CT561: Systems Modelling and Simulation

Week 11: Sensitivity Analysis and Statistical Screening in R

https://github.com/JimDuggan/CT561

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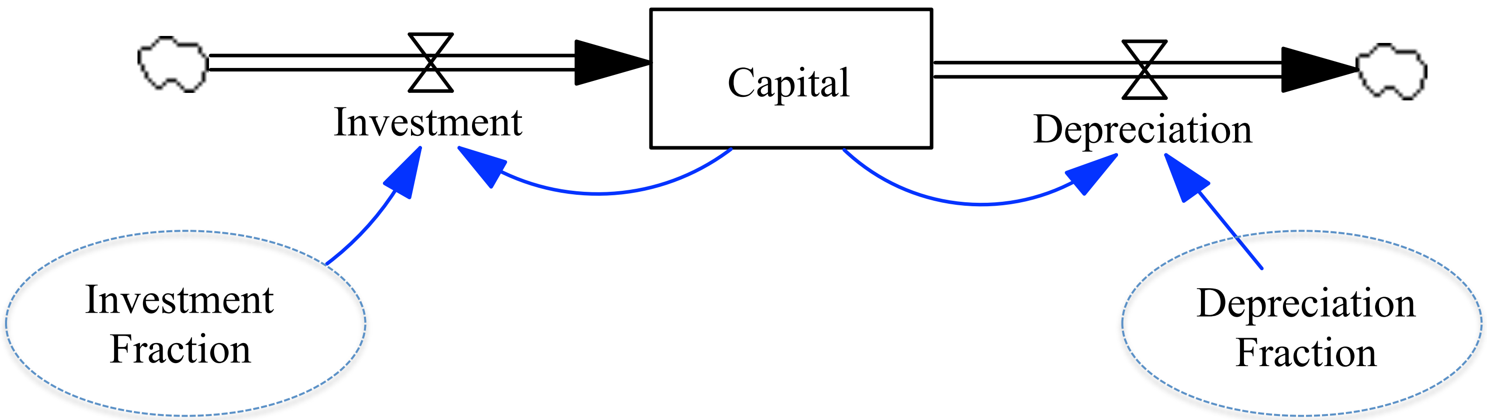
Sensitivity Analysis Process

- **Precondition:** Model equations finalised.
- Identify Uncertain Parameters, their distribution, and range of values
- Decide on number of runs (NRUNS)
- Let n = 1
- While n <= NRUNS
  - Generate Parameter Values (e.g. uniform distribution)
  - Run Simulation and store results
  - Let n =n + 1
- Plot results and analyse

Capital Model:
Experiment by varying the parameters



Capital = INTEG(Investment - Depreciation , 100)
Depreciation = Capital * Depreciation Fraction
Depreciation Fraction = 0.02
Investment = Capital * Investment Fraction
Investment Fraction = 0.07 - step (0.06, 10)

R Code – Notice the parameter ranges (Min and Max)

```r
library(deSolve)
library(ggplot2)
library(scales)

START<-2015; FINISH<-2035; STEP<-0.25;

simtime <- seq(START, FINISH, by=STEP)

INV_FRACT_MIN <-0.03; INV_FRACT_MAX <-0.12
DEP_FRACT_MIN <-0.01; DEP_FRACT_MAX <-0.04
INIT_CAP_MIN  <-9000; INIT_CAP_MAX  <-10100

NRUNS<-100
```

Need an outer function

```
sensRun<-function(run){
  model <- function(time, stocks, auxs)
  {...}
  stocks  <- c(sCapital=runif(1,INIT_CAP_MIN,INIT_CAP_MAX))

  # create the vector of auxiliaries
  auxs    <- c(aInvestmentFraction=
                runif(1, INV_FRACT_MIN,
                         INV_FRACT_MAX),
             aDepreciationFraction=
                runif(1,DEP_FRACT_MIN,
                        DEP_FRACT_MAX))

  o<-data.frame(ode(y=stocks, simtime, func = model,
                    parms=auxs, method="euler"))
  o$run<-run
  return (o)
}
```

The model file

```r
model <- function(time, stocks, auxs)
{
    with(as.list(c(stocks, auxs)),{

    fInvestment <- sCapital * aInvestmentFraction
    fDepreciation <- sCapital * aDepreciationFraction

    dC_dt  <- fInvestment - fDepreciation

    return (list(c(dC_dt),
                 Investment=fInvestment,
                 Depreciation=fDepreciation,
                 InvFraction=aInvestmentFraction,
                 DepFraction=aDepreciationFraction))
  })
}
```

The controlling loop

```
o1<-sensRun(1)
for(n in 2:NRUNS)
{
  o1<-rbind(o1,sensRun(n))
}
```

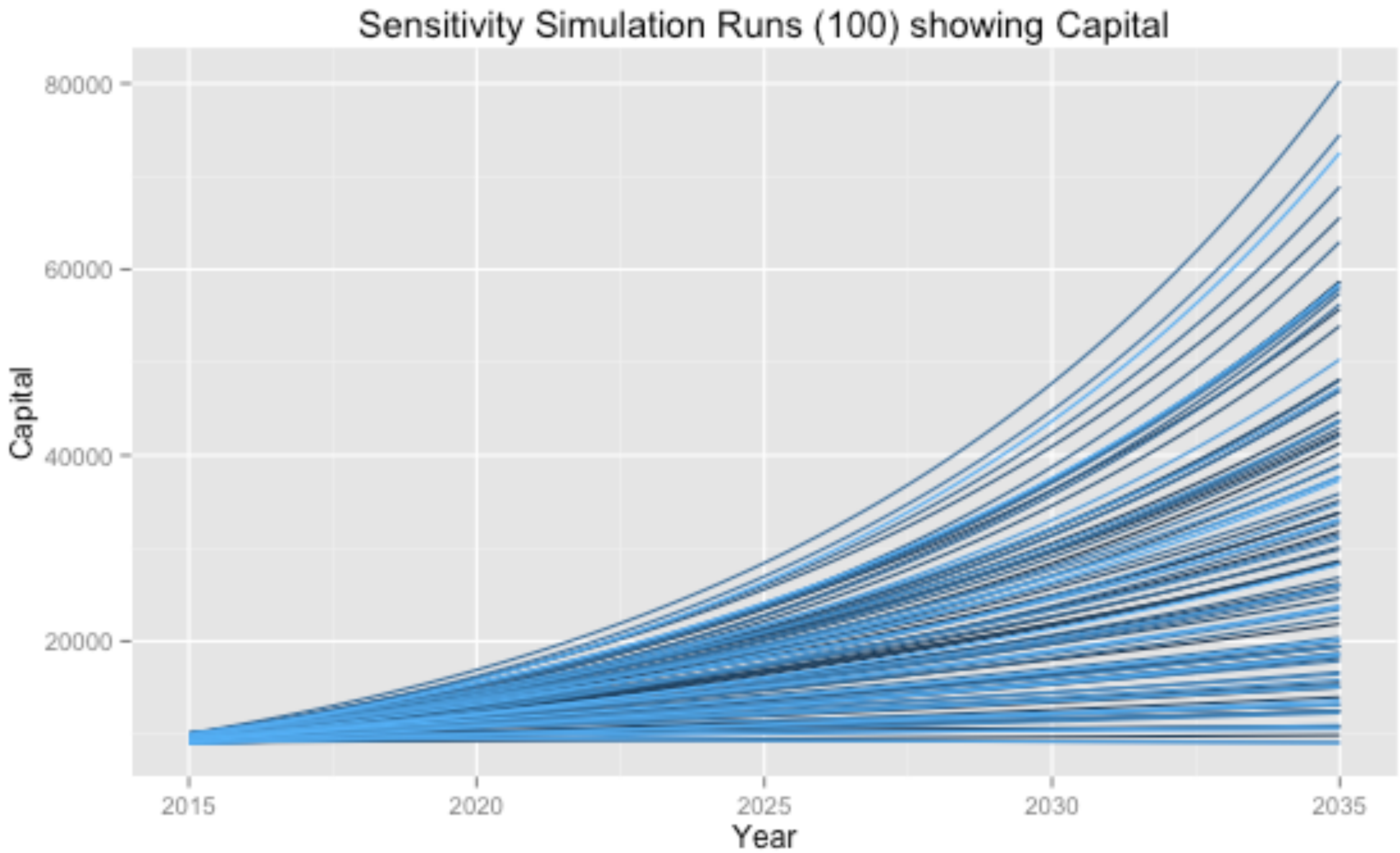
```
> dim(o1)
[1] 8100    7
> head(o1)
     time   sCapital Investment Depreciation InvFraction DepFraction run
1 2015.00  9206.398   947.8448     250.2711    0.102955  0.02718447   1
2 2015.25  9380.791   965.7995     255.0119    0.102955  0.02718447   1
3 2015.50  9558.488   984.0943     259.8425    0.102955  0.02718447   1
4 2015.75  9739.551  1002.7356     264.7646    0.102955  0.02718447   1
5 2016.00  9924.044  1021.7301     269.7799    0.102955  0.02718447   1
6 2016.25 10112.031  1041.0843     274.8902    0.102955  0.02718447   1
```

Data snapshot

```
> o1[3000:3005,]
        time sCapital Investment Depreciation InvFraction DepFraction run
3000 2015.50 10352.49   875.0018     112.8731  0.08452086  0.01090299  38
3001 2015.75 10543.03   891.1057     114.9505  0.08452086  0.01090299  38
3002 2016.00 10737.07   907.5060     117.0661  0.08452086  0.01090299  38
3003 2016.25 10934.68   924.2082     119.2206  0.08452086  0.01090299  38
3004 2016.50 11135.92   941.2178     121.4148  0.08452086  0.01090299  38
3005 2016.75 11340.87   958.5404     123.6494  0.08452086  0.01090299  38
```

```
> tail(o1)
        time sCapital Investment Depreciation InvFraction DepFraction run
8095 2033.75 26370.07   1970.675     473.7822  0.07473148  0.01796666 100
8096 2034.00 26744.30   1998.641     480.5057  0.07473148  0.01796666 100
8097 2034.25 27123.83   2027.004     487.3247  0.07473148  0.01796666 100
8098 2034.50 27508.75   2055.770     494.2404  0.07473148  0.01796666 100
8099 2034.75 27899.13   2084.944     501.2543  0.07473148  0.01796666 100
8100 2035.00 28295.06   2114.531     508.3677  0.07473148  0.01796666 100
```

Visualise Output (Stock)




```
p1<-ggplot(o1,aes(x=time,y=sCapital,color=run,group=run)) + geom_line() +
  ylab("Capital") +
  xlab("Year")  + guides(color=FALSE) +
  ggtitle("Sensitivity Simulation Runs (100) showing Capital")
```

Statistical Screening

- A pragmatic method to search for the most important of uncertain parameters.
- The goal is to learn which of the many uncertain inputs stands out as most influential

$$r = \frac{\sum(X - \bar{X})(Y - \bar{Y})}{\sqrt{[\sum(X - \bar{X})^2][\sum(Y - \bar{Y})^2]}}$$

Measures of Association
Sample Pearson Correlation Coefficient

- The quantitative measure of the strength in the linear relationship between two variables
- The values of *r* can range from a perfect positive correlation of 1, to a perfect negative correlation of -1
- Care should be taken when interpreting results, just because two variables are correlated does not guarantee a cause and effect situation
- Assumption that data follows a normal distribution

$$r = \frac{\sum(X - \bar{X})(Y - \bar{Y})}{\sqrt{[\sum(X - \bar{X})^2][\sum(Y - \bar{Y})^2]}}$$

Example (where r should be = 1)

$$r = \frac{\sum(X - \bar{X})(Y - \bar{Y})}{\sqrt{[\sum(X - \bar{X})^2][\sum(Y - \bar{Y})^2]}}$$

```
> x
[1] 1 2 3
> y
[1] 3 6 9
> sum((x-mean(x))*(y-mean(y)))
[1] 6
```

```
> t1<-sum((x-mean(x))^2)
> t1
[1] 2
> t2<-sum((y-mean(y))^2)
> t2
[1] 18
```

```
> sum((x-mean(x))*(y-mean(y)))/sqrt(t1*t2)
[1] 1
```

Process

- Identify the variable of interest
- Run a suite of n sensitivity runs, with parameters taking on a range of value
  - Organise the full data set by time (i.e. n observations for each variable at a given time)
  - Use the cor() function to calculate the correlation coefficient
  - Repeat for each time step
- Evaluate the overall result

From earlier…

```
> dim(o1)
[1] 8100    7
> head(o1)
     time  sCapital Investment Depreciation InvFraction DepFraction run
1 2015.00  9206.398   947.8448     250.2711    0.102955  0.02718447   1
2 2015.25  9380.791   965.7995     255.0119    0.102955  0.02718447   1
3 2015.50  9558.488   984.0943     259.8425    0.102955  0.02718447   1
4 2015.75  9739.551  1002.7356     264.7646    0.102955  0.02718447   1
5 2016.00  9924.044  1021.7301     269.7799    0.102955  0.02718447   1
6 2016.25 10112.031  1041.0843     274.8902    0.102955  0.02718447   1
.

> tail(o1)
        time sCapital Investment Depreciation InvFraction DepFraction run
8095 2033.75 26370.07   1970.675     473.7822  0.07473148  0.01796666 100
8096 2034.00 26744.30   1998.641     480.5057  0.07473148  0.01796666 100
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8100 2035.00 28295.06   2114.531     508.3677  0.07473148  0.01796666 100
```

The split() function

Description

`split` divides the data in the vector `x` into the groups defined by `f`. The replacement forms replace values corresponding to such a division. `unsplit` reverses the effect of `split`.

```
> runs<-split(o1,o1$time)
> head(runs[[1]])
    time  sCapital Investment Depreciation InvFraction DepFraction run
1   2015  9206.398   947.8448     250.2711  0.10295501  0.02718447   1
82  2015  9535.836   334.9849     117.4564  0.03512906  0.01231737   2
163 2015  9699.243   694.6379     114.4674  0.07161774  0.01180168   3
244 2015  9647.232  1070.9941     190.2613  0.11101568  0.01972185   4
325 2015  9962.860   907.6735     381.1013  0.09110572  0.03825220   5
406 2015 10014.443   384.6826     347.9108  0.03841278  0.03474091   6
> head(runs[[2]])
       time  sCapital Investment Depreciation InvFraction DepFraction run
2   2015.25  9380.791   965.7995     255.0119  0.10295501  0.02718447   1
83  2015.25  9590.218   336.8953     118.1262  0.03512906  0.01231737   2
164 2015.25  9844.286   705.0255     116.1791  0.07161774  0.01180168   3
245 2015.25  9867.416  1095.4379     194.6037  0.11101568  0.01972185   4
326 2015.25 10094.503   919.6670     386.1370  0.09110572  0.03825220   5
407 2015.25 10023.636   385.0358     348.2302  0.03841278  0.03474091   6
```

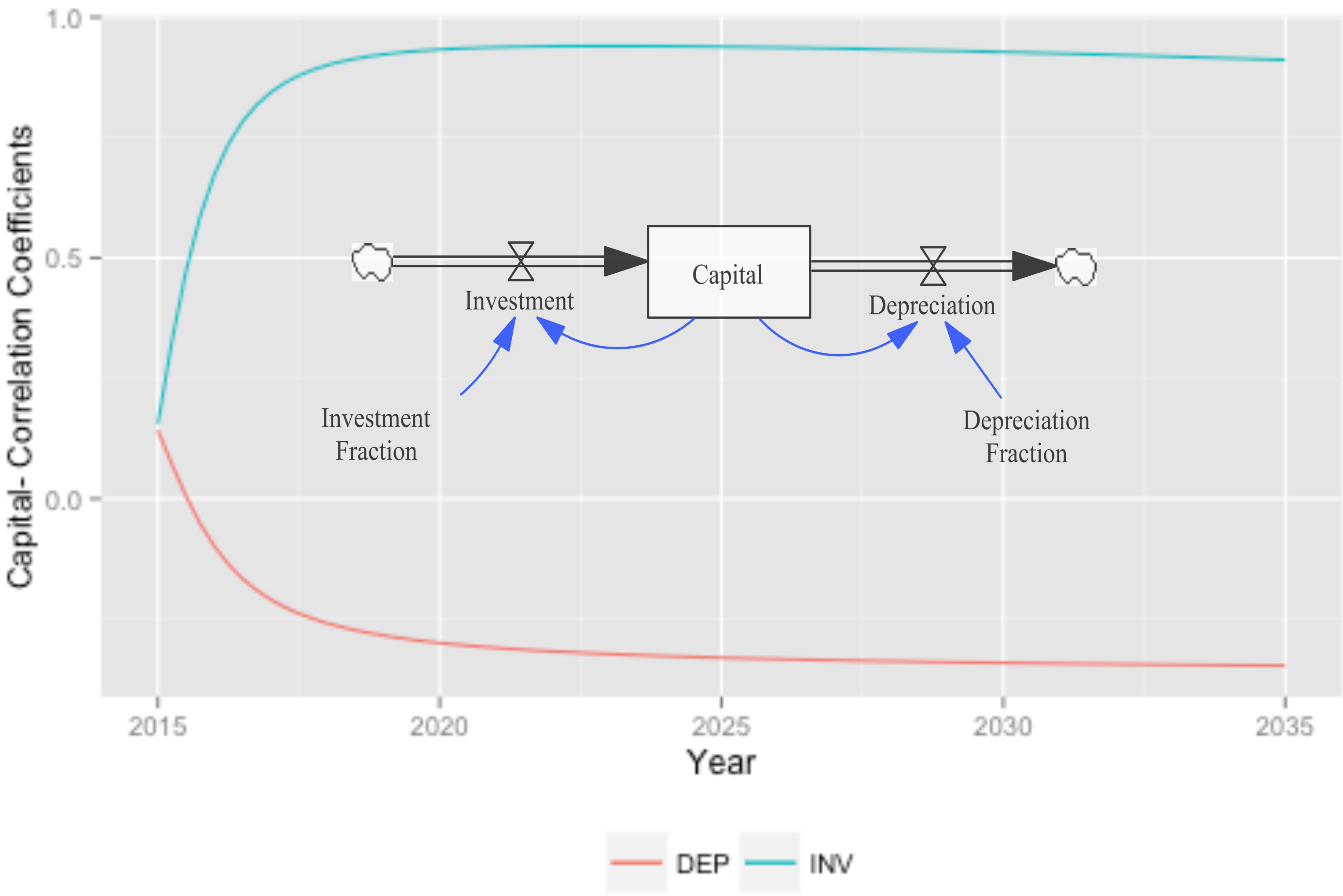
sapply()... to process the list.

```
corInv<-sapply(runs,function(l){cor(l$sCapital, l$InvFraction )})
corDep<-sapply(runs,function(l){cor(l$sCapital, l$DepFraction )})
```

```
> corDep[1:7]
       2015     2015.25      2015.5     2015.75        2016     2016.25      2016.5
 0.142374549  0.071538932  0.005488386 -0.051454845 -0.098256220 -0.135845238 -0.165839818
> corInv[1:7]
     2015   2015.25    2015.5   2015.75      2016   2016.25    2016.5
0.1553017 0.3271642 0.4723357 0.5864322 0.6723759 0.7359221 0.7827813
```

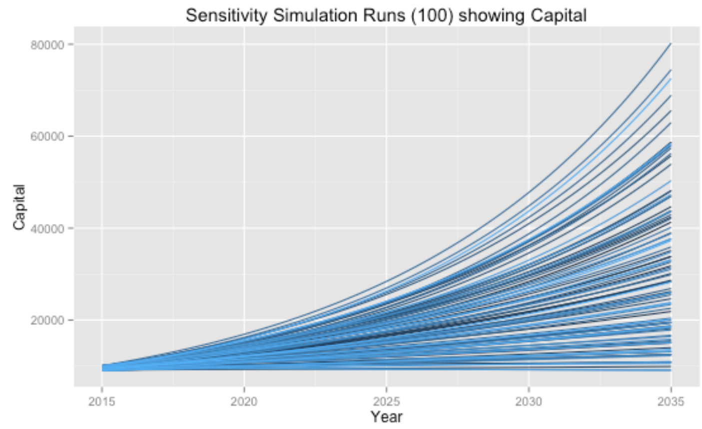
```
p2<-ggplot()+
  geom_line(aes(simtime,corInv,color="INV"))+
  geom_line(aes(simtime,corDep,color="DEP"))+
  scale_y_continuous(labels = comma)+
  ylab("Capital- Correlation Coefficients")+
  xlab("Year") +
  labs(color="")+
  theme(legend.position="bottom")
```

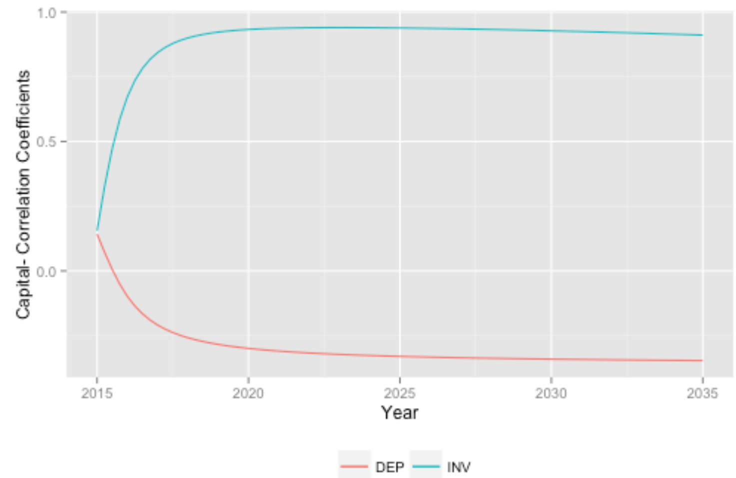
Analysis of Output: Investment Fraction more influential



Summary

- Sensitivity Analysis supports exploring parameter uncertainty
- R supports generating large data sets of runs
- **split()** and **cor()** functions can be used to calculate correlation coefficients
- Analysis shows most influential parameters





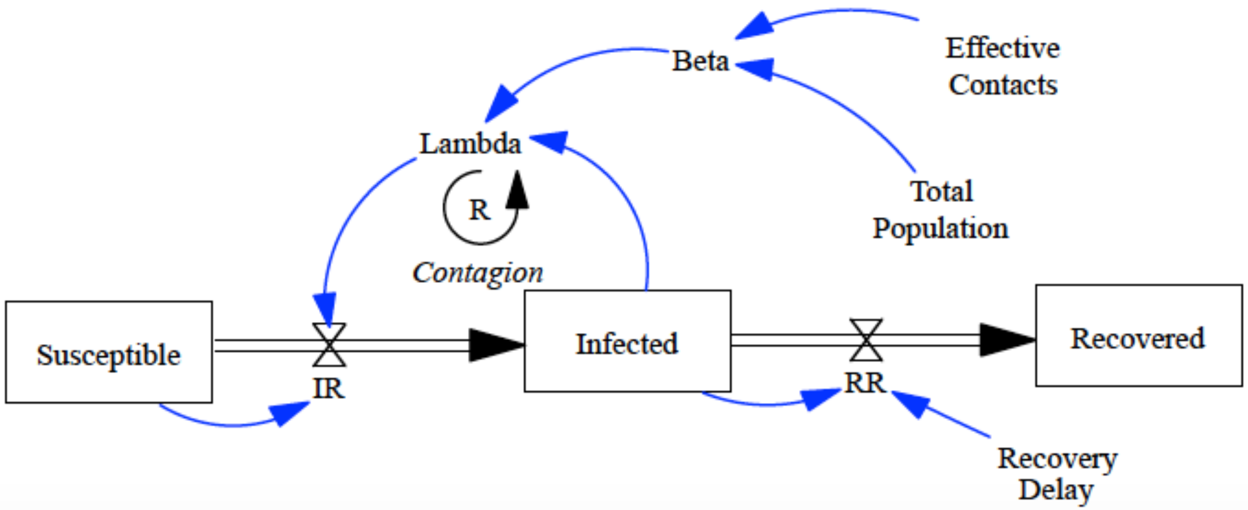
Final Assignment

CT561: Systems Modelling and Simulation

Final Assignment

Analysis of Parameter Influence – Using R - for the SIR Model

The goal of this assignment is to build an SIR model and test the influence of key parameters using sensitivity analysis and statistical screening (these will be covered in Lecture 11). The following is the SIR Structure.



The steps are:

- First, Implement and test the SIR model in a function. Use initial values S=9999, I=1, R=0. Assume C_E=2 and Recovery Delay (D) =2. Total population is always 10,000.
- Create an outer sensitivity function that can be called with the number of simulation runs as a parameter. Each time the model is run, the values for C_E and D must be obtained from a uniform distribution (see ranges below)
- Have a check to ensure that no stock value is negative. If this happens, the time step will have to be halved.
- Run the simulation 50 times and plot each simulation result for the variable **Infected.**
- Calculate the maximum, minimum and average peak of the epidemic.
- Plot the correlation coefficients of the two parameters against the variable of interest (Infected), and comment on the results.

The ranges of parameters, and initial stock of infected are as follows:

Variable	**Minimum**	**Maximum**
Infected	1.0	20.0
Recovery Delay	1.0	10.0
C_E	0.75	8.0

Whatever value of Infected is generated, always ensure that the total population stays at 10,000.

Reference

Statistical screening of system dynamics models

Andrew Ford[a]* and Hilary Flynn[b]

Abstract

This paper describes a pragmatic method of searching for the key inputs to a system dynamics model. This analysis is known as screening. The goal is to learn which of the many uncertain inputs stand out as most influential. The method is implemented with readily available software and relies on the simple correlation coefficient to indicate the relative importance of model inputs at different times in the simulation. The screening is demonstrated with two examples with step-by-step instructions. The paper recommends that screening analysis be used in an iterative process of screening and model expansion to arrive at tolerance intervals on model results. The appendices compare screening analysis with analytical methods to identify the key inputs to system dynamics models. Copyright © 2005 John Wiley & Sons, Ltd.

Syst. Dyn. Rev. **21**, 273–303, (2005)