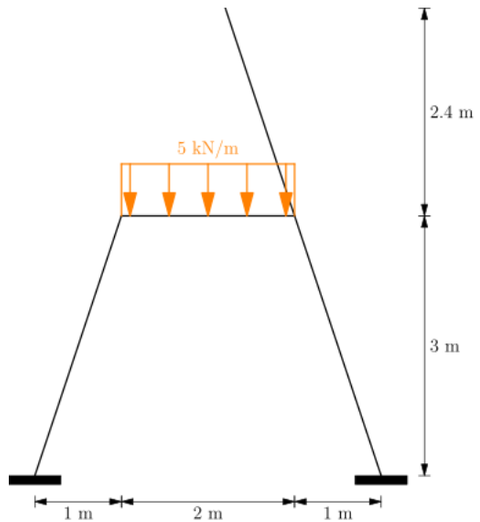
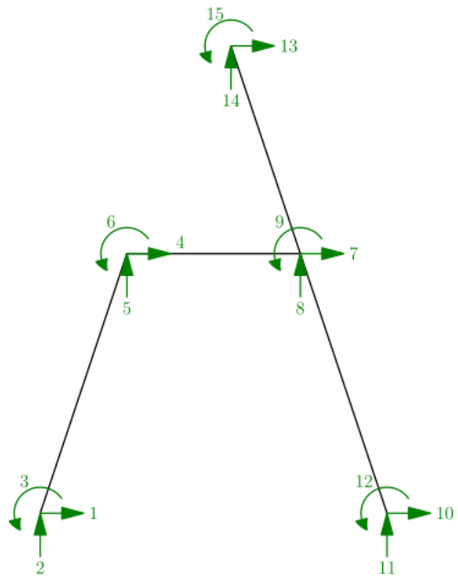
Introducción a elementos finitos
Tarea 3 I-2016

Resolver mediante matriz de rigidez



Coordenadas de los nodos

$$\textcircled{1} = [0, 0]$$
$$\textcircled{2} = [1, 3]$$
$$\textcircled{3} = [3, 3]$$
$$\textcircled{4} = [4, 0]$$
$$\textcircled{5} = [2.2, 5.4]$$



Propiedades geométricas

Elemento [1], tiene la dirección ① - ②

$$A = 0.3 \cdot 0.4 = 0.12 \text{ m}^2$$
$$I = \frac{0.3 \cdot 0.4^3}{12} = 0.0016\text{m}^4$$
$$L = \sqrt{(1-0)^2 + (3-0)^2} = 3.162 \text{ m}$$
$$\lambda_x = \frac{1-0}{3.162} = 0.316$$
$$\lambda_y = \frac{3-0}{3.162} = 0.949$$

Elemento [2], tiene la dirección ② - ③

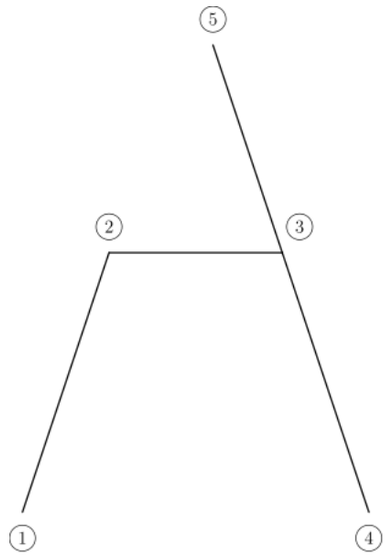
$$A = 0.25 \cdot 0.4 = 0.1 \text{ m}^2$$
$$I = \frac{0.25 \cdot 0.4^3}{12} = 0.0013\text{m}^4$$
$$L = \sqrt{(3-1)^2 + (3-.)^2} = 2 \text{ m}$$
$$\lambda_x = \frac{3-1}{2} = 1$$
$$\lambda_y = \frac{3-3}{2} = 0$$

Elemento $\boxed{3}$, tiene la dirección ③ - ④

$$A = 0.3 \cdot 0.4 = 0.12 \text{ m}^2$$
$$I = \frac{0.3 \cdot 0.4^3}{12} = 0.0016\text{m}^4$$
$$L = \sqrt{(4-3)^2 + (0-3)^2} = 3.162 \text{ m}$$
$$\lambda_x = \frac{4-3}{3.162} = 0.316$$
$$\lambda_y = \frac{0-3}{3.162} = -0.949$$

Elemento $\boxed{4}$, tiene la dirección ③ - ⑤

$$A = 0.3 \cdot 0.4 = 0.12 \text{ m}^2$$
$$I = \frac{0.3 \cdot 0.4^3}{12} = 0.0016\text{m}^4$$
$$L = \sqrt{(2.2-3)^2 + (5.4-3)^2} = 2.53 \text{ m}$$
$$\lambda_x = \frac{2.2-3}{2.53} = -0.316$$
$$\lambda_y = \frac{5.4-3}{2.53} = 0.949$$



Matriz de rigidez local de cada elemento

Elemento 1

$$k' = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 79696.39 & 0 & 0 & -79696.39 & 0 & 0 \\ 0 & 1275.37 & 2016.35 & 0 & -1275.37 & 2016.35 \\ 0 & 2016.35 & 4250.47 & 0 & -2016.35 & 2125.24 \\ -79696.39 & 0 & 0 & 79696.39 & 0 & 0 \\ 0 & -1275.37 & -2016.35 & 0 & 1275.37 & -2016.35 \\ 0 & 2016.35 & 2125.24 & 0 & -2016.35 & 4250.47 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Elemento 2

$$k' = \begin{array}{c} \begin{matrix} 4 & 5 & 6 & 7 & 8 & 9 \end{matrix} \\ \begin{bmatrix} 105000 & 0 & 0 & -105000 & 0 & 0 \\ 0 & 4095 & 4095 & 0 & -4095 & 4095 \\ 0 & 4095 & 5460 & 0 & -4095 & 2730 \\ -105000 & 0 & 0 & 105000 & 0 & 0 \\ 0 & -4095 & -4095 & 0 & 4095 & -4095 \\ 0 & 4095 & 2730 & 0 & -4095 & 5460 \end{bmatrix} \end{array} \begin{matrix} 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{matrix}$$

Elemento 3

$$k' = \begin{array}{c} \begin{matrix} 7 & 8 & 9 & 10 & 11 & 12 \end{matrix} \\ \begin{bmatrix} 79696.39 & 0 & 0 & -79696.39 & 0 & 0 \\ 0 & 1275.37 & 2016.35 & 0 & -1275.37 & 2016.35 \\ 0 & 2016.35 & 4250.47 & 0 & -2016.35 & 2125.24 \\ -79696.39 & 0 & 0 & 79696.39 & 0 & 0 \\ 0 & -1275.37 & -2016.35 & 0 & 1275.37 & -2016.35 \\ 0 & 2016.35 & 2125.24 & 0 & -2016.35 & 4250.47 \end{bmatrix} \end{array} \begin{matrix} 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{matrix}$$

Elemento 4

$$k' = \begin{array}{c} \begin{matrix} 7 & 8 & 9 & 13 & 14 & 15 \end{matrix} \\ \begin{bmatrix} 99604.74 & 0 & 0 & -99604.74 & 0 & 0 \\ 0 & 2489.77 & 3149.56 & 0 & -2489.77 & 3149.56 \\ 0 & 3149.56 & 5312.25 & 0 & -3149.56 & 2656.13 \\ -99604.74 & 0 & 0 & 99604.74 & 0 & 0 \\ 0 & -2489.77 & -3149.56 & 0 & 2489.77 & -3149.56 \\ 0 & 3149.56 & 2656.13 & 0 & -3149.56 & 5312.25 \end{bmatrix} \end{array} \begin{matrix} 7 \\ 8 \\ 9 \\ 13 \\ 14 \\ 15 \end{matrix}$$

Matriz de rotación de cada elemento

Elemento 1

$$T = \begin{bmatrix} 0.316 & 0.949 & 0 & 0 & 0 & 0 \\ -0.949 & 0.316 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.316 & 0.949 & 0 \\ 0 & 0 & 0 & -0.949 & 0.316 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Elemento $\boxed{2}$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Elemento $\boxed{3}$

$$T = \begin{bmatrix} 0.316 & -0.949 & 0 & 0 & 0 & 0 \\ 0.949 & 0.316 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.316 & -0.949 & 0 \\ 0 & 0 & 0 & 0.949 & 0.316 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Elemento $\boxed{4}$

$$T = \begin{bmatrix} -0.316 & 0.949 & 0 & 0 & 0 & 0 \\ -0.949 & -0.316 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.316 & 0.949 & 0 \\ 0 & 0 & 0 & -0.949 & -0.316 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Matriz de rigidez global de cada elemento

Elemento $\boxed{1}$

$$k = T^T k' T = \begin{bmatrix} 0.316 & -0.949 & 0 & 0 & 0 & 0 \\ 0.949 & 0.316 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.316 & -0.949 & 0 \\ 0 & 0 & 0 & 0.949 & 0.316 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 79696.39 & 0 & 0 & -79696.39 & 0 & 0 \\ 0 & 1275.37 & 2016.35 & 0 & -1275.37 & 2016.35 \\ 0 & 2016.35 & 4250.47 & 0 & -2016.35 & 2125.24 \\ -79696.39 & 0 & 0 & 79696.39 & 0 & 0 \\ 0 & -1275.37 & -2016.35 & 0 & 1275.37 & -2016.35 \\ 0 & 2016.35 & 2125.24 & 0 & -2016.35 & 4250.47 \end{bmatrix} \begin{bmatrix} 0.316 & 0.949 & 0 & 0 & 0 & 0 \\ -0.949 & 0.316 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.316 & 0.949 & 0 \\ 0 & 0 & 0 & -0.949 & 0.316 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9116.47 & 23524.31 & -1912.55 & -9116.47 & -23524.31 & -1912.55 \\ 23524.31 & 71847.96 & 637.52 & -23524.31 & -71847.96 & 637.52 \\ -1912.55 & 637.52 & 4250.1 & 1912.55 & -637.52 & 2125.05 \\ -9116.47 & -23524.31 & 1912.55 & 9116.47 & 23524.31 & 1912.55 \\ -23524.31 & -71847.96 & -637.52 & 23524.31 & 71847.96 & -637.52 \\ -1912.55 & 637.52 & 2125.05 & 1912.55 & -637.52 & 4250.1 \end{bmatrix}$$

Elemento $\boxed{2}$

$$k = T^T k' T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 105000 & 0 & 0 & -105000 & 0 & 0 \\ 0 & 4095 & 4095 & 0 & -4095 & 4095 \\ 0 & 4095 & 5460 & 0 & -4095 & 2730 \\ -105000 & 0 & 0 & 105000 & 0 & 0 \\ 0 & -4095 & -4095 & 0 & 4095 & -4095 \\ 0 & 4095 & 2730 & 0 & -4095 & 5460 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 105000 & 0 & 0 & -105000 & 0 & 0 \\ 0 & 4095 & 4095 & 0 & -4095 & 4095 \\ 0 & 4095 & 5460 & 0 & -4095 & 2730 \\ -105000 & 0 & 0 & 105000 & 0 & 0 \\ 0 & -4095 & -4095 & 0 & 4095 & -4095 \\ 0 & 4095 & 2730 & 0 & -4095 & 5460 \end{bmatrix}$$

Elemento $\boxed{3}$

$$k = T^T k' T = \begin{bmatrix} 0.316 & 0.949 & 0 & 0 & 0 & 0 \\ -0.949 & 0.316 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.316 & 0.949 & 0 \\ 0 & 0 & 0 & -0.949 & 0.316 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 79696.39 & 0 & 0 & -79696.39 & 0 & 0 \\ 0 & 1275.37 & 2016.35 & 0 & -1275.37 & \\ 0 & 2016.35 & 4250.47 & 0 & -2016.35 & \\ -79696.39 & 0 & 0 & 79696.39 & 0 & 0 \\ 0 & -1275.37 & -2016.35 & 0 & 1275.37 & -2016.35 \\ 0 & 2016.35 & 2125.24 & 0 & -2016.35 & 4250.47 \end{bmatrix} \begin{bmatrix} 0.316 & -0.949 & 0 & 0 & 0 & 0 \\ 0.949 & 0.316 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.316 & -0.949 & 0 \\ 0 & 0 & 0 & 0.949 & 0.316 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9116.47 & -23524.31 & 1912.55 & -9116.47 & 23524.31 & 1912.55 \\ -23524.31 & 71847.96 & 637.52 & 23524.31 & -71847.96 & 637.52 \\ 1912.55 & 637.52 & 4250.1 & -1912.55 & -637.52 & 2125.05 \\ -9116.47 & 23524.31 & -1912.55 & 9116.47 & -23524.31 & -1912.55 \\ 23524.31 & -71847.96 & -637.52 & -23524.31 & 71847.96 & -637.52 \\ 1912.55 & 637.52 & 2125.05 & -1912.55 & -637.52 & 4250.1 \end{bmatrix}$$

Elemento $\boxed{4}$

$$k = T^T k' T = \begin{bmatrix} -0.316 & -0.949 & 0 & 0 & 0 & 0 \\ 0.949 & -0.316 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.316 & -0.949 & 0 \\ 0 & 0 & 0 & 0.949 & -0.316 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 99604.74 & 0 & 0 & -99604.74 & 0 & 0 \\ 0 & 2489.77 & 3149.56 & 0 & -2489.77 & \\ 0 & 3149.56 & 5312.25 & 0 & -3149.56 & 2656.13 \\ -99604.74 & 0 & 0 & 99604.74 & 0 & 0 \\ 0 & -2489.77 & -3149.56 & 0 & 2489.77 & -3149.56 \\ 0 & 3149.56 & 2656.13 & 0 & -3149.56 & 5312.25 \end{bmatrix} \begin{bmatrix} -0.316 & 0.949 & 0 & 0 & 0 & 0 \\ -0.949 & -0.316 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.316 & 0.949 & 0 \\ 0 & 0 & 0 & -0.949 & -0.316 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 12202.44 & -29136.44 & -2988.35 & -12202.44 & 29136.44 & -2988.35 \\ -29136.44 & 89899.6 & -996.12 & 29136.44 & -89899.6 & -996.12 \\ -2988.35 & -996.12 & 5312.63 & 2988.35 & 996.12 & 2656.31 \\ -12202.44 & 29136.44 & 2988.35 & 12202.44 & -29136.44 & 2988.35 \\ 29136.44 & -89899.6 & 996.12 & -29136.44 & 89899.6 & 996.12 \\ -2988.35 & -996.12 & 2656.31 & 2988.35 & 996.12 & 5312.63 \end{bmatrix}$$

Matriz de rigidez de la estructura

$$K = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \end{matrix} \\ \begin{bmatrix} 9116.47 & 23524.31 & -1912.55 & -9116.47 & -23524.31 & -1912.55 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 23524.31 & 71847.96 & 637.52 & -23524.31 & -71847.96 & 637.52 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1912.55 & 637.52 & 4250.1 & 1912.55 & -637.52 & 2125.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -9116.47 & -23524.31 & 1912.55 & 114116.47 & 23524.31 & 1912.55 & -105000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -23524.31 & -71847.96 & -637.52 & 23524.31 & 75942.96 & 3457.48 & 0 & -4095 & 4095 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1912.55 & 637.52 & 2125.05 & 1912.55 & 3457.48 & 9710.1 & 0 & -4095 & 2730 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -105000 & 0 & 0 & 126318.91 & -52660.75 & -1075.81 & -9116.47 & 23524.31 & 1912.55 & -12202.44 & 29136.44 & -2988.35 \\ 0 & 0 & 0 & 0 & -4095 & -4095 & -52660.75 & 165842.56 & -4453.6 & 23524.31 & -71847.96 & 637.52 & 29136.44 & -89899.6 & -996.12 \\ 0 & 0 & 0 & 0 & 4095 & 2730 & -1075.81 & -4453.6 & 15022.73 & -1912.55 & -637.52 & 2125.05 & 2988.35 & 996.12 & 2656.31 \\ 0 & 0 & 0 & 0 & 0 & 0 & -9116.47 & 23524.31 & -1912.55 & 9116.47 & -23524.31 & -1912.55 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 23524.31 & -71847.96 & -637.52 & -23524.31 & 71847.96 & -637.52 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1912.55 & 637.52 & 2125.05 & -1912.55 & -637.52 & 4250.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -12202.44 & 29136.44 & 2988.35 & 0 & 0 & 0 & 12202.44 & -29136.44 & 2988.35 \\ 0 & 0 & 0 & 0 & 0 & 0 & 29136.44 & -89899.6 & 996.12 & 0 & 0 & 0 & -29136.44 & 89899.6 & 996.12 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2988.35 & -996.12 & 2656.31 & 0 & 0 & 0 & 2988.35 & 996.12 & 5312.63 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \end{matrix}$$

Cargas nodales

$$Q^T = \begin{bmatrix} Q_1 & Q_2 & Q_3 & 0 & -5 & -1.667 & 0 & -5 & 1.667 & Q_{10} & Q_{11} & Q_{12} & 0 & 0 & 0 \end{bmatrix}$$

Desplazamientos nodales

$$D^T = \begin{bmatrix} 0 & 0 & 0 & D_4 & D_5 & D_6 & D_7 & D_8 & D_9 & 0 & 0 & 0 & D_{13} & D_{14} & D_{15} \end{bmatrix}$$

Sistema de ecuaciones y solución

$$
\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ 0 \\ -5 \\ -1.667 \\ 0 \\ -5 \\ 1.667 \\ Q_{10} \\ Q_{11} \\ Q_{12} \\ 0 \\ 0 \\ 0 \end{bmatrix} =
\begin{bmatrix}
9116.47 & 23524.31 & -1912.55 & -9116.47 & -23524.31 & -1912.55 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
23524.31 & 71847.96 & 637.52 & -23524.31 & -71847.96 & 637.52 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1912.55 & 637.52 & 4250.1 & 1912.55 & -637.52 & 2125.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-9116.47 & -23524.31 & 1912.55 & 114116.47 & 23524.31 & 1912.55 & -105000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-23524.31 & -71847.96 & -637.52 & 23524.31 & 75942.96 & 3457.48 & 0 & -4095 & 4095 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1912.55 & 637.52 & 2125.05 & 1912.55 & 3457.48 & 9710.1 & 0 & -4095 & 2730 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -105000 & 0 & 0 & 126318.91 & -52660.75 & -1075.81 & -9116.47 & 23524.31 & 1912.55 & -12202.44 & 29136.44 & -2988.35 \\
0 & 0 & 0 & 0 & -4095 & -4095 & -52660.75 & 165842.56 & -4453.6 & 23524.31 & -71847.96 & 637.52 & 29136.44 & -89899.6 & -996.12 \\
0 & 0 & 0 & 0 & 4095 & 2730 & -1075.81 & -4453.6 & 15022.73 & -1912.55 & -637.52 & 2125.05 & 2988.35 & 996.12 & 2656.31 \\
0 & 0 & 0 & 0 & 0 & 0 & -9116.47 & 23524.31 & -1912.55 & 9116.47 & -23524.31 & -1912.55 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 23524.31 & -71847.96 & -637.52 & -23524.31 & 71847.96 & -637.52 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1912.55 & 637.52 & 2125.05 & -1912.55 & -637.52 & 4250.1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -12202.44 & 29136.44 & 2988.35 & 0 & 0 & 0 & 12202.44 & -29136.44 & 2988.35 \\
0 & 0 & 0 & 0 & 0 & 0 & 29136.44 & -89899.6 & 996.12 & 0 & 0 & 0 & -29136.44 & 89899.6 & 996.12 \\
0 & 0 & 0 & 0 & 0 & 0 & -2988.35 & -996.12 & 2656.31 & 0 & 0 & 0 & 2988.35 & 996.12 & 5312.63
\end{bmatrix}
\begin{bmatrix} 0 \\ 0 \\ 0 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \\ 0 \\ 0 \\ 0 \\ D_{13} \\ D_{14} \\ D_{15} \end{bmatrix}
$$

Resolviendo

$$
\begin{array}{lll}
Q_1 = 2.15 \text{ kN} & Q_2 = 5.00 \text{ kN} & Q_3 = -0.46 \text{ kN} \cdot \text{m} \\
D_4 = 1.02 \times 10^{-5} \text{ m} & D_5 = -7.51 \times 10^{-5} \text{ m} & D_6 = -2.48 \times 10^{-4} \text{ rad} \\
D_7 = -1.02 \times 10^{-5} \text{ m} & D_8 = -7.51 \times 10^{-5} \text{ m} & D_9 = 2.48 \times 10^{-4} \text{ rad} \\
Q_{10} = -2.15 \text{ kN} & Q_{11} = 5.00 \text{ kN} & Q_{12} = 0.46 \text{ kN} \cdot \text{m} \\
D_{13} = -6.07 \times 10^{-4} \text{ m} & D_{14} = -2.74 \times 10^{-4} \text{ m} & D_{15} = 2.48 \times 10^{-4} \text{ rad}
\end{array}
$$

Diagrama de esfuerzos

Elemento $\boxed{1}$

$$q = k'TD = \begin{bmatrix} 79696.39 & 0 & 0 & -79696.39 & 0 & 0 \\ 0 & 1275.37 & 2016.35 & 0 & -1275.37 & 2016.35 \\ 0 & 2016.35 & 4250.47 & 0 & -2016.35 & 2125.24 \\ -79696.39 & 0 & 0 & 79696.39 & 0 & 0 \\ 0 & -1275.37 & -2016.35 & 0 & 1275.37 & -2016.35 \\ 0 & 2016.35 & 2125.24 & 0 & -2016.35 & 4250.47 \end{bmatrix} \begin{bmatrix} 0.316 & 0.949 & 0 & 0 & 0 & 0 \\ -0.949 & 0.316 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.316 & 0.949 & 0 \\ 0 & 0 & 0 & -0.949 & 0.316 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.02 \times 10^{-5} \\ -7.51 \times 10^{-5} \\ -2.48 \times 10^{-4} \end{bmatrix} = \begin{bmatrix} 5.423 \\ -0.458 \\ -0.460 \\ -5.423 \\ 0.458 \\ -0.989 \end{bmatrix}$$

Elemento $\boxed{2}$

$$q = k'TD + q_0 = \begin{bmatrix} 105000 & 0 & 0 & -105000 & 0 & 0 \\ 0 & 4095 & 4095 & 0 & -4095 & 4095 \\ 0 & 4095 & 5460 & 0 & -4095 & 2730 \\ -105000 & 0 & 0 & 105000 & 0 & 0 \\ 0 & -4095 & -4095 & 0 & 4095 & -4095 \\ 0 & 4095 & 2730 & 0 & -4095 & 5460 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.02 \times 10^{-5} \\ -7.51 \times 10^{-5} \\ -2.48 \times 10^{-4} \\ -1.02 \times 10^{-5} \\ -7.51 \times 10^{-5} \\ 2.48 \times 10^{-4} \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \\ 1.667 \\ 0 \\ 5 \\ -1.667 \end{bmatrix} = \begin{bmatrix} 2.15 \\ 5 \\ 0.989 \\ -2.15 \\ 5 \\ -0.989 \end{bmatrix}$$

Elemento $\boxed{3}$

$$q = k'TD = \begin{bmatrix} 79696.39 & 0 & 0 & -79696.39 & 0 & 0 \\ 0 & 1275.37 & 2016.35 & 0 & -1275.37 & 2016.35 \\ 0 & 2016.35 & 4250.47 & 0 & -2016.35 & 2125.24 \\ -79696.39 & 0 & 0 & 79696.39 & 0 & 0 \\ 0 & -1275.37 & -2016.35 & 0 & 1275.37 & -2016.35 \\ 0 & 2016.35 & 2125.24 & 0 & -2016.35 & 4250.47 \end{bmatrix} \begin{bmatrix} 0.316 & -0.949 & 0 & 0 & 0 & 0 \\ 0.949 & 0.316 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.316 & -0.949 & 0 \\ 0 & 0 & 0 & 0.949 & 0.316 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1.02 \times 10^{-5} \\ -7.51 \times 10^{-5} \\ 2.48 \times 10^{-4} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5.423 \\ 0.458 \\ 0.989 \\ -5.423 \\ -0.458 \\ 0.461 \end{bmatrix}$$

Elemento $\boxed{4}$

$$q = k'TD = \begin{bmatrix} 99604.74 & 0 & 0 & -99604.74 & 0 & 0 \\ 0 & 2489.77 & 3149.56 & 0 & -2489.77 & 3149.56 \\ 0 & 3149.56 & 5312.25 & 0 & -3149.56 & 2656.13 \\ -99604.74 & 0 & 0 & 99604.74 & 0 & 0 \\ 0 & -2489.77 & -3149.56 & 0 & 2489.77 & -3149.56 \\ 0 & 3149.56 & 2656.13 & 0 & -3149.56 & 5312.25 \end{bmatrix} \begin{bmatrix} -0.316 & 0.949 & 0 & 0 & 0 & 0 \\ -0.949 & -0.316 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.316 & 0.949 & 0 \\ 0 & 0 & 0 & -0.949 & -0.316 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1.02 \times 10^{-5} \\ -7.51 \times 10^{-5} \\ 2.48 \times 10^{-4} \\ -6.07 \times 10^{-4} \\ -2.74 \times 10^{-4} \\ 2.48 \times 10^{-4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$