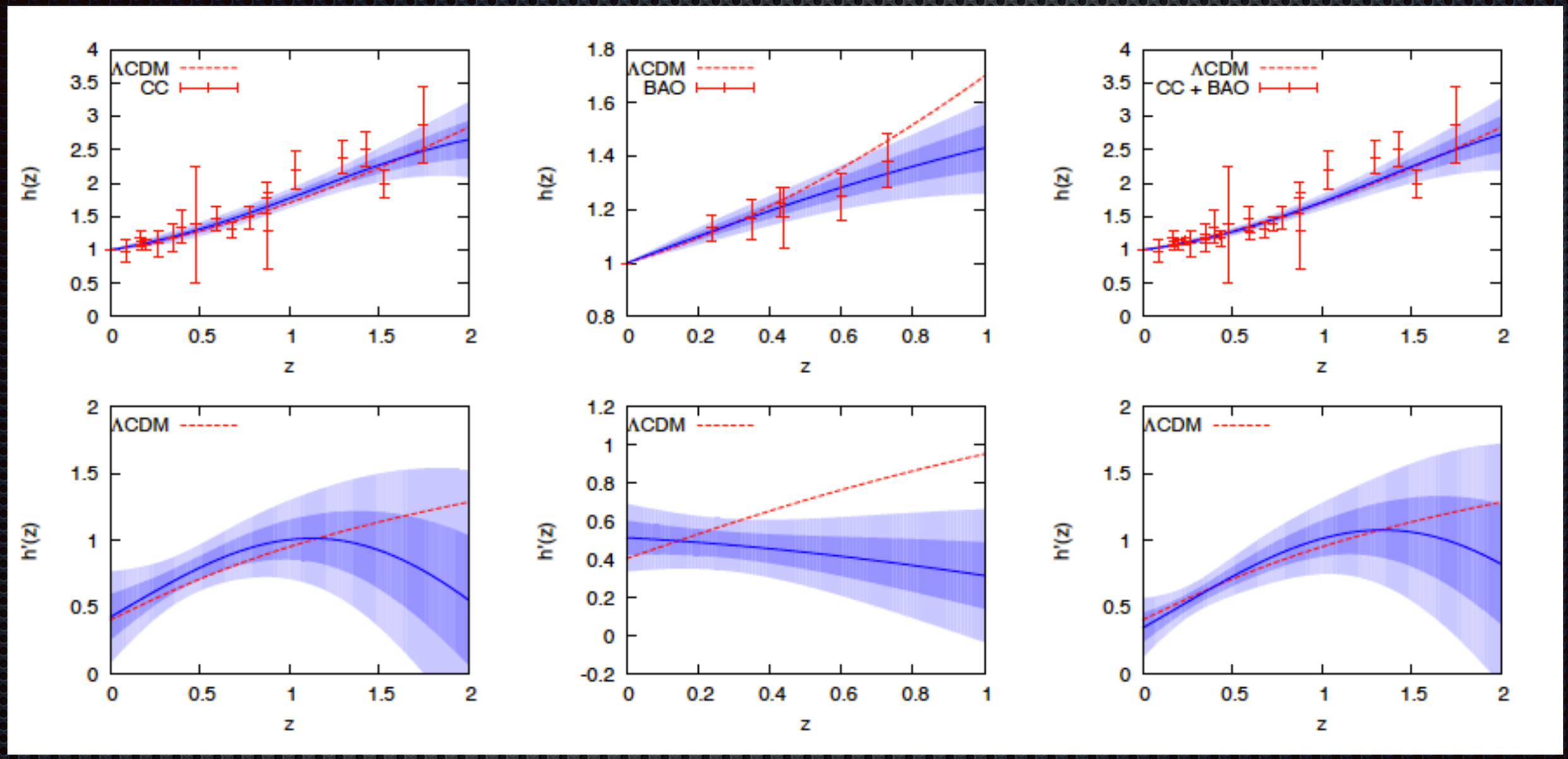
Marginalise the likelihood with respect to f. Then, maximise the likelihood

Step 6: Calculate f* and Cov(f*)

Reconstruction of the derivatives

$$f(x) \approx \mathcal{GP}(\mu(x), k(x, \tilde{x})),$$

$$f(x') \approx \mathcal{GP}\left(\mu'(x), \frac{\partial^2 k(x, \tilde{x})}{\partial x \partial \tilde{x}}\right)$$



M. Seikel et al. Phys. Rev. D 86, 083001 (2012)

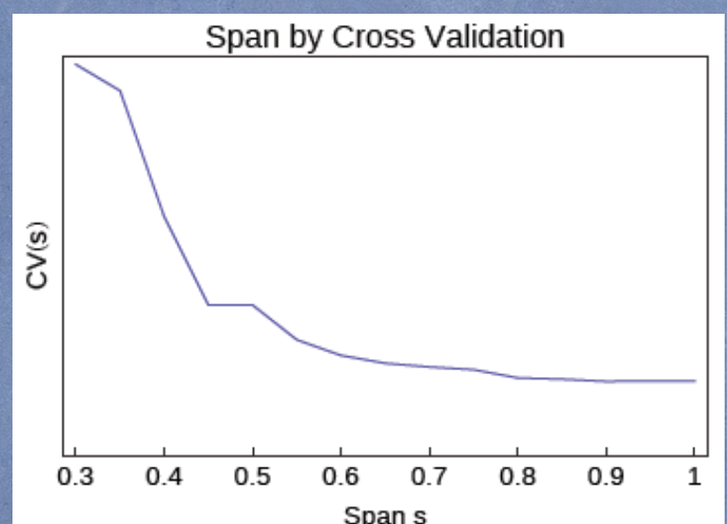
Phys.Rev. D89 (2014).

LOESS-SIMEX factory

Locally Weighted Scatterplot Smooting
+
Simulation and Extrapolation

Window size (Cross validation)

$$CV(s) = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i(s) - y_i)^2$$



Data entry

Weighted least square fit

$$W(\bar{z}_i) = \begin{cases} \left(1 - |\bar{z}_i|^3\right)^3 & \text{for} \quad |\bar{z}_i| < 1, \\ 0 & \text{for} \quad |\bar{z}_i| \geq 1, \end{cases}$$

SIMEX

$$\begin{pmatrix} y_1 & y_2 & \cdots & \boxed{y_i} & \cdots & y_n \end{pmatrix}$$

$$\Big\downarrow \; Simex \; (add\ \sqrt{\lambda_j}\sigma_i)$$

$$\begin{pmatrix} \eta_1(\lambda_1) & \eta_2(\lambda_1) & \cdots & \eta_i(\lambda_1) & \cdots & \eta_n(\lambda_1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \eta_1(\lambda_j) & \eta_2(\lambda_j) & \cdots & \eta_i(\lambda_j) & \cdots & \eta_n(\lambda_j) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \eta_1(\lambda_N) & \eta_2(\lambda_N) & \cdots & \eta_i(\lambda_N) & \cdots & \eta_n(\lambda_N) \end{pmatrix}$$

$$\Big\downarrow \; Loess \; (\eta \to \hat{\eta})$$

$$\begin{pmatrix} \hat{\eta}_1(\lambda_1) & \hat{\eta}_2(\lambda_1) & \cdots & \hat{\eta}_i(\lambda_1) & \cdots & \hat{\eta}_n(\lambda_1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{\eta}_1(\lambda_j) & \hat{\eta}_2(\lambda_j) & \cdots & \hat{\eta}_i(\lambda_j) & \cdots & \hat{\eta}_n(\lambda_j) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{\eta}_1(\lambda_N) & \hat{\eta}_2(\lambda_N) & \cdots & \hat{\eta}_i(\lambda_N) & \cdots & \hat{\eta}_n(\lambda_N) \end{pmatrix}$$

$$\Big\downarrow \; Simex \; (\lambda \to -1)$$

$$\begin{pmatrix} \hat{y}_1 & \hat{y}_2 & \cdots & \boxed{\hat{y}_i} & \cdots & \hat{y}_n \end{pmatrix}$$

Reconstruction of h(z)

Algorithm steps
Tools (Equations)
Important tips

Step 1: Smooting parameter and windows and subsample selection.

$k = ns$
s: span
n: total number of observations
k: number of observations per window and rounded to the next largest integer

Important!

Cross validation method

$$CV(s) = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i(s) - y_i)^2$$

Data entry

Step 2: Weighted subsamples.

$$W(\bar{z}_i) = \begin{cases} \left(1 - |\bar{z}_i|^3\right)^3 & \text{for} \quad |\bar{z}_i| < 1, \\ 0 & \text{for} \quad |\bar{z}_i| \geq 1, \end{cases} \qquad \bar{z}_i = (z_i - z_0)/d$$

Step 3: Regression analysis.

Step 4: Simulated data sample.

Step 5: Starting the reconstruction.

Step 6: About the confidence regions

Important!

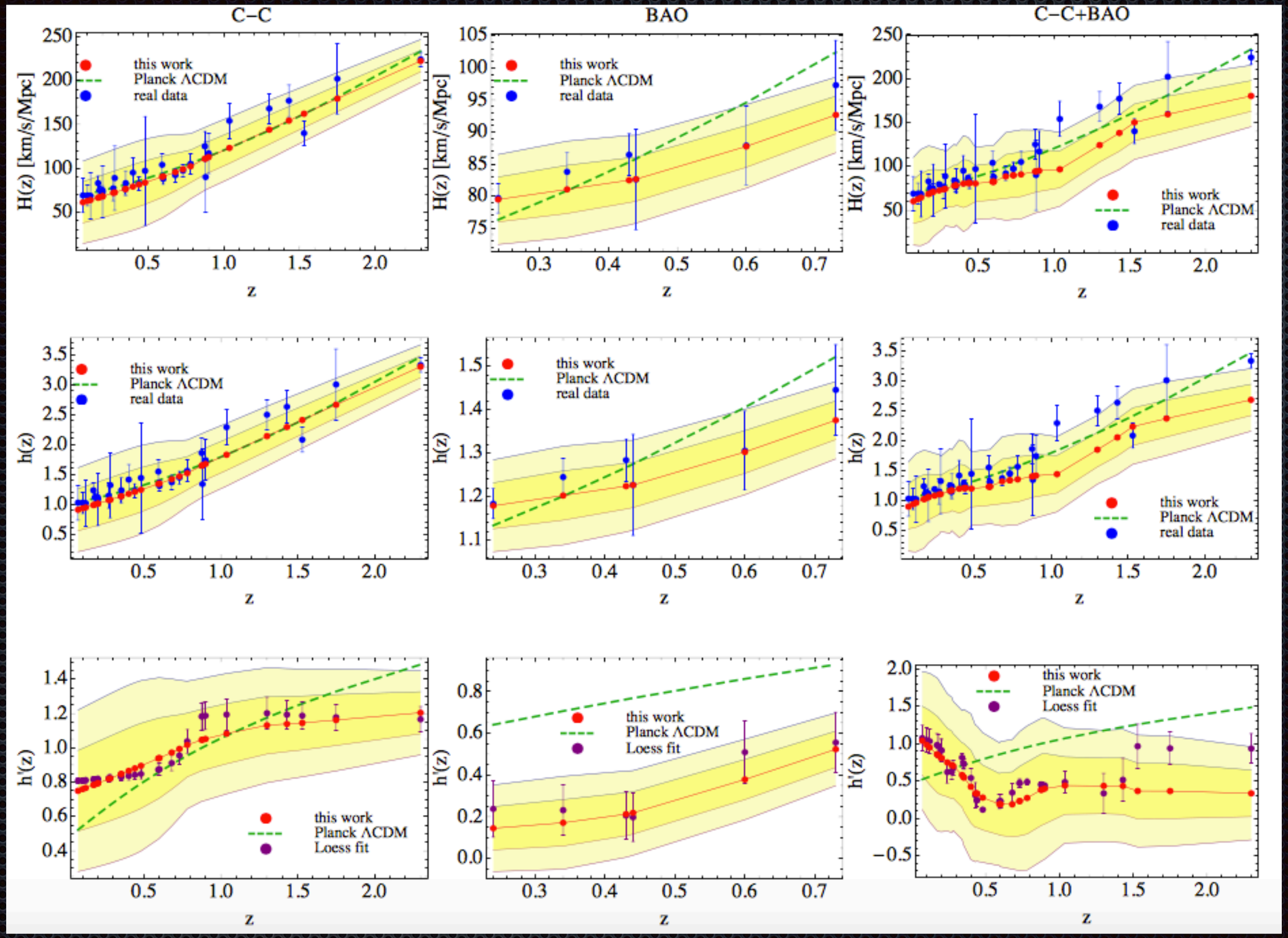
Error propagation:

$$\sigma_h{}^2 = \left(\frac{\sigma_H}{H_0}\right)^2 + \left(\frac{H^2}{H_0{}^4}\right)\sigma^2_{H_0}$$

$$S^2 = \frac{1}{n - df_{mod}} \sum_{i=1}^{n} r_I^2, \quad \text{where} \quad r_i = H_i - \hat{H}_i \qquad \hat{V}(\hat{H}_i) \equiv \sigma^2_{\hat{H}_i} = S^2 \sum_{j=1}^{n} W_{ij}^2$$

$$68\% \quad \text{confidence} \quad \text{interval} \quad h_I \pm \sqrt{\hat{V}(\hat{H}_I)}$$

$$95\% \quad \text{confidence} \quad \text{interval} \quad h_I \pm 2\sqrt{\hat{V}(\hat{H}_i)}$$



C. Escamilla-Rivera and J. Fabris. Galaxies 4 (2016) 76.

- To take away.... and ready to write/test codes

- We performed cosmological tests - Tension between data sets
- Designed the confidence regions
- Bayesian point-of-view
- Quantities to compute: Best fits, Figure-of-merit, Bayes factor/Evidence, …

- Bayesian statistics: Akaike information criteria (AIC) and Bayesian information criteria (BIC) for more parameters

$$\mathrm{AIC} \equiv -2 \ln \mathcal{L}_{\max} + 2k$$
$$\mathrm{BIC} \equiv -2 \ln \mathcal{L}_{\max} + k \ln N$$

- Dynamics of dark energy models with a scalar field.
- Treatment of linear and non-linear perturbations.