

Introducción a elementos finitos

Tarea 1 I-2016

El factor de forma en general es

$$k = \int_A \frac{Q^2 A}{b^2 I^2} dA$$

si la sección es constante se transforma en

$$k = \frac{A}{I^2} \int_A \frac{Q^2}{b^2} dA$$

Sección rectangular

El momento estático es

$$Q = \int_A y dA = \int_y^{\frac{h}{2}} y b dy = b \int_y^{\frac{h}{2}} y dy$$

Integrando

$$Q = b \frac{y^2}{2} \Big|_y^{\frac{h}{2}} = \frac{b}{2} \left(\frac{h^2}{4} - y^2 \right)$$

Factor de forma

$$k = \frac{A}{I^2} \int_A \frac{Q^2}{b^2} dA = \frac{A}{I^2} \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{Q^2}{b^2} b dy = \frac{A}{b I^2} \int_{-\frac{h}{2}}^{\frac{h}{2}} Q^2 dy$$

Reemplazando valores

$$k = \frac{bh}{b \left(\frac{bh^3}{12} \right)^2} \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\frac{b}{2} \left(\frac{h^2}{4} - y^2 \right) \right]^2 dy$$

Integrando

$$k = \frac{36}{h^5} \left(\frac{h^4}{16} y - \frac{h^2}{6} y^3 + \frac{1}{5} y^5 \right) \Big|_{-\frac{h}{2}}^{\frac{h}{2}}$$

Simplificando

$$k = \frac{36}{h^5} \left(\frac{h^5}{30} \right) = \frac{6}{5}$$

Sección triangular

El momento estático es

$$Q = \int_A y dA = \int_0^{\frac{2}{3}b - \frac{b}{h}y} \int_y^{\frac{2}{3}h - \frac{b}{b}x} y dy dx$$

Integrando respecto de y

$$Q = \int_0^{\frac{2}{3}b - \frac{b}{h}y} \frac{y^2}{2} \Big|_y^{\frac{2}{3}h - \frac{b}{b}x} dx = \int_0^{\frac{2}{3}b - \frac{b}{h}y} \frac{2h^2}{9} - \frac{y^2}{2} - \frac{2h^2}{3b}x + \frac{h^2}{2b^2}x^2 dx$$

Integrando respecto de x

$$Q = \left[\left(\frac{2h^2}{9} - \frac{y^2}{2} \right) x - \frac{h^2}{3b}x^2 + \frac{h^2}{6b^2}x^3 \right] \Big|_0^{\frac{2}{3}b - \frac{b}{h}y} = \frac{4bh^2}{81} - \frac{b}{3}y^2 + \frac{b}{3h}y^3$$

Factor de forma

$$k = \frac{A}{I^2} \int_A \frac{Q^2}{b^2} dA = \frac{A}{I^2} \int_{-\frac{h}{3}}^{\frac{2h}{3}} \frac{Q^2}{x^2} x dy = \frac{A}{I^2} \int_{-\frac{h}{3}}^{\frac{2h}{3}} \frac{Q^2}{x} dy$$

Reemplazando valores

$$k = \frac{\frac{bh}{2}}{\left(\frac{bh^3}{36} \right)^2} \int_{-\frac{h}{3}}^{\frac{2h}{3}} \frac{\left(\frac{4bh^2}{81} - \frac{b}{3}y^2 + \frac{b}{3h}y^3 \right)^2}{\frac{2}{3}b - \frac{b}{h}y} dy$$

Integrando

$$k = \frac{648}{bh^5} \left(\frac{8bh^4}{2187}y + \frac{2bh^3}{729}y^2 - \frac{10bh^2}{729}y^3 - \frac{bh}{324}y^4 + \frac{4b}{135}y^5 - \frac{b}{54h}y^6 \right) \Big|_{-\frac{h}{3}}^{\frac{2h}{3}}$$

Simplificando

$$k = \frac{648}{bh^5} \left(\frac{bh^5}{540} \right) = \frac{6}{5}$$

Sección circular

El momento estático es

$$Q = \int_A y \, dA = 2 \int_0^{\sqrt{r^2-y^2}} \int_y^{\sqrt{r^2-x^2}} y \, dy \, dx$$

Integrando respecto de y

$$Q = 2 \int_0^{\sqrt{r^2-y^2}} \left. \frac{y^2}{2} \right|_y^{\sqrt{r^2-x^2}} dx = 2 \int_0^{\sqrt{r^2-y^2}} (r^2 - y^2 - x^2) dx$$

Integrando respecto de x

$$Q = \left[(r^2 - y^2)x - \frac{1}{3}x^3 \right]_0^{\sqrt{r^2-y^2}} = \frac{2}{3}(r^2 - y^2)^{\frac{3}{2}}$$

Factor de forma

$$k = \frac{A}{I^2} \int_A \frac{Q^2}{b^2} \, dA = \frac{A}{I^2} \int_{-r}^r \frac{Q^2}{(2x)^2} 2x \, dy = \frac{A}{I^2} \int_{-r}^r \frac{Q^2}{2x} \, dy$$

Reemplazando valores

$$k = \frac{\pi r^2}{\left(\frac{\pi r^4}{4}\right)^2} \int_{-r}^r \frac{\left[\frac{2}{3}(r^2 - y^2)^{\frac{3}{2}}\right]^2}{2\sqrt{r^2 - y^2}} \, dy$$

Simplificando

$$k = \frac{32}{9\pi r^6} \int_{-r}^r (r^2 - y^2)^{\frac{5}{2}} \, dy$$

Integrando

$$k = \frac{32}{9\pi r^6} \left[\frac{33r^4y - 26r^2y^3 + 8y^5}{48} \sqrt{r^2 - y^2} + \frac{5r^6}{16} \arcsin\left(\frac{y}{r}\right) \right] \Big|_{-r}^r$$

Simplificando

$$k = \frac{32}{9\pi r^6} \left(\frac{5\pi r^6}{16} \right) = \frac{10}{9}$$