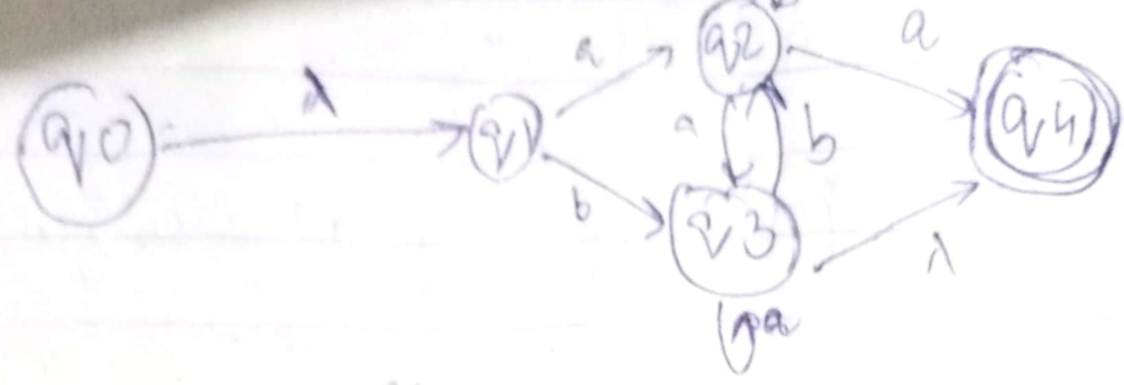
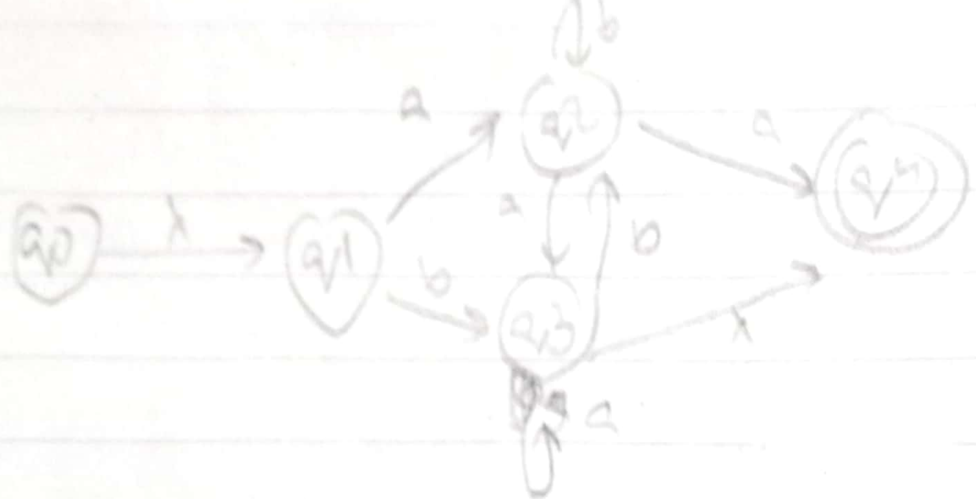
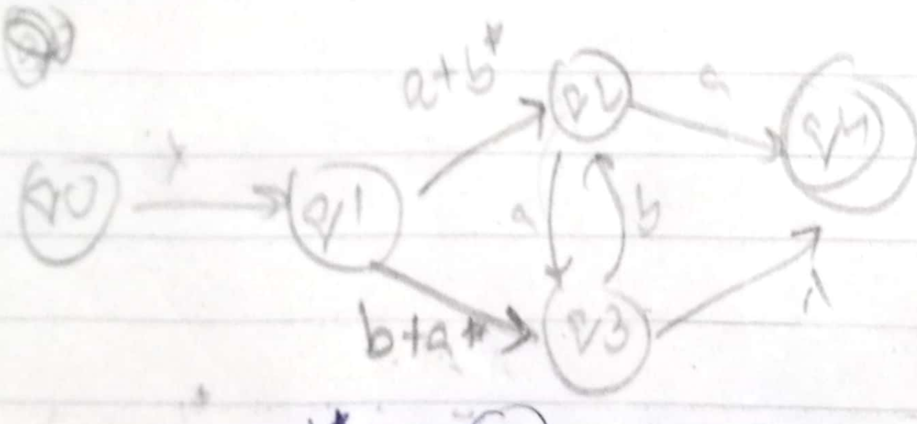
ICA assignment 4

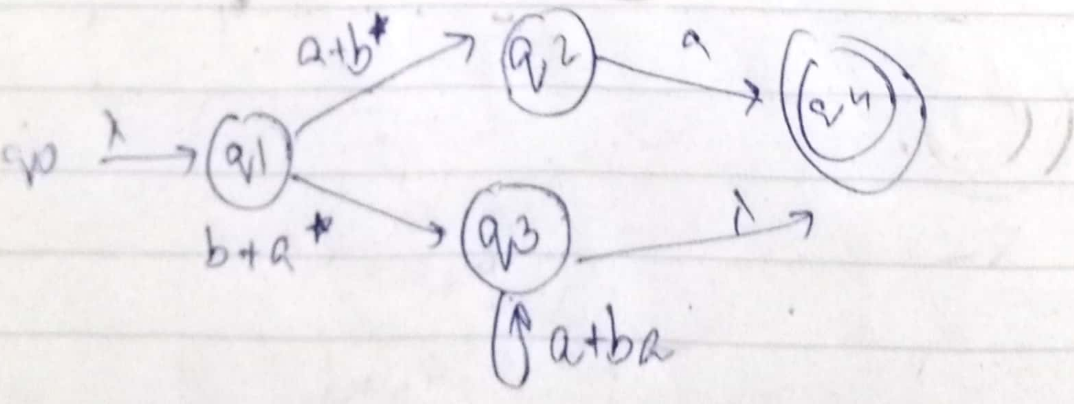
21K-3153

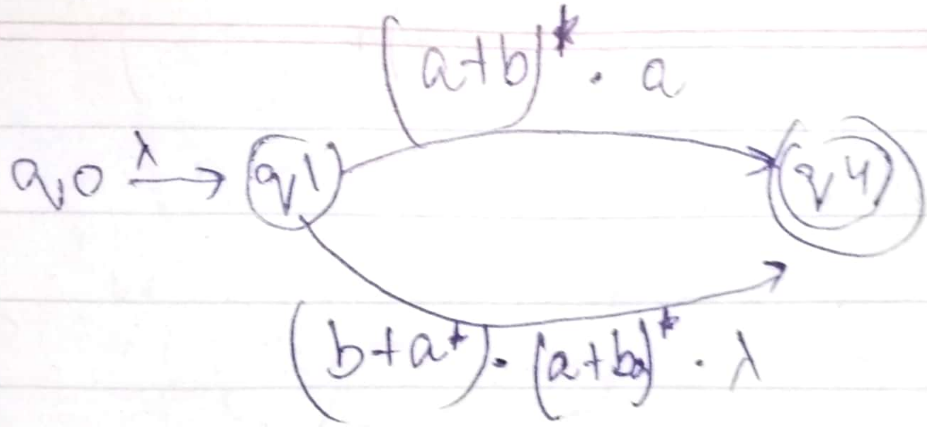
(i)

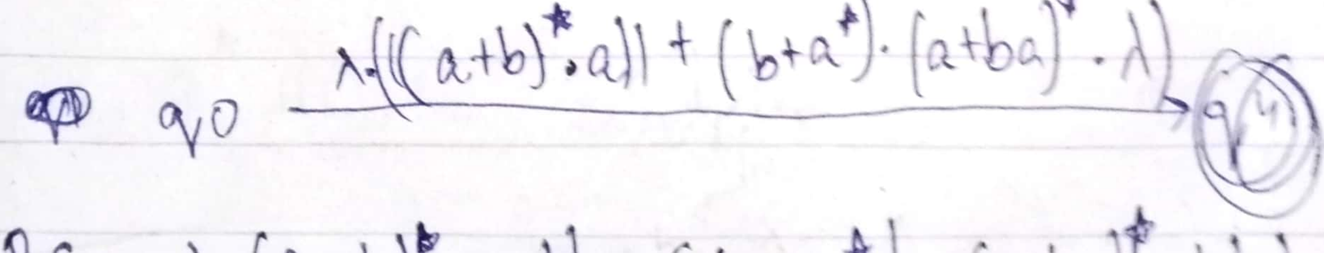




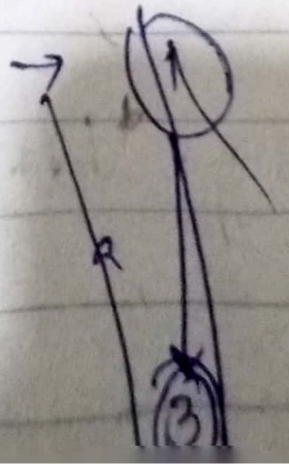




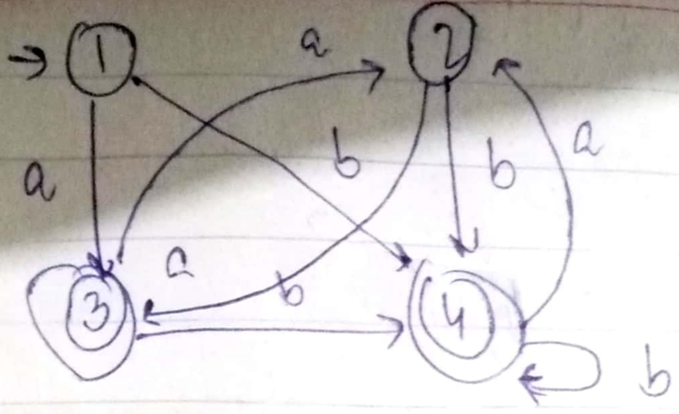




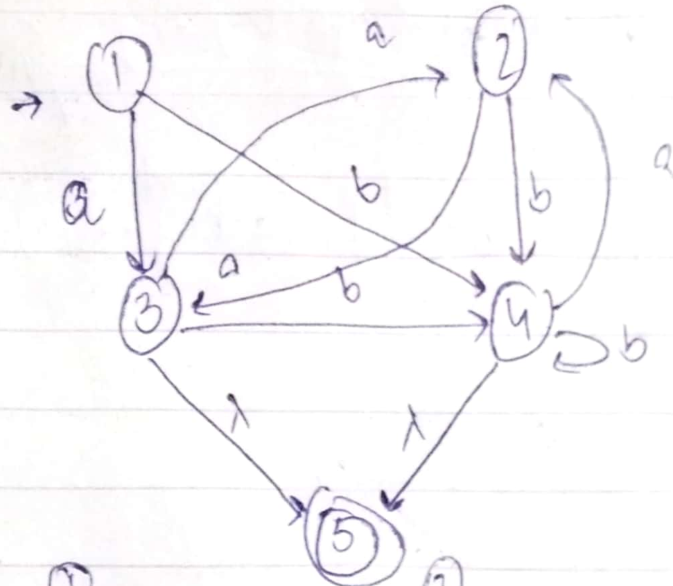
$$RE = \lambda \cdot ((a+b)^* \cdot a)) + (b+c^*) \cdot (a+ba)^* \cdot \lambda))$$



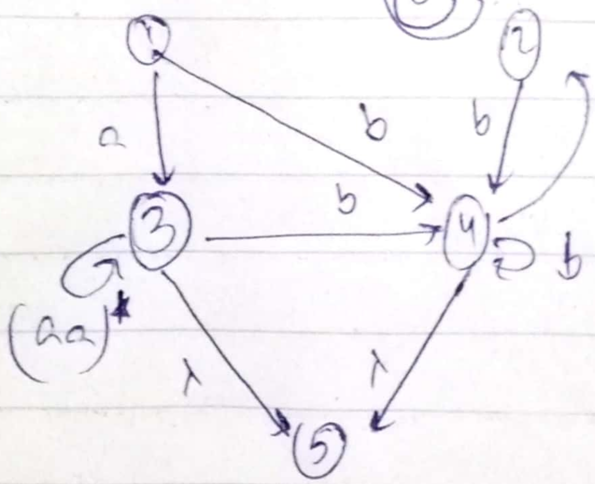
(ii)



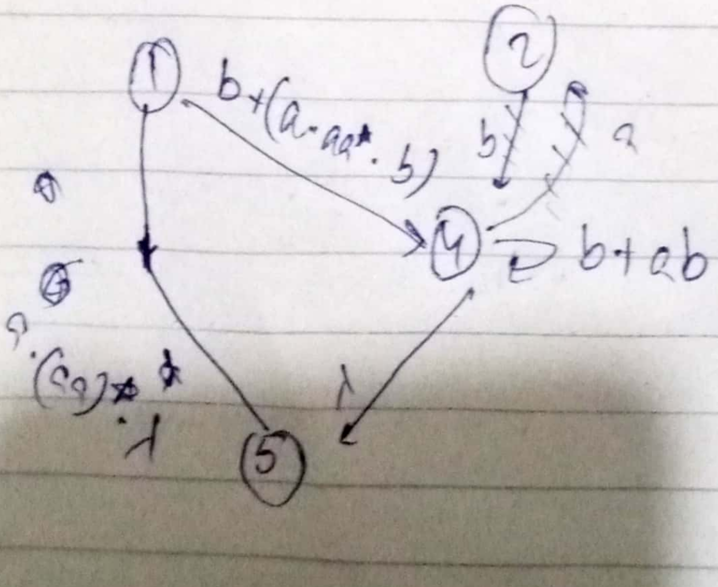
1
2
3
4
a
a
b
b
a
a
b
b



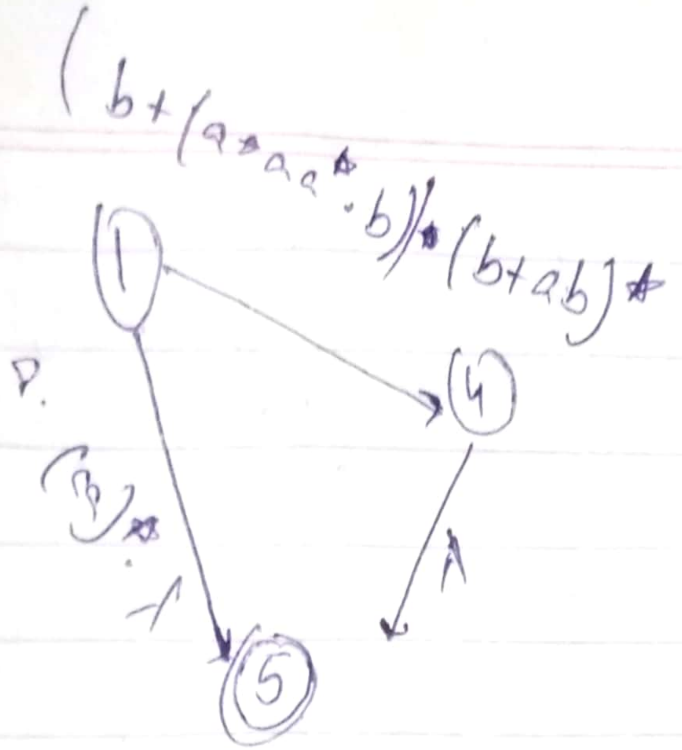
1
2
3
4
5
a
a
b
b
a
a
b
b
λ
λ

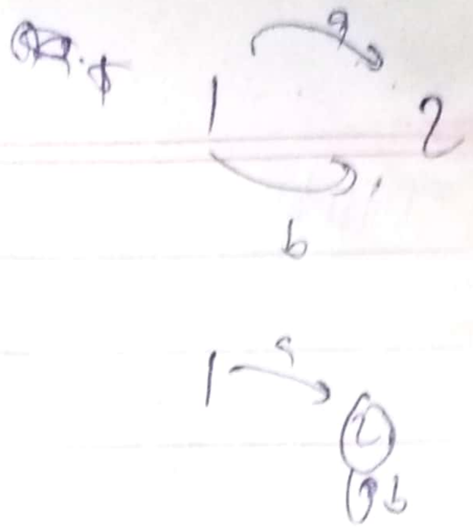


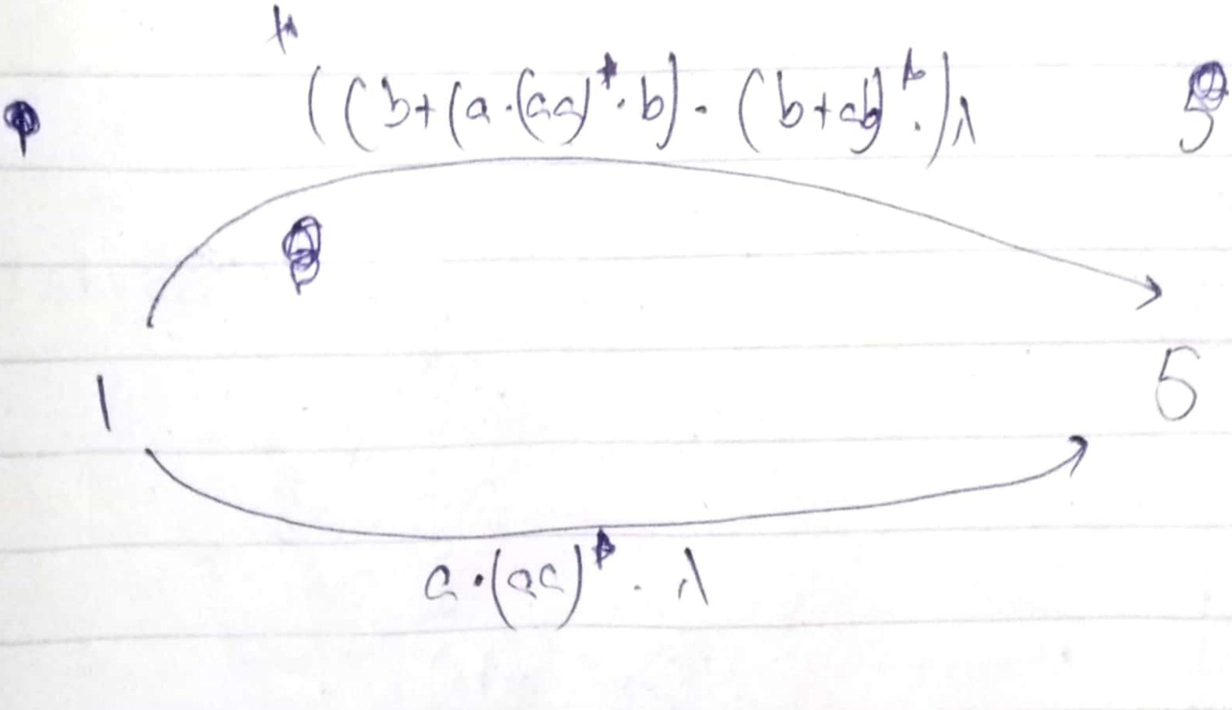
1
2
3
4
5
a
b
b
a
b
b
(aa)*
λ
λ



1
2
4
5
b+(a.aa*.b)
b
a
b+ab
a.(aa)*.λ
λ

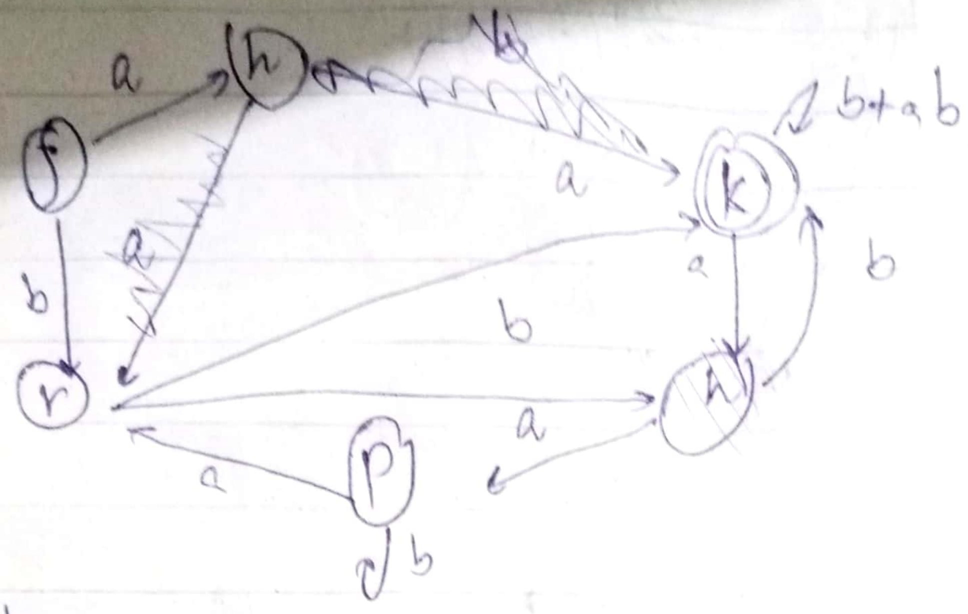




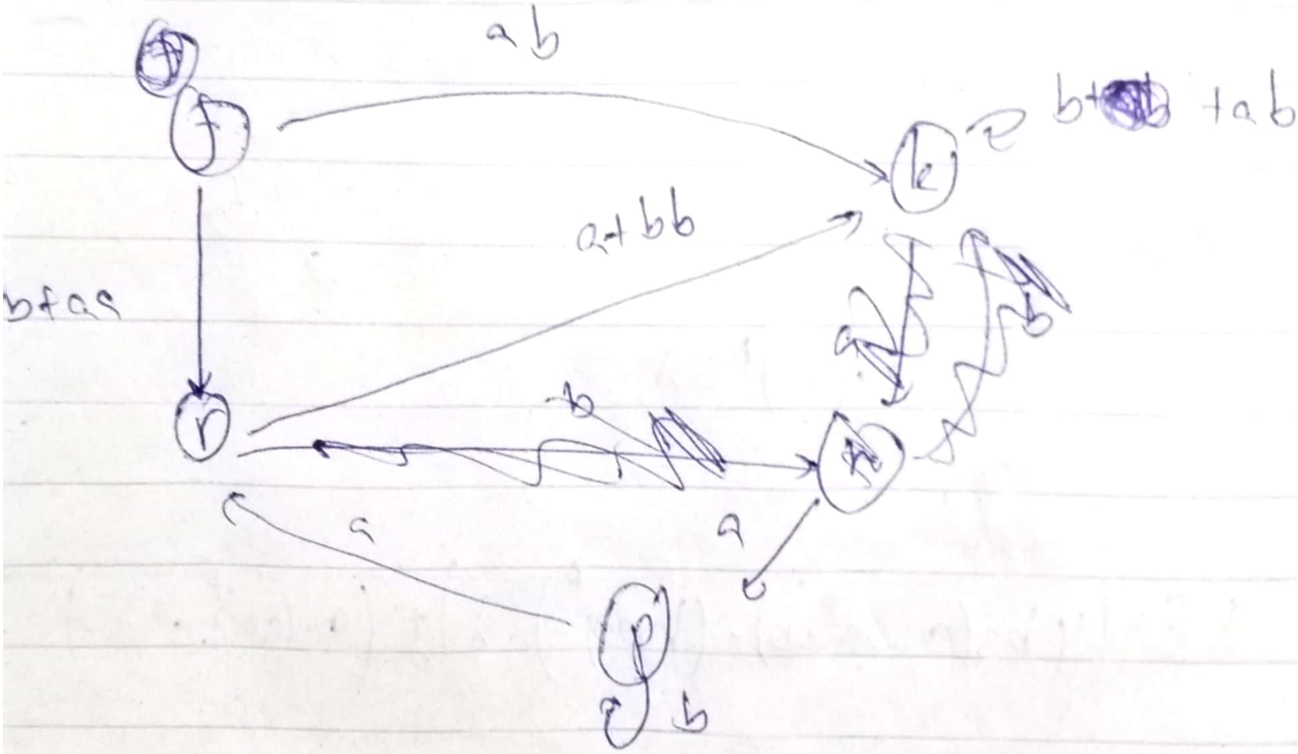


$RE = (((b+(a\cdot aa^*\cdot b)\cdot(b+ab)^*)\cdot\lambda)+(a\cdot(aa)^*\cdot\lambda)$

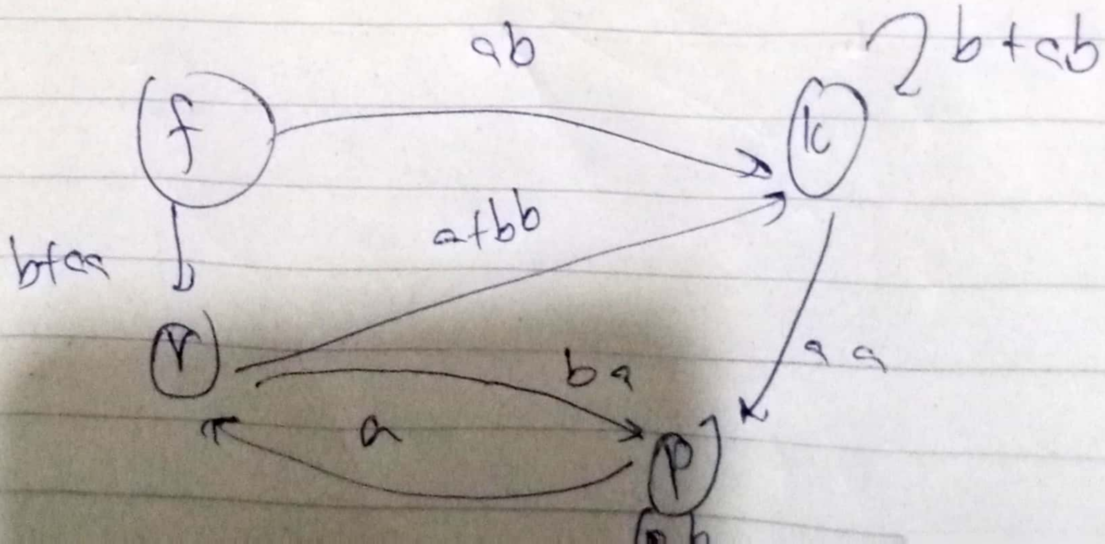
(iii)

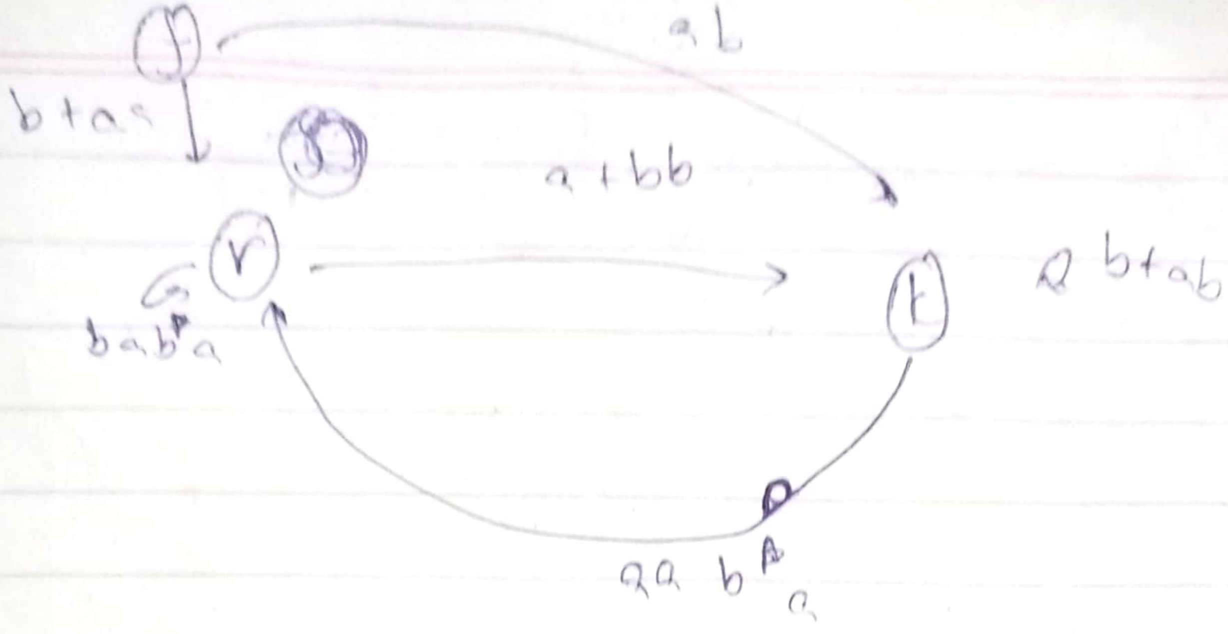


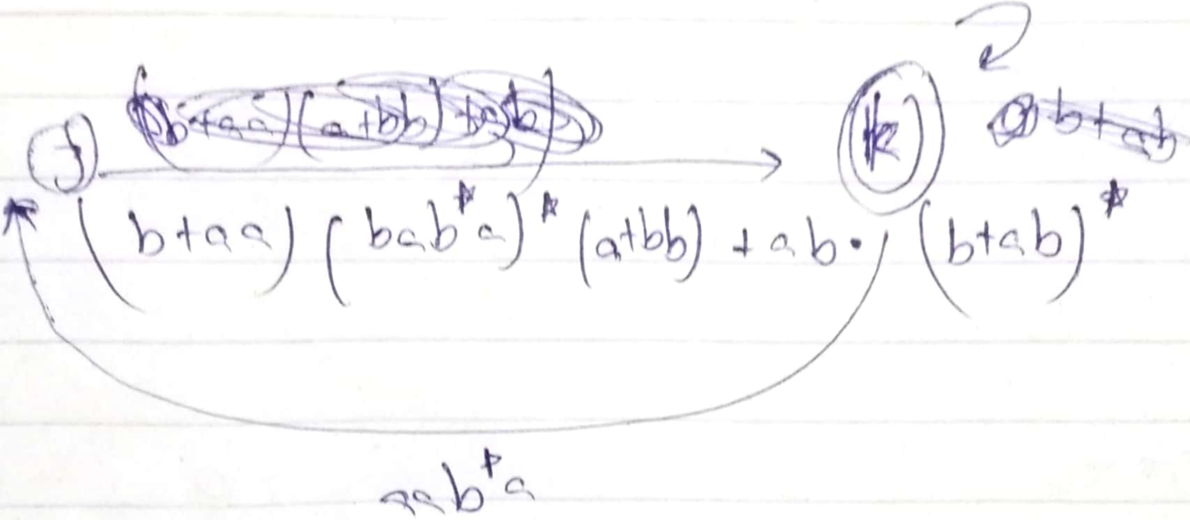
h::



n:

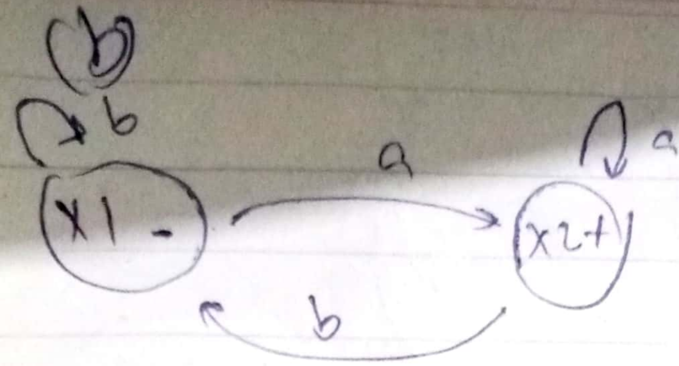


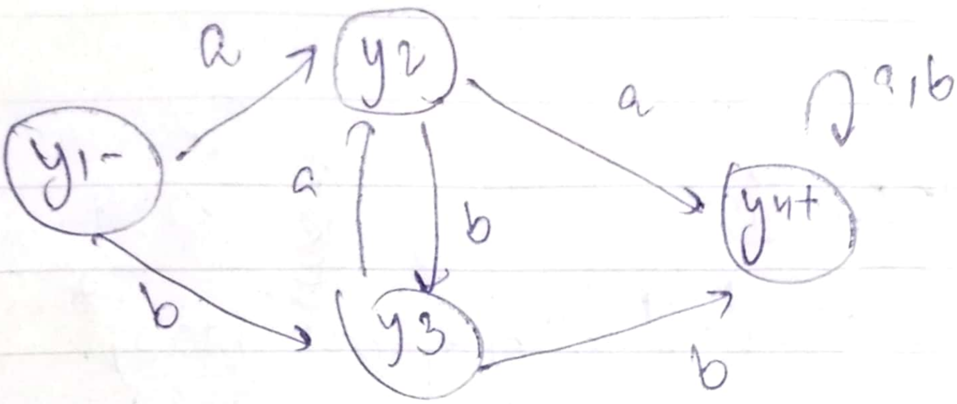




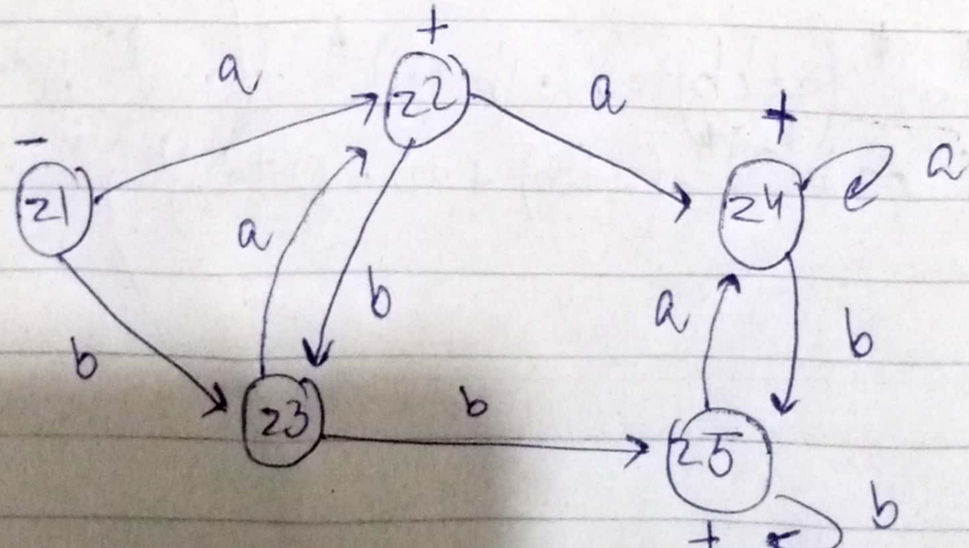
RE:

$$(b+aa)(bab^*a)^*(a+bb)+ab)\cdot(b+ab)^*\cdot\big((aab^*a)\cdot(b+aa)(bab^*a)^*(a+bb)+ab\cdot(b+ab)^*\big)^*$$

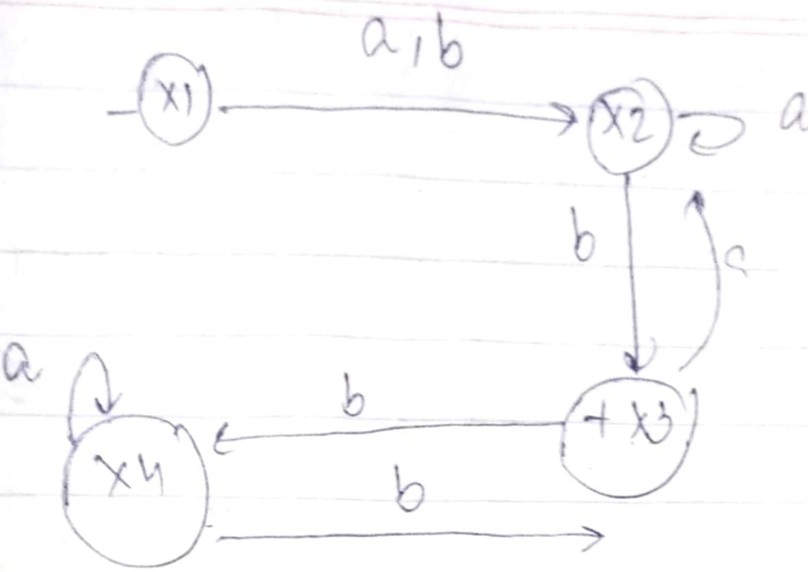


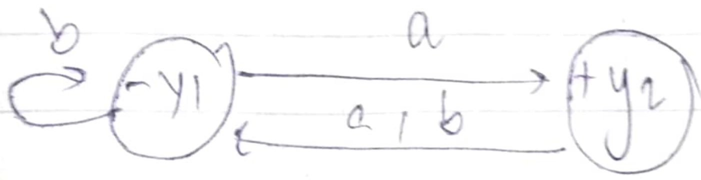


states	a	b
z1= x1,y1	z2= x2,y2	z3= x1,y3
z2= x2,y2	z4 = x2,y4	z3= x1,y3
z3= x1,y3	z2= x2,y2	z5= x1,y4
z4= x2,y4	z4= x2,y4	z5 x1,y4
z5= x1,y4	z4= x2,y4	z5= x1,y4

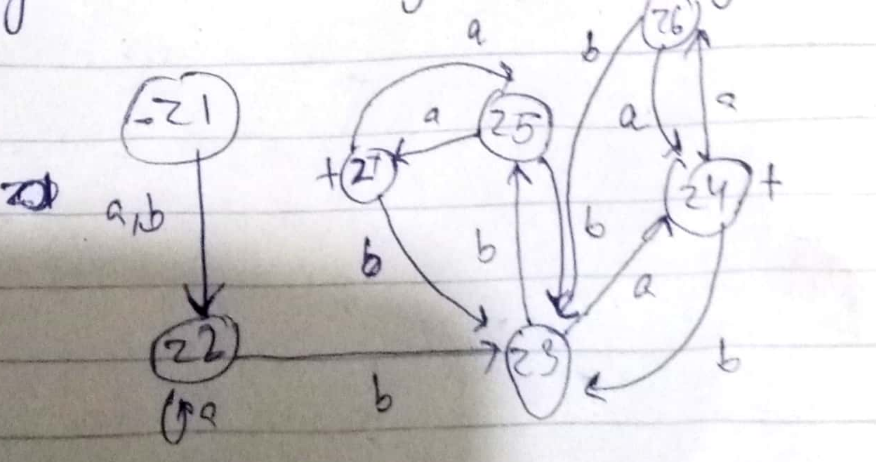


(c)





state	a	b
z1 = x1	z2 = x2	z2 = x2
z2 = x2	z2 = x2	z3 = x3, y1
z3 = x3, y1	z4 = x2, y2	z5 = x4, y1
z4 = x2, y2 +	z6 = x2, y1	z3 = x3, y1
z5 = x4, y1	z7 = x4, y2	z3 = x3, y1
z6 = x2, y1	z4 = x2, y2	z3 = x3, y1
z7 = x4, y2 +	z5 = x4, y1	z3 = x3, y1



(d) Probabilistic Kleene Theorem:

Paper aims at a probilistic conterpart of Kleene's Theorem. Assume that expressions an automata represent quantitative properties of words

Establishes Probabilistic Finite Automata (PFA).
PFAs are reactive automata, meaning probabilistic choice depends on input letter. Assume that final state has no out going transitions.

Then aims to make Probalistic Regular Expressions (PREs) by modifying WEs (Weighted expressions).
PREs are fragments of WEs built inductively
e.g
If E+F is PRE, then $E^*.F$ is PRE
To construct PRE equivalent to PFA, propositions are
(1) E+F & G = PRE then E+F . G = PRE
(2) $f: A^+ \to [0,1]$ if (f) recognized by PFA then recognized by PRE

Theorem is that PFAs and PREs are effectively equivalent

Then Two-way navigations (for traversing word) and Pebbles (for recovering later) are added

(3) Probabilistic Pebble Automata is constructed, then Probabilistic Pebble Expression. Both are shown to be equivalent

Probalistic Kleene Theorem

PPAs are constructed from PPE

Theorem 2: from any PPE can a PPA be constructed
equivalent

Theorem 3: From A ~~be~~, a PPA with p pebbles,
can be construct a ~~co~~ PPE. $E_i = \sum_{q \in Acc} E_{i,q}$

Corrolary: for every PPE E and $w \in A^+$
we have $[[E]](w) \in [0,1]$

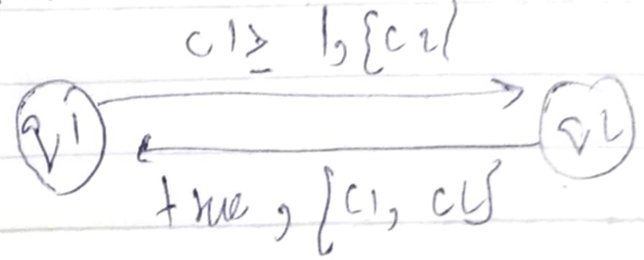
Kleene Timed Automata:

Aims to define timed REs, an extension of REs to specify sets of dense time discrete-valued signals

Times REs:

set of timed REs over an alphabet Σ is defined recursively as either a, $\varphi_1 \cdot \varphi_2$, $\varphi_1 \vee \varphi_2$, $\varphi_1 \wedge \varphi_2$, φ^* or $\langle \varphi \rangle$ when $a \in \Sigma$, φ, φ_1, φ_2.

Timed Automata:



A TA is a tuple $A = (Q, C, \Delta, \Sigma, \lambda, S, F)$
where ~~Q is a finite~~

Q: finite set of states
C: finite clocks
Σ: output state alphabet
Δ = transition relation
λ: $Q \to \Sigma$ is an output map
$S \subseteq Q$ initial set
$F \subseteq Q$ accepting set

1,2,3,4,5,6,7,

8,9,10,11

I : i
II : f

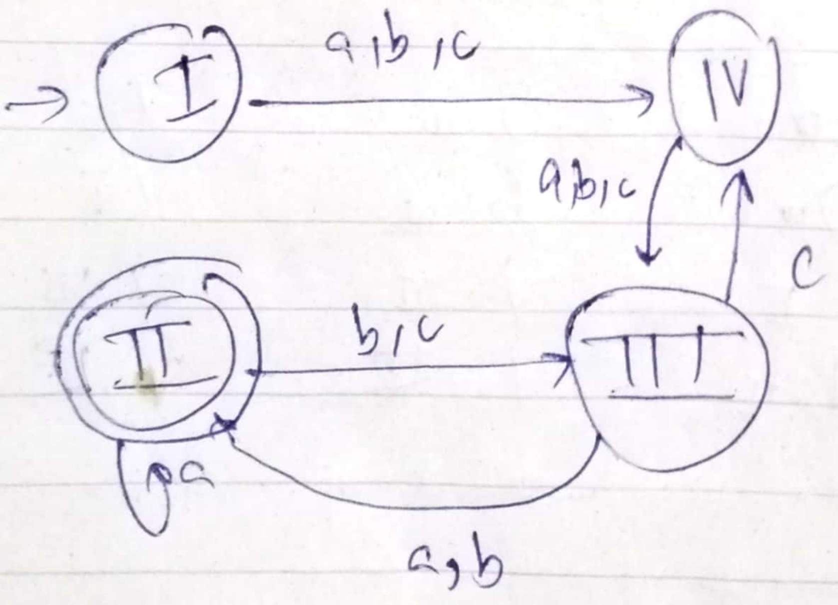
	state	a	b	c
	1	3 IV	4 IV	2 IV
IV	2	5 III	5 III	5 III
	3	6 III	7 III	7 III
	4	7 III	7 III	5 III
III	5	11 II	8 II	2 IV
	6	9 II	10 II	3 IV
	7	10 II	11 II	4 IV
II	8	11 II	5 III	7 III
	9	10 II	6 III	7 III
	10	9 II	7 III	7 III
	11	11 II	7 III	7 III

final:

	S	a	b	c
I	1	3 IV	4 IV	2 IV
IV	2	5 III	5 III	5 III
	3	6 III	7 III	7 III
	4	7 III	7 III	5 III
III	5	11 II	8 II	2 IV
	6	9 II	10 II	3 IV
	7	10 II	11 II	4 IV
II	8	11 "	5 III	7 III
	9	10 "	6 III	7 III
	10	9 II	7 III	7 III
	11	11 II	7 III	7 III

Diagram

	a	b	c
I	IV	IV	IV
II	II	III	III
III	II	II	IV
IV	III	III	III



Q3:

(a) $L1 = \{0^i 1^j 2^k \mid i,j,k \geq 0 \quad i = j \text{ or } i = k\}$

states = 5

$i = j = 3 \quad k = 2$

~~0000~~, ~~000 111, 22,~~

~~000 = x 111 = y 22 = z~~

~~000111 22~~

000 111 22

825 state

(1) $|w| \geq$ state ✓

$\underbrace{000}_{x} \; \underbrace{111}_{y} \; \underbrace{22}_{z}$

(2) $y \neq \lambda$ ✓

(3) $|xy| \leq$ state

$|000111| \leq 5$

$6 \leq 5$ X

thus not regular

(b) $L_2 = \{ w \in \{0,1,2\}^* \mid 0s = 2s$

states = 5

(5)

00122

(1) $|w| \geq k$ ✓

$\underbrace{00}_{x} \underbrace{1}_{y} \underbrace{22}_{z}$

(2) $y \neq \lambda$ ✓

0 01

(3) $|xy| \leq 5$

$3 \leq 5$ ✓

01 = y

(4) 0 01 01 0101 22

number of 0s = 5

number of 2s = 2

X

thus not regular

(b) $L_2 = \{w \in \{0,1,2\}^* \mid 0s = 2s$

states $= 5$

(5)

00122

(1) $|w| \geq k$ ✓

$\underbrace{00}_{x}\ \underbrace{1}_{y}\ \underbrace{22}_{z}$

(2) $y \neq \lambda$ ✓

0 01

(3) $|xy| \leq 5$

$3 \leq 5$ ✓

$01 = y$

(4) 0 01 01 0101 22

numbers of 0s $= 5$

number of 2s $= 2$

X

thus not regular

(c) $L_3 = \{0^n 1^m \mid n \le m\}$

states = 5

m = 3 n = 2

00111

(a) $|w| \ge k$ ✓

(b) $y \ne \lambda$

(c) $|00| \le 5$

$2 \le 5$ ✓

~~0 0101 0101 ...~~

~~0 011 011 011~~

$x = \lambda$ $y = 00$ $z = 111$

(d) 0 $(00)^i$

$i = 4$ 0000 111

$4 < 3$ ✗

(d) $L_4 = \{w \in \{0,1\}^* \mid w \text{ has more 1s than 0s}\}$

states = 5

00111

$\underbrace{0}_{x}\ \underbrace{0}_{y}\ \underbrace{111}_{z}$

(a) $|w| \ge k$ ✓

(b) $y \ne \lambda$ ✓

(c) $|00| \le 5$ ✓

0 $(0)^i$ 111

(d) for $i = 5$

0 00000 111

[1s are not more than 0s]

(c) $L_3 = \{0^n 1^m \mid n \le m\}$

state = 5

m = 3 n = 2

00111

(a) $|w| \ge k$ ✓

(b) $y \ne \lambda$

(c) $|00| \le 5$

$2 \le 5$ ✓

(d) 0 $(00)^i$

i = 4 0000 111~~0~~

$4 < 3$ ✗

~~0 0101 01 01~~

0 ~~011~~ 011 011

$x = \lambda$ $y = 00$ $z = 111$

(Q) $L_4 = \{w \in \{0,1\}^* \mid w$ has not more 1s than 0s$\}$

state = 5

00111

$\underbrace{0}_{x} \underbrace{0}_{y} \underbrace{111}_{z}$

(a) $|w| \ge k$ ✓

(b) $y \ne \lambda$ ✓

(c) ~~0~~ $|00| \le 5$ ✓

0 $(0)^i$ 111

(d) for i = 5

0 00000 111

[1s are not more than 0s]

(e) $L_5 = \{0^{n^2} \mid n \geq 0\}$

states $= 4$

$n = 2$

$\underbrace{0}_{x}\underbrace{0}_{y}\underbrace{0\,0}_{z}$

(a) $|w| \geq$ states

(b) $y \neq \lambda$

(c) $|xy| \leq$ states

$2 \leq 4$

(d) $(0)(0)^i(00)$

for $i = 2$

$0 \quad 00 \quad 00 \quad \notin L$

5 0s are not in L

thus not regular