CT561: Systems Modelling and Simulation

Week 8:
Effects and Limits to Growth

https://github.com/JimDuggan/CT561

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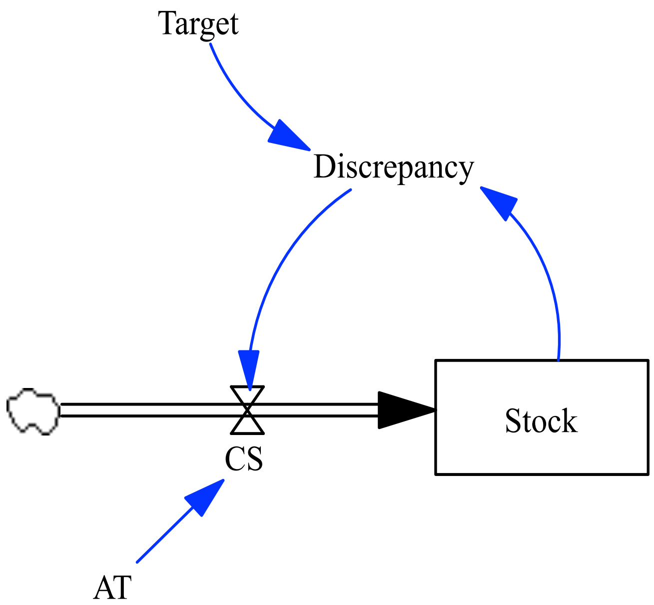
Recap – deSolve



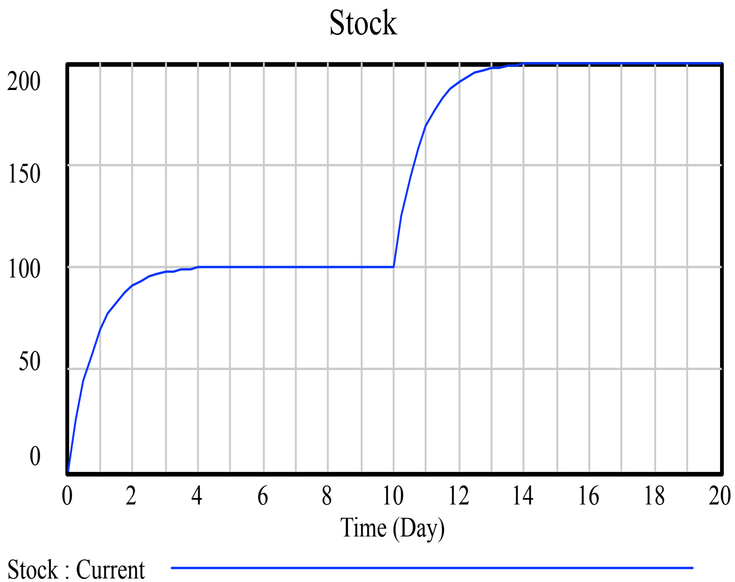
(01) AT= 1

(02) CS= Discrepancy/AT

(03) Discrepancy=Target-Stock

(04) Stock= INTEG (CS, 0)

(05) Target = 100+step(100,10)



Setting up key variables

```
library(deSolve)
library(ggplot2)


START<-0; FINISH<-20; STEP<-0.25;

simtime <- seq(START, FINISH, by=STEP)

# Setup the step function in a global data frame
target <- rep(NA,length(simtime))
target[1:(10/STEP)]<-100
target[((10/STEP)+1):length(simtime)]<-200

simData<-data.frame(time=simtime, aTarget=target)
```

Simulating the step function…

```
> simData[seq(1,80,by=8),]
   time aTarget
1     0     100
9     2     100
17    4     100
25    6     100
33    8     100
41   10     200
49   12     200
57   14     200
65   16     200
73   18     200
```

The model

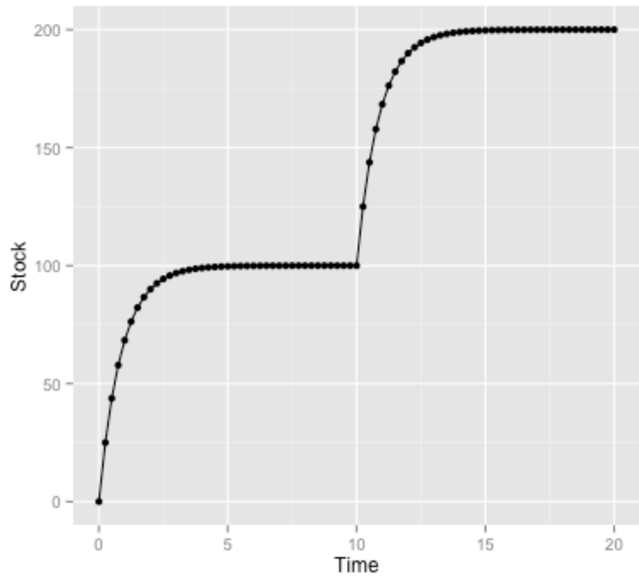
```
model <- function(time, stocks, auxs)
{
  with(as.list(c(stocks, auxs)),{

    index<-which(simData$time==time)

    aDisrepancy<-simData$aTarget[index] - sStock

    fRI<- aDisrepancy/ aAT

    dS_dt  <- fRI

    return (list(c(dS_dt),
                 Target=simData$aTarget[index],
                 Inflow=fRI,
                 AT=aAT))
  })
}
```

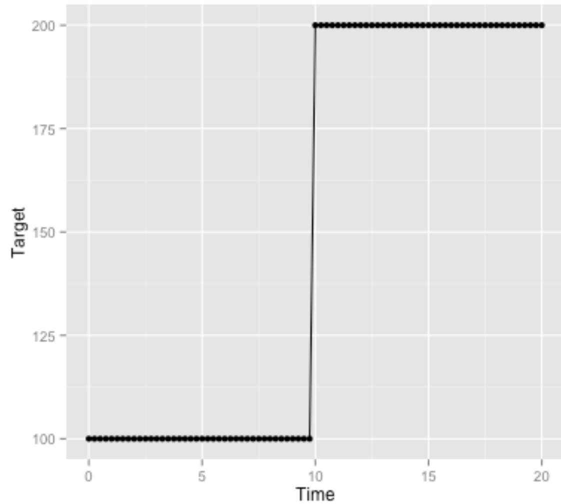
Running the simulation…

```r
stocks  <- c(sStock=0)
auxs    <- c(aAT=1)


o<-data.frame(ode(y=stocks, simtime, func = model,
                  parms=auxs, method="euler"))

p1<-qplot(data=o,x=o$time,y=o$sStock,
      geom=c("line","point"),xlab="Time",ylab="Stock")
```

Data and Plots

```
> o[seq(1,80,by=8),]
   time    sStock Target       Inflow AT
1     0   0.00000    100 100.00000000  1
9     2  89.98871    100  10.01129150  1
17    4  98.99774    100   1.00225958  1
25    6  99.89966    100   0.10033913  1
33    8  99.98995    100   0.01004524  1
41   10  99.99899    200 100.00100566  1
49   12 189.98861    200  10.01139218  1
57   14 198.99773    200   1.00226966  1
65   16 199.89966    200   0.10034014  1
73   18 199.98995    200   0.01004534  1
```





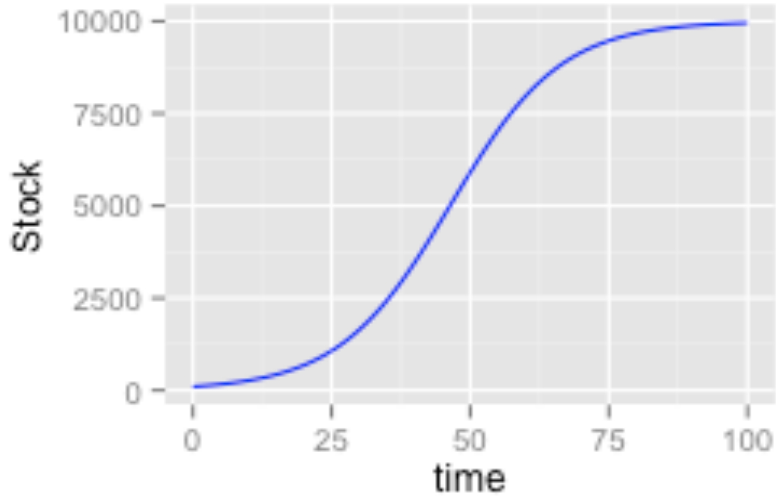
Overview

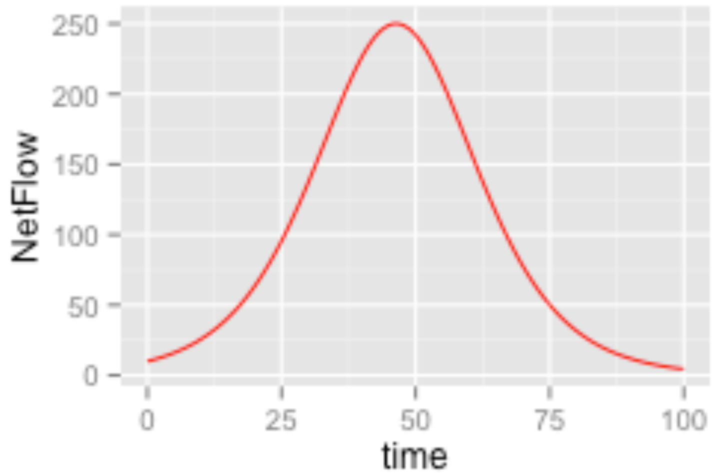
There will always be limits to growth.

They can be self-imposed.

If they aren't, they will be system-imposed.

Donella H. Meadows, Thinking in Systems: A Primer (2008), p.103





Formulating Effects

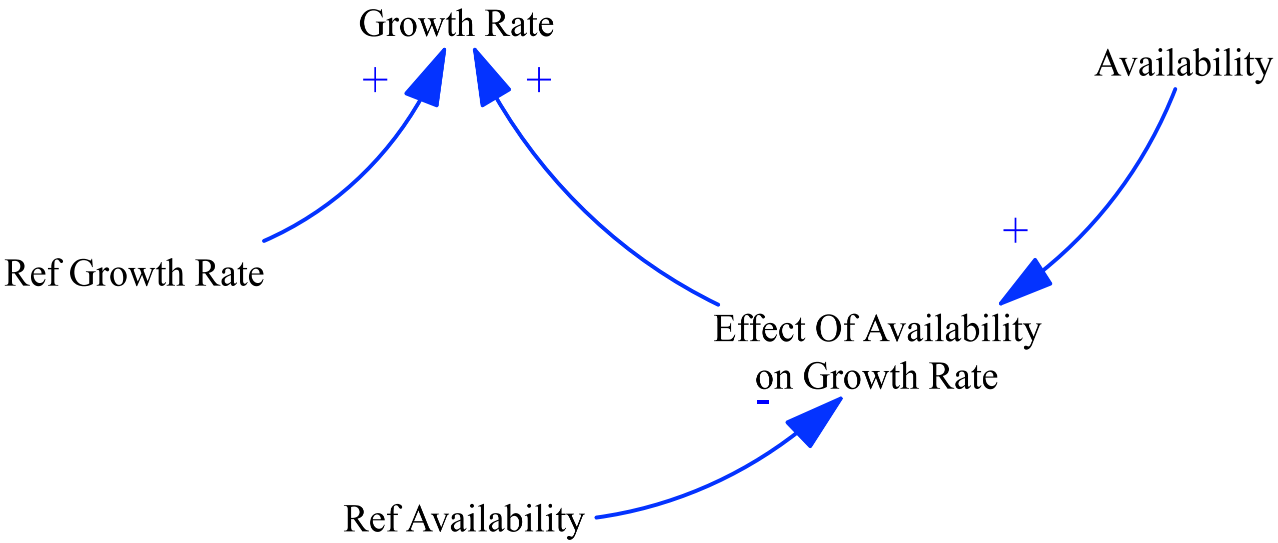
- An important building block for system dynamics modeling is to capture how variables influence one another over time.
- While some of these may be simple linear relationships, the reality is that real-world effects between variables can also be non-linear, and may also involve multiple variables.
- System dynamics offers a convenient structure for modeling effect variables (Sterman 2000).

$$Y = Y^* \times Effect\,(X_1 on\, Y) \times \ldots \times Effect(X_n on\, Y) \tag{4-1}$$

$$Effect\,(\,X_i\; on\; Y) = {X_i}/{X_i^*} \tag{4-2}$$

- There is a variable *Y* that is the dependent variable of a causal relationship, and this depends on a set of *n* independent variables ($X_1, X_2, \ldots, X_n$)
- The variable *Y* has a reference value Y^*, and this is multiplied by a sequence of *effect functions* that are calculated based on the normalized ratio of (X_i/X_i^*), where X_i^* is the reference value, and X_i is the actual value.
- The effect function (y-axis) has the normalized ratio (X/X^*) on its x-axis, and always contains the point (1,1) although the function itself can be either linear or non-linear around this point.
- This point (1,1) is important for the following reason: if *X* equals its reference value X^*, then the effect function will be 1, and therefore *Y* will then equal its reference value Y^* (from equation 4-1).

Growth Rate Example



$$Growth\ Rate = Ref\ Growth\ Rate \times Effect\ of\ Availability\ on\ Growth\ Rate \quad (4\text{-}3)$$

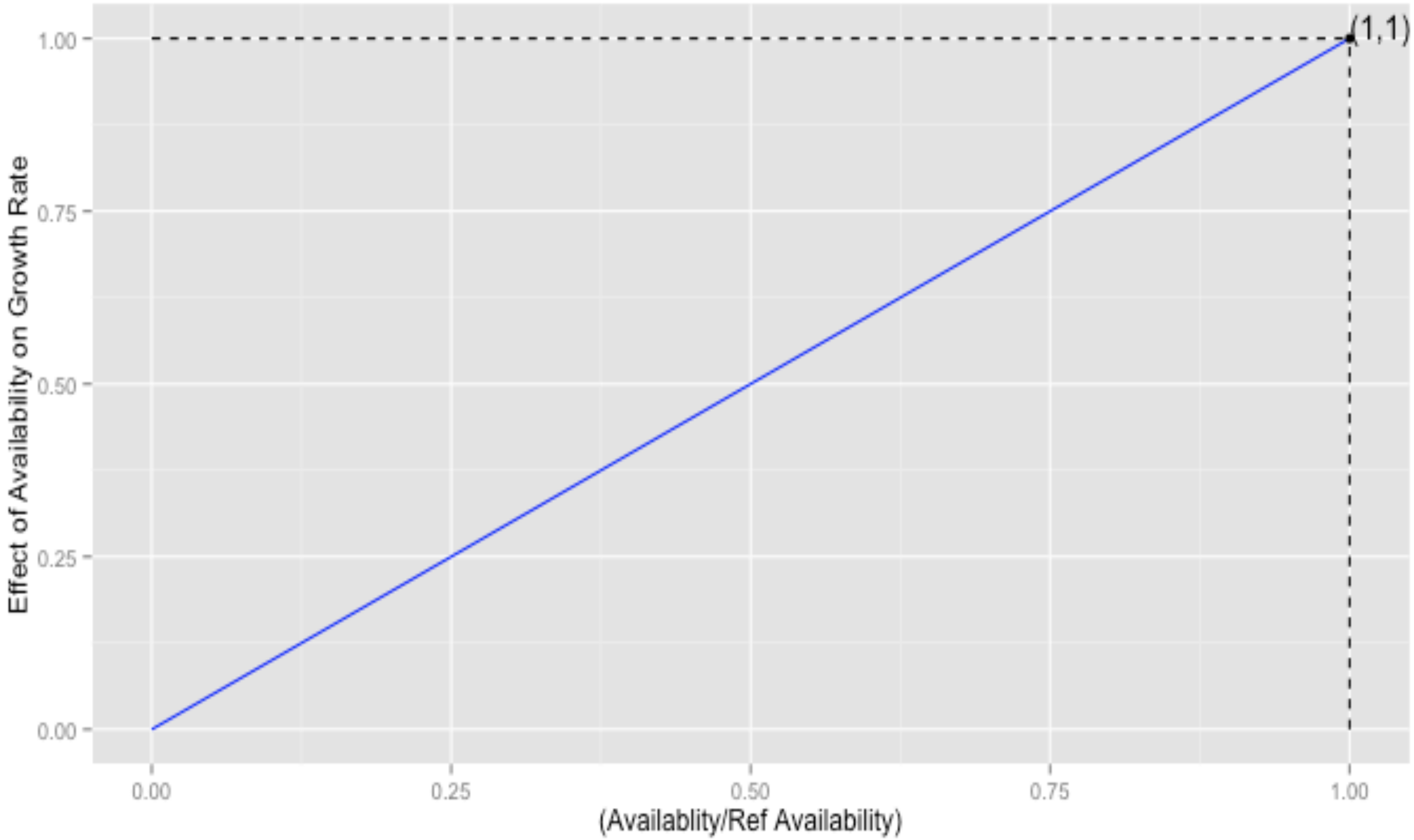
$$Effect\ of\ Availability\ on\ Growth\ Rate = f\left(\frac{Availability}{Ref\ Availability}\right) \quad (4\text{-}4)$$

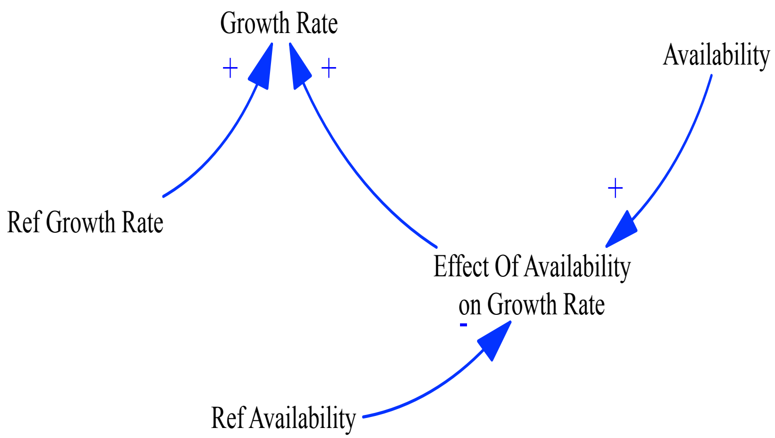
$$Ref\ Growth\ Rate = 0.10 \quad (4\text{-}5)$$

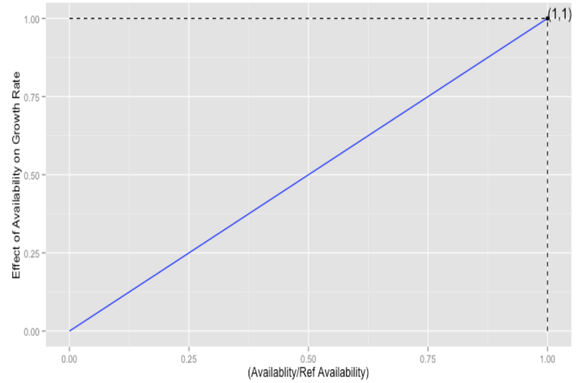
$$Ref\ Availability = 1.0 \quad (4\text{-}6)$$

The system grows at a rate of 10% if the availability stays at 1.0

Effect equation y = mx + c



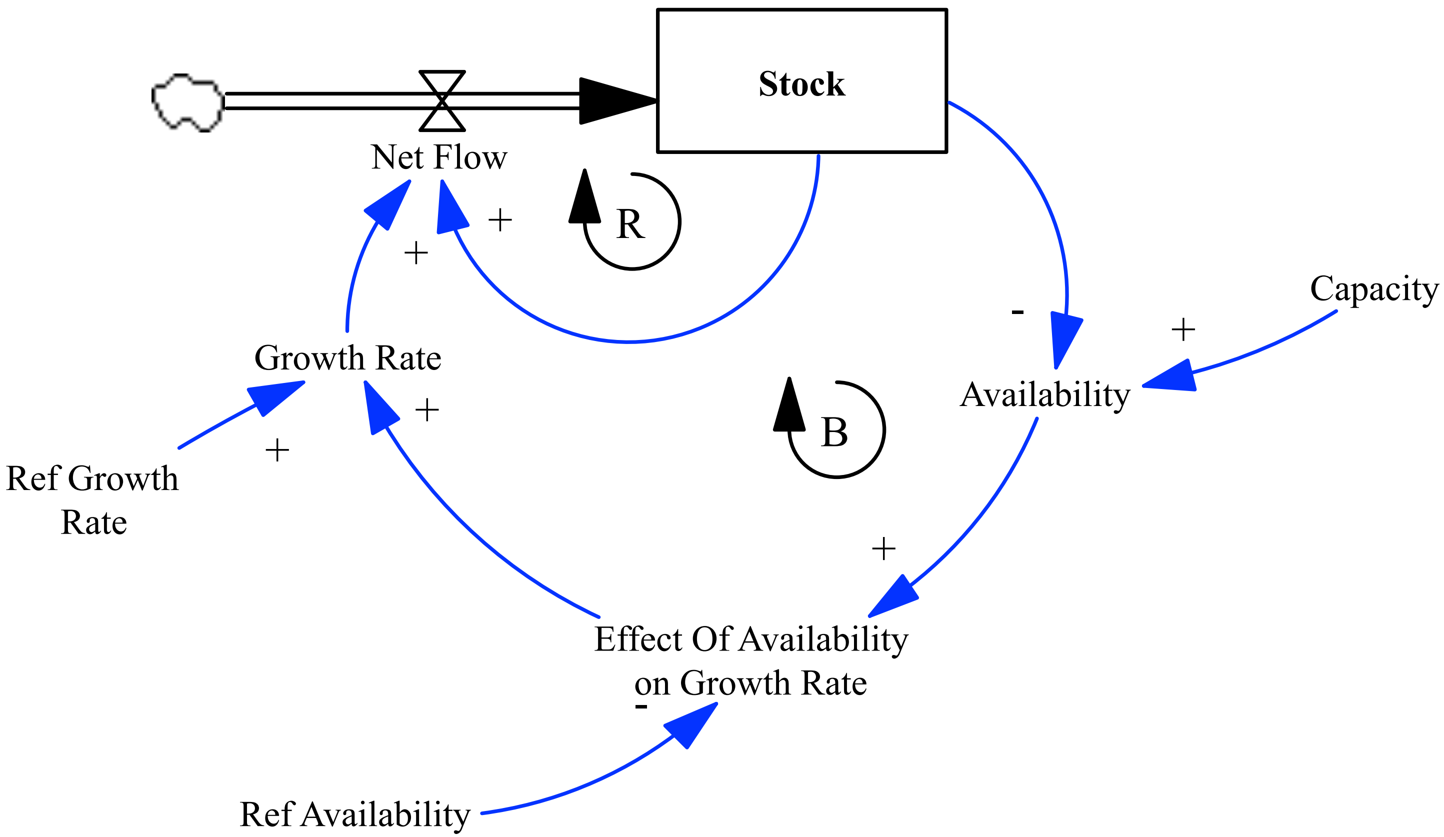




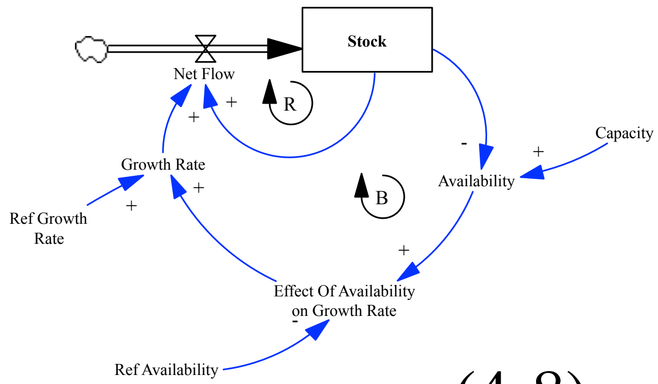
$$Effect\ of\ Availability\ on\ Growth\ Rate = \frac{Availability}{Ref\ Availability} \tag{4-7}$$

Ref Availability	*Availability*	*Effect of Availability on Growth Rate*	*Ref Growth Rate*	*Growth Rate*
1.0	1.0	1.0	0.10	0.10
1.0	0.5	0.5	0.10	0.05
1.0	0.0	0.0	0.10	0.00

Model of S-Shaped Growth



Equations



$$Stock = INTEGRAL(Net\ Flow, 100) \quad (4\text{-}8)$$

$$Net\ Flow = Stock \times Growth\ Rate \quad (4\text{-}9)$$

$$Availability = 1 - \frac{Stock}{Capacity} \quad (4\text{-}10)$$

$$Capacity = 10000 \quad (4\text{-}11)$$

$$Effect\ of\ Availability\ on\ Growth\ Rate = \frac{Availability}{Ref\ Availability} \quad (4\text{-}7)$$

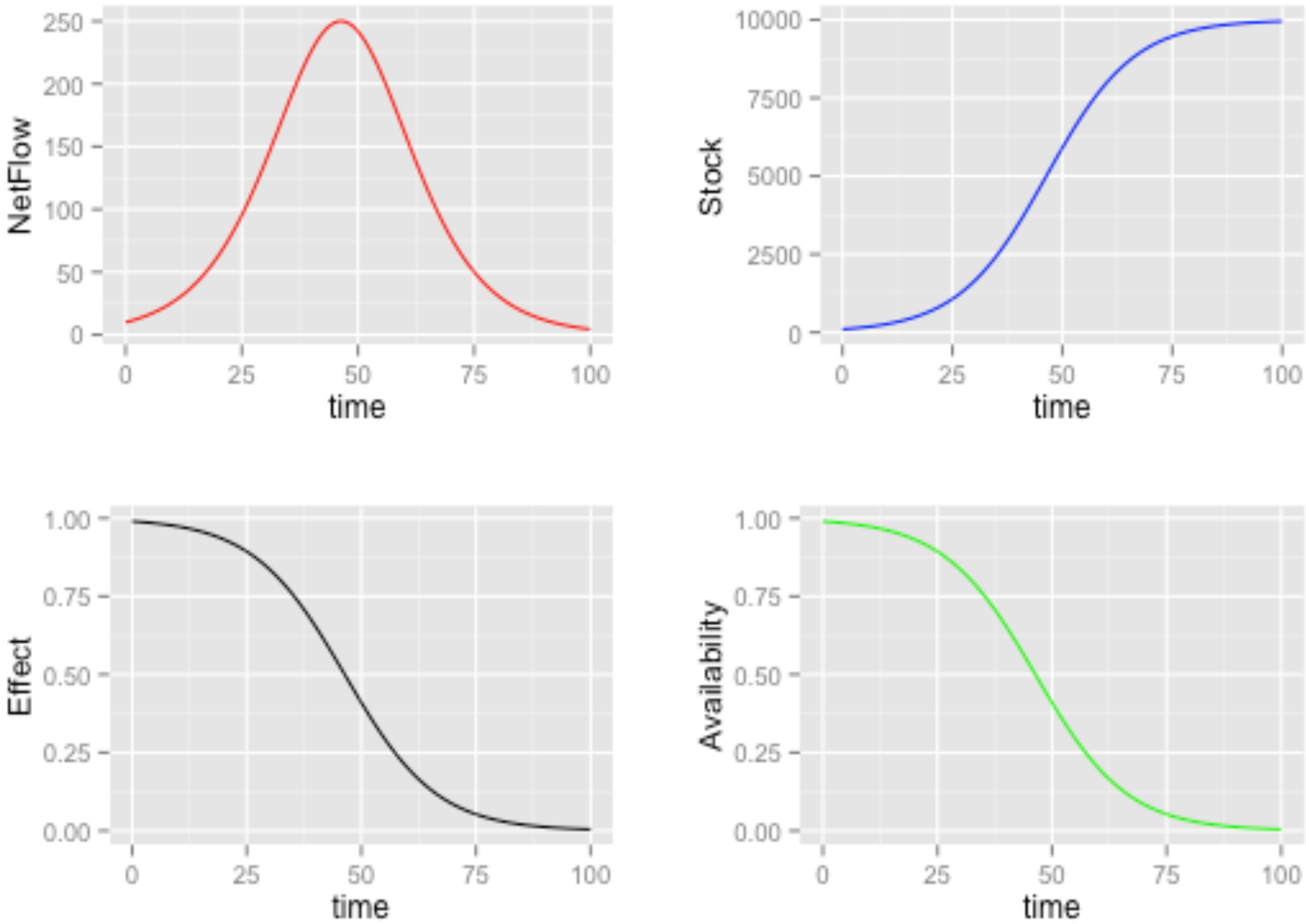
$$Growth\ Rate = Ref\ Growth\ Rate \times Effect\ of\ Availability\ on\ Growth\ Rate \quad (4\text{-}3)$$

$$Effect\ of\ Availability\ on\ Growth\ Rate = f\left(\frac{Availability}{Ref\ Availability}\right) \quad (4\text{-}4)$$

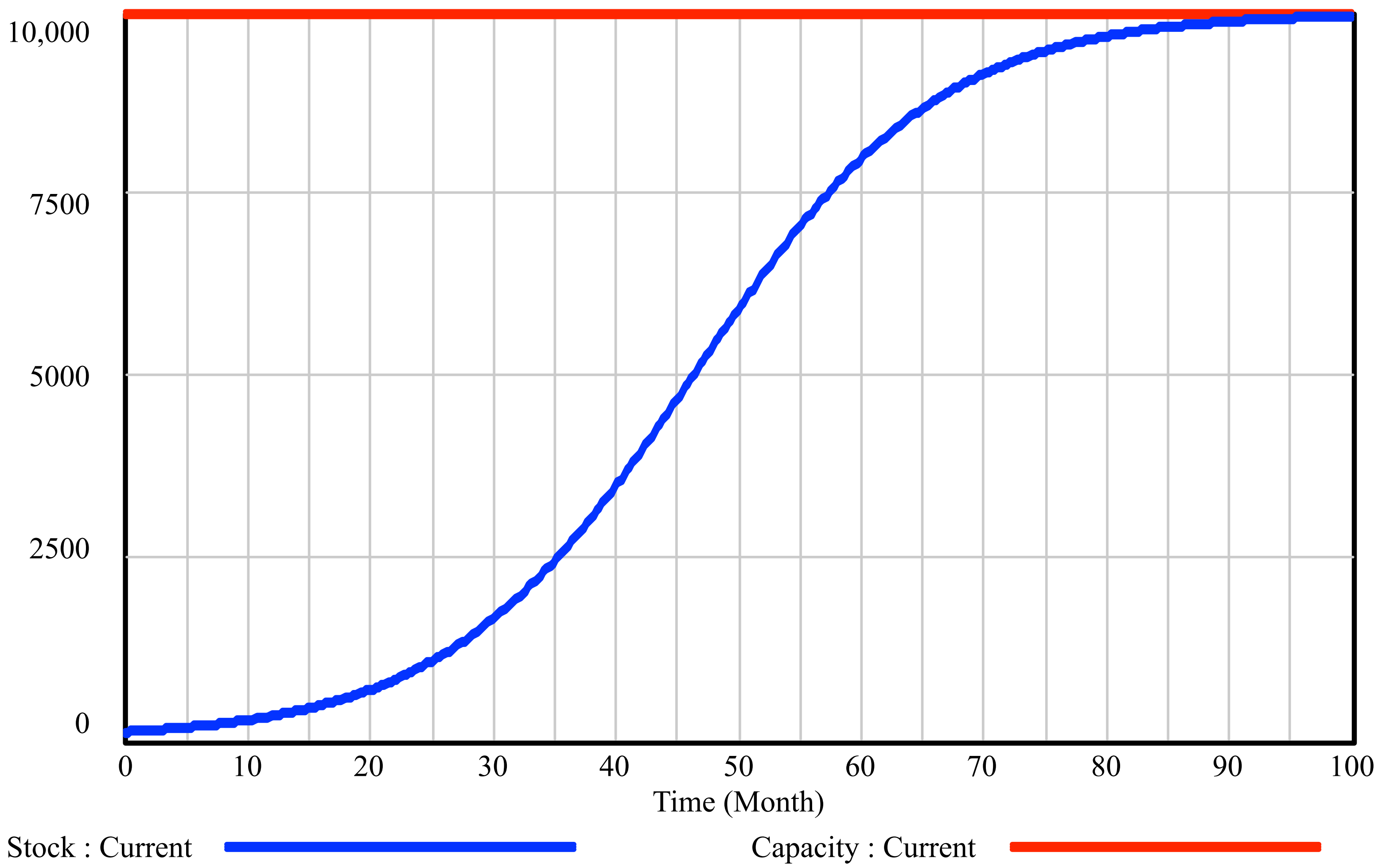
$$Ref\ Growth\ Rate = 0.10 \quad (4\text{-}5)$$

$$Ref\ Availability = 1.0 \quad (4\text{-}6)$$

Simulation Output



Limit constant, stock approaches



Verhulst Equations (1838)

In ecology: modeling population growth [edit]

A typical application of the logistic equation is a common model of population growth, originally due to Pierre-François Verhulst in 1838, where the rate of reproduction is proportional to both the existing population and the amount of available resources, all else being equal. The Verhulst equation was published after Verhulst had read Thomas Malthus' *An Essay on the Principle of Population*. Verhulst derived his logistic equation to describe the self-limiting growth of a biological population. The equation was rediscovered in 1911 by A. G. McKendrick for the growth of bacteria in broth and experimentally tested using a technique for nonlinear parameter estimation.[4] The equation is also sometimes called the *Verhulst-Pearl equation* following its rediscovery in 1920 by Raymond Pearl (1879–1940) and Lowell Reed (1888–1966) of the Johns Hopkins University.[5] Another scientist, Alfred J. Lotka derived the equation again in 1925, calling it the *law of population growth*.

Pierre-François Verhulst (1804–1849)

Letting P represent population size (N is often used in ecology instead) and t represent time, this model is formalized by the differential equation:

$$\frac{dP}{dt} = rP \cdot \left(1 - \frac{P}{K}\right)$$

where the constant r defines the growth rate and K is the carrying capacity.

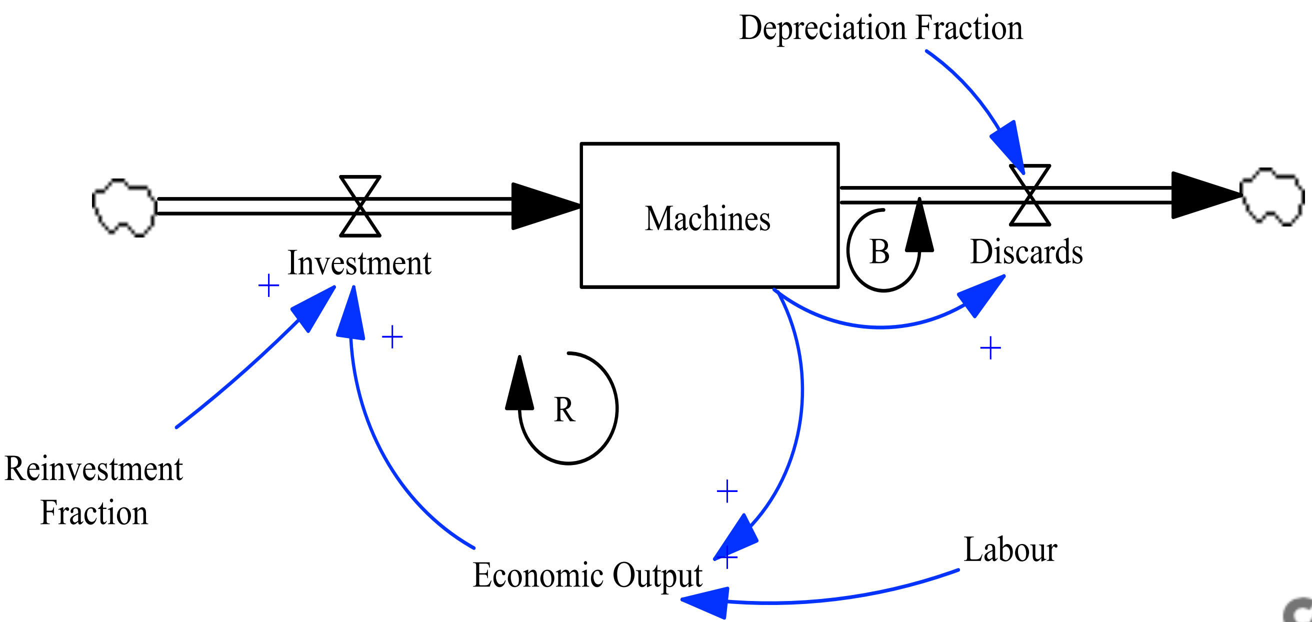
https://en.wikipedia.org/wiki/Logistic_function

Challenge 8.1

- Consider a software development team
- It contains:
  - Experienced Engineering
  - Rookie Engineers
- The normal productivity of experienced engineering is 200 LOC/Day, and this occurs when the Rookie percentage is 20%
- Construct an effects equation to model this, explain any assumptions made.

Model of Economic Growth

- In presenting the model, Page (2015) imagines a scenario where a self-contained civilization supports itself through harvesting coconuts using machines.
- A percentage of these resources can be reinvested to produce more machines, and therefore increase the economic output.



Model Thinking
coursera

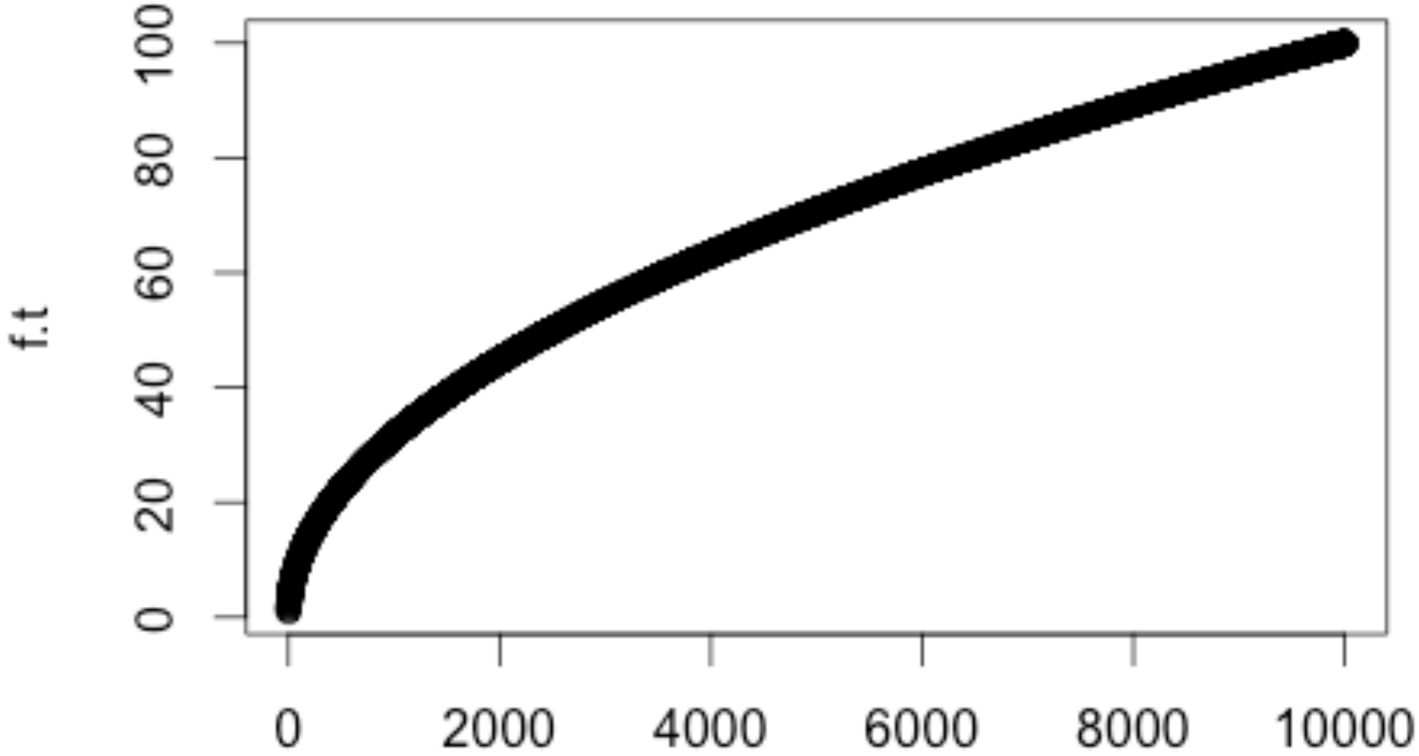
Equations

$$Machines\ (M) = INTEGRAL(Investment - Disards, 100) \quad (4\text{-}12)$$
$$Investment\ = Economic\ Output \times Reinvestment\ Fraction \quad (4\text{-}13)$$
$$Discards = Machines \times Depreciation\ Fraction \quad (4\text{-}14)$$
$$Reinvestement\ Fraction\ (R) = 0.20 \quad (4\text{-}15)$$
$$Depreciation\ Fraction(D) = 0.10 \quad (4\text{-}16)$$

$$Economic\ Output(O) = Labour \times \sqrt{Machines} \quad (4\text{-}17)$$
$$Labour\ \ (L) = 100 \quad (4\text{-}18)$$

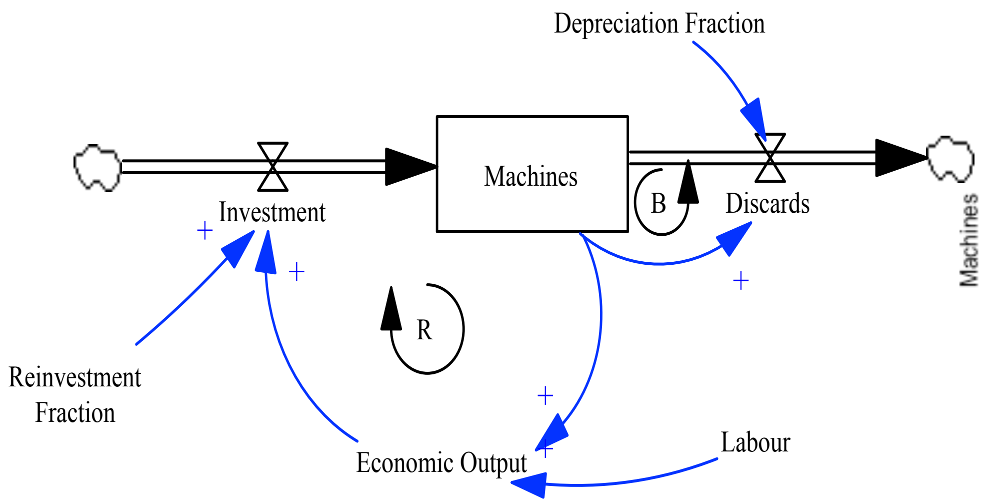
Economic output is based on the laws of diminishing returns (concave function)

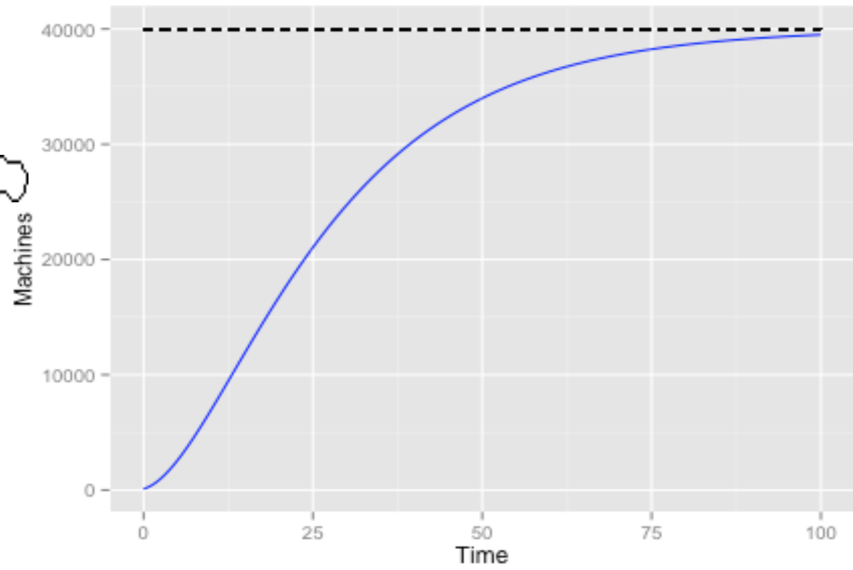
Concave Function




```
> t<-1:10000
> f.t<-t^0.5
> plot(f.t)
```

Simulation Output





Model based on…

A CONTRIBUTION TO THE THEORY OF ECONOMIC GROWTH

By Robert M. Solow

I. Introduction

All theory depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive.[1] A "crucial" assumption is one on which the conclusions do depend sensitively, and it is important that crucial assumptions be reasonably realistic. When the results of a theory seem to flow specifically from a special crucial assumption, then if the assumption is dubious, the results are suspect.

Challenge

- Find the steady state (equilibrium) value for the number of machines, given that the following values are known:
  - Reinvestment Fraction (R) = 0.20
  - Total Labour (L) = 100
  - Depreciation Fraction (D) = 0.1