CT561: Systems Modelling and Simulation

Week 3: Formulating Flows

https://github.com/JimDuggan/CT561

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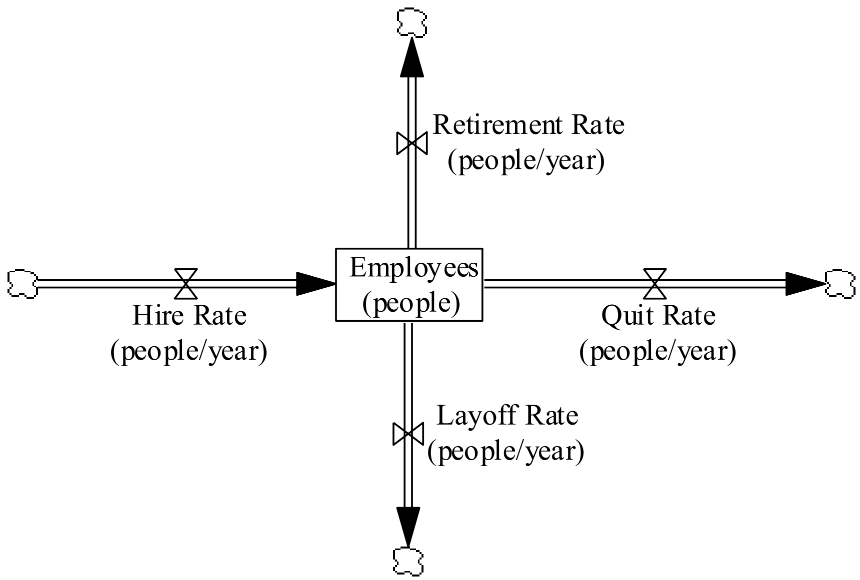
Stock and Flow Systems

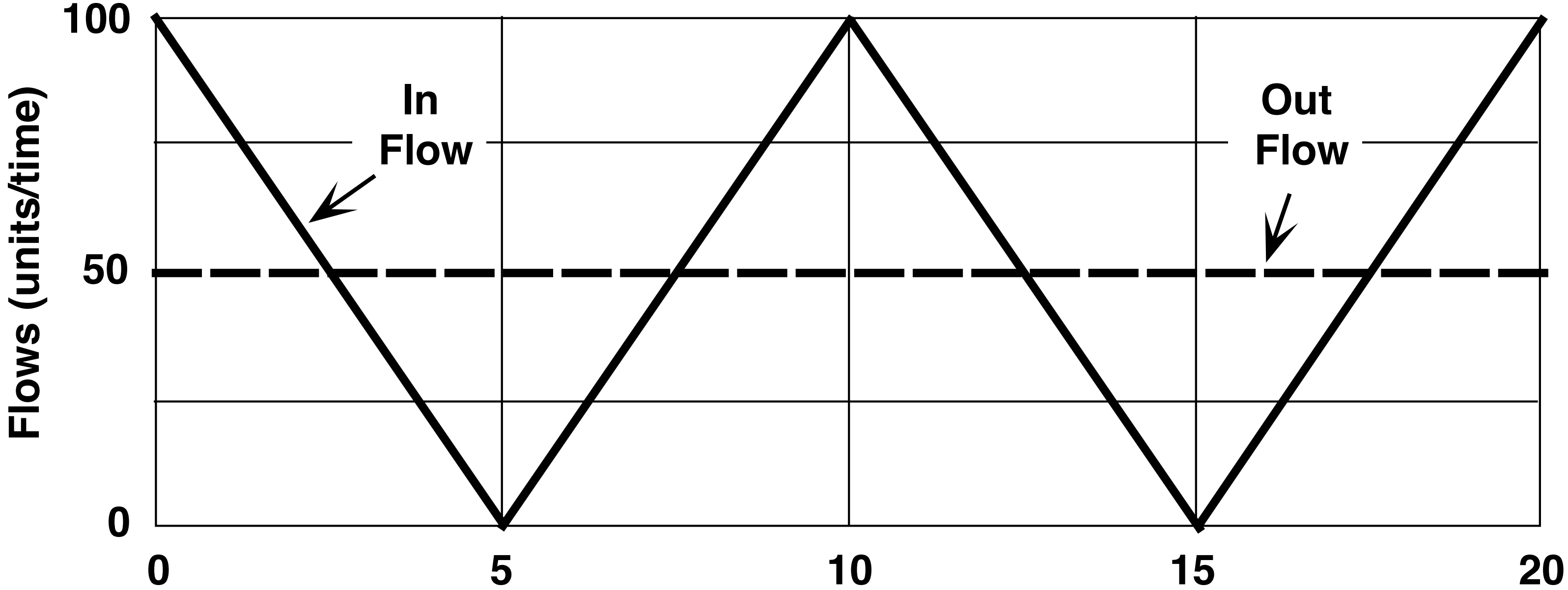
- As long as the sum of all inflows exceeds the sum of all outflows, the level of the stock will **rise**.
- As long as the sum of all outflows exceeds the sum of all inflows, the level of the stock will **fall**.
- If the sum of all outflows equals the sum of all inflows, the stock level **will not change**; it will be held in dynamic equilibrium at whatever level it happened to be when the two sets of flows became equal.



Meadows, Donella H. *Thinking in systems: A primer*. Chelsea Green Publishing, 2008.

Excel Model: Integration (DT=0.25)

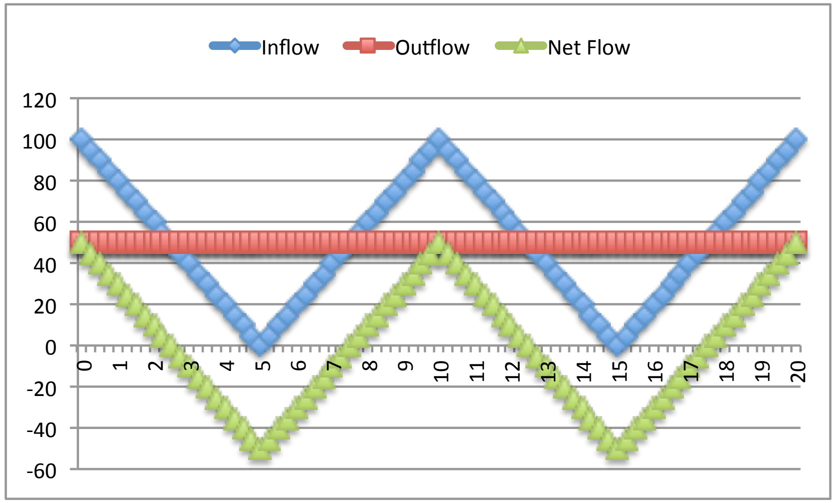
$$S_t = S_{t-dt} + NF_{t-dt} \times DT$$

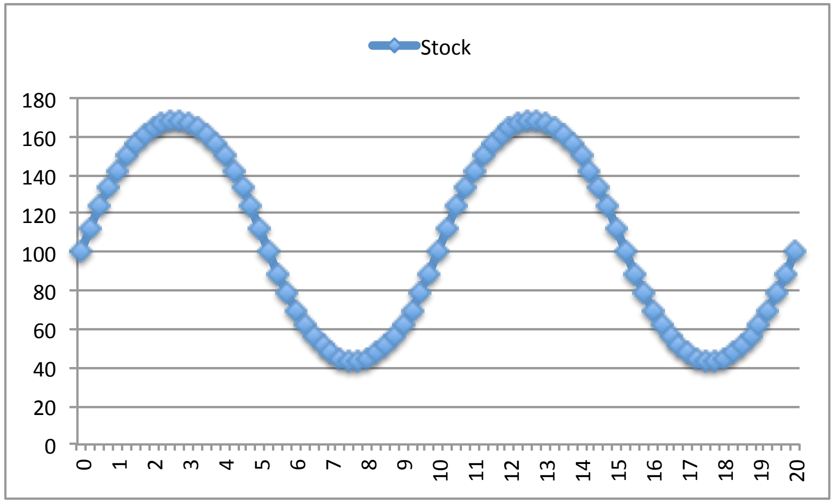


Excel Output

Time	Stock	Inflow	Outflow	Net Flow
0	100	100	50	50
0.25	112.5	95	50	45
0.5	123.75	90	50	40
0.75	133.75	85	50	35
1	142.5	80	50	30
1.25	150	75	50	25
1.5	156.25	70	50	20
1.75	161.25	65	50	15
2	165	60	50	10
2.25	167.5	55	50	5
2.5	168.75	50	50	0
2.75	168.75	45	50	-5
3	167.5	40	50	-10
3.25	165	35	50	-15
3.5	161.25	30	50	-20
3.75	156.25	25	50	-25
4	150	20	50	-30
4.25	142.5	15	50	-35
4.5	133.75	10	50	-40
4.75	123.75	5	50	-45
5	112.5	0	50	-50

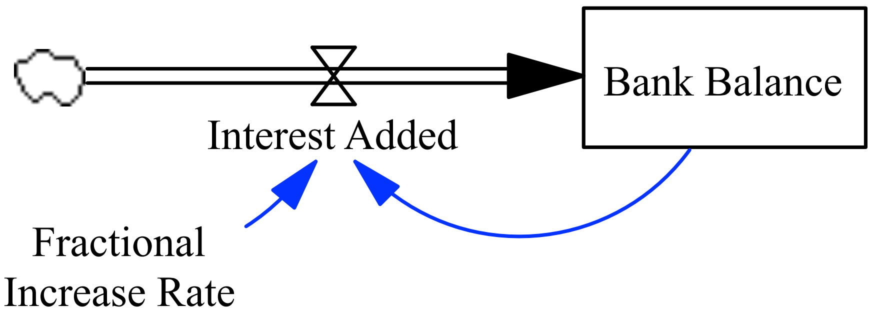
$$S_t = S_{t-dt} + NF_{t-dt} \times DT$$





Fractional Increase Rate

- Consider a stock *S* with inflow rate R_I
- The inflow is proportional to the size of *S*
- The fractional increase rate is a constant *g*



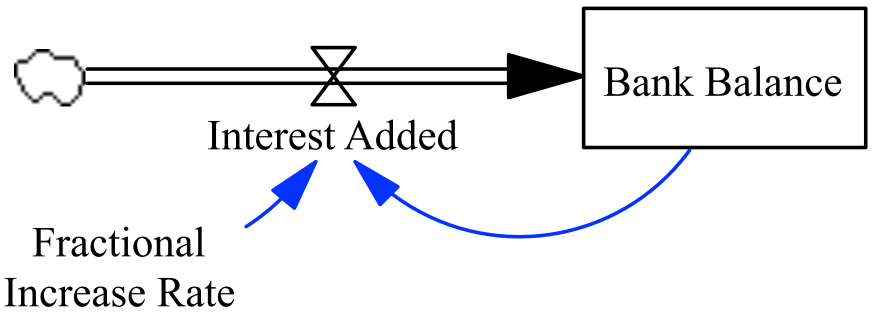
Bank Balance = INTEG(Interest Added , 1000)

Fractional Increase Rate = 0.01

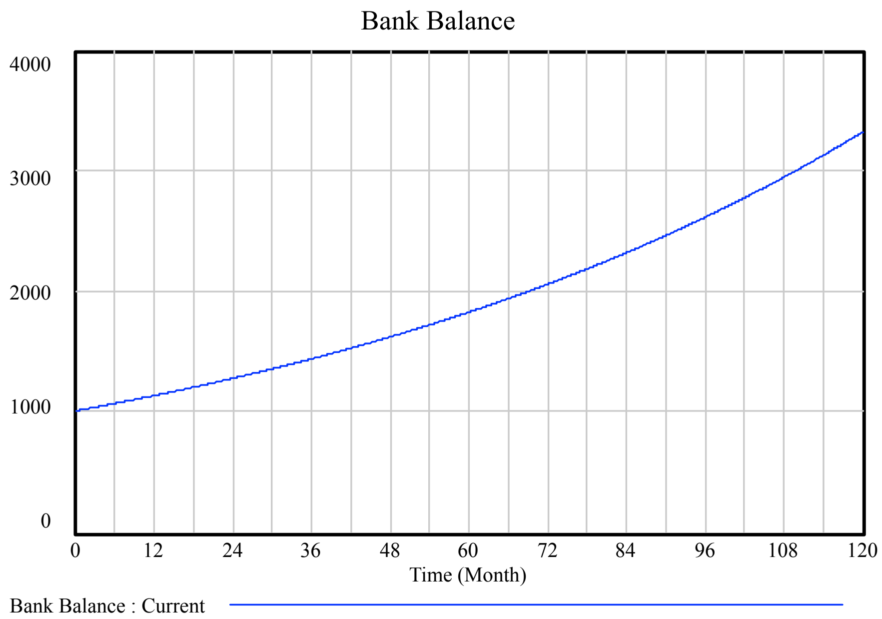
Interest Added = Bank Balance * Fractional Increase Rate

System behaviour
Exponential growth

$$\frac{dS}{dt} = gS$$



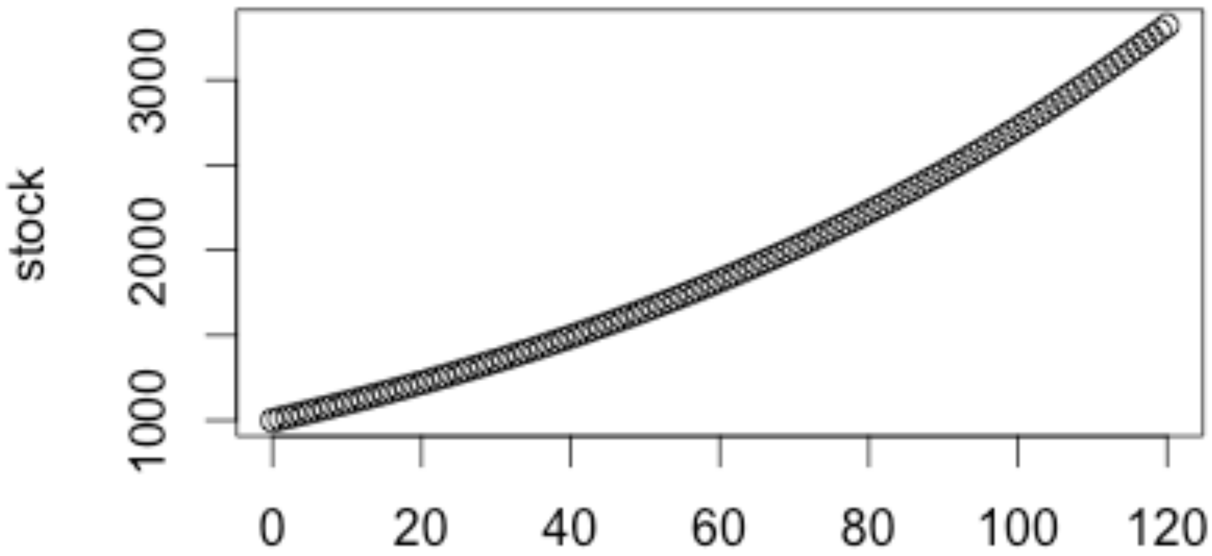
$$S_t = S_0 \, . \, e^{gt}$$



Analytic Model in R

$$S_t = S_0 \,.\, e^{gt}$$

```
> t<-seq(0,120)
> stock<-1000*exp(.01*t)
> plot(t,stock)
```

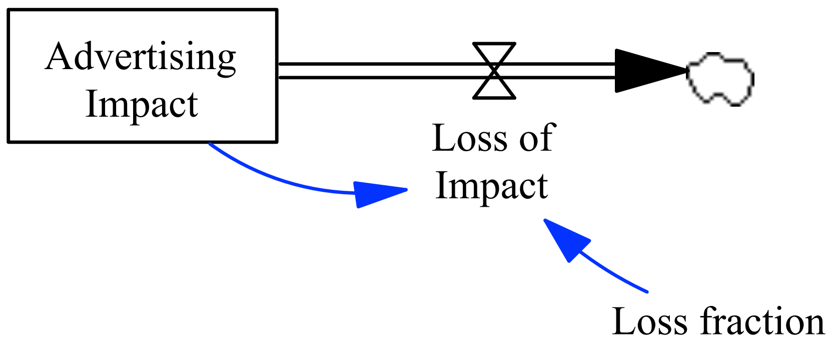


Challenge 3.1

- Build a stock and flow model of Population, where the inflow is represented and calculated using a fraction
- Include all equations
- Assume the growth fraction is 0.015
- On average, there is an increase of 1.5% in the population each year
- If the world population is 3,000,000,000 in 1960, predict its value after 50 years using an analytical approach

Fractional Decrease Rate

- Consider a stock *S* with outflow rate R_O
- The outflow is proportional to the size of *S*
- The fractional decrease rate is a constant *d*



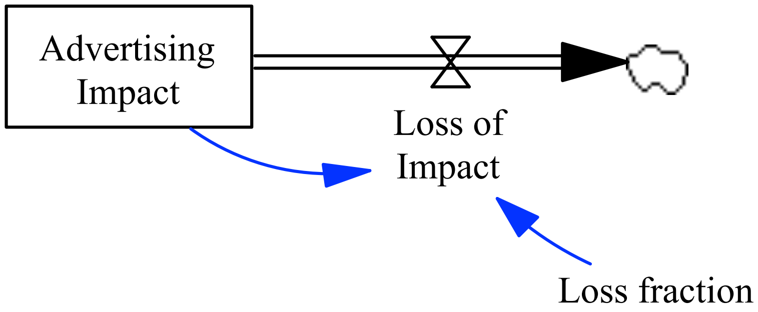
Advertising Impact = INTEG(- Loss of Impact , 100)

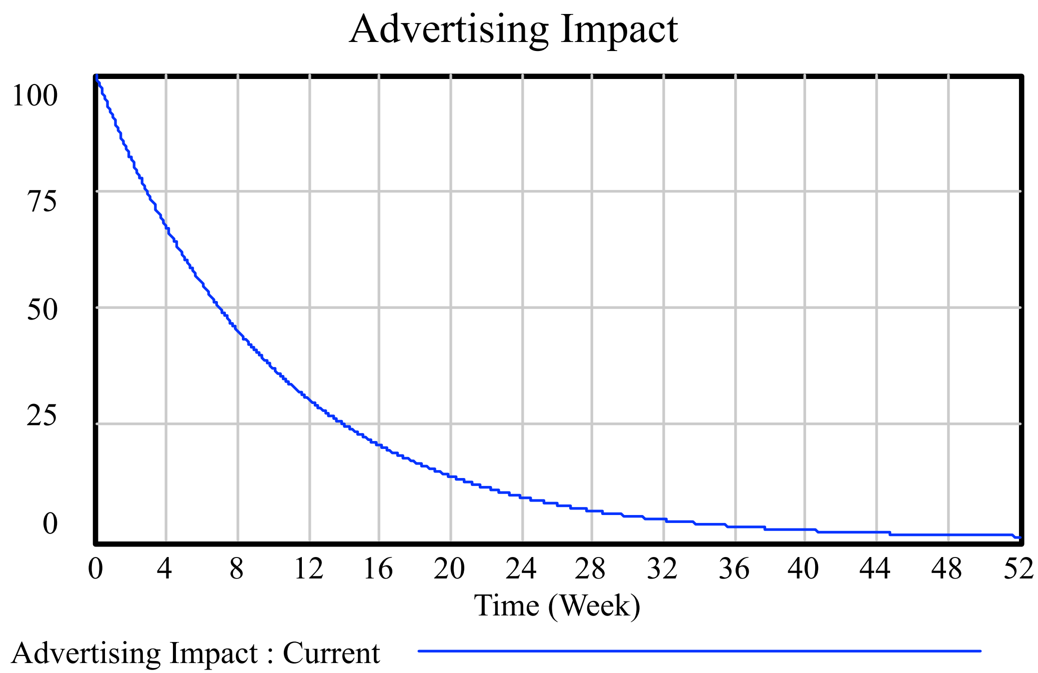
Loss fraction = 0.1

Loss of Impact = Loss fraction * Advertising Impact

System behaviour
Exponential decay

$$\frac{dS}{dt} = -dS$$



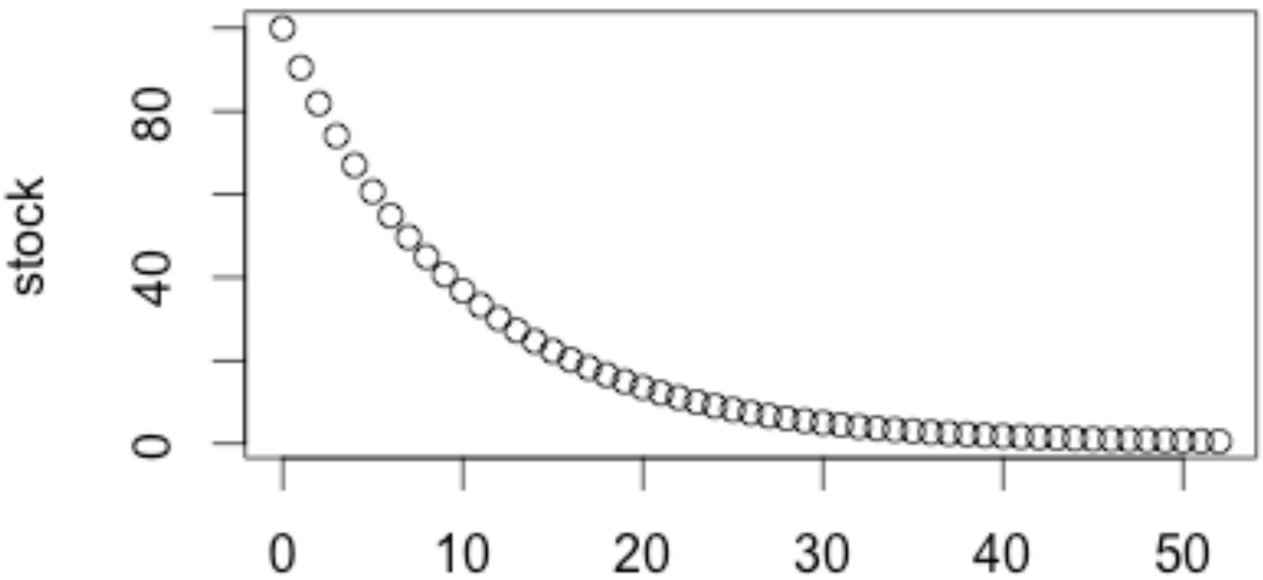


$$S_t = S_0 \,.\, e^{-dt}$$

Analytic Model in R

$$S_t = S_0 \,.\, e^{-dt}$$

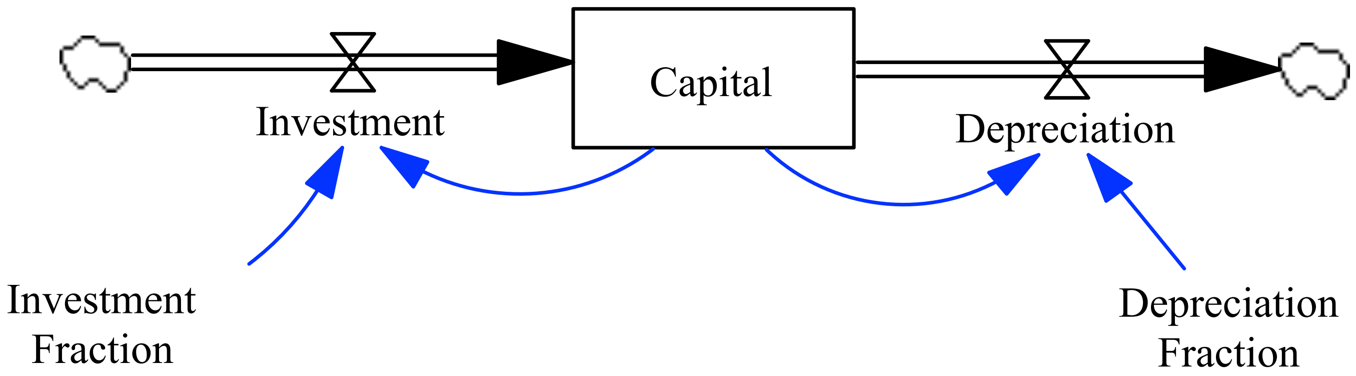
```
> t<-seq(0,52)
> stock<-100*exp(-.1*t)
> plot(t,stock)
```



Challenge 3.2

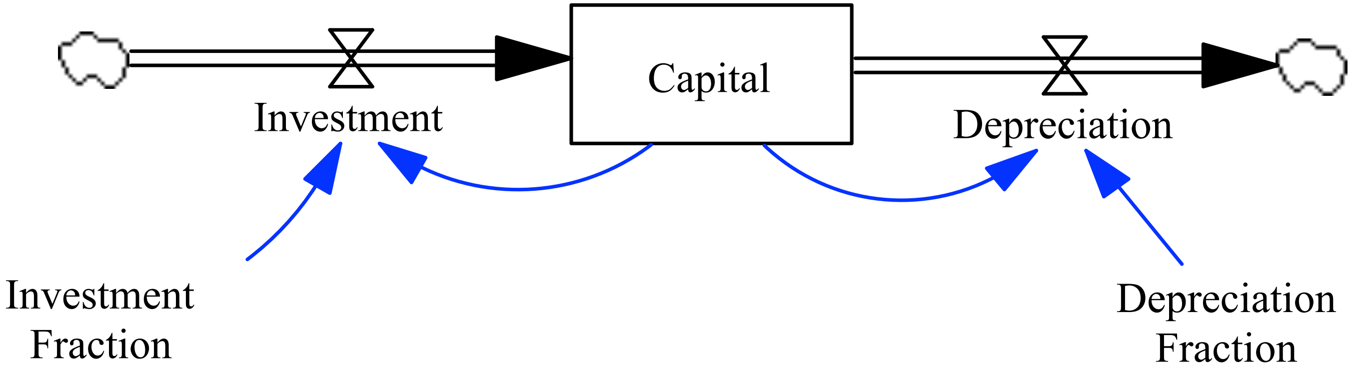
- Build a stock and flow model of letters in transit, with a simplifying assumption that there is no inflow
- Include all equations
- Assume 1000 letters are in the system
- Assume the delivery fraction is 0.75 per day
- How many letters remain after 3 days (use the analytical model for this).

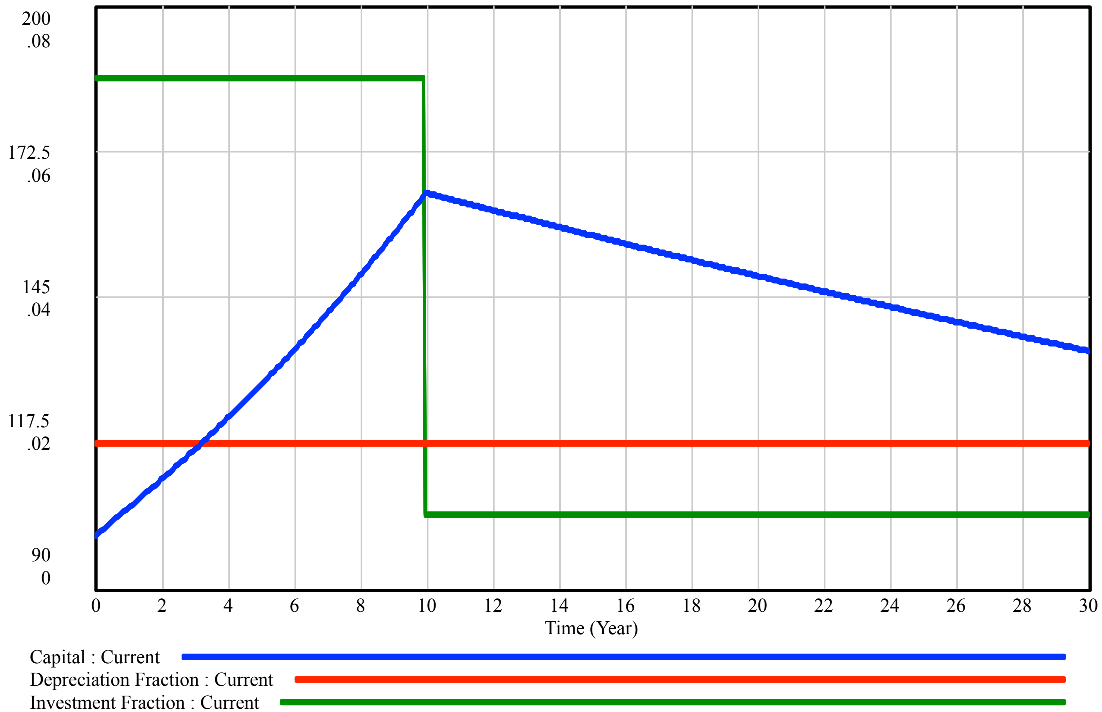
Combining Both Flows
A model of Capital



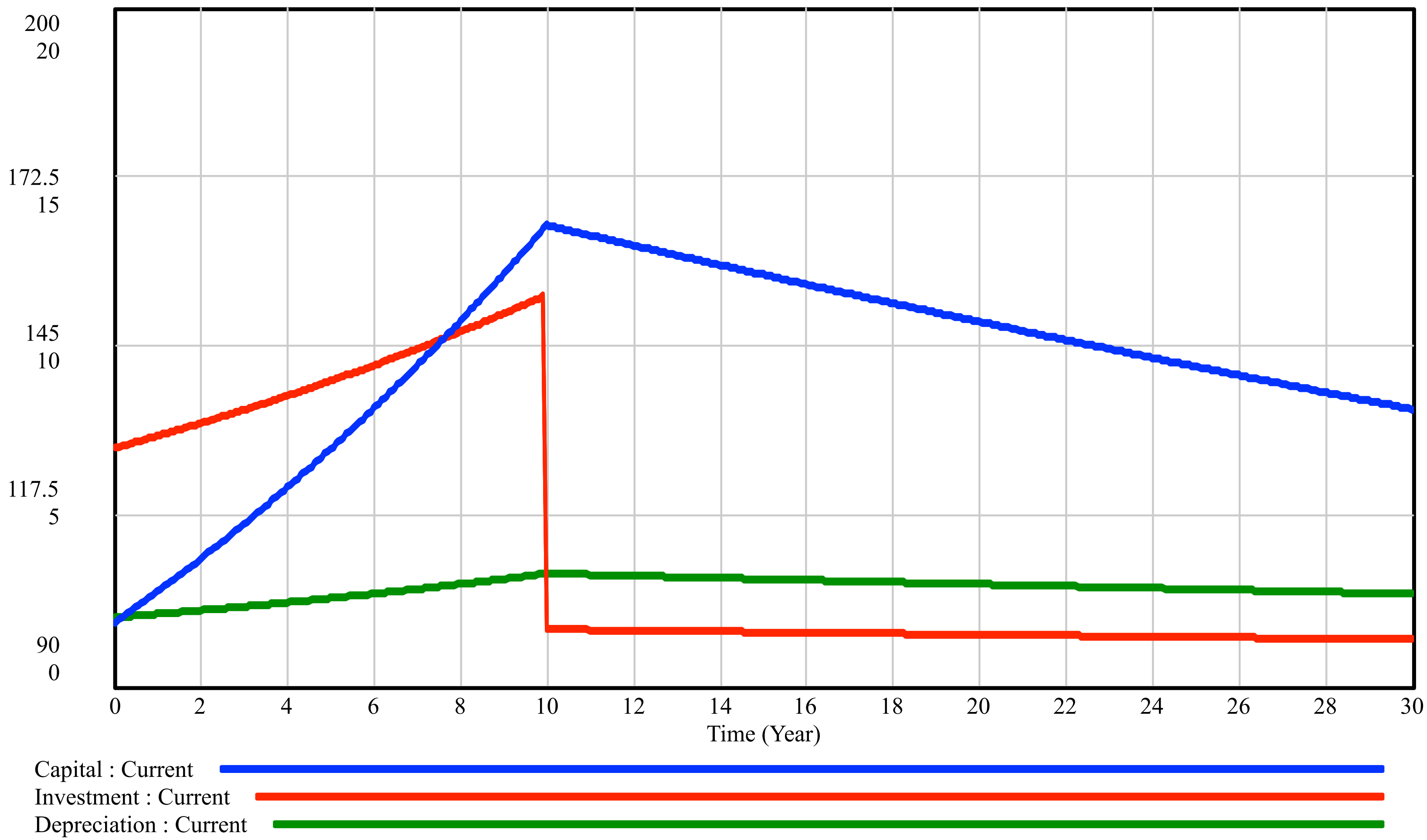
Capital = INTEG(Investment - Depreciation , 100)
Depreciation = Capital * Depreciation Fraction
Depreciation Fraction = 0.02
Investment = Capital * Investment Fraction
Investment Fraction = 0.07 - step (0.06, 10)

Simulation Run





Stocks and Flows (3 Rules)



Challenge 3.3

- Build a stock and flow model of a customers, where customers join and leave an organisation (e.g. a utility or subscription based service)
- Use fractional rates to model both inflows and outflow
- How might the model be extended to include factors that would impact the inflow and outflow?

Challenge 3.4

- For a University, identify the stocks that impact students flowing through the system
- Draw a high level model with a minimum number of stocks
- How would this model be disaggregated, and construct a detail model of students flowing through the CoEI from entry to exit.
- Add staff to the model, how might the stocks interact?

Challenge 3.5

- Build a model of University Competition
- Assume that students flow into a University in proportion to an attractiveness value
- This value is based on the student-staff ratio.
- The higher this value, the less attractive the University
- For example, a student staff ratio of 20:10 would lead to an attractiveness of ~33% to 67%
- Start the model in equilibrium and the “step up” the resources in one of the Universities.
- How might the model be extended to make it more realistic?