Incorporating Expert Knowledge in Structural Equation Models: Applications in Psychological Research

Integrare il Parere degli Esperti con i Modelli di Equazioni Strutturali: Applicazioni nella Ricerca Psicologica

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Abstract The abstract should be written in English and in Italian, each paper should be preceded by an abstract (no more than 10 lines long) that summarizes the content. Please insert your abstract here.
Abstract *Abstract in Italian*

Key words: Expert elicitation, Informative Priors, Structural Equation Models (SEM), Small sample sizes, Psychological research

1 Introduction

Structural Equation Modeling (SEM) encompasses a range of multivariate statistical techniques as, for example, confirmatory factor analysis, path analysis, or latent growth modeling. Generally, SEMs are composed of two parts: a *measurement model* and a *structural model* (see Fig. **). The measurement model defines unobserved constructs (*latent variables*, represented in Fig. ** as circles) according to a

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set of measured outcomes (*observed variables*, represented in Fig. ** as squares), whereas the structural model describes the relationships between latent variables.

In SEM, parameters are estimated by minimizing a discrepancy function between the sample covariance matrix and the covariance matrix implied by the model. Considering the covariance structure (instead of modeling all the observed data) allows to model complex relations between variables taking into account also the measurement error.

Given their great flexibility, SEMs are widely used in psychology for various purposes such as validating psychological tests and questionnaires or evaluating hypotheses and theoretical models that involve complex relations between different latent psychological constructs.

However, as the complexity of the model increases, more data are required to obtain accurate parameter estimates and model fit statistics. Recent concerns about the replicability of psychological results have raised awareness on the importance of properly define an adequate sample size. Nevertheless, in many research settings, the number of participants may be limited due to financial restriction, strict inclusion criteria, or clinical samples. In the case of small sample sizes, appropriate statistical techniques are required to enhance the reliability of the results.

Often in the literature, the Bayesian approach is suggested over frequentist estimation when limited data are available. In the context of small sample, the inclusion of prior information can help in the parameter estimation, but researchers have to carefully consider priors choice, as estimates are highly sensitive to the prior specification (or misspecification).

The use of Bayesian statistical approach is increasing and availability of softwares, such as R-package `blavaan`, provide researchers with flexible tools for estimating even Bayesian structural equation models. However, most of the studies are unlikely to carefully consider priors choices, and often they rely on default software prior settings. A recent review, considering the performance of Bayesian estimation for structural equation models with small sample sizes, underlined that the use of diffuse default priors can result in severely biased estimates, and this bias can be decreased only by incorporating informative priors. Thus, authors warn against the use of *naive prior* (i.e., diffuse default priors) when samples are small, and encourage researchers to incorporate *thoughtful priors* (i.e., informative priors).

Informative priors allow researchers to include in the analysis relevant knowledge in the field, such as previous studies results, meta-analyses or expert opinions. Priors choice should be clearly discussed and researchers are required to evaluate the generalizability of external information to the specific characteristics of their study. When the number of available studies is limited or their quality is judged not adequate, relying only on previous results in the literature could be misleading. Instead, researchers could consider to include opinions of experts as well. On the base of their experience in the field, experts can evaluate relevant information and help researchers in the definition of a plausible range of values and prior choices.

The remainder of this article is structured as follows. In Sec. 2, we briefly consider how informative priors can be defined according to experts' opinions. In Sec. 3, we present a simulation study to evaluate the influence of different prior specifica-

tion in the case of structural equation models with small sample size, considering an applied example of a mediation model in psychology. Finally, in Sec. 4, we discuss the obtained results highlighting limits and future developments in the use of structural equation models with small sample sizes.

2 Experts' knowledge elicitation

Elicitation is a structured procedure that allows experts to express their knowledge and uncertainty about quantities of interest in the form of probability distributions. Elicitation can be used to define informative priors according to experts' judgement.

Different elicitation methods have been proposed in the literature such as the *Cooke protocol*, the *Delphi method* or the *SHELF protocol*. The common aim of these methods is to make a subjective judgement as much objective as possible by limiting potential sources of bias, forcing the experts to carefully reason about the answer, and by documenting and transparently reporting all the procedure. In general, elicitation is composed of three phases:

- *Preparation and training* - experts are informed about the aim of the elicitation and the parameters and quantities to elicit are clearly defined. All the relevant information about the topic of interest is collected and made available to experts. Next, experts are trained to make probabilistic judgements and they are familiarize with the elicitation process in a practice example to avoid misunderstandings.
- *Individual judgements elicitation* - experts express their own judgement for each parameter or quantity of interest according to the elicitation technique they were trained before. Two of the main elicitation technique are the *quartile method* and the *roulette method*. In the former, experts provide values for the medians and the quartile of the distribution according to their expectation. In the latter, the range of plausible values is divided into different intervals and the experts can place a given number of tokens to allocate probabilities according to their expectation.
- *Aggregation of individual judgements* - the individual judgements of the different experts are aggregate to obtain a unique final distribution. The two principal approaches are *mathematical aggregation*, where distributions are combined mathematically according to a pooling rule, and *behavioral aggregation*, where experts are required to discuss together their opinions and reach a consensus judgement from which the aggregate distribution is obtained.

Elicitation methods differ in the solutions adopted during each phase and in the emphasis given to the different aspects of the elicitation. For example, in the Cooke protocol experts are not required to meet each other. Experts return a form with their individual answer and mathematical aggregation is used to summarize the results. In the SHELF protocol, instead, much more emphasis is given to the collaboration between experts. After the individual judgments, experts discuss and share their opinions to then reach a common final answer.

Researcher willing to conduct an elicitation process should evaluate the pros and cons of each methods considering their own needs and constraints such as availability of experts, the possibility to group experts together, number of quantities to elicit or financial and time constraints.

3 Simulation

4 Discussion and conclusions

previous results on their own analysis, avoiding to rely on limited previous studies. Within this aspects, expert opinions are a valuable source of information although it is often overlooked. When limited information are available expert elicitation could be a useful source of information defining plausible and unplausible range of values. Formalization of expert knowledge is based on different approach cooks shelf delphi method.

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when previous results are limited, relying on a restricted number of studies or low quality studies could be misleading.

5 Section

Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Further on please use the LATEX automatism for all your cross-references and citations. And please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

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$$a \times b = c \,, \tag{1}$$

however, for multiline equations we recommend to use the `eqnarray` environment[1].

$$a \times b = c$$
$$\mathbf{a} \cdot \mathbf{b} = \mathbf{c} \tag{2}$$

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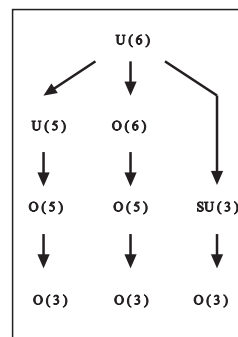
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1. Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.

   a. Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.
   b. Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.

2. Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.

Subparagraph Heading

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• Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development, cf. Table 1.

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Fig. 2 If the width of the figure is less than 7.8 cm use the `sidecapion` command to flush the caption on the left side of the page. If the figure is positioned at the top of the page, align the sidecaption with the top of the figure – to achieve this you simply need to use the optional argument `[t]` with the `sidecaption` command
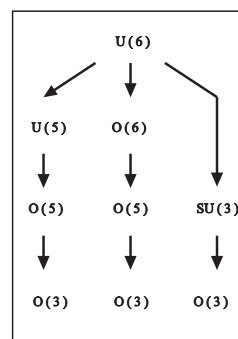
Table 1 Please write your table caption here

Classes	Subclass	Length	Action Mechanism
Translation	mRNA[a]	22 (19–25)	Translation repression, mRNA cleavage
Translation	mRNA cleavage	21	mRNA cleavage
Translation	mRNA	21–22	mRNA cleavage
Translation	mRNA	24–26	Histone and DNA Modification

[a] Table foot note (with superscript)

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Theorem 1. *Theorem text goes here.*

Definition 1. Definition text goes here.

Proof. Proof text goes here. □

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Definition 2. Definition text goes here.

Proof. Proof text goes here. □

Appendix

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$$a \times b = c \tag{3}$$