POLITECNICO
MILANO 1863

Solvency II - Final Project

Group 9

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Academic Year: 2023/2024

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Text of the project

FINAL PROJECT - SII

Consider a simplified insurance company whose assets and liabilities sides are characterized as follows:

ASSETS

- There is a single fund made of equity (80%) and property (20%): $F_t = EQ_t + PR_t$.
- At the beginning ($t = 0$), the value of the fund is equal to the invested premium: $F_0 = C_0 = 100{,}000$.
- **Equity features**:
  - Listed in the regulated markets in the EEA.
  - No dividend yield.
  - To be simulated with a Risk Neutral GBM ($\sigma = 20\%$) and a time-varying instantaneous rate r.
- **Property features**:
  - Listed in the regulated markets in the EEA.
  - No dividend yield.
  - To be simulated with a Risk Neutral GBM ($\sigma = 10\%$) and a time-varying instantaneous rate r.

LIABILITIES

- **Contract terms**:
  - Whole Life policy.

- **Benefits**:
  * In case of lapse, the beneficiary gets the value of the fund at the time of lapse, with 20 euros of penalties applied.
  * In case of death, the beneficiary gets the maximum between the invested premium and the value of the fund.
- **Others**:
  * Regular Deduction (RD) of 2.20%.
  * Commissions to the distribution channels (COMM or trailing) of 1.40%.

- **Model points**:
  - Just 1 model point.
  - Male with insured aged $x = 60$ at the beginning of the contract.
- **Operating assumptions**:
  - Mortality: rates derived from the life table SI2022 (`https://demo.istat.it/index_e.php`).
  - Lapse: flat annual rates $l_t = 15\%$.
  - Expenses: constant unitary (i.e., per policy) cost of 50 euros per year, that grows following the inflation pattern.
- **Economic assumption**:
  - risk free: rate r derived from the yield curve (EIOPA IT without VA 31.03.24)
  - inflation: flat annual rate of 2 %

Other specifications:

- time horizon for the projection: 50 years. In case of outstanding portfolio in T=50, let all the people leave the contract with a massive surrender
- the interest rates dynamic is deterministic, while the equity and property ones are stochastic.

QUESTIONS

1. Code a Matlab/Python script to compute the Basic Solvency Capital Requirement via Standard Formula and provide comments on the results obtained. The risks to be considered are:

- Market Interest
- Market equity
- Market property
- Life mortality
- Life lapse
- Life cat
- Expense

2. Split the BEL value into its main PV components: premiums (=0), death benefits, lapse benefits, expenses, and commissions.

3. Replicate the same calculations in an Excel spreadsheet using a deterministic projection.

    - Do the results differ from 1? If so, what is the reason behind?
    - For the base case only
        (a) calculate the Macaulay duration of the liabilities;
        (b) calculate the sources of profit for the insurance company, deriving its PVFP
        (c) check the magnitude of leakage by verifying the equation MVA = BEL + PVFP (i.e. MVA = BEL + PVFP + LEAK)
        (d) sense check the PVFP using a proxy calculation, based on the annual profit and the duration of the contract

4. Open questions:

    - what happens to the asset and liabilities when the risk-free rate increases/decreases with a parallel shift of, say, 100bps? Describe the effects for all the BEL components;
    - what happens to the liabilities if the insured age increases? What if there were two model points, one male and one female?

Summary Tables

EXCEL: Deterministic projection

Results:	**MVA**	**BEL**	**BOF**	**dBOF**	**Duration** (Liabilities)
Base	100000	95801,9383	4198,0617	-	5,6135
IR _up	100000	95782,5364	4217,4636	0	5,6129
IR _down	100000	96164,6547	3835,3453	362,7163	5,6402
Equity	64600	64121,8682	478,1318	3719,9299	5,7167
Property	95000	91208,4671	3791,5329	406,5287	5,6156
Mortality	100000	95832,5035	4167,4965	30,5651	5,5728
Lapse_up	100000	96981,0181	3018,9818	1179,0798	4,0160
Lapse_down	100000	93325,6459	6674,3541	0	9,0348
Lapse_mass	100000	97415,1780	2584,8220	1613,23967	3,4020
CAT	100000	95807,1430	4192,8570	5,20465863	5,6063
Expenses	100000	95851,8424	4148,1576	49,9040	5,6145
BSCR	4905,3106				

MATLAB: Stochastic projection

Results:	**MVA**	**BEL**	**BOF**	**dBOF**	**Duration** (Liabilities)
Base	100000	96933,2949	3066,7051	-	5,6863
IR_up	100000	96588,9288	3411,0712	0	5,6595
IR_down	100000	97260,7106	2739,2894	327,4155	5,7113
Equity	64600	64396,6859	203,3140	2863,3910	5,7604
Property	95000	92319,0733	2680,9267	385,7782	5,6955
Mortality	100000	97097,4637	2902,5363	164,1687	5,6504
Lapse_up	100000	97486,9724	2513,0276	553,6774	4,0351
Lapse_down	100000	96731,7826	3268,2174	0	9,3592
Lapse_mass	100000	98038,4567	1961,5433	1105,1618	3,4530
CAT	100000	96946,4323	3053,5676	13,1374	5.6786
Expenses	100000	96983,1991	3016,8009	49,9040	5,6873
BSCR	3796,6240				

Formulas adopted for the calculations

3.1 Interest Rates

We started by looking for the EIOPA yield curve (31/03/24 - no Volatility Adjustment) and selecting the European spot risk-free rates for the base case and the ones stressed up and down.
Then we computed the zero rates using:

$$r_0(t) = ln(1 + r(t))$$

the discount factors with the following capitalization rule:

$$B(t) = e^{-r_0(t)t}$$

and the forward rates, using the forward discount factors :

$$f(t-1,t) = ln(\frac{B(t)}{B(t-1)})$$

3.2 Assets

As an Insurance company, there is the need to manage the Asset side. We had to deal with a single fund made of an equity part (80 %) and a property part (20 %).

$$F_t = EQ_t + PR_t$$

In both cases, for property and equity, we had to simulate a Risk Neutral Geometric Brownian Motion, with $\sigma_E = 0.2$ and $\sigma_{PR} = 0.1$ respectively.
We took into account the following SDE for the stock dynamics:

$$dS_t = S_t(r_t dt + \sigma dW_t)$$

and we solved it, getting:

$$dS_{t+\Delta t} = S_t e^{(f(t,t+\Delta t)-0.5\sigma^2)\Delta t+\sigma(W_{t+\Delta t}-W_t)}$$

In front of which we considered the Regular Deduction term **(1-RD)**.

3.2.1 Simulation Insight:

In order to simulate these dynamics we implemented a Matlab function that uses the Monte Carlo approach to simulate a standard gaussian random variable 'g' ($g \sim N(0,1)$), for the dynamics of the Brownian Motion. Then we computed the underlying at each iteration and we created a matrix with 51 columns: each one represented the year in which the simulation takes place (including the value at time t_0); in the rows we stored the simulated values.
The function was built in order to compute the 97.5 % confidence interval too. The number of MC simulations had to be selected in terms of trade-off between accuracy and elapsed time of the code. Eventually, we chose 10^6. At the end we wanted to verify if our simulation satisfied the no arbitrage constraints, so we also implemented a Martingale Test. Thanks to that we were also able to test what happened in a monthly framework. In **Section 3.7** we adequately explain the test and the results we obtained.

3.3 Liabilities

In order to investigate the liabilities side of our insurance company we had to import some more information, such as the mortality rates for men (q_x), that we were able to find in the *SI2022 life tables*, given by ISTAT. Moreover we fixed the lapse rate l_t as constant and equal to 15 %, except for the last year where we decided to set it at 100 % to represent massive surrender. This way, we first focused on the probability of staying into the contract that we computed as:

$$P_t = \prod_{i=1}^{t} \left((1 - q_x(i-1)) \cdot (1 - l_t(i-1)) \right)$$

We explicitly set this probability to 1 in t_0 and to 0 in the year when the contract ends, t_{50}.

In order to compute benefits cash flow in case of lapse or death, we respectively used the following formulas:

$$L_t = (F_t - 20) \cdot \left(1 - q_{t-1}^{x}\right) \cdot l_{t-1}^{t}$$

$$D_t = (\max\{C_0, F_t\}) \cdot q_{t-1}^{x}$$

We computed the cash flows for the whole set of Monte Carlo simulations that represented the Fund; then we computed the mean for each year.
During this procedure we kept F_t as a matrix of 10^6 simulations in 51 columns for each year and later we obtained a row vector of 51 columns, representing the average payoff. Let us remark that this is the main reason why Excel deterministic results and Matlab Stochastic ones differ so much, as would be better explained later on.

We decided to compute the average at the end of the procedure because we want to consider the liabilities as an option for which we have to compute every possibility for each simulated path. However, following this method we are making calculations on 10^6 values of the fund each year instead of considering just one single value as in the deterministic scenario: that's why we get different cash flows and consequently different liabilities in the two cases.

Finally, by using the appropriate discount factors we were able to sum all the discounted cash flows, multiplied by the probability of remaining in the contract throughout the 50 years where our projection is done.

$$\text{Liabilities } = \sum_{t=0}^{50} B_t \cdot P_t \cdot (L_t + D_t + \text{comm}_t + \text{ exp}_t)$$

Where B_t is the discount factor at time t, comm represent the commission, and exp represent the expenses at given time.

3.4 BEL

In order to compute the Best Estimate Lability components we split them as follows:

$$\text{BEL} = BEL_{Lapse} + BEL_{Death} + BEL_{Expenses} + BEL_{Commissions}$$

- $BEL_{Lapse} = \sum_{i=0}^{50} P_t \cdot Lapse_{cf_i} \cdot B_t$
- $BEL_{Death} = \sum_{i=0}^{50} P_t \cdot Death_{cf_i} \cdot B_t$
- $BEL_{Expenses} = \sum_{i=0}^{50} P_t \cdot expenses_i \cdot B_t$
- $BEL_{Commissions} = \sum_{i=0}^{50} P_t \cdot F_{ti} \cdot \frac{COMM}{1-RD} \cdot B_t$

Liabilities	**BEL death**	**BEL lapse**	**BEL expenses**	**BEL commissions**
96933.2949	7584.4580	81204.775	300.9283	7843.1331

3.5 Basic Own Fund

For each stressed scenarios, after computing the Liabilities, we computed the BOF and the dBOF in order to compute the SCR.
The Basic Own fund is defined as $BOF = F_0 - Liabilities$.
For the base case we got **BOF=3066.7050**.
Let us now define the dBOF as $dBOF = min(BOF - BOF_{stressedcase}, 0)$. It's important to consider only positive values for the sake of the BSCR computation because we want to account only for risky cases where we suffer a loss (not a gain as would happen with a positive dBOF).

3.6 Basic Solvency Capital Requirement

Let us now analyze, one by one, each risk needed to compute the Basic Solvency Capital Requirement using for our computations the Standard Formula.

3.6.1 Market Interest

The first risk that we considered is Market Interest, that takes into account possible fluctuations in interest rates and how they can affect the value of the fund and of the liabilities. In particular, from our EIOPA table we derived the two stress scenarios IR_{up} and IR_{down}, respectevely *EIOPA UE shock up without VA 31.03.24* and *EIOPA UE shock down without VA 31.03.24* .
SCR is then computed taking the largest ΔBOF between the up and the down case.

We could also have derived the two situations on our own, deriving the stress factors from EIOPA and then by computing:

$$\text{IR}_{\text{down}} = \text{IR} \cdot (1 - \text{stress}_{\text{down}}) \cdot 1_{IR>0} + \text{IR} \cdot 1_{IR\leq=0}$$

$$\text{IR}_{\text{up}} = \text{IR} + \max\left(1\%, \text{stress}_{\text{up}} \cdot |\text{IR}|\right)$$

For the stressed scenario IR_{up} the value of the liabilities decreased, as we could have expected, since we were applying heavier discount factors.
On the other hand in the downward case IR_{down} discount factors and liabilities increase as rates go down.
We then computed BOF_{up}=**3411.0712** and BOF_{down} =**2739.2894**.

Moreover we observe $\Delta BOF_{up} = \mathbf{0}$ and $\Delta BOF_{down} = \mathbf{327.4155}$

It is obviously impossible to be exposed to both IR up and down risks at the same time and we confirmed it since the first one is null.

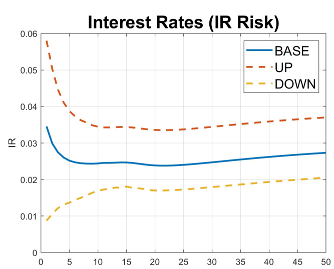

Figure 3.1: Interest rates

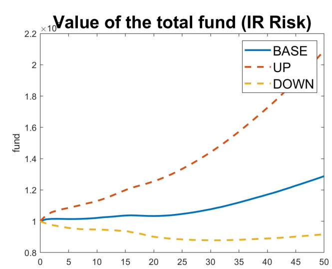

Figure 3.2: Fund value

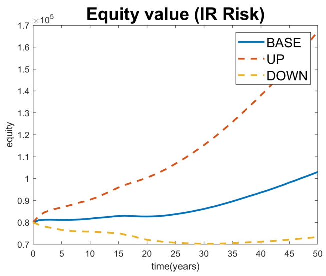

Figure 3.3: Equity value

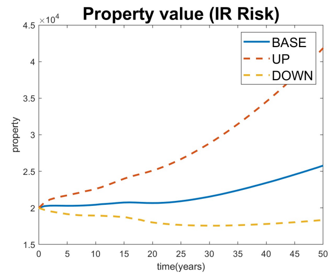

Figure 3.4: Property value

	BEL death	BEL lapse	BEL expenses	BEL commissions
IR up	7259.4937	81205.7993	280.5024	7843.1330
IR down	7894.8209	81203.8676	318.8888	7843.1330

3.6.2 Market equity

As insurance companies are used to hold investments such as stocks as part of their portfolio, fluctuations in the equity market can hold a large impact. Considering the Solvency II framework, equities are divided into two categories, type 1 and type 2. The ones that we were dealing with in this case belonged to type 1, as they were listed in regulated markets of a country of the EEA; according to the regulation the equity shock was therefore the following:

$$EQ_0^{shock} = (1 - 0.39 - 0.0525)EQ_0$$

We also considered the symmetric equity adjustment, that employs an additional factor of 0.0525, added to 0.39, as derived from EIOPA tables for the month of March.

The substantial decrease of the equity value (80% of the fund) consequently caused a huge decrease of the liabilities value: we got the lowest BEL between all the shocked scenarios, the only one that drops way below $9 \cdot 10^5$.
We also computed:
BOF = **203.3140**, ΔBOF = **2863.3910**

From the plot below we can observe the base case compared to the stressed one. The dynamics appear to be the same, but with a substantial difference in the price value.

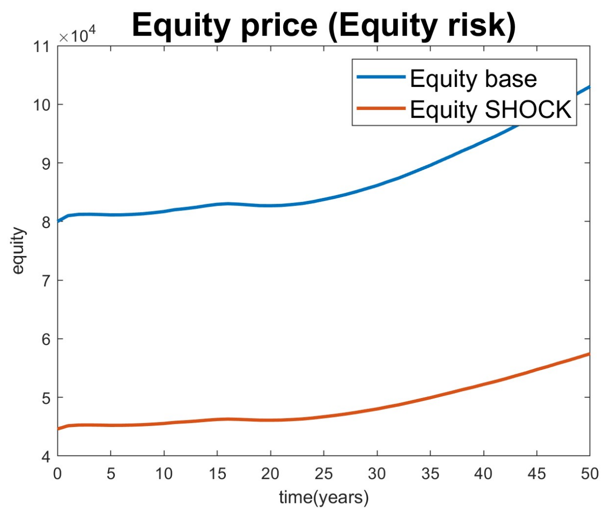

Figure 3.5: Equity price dynamics

	BEL death	BEL lapse	BEL expenses	BEL commissions
Equity Shock	6575.1622	52453.8171	300.9282	5066.7783

3.6.3 Market property

Insurance companies are used to invest in real estate properties and other tangible assets too, therefore market property concerns the exposure to fluctuations to these markets.

The stress scenario is computed as an instantaneous decrease in the market value of assets by 25%:

$$PR_0^{shock} = (1 - 0.25)PR_0$$

Comparing this case with the previous one, we noticed that here we had a much smaller reduction of the Liabilities side since the property represents only the 20% of the fund.
Computing the BOF and ΔBOF we obtain:
BOF = **2680.9267** , ΔBOF = **385.7782** .

As the equity case, from the plot we observe similar dynamics in the property evolution but very different prices.

Figure 3.6: Property price dynamics

	BEL death	BEL lapse	BEL expenses	BEL commissions
Property Shock	7424.1649	77143.0682	300.9282	7450.9118

3.6.4 Life mortality

Unexpected deaths of policyholders who subscribed a life insurance policy must be considered when computing the Basic Solvency Capital Requirement. They are taken into account by life mortality risk, which, refers to the uncertainty of mortality: both its timing and frequency.
The computation was done considering again a stressed scenario, that in this case is an immediate increase of 15% of the mortality rates, applied to the whole life table.

$$q_x^{mor} = (1 + 0.15)q_x$$

In this stressed scenario, the shock did not impact the value of the fund, while there was a very small change in liabilities value. We tried to understand this considering the fact that as more people die, less people lapse and the two effects almost balance each other.
In the mortality risk case we obtained $BOF =$**2902.5363** and $\Delta BOF =$**164.1687** which, as already observed, is a very modest value.

From the plot we are able to observe that as time goes on, the difference between the mortality rates curves widens.

Figure 3.7: Mortality rates dynamics

	BEL death	**BEL lapse**	**BEL expenses**	**BEL commissions**
Mortality Shock	8535.9648	80477.6832	298.3775	7785.4380

3.6.5 Life lapse

Life lapse risk is about the possibility of policyholders terminating or surrendering their life insurance policies before the maturity or expected duration. Our main focus was the financial impact that may be caused on the insurance company. We firstly needed to analyse three different stressed scenarios, and then consider the one with the highest SCR for the final computation.

3.6.6 Lapse up

The first shock considered is the so called lapse up shock, an instantaneous increase of 50% that continues until the end of our time frame, assuming that such lapse rate does not exceed 100%.

$$l_{up} = min(150\%R, 100\%)$$

$BOF_{lapseup} = 2513.0276$; $\Delta BOF_{lapseup} = 553.6774$

3.6.7 Lapse down

Secondly we considered the lapse down shock, an instantaneous decrease of 50% until the end of the time frame, assuming it doesn't exceed the value of the initial lapse minus 20 %.

$$l_{down} = max(50\%R, R - 20\%)$$

$BOF_{lapsedown} = 3268, 2174$; $\Delta BOF_{lapsedown} = 0$

3.6.8 Lapse mass

We eventually considered lapse mass risk, in which we have to consider a discontinuance in the first year of 40% due to the nature of the insurance policies that we are considering. $BOF_{lapsemass} = 1961.5432$; $\Delta BOF_{lapsemass} = 1105.1618$

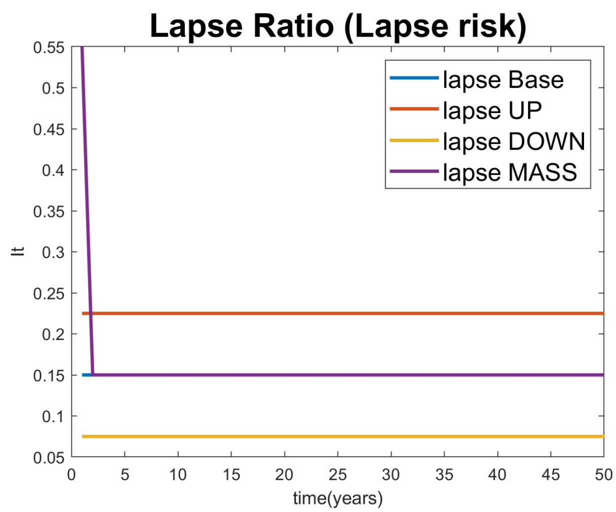

Figure 3.8: Lapse ratio dynamics

Considering lapse up case, we observed a decrease in the liabilities, coherent with the fact that more people left the contract. The opposite scenario appeared in the lapse down case, where as a smaller number of people left the contract, the liabilities tended to increase. The lapse mass case is similar to the lapse up: in both cases the BOF computed is higher than the base-scenario. Finally, since the lapse mass is the one with the biggest impact, we concluded $SCR_{lapse} = SCR_{lapse.mass}$ =**1105.1618**.

	BEL death	BEL lapse	BEL expenses	BEL commissions
Lapse mass	4332.4112	88712.4053	182.5002	4811.1399

3.6.9 Life CAT

Life catastrophe risk stems from extreme or irregular events that can increase rapidly and affect mortality risk in an unenforceable way; as this affects the number of life insurance claims, companies need to assess this risk. We considered the stressed scenario as an absolute 1.5/100 increase in the rate of policyholders dying over the following year.

$$q_x^{cat}(1) = q_x(1) + \frac{1.5}{1000}$$

We computed BOF =**3053.5676** , $\Delta BOF_{lapseup}$ =**13.1374** and observed a slight change in the liabilities: catastrophe risk affects their value due to the presence of mortality-sensitive financial instruments, but the stress is only related to the first year and therefore the impact is not huge. This is probably due to the fact that while lots of people die and make the death benefit increase, on the other hand the lapse benefits decrease. This, joint to the fact that as the first year more death payments are made a little less payments may be done in the remaining years, can explain this fact. As it can be seen by the plot, the difference with respect to the base case can be observed in the first value of the mortality rate.

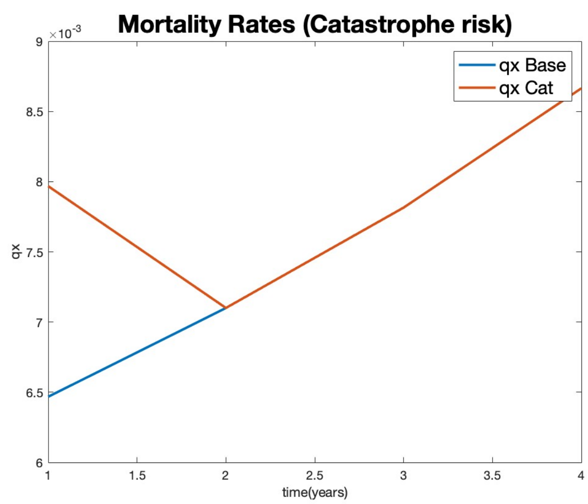

Figure 3.9: Mortality rates dynamics

	BEL death	BEL lapse	BEL expenses	BEL commissions
Catastrophe	7730.3030	81082.1753	300.5483	7833.4056

3.6.10 Expense

We eventually considered expenses risk, that regards the impact of operational costs of an insurance company's solvency positions, like administrative costs, marketing expenses, claims handling costs and commissions paid to intermediaries. The stressed scenario that we considered was an increase of 10% of the predicted future expenses and an increase by an annual 1% of the predicted expense inflation rate.

$$exps^{shock} = exps(1+0.1)(1+(infl+0.01))^t$$

A small increase in liabilities can be noticed since only the expenses part of the BEL changed, while the rest remained the same. We obtained the following computations: BOF=**3016.8009** ; ΔBOF=**49.9040**

Figure 3.10: Expenses value dynamics

	BEL death	BEL lapse	BEL expenses	BEL commissions
Expenses Shock	7584.4581	81204.7754	350.8323	7843.1330

3.6.11 Capital requirement computation

The final step was to compute the Basic Solvency Capital Requirement after aggregating all the previously obtained computations, for each risk considered.

- **Market SCR:**

  After aggregating the interest rate risk, the equity risk and the property risk we obtained the market SCR. For what concerned interest rate risk,

we observed that the SCR in case of a negative parallel shift was larger than the opposite case; according to theory, this meant that the correlation table between risks we had to use for computations was the following:
$CorrMatrix_{market}$ =

$$\begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.75 \\ 0.5 & 0.75 & 1 \end{bmatrix}$$

$$SCR_{market} = \sqrt{\sum_{r,c} CorrIndex_{mkt}^{r,c} SCR^S CR^c}$$

with r,c varying between interest, equity and property.

We obtained: SCR_{market}=**3343.0246**.

- **Life SCR:**

  After aggregating the mortality risk, the lapse risk, the expense risk and the CAT risk we obtained the life SCR. According to Solvency II, the correlation matrix needed is the following
  $CorrMatrix_{life}$ =

$$\begin{bmatrix} 1 & 0 & 0.25 & 0.25 \\ 0 & 1 & 0.5 & 0.25 \\ 0.25 & 0.5 & 1 & 0.25 \\ 0.25 & 0.25 & 0.25 & 1 \end{bmatrix}$$

$$SCR_{life} = \sqrt{\sum_{r,c} CorrIndex_{life}^{r,c} SCR^r SCR^c}$$

  We obtained: SCR_{life}=**1148.4385**

- **BSCR:** We finally aggregated market and life risks using the following matrix:

$$\begin{bmatrix} 1 & 0.25 \\ 0.25 & 1 \end{bmatrix}$$

  We obtained the final BSCR with the formula:

$$BSCR = \sqrt{\sum_{r,c} Corr^{r,c} SCR^r SCR^c}$$

  with r,c varying between market and life. The final result was :**3796.6240**

  The value seems a reasonable request compared to the value of the invested premium considered.

3.7 Martingale Test

Before explaining the deterministic calculation, we thought it was best to verify the martingale assumption we did for all the computations upwards, through a Martingale Test. In this section we aimed to verify if the annual step assumption was better than, for example, a monthly one and to prove that our simulation was driftless, in order to preserve the arbitrage free assumption of the market. Let us remind that the mean of a martingale is constant and that:

$$E[S_t|F_0] = S_0$$

To have a visual representation of this property, we plotted the value of both property and equity discounted, in the case of a monthly step (in this case we has to linearly interpolate to obtain the rates and then the discount factors corresponding to each month) or an annual one, as shown in the following plots:

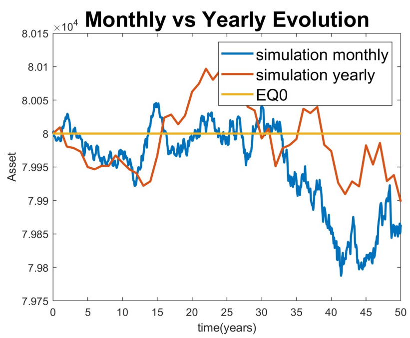

Figure 3.11: Equity discounted simulated yearly and monthly, against EQ0 initial value

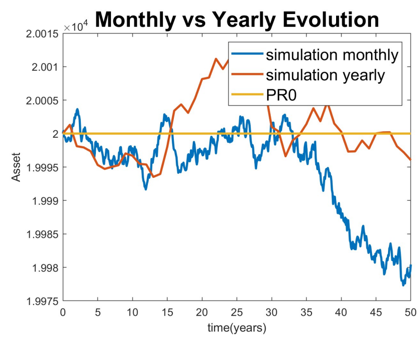

Figure 3.12: Property discounted simulated yearly and monthly, plotted against PR0 initial value

From these plot we can visually observe that a yearly simulation is closer to a martingale, as its evolution seems to remain closer to the initial value, for both equity and property. However, as confirmed by computations on the deterministic case versus the stochastic ones and by the plot above, the property seems to follow better the martingale condition.

Our intuition was later confirmed by the computation of the errors. The MonteCarlo error is defined as in the following expression:

$$error_{MC} = \frac{std(S_t)}{\sqrt{NMC}}$$

NMC=number of simulations adopted , t = annual/monthly

We noticed smaller errors for both property and equity in the annual case, rather than the monthly, confirming our visual considerations. In particular the property one was way smaller. We also tried different numbers of simulations, and we observed that as they increased, the error terms shrinked down.

We also performed another evaluation of the error, simply taking the absolute value of the difference between the simulated value and the deterministic quantity. In the above tables we plotted the results, for both equity and property, for different NMC-number of Monte Carlo simulations.

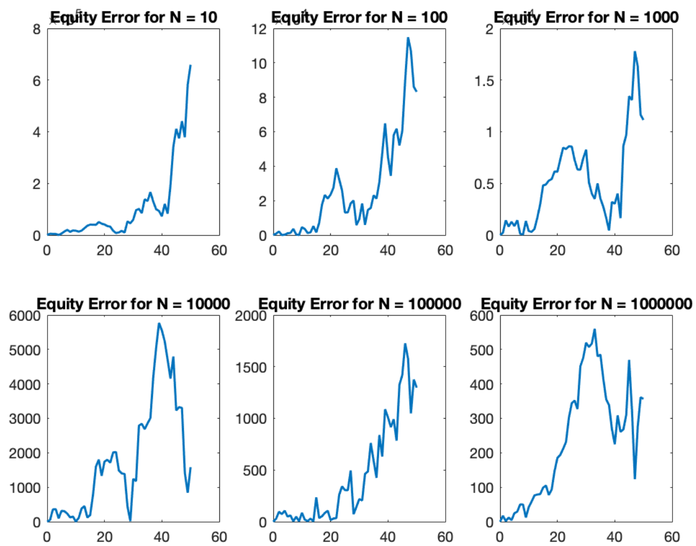

Figure 3.13: Error between a simulated and a deterministic Equity, for different N= number of simulations. Even if it's difficult to see, in the first three graphs the error terms are multiplied by $10^5, 10^4, 10^4$ respectively.

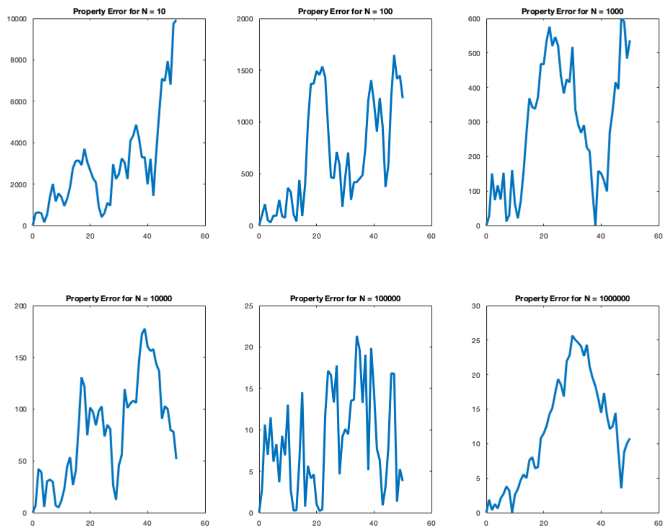

Figure 3.14: Error between a simulated and a deterministic Property, for different N= number of simulations

In the 10^6 case the martingale hypothesis is reasonable, as the error for both equity and property is under 1% of the value of the Fund. For running time purposes we did not increase any further the number of simulations.

Deterministic calculations and comments on the results

4.0.1 BSCR result

For the deterministic part of the computations, performed in Excel, we had to exclude randomness since this time we were not able to accomplish a Monte Carlo simulation. Recalling the underlying dynamics, shown in **Section 3.2**, we wanted to exclude the Brownian Motion part, by setting $\sigma = 0$ and then discretizing the time. What we got for the Equity and Property dynamics is:

$$dS_{t+\Delta t} = S_t e^{f(t,t+\Delta t)\Delta t}$$

In front of which we always consider the Regular Deduction term (1-RD). For what concerns the analysis of the risks we repeated the same exact evaluations previously explained to which we added also an overview about the different values of Duration and the PFPV computations.

We got **BSCR=4905,3106** as a result.

4.0.2 Stochastic VS Deterministic case

As we expected, we got a very different result with respect to the Matlab one, since by removing the stochastic part of the equation, we got more inaccurate and less realistic results. We could infer that the reason why this happens is the non-linearity of the payoffs: in this way taking the mean after or before computing the cashflows is different. In fact, we saw that this concernes mainly the payments in case of death, which expression is represented by:

$$D = max(C_0, F_{ti}) \cdot q_x^i$$

When we simulate randomly via a Monte Carlo, some of the F_{ti} are going to be under the "benchmark" C_0. Instead in the case of deterministic simulation, as we consider the fund evolving as the risk free rate, the fund evolving in time is always bigger than 0. This leads to having bigger Liabilities in the deterministic case, and therefore an higher BSCR. We suppose this is what makes the most difference between the two different computations.
If we repeat the same deterministic calculations in Matlab without simulating the Brownian Motion or taking the mean directly in the MC simulations and not when we compute the Liabilities cash flows, we have almost equal results to

the Excel's. However, as said before, this result is much more inaccurate and irrealistic, so, if we were the insurance company we would probably trust the stochastic result.

4.0.3 Duration

Regarding the base case, we conducted additional calculations. We began by computing the Macaulay Duration, as the sum of the products of discounted cash flows and their respective years, divided by the total liabilities.

$$\text{Duration} = \frac{\sum_{t=0}^{50} t \cdot B_t \cdot P_t \cdot (L_t + D_t + commissions_t + expenses_t)}{\text{liabilities}}$$

The Duration of an investment represents in how much time we expect our debt to be repayed. In this sense it was interesting to compute the duration for each stressed scenarios since it was also requested in the summary table. This way, we noticed, for example, that mortality risk results had a lower duration with respect to the base case since probably the market expects to repay us before. The same coherently happens in case of a catastrophy.

It can also be observed by the **Summary Tables** that lapse risk is the one that makes the duration fluctuate the most, indicating the most exposure to risk in our investment: increasing when liabilities go down and viceversa. In particular lapse down risk makes the duration peak since a smaller number of people leaves the contract, while Lapse up and most of all Lapse mass make it fall down, coherently with the fact that this last one is the heaviest scenario of the three.

Another relevant change in the duration is observed when we stress equity, since, as already pointed out when commenting the big drop in the liabilities, they represent a huge part of our investment.

4.0.4 PVFP check and proxy check

In order evaluate the Present Value of Future Profits (PVFP), we started by determining the industrial profit for each unit of time as the difference between regular deduction and commissions, to which we also removed expenses. The PVFP was then calculated as the sum of these discounted differences, resulting in a value of 4182.0909.

$$PFPV_t = \frac{(RD - COMM)}{(1 - RD)} \cdot P_t \cdot B_t \cdot F_t - exp_{cf} \cdot B_t$$

This was particularly useful to check the magnitude of the leakage using the formula MVA = BEL + PVFP + LEAK and so to verify our deterministic computations. By rearranging the sum, we derived a leakage value of 15.9707. This small value for the leakage confirms our assumption that MVA = BEL + PVFP holds true.
This implies also the equation BOF $\approx PVFP$.

We also sense checked the PVFP by using the duration, obtaining a very satisfactory result. Our idea was to compute the gains of each year, multiplied by the duration which refers to the expected timeframe of the contract. Therefore we considered our gains and subtracted regular deduction and commission, multiplied for the fund and then subtracted the expenses. We multiplied everything for the duration and got: 4210.1737, that is very close to the actual value of PVFP. The formula used for the proxy calculation was the following:

$$((RD - COMM) \cdot F_0 - exp_{cf}) \cdot Duration$$

Open questions

5.1 Question 1

What happens to the asset and liabilities when the risk-free rate increases/decreases with a parallel shift of, say, 100bps? Describe the effects for all the BEL components.

When the risk-free rates increase, it affects both equity and liabilities, influencing asset dynamics. Referring to section **3.2** it can clearly be seen by the formulas that as rates go up, also the market stock dynamics move in the same direction, leading to an overall asset growth on the equity and property side. In fact an increase in the risk-free rate leads to a greater drift coefficient that makes their value to do the same.

As far as liabilities are concerned, referring to section **3.3** we can infer that:

- Mortality and lapse rates are obviously not related with rates, so they are not affected. Then the probability of staying in the contract remains unchanged. So the corresponding BEL components will rise up only due to the increasing in the assets.
- The value of the fund increases with rates, as the equity and property value increase, based on the observations above. Then the benefit cash flows in case of lapse and death rise accordingly and so does the liability value. In particular this is due to the fact that the BEL components related to the fund rise up.
- The discount factor applied to the insured capital decreases as risk free rates go up, resulting in a reduction of liability value.

In essence, we observed that while the fund is only affected positively by a shift upwards of the risk free rate curve, the liability part is influenced both positively and negatively. For this reason a qualitative analysis is not enough, and we had to test numerically the shift. What we observed is that the negative impact outweights the other since overall the liabilities decrease. We expected this result since we noticed a similar behaviour in the IR up case, even if here we are performing a parallel shift of the curve.

In case of a downward parallel shift of the risk-free rate curve, we repeated the same evaluations but in the opposite way. We would observe that discounts increase, resulting in an overall increase of the liabilities, even if equity and property values would go down.

5.2 Question 2

What happens to the liabilities if the insured age increases? What if there were two model points, one male and one female?

For this second question, differently from the previous one, we are asked to consider the risk-free rates fixed and to deal with mortality rates. In particular if the insured age increases also mortality rates become bigger.
In this case the asset side will remain unchanged and we'll focus only on the liability side.
If the insured age increases, the same will happen to mortality rates and so the probability of staying in the contract will decrease.
This affects negatively the liabilities that will drop down. In particular, this is due to the fact that lapse benefits decrease since they are less likely to be paid, as the probability of surviving is less. The same happens to expenses and commissions benefits.
On the other hand, death benefits are the only ones that increase since people are likely to die before, payments will be made earlier and the duration of the contract will shorten. We checked our assumptions by selecting in Matlab the mortality rates corresponding to people aged from 65 to 115 (from the EIOPA life tables) years to maintain the same length of the contract while increasing mortality rates based on real data.

Let us now consider the possibility of having two model points, one for females and one for males. Since data shows that females have a longer life expectancy than men, if we consider two people of the same age but from the two different genders, we can say that the first is more likely to get a higher final benefit by the insurance company. In fact for women death benefits will decrease since they are less likely to be paid, while lapse, expenses and commissions benefits will increase as the probability of staying in the contract increases. Overall with two model points we observe that for males liabilities decrease while for women increase.

Annex - Matlab Code

6.1 Script

```
% Solvency II - Final project  2023/2024
%
% GROUP 9 : Michela Galletti, Margherita Bencini
%           Sara Fregnan, Alice Flamigni

clear all
clc
rng(42) % setting the seed for MC simulations

% setting  given parameters
F0 = 100000;  % initial value of the fund
C0 = 100000;  % initial value of

equityp = 0.8;           % percentage of equity in the fund
propertyp = 0.2;         % percentage of property in the fund
sigma_equity = 0.20;     % equity volatility
sigma_property = 0.10;   % property volatility

EQ0 = F0*equityp;     % computing the initial value of equity
PR0 = F0*propertyp;   % computing the initial value of property

RD = 2.2/100;         % Regular Deduction
COMM = 1.4/100;       % Commissions to the distribution channels
T = 50;               % Time horizon for projections

lt = 15/100;                    % costant annual lapse rate
lapse = [ones(T-1,1)*lt; 1]; % setting the lapse rate as vector (for easier computations in the implemented functions)
                                     % we set the last value at 1 for massive surrender after 50 years
lapse_penalties = 20;           % penalty lapse
inflation_rate = 2/100;         % annual inflation rate
expenses = 50;
```

```
expenses_inflation = [expenses ,expenses.*(1+inflation_rate)
    .^[1:T]]'; % cost of 50 euros per year growing for
    inflation factor
%% Importing the rates from EIOPA tables
zero_RFR = xlsread("EIOPA_RFR_20240331_Term_Structures","
    RFR_spot_no_VA","S11:S60");
zero_RFR_up = xlsread("EIOPA_RFR_20240331_Term_Structures"
    ,"Spot_NO_VA_shock_UP","S11:S60") ; % shock up
zero_RFR_down = xlsread("EIOPA_RFR_20240331_Term_Structures"
     , "Spot_NO_VA_shock_DOWN","S11:S60") ; %shock down

% setting time discretization
t = [1:T]';
% computing zero rates
zero_RFR= log(1+ zero_RFR) ;
zero_RFR_up= log(1+ zero_RFR_up) ;
zero_RFR_down= log(1+ zero_RFR_down) ;

%% computing discounts
B = exp(-zero_RFR.*t);
B_up = exp(-zero_RFR_up.*t);
B_down = exp(-zero_RFR_down.*t);

% forward discounts
B_fwd = B(2:end)./B(1:(end-1));
B_fwd_up = B_up(2:end)./B_up(1:end-1); % up scenario
B_fwd_down = B_down(2:end)./B_down(1:end-1) ; % down
    scenario

% forward rates
R_fwd = [zero_RFR(1); -log(B_fwd)]
R_fwd_up = [zero_RFR_up(1); -log(B_fwd_up)];
R_fwd_down = [zero_RFR_down(1); -log(B_fwd_down)];

% FUND computations
lifetable = xlsread('Lifetables');           % importing
    life tables
qx = xlsread("Lifetables","E69:E118")/1000;  % importing
    mortality rates

NMC = 1e6; % number of simulations for MC
EQ = MonteCarlo(EQ0 ,RD , R_fwd , sigma_equity ,T,NMC); %
    simulating equity
PR = MonteCarlo(PR0, RD, R_fwd, sigma_property ,T,NMC);   %
    simulating property
F = EQ + PR;  % computing the fund

% up scenario
EQ_up = MonteCarlo(EQ0 ,RD , R_fwd_up , sigma_equity ,T,NMC)
    ;  % simulating equity
```

```
PR_up = MonteCarlo(PR0, RD, R_fwd_up, sigma_property ,T,NMC)
    ;    % simulating property
F_up = EQ_up + PR_up;  % computing the fund

%down scenario
EQ_down = MonteCarlo(EQ0 ,RD , R_fwd_down , sigma_equity ,T,
    NMC);  % simulating equity
PR_down = MonteCarlo(PR0, RD, R_fwd_down, sigma_property ,T,
    NMC);    % simulating property
F_down = EQ_down + PR_down;  % computing the fund

%% Liabilities and BEL computations
[Liabilities, BEL_D, BEL_L, BEL_Exp, BEL_Comm] = BEL_func(F,
     T, B , lapse , qx , lapse_penalties, expenses_inflation
    , RD, COMM);
% BOF base case
BOF = F0 - Liabilities;

%% Market interest risk
Liabilities_up = BEL_func(F_up, T, B_up , lapse , qx ,
    lapse_penalties, expenses_inflation , RD, COMM);
Liabilities_down = BEL_func(F_down,T, B_down , lapse , qx ,
    lapse_penalties, expenses_inflation , RD, COMM);

BOF_IR_up = F0-Liabilities_up;
BOF_IR_down = F0-Liabilities_down;

SCR_IR_up = max(BOF-BOF_IR_up,0);
SCR_IR_down = max(BOF-BOF_IR_down,0);

SCR_IR = max(SCR_IR_up,SCR_IR_down);

%plots

%plot of the interest rates
figure()
grid on
plot(t,zero_RFR,'LineWidth',2)
grid on
hold on
plot(t,zero_RFR_up,'--','LineWidth',2)
plot(t,zero_RFR_down,'--','LineWidth',2)
legend('BASE', 'UP', 'DOWN','FontSize',15)
xlabel('time(years)')
ylabel('IR')
title('Interest Rates (IR Risk)','FontSize',20)

%plot of the equity
figure()
t_p = [0;t]
```

```
plot(t_p,mean(EQ,1),'LineWidth',2)
hold on
plot(t_p,mean(EQ_up,1),'--','LineWidth',2)
plot(t_p, mean(EQ_down,1),'--','LineWidth',2)
legend('BASE', 'UP', 'DOWN','FontSize',15)
xlabel('time(years)')
ylabel('equity')
title('Equity value (IR Risk)','FontSize',20)


%plot of the property
figure()
plot(t_p,mean(PR,1),'LineWidth',2)
hold on
plot(t_p,mean(PR_up,1),'--','LineWidth',2)
plot(t_p,mean(PR_down,1),'--','LineWidth',2)
legend('BASE', 'UP', 'DOWN','FontSize',15)
title('Property value (IR Risk)','FontSize',20)
ylabel('property')
xlabel('time(years)')

%plot of the fund
figure()
plot(t_p,mean(F,1),'LineWidth',2)
hold on
plot(t_p,mean(F_up,1),'--','LineWidth',2)
plot(t_p,mean(F_down,1),'--','LineWidth',2)
legend('BASE', 'UP', 'DOWN','FontSize',15)
title('Value of the total fund (IR Risk)','FontSize',20)
ylabel('fund')
xlabel('time(years)')



%% Market equity risk
shock_type1 = 0.39; % this is type 1, so this is the shock
symm_adj= 0.0525; % for march 2024 provided by EIOPA
equity_shock = (1-shock_type1-symm_adj)*EQ0;
NMC = 1e6; % number of MC simulations
simulation_equity = MonteCarlo(equity_shock ,RD , R_fwd ,
    sigma_equity ,T,NMC);
liabilities_equity = BEL_func(simulation_equity + PR ,T, B ,
     lapse , qx , lapse_penalties, expenses_inflation , RD,
    COMM);
F0_shock = equity_shock + PR0;
BOF_equity_shock = F0_shock - liabilities_equity;
SCR_equity_shock = max(BOF-BOF_equity_shock,0);

%plotting the equity price
figure()
```

```
grid on
plot(t_p,mean(EQ,1),'LineWidth',2)
hold on
plot(t_p,mean(simulation_equity,1),'LineWidth',2)
legend('Equity base', 'Equity SHOCK','FontSize',15)
title('Equity price (Equity risk)','FontSize',20)
ylabel('equity')
xlabel('time(years)')


%% Property risk
shock_property = 0.25;
property_shock = (1-shock_property)*PR0;
NMC = 1e6; % number of MC simulations
simulated_property = MonteCarlo(property_shock ,RD , R_fwd ,
    sigma_property ,T,NMC);
liabilities_property = BEL_func(simulated_property + EQ ,T,
    B , lapse , qx , lapse_penalties, expenses_inflation , RD
    , COMM);
F0_shock_p = EQ0 + property_shock;
BOF_property_shock = F0_shock_p - liabilities_property;
SCR_property_shock = max(BOF-BOF_property_shock,0);

%plotting the property value
figure()
grid on
plot(t_p,mean(PR,1),'LineWidth',2)
hold on
plot(t_p,property_shock*ones(T+1),'LineWidth',2)
legend('Property base', 'Property SHOCK','FontSize',15)
title('Property price (Property risk)','FontSize',20)
ylabel('property')
xlabel('time(years)')

%% Expenses risk
increase_expense = 0.1; % shock
expense_shock = (1 + increase_expense)* expenses;
NMC = 1e6; % number of MC simulations
expenses_increased = [expense_shock; expense_shock*(1+(
    inflation_rate + 0.01)).^t];
liabilities_expense = BEL_func(PR + EQ ,T, B , lapse , qx ,
    lapse_penalties, expenses_increased , RD, COMM);
BOF_expense_shock = F0 - liabilities_expense;
SCR_expense_shock = max(BOF-BOF_expense_shock,0);


figure()
grid on
plot(t_p,expenses_inflation,'LineWidth',2)
hold on
```

```
plot(t_p,expenses_increased,'LineWidth',2)
legend('Expenses base', 'Expenses SHOCK','FontSize',15)
title('Expenses (Expenses risk)','FontSize',20)
ylabel('expenses')
xlabel('time(years)')

%% Mortality risk
increase_mortality = 0.15; % shock value
qx_mort = (1 + increase_mortality)*qx;
liabilities_mortality = BEL_func(PR + EQ ,T, B , lapse ,
    qx_mort , lapse_penalties, expenses_inflation , RD, COMM)
    ;
BOF_mort_shock = F0 - liabilities_mortality;
SCR_mort_shock = max(BOF-BOF_mort_shock,0);

figure()
grid on
plot(t,qx,'LineWidth',2)
hold on
plot(t,qx_mort,'LineWidth',2)
legend('qx Base', 'qx SHOCK','FontSize',15)
title('Mortality Rates (Mortality risk)','FontSize',20)
ylabel('qx')
xlabel('time(years)')

%% Lapse risk
% lapse up
lapse_up = min(1.5*lt,1).*ones(T,1);
liabilities_lapse_up = BEL_func(PR + EQ ,T, B , lapse_up ,
    qx , lapse_penalties, expenses_inflation , RD, COMM);
BOF_lapse_up = F0 - liabilities_lapse_up;
dBOF_lapse_up = max(BOF-BOF_lapse_up,0);

% lapse down
lapse_down = max(0.5*lt,lt-0.2).*ones(T,1);
liabilities_lapse_down = BEL_func(PR + EQ ,T, B , lapse_down
     , qx , lapse_penalties, expenses_inflation , RD, COMM);
BOF_lapse_down = F0 - liabilities_lapse_down;
dBOF_lapse_down = max(BOF-BOF_lapse_down,0);

% lapse mass
lapse_mass = [lt+0.4; lt.*ones(T-1,1)];
liabilities_lapse_mass = BEL_func(PR + EQ ,T, B , lapse_mass
     , qx , lapse_penalties, expenses_inflation , RD, COMM);
BOF_lapse_mass = F0 - liabilities_lapse_mass;
dBOF_lapse_mass = max(BOF-BOF_lapse_mass,0);

figure()
grid on
plot(t,lt*ones(T),'LineWidth',2)
```

```
hold on
plot(t,lapse_up,'LineWidth',2)
plot(t,lapse_down,'LineWidth',2)
plot(t,lapse_mass,'LineWidth',2)
legend('lapse Base', 'lapse UP','lapse DOWN','lapse MASS','
    FontSize',15)
title('Lapse Ratio (Lapse risk)','FontSize',20)
ylabel('lt')
xlabel('time(years)')

% taking the maximum to compute the SCR
SCR_lapse = max([dBOF_lapse_mass,dBOF_lapse_down,
    dBOF_lapse_up]);

%% Catastrophe risk
incr = 1.5/1000;  % shock value
qx_cat = [qx(1) + incr; qx(2:end)];
liabilities_cat =  BEL_func(PR + EQ ,T, B , lapse , qx_cat ,
     lapse_penalties, expenses_inflation , RD, COMM);
BOF_cat = F0 - liabilities_cat;
SCR_cat = max(BOF-BOF_cat,0);

figure()
grid on
plot(t,qx,'LineWidth',2)
hold on
plot(t,qx_cat,'LineWidth',2)
legend('qx Base', 'qx Cat','FontSize',15)
title('Mortality Rates (Catastrophe risk)','FontSize',20)
ylabel('qx')
xlabel('time(years)')

%% Computation of BSCR
% SCR market
market = [SCR_IR;SCR_equity_shock;SCR_property_shock];
if SCR_IR == SCR_IR_up
    corr_mkt = [1 0 0; 0 1 0.75; 0 0.75 1];
else
    corr_mkt = [1 0.5 0.5; 0.5 1 0.75; 0.5 0.75 1]
end
SCR_mkt = sqrt(market'*corr_mkt*market);

%SCR life
life = [SCR_mort_shock; SCR_lapse ;SCR_expense_shock;SCR_cat
    ];
corr_life = [1 0 0.25 0.25; 0 1 0.5 0.25; 0.25 0.5 1 0.25;
    0.25 0.25 0.25 1];
SCR_life = sqrt(life'*corr_life*life);

% BSCR
```

```
SCR = [SCR_mkt;SCR_life];
corr = [1 0.25; 0.25 1];
BSCR = sqrt(SCR'*corr*SCR);

%% Martingale Test
% Adding the discount at time 0
discounts = [1;B];
NMC = 1e6; % setting number of MC simulations
% monthly and yearly MC errors for assets for time step
    selection
[err_monthly_equity,err_yearly_equity] = mtg_test(T,R_fwd,
    EQ0,sigma_equity, RD,NMC);
[err_monthly_property,err_yearly_property] = mtg_test(T,
    R_fwd,PR0,sigma_property, RD,NMC);

% Plotting the results for different values
 for ii = 1:6
     NMC = 10^ii
     figure()
     EQ = mean(MonteCarlo(EQ0 , RD, zero_RFR ,sigma_equity ,
         T, NMC))./((1-RD).^([0:T]));
     EquityDeterministic = mean(MonteCarlo(EQ0 , RD,
        zero_RFR ,0 , T, NMC))./((1-RD).^([0:T]));
     errorDet = abs(EquityDeterministic-EQ);
     plot(t_p,errorDet)
     hold on
     % plot(t_p,EquityDeterministic);
     plot(t_p,zeros(size(t_p)))
     PR = mean(MonteCarlo(PR0 , RD, zero_RFR ,sigma_equity ,
         T, NMC));
     PropertyDeterministic = mean(MonteCarlo(PR0 , RD,
        zero_RFR ,0 , T, NMC));
     errorProp = abs(PropertyDeterministic-PR);
     figure()
    plot(t_p,errorProp);
 end
```

6.2 Monte Carlo simulation Function

```
function [St,CFInt] = MonteCarlo ( S0 , RD, R_fwd , sigma ,
    T, NMC)

% INPUT
% S0:      initial value of the asset to be simulated
% RD:      Regular Deductio
% R_fwd    forward rate
% sigma:   volatility
% T:       time to maturity
```

```
% NMC:      number of MC simulations
%
% OUTPUT
% St:             expected value of the simulated asset
% CFInt:          confidence intervals 97.5%

% The function which simulates assets using the Geometric
    Brownian Motion
% model

rng(42)  % setting the seed
n = length(R_fwd);
dt = T/n; % delta time
g = randn(NMC, n);  % matrix with random values extracted
    from a standard normal distribution

St = zeros(NMC, n+1);  % initialization
St(:,1) = S0;          % sttin the initial value
for ii = 2:n+1
    St(:,ii) = (1-RD).*St(:,ii-1).*exp((R_fwd(ii-1)-0.5*
        sigma^2)*dt+sigma*sqrt(dt).*g(:, ii-1));
    % actual simulation of the asset according to a GBM
end

% computing confidence intervals 97.5% for MC
StMean = mean(St,1);
StVar = std(St,1);
CFInt(:,1) = StMean + StVar./sqrt(NMC)*normcdf(0.975);
CFInt(:,2) = StMean - StVar./sqrt(NMC)*normcdf(0.975);
```

6.3 Liabilities Function

```
function [Liabilities, BEL_D, BEL_L, BEL_Exp, BEL_Comm,
    Duration] = BEL_func(F,T, B , lt , qx , lapse_penalties,
    expenses_inflation , RD, COMM)

% INPUT
% F:                    Fund matrix with simulated EQ and PR
% T:                    Time horizon
% B:                    discount factors
% lt:                   flat annual spot lapse (cost. vector
    )
% qx:                   annual mortality rate
% lapse_penalties       penalty in case of lapse
% expenses_inflation    expenses following inflation
% RD:                   Regular deduction
% COMM:                 Commissions to the distribution
    channels
```

```
%
% OUTPUT
% BEL:      Total BEL value = Liabilities
% BEL_D:    Death BEL
% BEL_L:    Lapse BEL
% BEL_Exp:  Expenses BEL
% BEL_Comm: Commissions BEL


% The function computes the BEL value and its components

F0 = F(:,1); % invested premium

Prob_stay = cumprod ([1;(1- qx(1:end-1)).*(1-lt(1:end-1)) ;
    0]); % probability of staying into the contract

% initialization of the vectors created in the for loop
c_f = zeros(T,1);
L_cf = zeros(T,1);
D_cf = zeros(T,1);
Exp_cf = zeros(T,1);
Comm_cf = zeros(T,1);

% computing the different cash flows at each time instant
for ii = 1:T

    D = mean((max(100000, F(:,ii+1))))*qx(ii); % case of
        death
    L = mean((F(:,ii+1) - lapse_penalties))*lt(ii)*(1-qx(ii)
        ); % case of lapse
    commissions = mean(COMM*F(:,ii+1)/(1-RD));   %
        commissions

    c_f(ii) = (D + L + commissions + expenses_inflation(ii
        +1))*Prob_stay(ii);
    L_cf(ii) = L*Prob_stay(ii);
    D_cf(ii) = D*Prob_stay(ii);
    Exp_cf(ii) = expenses_inflation(ii+1)*Prob_stay(ii);
    Comm_cf(ii) = commissions*Prob_stay(ii);
end


% computing BEL components and total BEL
BEL_D = sum(D_cf .*B);
BEL_L = sum(L_cf .*B);
BEL_Exp = sum(Exp_cf .*B)
BEL_Comm= sum( Comm_cf.*B)

Liabilities = BEL_D + BEL_L + BEL_Exp + BEL_Comm;

```

```
% computing the duration
t = linspace(1,50,50)';
c_f_disc=c_f.*B; % discounting the cash flows
tot = sum(c_f_disc.*t) ;
Duration = tot/Liabilities ;
end
```

6.4 Martingale Test Function

```
function [err_monthly,err_yearly] = mtg_test(T,fwd,S0,sigma,RD,NMC)

% INPUT
% T:        Time horizon
% fwd:      forward rates
% S0:       initial value of the asset
% sigma:    volatility of the asset
% RD:       regular deduction
% NMC:      number of MC simulations
% OUTPUT
% err_monthly:  MC error on a monthly basis
% err_yearly:   MC error on an annual basis

% The function verifies the Martingale property of the Stock
t_p = 1:T;
times_m = 1/12:1/12:T % setting monthly time fractions

fwd_m = interp1(0:T,[1;fwd],times_m); % getting missing fwd rates for the months via linear interpolation
B_mfwd = exp(-fwd_m*(1/12)); % getting monthly forward discounts
B_m = [1,cumprod(B_mfwd)]; % getting monthly

B_fwd = exp(-fwd.*1); % getting yearly forward discounts
B_yearly = [1;cumprod(B_fwd)]'; % getting yearly discounts

S_m = mean(MonteCarlo(S0,RD,fwd_m,sigma,T,NMC),1); % MC simulation on months
S_y = mean(MonteCarlo(S0,RD,fwd,sigma,T,NMC),1);   % MC simulation yearly

%err_monthly = (mean(S_m,1)-F0).^2  %MC error on a monthly basis
%err_yearly = (mean(S_y,1)-F0).^2   %MC error on a monthly basis
err_monthly = std(S_m)./sqrt(NMC)%MC error on a monthly basis
err_yearly = std(S_y)./sqrt(NMC)%MC error on a monthly basis
```

```matlab

FDiscountedmonthly = S_m.*B_m; %Compute the value of the asset yearly simulated and discounted
FDiscountedyearly = S_y.*B_yearly; % Compute the value of the asset monthly simulated and discounted

% After discounting again wrt the RD, we plotted the results
figure()
plot([0,times_m],FDiscountedmonthly./((1-RD).^([0,times_m]*12)),'LineWidth',2);
legend('simulation monthly','simulation yearly','EQ0','FontSize',15)
title('Monthly vs Yearly Evolution','FontSize',20)
ylabel('Asset')
xlabel('time(years)')
hold on
plot([0,t_p],FDiscountedyearly./((1-RD).^([0,t_p])),'LineWidth',2);
plot([0,t_p],ones(size([0,t_p])).*S0,'LineWidth',2);
legend('simulation monthly','simulation yearly','PRO','FontSize',15)
title('Monthly vs Yearly Evolution','FontSize',20)
ylabel('Asset')
xlabel('time(years)')
end
```