QC
Assignment 1
21K-3153

(Q1)

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$= Y\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$$

$$= \left(\frac{1}{\sqrt{2}} i|1\rangle + \frac{1}{\sqrt{2}} -i|0\rangle\right)$$

$$\left(\frac{1}{\sqrt{2}} -i|1\rangle + \frac{1}{\sqrt{2}} -i|0\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(-i|0\rangle - i|1\rangle\right)$$

$$\frac{-i}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$-i|+\rangle$$

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(2) $HYTHX|0\rangle = |1\rangle$

$HXTH|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

T gate:

$$\frac{1}{\sqrt{2}}\left(|0\rangle - e^{i\pi/4}|1\rangle\right)$$

Y gate:

$$\frac{1}{\sqrt{2}}\left(i|1\rangle + ie^{i\pi/4}|0\rangle\right)$$

H gate:

$$\frac{1}{\sqrt{2}}\left(i\left(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right) + ie^{i\pi/4}\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)\right)$$

$$\frac{1}{2}\left(i(|0\rangle - |1\rangle) + ie^{i\pi/4}(|0\rangle + |1\rangle)\right)$$

~~$i+ie^{i\pi/4}|0\rangle + i-e^{i\pi/4}|1\rangle$~~

$$\frac{i+ie^{i\pi/4}}{2}|0\rangle + \frac{i-ie^{i\pi/4}}{2}|1\rangle$$

$$\left|\frac{i(1+e^{i\pi/4})}{2}\right|^2$$

$$(i^2)\frac{(1+e^{i\pi/4})^2}{4} = \frac{(-1)\left(\frac{2+\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2}{4} =$$

$H\ \frac{\sqrt{2}}{2}$

$i\frac{\sqrt{2}}{2}$

$$\frac{-1-\sqrt{2}-(1+\sqrt{2})i}{4}$$ $|0\rangle$

$\rightarrow$ prob of $|0\rangle$

prob of $|1\rangle$:

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$$\frac{-1-\sqrt{2}-(1+\sqrt{2})i}{4} \quad |0\rangle$$

$\downarrow$

$$\boxed{\begin{array}{cc} 0.85355 & \\ 85.3\% & |0\rangle \\ 14.6\% & |1\rangle \end{array}}$$

$\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}() + \frac{1}{\sqrt{2}}()\right)$

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(3) $HTHT\cancel{H}|0\rangle =$

H gate:

$$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

T gate:

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\pi/4}|1\rangle\right)$$

2nd H gate:

$$\frac{1}{\sqrt{2}}\left\{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + \frac{1}{\sqrt{2}}e^{i\pi/4}(|0\rangle - |1\rangle)\right\}$$

$$\frac{1}{2}\left\{(|0\rangle + |1\rangle) + e^{i\pi/4}(|0\rangle - |1\rangle)\right\}$$

$$\frac{1}{2}\left\{(1+e^{i\pi/4})|0\rangle + (1-e^{i\pi/4})|1\rangle\right\}$$

$$(1-e^{i\pi/4})(e^{i\pi/4})$$

2nd T gate

$$\frac{1}{2}\left\{(1+e^{i\pi/4})|0\rangle + (1 - \cancel{e^{i\pi/4}} + e^{i\pi/4})|1\rangle\right\}$$

3rd ~~H gate:~~

$$\cancel{\frac{1}{}\left\{1+e^{i\pi/4}\left(\frac{1}{}(|0\rangle+|1\rangle)\right) + \frac{1}{}(|0\rangle - |1\rangle)\right\}}$$

$$\left| \frac{1+2\sqrt{2}}{4} + \frac{1}{4} i \right|^2 \approx 97.8\%$$

$1 + e^{i\pi/4}\,0 \quad + 1 + e^{i\pi/4}\,1 \quad + 1\,e^{i\pi/4} e^{i\pi/4}\,0 \quad - (1-e^{i\pi/4})(e^{i\pi/4})\,1 0$

$1 + e^{i\pi/4}\,0 \,(1-e)^{i\pi/4} e^{i\pi/4}\,1 0$

3rd H gate:

$$\frac{1}{2}\left\{ \frac{1}{\sqrt{2}}(1+e^{i\pi/4})(|0\rangle + |1\rangle) + \frac{1}{\sqrt{2}}(1-e^{i\pi/4})(e^{i\pi/4})(|0\rangle - |1\rangle) \right\}$$

$$\frac{1}{2\sqrt{2}}\Big[(1+e^{i\pi/4})(1-e^{i\pi/4})(e^{i\pi/4}))\,|0\rangle + (1+e^{i\pi/4})(1-e^{i\pi/4})$$

$$(1+e^{i\pi/4}) - ((1-e^{i\pi/4})(e^{i\pi/4}))\,|1\rangle$$

Prob of $|0\rangle$:

$$\frac{1}{2\sqrt{2}}\left\{ \sqrt{2}\,|0\rangle - \sqrt{2}\,|1\rangle \right\}$$

$$\boxed{\left| 1+e^{i\pi/4} + (1-e^{i\pi/4})(e^{i\pi/4}) \right|^2 \approx 0.75}$$

Prob g 11 7:

15% or 0.15

10) 10 7:

75%

$\frac{1}{2}$

④ $H X H = Z$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$XH = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} =$$

$H:$

$$\frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \rightarrow Z}$$

(5) $|0\rangle$:

X gate: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\begin{matrix} 1 \\ 0 \end{matrix}$

$|1\rangle$

$1\times0+0\times1$

Z gate:

$-|1\rangle$

$\rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$\boxed{-\begin{pmatrix} 0 \\ 1 \end{pmatrix}}$

$\boxed{-|1\rangle}$

Task 2:

$$U|0\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{\sqrt{3}+i}{4}|1\rangle$$

$$U|1\rangle = \frac{\sqrt{3}+i}{4}|0\rangle - \frac{\sqrt{3}+3i}{4}|1\rangle$$

$$U(a|0\rangle + B|1\rangle)$$

$$aU|0\rangle + BU|1\rangle$$

$$a\left(\frac{\sqrt{3}}{2}|0\rangle + \frac{\sqrt{3}+i}{4}|1\rangle\right) + B\left(\frac{\sqrt{3}+i}{4}|0\rangle - \frac{\sqrt{3}+3i}{4}|1\rangle\right)$$

$$\left|\left(a\frac{\sqrt{3}}{2} + B\frac{\sqrt{3}+i}{4}\right)|0\rangle\right|^2 + \left|\left(a\frac{\sqrt{3}+i}{4} - B\frac{\sqrt{3}+3i}{4}\right)|1\rangle\right|^2$$

$$\left|a\frac{\sqrt{3}}{2} + B\frac{\sqrt{3}+i}{4}\right|^2:$$

$$\left(a\frac{\sqrt{3}}{2} + B\frac{\sqrt{3}+i}{4}\right)\left(a^*\frac{\sqrt{3}}{2} + B^*\frac{\sqrt{3}-i}{4}\right)$$

$$+$$

$$\left(a\frac{\sqrt{3}+i}{4} - B\frac{\sqrt{3}+3i}{4}\right)\left(a^*\frac{\sqrt{3}-i}{4} - B^*\frac{\sqrt{3}-3i}{4}\right)$$

$$\rightarrow |a|^2 \frac{3}{4} + aB^* \frac{3-\sqrt{3}i}{8} + Ba^* \frac{3+\sqrt{3}i}{8} + |B|^2 \frac{(\sqrt{3}+i)(}{4}$$

$$+$$

$$|a|^2 \frac{(\sqrt{3}+i)(\sqrt{3}-i)}{4} - aB^* \frac{3-\sqrt{3}i}{8}$$

$$-Ba^* \frac{-3-\sqrt{3}i}{8} + |B|^2 \frac{(\sqrt{3}+3i)(\sqrt{3}-3i)}{4}$$

$$\cancel{|a|^2 \frac{3}{4} + (1)}$$

$$|a|^2 \frac{7}{4} + \cancel{aB^*}$$

$$|a|^2 \frac{3}{4} + \cancel{aB^* \frac{3-\sqrt{3}i}{8}} + \cancel{Ba^* \frac{3+\sqrt{3}i}{8}} + |B|^2 \frac{1}{4}$$

$$+$$

$$|a|^2 \frac{1}{4} - \cancel{aB^* \frac{3-\sqrt{3}i}{8}} - \cancel{Ba^* \frac{-3-\sqrt{3}i}{8}} + |B|^2 \frac{3}{4}$$

$$|a|^2 \frac{3}{4} + |B|^2 \frac{1}{4} + |a|^2 \frac{1}{4} + |B|^2 \frac{3}{4}$$

$$|a|^2 \frac{4}{4} + |B|^2 \frac{4}{4} = \boxed{1}$$ → thus U is a valid gate

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Task3 : $\frac{1+i\sqrt{3}}{3}|0\rangle + \frac{2-i}{3}|1\rangle$

① $XRx(30)\ HXTRy(45)\ Y$:

Y gate:

$\frac{1+i\sqrt{3}}{3}(i)|1\rangle + \frac{2-i}{3}(-i)|0\rangle$

$\underbrace{\frac{\sqrt{3}}{3}+\frac{1}{3}i}_{A}|1\rangle - \underbrace{\frac{1}{3}-\frac{2}{3}i}_{B}|0\rangle$

Ry(45)

$\begin{pmatrix} \cos 45/2 & -\sin 45/2 \\ \sin 45/2 & \cos 45/2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\sin 45/2 \\ \cos 45/2 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos 45/2 \\ \sin 45/2 \end{pmatrix}$

$A\{-\sin(45/2)|0\rangle + \cos(45/2)|1\rangle\} + B\{\cos(45/2)|0\rangle + \sin(45/2)|1\rangle\}$

$A(-\sin 45/2)|0\rangle + A(\cos 45/2)|1\rangle) + B\cos(45/2) + B(\sin 45/2)|1\rangle$

$\underbrace{(A)(-\sin(45/2)) + B\cos(45/2)}|0\rangle + \underbrace{A(\cos 45/2) + B\sin(45/2)}|1\rangle$

T gate:

$\underbrace{(A)(-\sin 45/2) + B\cos(45/2)}_{C}|0\rangle + \underbrace{A\cos(45/2) + B(\sin 45/2)\,(e^{i\pi/4})}_{D}|1\rangle$

Y gate:

$$\underbrace{C\,(i)}_{E}\,|1\rangle + \underbrace{D\,(-i)}_{F}\,|0\rangle$$

H gate:

$$\frac{E}{\sqrt{2}}\left[|0\rangle - |1\rangle\right] + \frac{F}{\sqrt{2}}\left[|0\rangle + |1\rangle\right]$$

$$\underbrace{\frac{E}{\sqrt{2}} + \frac{F}{\sqrt{2}}\,|0\rangle}_{C_2} + \underbrace{\left(-\frac{E}{\sqrt{2}} + \frac{F}{\sqrt{2}}\right)}_{D_2}|1\rangle$$

Rx(30):

$$\begin{pmatrix} \cos 30/2 & -i\sin 30/2 \\ -i\sin 30/2 & \cos 30/2 \end{pmatrix}$$

$$|0\rangle = \begin{pmatrix} \cos 30/2 \\ i\sin 30/2 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} -i\sin 30/2 \\ \cos 30/2 \end{pmatrix}$$

$$C_2\left\{\cos(30/2)\,|0\rangle - i\sin(30/2)\,|1\rangle\right\}$$

$$+$$

$$D_2\left[-i\sin(30/2)\,|0\rangle + \cos(30/2)\,|1\rangle\right]$$

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$$\overbrace{\left[C_2\cos(30/2) - D_2 i\sin(30/2)\right]|0\rangle}^{A_2}$$

$$+$$

$$\underbrace{\left[-C_2 i\sin(30/2) + D\cos(30/2)\right]}_{B_2}|1\rangle$$

X :

$A_2|1\rangle + B_2|0\rangle$

$B_2|0\rangle + A_2|1\rangle$

$|A_2|^2 =$

$$\boxed{\begin{aligned}&-0.7200693802 + 0.6902566782i\ |0\rangle \\ &+ \\ &0.07085274612 - 5.068946863\times10^{-3}i\ |1\rangle\end{aligned}}$$

$|A_2|^2 = 5.045\times10^{-3}$

0.005045

$|0\rangle = 99\%$

$|1\rangle = 1\%$

$|B_2|^2 = 0.994954941$

$|A_2|^2 + |B_2|^2 = \boxed{1}$

(7) $S\,R_z(170)\,H\,Y\,R_z(45)\,X\,H$

$$\frac{1+i\sqrt{3}}{3}|0\rangle + \frac{2-i}{3}|1\rangle$$

Hgate:

$$\underbrace{\frac{1+i\sqrt{3}}{3\sqrt{2}}}_{A}\left[|0\rangle+|1\rangle\right] + \underbrace{\frac{2-i}{3\sqrt{2}}}_{B}\left[|0\rangle-|1\rangle\right]$$

$$A|0\rangle + A|1\rangle + B|0\rangle - B|1\rangle$$

$$A+B\;|0\rangle + A-B\;|1\rangle$$

$$\underbrace{\frac{\sqrt{2}}{2} + \frac{\sqrt{6}-\sqrt{2}}{6}i}_{C}|0\rangle\;\underbrace{-\frac{\sqrt{2}}{6} + \frac{\sqrt{6}+\sqrt{2}}{6}i}_{D}|1\rangle$$

Xgate on $C|0\rangle + D|1\rangle$:

$$\cancel{\boxed{D|0\rangle + C|1\rangle}}$$

$$C|1\rangle + D|0\rangle$$

$$\boxed{D|0\rangle + C|1\rangle}$$

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$R_z(45)$:

$R_z|0\rangle = |0\rangle$

$\cancel{R_z|1\rangle} = R_z(45)|1\rangle = e^{i45}|1\rangle$

$D|0\rangle + \underbrace{Ce^{i45}}_{E}|1\rangle$

$D|0\rangle + \underbrace{0.2246387155 + 0.692321876i}_{E}|1\rangle$

$D|0\rangle + E|1\rangle$

Y on $D|0\rangle + E|1\rangle$:

$\underbrace{Di}_{F}|1\rangle + \underbrace{E(-i)}_{X}|0\rangle$

$\underbrace{\frac{-\sqrt{6}+\sqrt{2}}{6} - \frac{\sqrt{2}}{6}i}_{F}|1\rangle + \underbrace{0.692321876 - 0.2246387155i}_{X}|0\rangle$

$\boxed{X|0\rangle + F|1\rangle}$

H on $X|0\rangle + F|1\rangle$:

$$\frac{X}{\sqrt{2}}\{|0\rangle + |1\rangle\} + \frac{F}{\sqrt{2}}\{|0\rangle - |1\rangle\}$$

$$\frac{X}{\sqrt{2}}|0\rangle + \frac{X}{\sqrt{2}}|1\rangle + \frac{F}{\sqrt{2}}|0\rangle - \frac{F}{\sqrt{2}}|1\rangle$$

$$\frac{X}{\sqrt{2}} + \frac{F}{\sqrt{2}}|0\rangle + \frac{X}{\sqrt{2}} - \frac{F}{\sqrt{2}}|1\rangle$$

$$\downarrow$$

$$\underbrace{0.0342038 9203 - 0.3755102257i}_{Y}|0\rangle$$

$$+$$

$$\underbrace{0.9448872946 + 7.8231076 41 \times 10^{-3} i}_{Z}|1\rangle$$

$$Y|0\rangle + Z|1\rangle$$

$R_Z(120)$ on $Y|0\rangle + Z|1\rangle$:

$R_Z|0\rangle = |0\rangle$
$R_Z|1\rangle = e^{i120}$

$$Y|0\rangle + Z(e^{i120})|1\rangle$$

$$Y|0\rangle + \underbrace{0.7647676707 + 0.5549815564i}_{A2}|1\rangle$$

$$Y|0\rangle + A2|1\rangle$$

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S on $Y|0\rangle + A_2|1\rangle$:

$S|0\rangle = |0\rangle$
$S|1\rangle = i|1\rangle$

$$Y|0\rangle + \overbrace{A_2(i)}^{B_2}|1\rangle$$

$\downarrow$

$Y|0\rangle$ $-0.5549815564+0.7647670707i$ $|1\rangle$

$\downarrow$

final

$\downarrow$

$0.0342036 9203-0.3255102257i\ |0\rangle$

$+$

~~0.9448 79 46 7 833~~

$-0.5549815564+0.7647670707i\ |1\rangle$

~~Probabily~~ Probability:

$|\alpha|^2$ $|0\rangle = 0.1071267996 \approx 10.7\%$

$|B_2|^2$ $|1\rangle = 0.8928732004 \approx 89.3\%$

$$|0|^2 + |1|^2 = \boxed{1}$$

thus correct

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③ HH Ry(150) H y x H :

$$\frac{1+i\sqrt{3}}{3}|0\rangle + \frac{2-i}{3}|1\rangle$$

H :

$$\underbrace{\frac{1+i\sqrt{3}}{3\sqrt{2}}}_{A}\{|0\rangle+|1\rangle\} + \underbrace{\frac{2-i}{3\sqrt{2}}}_{B}\{|0\rangle-|1\rangle\}$$

$$A|0\rangle + A|1\rangle + B|0\rangle - B|1\rangle$$

$$A|0\rangle + B|0\rangle + A|1\rangle - B|1\rangle$$

C

$$\boxed{\frac{\sqrt{2}}{2} + \frac{\sqrt{6} - \sqrt{2}\,i}{6}}|0\rangle$$

+

$$\underbrace{-\frac{\sqrt{2}}{6} + \frac{\sqrt{6} + \sqrt{2}\,i}{6}}_{D}|1\rangle$$

$$C|0\rangle + D|1\rangle$$

X on $C|0\rangle + D|1\rangle$:

$$C|1\rangle + D|0\rangle$$

$$\downarrow$$

$$\boxed{D|0\rangle + C|1\rangle}$$

Y on $D|0\rangle + C|1\rangle$:

$$\overbrace{D(i)}^{E}|1\rangle + \overbrace{C(-i)}^{F}|0\rangle$$

$$\downarrow \quad E \qquad\qquad \downarrow$$

$$\left(\frac{-\sqrt{6}+\sqrt{2}}{6} - \frac{\sqrt{2}}{6}i\right)|1\rangle$$

$$+$$

$$\underbrace{\left(\frac{\sqrt{6}-\sqrt{2}}{6} - \frac{\sqrt{2}}{2}i\right)}_{F}|0\rangle$$

$$E|1\rangle + F|0\rangle$$

$$F|0\rangle + E|1\rangle$$

H on $F|0\rangle + E|1\rangle$

$$\frac{F}{\sqrt{2}}\left[|0\rangle + |1\rangle\right] + \frac{E}{\sqrt{2}}\left[|0\rangle - |1\rangle\right]$$

$$\frac{F}{\sqrt{2}}|0\rangle + \frac{F}{\sqrt{2}}|1\rangle + \frac{E}{\sqrt{2}}|0\rangle - \frac{E}{\sqrt{2}}|1\rangle$$

$$\frac{F}{\sqrt{2}}|0\rangle + \frac{E}{\sqrt{2}}|0\rangle \quad + \frac{F}{\sqrt{2}} - \frac{E}{\sqrt{2}}|1\rangle$$

$$\underbrace{\left(\frac{F}{\sqrt{2}} + \frac{E}{\sqrt{2}}\right)}_{x} |0\rangle + \underbrace{\left(\frac{F}{\sqrt{2}} - \frac{E}{\sqrt{2}}\right)}_{y} |1\rangle$$

$$\downarrow$$

$$\underbrace{\left(-\frac{1}{3} - \frac{2}{3}i\right)}_{x} |0\rangle + \underbrace{\left(\frac{\sqrt{3}}{3} - \frac{1}{3}i\right)}_{y}$$

$$x|0\rangle + y|1\rangle$$

$Ry(150)$ on $x|0\rangle + y|1\rangle$:

$$\begin{pmatrix} \cos 150/2 & -\sin 150/2 \\ \sin 150/2 & \cos 150/2 \end{pmatrix} \begin{matrix} 0 \\ 1 \end{matrix}$$

$$|0\rangle = \begin{pmatrix} \cos 150/2 \\ \sin 150/2 \end{pmatrix}$$

$$|1\rangle : \begin{pmatrix} -\sin 150/2 \\ \cos 150/2 \end{pmatrix}$$

$$x\left[\cos(150/2)\,|0\rangle + \sin(150/2)\,|1\rangle\right]$$
$$+$$
$$y\left[-\sin(150/2)\,|0\rangle + \cos(150/2)\,|1\rangle\right]$$

$$x\cos(150/2)|0\rangle + x\sin(150/2)|1\rangle$$

$$+$$

$$-y\sin(150/2)|0\rangle + y\cos(150/2)|1\rangle$$

$$\overbrace{x\cos(150/2) - y\sin(150/2)}^{z}|0\rangle$$

$$+$$

$$\underbrace{x\sin(150/2) + y\cos(150/2)}_{A_2}|1\rangle$$

$$-0.0833645 9165 - 0.7437613916i \; |0\rangle$$

$$+$$

$$0.6614338888 - 0.0457729333297i \; |1\rangle$$

$$z|0\rangle + A_2|1\rangle$$

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Last 2 H gates cancel out so final equation remains same.

$$H \times H = I \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Final equation:

$$-0.08336459165 - 0.7437613916i\,|0\rangle$$
$$+$$
$$0.6614338888 - 0.04872933297i\,|1\rangle$$

Probabilities

$|Z|^2 = |0\rangle$: $0.5601306628 \simeq 56.01\%$

$|A|^2 = |1\rangle$: $0.4398693372 \simeq 43.9\%$

thus correct

Task 4

(1) $|0\rangle \rightarrow \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}i\,|1\rangle$

1st : apply H gate :

$H\,|0\rangle : \frac{1}{\sqrt{2}}\{|0\rangle + |1\rangle\}$

2nd : apply S gate

$SH|0\rangle : \frac{1}{2}\{|0\rangle + i\,|1\rangle\}$

3rd : apply Rx(30°) gate :

$Rx(30)\ SH|0\rangle :$

$$\begin{bmatrix} \cos 15 & -i\sin 15 \\ -i\sin 15 & \cos 15 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} i \end{bmatrix} = \begin{matrix} \frac{1}{\sqrt{2}}\cos 15 + \frac{1}{\sqrt{2}}(i)(-i\sin 15) \\ \frac{1}{\sqrt{2}}(-i\sin 15) + \left(\cos 15 \frac{1}{\sqrt{2}} i\right) \end{matrix}$$

$$\begin{pmatrix} 0.9659 & -0.2588i \\ -0.2588i & 0.9659 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2}\,i \end{pmatrix} = \begin{matrix} \left(\frac{1}{\sqrt{2}}\right)(0.9659) + (-0.2588i)\left(\frac{1}{\sqrt{2}}\right) \\ \left(-0.2588i \cdot \frac{1}{\sqrt{2}}\right) + \left(0.9659 \times \frac{1}{\sqrt{2}} i\right) \end{matrix}$$

$0.866 = \frac{\sqrt{3}}{2}$

$0.49i\ \cancel{0.5}$ $0.5i = \frac{1}{2} i$

$$\boxed{\frac{\sqrt{3}}{2} + \frac{1}{2} i}$$

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Task4 Q2

Step1:

Apply H gate

$$H\,|0\rangle \quad \frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right]$$

Step2: $R_y(20^\circ)$

$$\begin{pmatrix} \cos(20/2) & -\sin(20/2) \\ \sin(20/2) & \cos(20/2) \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} 0.5735764364 \\ 0.8191520445 \end{pmatrix}$$

$$0.573\,|0\rangle + 0.819\,|1\rangle$$

$$\boxed{\begin{array}{ll} \frac{1}{\sqrt{3}} \approx 0.57 & \sqrt{\frac{2}{3}} \approx 0.81 \\ \\ \frac{1}{\sqrt{3}}\,|0\rangle + \sqrt{\frac{2}{3}}\,|1\rangle & \end{array}}$$

Task 4 Q3:

Step 1: $R_y(45)$ on $|0\rangle$

$$\begin{pmatrix} \cos 45/2 & -\sin 45/2 \\ \sin 45/2 & \cos 45/2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\frac{45}{2}^{\circ} = \frac{\pi}{8}$

$$\begin{pmatrix} \cos 45/2 \\ \sin 45/2 \end{pmatrix} \rightarrow \cos\frac{\pi}{8}|0\rangle + \sin\frac{\pi}{8}|1\rangle$$

Step 2: T gate:

$$\boxed{\cos\frac{\pi}{8}|0\rangle + \sin\frac{\pi}{8}\, e^{i\pi/4}\, |1\rangle}$$

$3 \times \frac{180}{8}$ $\quad$ $\pi/3$

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Task 4 Q4

Step 1: $R_y(120)$

$$\begin{pmatrix} \cos 60 & -\sin 60 \\ \sin 60 & \cos 60 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \cos 60 \\ \sin 60 \end{pmatrix}$$

Step 2: $R_z(60)$:

$$\cos 60 |0\rangle + \sin 60 \, e^{i60} |1\rangle \qquad 60^\circ = \frac{\pi}{3}$$

$$\boxed{\frac{1}{2} |0\rangle \quad \frac{\sqrt{3}}{2} e^{i\pi/3} |1\rangle}$$

~~Task 5 Q5: R Ric~~

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Task 5 Q5

$\cos(3\pi/8)\,|0\rangle + \sin(3\pi/8)\,e^{i2\pi/3}\,|1\rangle$

① Apply $R_y(135)$

$$\begin{pmatrix} \cos 67.5 & -\sin 67.5 \\ \sin 67.5 & \cos 67.5 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\downarrow$

$$\begin{pmatrix} \cos 67.5 \\ \sin 67.5 \end{pmatrix}$$

$\cos 67.5 = \cos \frac{3\pi}{8} = 0.382$ $\qquad \frac{3\pi}{8} = 67.5$

~~cos 67~~ $\cos \frac{3\pi}{8}|0\rangle + \sin \frac{3\pi}{8}|1\rangle \quad \frac{3\pi}{8} = 67.5$

② Apply $R_z(2\pi/3)$

$$\boxed{\cos \frac{3\pi}{8}|0\rangle + \sin\left(\frac{3\pi}{8}\right) e^{i2\pi/3}|1\rangle}$$