CT561: Systems Modelling and Simulation

Week 10: Modelling Diffusion and the SIR Model

https://github.com/JimDuggan/CT561

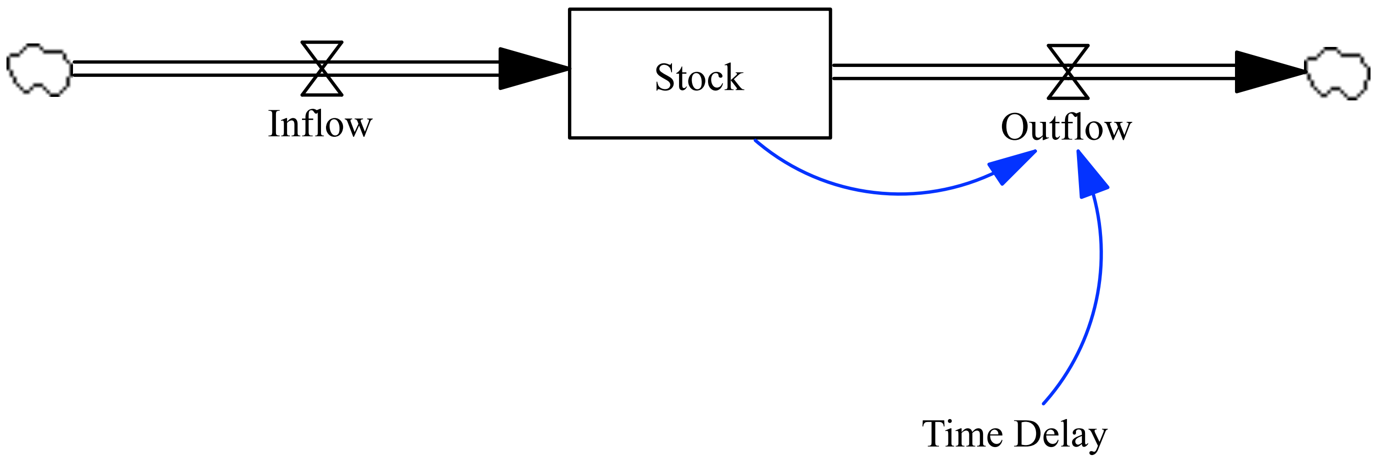
Dr. Jim Duggan,

Information Technology,

School of Engineering & Informatics

NUI Galway
OÉ Gaillimh

Recap: Tutorial Model (in R)

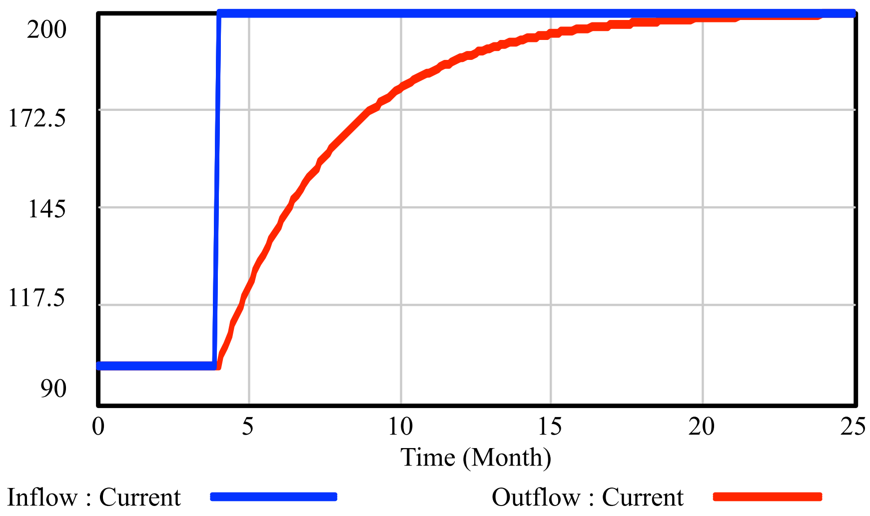


Inflow=100+step(100,4)

Outflow=Stock/Time Delay

Stock= INTEG (Inflow-Outflow, 400)

Time Delay=4



R Code

```r
library(deSolve)
library(ggplot2)

# Model equations (from Vensim)
# INITIAL TIME  = 0
# FINAL TIME  = 25
# Inflow=100+step(100,4)
# Outflow=Stock/Time Delay
# Stock= INTEG (Inflow-Outflow, 400)
# Time Delay=4
# TIME STEP  = 0.125

START<-0; FINISH<-25; STEP<-0.125;
simtime <- seq(START, FINISH, by=STEP)
```

Exploring vector simtime
time = 4 is at vector location 33

```
> simtime
  [1]  0.000  0.125  0.250  0.375  0.500  0.625  0.750  0.875  1.000  1.125  1.250  1.375
 [13]  1.500  1.625  1.750  1.875  2.000  2.125  2.250  2.375  2.500  2.625  2.750  2.875
 [25]  3.000  3.125  3.250  3.375  3.500  3.625  3.750  3.875  4.000  4.125  4.250  4.375
 [37]  4.500  4.625  4.750  4.875  5.000  5.125  5.250  5.375  5.500  5.625  5.750  5.875
 [49]  6.000  6.125  6.250  6.375  6.500  6.625  6.750  6.875  7.000  7.125  7.250  7.375
 [61]  7.500  7.625  7.750  7.875  8.000  8.125  8.250  8.375  8.500  8.625  8.750  8.875
 [73]  9.000  9.125  9.250  9.375  9.500  9.625  9.750  9.875 10.000 10.125 10.250 10.375
 [85] 10.500 10.625 10.750 10.875 11.000 11.125 11.250 11.375 11.500 11.625 11.750 11.875
 [97] 12.000 12.125 12.250 12.375 12.500 12.625 12.750 12.875 13.000 13.125 13.250 13.375
[109] 13.500 13.625 13.750 13.875 14.000 14.125 14.250 14.375 14.500 14.625 14.750 14.875
[121] 15.000 15.125 15.250 15.375 15.500 15.625 15.750 15.875 16.000 16.125 16.250 16.375
[133] 16.500 16.625 16.750 16.875 17.000 17.125 17.250 17.375 17.500 17.625 17.750 17.875
[145] 18.000 18.125 18.250 18.375 18.500 18.625 18.750 18.875 19.000 19.125 19.250 19.375
[157] 19.500 19.625 19.750 19.875 20.000 20.125 20.250 20.375 20.500 20.625 20.750 20.875
[169] 21.000 21.125 21.250 21.375 21.500 21.625 21.750 21.875 22.000 22.125 22.250 22.375
[181] 22.500 22.625 22.750 22.875 23.000 23.125 23.250 23.375 23.500 23.625 23.750 23.875
[193] 24.000 24.125 24.250 24.375 24.500 24.625 24.750 24.875 25.000
```

Setting up a global data frame

```r
# Setup the step function in a global data frame
input <- rep(NA,length(simtime))
input[1:(4/STEP)]<-100
input[((4/STEP)+1):length(simtime)]<-200

simData<-data.frame(time=simtime, aInput=input)
```

The input vector (step function) step happens at vector location 33

```
> input
  [1] 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100
 [18] 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 200 200
 [35] 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200
 [52] 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200
 [69] 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200
 [86] 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200
[103] 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200
[120] 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200
[137] 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200
[154] 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200
[171] 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200 200
[188] 200 200 200 200 200 200 200 200 200 200 200 200 200 200
```

The simData data frame

```
> simData[1:10,]
    time aInput
1  0.000    100
2  0.125    100
3  0.250    100
4  0.375    100
5  0.500    100
6  0.625    100
7  0.750    100
8  0.875    100
9  1.000    100
10 1.125    100
```

```
> simData[30:40,]
    time aInput
30 3.625    100
31 3.750    100
32 3.875    100
33 4.000    200
34 4.125    200
35 4.250    200
36 4.375    200
37 4.500    200
38 4.625    200
39 4.750    200
40 4.875    200
```

```
> simData[191:201,]
      time aInput
191 23.750    200
192 23.875    200
193 24.000    200
194 24.125    200
195 24.250    200
196 24.375    200
197 24.500    200
198 24.625    200
199 24.750    200
200 24.875    200
201 25.000    200
```

The model function

```
# model equations
model <- function(time, stocks, auxs)
{
  with(as.list(c(stocks, auxs)),{
    index<-which(simData$time==time)

    fInflow<-simData$aInput[index]
    fOutflow<-sStock/aTimeDelay

    dS_dt  <- fInflow - fOutflow

    return (list(c(dS_dt),
                 Inflow=fInflow,
                 Outflow=fOutflow))
  })
}
```

Inflow=100+step(100,4)

Outflow=Stock/Time Delay

Stock= INTEG (Inflow-Outflow, 400)

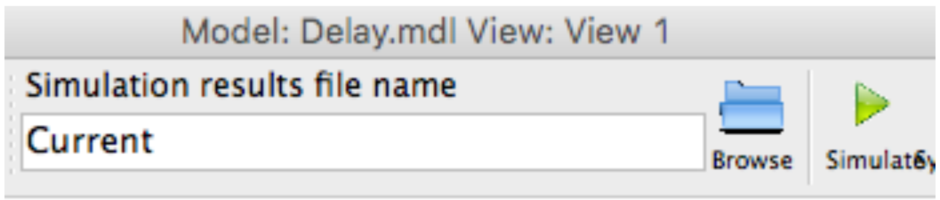
Setup and run

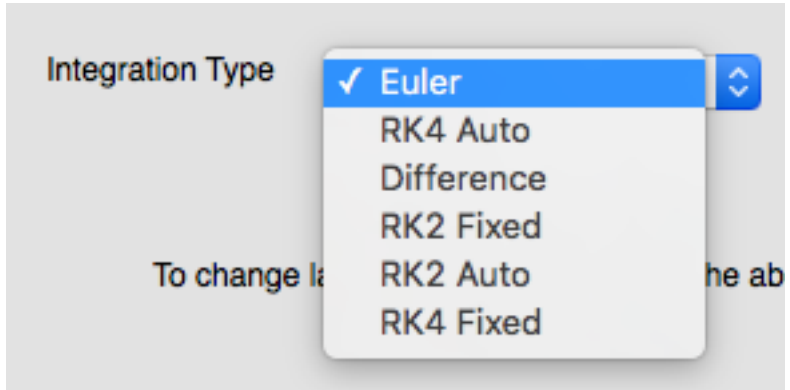
```
stocks   <- c(sStock=400)
auxs     <- c(aTimeDelay=4)


o<-data.frame(ode(y=stocks, simtime, func = model,
                  parms=auxs, method="euler"))
```

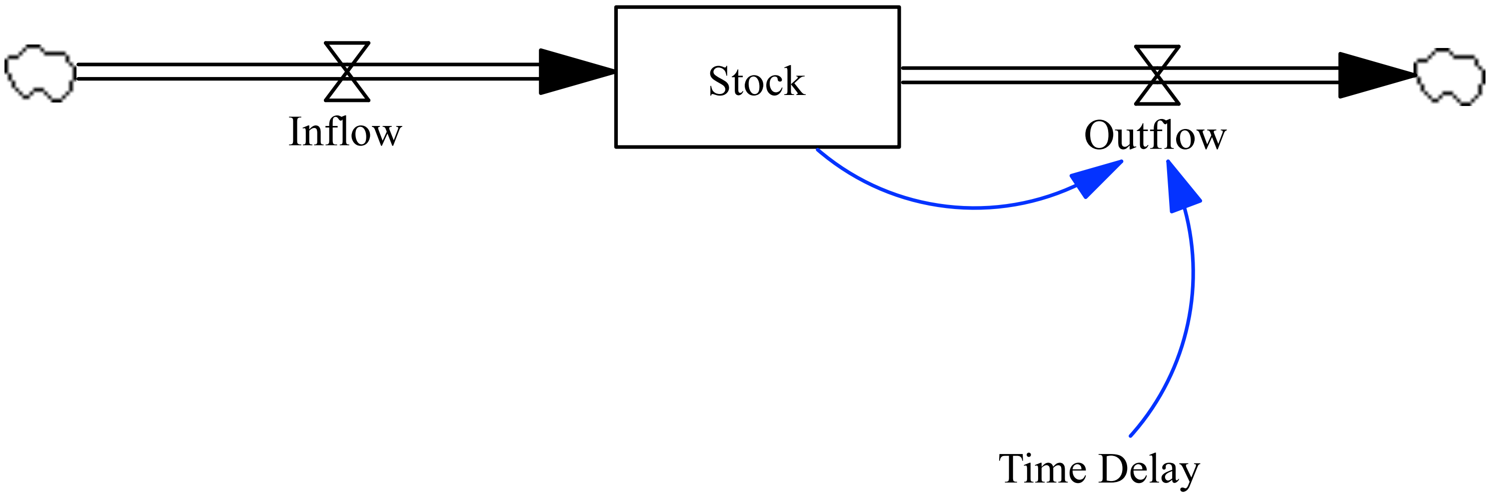
Stock= INTEG (Inflow-Outflow, **400**)

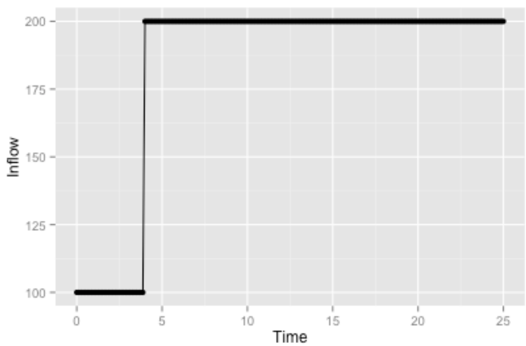
Time Delay=4

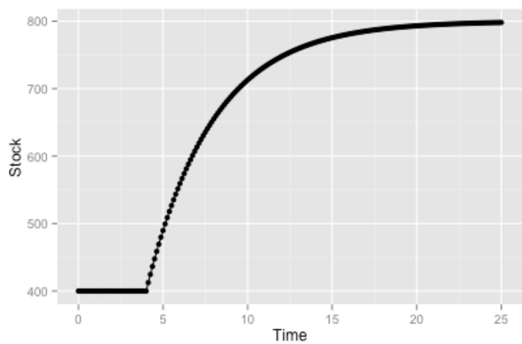


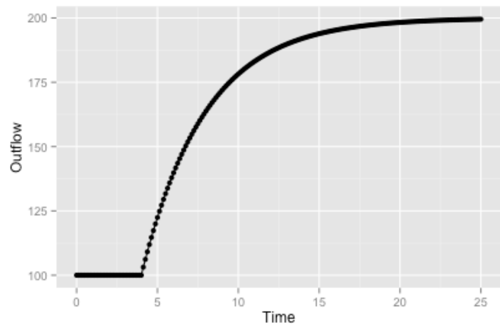


View the results



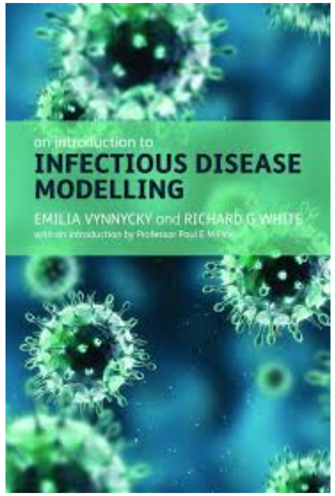


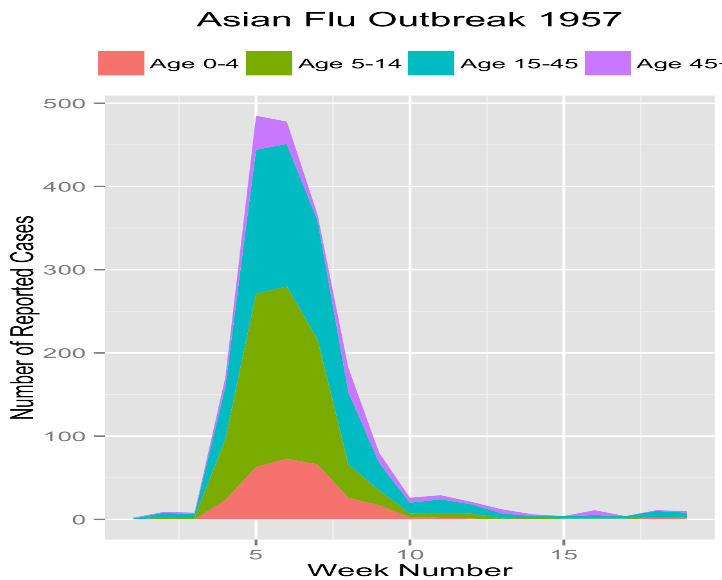




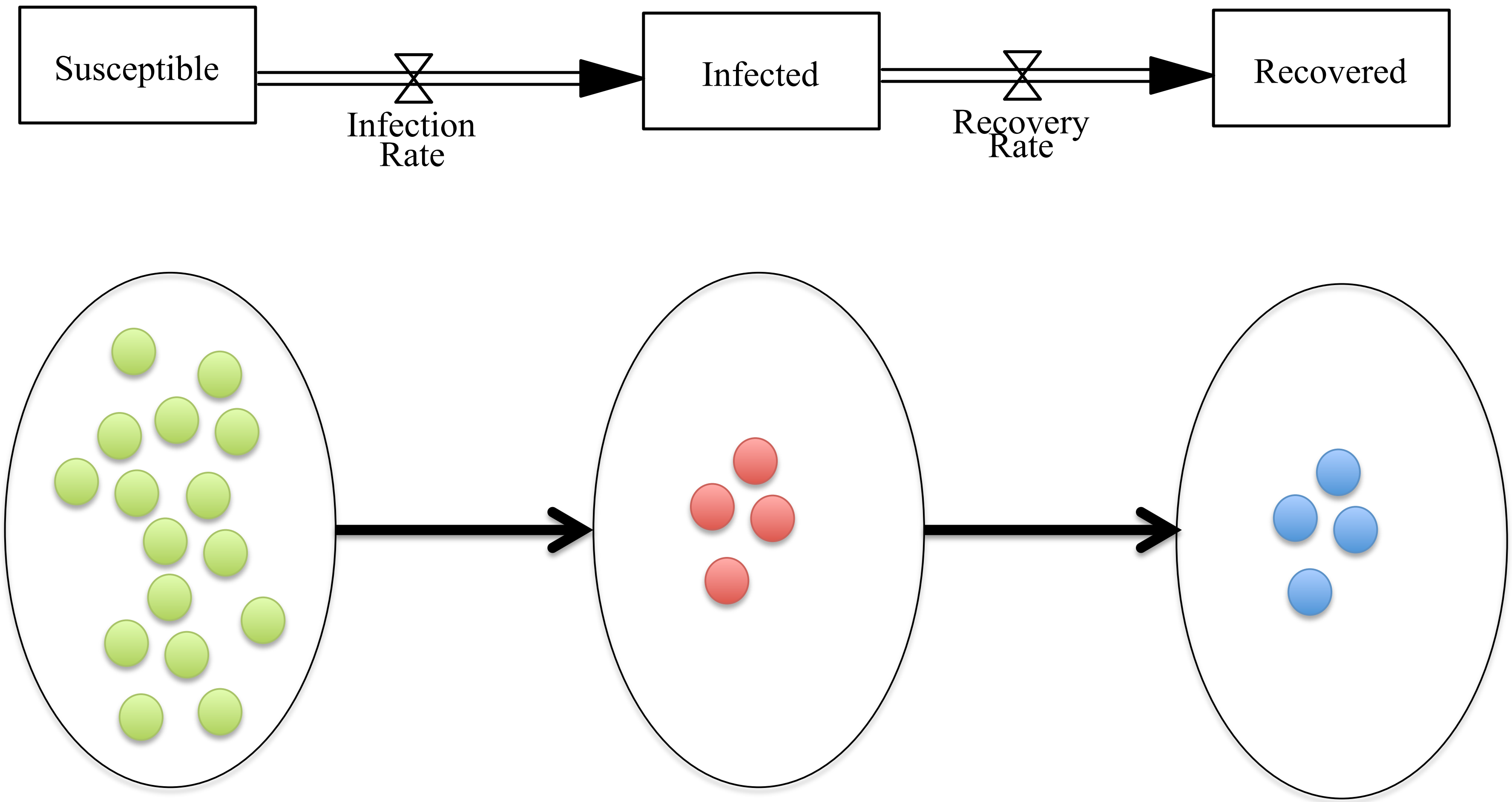
Public Health: Modelling Infectious Disease Outbreaks

- Public Health
- Modelling transmission of infectious diseases
- SIR Model (3 stocks, 2 flows)





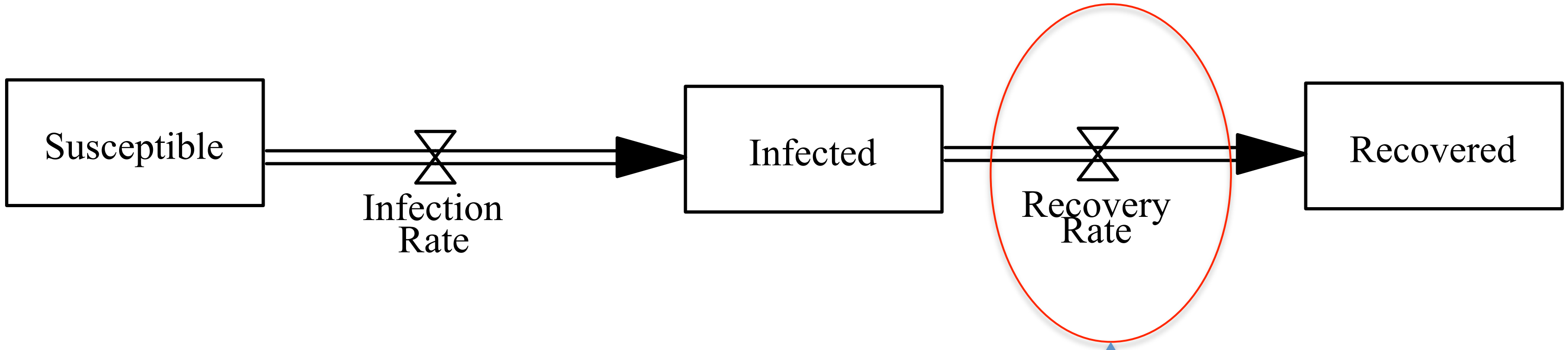
Need a model structure (Stocks and Flows)



Stock Equations

$$Susceptible\ (S) = \ INTEGRAL(-IR, 99999)$$
$$Infected\ (I) = \ INTEGRAL(IR - RR, 1)$$
$$Recovered(R) = \ INTEGRAL(RR, 0)$$

What about the flows?

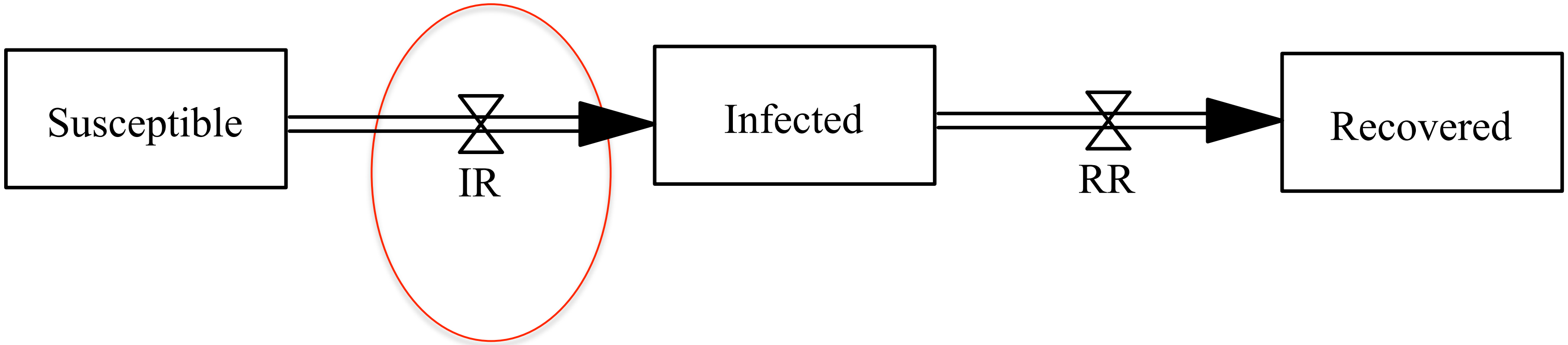


First Order Delay Structure

$$RR = \frac{I}{D}$$

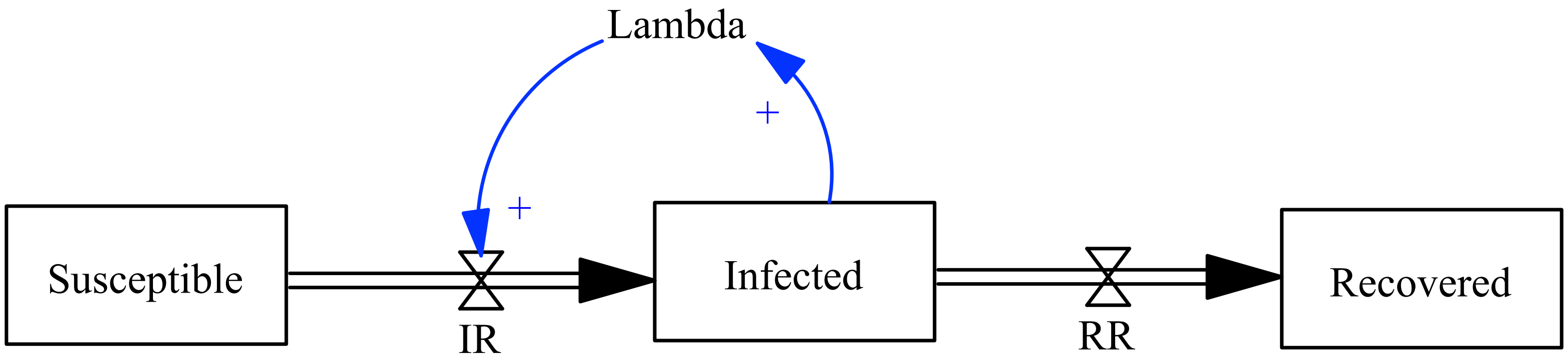
$$Delay\ (D) = 2$$

Infection Rate?



- Infection spreads through contact
- As the number of infected increase, so to does the infection rate
- A positive feedback process

Lambda – Force of Infection (Attack Rate)



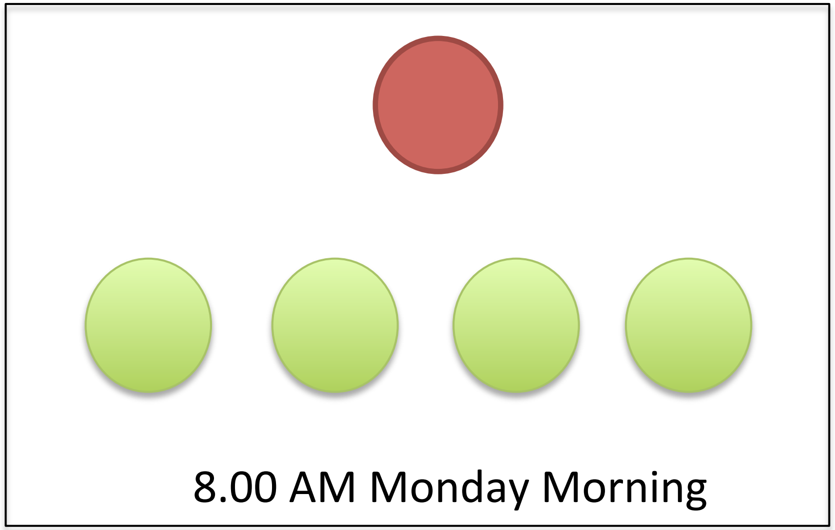
$\uparrow$	Infected	$\rightarrow$	Lambda	$\uparrow$
$\uparrow$	Lambda	$\rightarrow$	IR	$\uparrow$
$\uparrow$	IR	$\rightarrow$	Infected	$\uparrow$

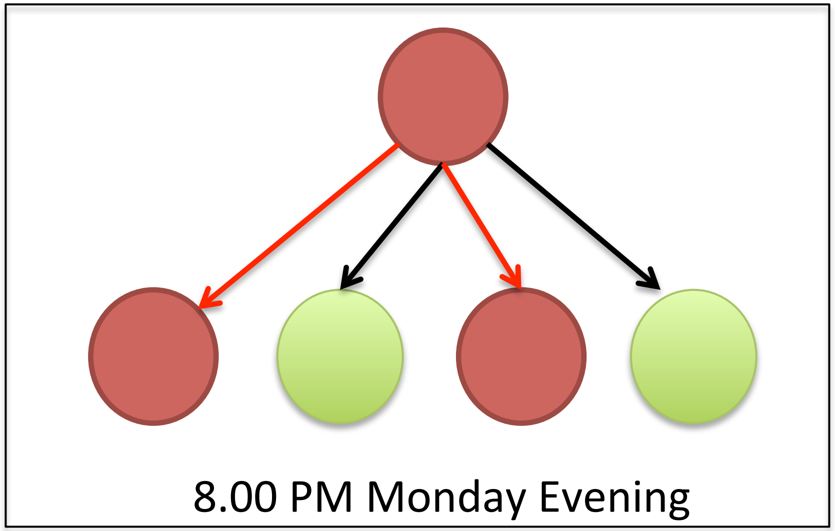
The rate at which susceptible individuals become infected per unit time

Proportional to the number infected

Effective Contact (C_e)

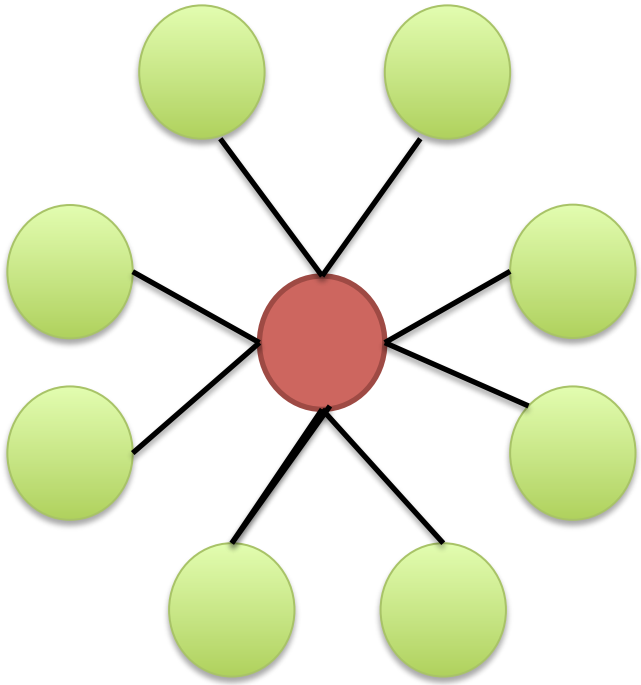
- Defined a one which is sufficient to lead to infection, were it to occur between a **susceptible** and **infectious** individuals
- For example, if $C_e = 2$
  - An infectious person will infect two susceptible people in one day
  - They could meet 4 people, and pass on the virus with probability (0.50)





Beta (β)

- **Per capita rate** at which two specific individuals come into effective contact per unit time
- An important parameter used to model interactions between infectious and susceptible individuals



$$\beta = \frac{c_e}{N}$$

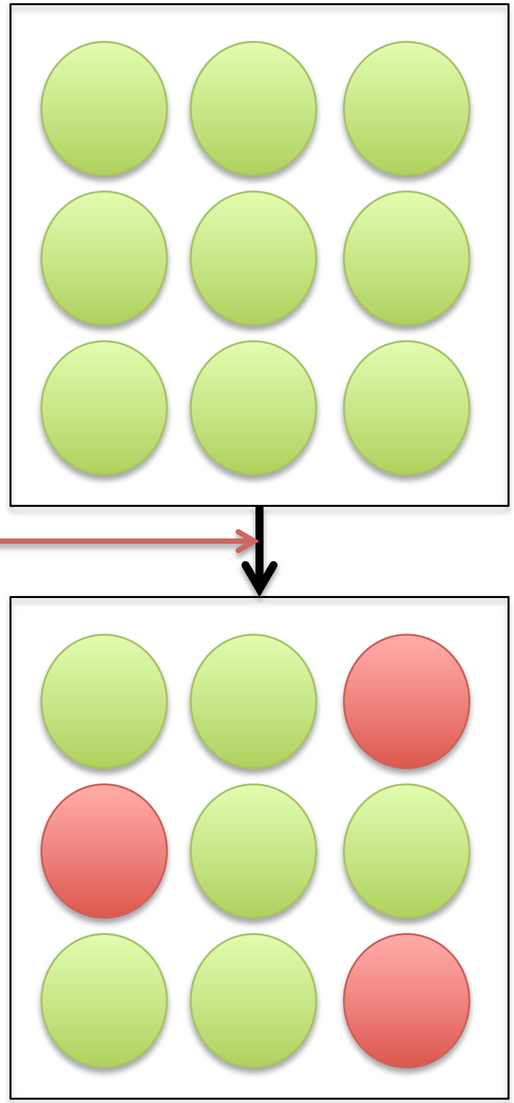
$$\beta = \frac{2}{10000} = 0.0002$$

Lambda – Force of Infection

- The rate at which susceptible individuals become infected per unit time
- Also know as the hazard rate or incidence rate

$$\lambda = \beta I$$

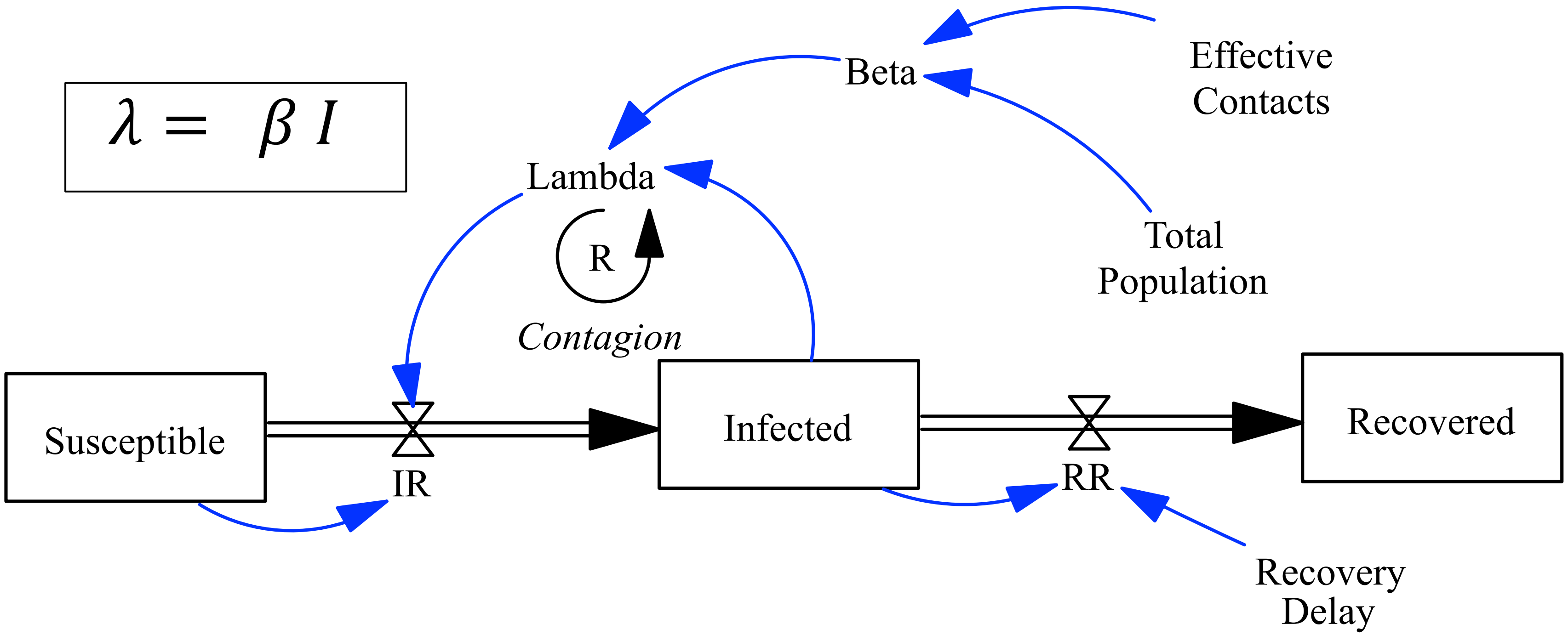
$$\lambda = \frac{1}{3}$$



A Stock and Flow Model

$$\beta = \frac{c_e}{N}$$

$$\lambda = \beta I$$



Model Equations

Total Population = 10000

Effective Contacts=2

Beta= Effective Contacts/Total Population

Lambda = Beta*Infected

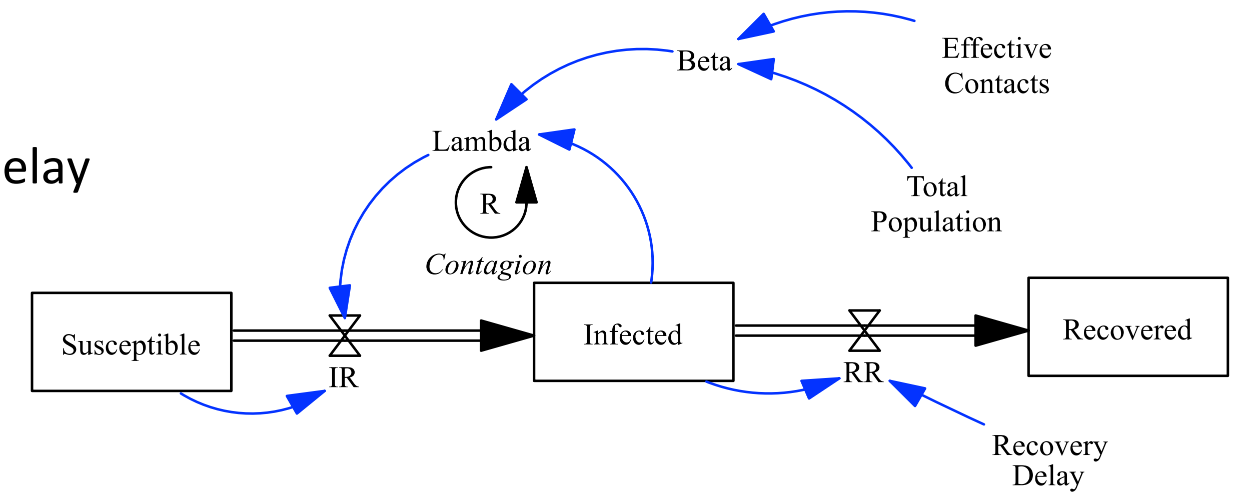
IR=Lambda*Susceptible

RR=Infected/Recovery Delay

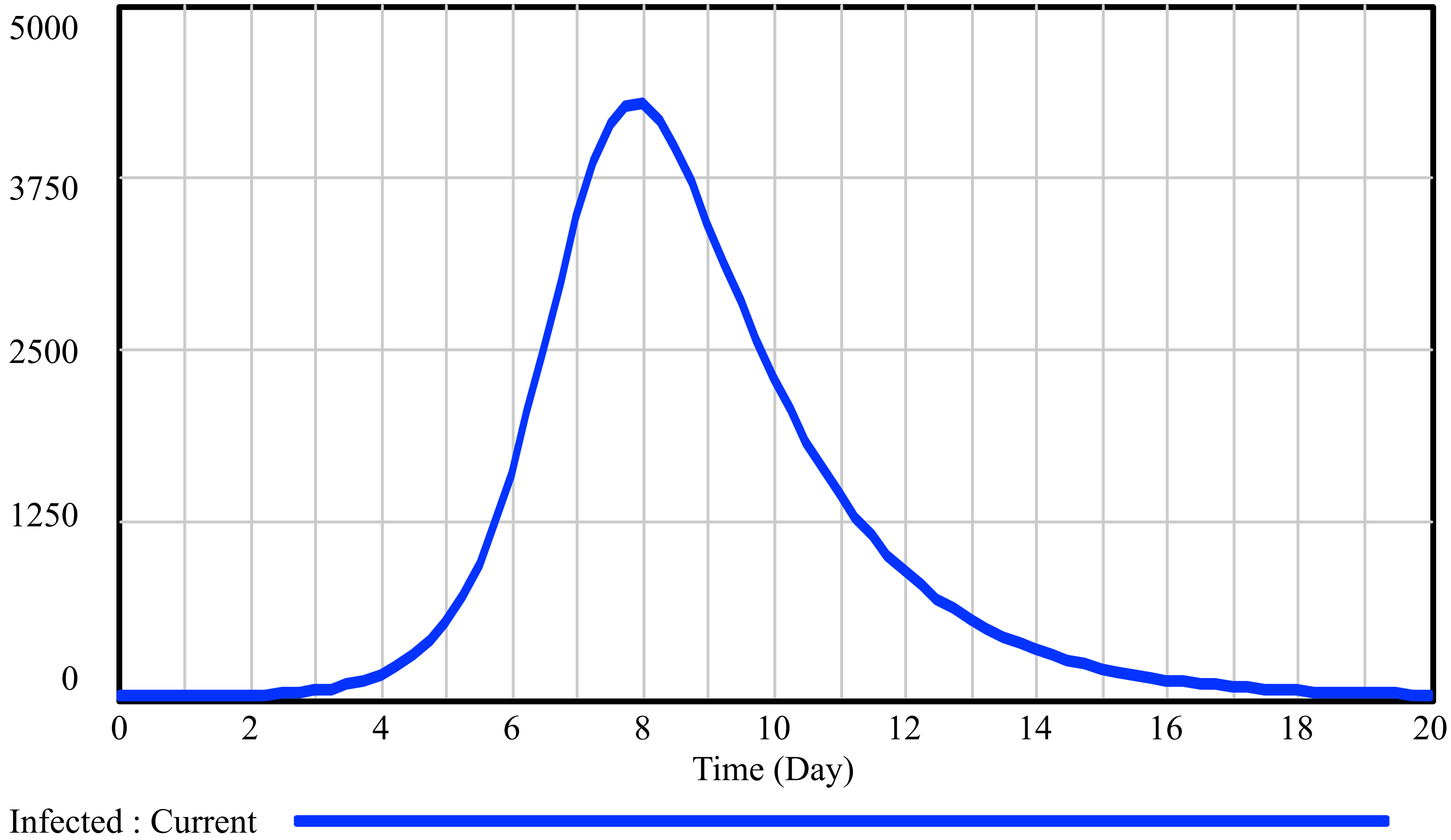
Susceptible= INTEG (-IR, 9999)

Infected= INTEG (IR-RR, 1)

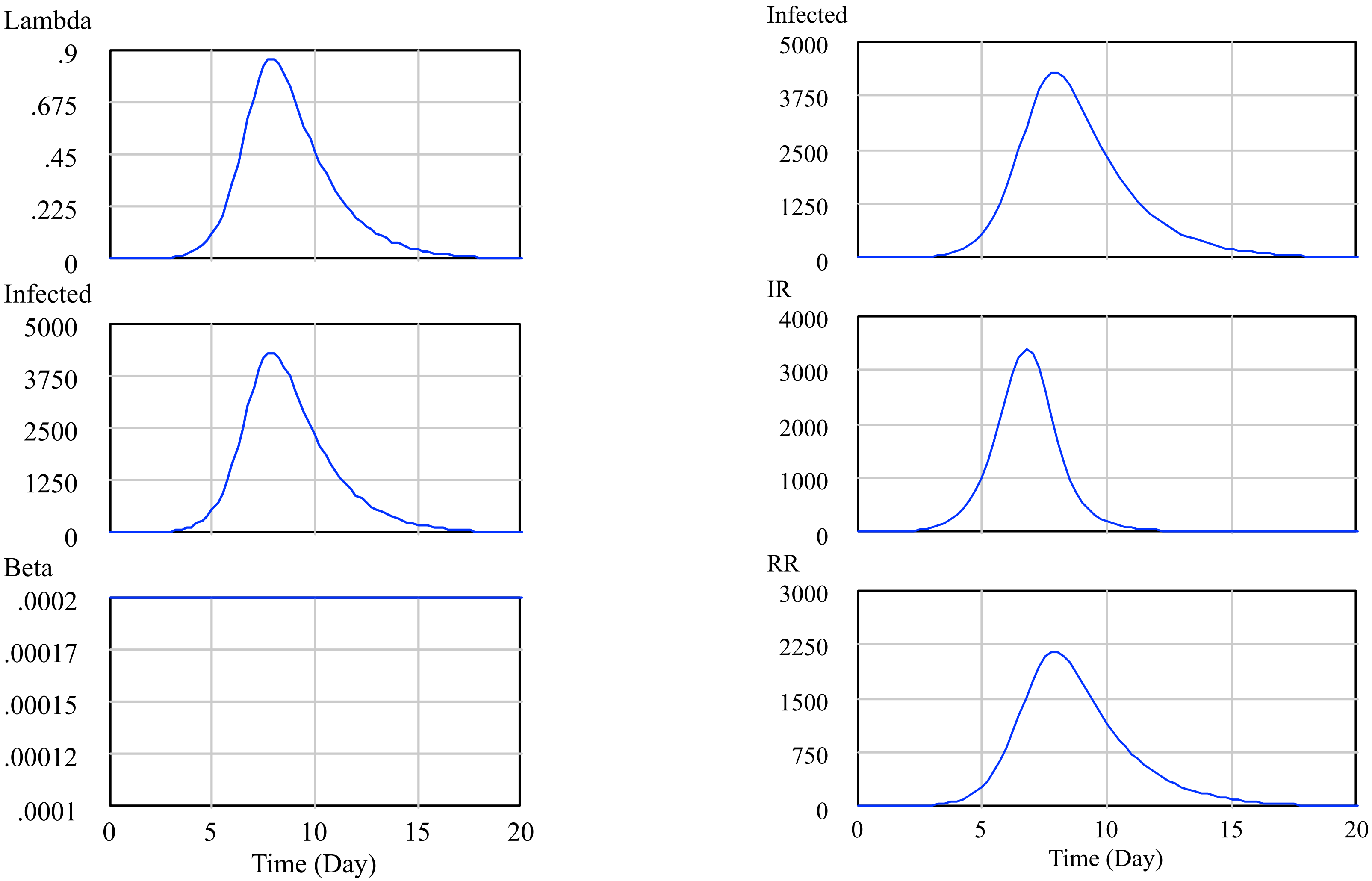
Recovered= INTEG (RR, 0)



Simulation Output



Exploring Variables…



Challenge 10.1

- Draw the stock and flow model the corresponds to the following equations

$$\frac{dS}{dt} = -\lambda S$$

$$\frac{dI}{dt} = fE - rI$$

$$\frac{dE}{dt} = \lambda S - fE$$

$$\frac{dR}{dt} = rI$$

The rate at which something occurs is 1 / Average time to the event
f and *r* are rates.

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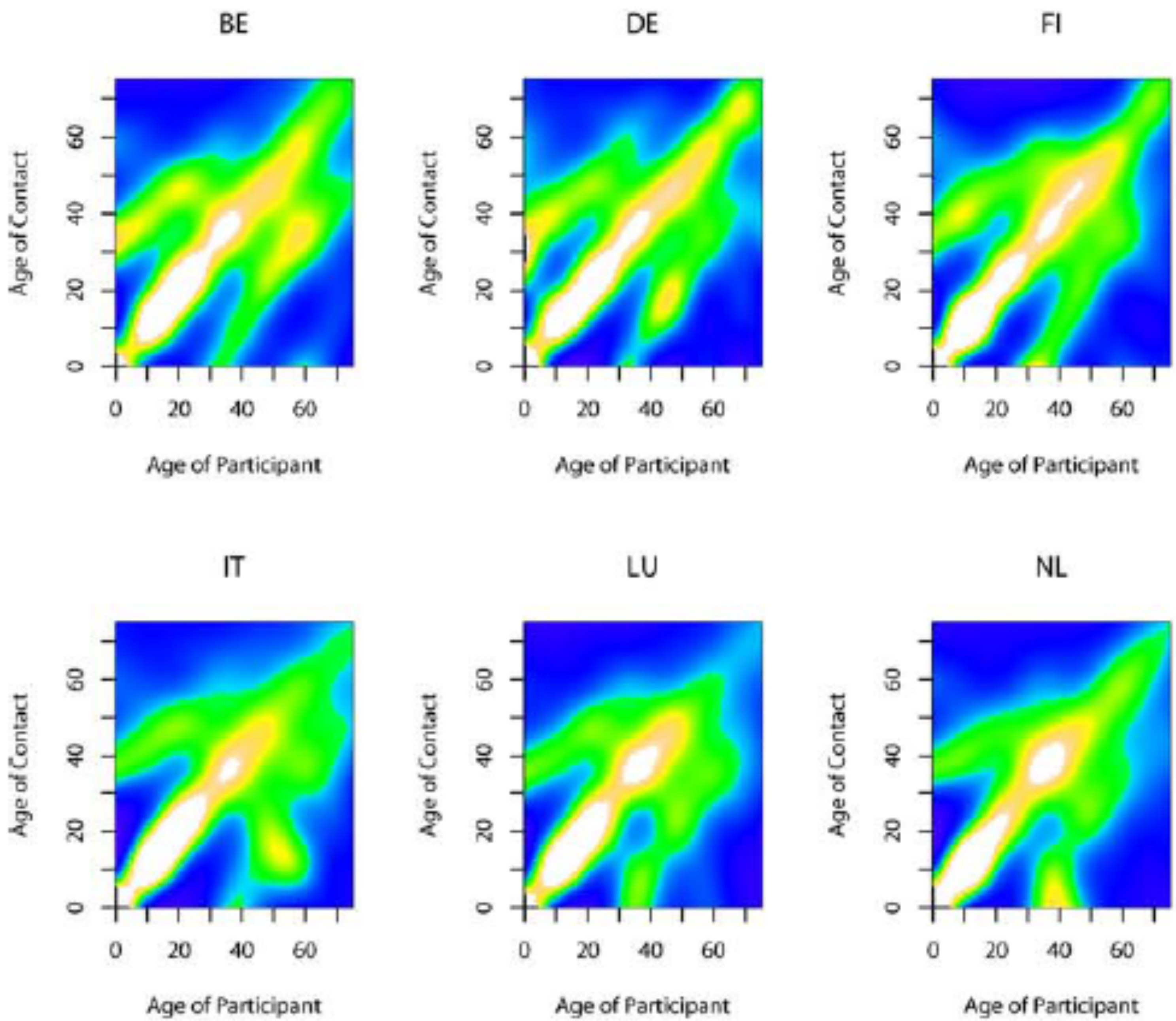
Social Contacts and Mixing Patterns Relevant to the Spread of Infectious Diseases

Joël Mossong[1,2*], Niel Hens[3], Mark Jit[4], Philippe Beutels[5], Kari Auranen[6], Rafael Mikolajczyk[7], Marco Massari[8], Stefania Salmaso[8], Gianpaolo Scalia Tomba[9], Jacco Wallinga[10], Janneke Heijne[10], Malgorzata Sadkowska-Todys[11], Magdalena Rosinska[11], W. John Edmunds[4]

Methods and Findings

7,290 participants recorded characteristics of 97,904 contacts with different individuals during one day, including age, sex, location, duration, frequency, and occurrence of physical contact. We found that mixing patterns and contact characteristics were remarkably similar across different European countries. Contact patterns were highly assortative with age: schoolchildren and young adults in particular tended to mix with people of the same age. Contacts lasting at least one hour or occurring on a daily basis mostly involved physical contact, while short duration and infrequent contacts tended to be nonphysical. Contacts at home, school, or leisure were more likely to be physical than contacts at the workplace or while travelling. Preliminary modelling indicates that 5- to 19-year-olds are expected to suffer the highest incidence during the initial epidemic phase of an emerging infection transmitted through social contacts measured here when the population is completely susceptible.


BE
DE
FI
IT
LU
NL
Age of Contact
Age of Participant
0
20
40
60

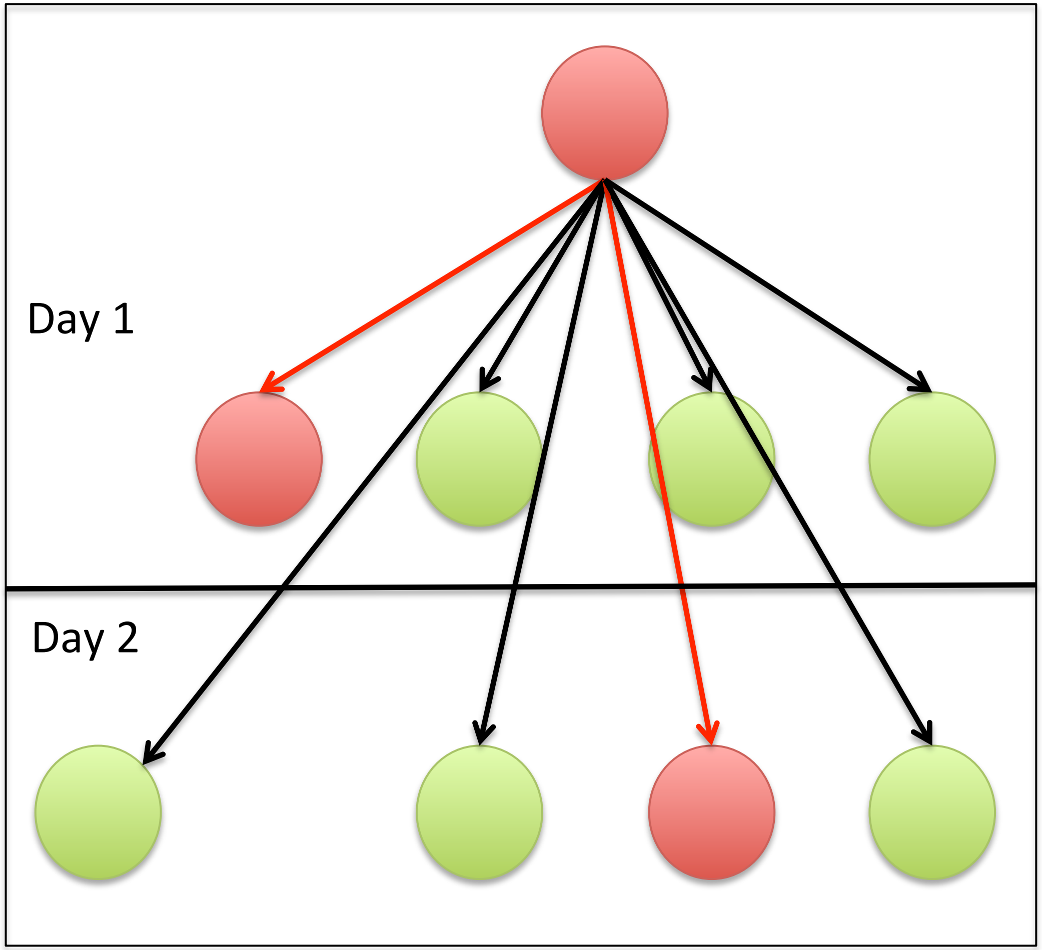
Three Additional Concepts

- Reproduction Number
- Herd Immunity
- Epidemic Thresholds

(1) Reproduction Number – R_0

- Formally defined as the average number of secondary infectious resulting from a typical infectious person being introduced to a totally susceptible population

$$\boxed{R_0 = c_e D}$$



$D = 2 \quad c_e = 1 \quad R_0 = 2$

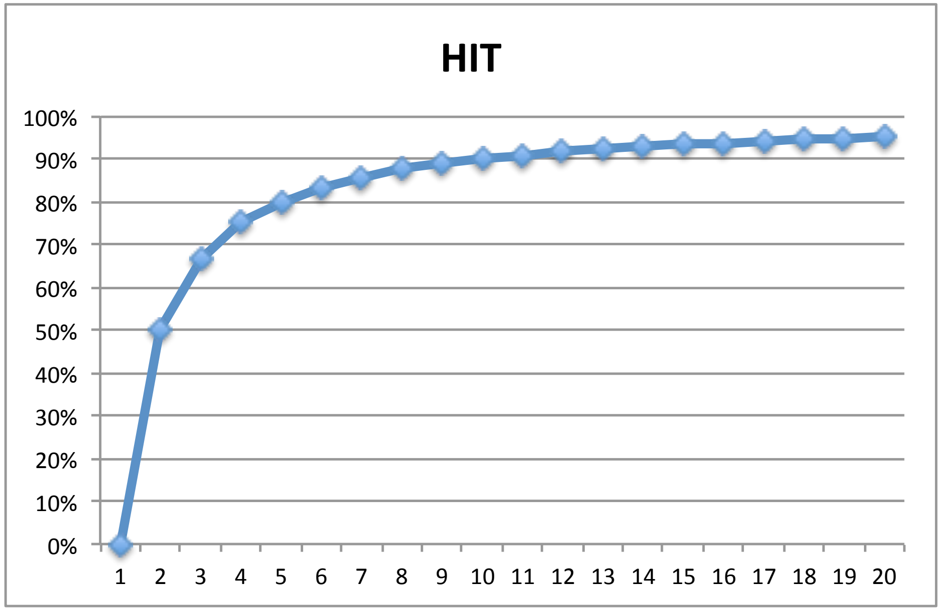
Challenge 10.2

- Suppose we have a town with 10,000 (=N) individuals, of which 1% were infectious with measles, with R_0 = 13 and D=7 Days
- Calculate the force of infection λ

(2) Herd Immunity Threshold

- Depends on R_0
- The proportion of the population which needs to be immune for the infection incidence to be stable
- To eradicate an infection, the proportion of the population that is immune must exceed this threshold value

$$HIT = 1 - \frac{1}{R_0}$$

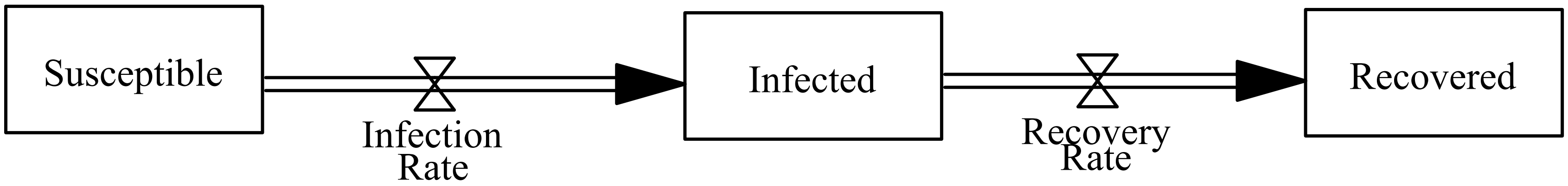


Approximate data for common potentially vaccine-preventable diseases

Infection	R_o	Herd Immunity
Diphtheria	6-7	85
Influenza	2-4	50-75
Malaria	5-100	80-99
Measles	12-18	83-94
Pertussis	12-17	92-94

(3) Threshold Dynamics for SIR Model

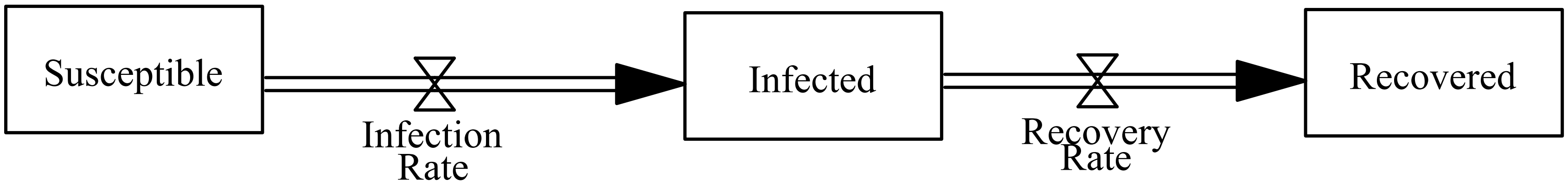
For the number of infectious people to increase, the inflow must be greater than the outflow.



$$IR > RR \longrightarrow \lambda S > \frac{I}{D} \longrightarrow \beta \, I \, S > \frac{I}{D}$$

(3) Threshold Dynamics for SIR Model

For the number of infectious people to increase, the inflow must be greater than the outflow.



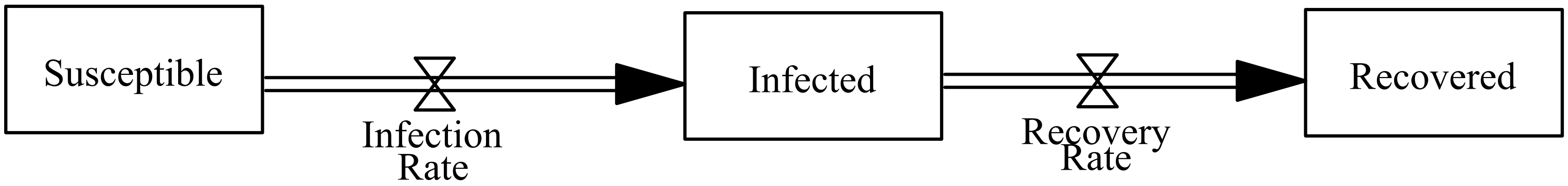
$$IR > RR \longrightarrow \lambda S > \frac{I}{D} \longrightarrow \beta\, I\, S > \frac{I}{D}$$

$$S = N \; for\ totally\ susceptible\ population$$

$$\beta\, N\, D > 1 \longrightarrow c_e\, D > 1 \qquad Given\ that\ \ \beta = \frac{c_e}{N}$$

(3) Threshold Dynamics for SIR Model

For the number of infectious people to increase, the inflow must be greater than the outflow.



$$IR > RR \longrightarrow \lambda S > \frac{I}{D} \longrightarrow \beta\, I\, S > \frac{I}{D}$$

$$S = N \; for\ totally\ susceptible\ population$$

$$\beta\, N\, D > 1 \longrightarrow c_e\, D > 1 \qquad Given\ that\ \ \beta = \frac{c_e}{N}$$

$$\boxed{R_0 > 1}$$

Challenge 10.3

- Introduce vaccinations and quarantine into the SIR model. Provide the appropriate rate equations.