CT561: Systems Modelling & Simulation

Lecture 4: Formulating Effects and Limits to Growth

Dr. Jim Duggan,

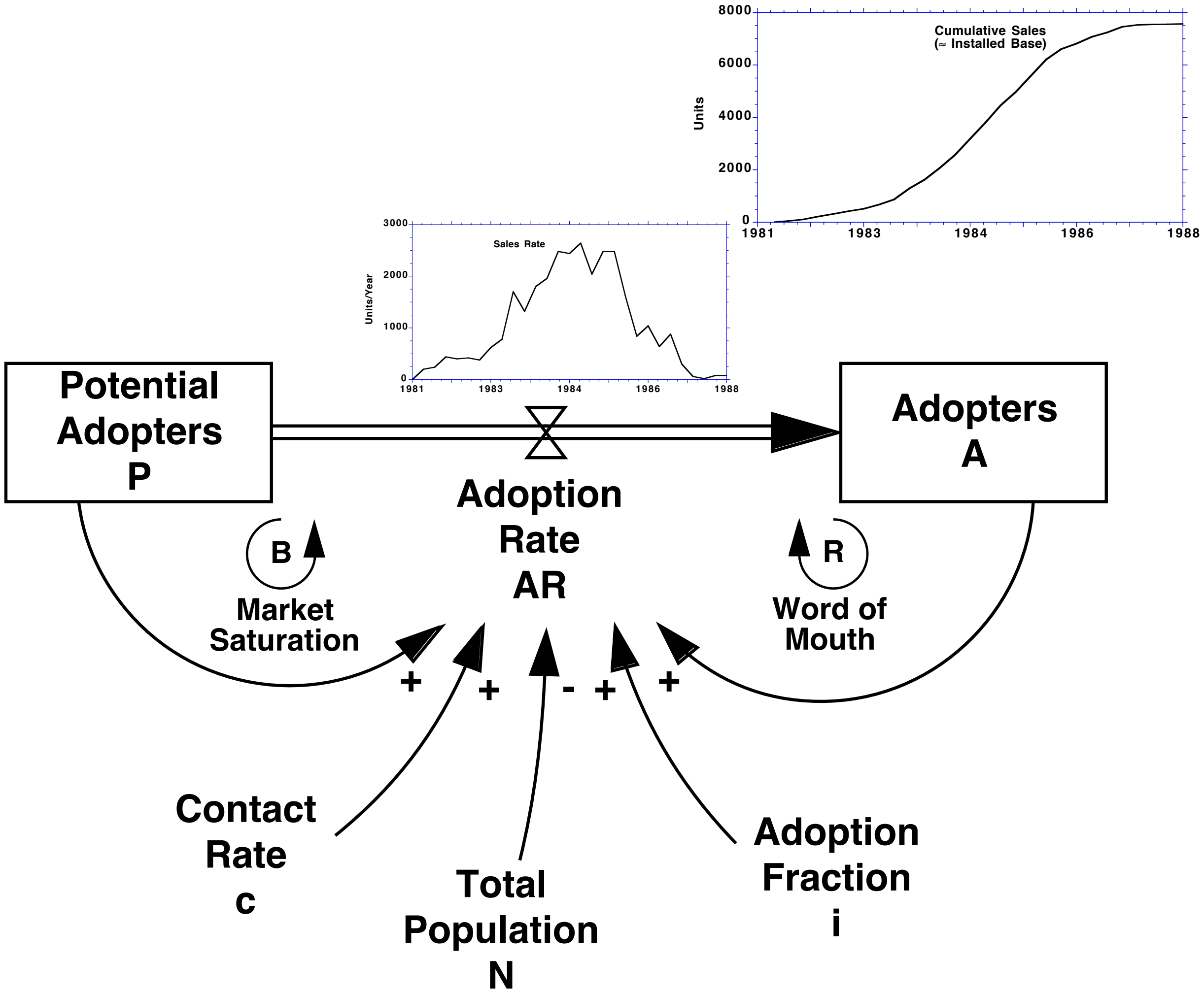
School of Engineering & Informatics

National University of Ireland Galway.

https://github.com/JimDuggan/SDMR

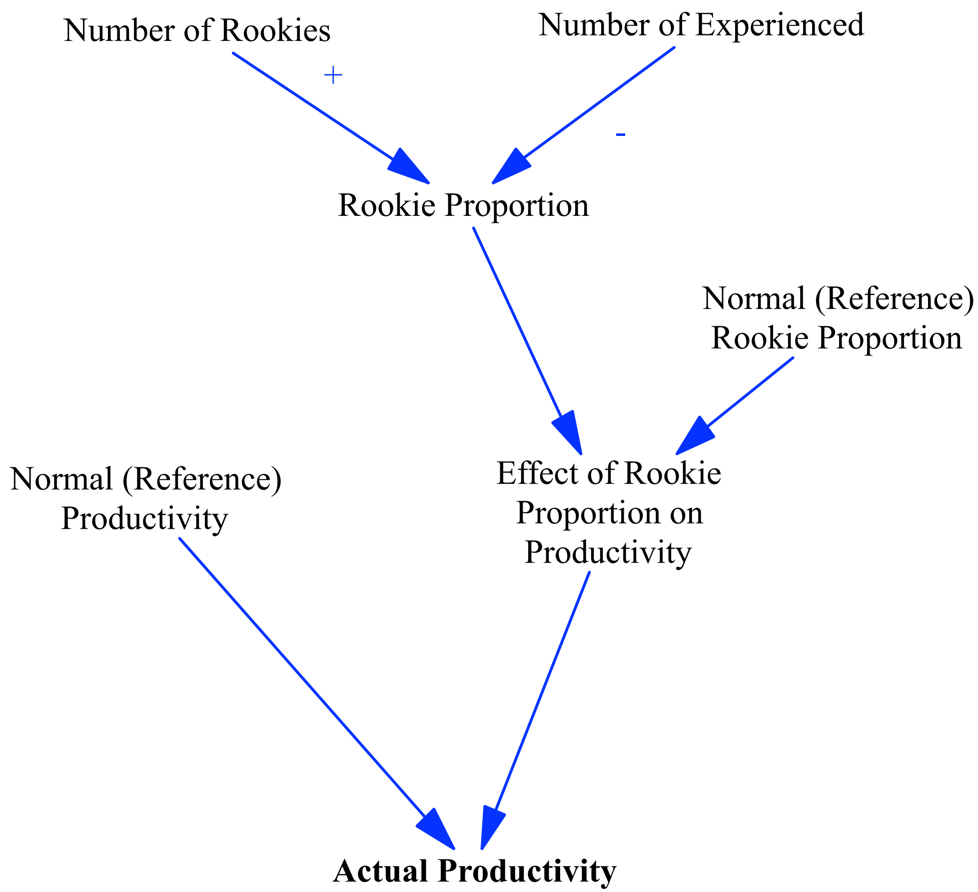
https://twitter.com/_jimduggan

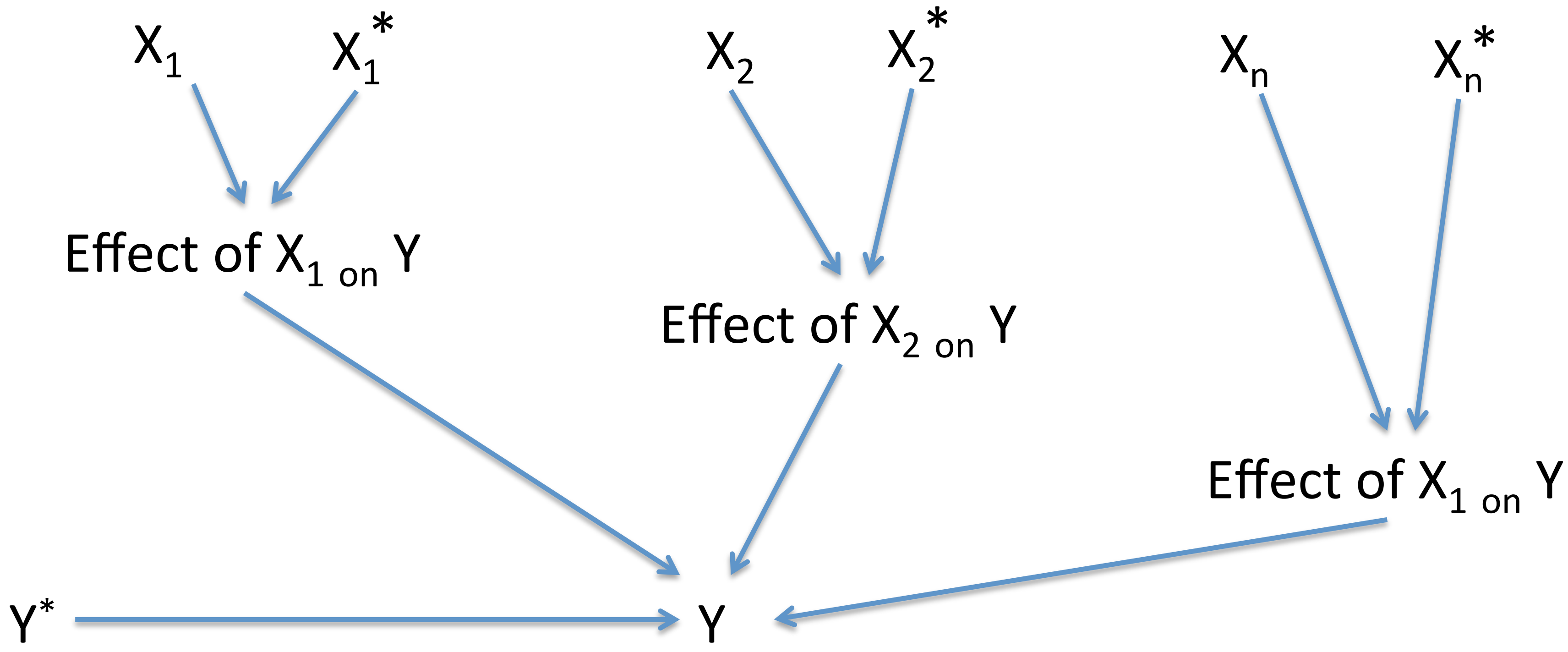
Recap: Feeback Example



Formulating Effects

- An important building block for models is to capture how variables influence one another over time.
- System dynamics offers a convenient structure for modeling effect variables (Sterman 2000).



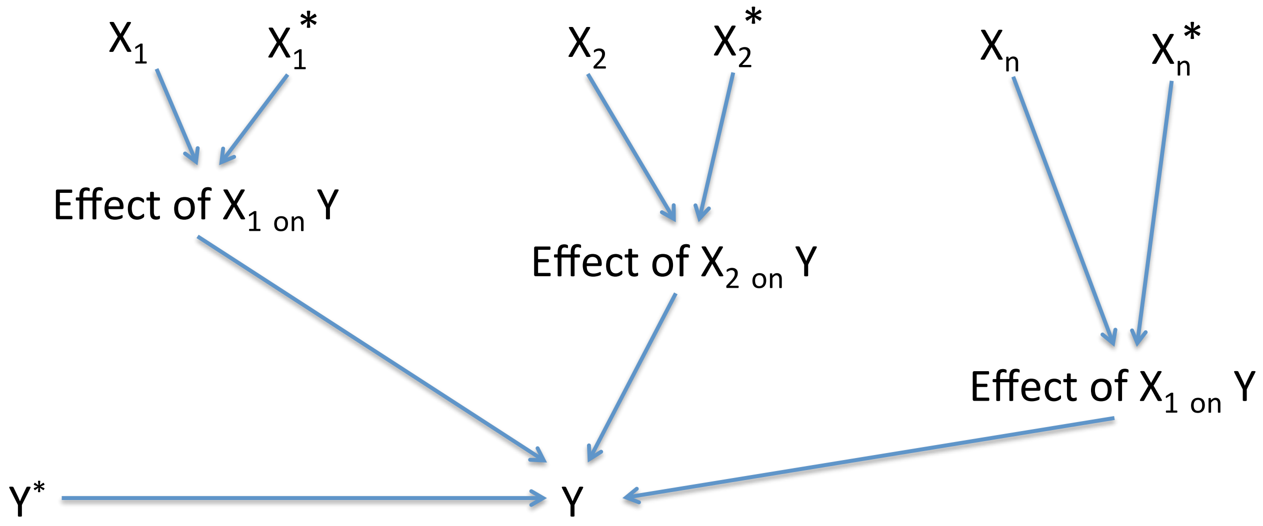


$$Y = Y^* \times Effect\,(X_1 on\, Y) \times \ldots \times Effect(X_n on\, Y)$$

$$Effect\,(\,X_i\ on\ Y) = f\left({}^{X_i}\!/_{X_i^*}\right)$$

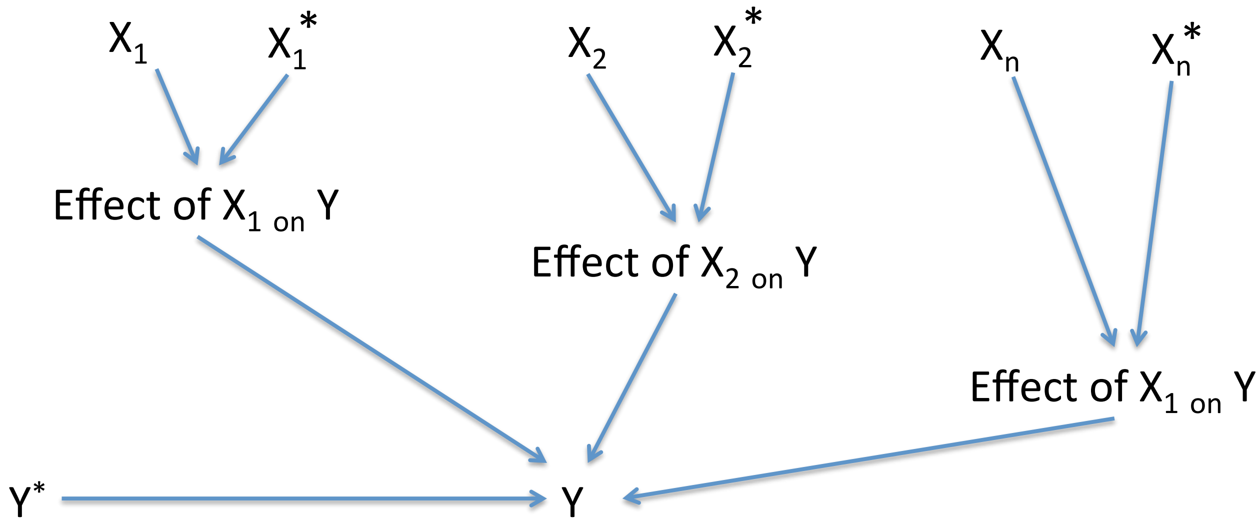
Effects structure (1)

- There is a variable Y that is the dependent variable of a causal relationship, and this depends on a set of n independent variables $(X_1, X_2, ..., X_n)$
- The variable Y has a reference value Y^*, and this is multiplied by a sequence of *effect functions* that are calculated based on the normalized ratio of (X_i/X_i^*), where X_i^* is the reference value, and X_i is the actual value.



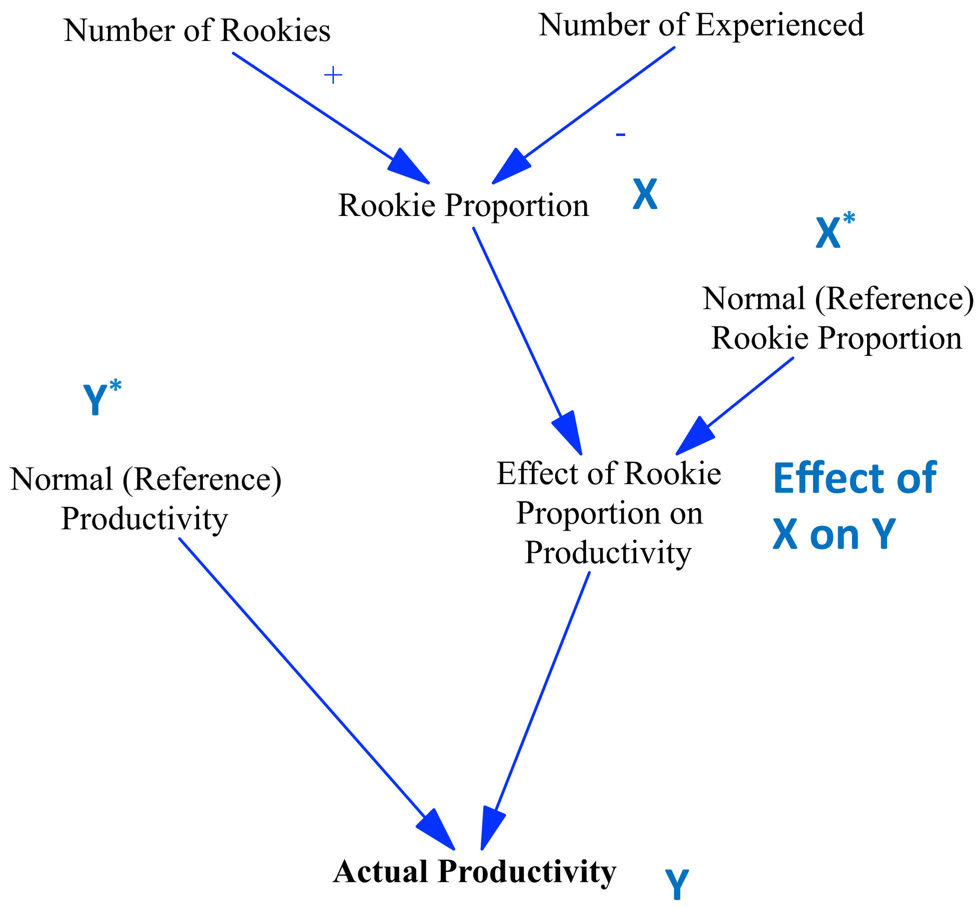
Effects structure (2)

- The effect function (y-axis) has the normalized ratio (*X/X**) on its x-axis, and always contains the point (1,1) although the function itself can be either linear or non-linear around this point.
- This point (1,1) is important for the following reason: if *X* equals its reference value *X**, then the effect function will be 1, and therefore *Y* will then equal its reference value *Y**.



Software Engineering Example

- Reference productivity is 100 loc/person/day
- This assumes a reference rookie proportion in the team (say 20%)
- If we have exactly 20% Rookies
  - Actual Productivity = Reference Productivity
- If we have > 20% Rookies
  - Actual Productivity < Reference Productivity
- If we have < 20% Rookies
  - Actual Productivity > Reference Productivity



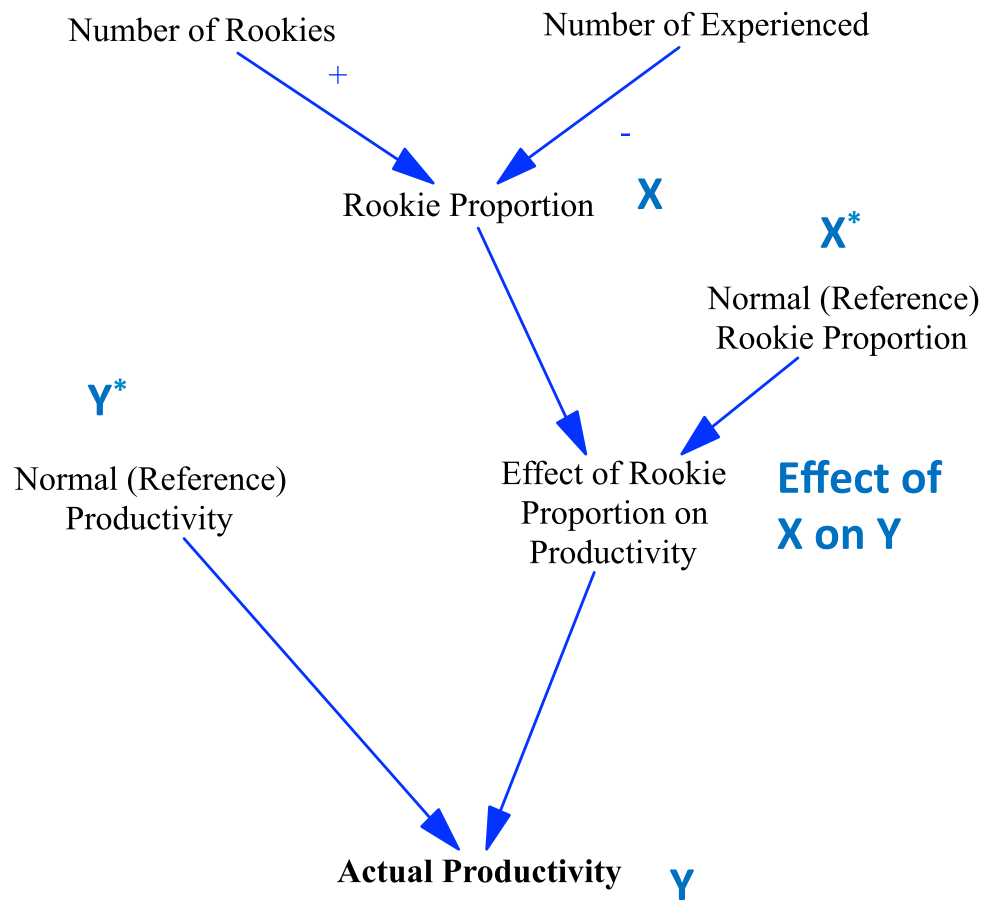
The equation

- Actual Productivity = Reference Productivity * Effect of Rookie Proportion on Productivity

Reference Productivity	Reference Rookie Proportion	Actual Rookie Proportion	Effect Multiplier	Actual Productivity
100	20%	20%	1	100
100	20%	40%	< 1	< 100
100	20%	10%	> 1	> 100

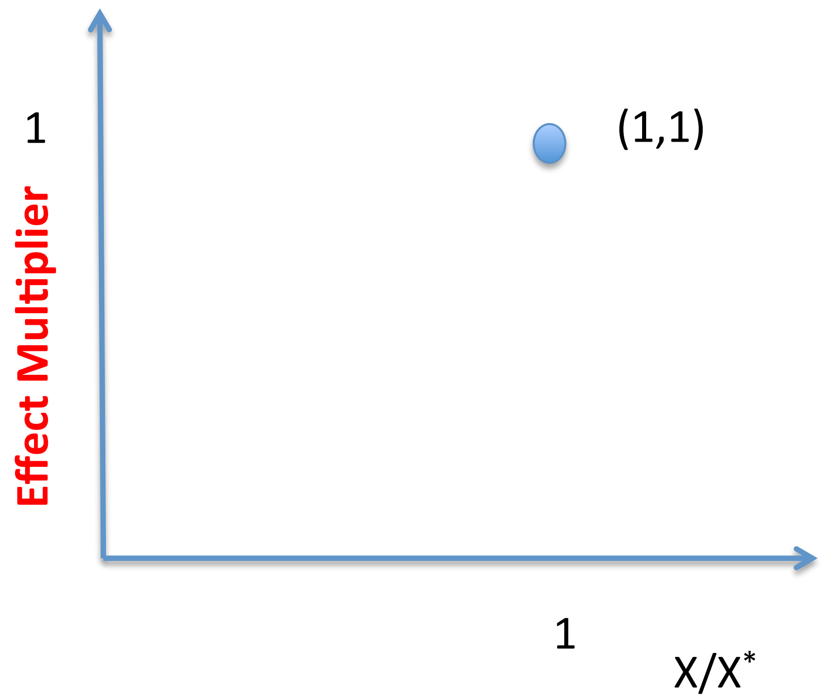
The Effect Equation

- Actual Productivity = Reference Productivity * Effect of Rookie Proportion on Productivity
- Effect of X on Y = $F(X/X^*)$
- Normalised Value
- *When X = X*, F(X) = 1*
- *X^* and Y^* are reference values*



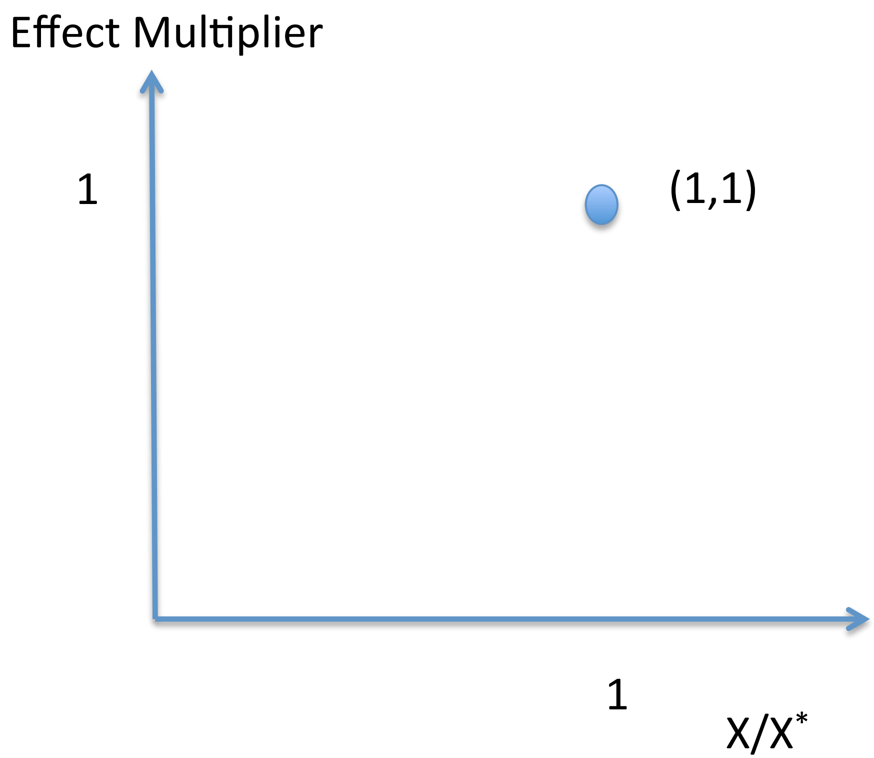
Example

- X = Rookie Proportion
- X^* = Reference Rookie Percentage
- Impact on productivity?
- (1,1) is always on the line



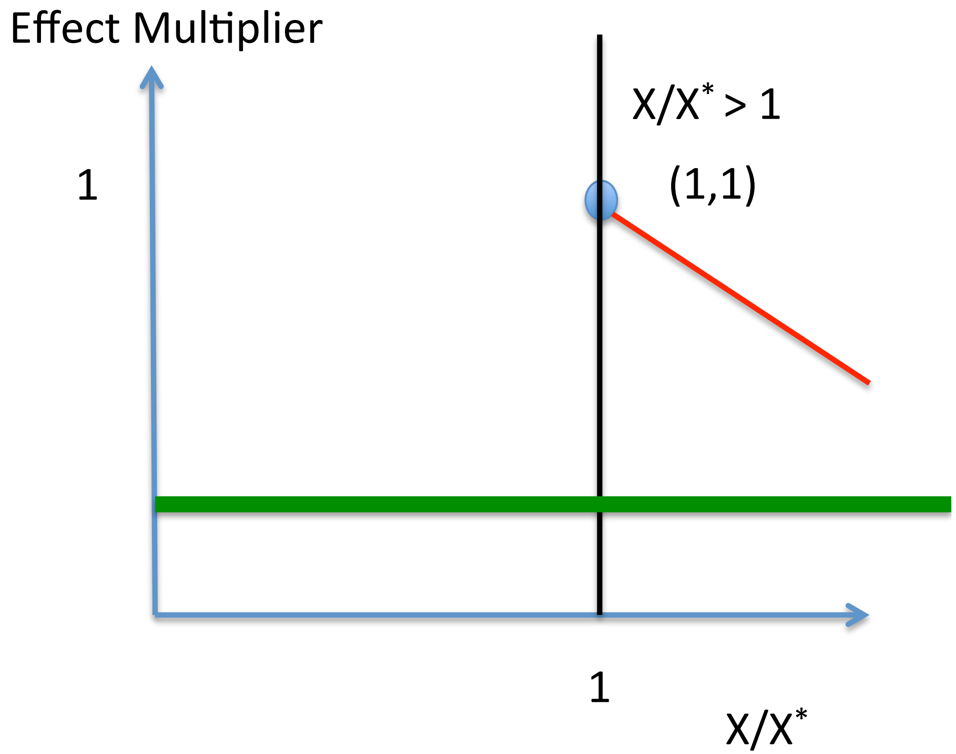
Thinking about the effects…

- X = Actual Rookie Proportion
- X^* = Reference Rookie Proportion (i.e. the number at which our experienced productivity is at its reference value)
- Scenarios:
  - If $X > X^*$, Effect?
  - If $X < X^*$, Effect?



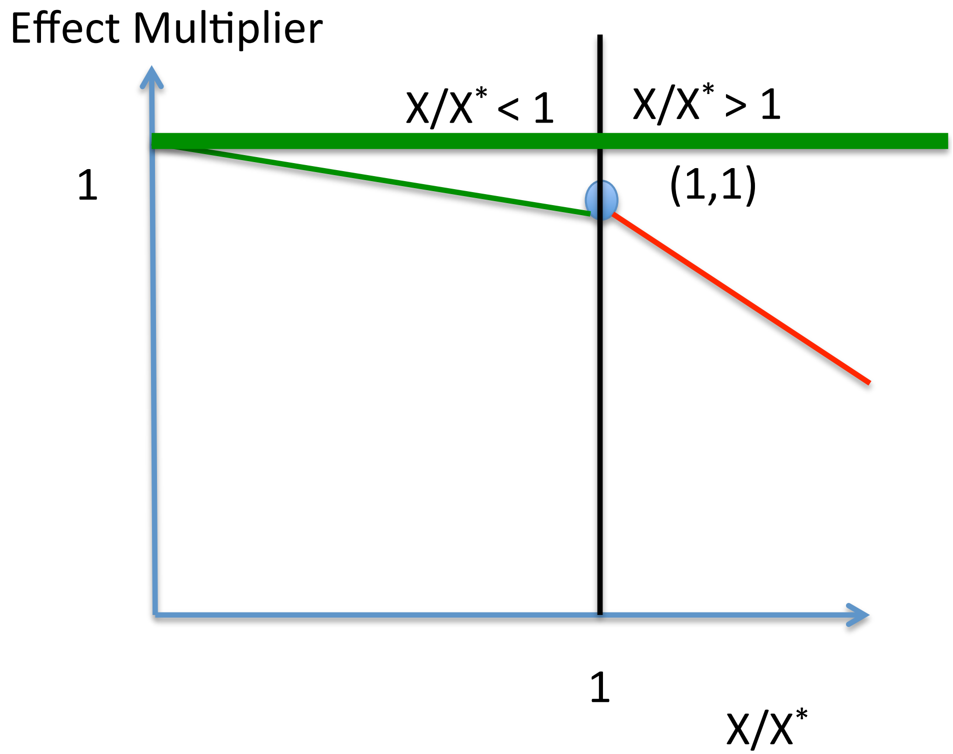
Sketching the relationship, More rookies than reference value

- $X > X^*$
  - We have more Rookies than our target level
  - This will reduce our experienced productivity
  - More work to train rookies
  - Effect will be lower than 1
  - Decide on minimum value (0.25)

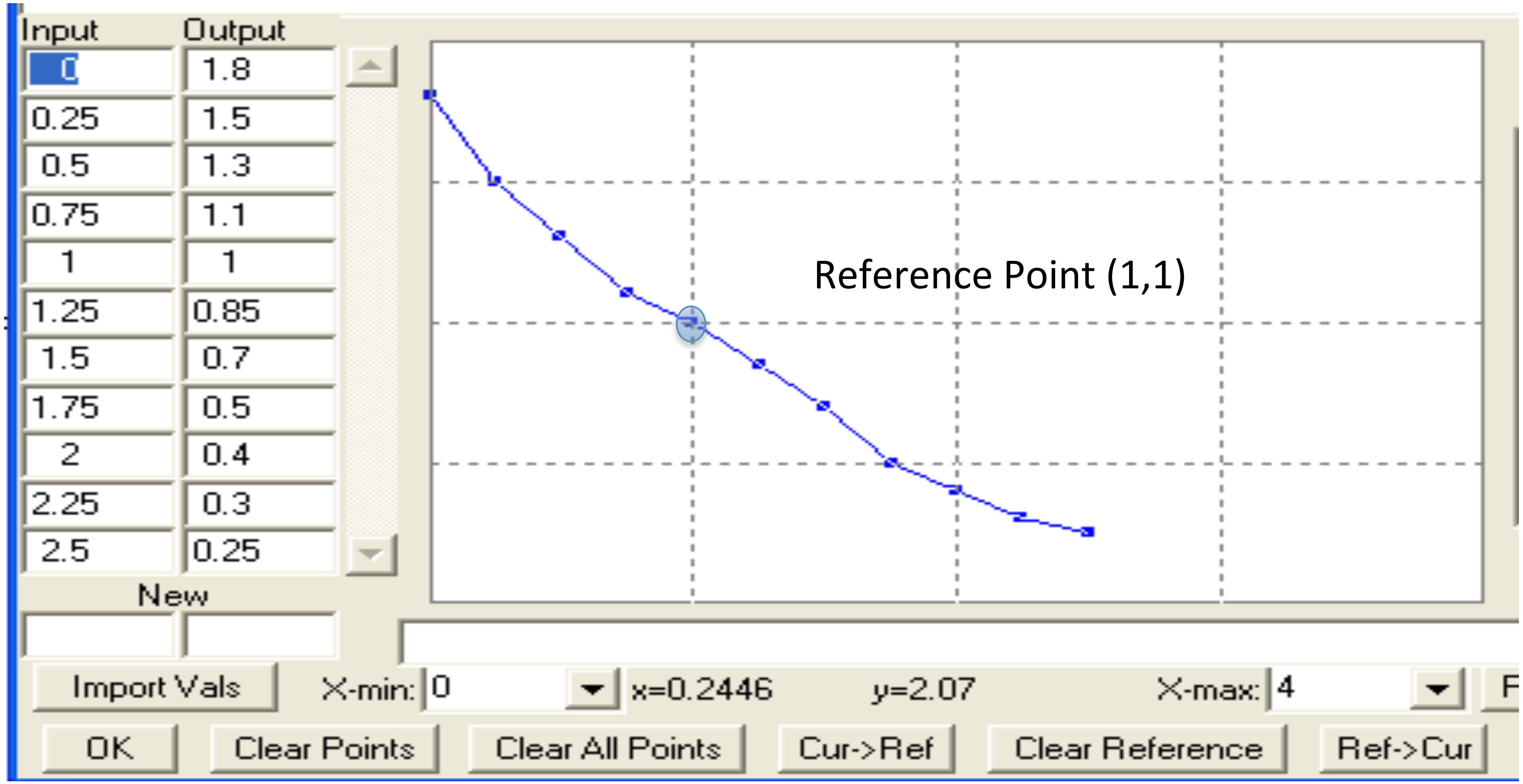


Sketching the relationship, Less rookies than reference value

- $X < X^*$
  - We have less Rookies than our target level
  - This will increase our experienced productivity
  - Less work to train rookies
  - Effect will be greater than 1
  - Decide on a maximum value (1.8)

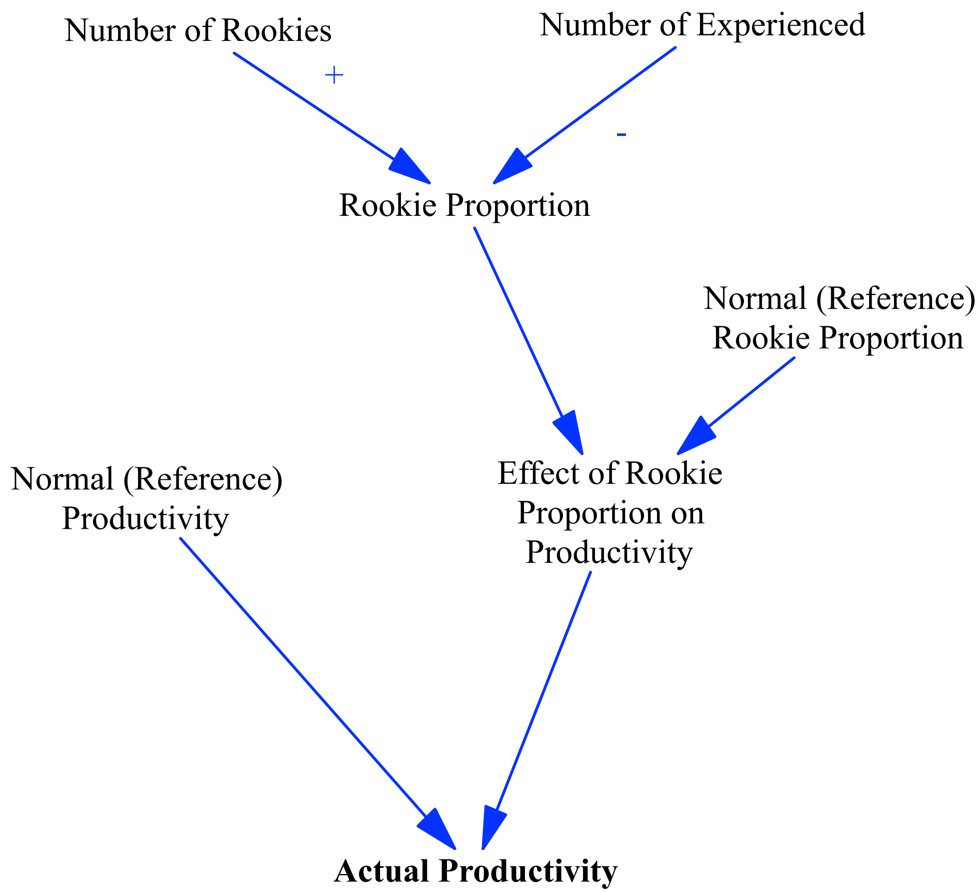


Effect equation represented as table lookup



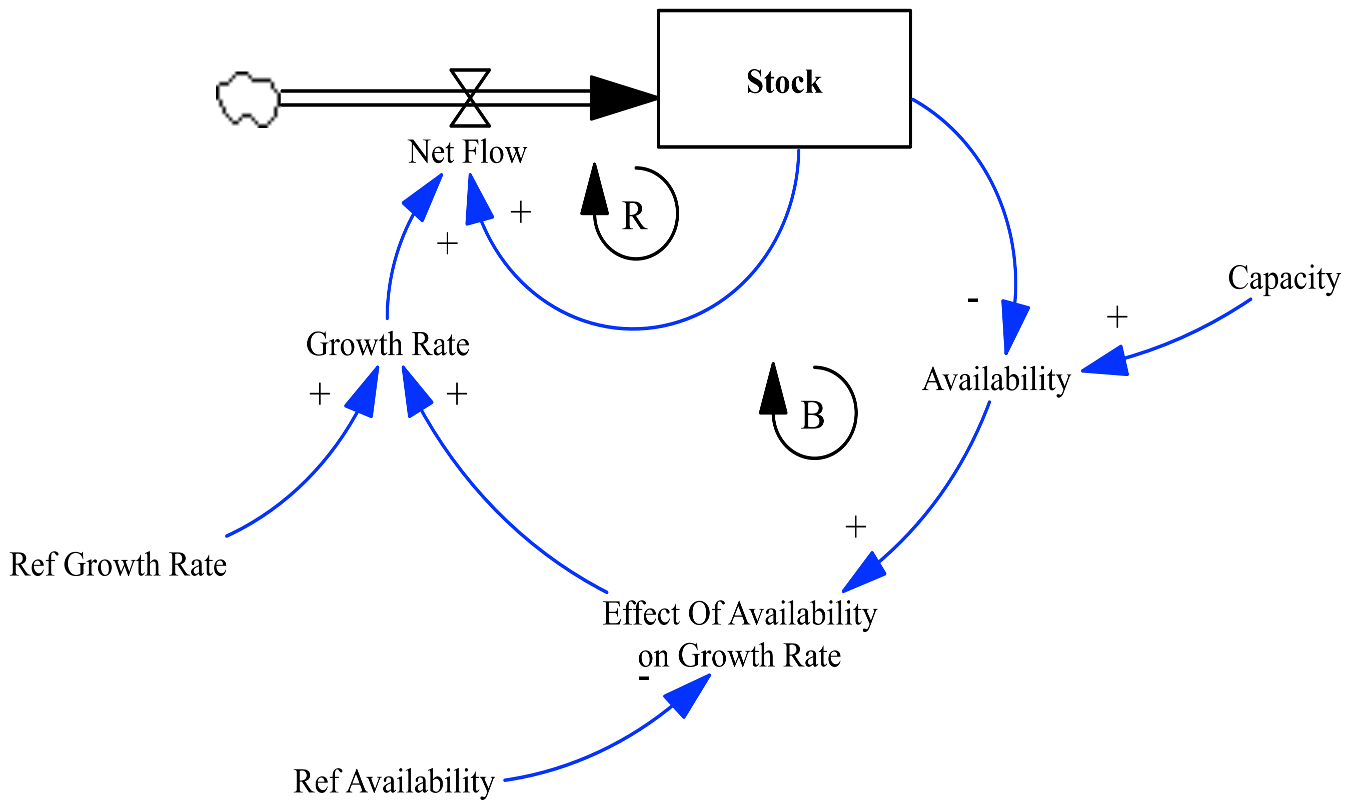
Challenge 4.1

- Extend the model to include the following variable:
  - Average time to promotion

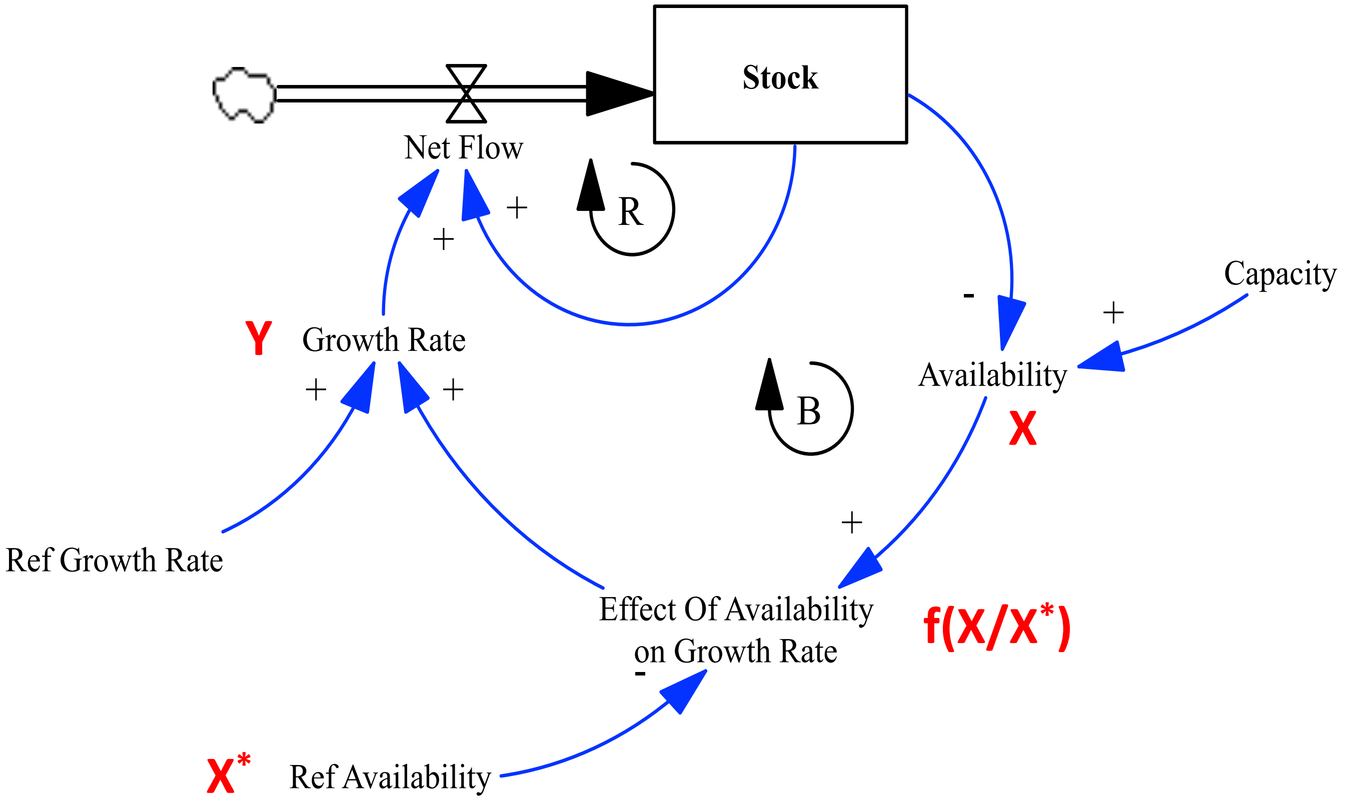


Limits to Growth Model

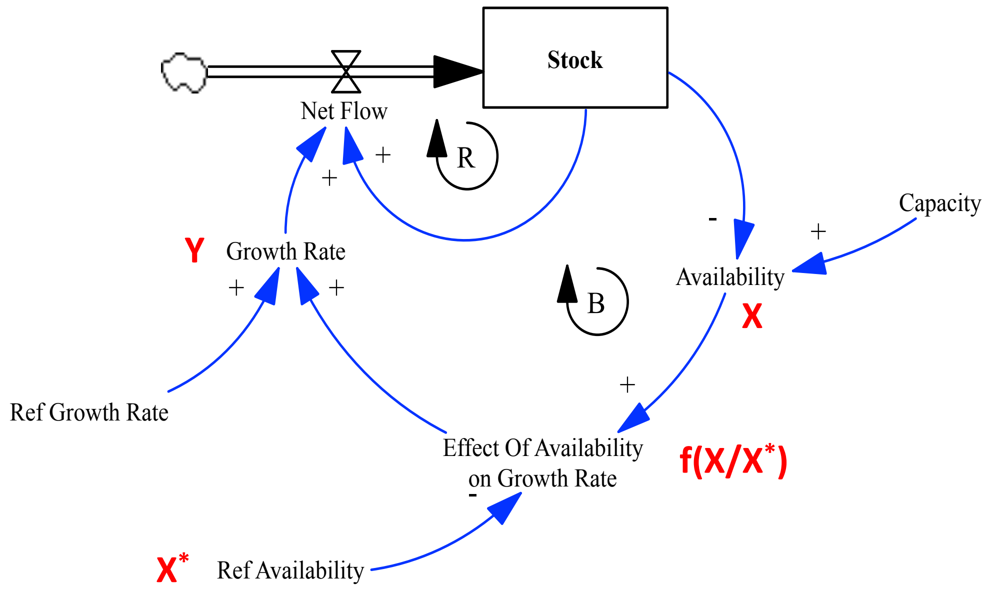
There will always be limits to growth. They can be self-imposed. If they aren't, they will be system-imposed.
Donella H. Meadows, Thinking in Systems: A Primer (2008), p.103



Formulating the Effect



Initial Equations



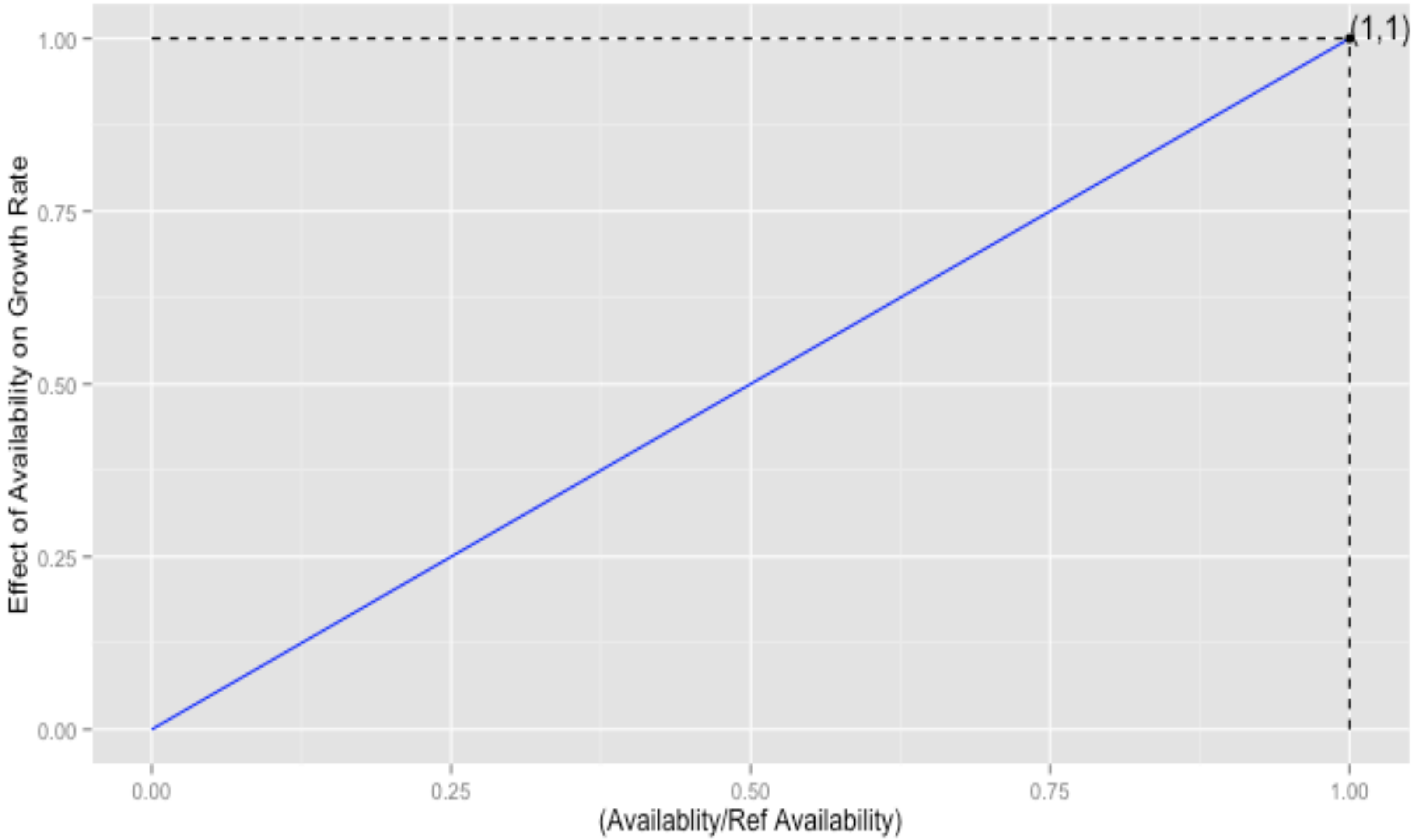
$$\text{Growth Rate} = \text{Ref Growth Rate} \times \text{Effect of Availability on Growth Rate}$$

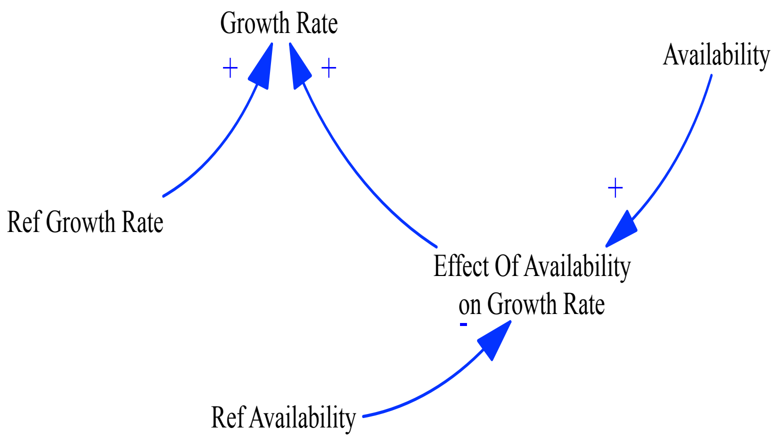
$$\text{Effect of Availability on Growth Rate} = f\left(\frac{\text{Availability}}{\text{Ref Availability}}\right)$$

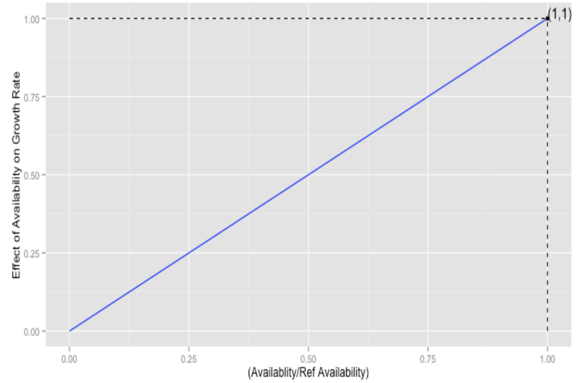
$$\text{Ref Growth Rate} = 0.10$$

$$\text{Ref Availability} = 1.0$$

Effect equation y = mx + c



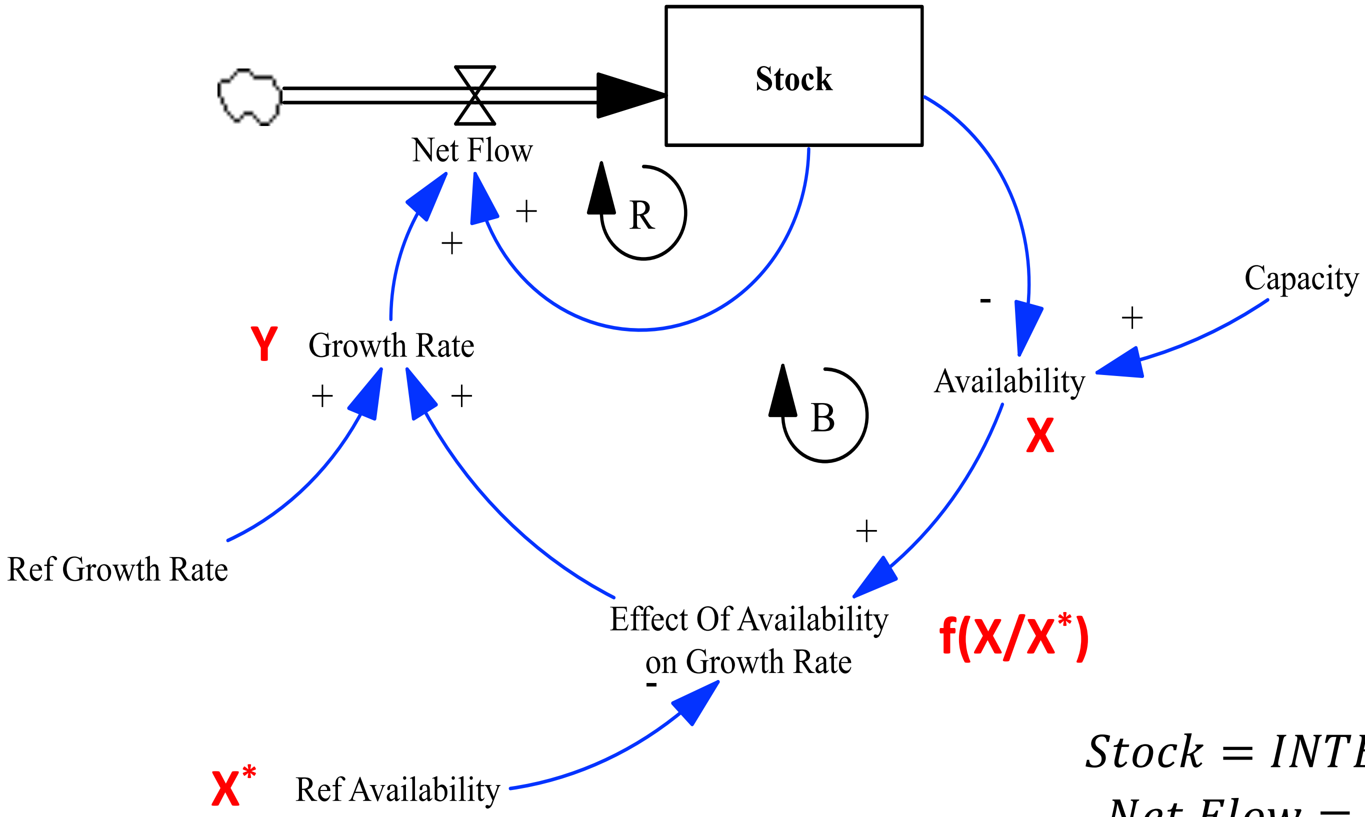




$$Effect\ of\ Availability\ on\ Growth\ Rate = \frac{Availability}{Ref\ Availability}$$

Ref Availability	*Availability*	*Effect of Availability on Growth Rate*	*Ref Growth Rate*	*Growth Rate*
1.0	1.0	1.0	0.10	0.10
1.0	0.5	0.5	0.10	0.05
1.0	0.0	0.0	0.10	0.00

Remaining Equations



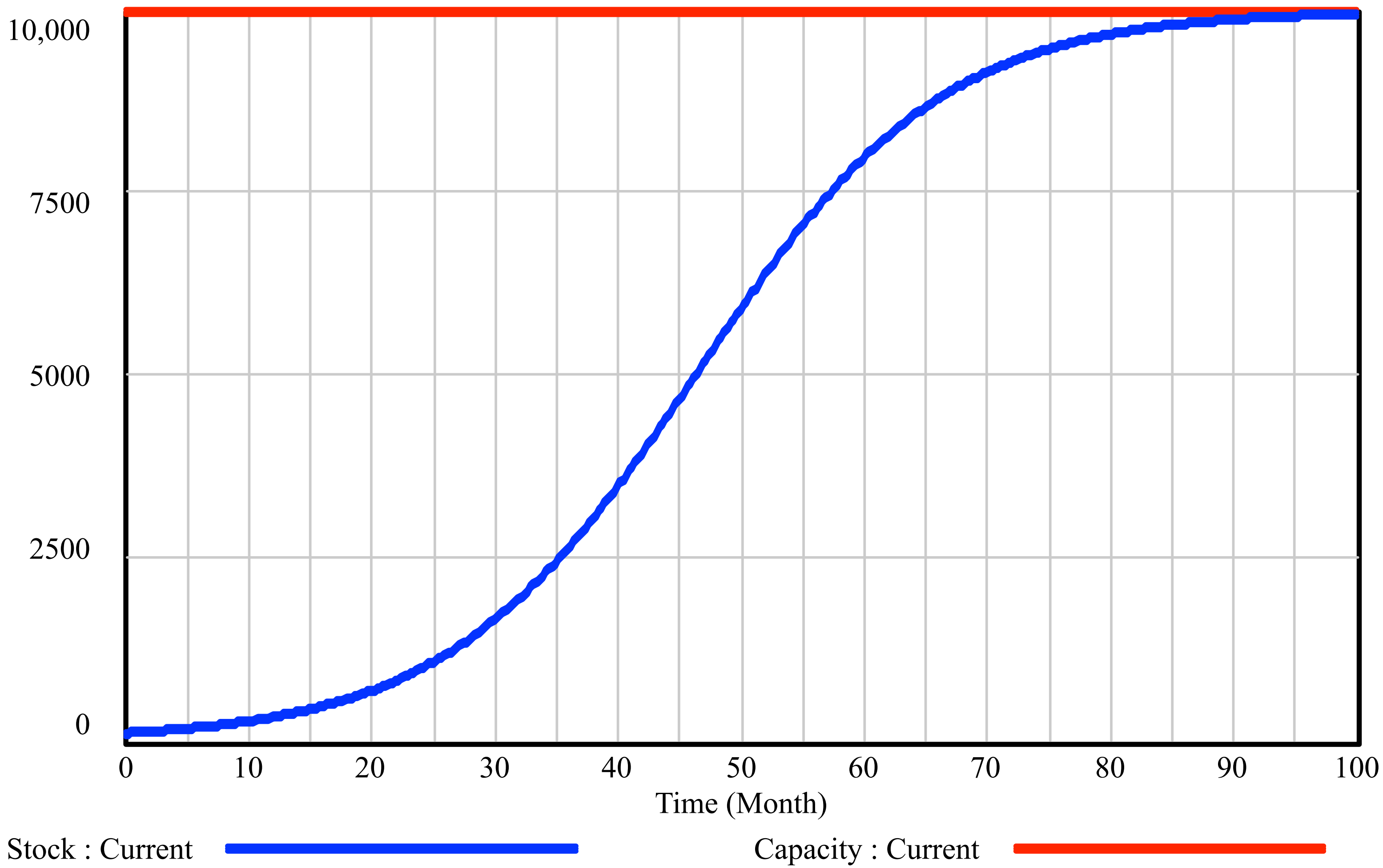
$$Stock = INTEGRAL(Net\ Flow, 100)$$

$$Net\ Flow = Stock \times Growth\ Rate$$

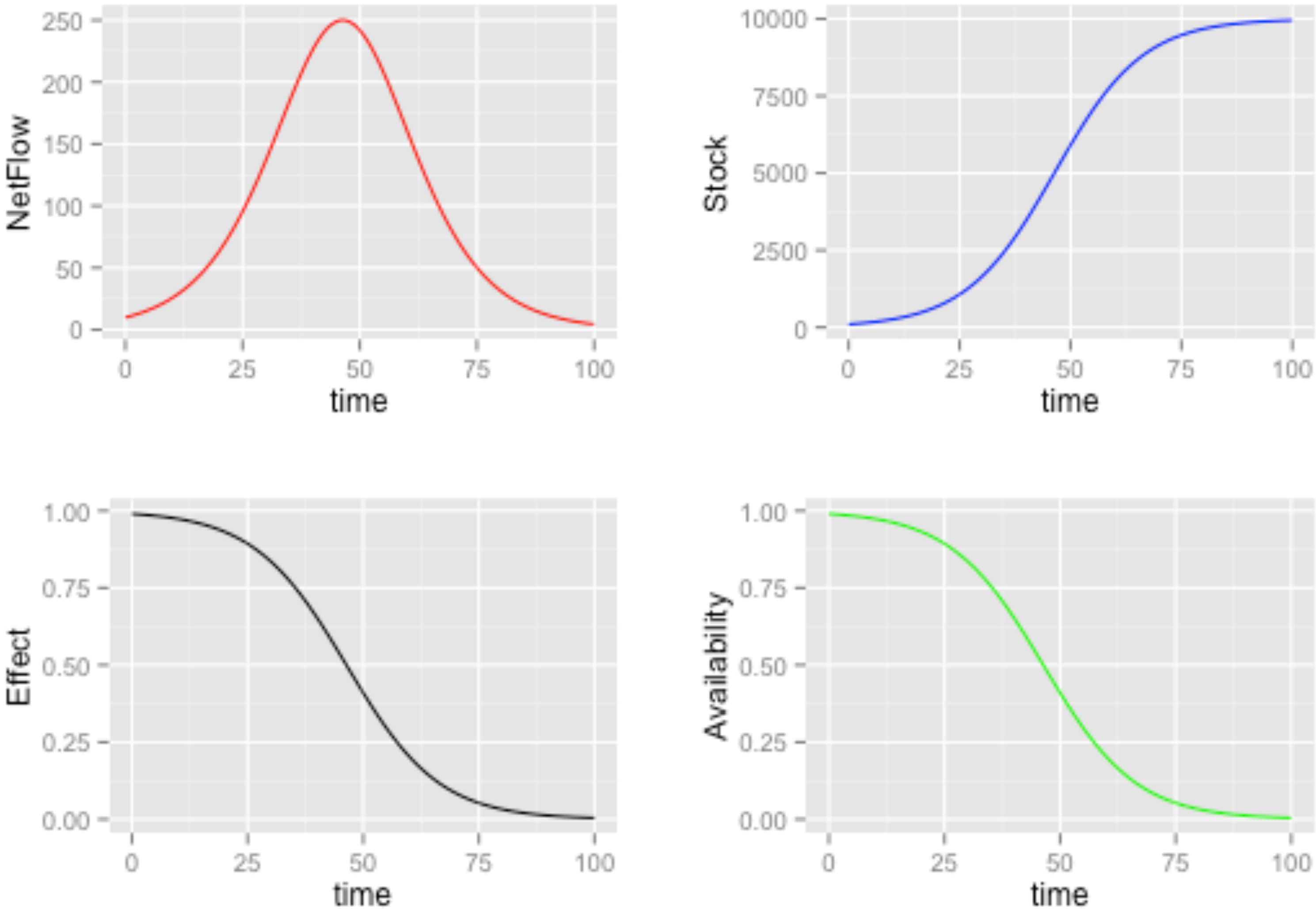
$$Availability = 1 - \frac{Stock}{Capacity}$$

$$Capacity = 10000$$

Limit constant, stock approaches



Simulation Output



Verhulst Equations (1838)

In ecology: modeling population growth [edit]

A typical application of the logistic equation is a common model of population growth, originally due to Pierre-François Verhulst in 1838, where the rate of reproduction is proportional to both the existing population and the amount of available resources, all else being equal. The Verhulst equation was published after Verhulst had read Thomas Malthus' *An Essay on the Principle of Population*. Verhulst derived his logistic equation to describe the self-limiting growth of a biological population. The equation was rediscovered in 1911 by A. G. McKendrick for the growth of bacteria in broth and experimentally tested using a technique for nonlinear parameter estimation.[4] The equation is also sometimes called the *Verhulst-Pearl equation* following its rediscovery in 1920 by Raymond Pearl (1879–1940) and Lowell Reed (1888–1966) of the Johns Hopkins University.[5] Another scientist, Alfred J. Lotka derived the equation again in 1925, calling it the *law of population growth*.

Pierre-François Verhulst (1804–1849)

Letting P represent population size (N is often used in ecology instead) and t represent time, this model is formalized by the differential equation:

$$\frac{dP}{dt} = rP \cdot \left(1 - \frac{P}{K}\right)$$

where the constant r defines the growth rate and K is the carrying capacity.

https://en.wikipedia.org/wiki/Logistic_function

Challenge 4.2

$$\frac{dP}{dt} = rP \cdot \left(1 - \frac{P}{K}\right)$$

where the constant r defines the growth rate and K is the carrying capacity.

- Map the Verhulst equation onto the stock and flow model.