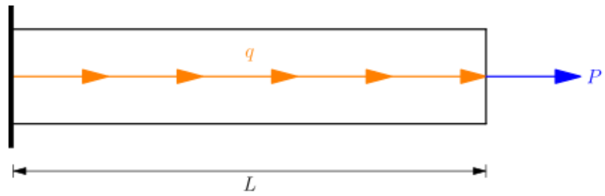
Ejemplo 1



Resolver

$$
\begin{aligned}
EA\,\frac{d^2u}{dx^2} + q = 0 \\
u(0) = 0 \\
u'(L) = \frac{P}{EA}
\end{aligned}
$$

Solución exacta

$$u(x) = -\frac{q}{2EA}x^2 + \frac{P + qL}{EA}x$$

Solución aproximada generalizada

La forma débil de la ecuación diferencial es

$$\int_0^L R(x)\,W(x)\,dx = \int_0^L \left(EA\,\frac{d^2\hat{u}}{dx^2} + q\right) W\,dx = 0$$

multiplicando

$$\int_0^L W\,EA\frac{d^2\hat{u}}{dx^2}\,dx + \int_0^L W\,q\,dx = 0$$

Usando el teorema de Gauss o integrando por partes

$$\left(W\,EA\frac{d\hat{u}}{dx}\right)\Bigg|_0^L - \int_0^L \frac{dW}{dx}EA\frac{d\hat{u}}{dx}\,dx + \int_0^L W\,q\,dx = 0$$

Reordenando

$$\int_0^L \frac{dW}{dx}EA\frac{d\hat{u}}{dx}\,dx = \int_0^L W\,q\,dx + \left(W\,EA\frac{d\hat{u}}{dx}\right)\Bigg|_0^L$$

Reemplazando $F = EA\frac{d\hat{u}}{dx}$

$$\int_0^L \frac{dW}{dx}EA\frac{d\hat{u}}{dx}\,dx = (W\,F)|_0^L$$