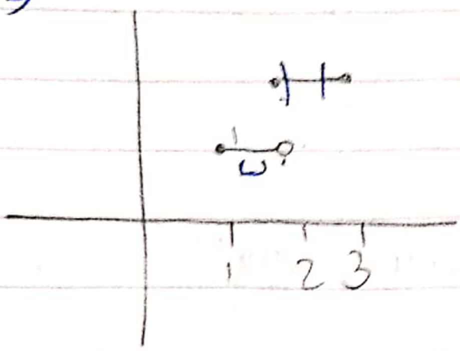
Calculus Assignment

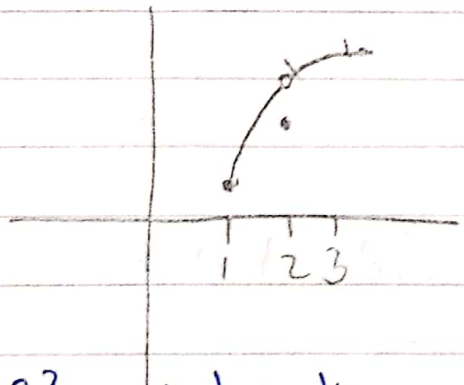
(Q1)

(1)



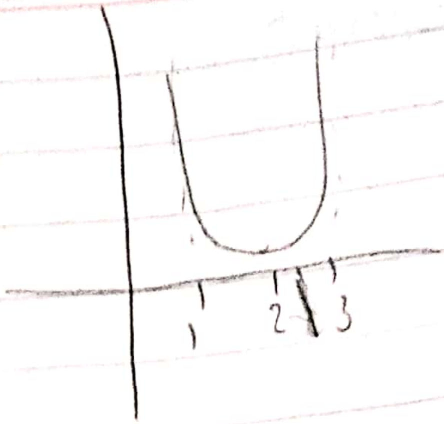
(a) [1,3] = not continuous because of a jump at x=2
(b) (1,3) = not continuous because of a jump at x=2
(c) [1,2] = not continuous because of a hole at x=2
(d) (1,2) = continuous
(e) [2,3] = ~~not~~ continuous
(f) (2,3) = continuous

(2)



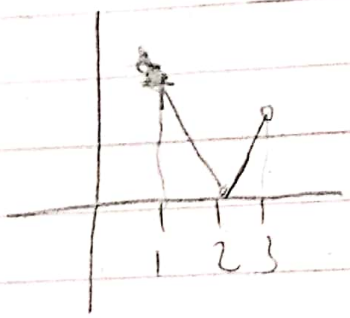
(a) [1,3] = not continuous, hole at x=2
(b) (1,3) = not continuous, hole at x=2
(c) [1,2] = not continuous, hole at x=2
(d) (1,2) = continuous
(e) [2,3] = not continuous, hole at x=2
(f) (2,3) = continuous

(3)



(a) [1,3] = not continuous, asymptote at x = 1,3
(b) (1,3) = continuous
(c) [1,2] = not continuous, asymptote at x = 1
(d) (1,2) = continuous
(e) [2,3] = not continuous, asymptoted at x = 3
(f) (2,3) = continuous

(4)



(a) [1,3] = not continuous, hole at x = 3
(b) (1,3) = continuous
(c) [1,2] = continuous
(d) (1,2) = continuous
(e) [2,3] = not continuous, hole at x = 3
(f) (2,3) = continuous

Q5) Consider the functions

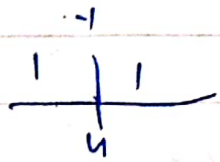
$$f(x) = \begin{cases} 1 & x \neq 4 \\ -1 & x = 4 \end{cases}$$

$$g(x) = \begin{cases} 4x-10 & x \neq 4 \\ -6 & x = 4 \end{cases}$$

Is function continuous at $x = 4$

(a) $f(x)$

$f(4) = -1$



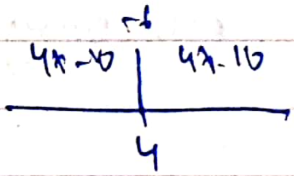
$\lim\limits_{x \to 4^-} f(x) = \lim\limits_{x \to 4^+} f(x)$

$-1 = -1$

$1 \neq 1$ not continuous

(b) $g(x)$

$f(4) = -6$



$\lim\limits_{x \to 4^-} f(x) = \lim\limits_{x \to 4^+} f(x)$

$4(4) - 10 = 4(4) - 10$

$\boxed{6 = 6}$

$f(x)$ exists

$-6 \neq 6$

not continuous

(c) 4 $-g(x)$

$$g(x) = \begin{cases} -4x+10 \\ 6 \end{cases}$$

$g(4) = 6$

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$$

$$-4(4)+10 = -4(4)+10$$

$$-6 = -6$$

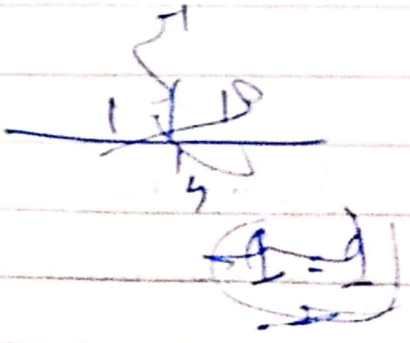
$f(x)$ exists

$6 \neq -6$ not continuous

(d) $|f(x)|$

$$\begin{cases} 1 & x \neq 4 \\ 1 & x = 4 \end{cases}$$

Continuous ~~for all real values of~~ bec every value is 1, including at $x=4$



(e) $f(x)\ g(x)$

$4x-10 \qquad x \neq 4$

$6 \qquad x = 4$

$f(4) = 6$

$4(4)-10 = 4(4)-10$

$\boxed{6 = 6} \Rightarrow f(x)$ exists

$\boxed{6 = 6} \rightarrow$ continuous

(f) $g((f(x)))$ $\qquad$ at $x=4$ $f(x)$ 6

$\boxed{-1}$

~~at $x \neq 4$~~:

$f(x)$ $\qquad$ $1 \quad x \neq 4$

$-1 \quad x = 4$

$g(x) = \begin{cases} 4x-10 & x \neq 4 \\ 4x-10 & x = 4 \end{cases}$

~~continuous since same at $x=4$ and $x \neq 4$~~

① $g(f(4)) = 4(1)-10 = -14$

② $x \to 4^- = x \to 4^+$

$4(1)-10 = 4(1)-10$

$-6 = -6 \rightarrow$

③ $-14 \neq -6$

∴ not continuous

(g) $g(x) - 6f(x)$

$$6f(x) \begin{cases} 6 & x \neq 4 \\ -6 & x = 4 \end{cases}$$

$4x-10-6 \qquad x \neq 4$

$-6+6) \qquad x = 4$

$$g(x) - 6(f(x)) \begin{cases} 4x-16 & x \neq 4 \\ 0 & x = 4 \end{cases}$$

$x=4=0$ ⓐ $x=4 \to 0$

$4x-16 = 4x-16$

$4(4)-16 = 4(4)-16$

$0=0$ $f(x)$ exist

$\boxed{0 = 0} \to$ continuous

(Q5) $g(-x)$

(Q6) $f(x) = \begin{cases} 1, & 0 \le x \\ 0, & x < 0 \end{cases}$

$g(x) = \begin{cases} 0, & 0 \le x \\ 1, & x < 0 \end{cases}$

Is function continuous at $x=0$

(a) $f(x)$

$f(0) = 1$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$\boxed{0 \neq 1} \Rightarrow$ not continuous

(b) $g(x)$

$g(0) = 0$

$\lim_{x \to 0^-} g(x) \quad \lim_{x \to 0^+} g(x)$

$\boxed{1 \neq 0} \Rightarrow$ not continuous

(c) $f(-x)$ $\quad 1 \quad 0 \le x$

$0 \quad x < 0$

$f(0) = 1$

$\boxed{0 \neq 1} \Rightarrow$ not continuous

$x \to 0^- \quad x \to 0^+$

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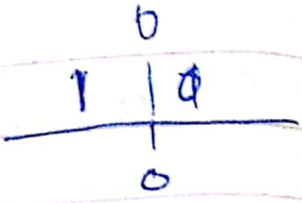
(d) $|g(x)|$

$$g(x)\begin{cases} 0 , & 0 \le x \\ 1 , & x < 0 \end{cases}$$



$g(0) = 0$

$$\underset{x\to 0^-}{1} = \underset{x \to 0^+}{0} \Rightarrow \text{not continuous}$$

(e) $f(x)\, g(x)$

$$\begin{cases} 0 & , \ 0 \le x \\ 0 & \ \ x < 0 \end{cases}$$

$f(0) = 0$

$\underset{x\to 0^+}{0} = \underset{x\to 0^-}{0}$

$\boxed{0 = 0}$

continuous because same at every point

(f) $g(f(x))$

$$\begin{cases} 0 & 0 \le x \\ 1 & x < 0 \end{cases}$$

$gf(0) = 0$

$x \to 0^+ ,\ x \to 0^-$

$\boxed{1 = 0} \Rightarrow$ not continuous

(9) $f(x) + g(x)$

$$\begin{cases} 1, & 0 \le x \\ 1, & x < 0 \end{cases}$$

continue $f(0) = 1$

$$\underset{x \to 0^+}{1} = \underset{x \to 0^-}{1}$$

$\boxed{1 = 1} \to$ continuous

(Q11) Find values of x, if any, where f is not continuous

(11) $f(x) = 5x^4 - 3x + 7$ = continuous because polynomials are always continuous

(12) $f(x) = \sqrt[3]{x-8}$ = continuous because it can exist ~~at~~ at any point

(13) $f(x) = \dfrac{x+2}{x^2+4}$ x^2+4 will never be 0 thus it is continuous

(22) $f(x) = \begin{cases} \dfrac{3}{x-1} & x \neq 1 \\ 3 & x = 1 \end{cases}$

$f(1) = 3$

$$\begin{array}{c|c} & 3 \\ \frac{3}{x-1} & \frac{3}{x-1} \\ \hline & 1 \end{array}$$

~~$x \to 1^+ = x \to 1^-$~~

~~$\dfrac{3}{3-1} = \dfrac{3}{3-1}$~~

~~$\dfrac{3}{2} = \dfrac{3}{2} \to f(x)$ exists~~

~~$f(c) = f(x)$~~

~~$3 \neq \dfrac{3}{2} \to$ not continuous~~

$x \to 1^- = x \to 1^+$

$\dfrac{3}{1-1} = \dfrac{3}{1-1}$

$\boxed{\text{undefined}}$

$\frac{3}{0}$

$\lim f(x)$ does not exist

& not continuous

Find k that will make function continuous

(29) (a) $f(x) = \begin{cases} 7x-2 & x \le 1 \\ kx^2 & x > 1 \end{cases}$

$f(1) = 7x-2$

$7-2$

$\boxed{5}$

5

5 | kx^2

1

$x \to 1^- = x \to 1^+$

$5 = k(1)^2$

$\boxed{5 = k}$

if $k = 5$ $f(x)$ will be continuous

(b) $f(x) \begin{cases} kx^2 & x \le 2 \\ 2x+k & x > 2 \end{cases}$

$f(2) = 4k$

4k

4k | 2x+k

2

$x \to 2^- = x \to 2^+$

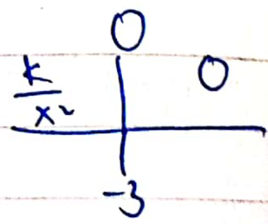
$$4k = 2(2) + k$$
$$4k = 4 + k$$

$$3k = 4$$
$$\boxed{k = \frac{4}{3}}$$

if $k = \frac{4}{3}$ then $f(x)$ continuous

(30) (9) $f(x) \begin{cases} 9 - x^2 & x \geq -3 \\ \frac{k}{x^2} & x < -3 \end{cases}$

$$f(-3) = 9 - (-3)^2$$
$$9 - (9)$$
$$\boxed{0}$$



$x \to -3^- = x \to -3^+$

$$\frac{k}{(-3)^2} = 0$$

$$\frac{k}{9} = 0$$

$\boxed{k=0} \to$ if $k=0$ then contin

~~no value of k where the function is continuous~~

(b) $\begin{cases} 9-x^2 & x \geq 0 \\ \dfrac{k}{x^2} & x < 0 \end{cases}$

$f(0) = 9$

$x \to 0^+ = x \to 0^-$

$9 = \dfrac{k}{0^2}$

no value of k where x is continuous

Find the values of x where f is not continuous and determine whether each such value is a removable discontinuity

(35) (a) $f(x) = \dfrac{|x|}{x}$ $x = 0$, x cannot be removed

(b) $f(x) = \dfrac{x^2+3x}{x+3}$ $\dfrac{x(x+3)}{x+3}$ $x = -3$, x can be ...

$f(-3) = -3$

Q35 Find discontinuity and place if removable

(a)

(a) $\frac{|x|}{x}$

$\frac{\pm x}{x}$

discontinuity at $x = 0$, not removable

(b) $\frac{x^2+3x}{x+3}$

discontinuity at $x=-3$, removable

$\frac{x\cancel{(x+3)}}{\cancel{x+3}} = \boxed{x}$

$\boxed{f(-3) = -3}$

(c) $\frac{x-2}{|x|-2}$

discontinuous at $= \pm 2$, ~~not removable~~

$\frac{\cancel{x-2}}{\cancel{x-2}}$

$\boxed{1}$

~~no discontinuity~~

$f(2) = \frac{2-2}{2-2}$

$\frac{0}{0}$ = removable

$\frac{\cancel{x-2}}{\cancel{-x-2}}$

$\frac{-(-x+2)}{-(x+2)}$

$\frac{2-x}{x+2}$

discontinuous at -2, removable

(Q36) (a) $\frac{x^2-4}{x^3-8}$ → discontinuity at $x = +2$, removable

$$\frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}\,\cancel{(x^2+4x+4)}}$$

x^2+2x+4

$\frac{\cancel{x+2}}{\cancel{x^2+4x+2}}$ → $\frac{1}{x+2}$

discontinuity at $x = 0$, removable

$$\frac{x+2}{x^2+2x+4} \to \frac{4}{12} = \frac{1}{3} \qquad f(2) = \frac{1}{3}$$

f be come continuous at $f(2)$

(b) $f(x) = \begin{cases} 2x-3 & x \le 2 \\ x^2 & x > 2 \end{cases}$

$f(2) = 1$

1 | x^2

2

$\boxed{1 \neq 4}$ → not continuous

non removable continuity at $x = 2$

1 (c) $f(x) \begin{cases} 3x^2+5 & x \neq 1 \\ 6 & x=1 \end{cases}$

$f(1) = 6$

$3x^2+5 = 3x^2+5$

$8 = 8$

$6 \neq 8$

thus removable discontinuity at $x=1$

(14) $f(x) = \dfrac{x+2}{x^2-4}$

discontinuity at ~~x=2~~ $x = \pm 2$

(15) $f(x) \dfrac{x}{2x^2+x} =$

$2x^2+x=0$

$x(2x+1)=0$

$x=0$

$x=-\frac{1}{2}$

discontinuity at $x=0$ and $x=-\frac{1}{2}$

(16) $f(x) = \dfrac{2x+1}{4x^2+4x+5}$ continuous because polynomials are always continuous

(17) $f(x) = \dfrac{3}{x} + \dfrac{x-1}{x^2-1}$

$x^2-1=0$

$x^2=1$

$x=\pm 1$

$\dfrac{3x^2-3+x^2-x}{x^3-x} \quad \dfrac{4x^2-x-3}{x^3-x}$

not continuous at $x=0$ and $x=\pm 1$

(18) $f(x) = \dfrac{5}{x} + \dfrac{2x}{x+4}$

not continuous at $x=0$ and $x=-4$

(19) $f(x) = \frac{x^2+6x+9}{|x|+3}$

continuous everywhere

(20) $f(x) = \left| 4 - \frac{8}{x^4+x} \right|$

$x^4+x=0$

$x(x^3+1)=0$

$\boxed{x=0}$

$x^3 \cdot \quad x=\sqrt[3]{-1}$

$\boxed{x=-1}$

not continuous at $x=0$ and $x=-1$

(21) $f(x) = \begin{cases} 2x+3 & x \le 4 \\ 7+\frac{16}{x} & x > 4 \end{cases}$

11 | $7+\frac{16}{x}$

4

$f(4) = 2x+3$

$8+3$

$\boxed{11}$

$x \to 4^-$ $\quad$ $x \to 4^+$

$11 = 7+\frac{16}{4}$

$11 = 7+4$

$\boxed{11=11} \Rightarrow f(x)$ exists

$f(c) = f(x)$

$\boxed{11=11} \Rightarrow$ continuous

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