

CT561: Systems Modelling and Simulation

Week 8: Effects and Limits to Growth

<https://github.com/JimDuggan/CT561>

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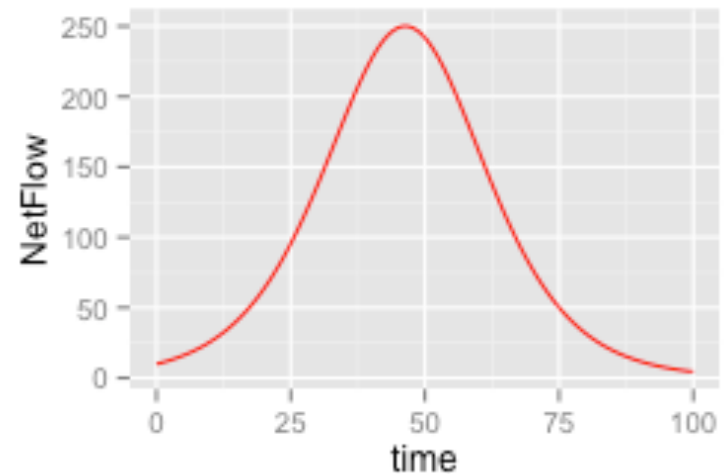
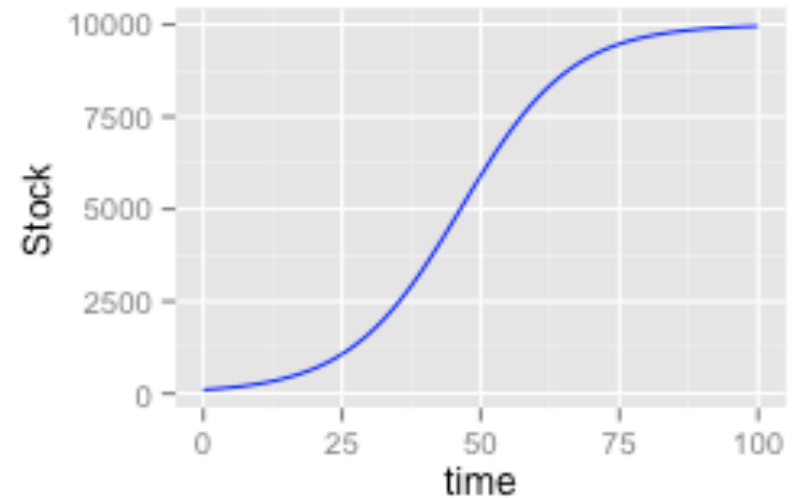
Overview

There will always be limits to growth.

They can be self-imposed.

If they aren't, they will be system-imposed.

Donella H. Meadows, Thinking in Systems: A Primer (2008), p.103



Formulating Effects

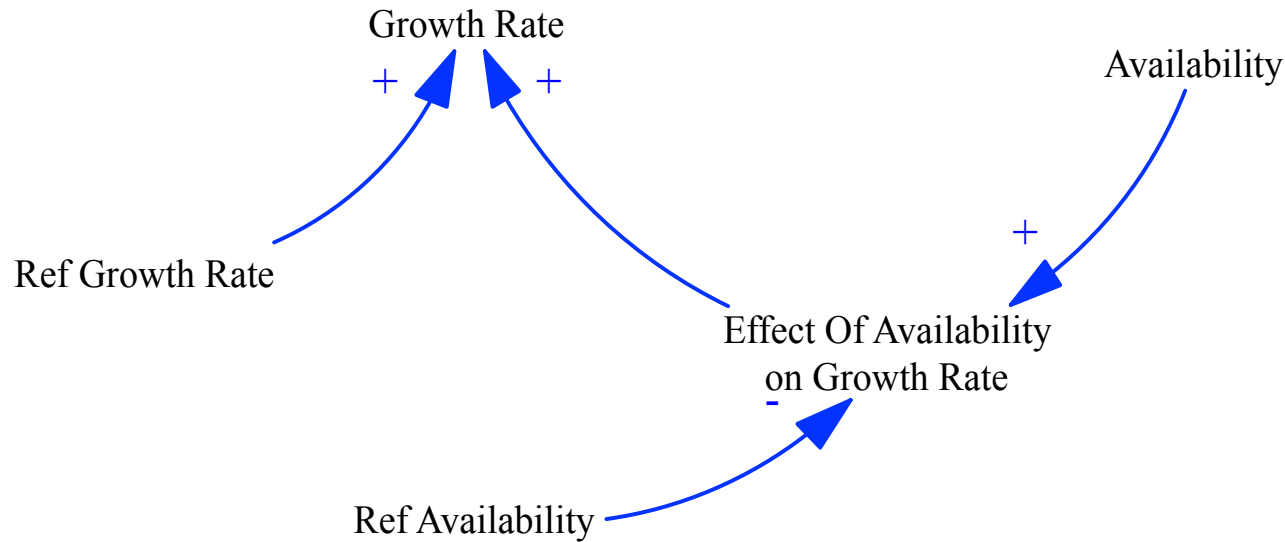
- An important building block for system dynamics modeling is to capture how variables influence one another over time.
- While some of these may be simple linear relationships, the reality is that real-world effects between variables can also be non-linear, and may also involve multiple variables.
- System dynamics offers a convenient structure for modeling effect variables (Sterman 2000).

$$Y = Y^* \times Effect(X_1 \text{ on } Y) \times \dots \times Effect(X_n \text{ on } Y) \quad (4-1)$$

$$Effect(X_i \text{ on } Y) = X_i / X_i^* \quad (4-2)$$

- There is a variable Y that is the dependent variable of a causal relationship, and this depends on a set of n independent variables (X_1, X_2, \dots, X_n)
- The variable Y has a reference value Y^* , and this is multiplied by a sequence of *effect functions* that are calculated based on the normalized ratio of (X_i/X_i^*) , where X_i^* is the reference value, and X_i is the actual value.
- The effect function (y-axis) has the normalized ratio (X/X^*) on its x-axis, and always contains the point (1,1) although the function itself can be either linear or non-linear around this point.
- This point (1,1) is important for the following reason: if X equals its reference value X^* , then the effect function will be 1, and therefore Y will then equal its reference value Y^* (from equation 4-1).

Growth Rate Example



$$\begin{aligned} \text{Growth Rate} &= \text{Ref Growth Rate} \\ &\times \text{Effect of Availability on Growth Rate} \end{aligned} \quad (4-3)$$

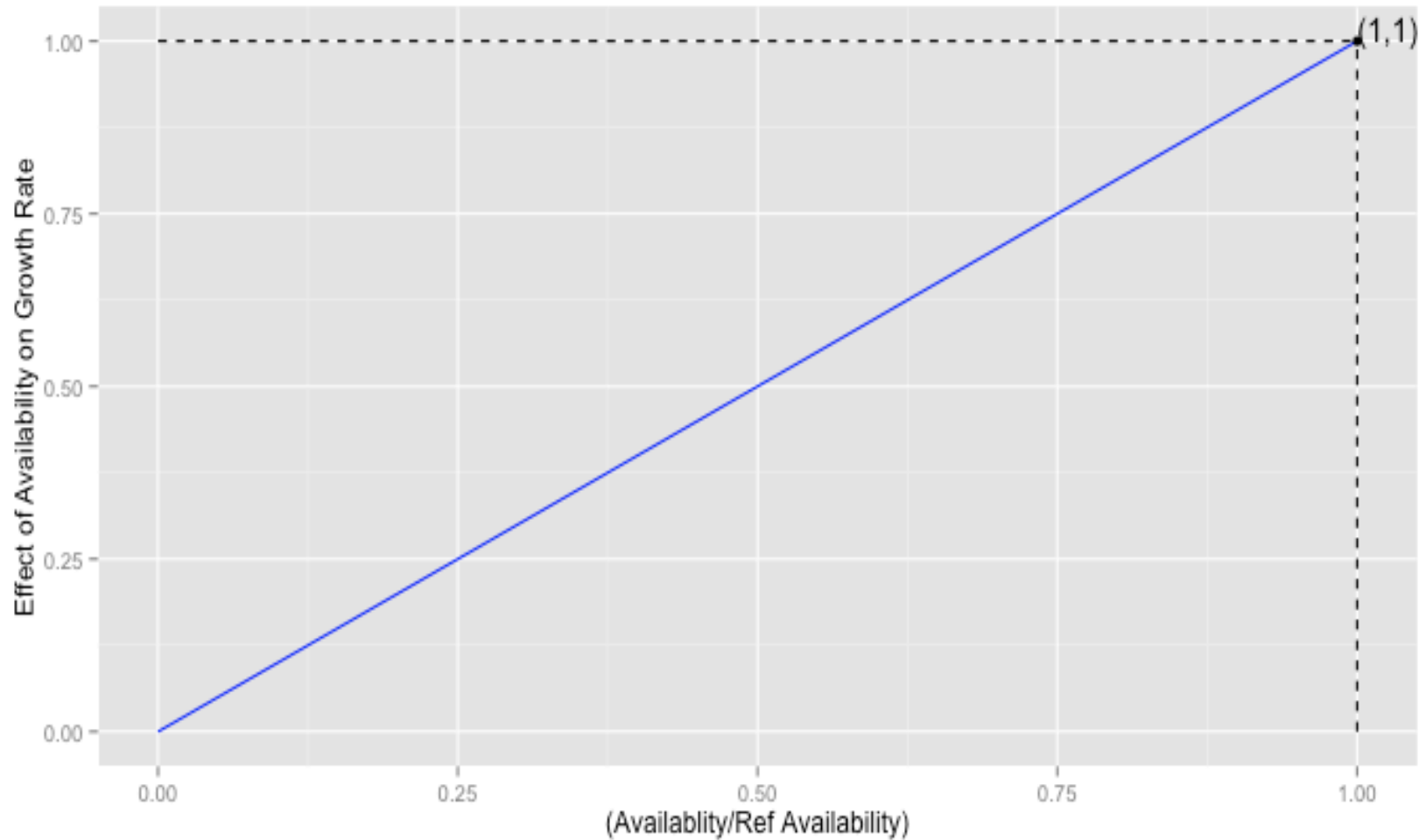
$$\text{Effect of Availability on Growth Rate} = f\left(\frac{\text{Availability}}{\text{Ref Availability}}\right) \quad (4-4)$$

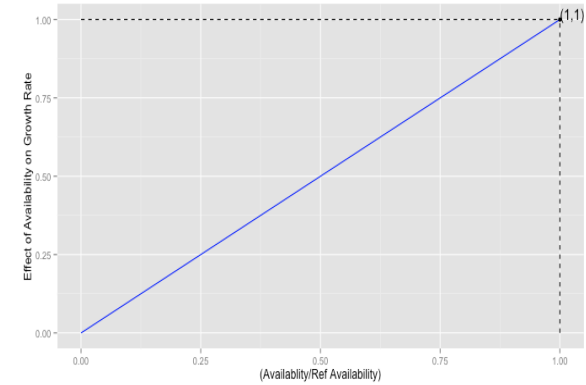
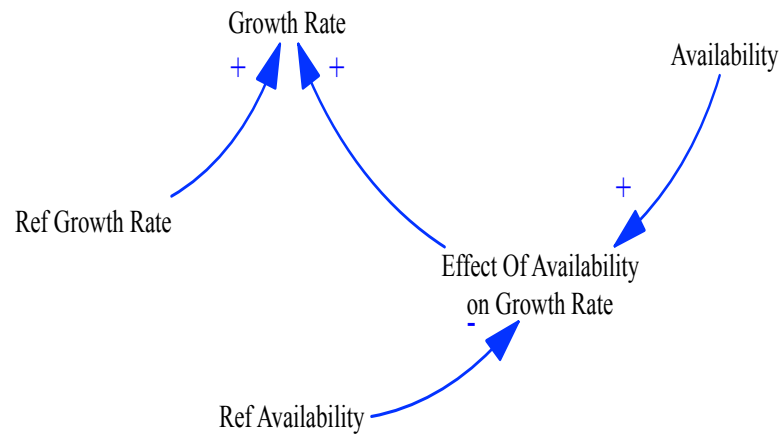
$$\text{Ref Growth Rate} = 0.10 \quad (4-5)$$

$$\text{Ref Availability} = 1.0 \quad (4-6)$$

The system grows at a rate of 10% if the availability stays at 1.0

Effect equation $y = mx + c$

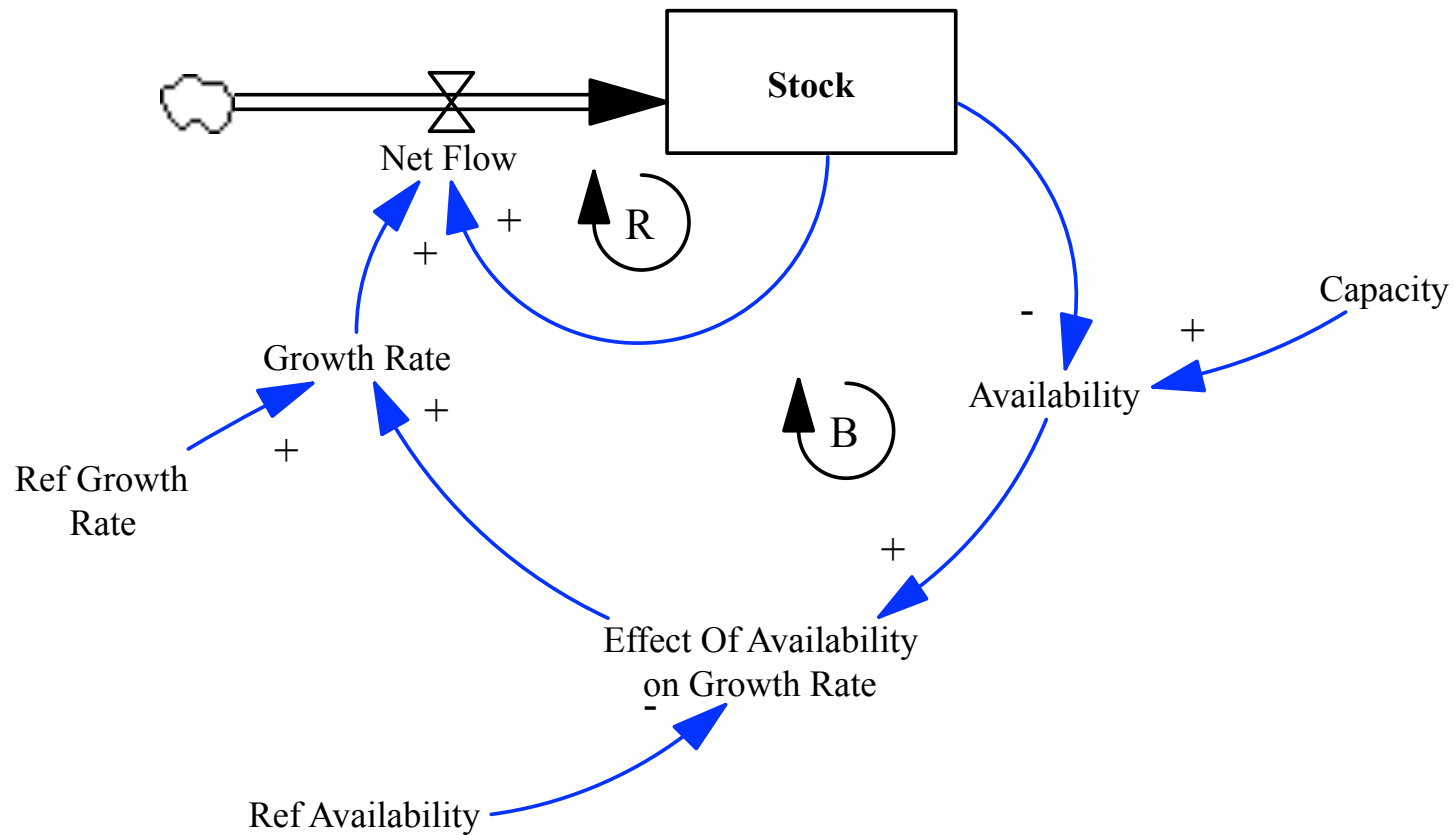




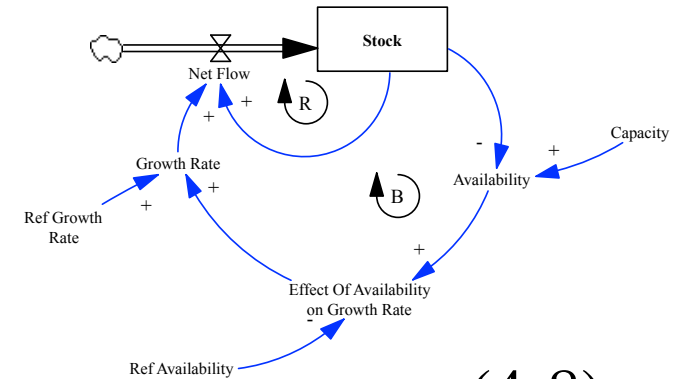
$$\text{Effect of Availability on Growth Rate} = \frac{\text{Availability}}{\text{Ref Availability}} \quad (4-7)$$

<i>Ref Availability</i>	<i>Availability</i>	<i>Effect of Availability on Growth Rate</i>	<i>Ref Growth Rate</i>	<i>Growth Rate</i>
1.0	1.0	1.0	0.10	0.10
1.0	0.5	0.5	0.10	0.05
1.0	0.0	0.0	0.10	0.00

Model of S-Shaped Growth



Equations



$$Stock = INTEGRAL(Net\ Flow, 100) \quad (4-8)$$

$$Net\ Flow = Stock \times Growth\ Rate \quad (4-9)$$

$$Availability = 1 - \frac{Stock}{Capacity} \quad (4-10)$$

$$Capacity = 10000 \quad (4-11)$$

$$Effect\ of\ Availability\ on\ Growth\ Rate = \frac{Availability}{Ref\ Availability} \quad (4-7)$$

$$Growth\ Rate = Ref\ Growth\ Rate \quad (4-3)$$

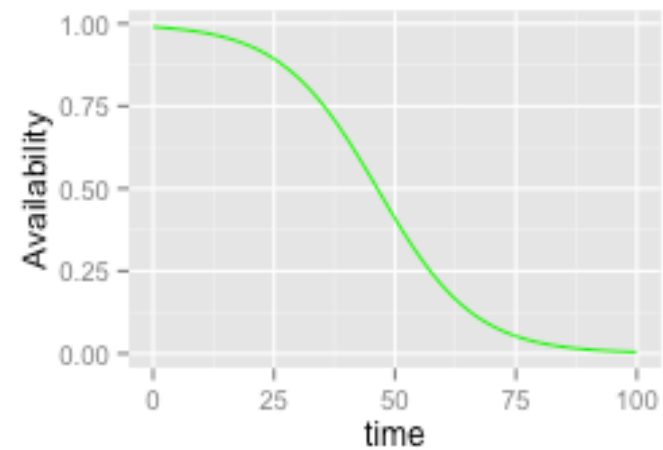
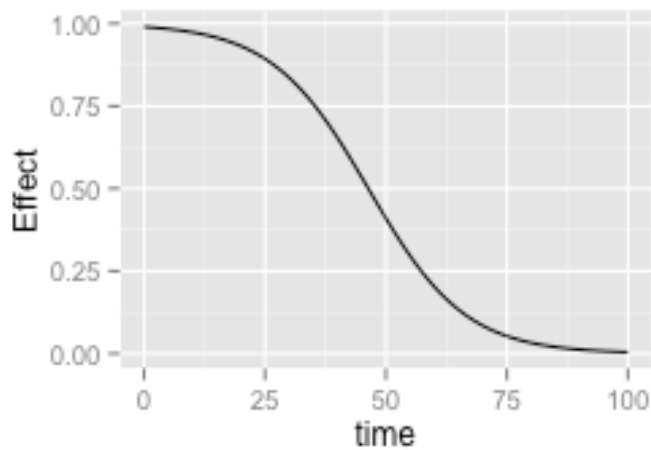
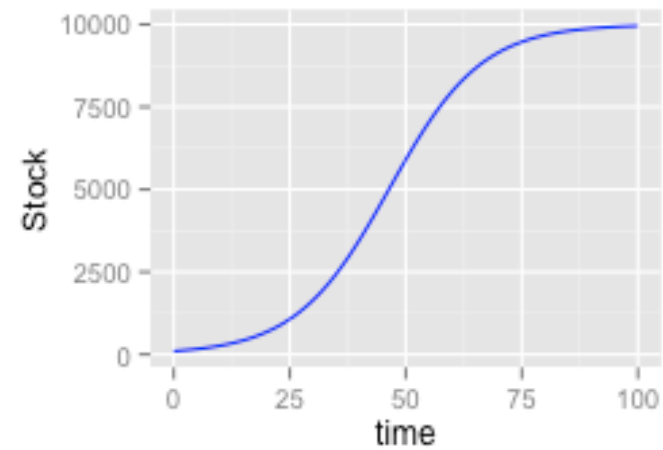
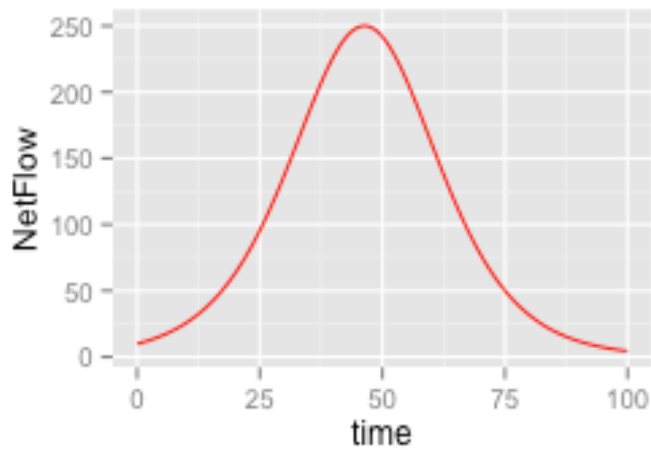
$$\times Effect\ of\ Availability\ on\ Growth\ Rate$$

$$Effect\ of\ Availability\ on\ Growth\ Rate = f\left(\frac{Availability}{Ref\ Availability}\right) \quad (4-4)$$

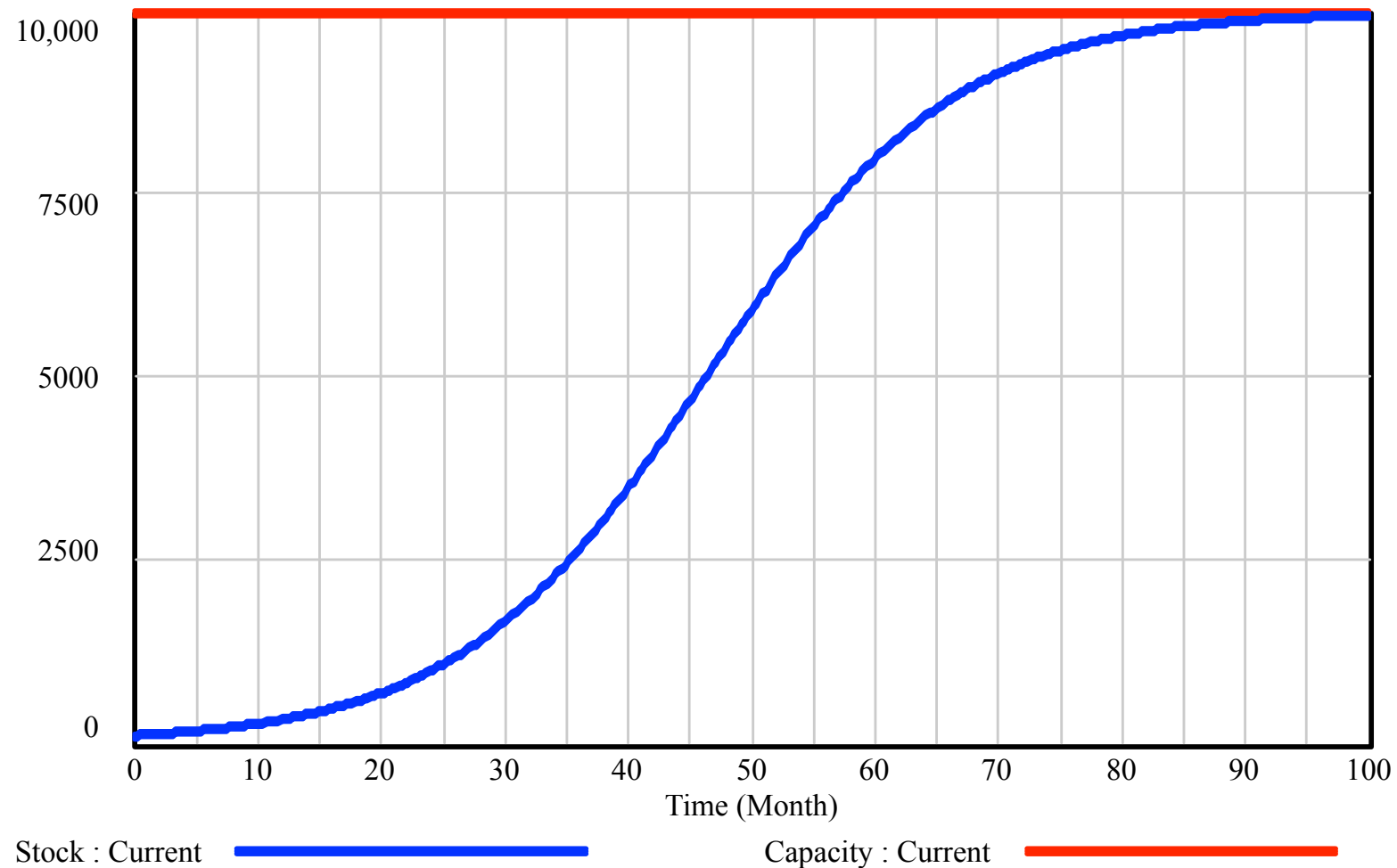
$$Ref\ Growth\ Rate = 0.10 \quad (4-5)$$

$$Ref\ Availability = 1.0 \quad (4-6)$$

Simulation Output



Limit constant, stock approaches



Verhulst Equations (1838)

In ecology: modeling population growth [\[edit \]](#)

A typical application of the logistic equation is a common model of [population growth](#), originally due to [Pierre-François Verhulst](#) in 1838, where the rate of reproduction is proportional to both the existing population and the amount of available resources, all else being equal. The Verhulst equation was published after Verhulst had read [Thomas Malthus' *An Essay on the Principle of Population*](#). Verhulst derived his logistic equation to describe the self-limiting growth of a [biological](#) population. The equation was rediscovered in 1911 by [A. G. McKendrick](#) for the growth of bacteria in broth and experimentally tested using a technique for nonlinear parameter estimation.^[4] The equation is also sometimes called the *Verhulst-Pearl equation* following its rediscovery in 1920 by [Raymond Pearl](#) (1879–1940) and [Lowell Reed](#) (1888–1966) of the [Johns Hopkins University](#).^[5] Another scientist, [Alfred J. Lotka](#) derived the equation again in 1925, calling it the *law of population growth*.

Letting P represent population size (N is often used in ecology instead) and t represent time, this model is formalized by the [differential equation](#):

$$\frac{dP}{dt} = rP \cdot \left(1 - \frac{P}{K}\right)$$

where the constant r defines the growth rate and K is the [carrying capacity](#).



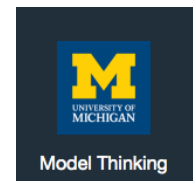
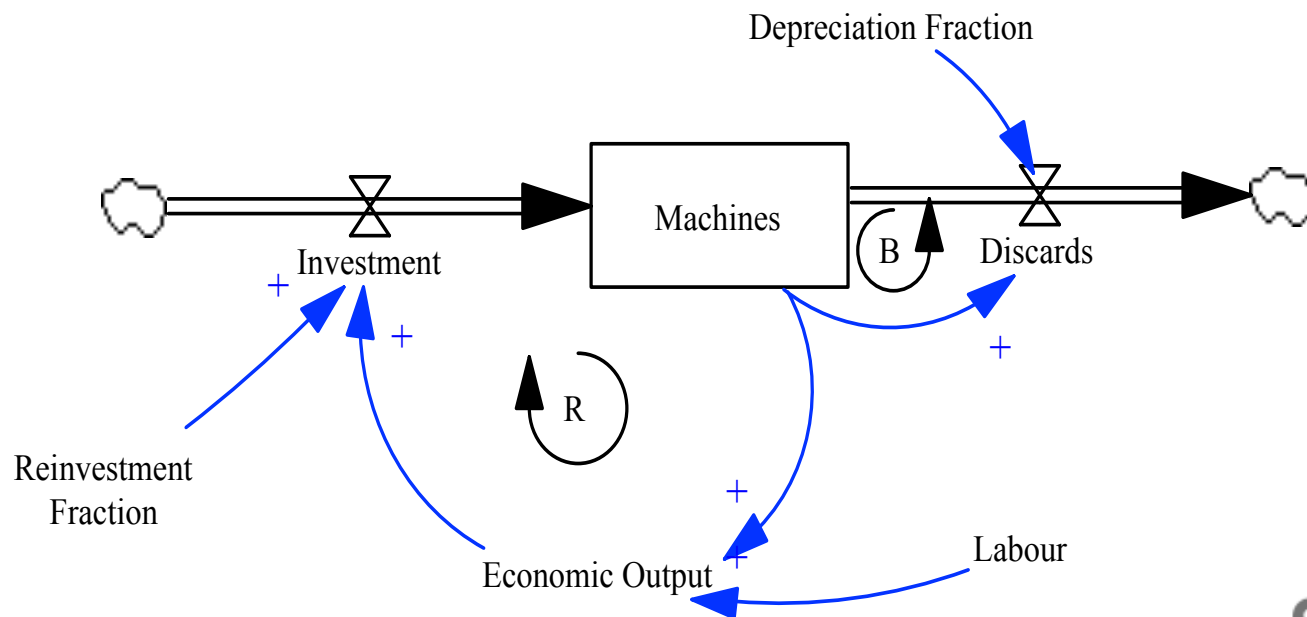
https://en.wikipedia.org/wiki/Logistic_function

Challenge 8.1

- Consider a software development team
- It contains:
 - Experienced Engineering
 - Rookie Engineers
- The normal productivity of experienced engineering is 200 LOC/Day, and this occurs when the Rookie percentage is 20%
- Construct an effects equation to model this, explain any assumptions made.

Model of Economic Growth

- In presenting the model, Page (2015) imagines a scenario where a self-contained civilization supports itself through harvesting coconuts using machines.
- A percentage of these resources can be reinvested to produce more machines, and therefore increase the economic output.



Equations

$$\text{Machines } (M) = \text{INTEGRAL}(\text{Investment} - \text{Discards}, 100) \quad (4-12)$$

$$\text{Investment} = \text{Economic Output} \times \text{Reinvestment Fraction} \quad (4-13)$$

$$\text{Discards} = \text{Machines} \times \text{Depreciation Fraction} \quad (4-14)$$

$$\text{Reinvestment Fraction } (R) = 0.20 \quad (4-15)$$

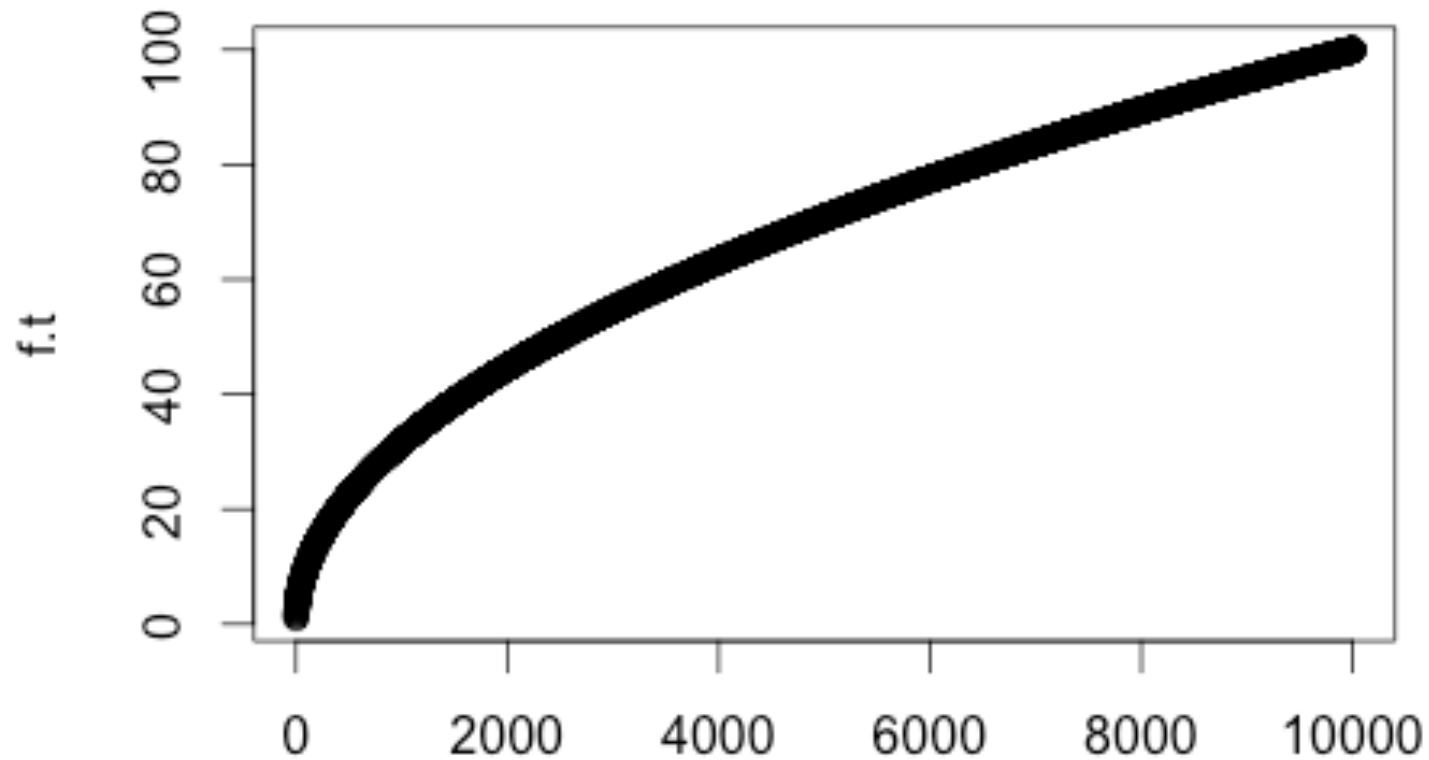
$$\text{Depreciation Fraction } (D) = 0.10 \quad (4-16)$$

$$\text{Economic Output } (O) = \text{Labour} \times \sqrt{\text{Machines}} \quad (4-17)$$

$$\text{Labour } (L) = 100 \quad (4-18)$$

Economic output is based on the laws of diminishing returns (concave function)

Concave Function



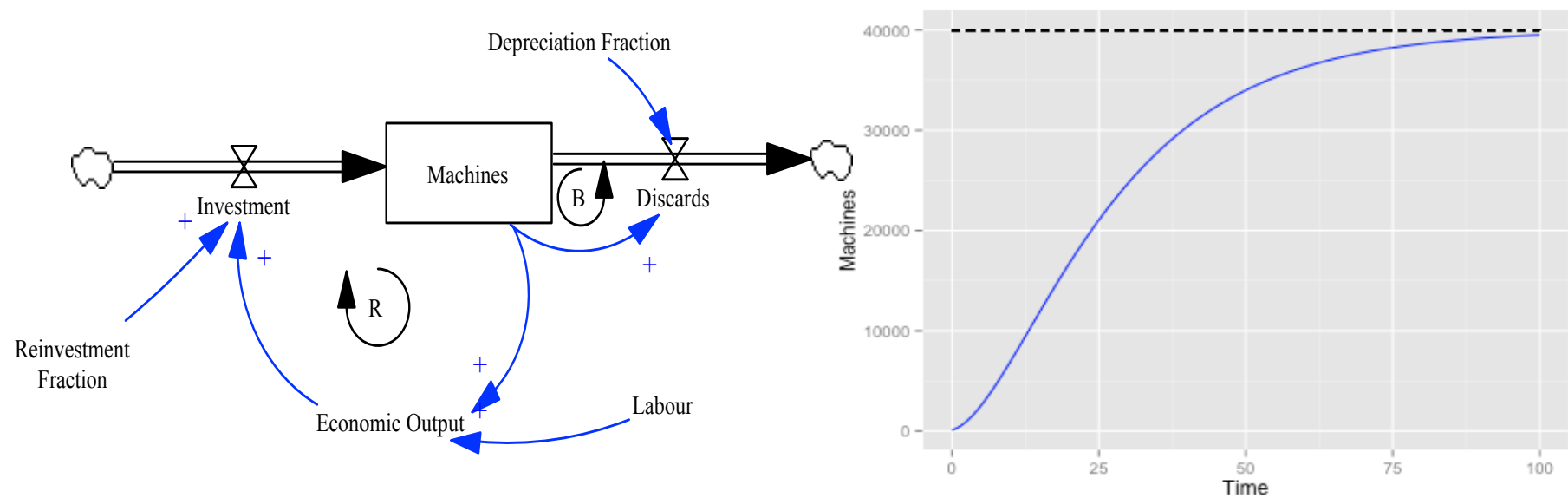
```
> t<-1:10000
```

```
> f.t<-t^0.5
```

```
> plot(f.t)
```

```
|
```


Simulation Output



Model based on...

A CONTRIBUTION TO THE THEORY OF ECONOMIC GROWTH

By ROBERT M. SOLOW

I. Introduction, 65. — II. A model of long-run growth, 66. — III. Possible growth patterns, 68. — IV. Examples, 73. — V. Behavior of interest and wage rates, 78. — VI. Extensions, 85. — VII. Qualifications, 91.

I. INTRODUCTION

All theory depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive.¹ A “crucial” assumption is one on which the conclusions do depend sensitively, and it is important that crucial assumptions be reasonably realistic. When the results of a theory seem to flow specifically from a special crucial assumption, then if the assumption is dubious, the results are suspect.

Challenge

- Find the steady state (equilibrium) value for the number of machines, given that the following values are known:
 - Reinvestment Fraction (R) = 0.20
 - Total Labour (L) = 100
 - Depreciation Fraction (D) = 0.1

