09-03-2022

Seminar 2

Problema → LOAD BALANCE (P.2)

N taskuri → t_i

M mașini

- 2 aproximare
- $2 - \frac{1}{m}$
- $\frac{3}{2}$
- $\frac{4}{3}$

de lop

de lop

$\nearrow \frac{3}{2} - \frac{1}{m}$ → (TEMĂ)

$\nearrow \frac{4}{3} - \frac{1}{m}$

OPT → soluția optimă

L_i → load-ul mașinii i

$L_j = \max_{i=1}^{m} L_i$

K → indicele ultimului task de pe mașina j

$t_{max} = \max_{i=1}^{m} t_i$

$OPT \geq t_{max}$ (1)

$OPT \geq \frac{1}{m} \cdot \sum_{i=1}^{m} t_i$ (2)

$$\left.\begin{array}{l} L_j - t_k \leq L_i, \forall i \in \{1..m\} \\ L_j - t_k \leq L_1 \\ \vdots \\ L_j - t_k \leq L_m \end{array}\right\} (+) \Rightarrow m(L_j - t_k) \leq \sum_{i=1}^{m} t_i$$

$$L_j - t_k \leq \frac{1}{m} \sum_{i=1}^{m} t_i$$

$$L_j = \underbrace{L_j - t_k}_{\leq \frac{1}{m}\sum_{i=1}^{m} t_i} + \underbrace{t_k}_{\leq t_{max}} \leq \frac{1}{m}\sum_{i=1}^{m} t_i + t_{max} \leq \underset{\text{din (1) și (2)}}{OPT + OPT} \leq 2\,OPT$$

$$L_j - t_k \leq L_1$$

$\vdots$

$$L_j - t_k \leq L_j - t_k$$

$\vdots$

$$L_j - t_k \leq L_m \quad (+)$$

$$m(L_j - t_k) \leq \sum_{i=1}^{m} t_i - t_k \Rightarrow L_j - t_k \leq \frac{1}{m}\left(\sum_{i=1}^{m} t_i - t_k\right)$$

$$L_j = L_j + t_k - t_k \leq \frac{1}{m}\left(\sum_{i=1}^{m} t_i - t_k\right) + t_k = \frac{1}{m}\sum_{i=1}^{m} t_i - \frac{1}{m} t_k + t_k$$

$$= \frac{1}{m}\sum_{i=1}^{m} t_i + t_k\left(1 - \frac{1}{m}\right)$$

$$\leq OPT + OPT\left(1 - \frac{1}{m}\right)$$

$$\leq OPT\left(2 - \frac{1}{m}\right)$$

$\frac{3}{2}$ aproximare

$t_1 \geq t_2 \ldots \geq t_n$

I) $n \leq m$

$L_j \leq t_{max} \leq OPT \leq \frac{3}{2} OPT$

II) $n \geq m+1$

$1, \ldots, m+1$

$OPT \geq 2 t_{m+1}$ $(t_m + t_{m+1})$

$(t_i + t_j \geq t_{m+1} + t_{m+1})$

$k < m+1$

$k \geq m+1 \Rightarrow t_k \leq t_{m+1}$

$$L_j = L_j - t_k + t_k \leq OPT + t_{m+1}$$

$$\leq OPT + \frac{1}{2} OPT$$

$$\leq \frac{3}{2} OPT$$

Problema → TSP NP HARD (p.3)

- prima problemă NP HARD: N SAT (pb. satisfiabilității): $(X_1 \vee X_2 \vee X_{11}) \wedge (\overline{X}_2 \vee X_8 \vee X_{10}) \wedge \dots$ → 3 SAT
  - → 3 SAT → demonstrată că e NP HARD
  - → 2 SAT → nu e NP HARD $\left((X_1 \vee X_2) \Rightarrow X_1 \to X_2\right)$
- HC → NP HARD (ciclu hamiltonian)

Fie $G = (V, E)$ o instanță de HC.

Construim $G' = (V', E')$ a.î. $V' = V$

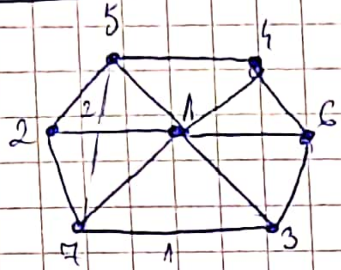
$$(x, y) \in E \Rightarrow (x, y, 1) \in E'$$

$$(x, y) \notin E \Rightarrow (x, y, 3) \in E'$$

G admite un HC $\Leftrightarrow$ G' are un TSP de lungime $|V| = n$

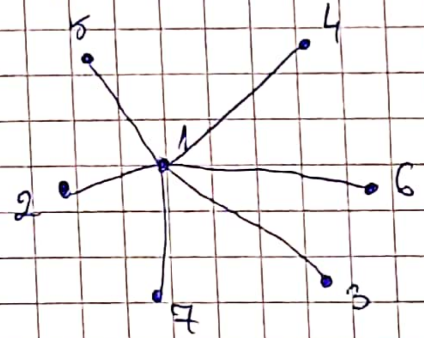
(pt temă trb să demonstrăm implicația „$\Leftarrow$")

$\boxed{2 - \text{aproximare}}$



1 7 2 5 4 6 3 1 → lungime 7

$\boxed{n}$



Prim

$O(m \log n)$

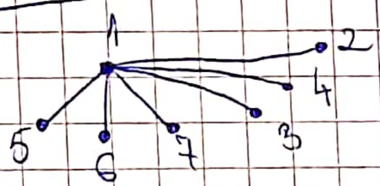
brut → $O(n^2)$

· pe graf complet, e bun

Kruskal

$O(n + m \log n)$

$\boxed{APM \leq OPT}$

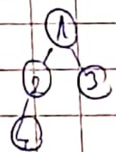


$\boxed{2n-2}$

DFS: 1 5 ~~1~~ 6 ~~1~~ 7 1 3 ~~1~~ 4 ~~1~~ 2 1

lungime / cost = 12

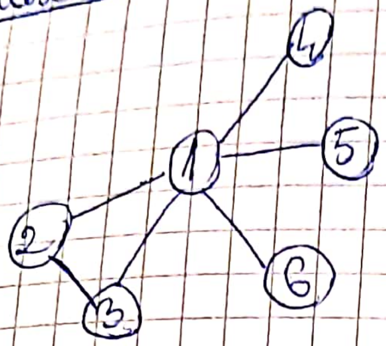
ALG ≤ 2OPT



LCA = 1 2 4 2 1 3 1

ALG ≤ 2 APM

Vertex Cover Problem



Ex: $\{1,2\}$ — mulțimea de vertex cov...

$\boxed{\text{2-aproximare}}$

$G=(V,E)$

$C=\emptyset$, $M=\emptyset$

while E not empty

  pick $e=(x,y)$

  $C=C\cup\{x,y\}$, $M=M\cup\{(x,y)\}$

  $G=\{V\setminus\{x,y\}, E\setminus(x,y)\}$

Fie $G=(V,E)$ un graf, $E^*\subset E$ a.î. oricum alegem 2 muchii din E^* ele să fie disjuncte (ex. $\{2,3\},\{1,4\}$). Atunci $OPT \geq |E^*|$

$|C| \leq 2|M| \leq 2\,OPT$ ($M\subset E$, M are muchii disjuncte)

Linear programming

$$f(x_1, x_2) = \alpha_1 x_1 + \alpha_2 x_2$$

$$x_1 + x_2 \leq 3$$

$$x_1 - x_2 \geq 1$$

$x_i \in \mathbb{R} \Rightarrow$ timp polinomial

$x_i \in \mathbb{Z} \Rightarrow$ NP HARD

Problema reducerii Vertex Cover Pb. la linear programming

$$V_i \rightarrow x_i \in [0,1]$$

$$\min \sum_{i=1}^{n} x_i$$

$$x_i + x_j \geq 1, \ \forall (i,j) \in E$$