

# CT561: Systems Modelling and Simulation

## Week 7: Functions, Data Frames and deSolve

<https://github.com/JimDuggan/CT561>

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# Functions

- A function is a group of instructions that:
  - takes input,
  - uses the input to compute other value, and
  - returns a result (Matloff 2009).
- Users of R should adopt the habit of creating simple functions which will make their work more effective and also more trustworthy (Chambers 2008).
- Functions are declared:
  - using the **function** reserved word
  - are objects

# Example

- Function declared as an object
- Takes arguments
- Local scope for function variables
- Generates result
- `return()` explicit
- Implicit – last expression evaluated.

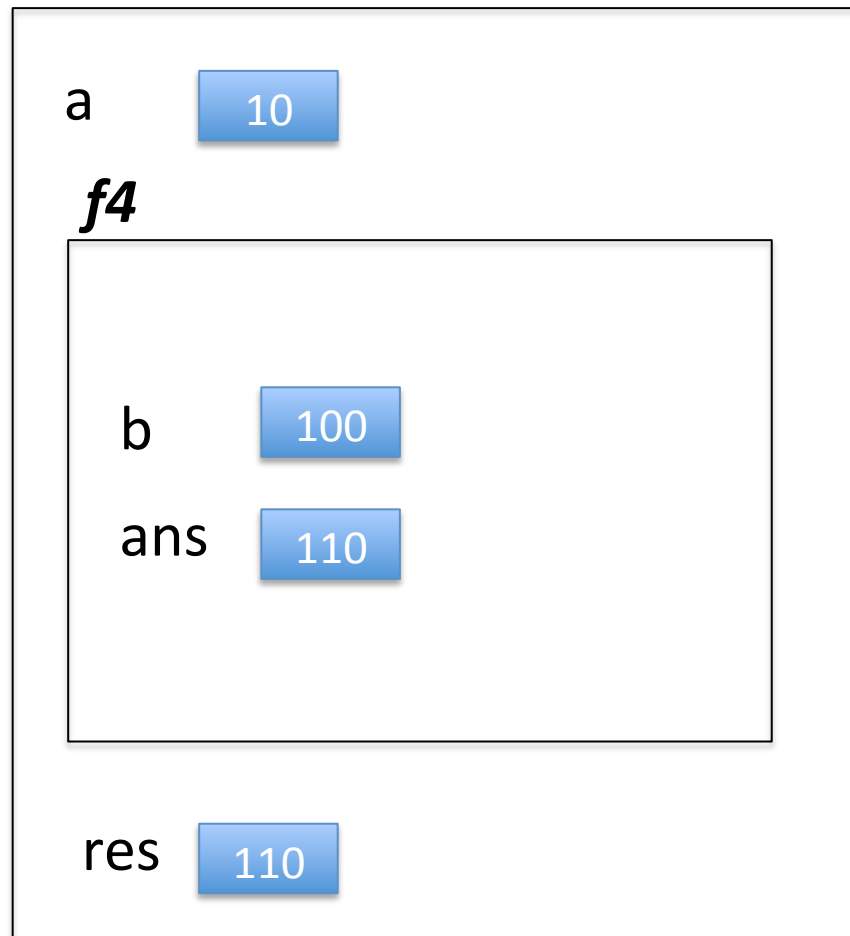
```
convC2F<-function(celsius)
{
  fahr<-celsius*9/5 + 32.0
  return(fahr)
}
```

 convC2F(celsius) ↕

```
>
>
> x<-convC2F(100)
> x
[1] 212
```

# Nested Functions: *Variable Scope*

*f3*



```
f3<-function(a){  
  f4<-function(b){  
    ans<-a+b  
  }  
  res<-f4(a^2)  
}
```

```
> f3(10)  
> r<-f3(10)  
> r  
[1] 110
```

# Function objects and arguments

- Functions are first-class objects, can be passed to other functions
- Arguments can be named directly in the call
- In that scenario, order of arguments not an issue

```
convert<-function(func, input)
{
  ans<-func(input)
  return (ans)
}
```

```
--
> convert(func=convC2F,input=100)
[1] 212
> convert(input=100,func=convC2F)
[1] 212
```

# Challenge 7.1

- Write a function that takes in a vector and returns the number of odd numbers
- Write a function that takes in a vector and returns a vector of even numbers
- Write a function that returns the unique values in a vector

`uplicated()` determines which elements of a vector or data frame are duplicates of elements with smaller subscripts, and returns a logical vector indicating which elements (rows) are duplicates.

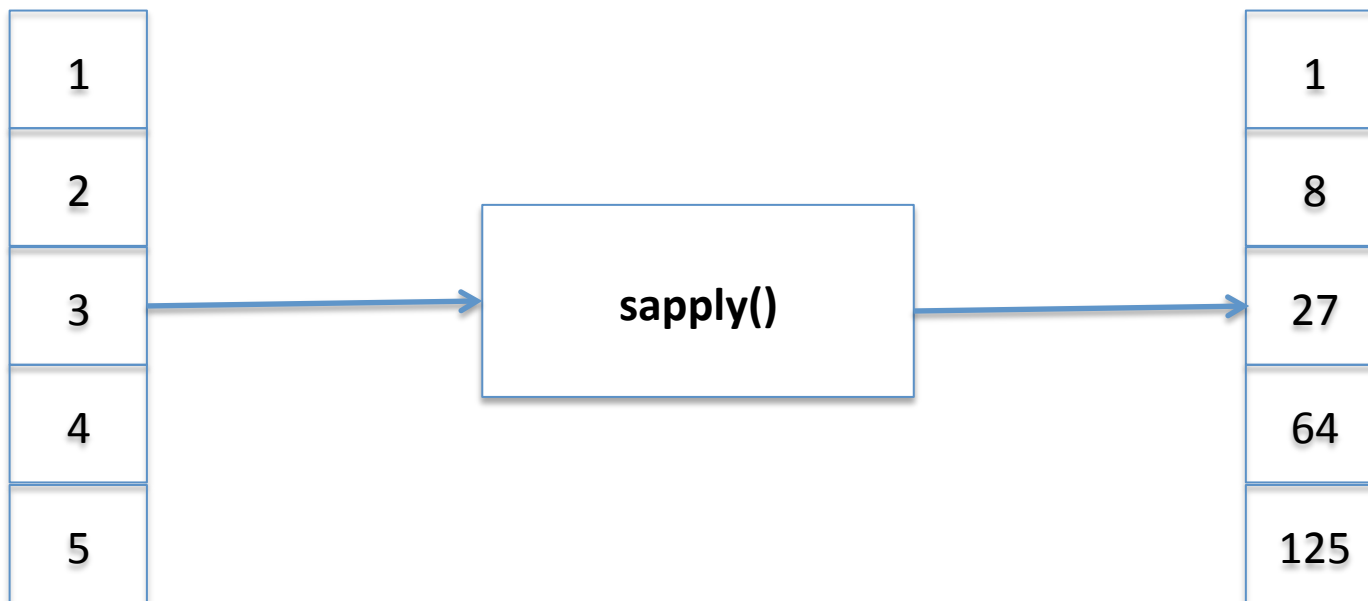
```
> duplicated(c(1,2,3,4,5,6,1))  
[1] FALSE FALSE FALSE FALSE FALSE FALSE  TRUE
```

# apply()

- Another use of user-defined functions in R is as parameters to the *apply* family of functions, which are one of the most famous and used features of R (Matloff 2009).
- The general form of the **apply(x,f,fargs)** function is as follows:
  - **x** is the target vector or list
  - **f** is the function to be called and applied to each element
  - **fargs** are the optional set of arguments that can be applied to the function **f**.

# Sample Problem

- To get the cube of numbers in a vector





# Method

- Embed function in call

```
> sapply(v1,function(x){x^3})  
[1]      1     64    729   4096  15625  
>
```

```
> v1  
[1]  1  4  9 16 25  
> sapply(v1,function(x,y){x^3+y},10)  
[1]   11   74  739 4106 15635  
>
```

# Challenge 7.2

- Use `apply()` for the following task
  - transform a vector according to the following function
    - $f(x) = 3x^2 + 5x + 10$

# Data Frames

- A data frame has a two-dimensional rows and columns structure.
- Can be viewed as a set of vectors, organised into a column format

```
> ids<-c("1234567","1234568")
> fName<-c("Jane","Matt")
> sName<-c("Smith","Johnson")
> ages<-c(21,25)
> ids
[1] "1234567" "1234568"
> fName
[1] "Jane" "Matt"
> sName
[1] "Smith" "Johnson"
> ages
[1] 21 25
```

# Creating a Data Frame

```
s<-data.frame(ID=ids,FirstName=fNames,Surname=sNames,  
              Age=ages,stringsAsFactors=FALSE)
```

```
> s
```

	ID	FirstName	Surname	Age
1	1234567	Jane	Smith	21
2	1234568	Matt	Johnson	25

```
> str(s)
```

```
'data.frame':  2 obs. of  4 variables:  
 $ ID          : chr  "1234567" "1234568"  
 $ FirstName: chr  "Jane" "Matt"  
 $ Surname   : chr  "Smith" "Johnson"  
 $ Age       : num  21 25
```

# Accessing Row Data

```
> s
```

	ID	FirstName	Surname	Age
1	1234567	Jane	Smith	21
2	1234568	Matt	Johnson	25

```
> s[1,]
```

	ID	FirstName	Surname	Age
1	1234567	Jane	Smith	21

```
> s[1:2,]
```

	ID	FirstName	Surname	Age
1	1234567	Jane	Smith	21
2	1234568	Matt	Johnson	25

```
> s[-1,]
```

	ID	FirstName	Surname	Age
2	1234568	Matt	Johnson	25

1

# Accessing Column Data

```
> s
      ID FirstName Surname Age
1 1234567      Jane   Smith  21
2 1234568      Matt Johnson  25
> s[, "Age"]
[1] 21 25
> s[, 1]
[1] "1234567" "1234568"
> s[, 2]
[1] "Jane" "Matt"
> s[, 3]
[1] "Smith" "Johnson"
> s[, 4]
[1] 21 25
```

# Accessing Column Data (using tags)

```
> s
      ID FirstName Surname Age
1 1234567      Jane   Smith  21
2 1234568      Matt Johnson  25
> s$ID
[1] "1234567" "1234568"
> s$FirstName
[1] "Jane" "Matt"
> s$Age[1:2]
[1] 21 25
> max(s$Age)
[1] 25
|
```

# subset() function

```
> s
```

	ID	FirstName	Surname	Age
1	1234567	Jane	Smith	21
2	1234568	Matt	Johnson	25

```
> sub<-subset(s,s$Age>21)
```

```
> sub
```

	ID	FirstName	Surname	Age
2	1234568	Matt	Johnson	25



# Adding extra information to a data frame - Vectorized

```
> s
```

	ID	FirstName	Surname	Age
1	1234567	Jane	Smith	21
2	1234568	Matt	Johnson	25

```
> s$Discount<-ifelse(s$Age<=21,0.25,0.10)
```

```
> s
```

	ID	FirstName	Surname	Age	Discount
1	1234567	Jane	Smith	21	0.25
2	1234568	Matt	Johnson	25	0.10

# Merging Data Frames

- For data analytics, opportunities often arise by merging different data sets into a single data frame, and the **merge()** function facilitates this
- In our student example, we could have a second data frame that stores examination results.

```
> ids<-c("1234567","1234568")
> subjects<-c("CT111","CT111")
> grade<-c(80,80)
> res<-data.frame(ID=ids,Subject=subjects,Grade=grade,stringsAsFactors=FALSE)
> res
```

	ID	Subject	Grade
1	1234567	CT111	80
2	1234568	CT111	80

|

# Merge Function

```
> s
```

	ID	FirstName	Surname	Age	Discount
1	1234567	Jane	Smith	21	0.25
2	1234568	Matt	Johnson	25	0.10

```
> res
```

	ID	Subject	Grade
1	1234567	CT111	80
2	1234568	CT111	80

```
> new<-merge(s,res,by="ID")
```

```
> new
```

	ID	FirstName	Surname	Age	Discount	Subject	Grade
1	1234567	Jane	Smith	21	0.25	CT111	80
2	1234568	Matt	Johnson	25	0.10	CT111	80

# deSolve package



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*Journal of Statistical Software*

February 2010, Volume 33, Issue 9.

<http://www.jstatsoft.org/>

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## Solving Differential Equations in R: Package deSolve

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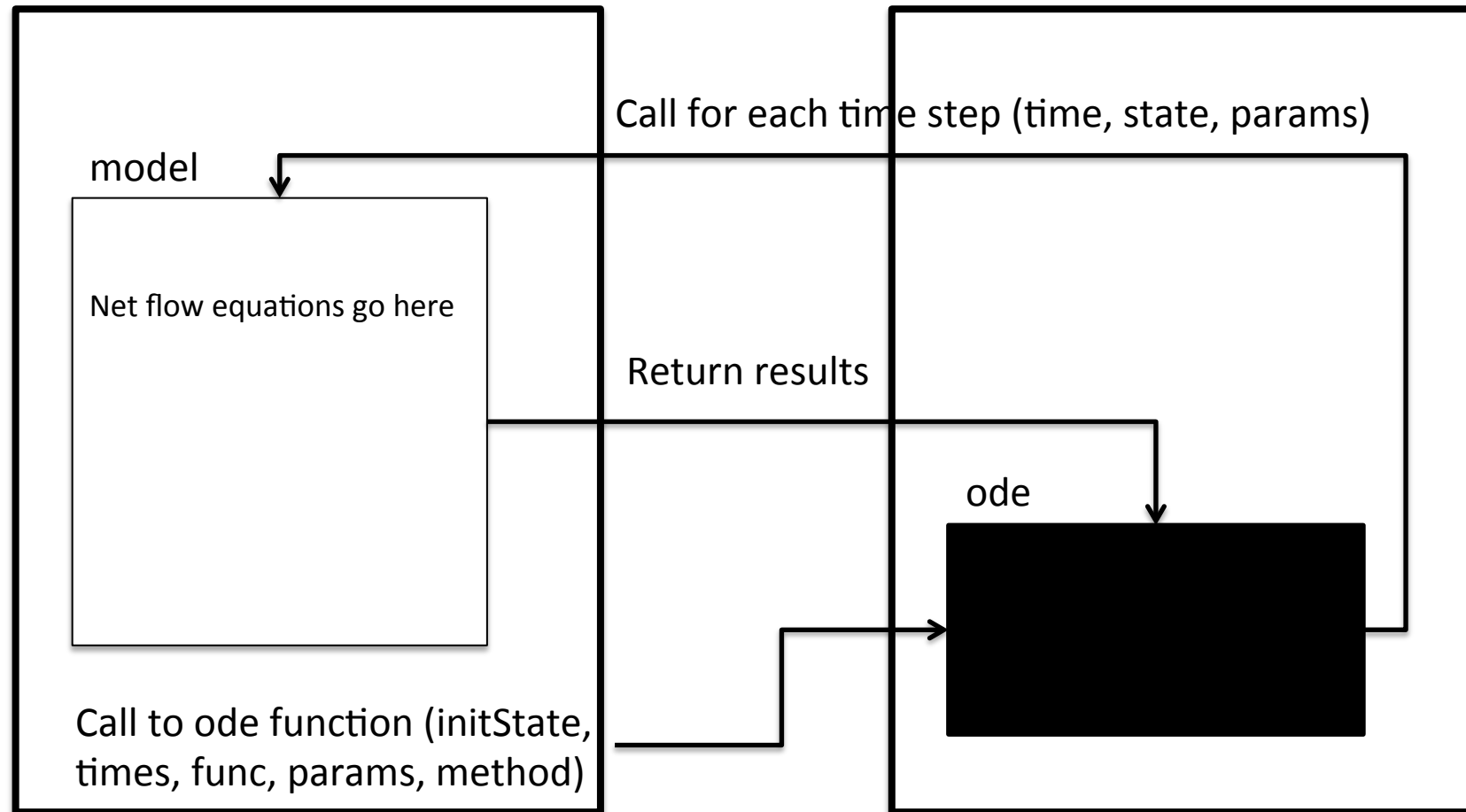
**R. Woodrow Setzer**  
US Environmental  
Protection Agency

---

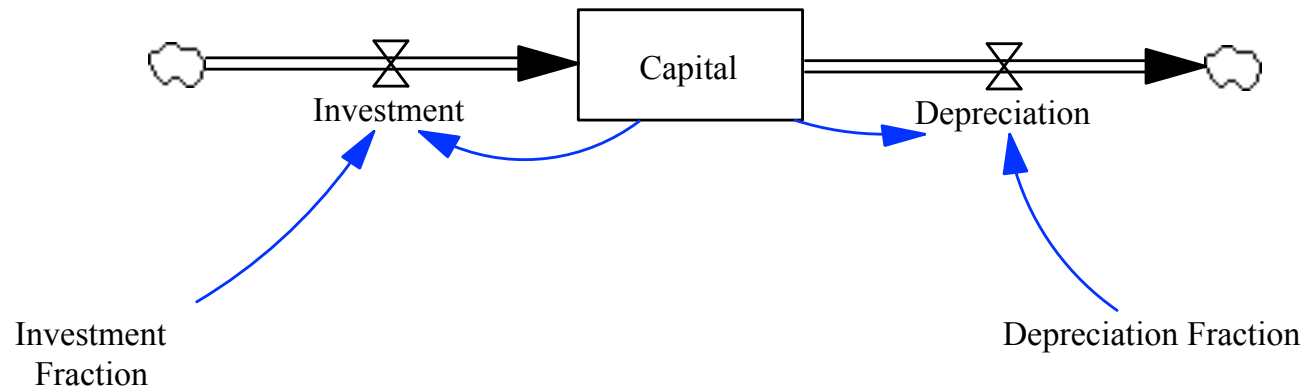
# Overall Idea

R Source mode

deSolve



# Capital Model



$\text{Capital} = \text{INTEG}(\text{Investment} - \text{Depreciation}, 10000)$

$\text{Investment Fraction} = 0.08$

$\text{Investment} = \text{Capital} * \text{Investment Fraction}$

$\text{Depreciation Fraction} = 0.05$

$\text{Depreciation} = \text{Capital} * \text{Depreciation Fraction}$

# R Implementation

```
1 library(deSolve) # include the deSolve library for odes
2 library(ggplot2) # include ggplot for displaying graphs
3
4 # setup simulation start and finish times, and time step
5 START<-2015; FINISH<-2035; STEP<-0.25;
6 |
7 # create a simulation time vector, needed by deSolve
8 simtime <- seq(START, FINISH, by=STEP)
9
```

```
> head(simtime)
[1] 2015.00 2015.25 2015.50 2015.75 2016.00 2016.25
> tail(simtime)
[1] 2033.75 2034.00 2034.25 2034.50 2034.75 2035.00
```

# The model function (1/2)

```
10 # create model function that contains all the equations
11 # this function receives
12 #   (1) the current time (time)
13 #   (2) a vectors of stocks (stocks)
14 #   (3) the auxiliary parameters (auxs)
15
16 model <- function(time, stocks, auxs)
17 {
18   # with(as.list) makes it easy to reference variable
19   with(as.list(c(stocks, auxs)),{
20     # Calculate the flows
21     fInvestment <- sCapital * aInvestmentFraction
22     fDepreciation <- sCapital * aDepreciationFraction
23
```



## The model function (2/2)

```
24 # Calculate the net flow
25 dC_dt <- fInvestment - fDepreciation
26
27 # return calculations as a list,
28 # first value is a vector of net flows
29 return (list(c(dC_dt),
30             Investment=fInvestment,
31             Depreciation=fDepreciation,
32             InvFraction=aInvestmentFraction,
33             DepFraction=aDepreciationFraction))
34 })
35 }
```

Create stocks and auxiliaries.  
Call solver, wrap in data frame

```
37 # create the vector of stocks and their initial values
38 stocks <- c(sCapital=10000)
39
40 # create the vector of auxiliaries
41 auxs <- c(aInvestmentFraction=0.08,
42           aDepreciationFraction=0.05)
43
44 # call the function ode, results returned as a data frame
45 o<-data.frame(ode(y=stocks, simtime, func = model,
46                  parms=auxs, method="euler"))
47
```

# Simulation Results (in data frame)

```
> str(o)
'data.frame':  81 obs. of  6 variables:
 $ time      : num  2015 2015 2016 2016 2016 ...
 $ sCapital   : num  10000 10075 10151 10227 10303 ...
 $ Investment : num  800 806 812 818 824 ...
 $ Depreciation: num  500 504 508 511 515 ...
 $ InvFraction : num  0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.08 ..
 $ DepFraction : num  0.05 0.05 0.05 0.05 0.05 0.05 0.05 0.05 0.05 0.05 ..

> head(o)
      time sCapital Investment Depreciation InvFraction DepFraction
1 2015.00 10000.00   800.0000    500.0000      0.08      0.05
2 2015.25 10075.00   806.0000    503.7500      0.08      0.05
3 2015.50 10150.56   812.0450    507.5281      0.08      0.05
4 2015.75 10226.69   818.1353    511.3346      0.08      0.05
5 2016.00 10303.39   824.2714    515.1696      0.08      0.05
6 2016.25 10380.67   830.4534    519.0334      0.08      0.05
```

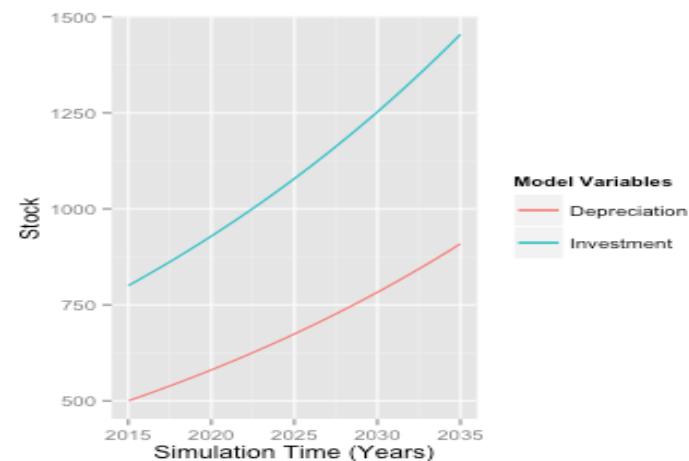
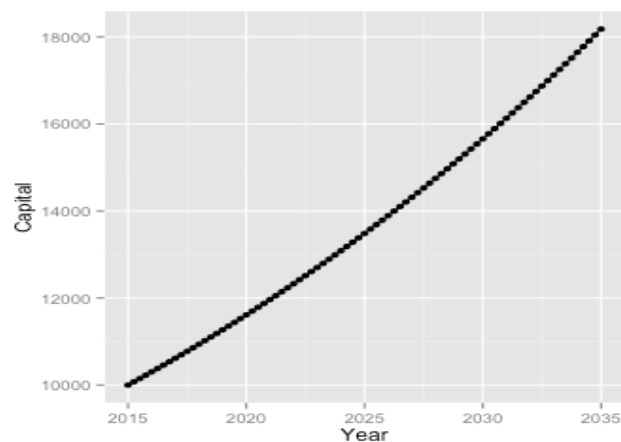
# Plot results

```
# plot the results
```

```
qplot(data=o,x=o$time,y=o$sCapital,  
      geom=c("line","point"),xlab="Year",ylab="Capital")
```

```
ggplot() +
```

```
  geom_line(data = o, aes(x = time, y = Investment, color = "Investment")) +  
  geom_line(data = o, aes(x = time, y = Depreciation, color = "Depreciation")) +  
  xlab('Simulation Time (Years)') +  
  labs(color="Model Variables")+  
  ylab('Flows')
```



# Challenge 7.3

- Build an R model for the stock adjustment structure
- Assume  $S_{\text{INIT}}=0$
- Assume  $S^*$  starts at 100 and increases to 200 after 10 time units
- Let  $AT=1$
- Run the simulation for 20 time units with  $DT=0.125$

