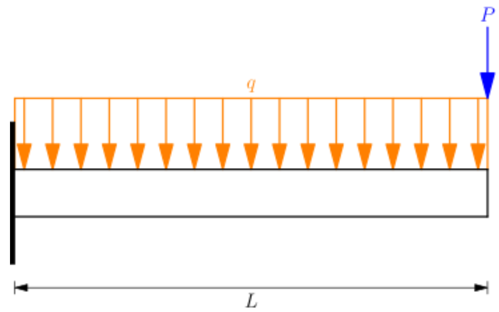
Ejemplo 4



Resolver

$$EI\,\frac{d^4v}{dx^4} + q = 0$$
$$v(0) = 0 \qquad EIv''(L) = 0$$
$$v'(0) = 0 \qquad EIv'''(L) = P$$

Solución exacta

$$v(x) = -\left(\frac{PL}{2EI} + \frac{qL^2}{4EI}\right)x^2 + \frac{P+qL}{6EI}x^3 - \frac{q}{24EI}x^4$$

Solución aproximada generalizada

La forma débil de la ecuación diferencial es

$$\int_0^L R(x)\,W(x)\,dx = \int_0^L \left(EI\,\frac{d^4\hat{v}}{dx^4} + q\right)W\,dx = 0$$

multiplicando

$$\int_0^L \left(EI\frac{d^4\hat{v}}{dx^4} + q\right)W\,dx = \int_0^L W\,EI\frac{d^4\hat{v}}{dx^4}\,dx + \int_0^L W\,q\,dx = 0$$

usando el teorema de Gauss o integrando por partes

$$\left(W\,EI\frac{d^3\hat{v}}{dx^3} - \frac{dW}{dx}EI\frac{d^2\hat{v}}{dx^2}\right)\bigg|_0^L + \int_0^L \frac{d^2W}{dx^2}EI\frac{d^2\hat{v}}{dx^2}\,dx + \int_0^L W\,q\,dx = 0$$

reordenando

$$\int_0^L \frac{d^2W}{dx^2}EI\frac{d^2\hat{v}}{dx^2}\,dx = -\int_0^L W\,q\,dx - \left(W\,EI\frac{d^3\hat{v}}{dx^3} - \frac{dW}{dx}EI\frac{d^2\hat{v}}{dx^2}\right)\bigg|_0^L$$

cortante y momento

$$V = EI\frac{d^3\hat{v}}{dx^3} \qquad M = EI\frac{d^2\hat{v}}{dx^2}$$

reemplazando

$$\int_0^L \frac{d^2W}{dx^2}EI\frac{d^2\hat{v}}{dx^2}\,dx = -\int_0^L W\,q\,dx - \left(W\,V - \frac{dW}{dx}\,M\right)\bigg|_0^L$$