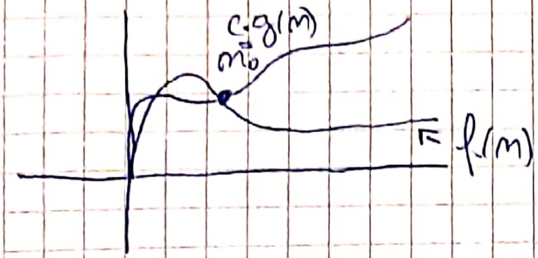
14-02-2022

Seminar 1

→ $\boxed{O}$

$$O(g(n)) = \{f(n) \mid \exists c>0\ \exists n_0>0 \text{ a.î. } f(n) \le c\cdot g(n), \forall n \ge n_0\}$$



$n \in O(n^2)$ ✓

$\lim_{n\to\infty} \frac{f(n)}{g(n)} < \infty \quad (n^3 \notin O(n^2))$

→ $\boxed{\Omega}$

$$\Omega(g(n)) = \{f(n) \mid \exists c>0\ \exists n_0>0 \text{ a.î. } f(n) \ge c\cdot g(n)\}$$

$n^3 \in \Omega(n^2 \log n)$ ✓

$\lim_{n\to\infty} \frac{f(n)}{g(n)} > 0$

→ $\boxed{\Theta}$

$$\Theta(g(n)) = \{f(n) \mid \exists c_1, c_2 > 0\ \exists n_0>0 \text{ a.î. } c_1 g(n) \le f(n) \le c_2 g(n)$$

$\exists \lim_{n\to\infty} \frac{f(n)}{g(n)} \in (0, \infty)\ (\in \mathbb{R}_+)$

Algoritmi → P (Polinomiali) → $O(n^c)$

→ NP (Nedeterminist polinomiali)
→ $O(n!)$...
(ex: ciclu hamiltonian)

$P \subset NP$
$(=?)$

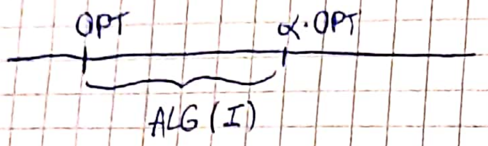
NP-complet

Scanned with CamScanner

Minimizare

Un algoritm are factorul de aproximare α dacă

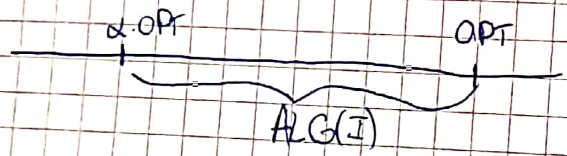
$$\text{output } ALG(I) \le \alpha \cdot OPT,\ \alpha \ge 1 \ ((1,\infty))$$



Maximizare

Un algoritm are factorul de aproximare α dacă

$$\text{output } ALG(I) \ge \alpha OPT,\ \alpha \in (0,1)$$



Problema rucsacului

G - greutate

n - obiecte

(v_i, g_i) - (valoare, greutate) pt obiectul i

$$\frac{v_i}{g_i} \ne \frac{v_j}{g_j},\ \forall i,j \in \{1,2,\dots,n\},\ i \ne j$$

$O(n \log n)$ - sortare $\Rightarrow$ complexitatea algoritmului e $O(n \log n)$

$(p_1, p_2, \dots, p_n)$

$p_1 = 1$ → iau tot ob., $p_2 = \frac{2}{3}$ → iau 2/3 din ob

Ex: $G = 10$

$v_1 = 4,\ v_2 = 5,\ v_3 = 7$

$g_1 = 4,\ g_2 = 6,\ v_3 = 8$

$\Longrightarrow \left(1, 0, \frac{6}{8}\right)$

$$4 \cdot 1 + 0 \cdot 5 + \frac{6}{8} \cdot 7$$

Demonstrație

Fie $ALG = (p_1, p_2, \dots, p_n)$
$1 \dots 1 \dots x_i \dots 0 \dots 0$

Pp că $\exists$ o soluție mai bună
$OPT = (g_1, g_2, \dots, g_n) \neq ALG$

Fie i primul indice pentru care $p_i \neq g_i$ $\Downarrow$ $p_i > g_i$

$$\sum_{i=1}^{n} p_i \cdot v_i < \sum_{i=1}^{n} g_i v_i$$

$ALG = (1, 1, 1, \dots, x_i, 0, 0, 0, \dots, 0)$

$\exists j > i$ a.î. $\boxed{p_j < g_j}$

$OPT' = [g_1', g_2', \dots, g_n']$
$g_k' = g_k, \forall k \in \{1, \dots, n\}$, $k \neq i, j$

$g_i' = g_i + \varepsilon$ $\leftarrow <0$
$g_j' = g_j - \varepsilon \dfrac{g_i}{g_j}$

$g_1' \cdot g_1 + g_2' \cdot g_2 + \dots + g_n' \cdot g_n = g_1/g_1 + g_2 \cdot g_2 + \dots + g_n/g_n$
$g_i' \cdot g_i + g_j' \cdot g_j = \cancel{g_i \cdot g_i + g_j \cdot g_j - \varepsilon}$
$= g_i \cdot g_i + \varepsilon/g_i + g_j \cdot g_j - \varepsilon/g_i$

$g_1' \cdot v_1 + g_2' \cdot v_2 + \dots + g_n' \cdot v_n = g_1 \cdot v_1 + g_2 v_2 + \dots + g_n v_n +$
$+ (g_i + \varepsilon) \cdot v_i + (g_j - \varepsilon \dfrac{g_i}{g_j}) g_j$

$= g_1 v_1 + g_2 v_2 + \dots + g_i v_i + g_j \cdot g_j + \dots + g_n v_n + \underbrace{\varepsilon v_i - \varepsilon \dfrac{g_i}{g_j} v_j}_{>0}$

$\varepsilon \left(v_i - \dfrac{g_i \cdot v_j}{g_j} \right) = \dfrac{\varepsilon}{v_j} \left(\dfrac{v_i g_j - g_i \cdot v_j}{g_j} \right) > 0$ $\dfrac{v_i}{g_i} > \dfrac{v_j}{g_j}$

Algo aprox (1/2 aproximativ) 20 ~~...~~

Fie $L \leftarrow$ lista obiectelor sortate descrescător după $\frac{v_i}{g_i}$

Fie $Op \leftarrow$ obiectul cu cea mai mare valoare

$S = 0$

$G \leftarrow$ capacitatea rucsacului

Pentru $O \in L$:

dacă greutate$(O) \leq G$

$S += $ valoare(O)

$G -= $ greutate(O)

$ALG = \max(S, Op)$

$\frac{1}{2}$ aproximare

$\left[\frac{1}{2} \cdot OPT, OPT\right]$

Problema rucsacului – dinamica

$$Dp[i][j] = \begin{cases} Dp[i-1][j], & g_i > j \\ \max(Dp[i-1][j], v_i + Dp[i-1][j-g_i]), & g_i \leq j \end{cases}$$

Complexitate: $O(\underbrace{n}_{\text{obiecte}} \underbrace{G}_{\text{greutate (cât de mare)}})$

Ex: (pe algo)

$v_1 = 60$, $v_2 = 100$, $v_3 = 120$

$g_1 = 10$, $g_2 = 20$, $g_3 = 30$

$G = 50$

$OPT = 220$ $(\overset{100}{2}$ și $\overset{120}{3})$

$60 + 100 = 160$

OPT

$ALG(I) \geq \frac{1}{2}$ OPT

Fie OPT soluția optimă

Fie O_j primul obiect care nu este ales

$$OPT \leq \sum_{i=1}^{j} O_i = \sum_{i=1}^{j-1} O_i + O_j \leq \sum_{i=1}^{j-1} O_i + O_p \leq ALG + ALG = 2ALG \Rightarrow$$

$$\Rightarrow ALG \geq \frac{1}{2} OPT$$

$$ALG = \max(S, O_p)$$

$$\sum_{i=1}^{j-1} O_i$$

Ex $G = 100,\ \varepsilon_1 > \varepsilon_2 > 0$

$$\frac{50+\varepsilon_1}{50+\varepsilon_2},\ \frac{50}{50},\ \frac{50}{50}$$

$OPT = 100$

$$\boxed{50+\varepsilon_1} \leq \frac{1}{2} 100$$