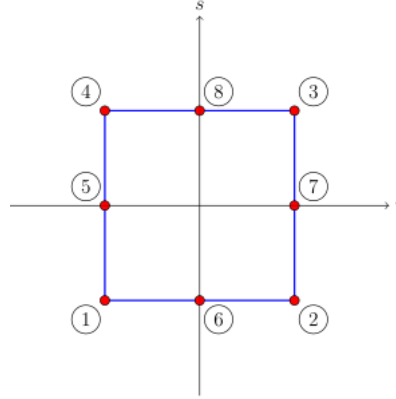# Introducción a elementos finitos
Tarea 4 I-2016

Calcular las funciones de forma del elemento bidimensional lagrangiano de ocho nodos en coordenadas naturales

Coordenadas de los nodos

$$\begin{aligned}
①&= [r_1, s_1] = [-1, -1] & ⑤&= [r_5, s_5] = [-1, 0] \\
②&= [r_2, s_2] = [1, -1] & ⑥&= [r_6, s_6] = [0, -1] \\
③&= [r_3, s_3] = [1, 1] & ⑦&= [r_7, s_7] = [1, 0] \\
④&= [r_4, s_4] = [-1, 1] & ⑧&= [r_8, s_8] = [0, 1]
\end{aligned}$$

Reemplazando valores

$$N_1 = \frac{r - r_6}{r_1 - r_6} \cdot \frac{r - r_2}{r_1 - r_2} \cdot \frac{s - s_5}{s_1 - s_5} \cdot \frac{s - s_4}{s_1 - s_4} = \frac{r - 0}{-1 - 0} \cdot \frac{r - 1}{-1 - 1} \cdot \frac{s - 0}{-1 - 0} \cdot \frac{s - 1}{-1 - 1} = \frac{1}{4} r(r - 1)s(s - 1)$$

$$N_2 = \frac{r - r_6}{r_2 - r_6} \cdot \frac{r - r_1}{r_2 - r_1} \cdot \frac{s - s_7}{s_2 - s_7} \cdot \frac{s - s_3}{s_2 - s_3} = \frac{r - 0}{1 - 0} \cdot \frac{r - (-1)}{1 - (-1)} \cdot \frac{s - 0}{-1 - 0} \cdot \frac{s - 1}{-1 - 1} = \frac{1}{4} r(r + 1)s(s - 1)$$

$$N_3 = \frac{r - r_8}{r_3 - r_8} \cdot \frac{r - r_4}{r_3 - r_4} \cdot \frac{s - s_7}{s_3 - s_7} \cdot \frac{s - s_2}{s_3 - s_2} = \frac{r - 0}{1 - 0} \cdot \frac{r - (-1)}{1 - (-1)} \cdot \frac{s - 0}{1 - 0} \cdot \frac{s - (-1)}{1 - (-1)} = \frac{1}{4} r(r + 1)s(s + 1)$$

$$N_4 = \frac{r - r_8}{r_4 - r_8} \cdot \frac{r - r_3}{r_4 - r_3} \cdot \frac{s - s_5}{s_4 - s_5} \cdot \frac{s - s_1}{s_4 - s_1} = \frac{r - 0}{-1 - 0} \cdot \frac{r - 1}{-1 - 1} \cdot \frac{s - 0}{1 - 0} \cdot \frac{s - (-1)}{1 - (-1)} = \frac{1}{4} r(r - 1)s(s + 1)$$

$$N_5 = \frac{r - r_7}{r_5 - r_7} \cdot \frac{s - s_1}{s_5 - s_1} \cdot \frac{s - s_4}{s_5 - s_4} = \frac{r - 1}{-1 - 1} \cdot \frac{s - (-1)}{0 - (-1)} \cdot \frac{s - 1}{0 - 1} = \frac{1}{2}(r - 1)(s + 1)(s - 1)$$

$$N_6 = \frac{r - r_2}{r_6 - r_2} \cdot \frac{r - r_1}{r_6 - r_1} \cdot \frac{s - s_8}{s_6 - s_8} = \frac{r - 1}{0 - 1} \cdot \frac{r - (-1)}{0 - (-1)} \cdot \frac{s - 1}{-1 - 1} = \frac{1}{2}(r - 1)(r + 1)(s - 1)$$

$$N_7 = \frac{r - r_5}{r_7 - r_5} \cdot \frac{s - s_2}{s_7 - s_2} \cdot \frac{s - s_3}{s_7 - s_3} = \frac{r - (-1)}{1 - (-1)} \cdot \frac{s - (-1)}{0 - (-1)} \cdot \frac{s - 1}{0 - 1} = -\frac{1}{2}(r + 1)(s + 1)(s - 1)$$

$$N_8 = \frac{r - r_3}{r_8 - r_3} \cdot \frac{r - r_4}{r_8 - r_4} \cdot \frac{s - s_6}{s_8 - s_6} = \frac{r - 1}{0 - 1} \cdot \frac{r - (-1)}{0 - (-1)} \cdot \frac{s - (-1)}{1 - (-1)} = -\frac{1}{2}(r - 1)(r + 1)(s + 1)$$