CT561: Systems Modelling & Simulation

Lecture 2: Formulating Flows

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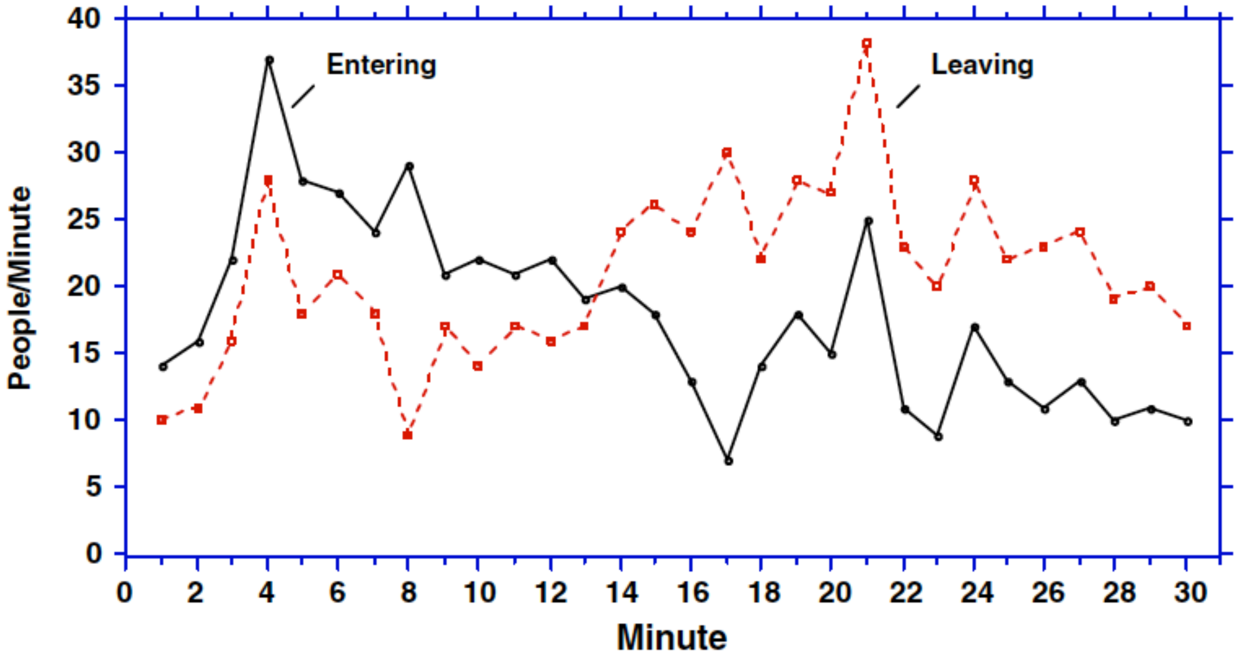
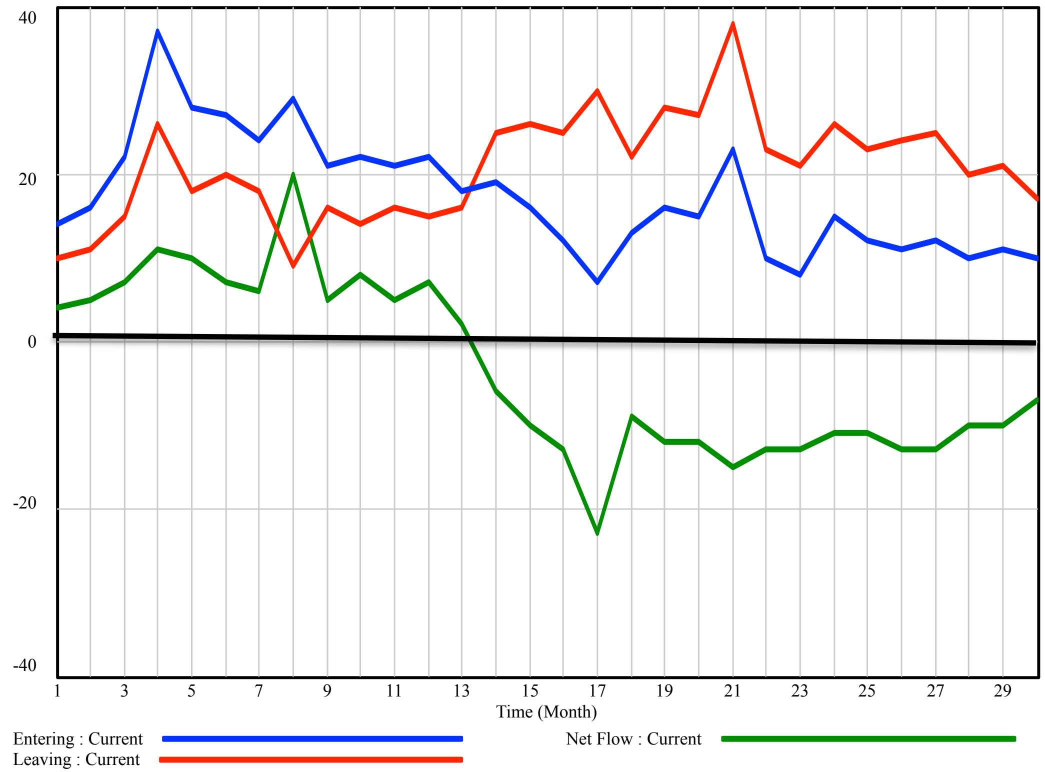
Review:
The Department Store Challenge

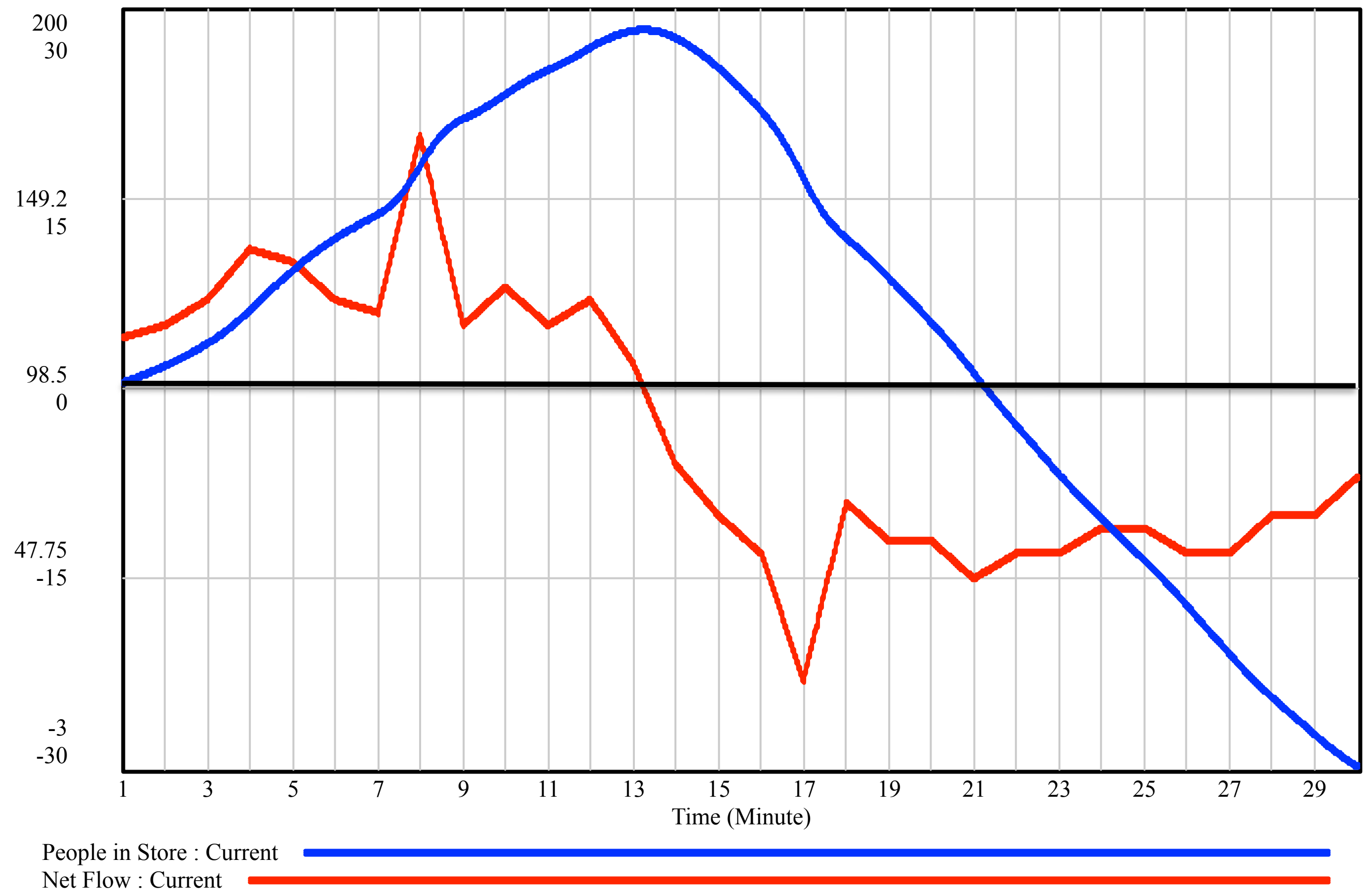


General Principle of Stock/Flow Systems

- As long as the sum of all inflows exceeds the sum of all outflows, the level of the stock will **rise**.
- As long as the sum of all outflows exceeds the sum of all inflows, the level of the stock will **fall**.
- If the sum of all outflows equals the sum of all inflows, the stock level **will not change**.



Verifying through Simulation



Equations

Entering = WITH LOOKUP (Time,
([(0,0)-(30,40)],(1,14),(2,16),(3,22),(4,37),(5,28),(6,27),(7,24),(8,29) ,(9,21),(10,22),
(11,21),(12,22),(13,18),(14,19),(15,16),(16,12),(17,7),(18 ,13),(19,16),(20,15),(21,23),
(22,10),(23,8),(24,15),(25,12),(26,11),(27,12),(28,10),(29,11),(30,10)))

Leaving= WITH LOOKUP (Time,
([(0,0)-(30,40)],(1,10),(2,11),(3,15),(4,26),(5,18),(6,20),(7,18),(8,9),(9,16),(10,14),
(11,16),(12,15),(13,16),(14,25),(15,26),(16,25),(17,30),(18,22),(19,28),(20,27),
(21,38),(22,23),(23,21),(24,26),(25,23),(26,24),(27,25),(28,20),(29,21),(30,17)))

People in Store = INTEG (Entering-Leaving,100)

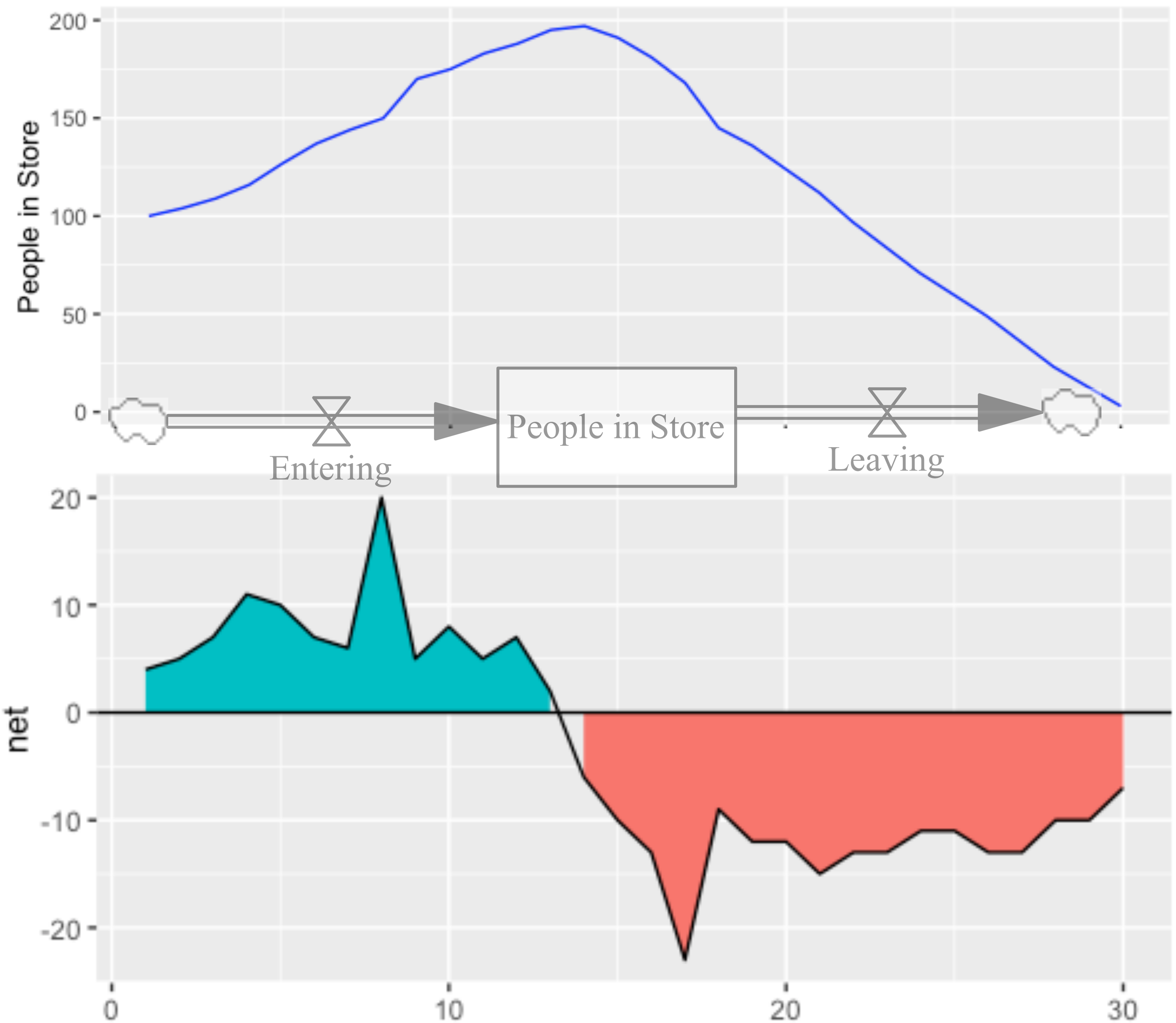
OR

$\text{People in Store}_t = \text{People in Store}_{t\text{-}dt} + (\text{Entering-Leaving}) * DT$

$\text{People in Store}_0 = 100$

Programming the Model (R)

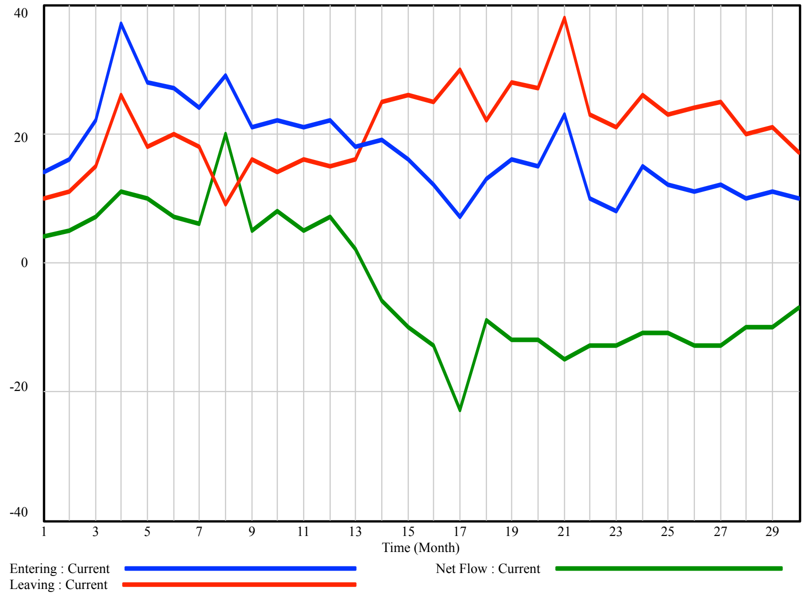
```r
# inflow
entering <- c(14, 16, 22, 37, 28, 27, 24, 29, 21, 22, 21, 22, 18, 19, 16,
              12,  7, 13, 16, 15, 23, 10,  8, 15, 12, 11, 12, 10, 11, 10)

#outflow
leaving <-  c(10, 11, 15, 26, 18, 20, 18,  9, 16, 14, 16, 15, 16, 25, 26,
              25, 30, 22, 28, 27, 38, 23, 21, 26, 23, 24, 25, 20, 21, 17)

people_in_store    <- vector(mode="numeric",length = 30)
people_in_store[1] <- 100 # initial stock

DT <- 1
for(i in 2:length(people_in_store)){
  people_in_store[i] <- people_in_store[i-1] +
                          (entering[i-1] - leaving [i-1]) * DT
}
```



People in Store
200
150
100
50
0
Entering
People in Store
Leaving
net
20
10
0
-10
-20
0
10
20
30

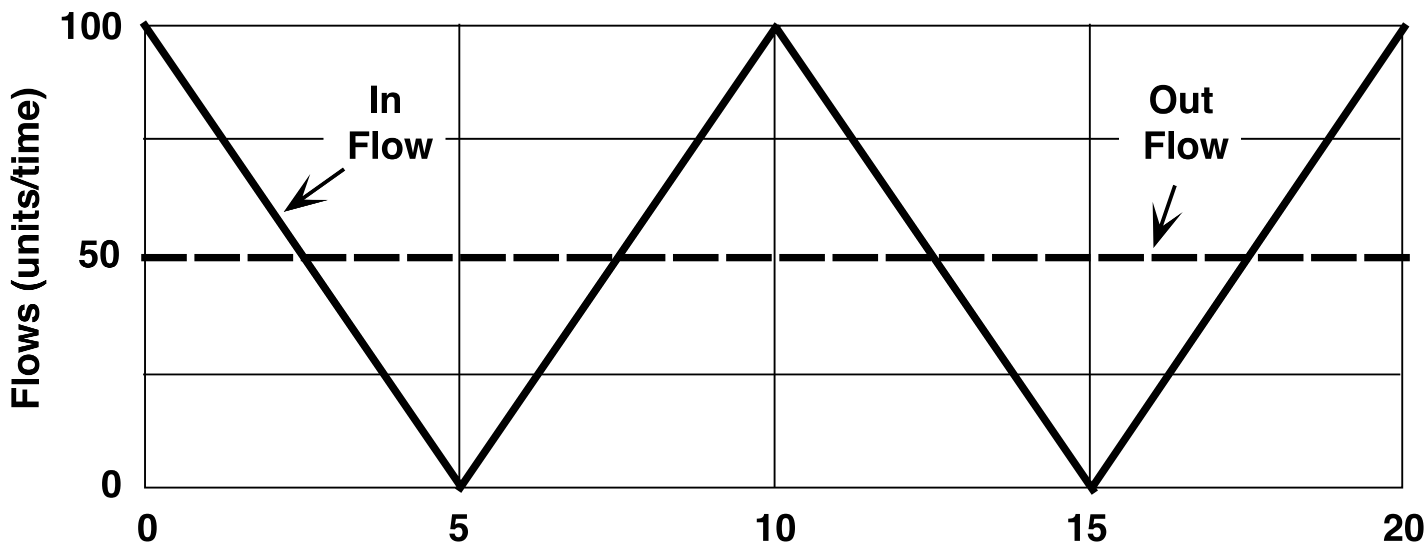
Challenge 2.1

- Implement the department store model in Excel, with DT=0.125
- Use Euler's Equation



Challenge 2.2

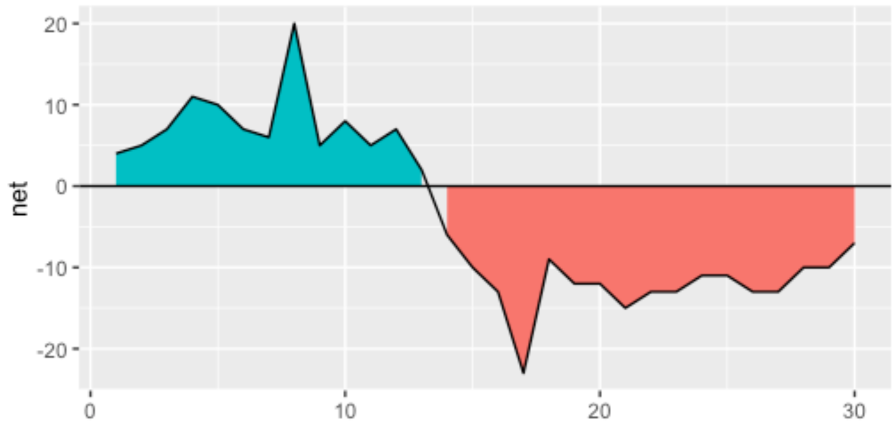
- Solve the following integration problem using Excel, where DT=0.25



$$S_t = S_{t-dt} + NF_{t-dt} \times DT$$

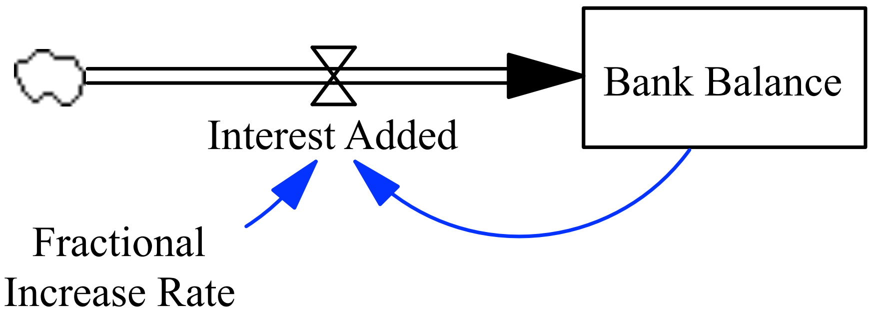
Flows

- Stocks change over time through the actions of a **flow**.
- Basic flow types:
  – Fractional increase
  – Fractional decrease
- Flow depends on the stock and a constant value (increase of decrease fraction)



Fractional Increase Rate

- Consider a stock S with inflow rate R_I
- The inflow is proportional to the size of S
- The fractional increase rate is a constant g

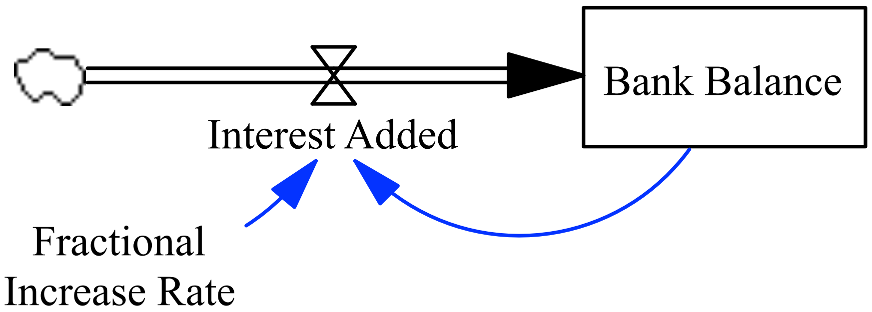


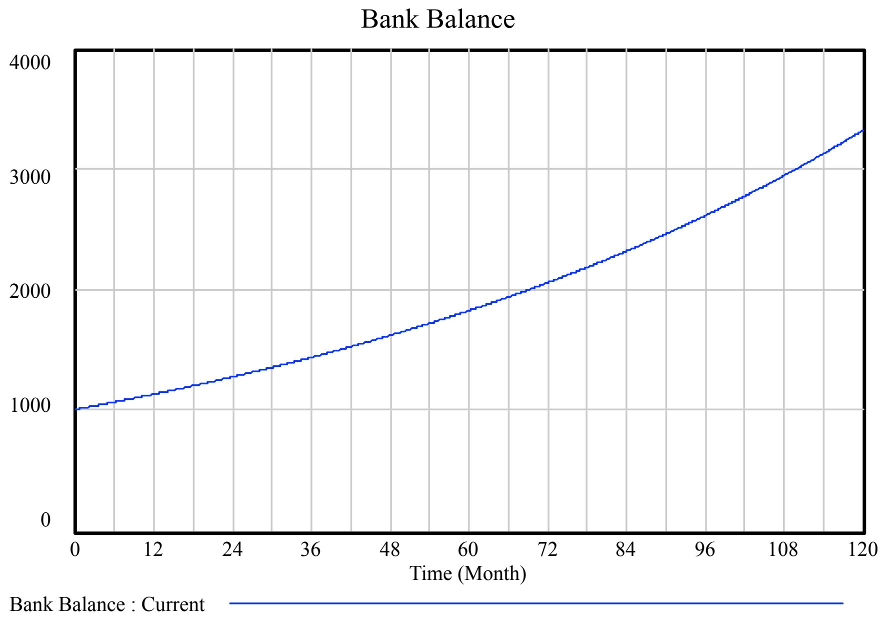
Bank Balance = INTEG(Interest Added , 1000)

Fractional Increase Rate = 0.01

Interest Added = Bank Balance * Fractional Increase Rate

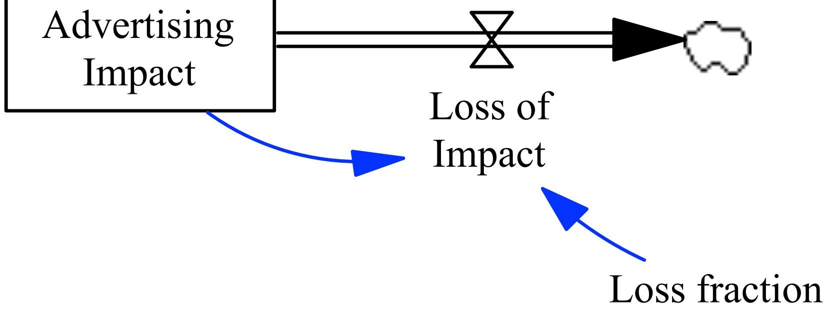
System behaviour
Exponential growth





Fractional Decrease Rate

- Consider a stock *S* with outflow rate R_O
- The outflow is proportional to the size of *S*
- The fractional decrease rate is a constant *d*

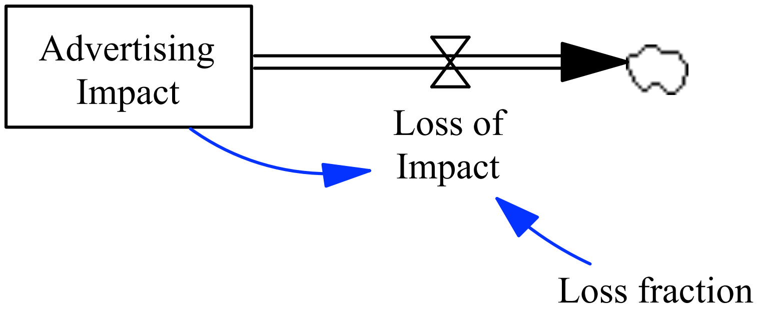


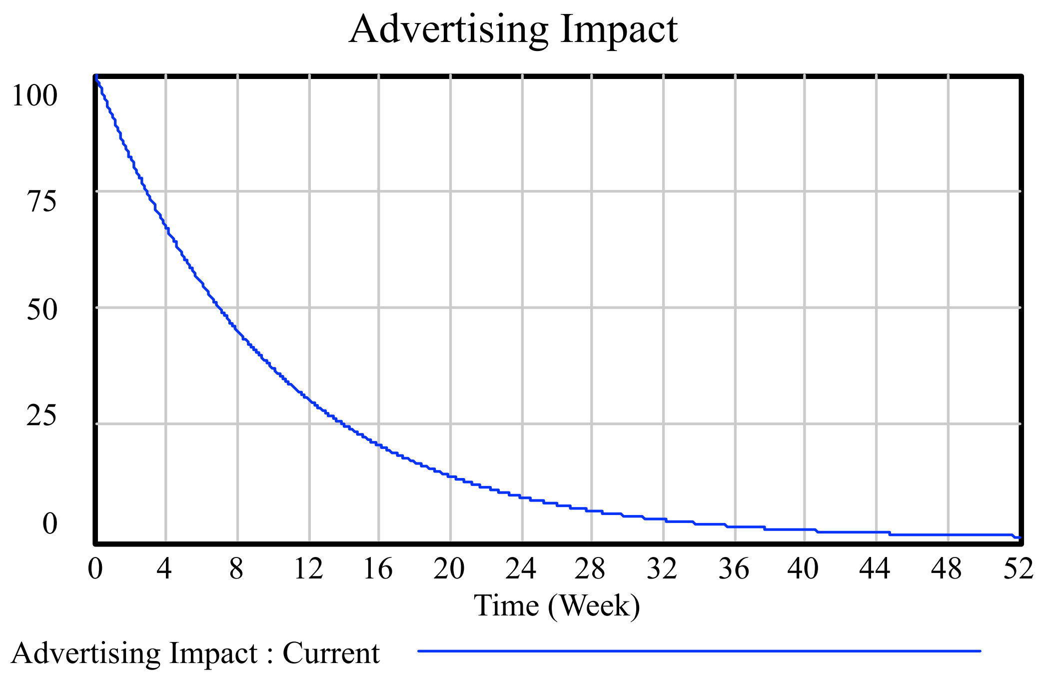
Advertising Impact = INTEG(- Loss of Impact , 100)

Loss fraction = 0.1

Loss of Impact = Loss fraction * Advertising Impact

System behaviour
Exponential decay

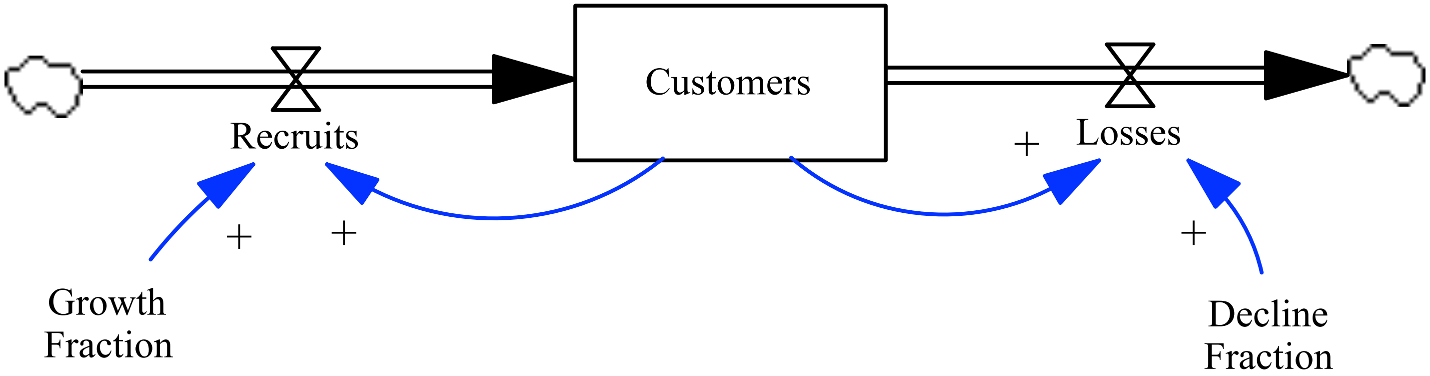




A model of Customers

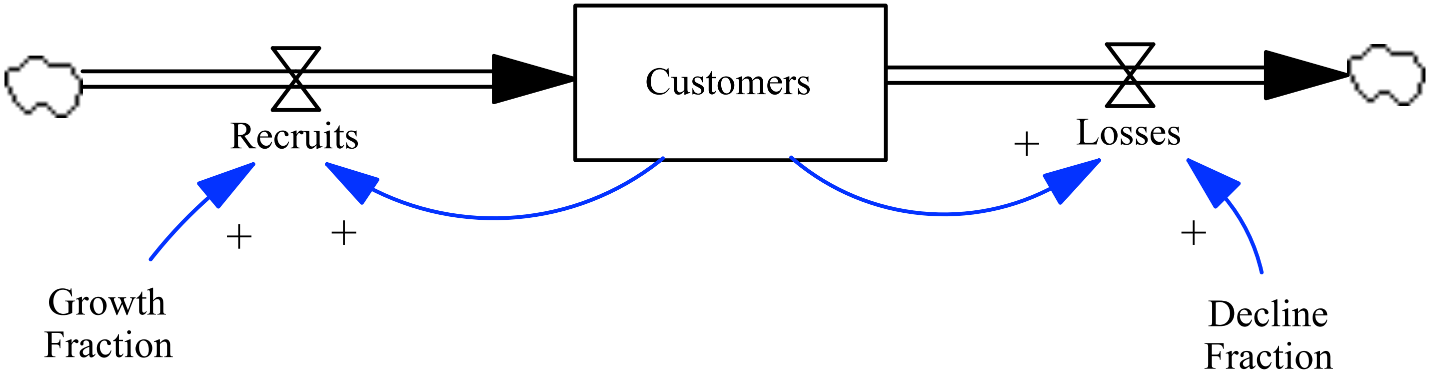
- Given that the customer base is an accumulation, it can be modeled as a stock (assume = 10,000)
- The inflow is recruits, and the outflow are losses, also known as the churn rate.
- The goal of organizations is to limit the losses and maximize the recruits, in order to maintain increasing customers levels, and therefore support company growth.

Stock and Flow Model



Customers= INTEG (Recruits-Losses, 10000)

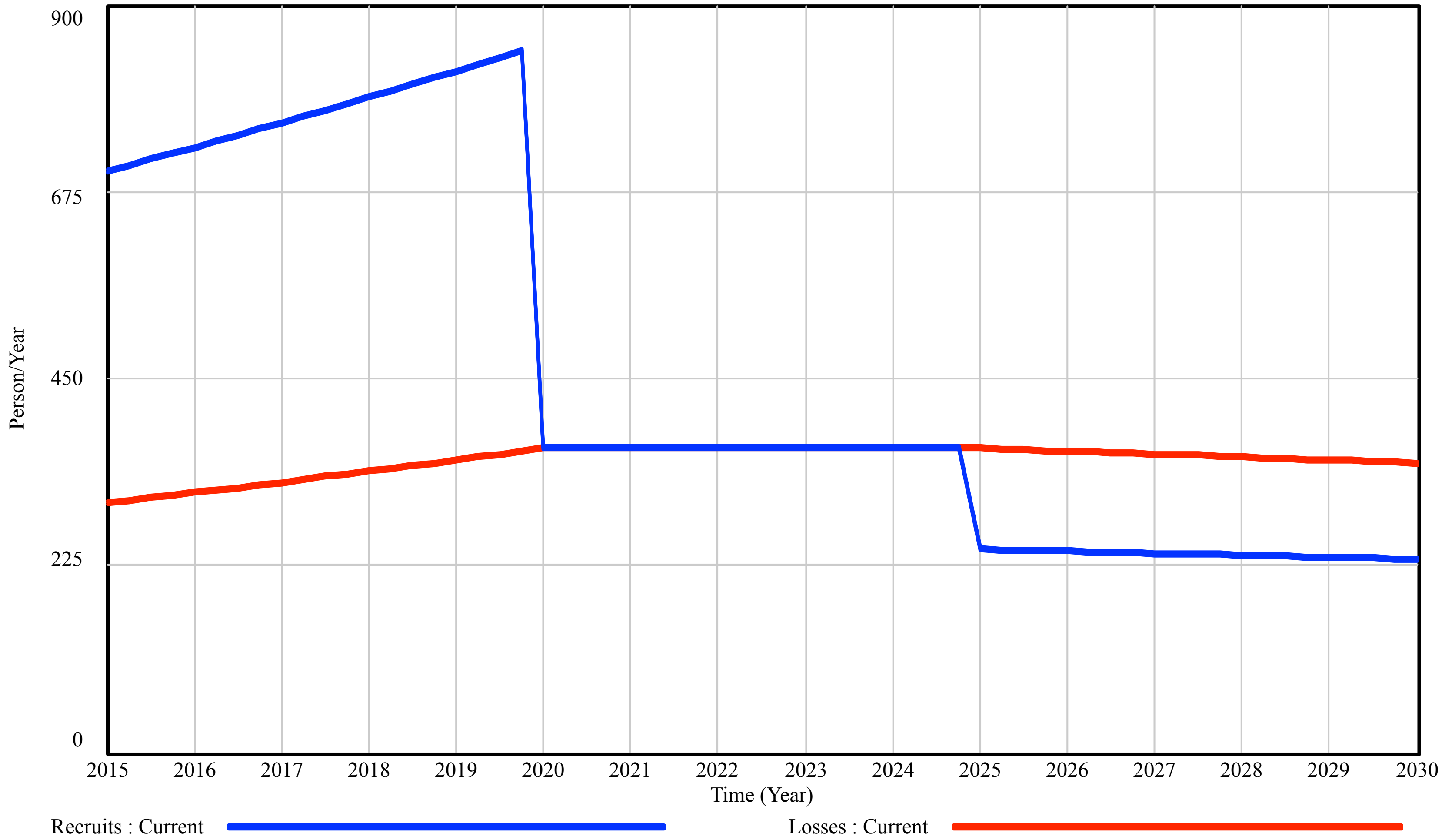
Flow equations



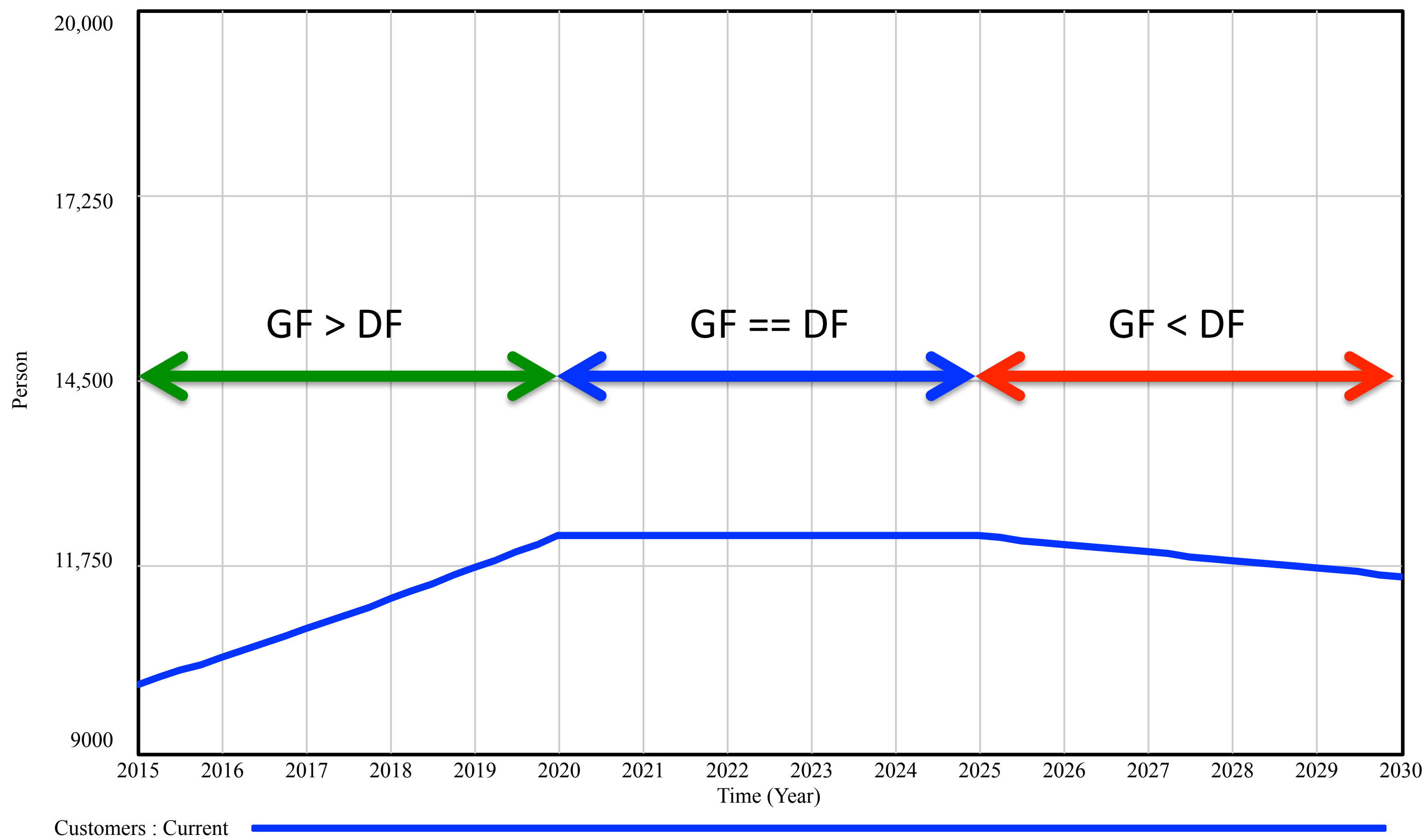
Recruits = Customers*Growth Fraction
Losses = Customers*Decline Fraction

Decline Fraction=0.03
Growth Fraction=0.07-step(0.04,2020)-step(0.01,2025)

Simulation – Flows

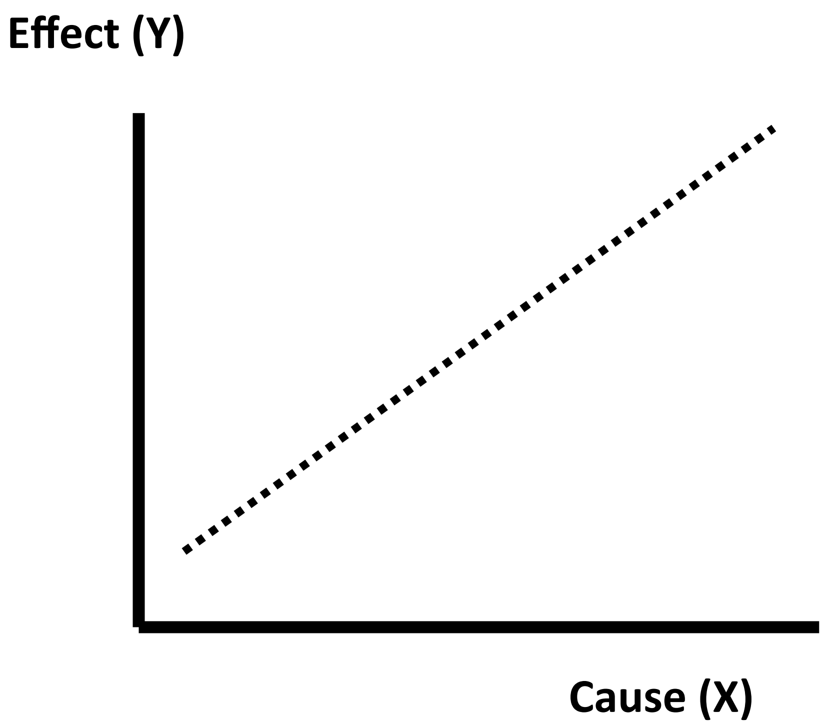


Stock: 3 Phases of behaviour

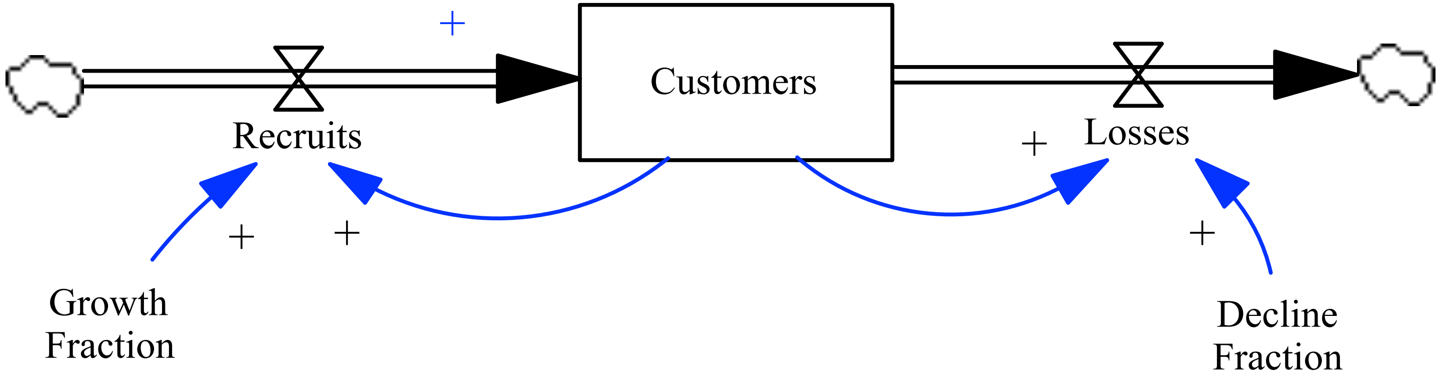


Link polarity – Positive Link

- A positive link means that if the cause **increases**, the effect **increases** *above what it otherwise would have been,* and if the cause **decreases**, the effect **decreases** below what it would otherwise have been.



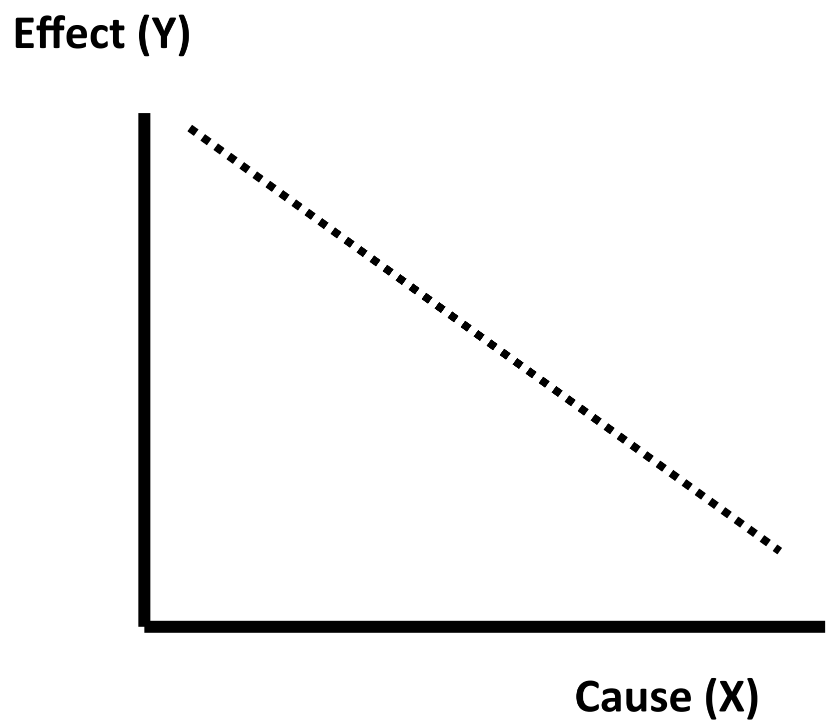
Examples



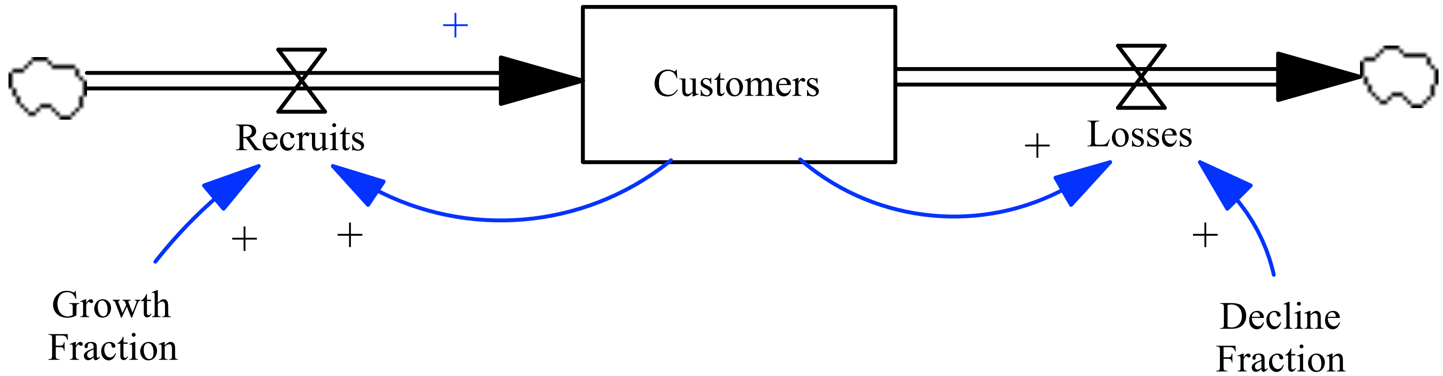
Recruits	↑	Customers	↑
Customers	↑	Recruits	↑
Growth Fraction	↑	Recruits	↑
Decline Fraction	↑	Losses	↑
Customers	↑	Losses	↑

Link polarity – Negative Link

- A negative link means that if the cause **increases**, the effect **decreases** *below what it would otherwise have been*, and if the cause **decreases**, the effect **increases** *above what it might otherwise have been*.



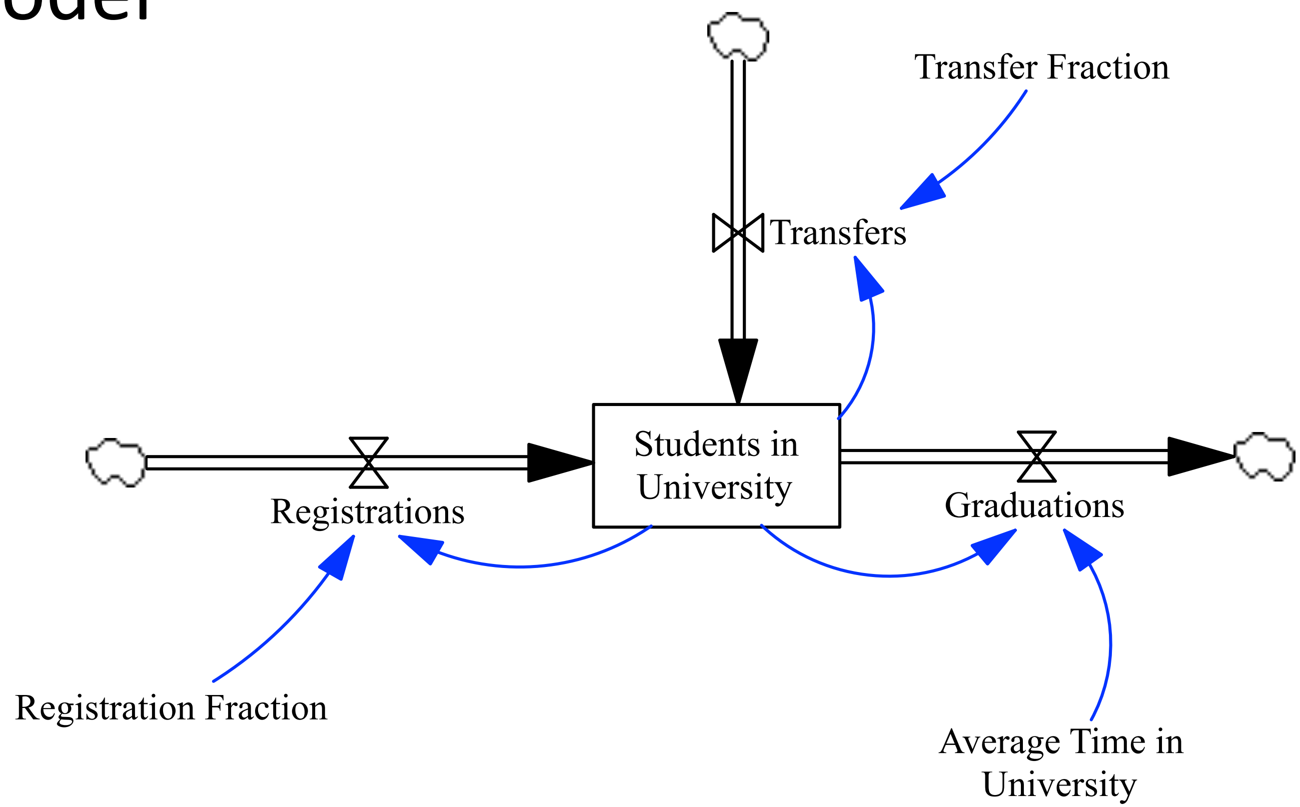
Example



Losses	↑	Customers	↓

Challenge 2.3

- Identify the link polarities for the following model



Challenge 2.4

- Build a stock and flow model of Population, where the inflow is represented and calculated using a fraction
- Include all equations
- On average, there is an increase of 1.5% in the population each year
- If the world population is 3,000,000,000 in 1960, predict its value after 50 years

Challenge 2.5

- Build a stock and flow model of letters in transit, with a simplifying assumption that the inflow is zero.
- Include all equations
- Assume 1000 letters are in the system
- Assume the delivery fraction is 0.75 per day
- How many letters remain after 3 days.