CT561: Systems Modelling & Simulation

Lecture 5: Delays and Resource Constraints

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Laboratory Work

- Starts next week, 18^{th}-19^{th} October
- Tuesday 10-11
  - 4BCT1 (Final Year Lab)
  - 3BSE1, 1CSD1, 1CSD2, 1MSME1 (IT102)
- Wednesday 9-10
  - 1MEEE1, 1MEES1 (IT102)

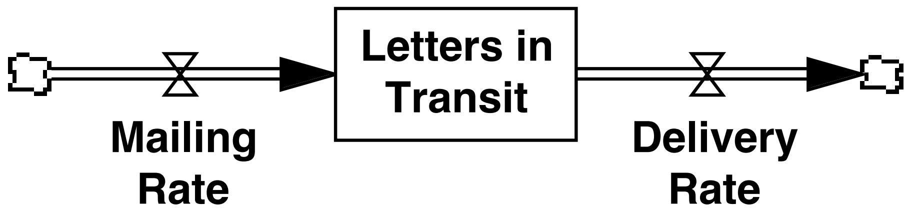
Delays

- "Delays are pervasive.
  - It takes time to measure and report information.
  - It takes time to make decisions.
  - It takes time for decisions to affect the state of the system" (Sterman 2000)
- We need to use delays in many of our models

The output of a delay lags behind the input:

General structure of a material delay:

The post office as a delay:



Delay Distributions

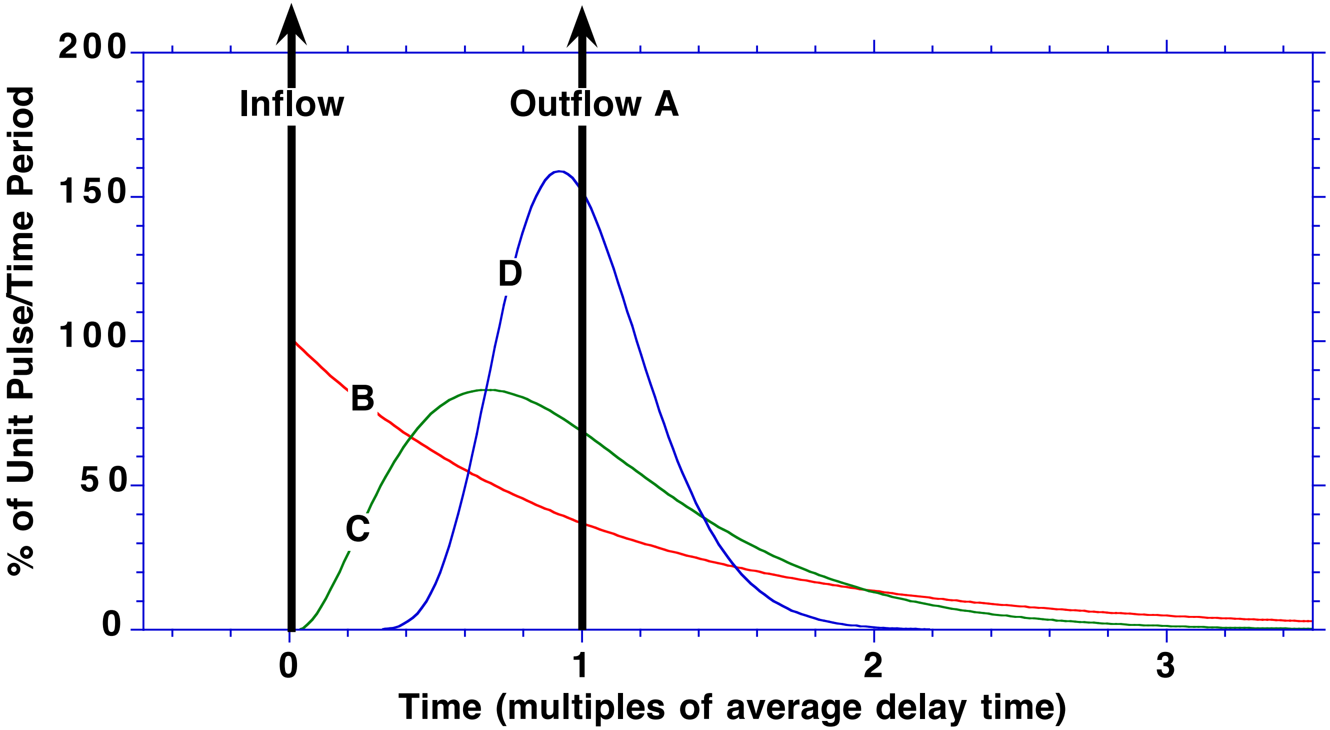
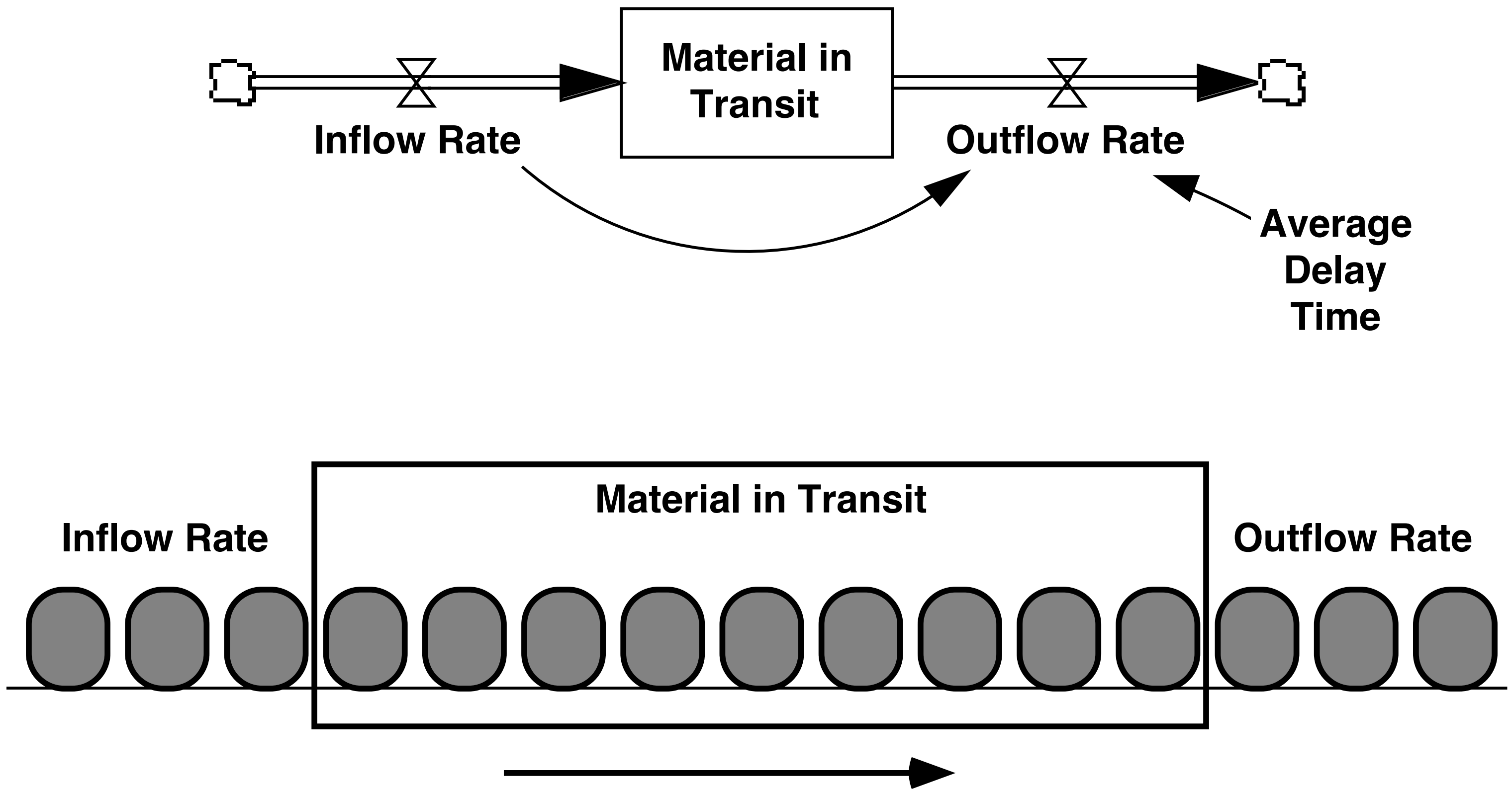


Figure 11-2 Some distributions of the outflow from a delay

The input I all cases is a unit pulse at time zero. Outflow A is a pipeline delay in which all items arrive together exactly 1 delay time after they enter. Outflow distributions B-D exhibit different degrees of variation in processing times for individual items so some arrive before and some after the average delay time. In all cases the average delay time is the same and the areas under each distribution are equal.



Material in Transit(t) = INTEGTRAL(Inflow(t) – Outflow(t), Material in Transit(0))

Outflow(t) = Inflow(t – Average Delay Time)

Figure 11-3 Pipeline delay: structure

In a pipeline delay individual items exit the delay in the same order and after exactly the same time, like widgets moving down an assembly line at a constant speed.

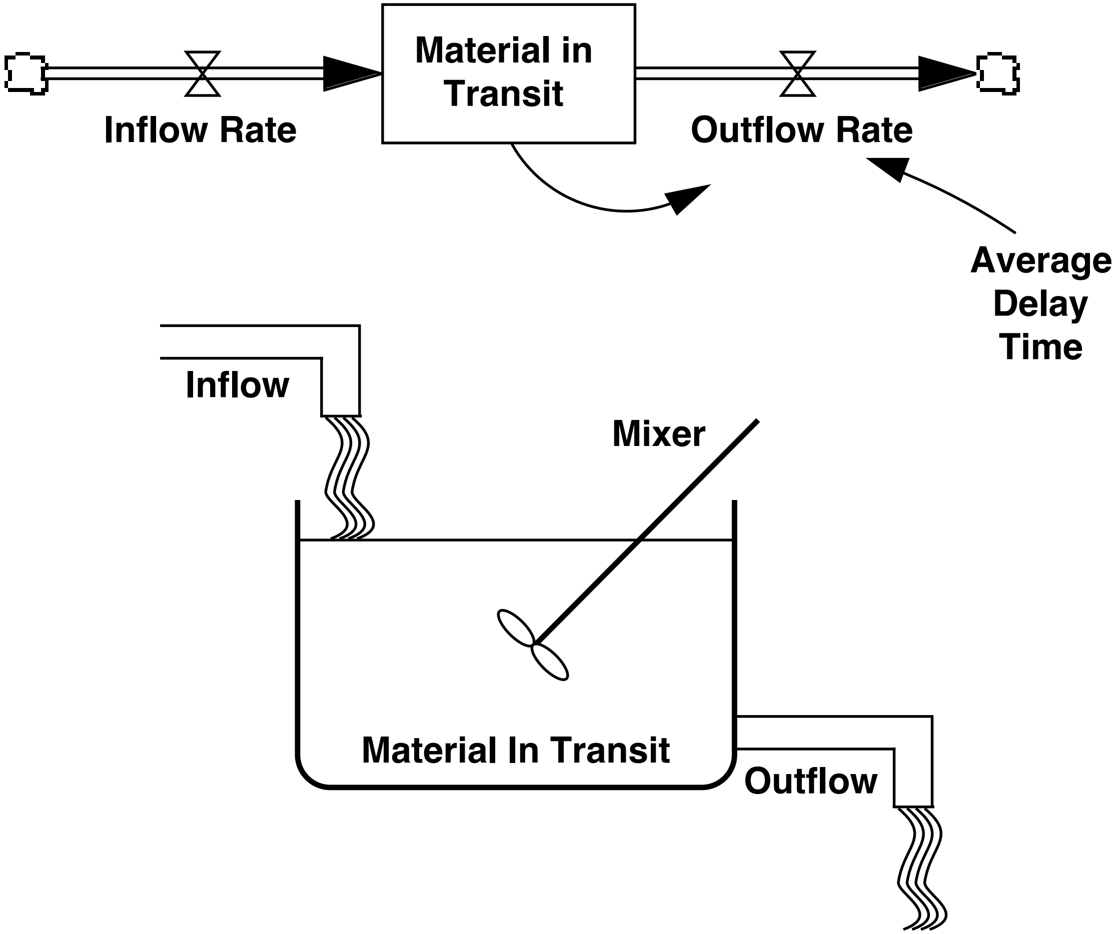


Figure 11-4 First-order material delay: structure

The outflow is proportional to the stock of material in transit. The contents of the stock are perfectly mixed at all times, so all items in the stock have the same probability of exit, independent of their arrival time.

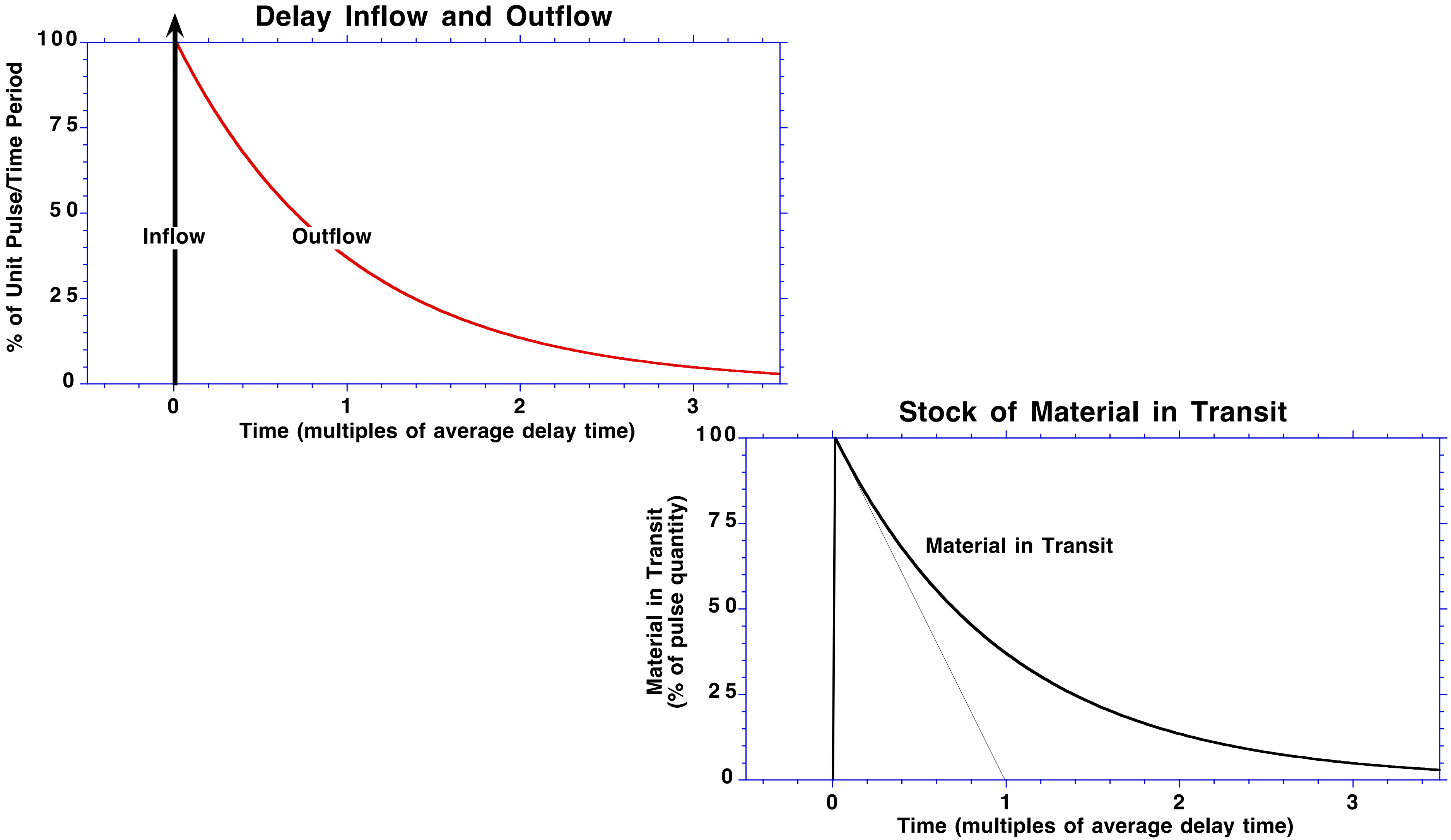
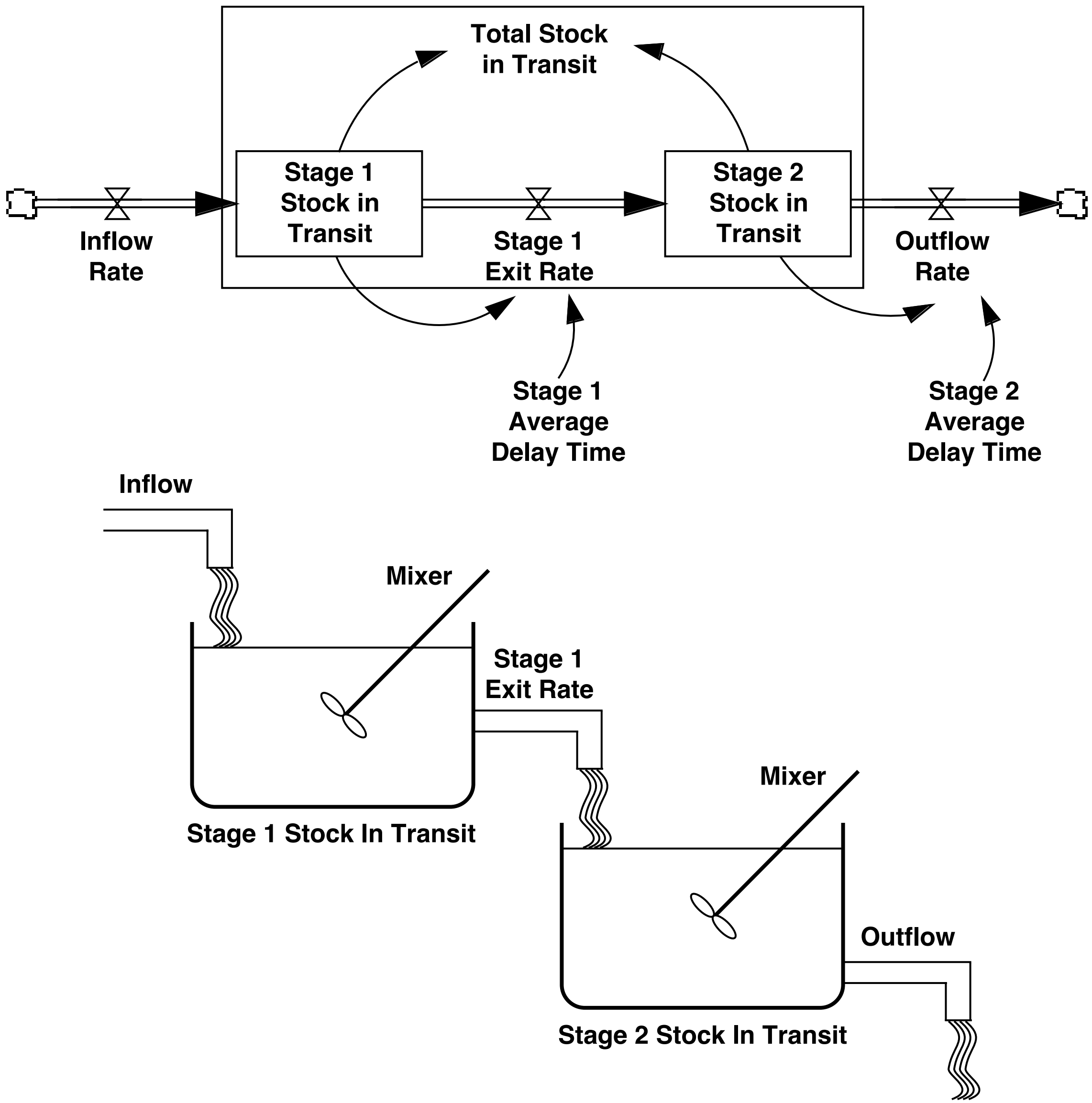


Figure 11-5 Pulse response of first-order material delay

The input to the delay is a unit pulse at time zero. The stock of material in transit instantly jumps to 100%, then decays exponentially with a time constant equal to the average delay time.

Figure 11-6 Higher-order delays are formed by cascading first-order delays together.



Stage 1 Exit Rate = Stage 1 Stock in Transit/Stage 1 Average Delay Time

Outflow Rate = Stage 2 Stock in Transit/Stage 2 Average Delay Time

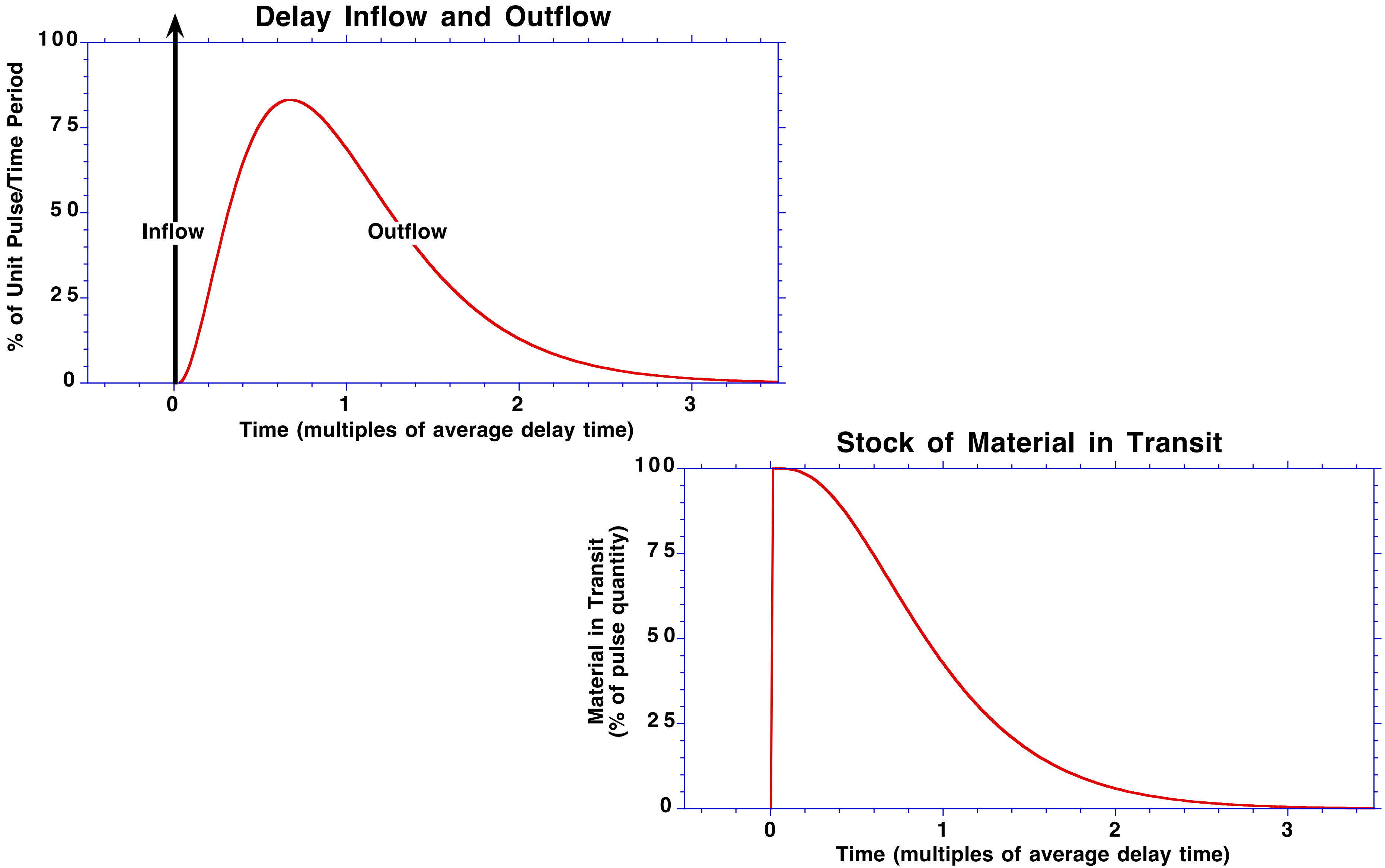
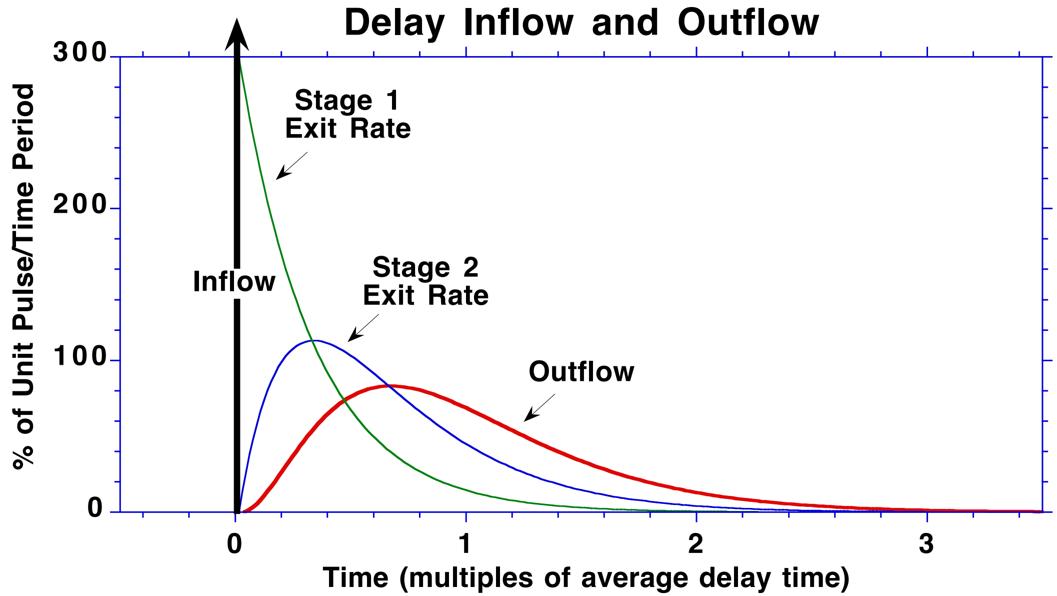


Figure 11-7 Pulse response of a third-order delay



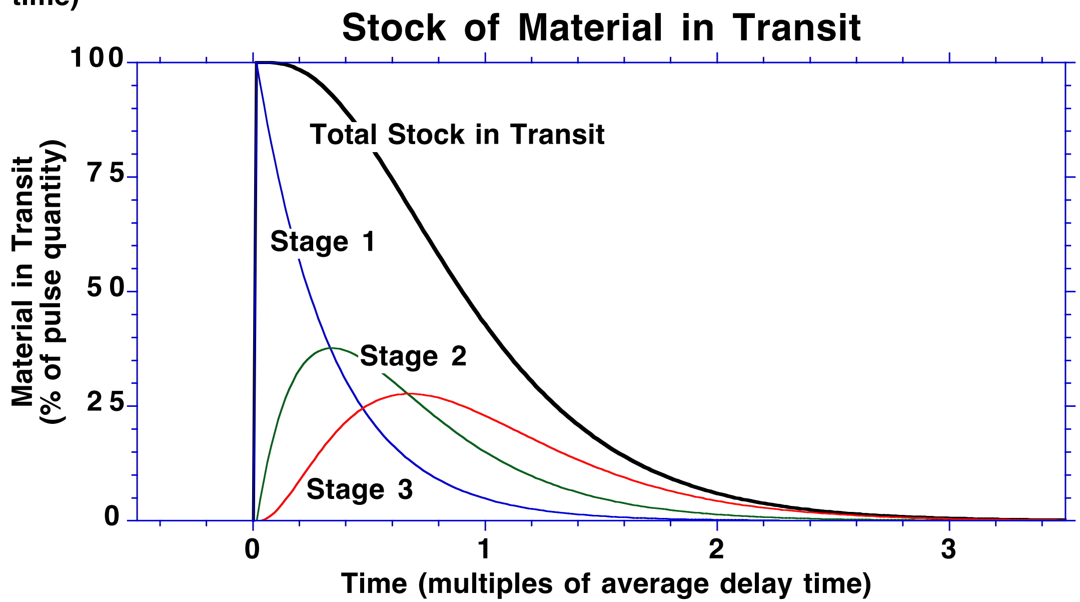
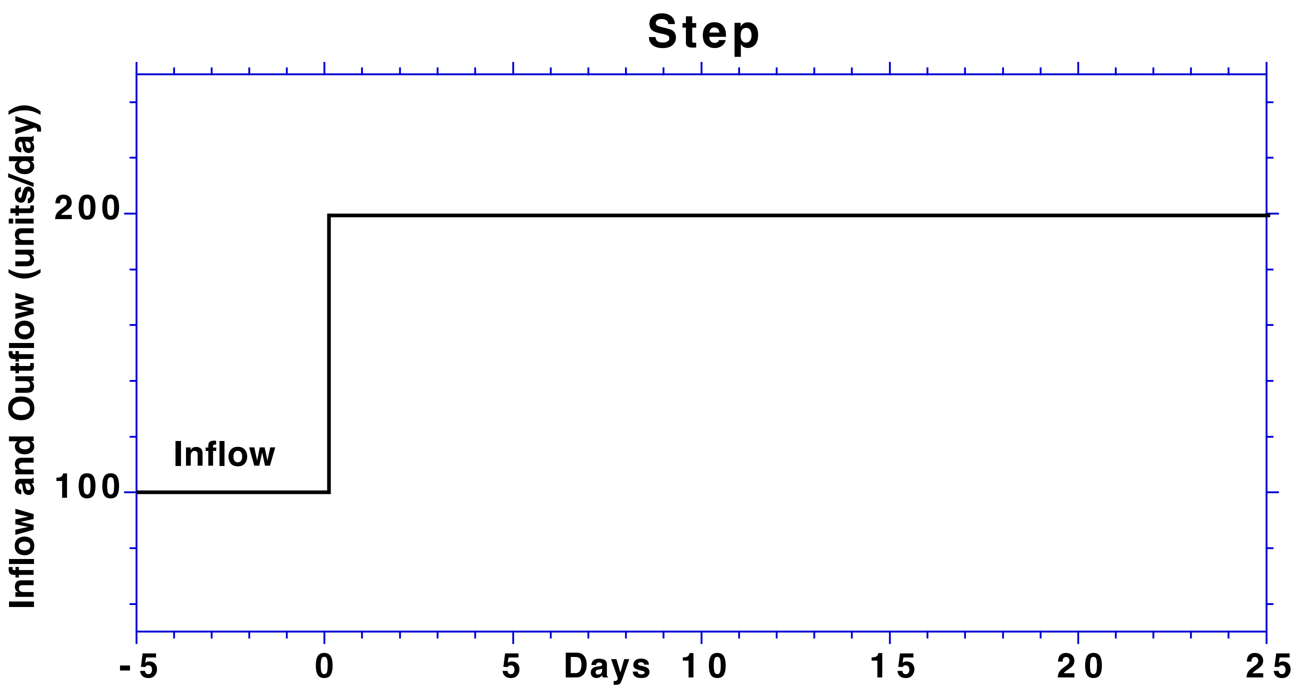


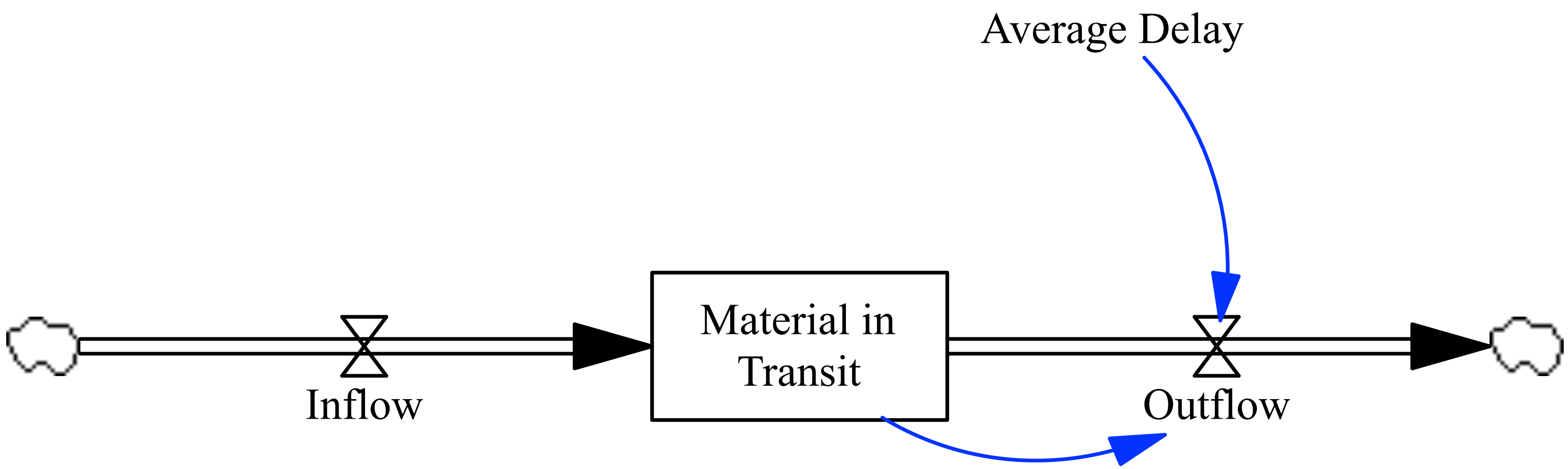
Figure 11-8 Pulse response of third-order delay by stage of processing

Challenge 5.1

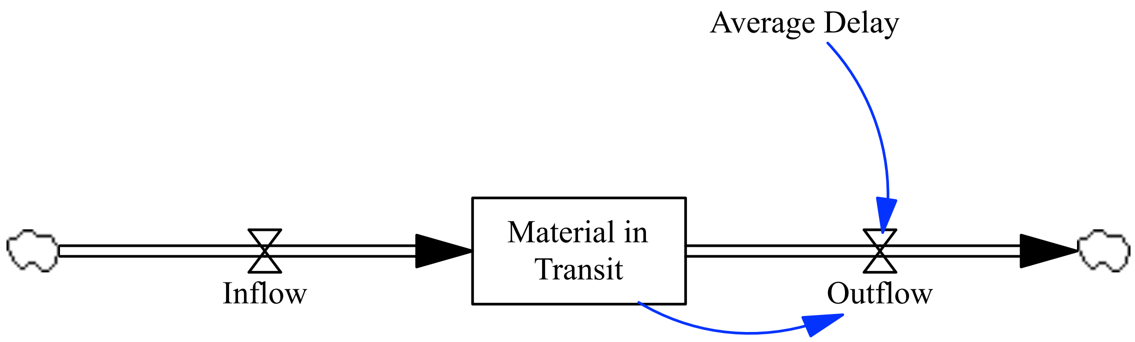
- Sketch the response of a first order delay to each of this input, assuming a 5 day average delay time



Solution – Step Function



Equations



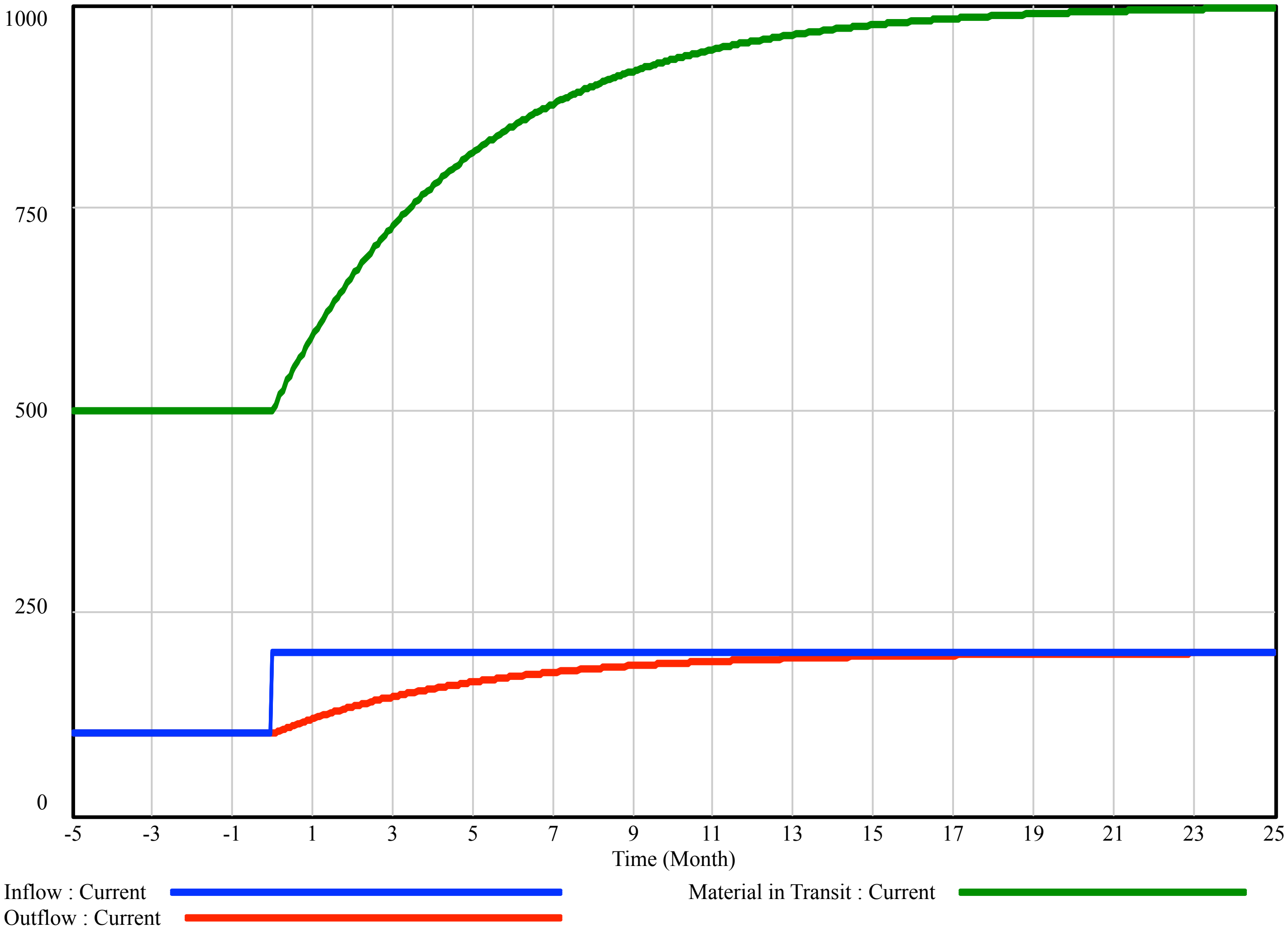
Average Delay = 5

Inflow = 100 + step (100, 0)

Material in Transit = INTEG(Inflow - Outflow , 500)

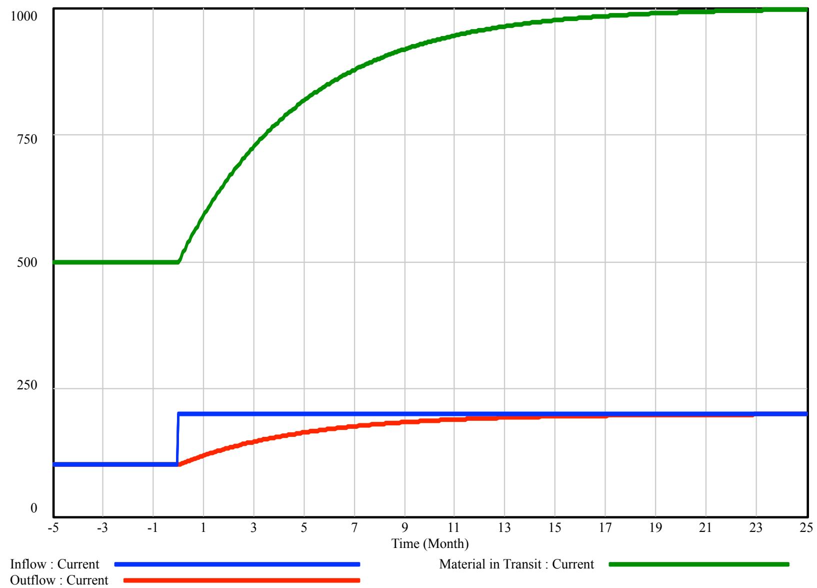
Outflow = Material in Transit / Average Delay

System Behaviour



Interesting relationship

- Outflow, delay and Material in Transit
- In steady state (equilibrium where inflow = outflow)
  - Material in Transit = Inflow Rate * Delay
- **Little's Law:**
  - "The stock in transit is fully characterised by the average delay time and the inflow rate" (Sterman 2000)

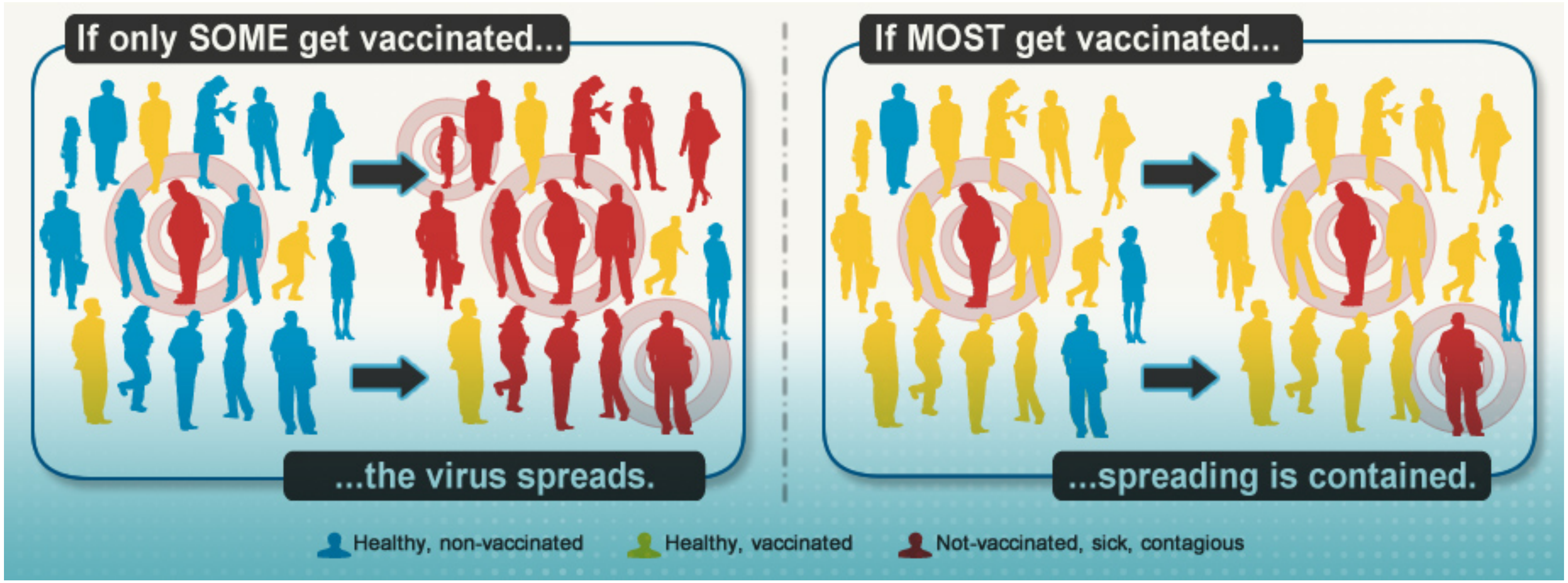


Challenge 5.2

- Build an aging chain model career progression, where software engineers have the following trajectory:
  - Graduate Engineers (24 months)
  - Software Engineers (36 months)
  - Senior Software Engineers (48 months)
  - Consulting Engineers (60 months)

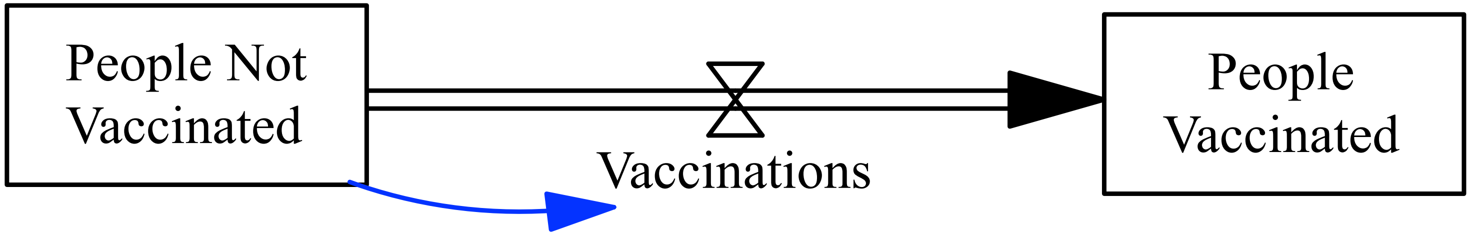
Resource Constrained Flows

- Some flows are resource constrained
- No resources, no flow
- Examples: Coders write code, Health workers administer vaccines



http://www.cdc.gov/vaccines/vac-gen/images/vaccines-protect.jpg

A simple model

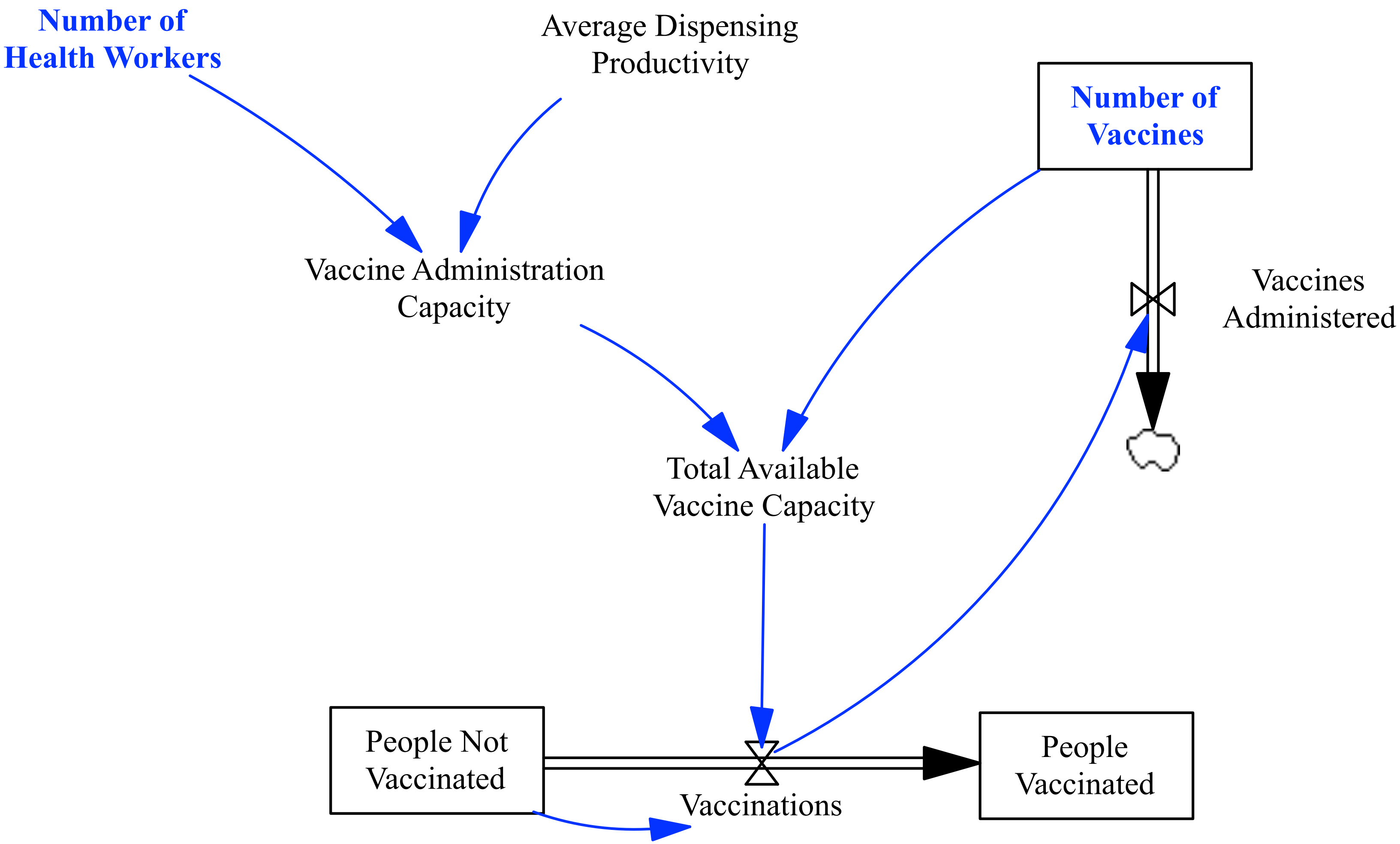


- However, in order for vaccinations to occur, two resources are needed:
  - The vaccine
  - The health worker to administer the vaccine

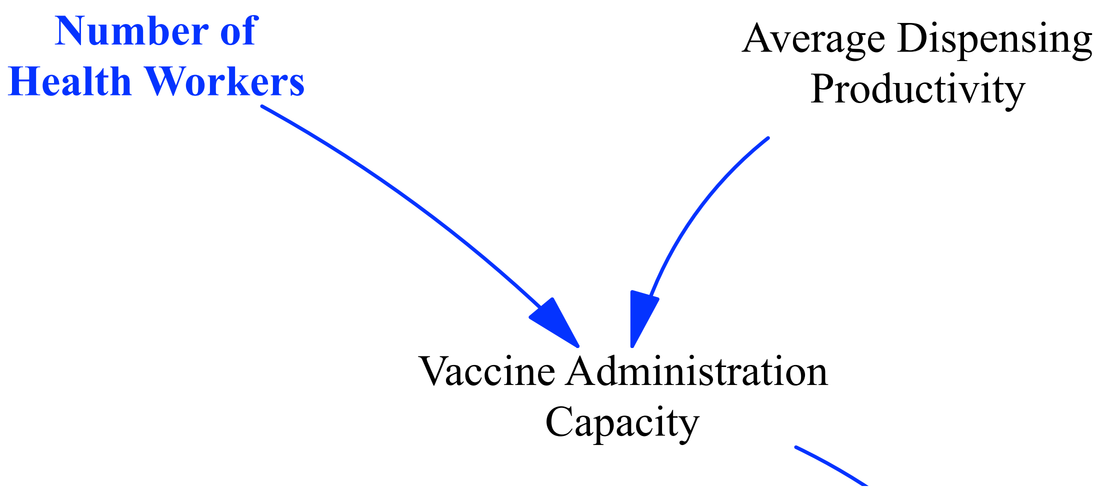
Overall resource constraining logic

Population Demand	Health Workers	Vaccines	Vaccines Administered
1,000	200	0	0
1,000	0	1,000	0
1,000	200	500	500
1,000	200	1,000	1,000

The stock and flow model



Vaccine Administration Capacity

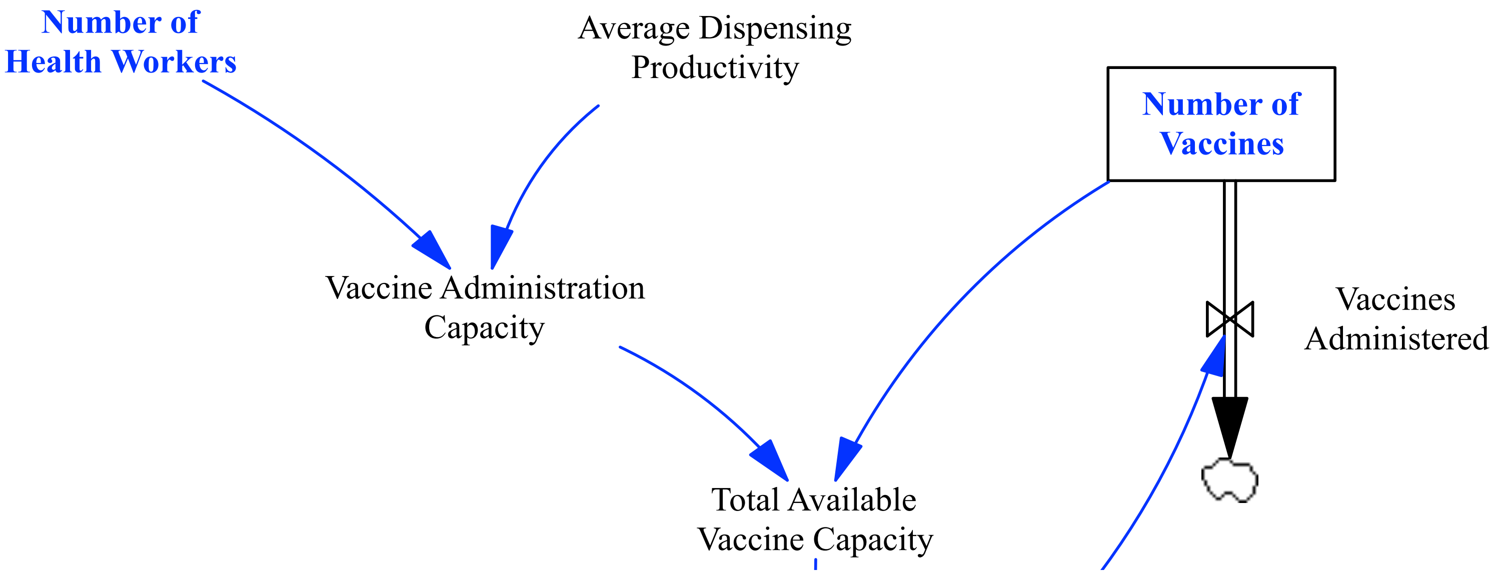


Vaccine Administration Capacity = Average Dispensing Productivity * Number of Health Workers

Average Dispensing Productivity = 200

Number of Health Workers = 100

Total available capacity

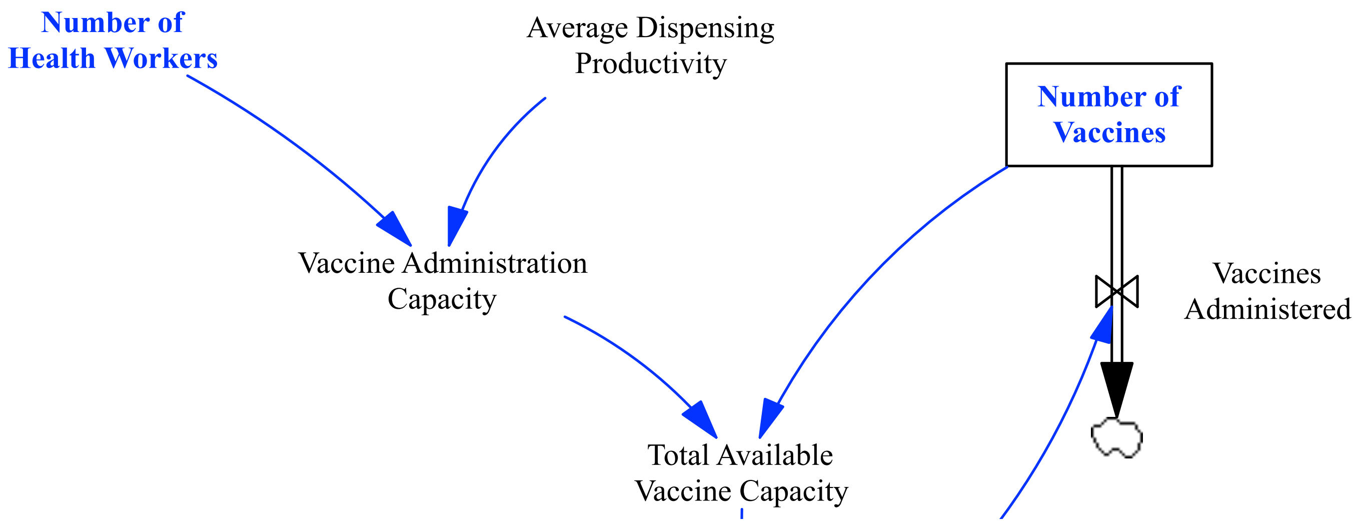


Number of Vaccines = INTEG(- Vaccines Administered , 50000)

Total Available Vaccine Capacity = min (Vaccine Administration Capacity ,
Number of Vaccines)

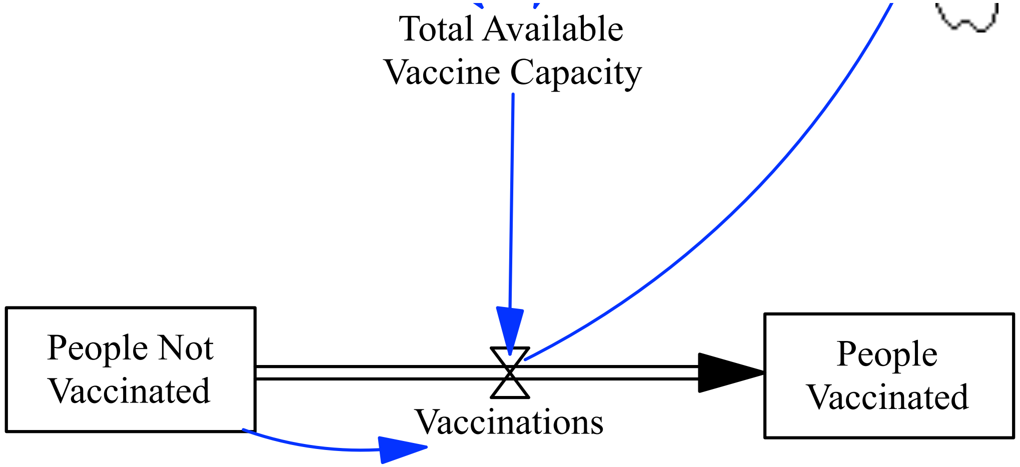
Vaccines Administered = Vaccinations

How the equations work…



Number of Health Workers	Average Dispensing Productivity	Vaccine Administrative Capacity	Number of Vaccines	Total Available Vaccine Capacity
100	200	20,000	40,000	20,000
0	200	0	100,000	0

Final flow equation… includes non-negative check for stock

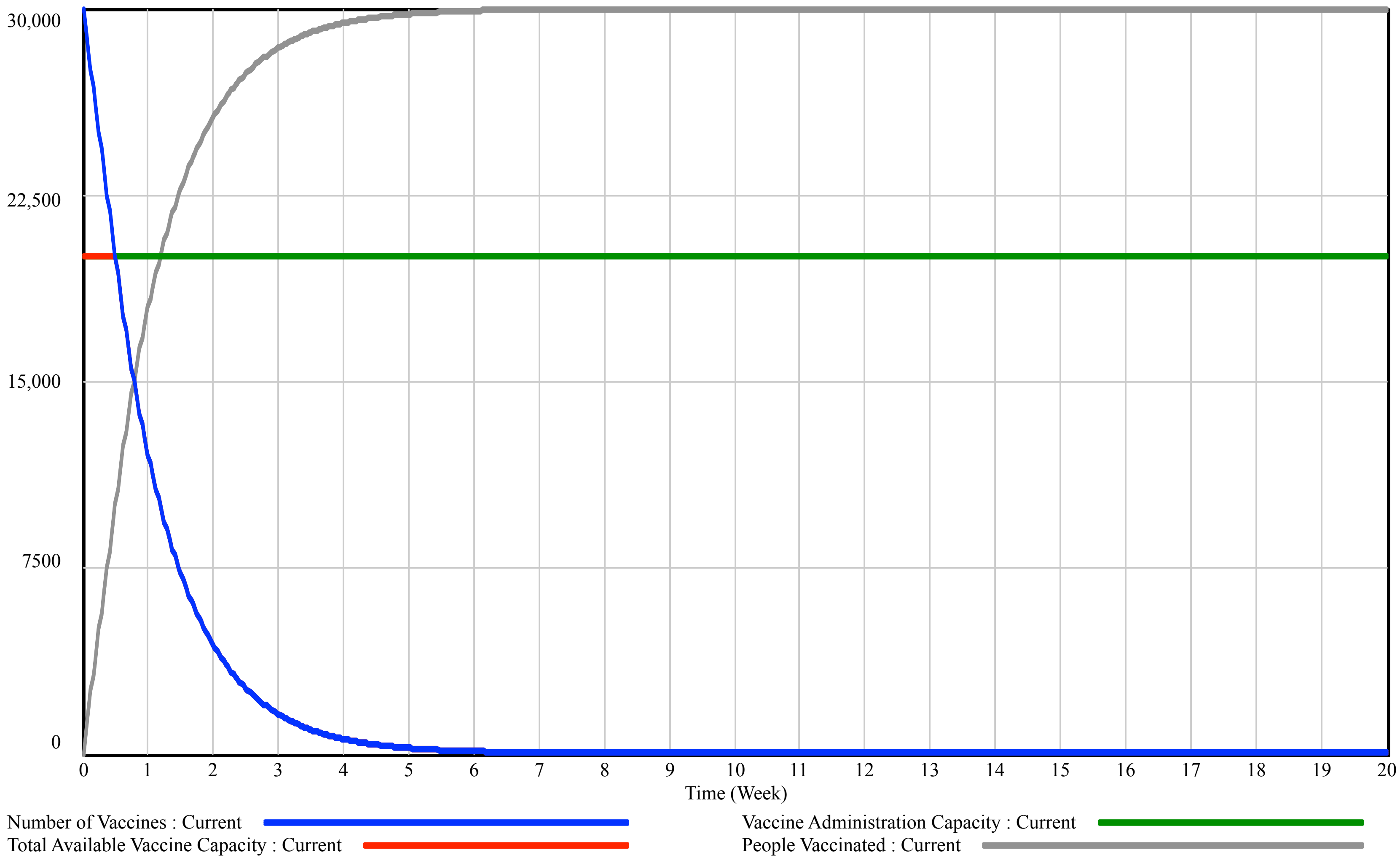


People Not Vaccinated = INTEG(- Vaccinations , 100000)

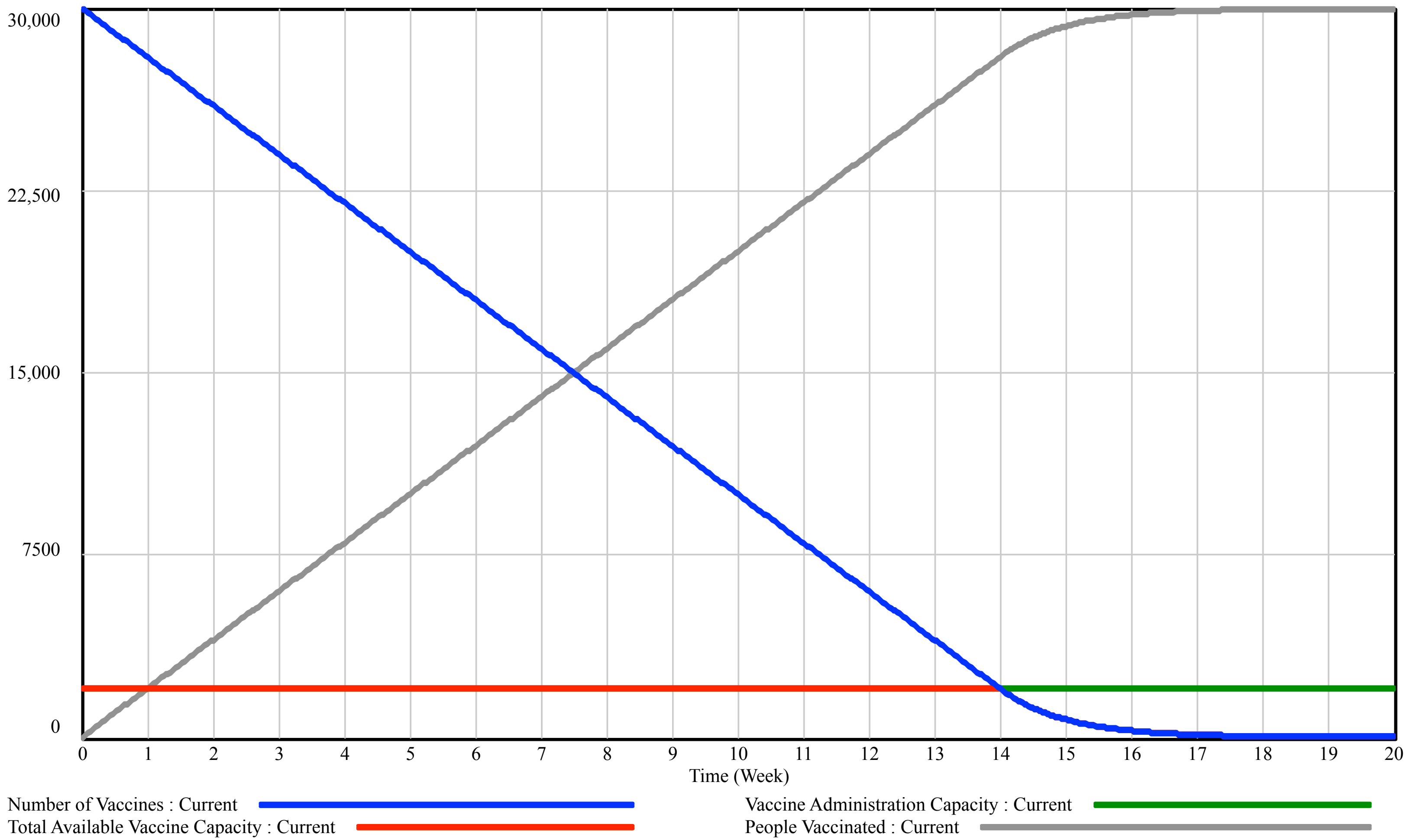
People Vaccinated = INTEG(Vaccinations , 0)

Vaccinations = min (People Not Vaccinated , Total Available Vaccine Capacity)

Vaccines = 30,000



Health Workers @ 10% Availability



Challenge 5.3

- Extend the Vaccine model in a number of ways
  – Introduce a vaccine supply chain, with a third order delay before vaccines are distributed, with the average transit time delay of 6 weeks
  – Introduce a stock and flow model for health workers, where they can become sick at a rate of 10% per week, and then recover after an average of two weeks.