Bilal Ali
2115-3153

Date: M T W T F S S

DISCRETE MATHS
ASSIGNMENT 1

(Q1)
(a) Proposition, true
(b) Proposition, false
(c) Proposition, true
(d) Proposition, false
(e) Not a proposition
(f) Not a proposition

(Q2) (a) p = Smartphone B has the most RAM.
The proposition is true.

p

(b) q = Smartphone C has more Rom than B
r = Smartphone C has a higher resolution camera than B
$(q \vee r)$ = hence TRUE
q → false, r → true

(c) p = B has more RAM than A
q = B has more ROM than A
r = B has a higher resolution camera than A
$p \wedge q \wedge r$ = hence FALSE
p → True, q → True, r → False

$\neg(p \wedge q) \vee r$
$\neg p \vee \neg q \vee r$

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(d) p = B has more RAM than C
q : B has more ROM than C
r = B has a higher resolution camera than C

$T \to f = F$

$(p \wedge q) \to r$ = hence FALSE

true true FALSE

$T \to F$

(e) p = A has more RAM than B
q = B has more RAM than A

$p \leftrightarrow q$ = hence FALSE

false true

$F \leftrightarrow T = f$

(Q3) ~~p~~ q = annual revenue of Acme Computer was 138 billion
q ~~q~~ = net profit of Acme Computer was 8 billion
r = annual revenue of Nadir was 87 billion

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(Q3) (a) p = Quixote had the largest annual revenue

p is FALSE

p

(b) p = Nadir had lowest net profit

q = Acme had largest annual revenue

$(p \wedge q)$ = hence TRUE

true true

(c) p = Acme had largest net profit

q = Quixote had largest net profite

$(p \vee q)$ = hence TRUE

False True

(d) p = Quixote had smallest net profit

q = Acme had largest ~~net~~ annual revenue

$(p \rightarrow q)$ = hence TRUE

false true

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(e)

p = Nadir had smallest net profit
q = Acme had largest annual revenue

$(p \leftrightarrow q)$ = hence TRUE

p → true, q → true

(Q4) (a) If you have the flu, then you will miss the final examination

~~(b) $\neg q$~~

(b) $\neg q \leftrightarrow r$ =

~~You will pas~~

You will not miss the final examination if and only if you pass the course

(d) $p \vee q \vee r$:

You have the flu or you miss the final examination or you pass the course

(c) ~~$p \rightarrow \neg r$~~ $q \rightarrow \neg r$:

If you miss the final examination, then you will not pass the course

(e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$:

If you have the flu, then you will not pass the course or if you miss the final examination then you will not pass the course.

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(f) $(p \wedge q) \vee (\neg q \wedge r)$:

You have the flue and you miss the final examination or you donot miss the final examination and you pass the course

(Q5) (a) $(r \wedge \neg q)$

(b) $(p \wedge q \wedge r)$

(c) ~~p → q~~ ~~p → p~~ $p \to r$

(d) $(p \wedge \neg q) \wedge r$

(e) $(p \wedge q) \to r$

(f) $r \leftrightarrow (q \vee p)$

(Q6) (a) If you send me an email message, then I will remember to send you the address $\boxed{\text{p only if q}}$

(b) If you are born in the United States, then you are a citizen of this country

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(c) If you keep your textbook, then it will be a useful reference in your future course

(d) If their goalie plays well, then the Red Wings will win the Stanley Cup

(e) If you ~~had~~ get the job, then you had the best credentials

(f) If there is a storm, then the beach erodes

(g) If you log on to the server, then you have a valid password

(h) If you do not begin your climb too late, then you will have reached the summit.

(Q7)(a)

(i) ~~Being~~ ~~A~~ It being sunny tomorrow is sufficient for me going for a walk in the woods

(ii) I will go for a walk in the woods if it is sunny tomorrow

~~(iii) A necessary condition for me going for a walk in the woods is it being sunny tomorrow~~

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(iv) ~~Me going~~ –

(iii) ~~It~~ Being sunny tomorrow implies I will go for a walk in the woods

(iv) A sufficient condition for me going for a walk in the woods ~~for~~ is it being sunny tomorrow

(v) I will go for a walk in the woods where it is sunny tomorrow

(b) Converse: ~~p→q~~ $q \to p$

~~If I go for a walk in the woods, then it is~~

If I go for a walk in the woods, then it is sunny tomorrow

inverse = $(\neg p \to \neg q)$

If it is not sunny tomorrow, then I will not go for a walk in the woods

Contrapositive : ~~$(\neg p \to \neg q)$~~ $(\neg q \to \neg p)$

If I do not go for a walk tomorrow, then it is not sunny

(c) $p \to q$

inverse of it's inverse:

$p \to q$ | $\neg p \to \neg q$ | $\neg(\neg p) \to \neg(\neg q)$ = $\boxed{p \to q}$

inverse 1

If it's sunny tomorrow, I will go for a walk in the woods

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inverse of converse:

$p \to q$ | $q \to p$ | $\neg q \to \neg p$

If I do not go for a walk in the woods, then it is not sunny tomorrow

inverse of contrapositive

$p \to q$ | $\neg q \to \neg p$ | $q \to p$

If I go for a walk in the woods then it is sunny tomorrow

(Q8) (a) Jan is poor or unhappy
(b) Carlos will not bicycle ~~and~~ and not run tomorrow
(c) The fan is fast (not slow) and it is not very hot
(d) Akram is fit (not unfit) or Saleem is ~~not~~ not injured

(Q9) (a) Exclusive or
(b) Inclusive or
(c) Inclusive or
(d) Inclusive or

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$(\neg p \ldots) \cdot (\neg q \to p)$ $P(P \neg q)$ $p \times (p \neg q)$ $\neg p \neg q + pq$ $(p \neg q) + pq$ $(p+q)(\neg p + \neg q)$ $p \neg p + p\neg q + q \neg p + q \neg q$

(10)

(a) $(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q) \equiv p$

$p \wedge (p \wedge \neg q)$

$p \wedge p \wedge \neg q$

$(p \wedge \neg q) \vee (p \wedge q)$

$p \wedge \neg q \vee p \wedge q$

$p \wedge (q \vee \neg q)$

$p \wedge (t) = \boxed{p}$

(b) $\neg (p \leftrightarrow q) \equiv (p \leftrightarrow \neg q) \equiv p \to \neg q \wedge \neg q \to p$

$(\neg p \vee \neg q) \wedge (q \vee p)$

$\neg (p \to q \wedge q \to p)$

$\neg ((\neg p \vee q) \wedge (\neg q \vee p))$

$\neg (\neg p \wedge \neg q \vee \underset{c}{\neg p \wedge p} \vee \underset{c}{q \wedge \neg q} \vee p \wedge q))$

$\neg (\neg p \wedge \neg q \vee p \wedge q)$

$(p \vee q) \wedge (\neg p \vee \neg q)$

~~$p \wedge \neg p \vee p \neg q \vee q \neg p q \vee$~~

$\underset{c}{p \wedge \neg p} \vee p \wedge \neg q \vee \neg p \wedge q \vee \underset{c}{\neg q \wedge q}$

$(\neg p \wedge q) \vee (p \wedge \neg q) = \boxed{p \leftrightarrow \neg q}$

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(c) $\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$

$(p \to \neg q) \wedge (\neg q \to p)$

$(\neg p \vee \neg q) \wedge (q \vee p)$

$(\neg p \to q) \wedge (q \to \neg p)$

$(p \vee q) \wedge (\neg q \vee \neg p) \equiv \boxed{\cancel{p \leftrightarrow \neg q}}$

$(q \vee p) \wedge (\neg p \vee \neg q)$

$\cancel{(\neg\neg q \vee p)}$ $(p \to \neg q)$

$\cancel{(\neg q \to p) \wedge (p}$ $p \to \neg q$

$(\neg\neg q \vee p)$

$(\neg q \to p) \wedge (p \to \neg q) \equiv \boxed{p \leftrightarrow \neg q}$

(d) $(p \wedge q) \to (p \to q) \equiv T$

$(p \wedge q) \to (\neg p \vee q)$

$\neg(p \wedge q) \vee (\neg p \vee q)$

$(\neg p \vee \neg q) \vee (\neg p \vee q)$

$\cancel{\neg p + \neg q + \neg p + q}$

$\cancel{\neg p + \neg q}$

$\neg p \vee \neg q \vee \neg p \vee q$

$\neg p \vee (q \vee \neg q)$

$\neg p \vee T \equiv \boxed{T}$

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(e) $\neg(p \vee \neg(p \wedge q)) \equiv f$

$\neg(p \vee (\neg p \vee \neg q))$

$p \vee \neg p \vee \neg q$

$\neg(t \vee \neg q)$

$\neg(t \vee \neg q)$

$\neg(t) = \boxed{F}$

(a)(a) $\underbrace{(p \to r)}_{A} \wedge \underbrace{(q \to r)}_{B}$ and $(p \vee q) \to r$

P	Q	R	P→R	Q→R	A∧B	P∨Q	P∨Q→R
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

The propositions are logically equivalent

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(b) $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$

P	Q	R	P→Q	P→R	P→Q ∨ P→R	Q∨R	P→(q∨r)
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

The proposition are logically equivalent

~~p → (q)~~ → next page

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(c) $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$

P	Q	R	S	P→Q	R→S	(P→Q)→(R→S)	P→R	Q→S	(P→R→Q→S)
T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	F	T	F	F
T	T	F	T	T	T	T	F	T	F
T	T	F	F	T	T	T	F	T	T
T	F	T	T	F	T	T	T	F	T
T	F	T	F	F	F	T	T	T	T
T	F	F	T	F	T	T	F	T	T
T	F	F	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	F	F	T	F	F
F	T	F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	F	F
F	F	T	T	T	T	T	T	T	T
F	F	T	F	T	F	F	T	T	T
F	F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T

The ~~propos~~ statements are not logically equivalent

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(14) m divides n = n/m = k

(a) P(4,5) = 5/4 ≠ k False

(b) P(2,4) = 4/2 = 2 → True

(c) $\forall m \forall n\ P(m,n)$ = FALSE

(d) $\exists m \forall n\ P(m,n)$ = True

(e) $\exists n \forall m\ P(m,n)$ = False

(f) $\forall n\ P(1,n)$ = True

(Q15)(a) $\exists x(x^2=2)$ = false

(b) $\exists x(x^2=-1)$ = false

(c) $\forall x(x^2+2 \geq 1)$ = True

(d) $\exists x(x^2=x)$ = True

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(14) (a) $\forall x\, F(x, Bob)$

(b) $\forall x\, F(Alice, x)$

(c) $\forall x \exists y\, F(x, y)$

(d) $\forall x \neg \exists y\, F(y, x)$ or $\neg \exists y\, \forall x\, F(y, x)$

(e) somebody can fool everybody

~~$\forall x\, \exists y\, F(y, x)$~~ $\exists x\, \forall y\, F(x, y)$

(15) $P(x)$ = x can speak Russian
$Q(x)$ = x knows C++

(a) $\forall x \exists y\, (P(y) \wedge Q(y))$
(b) $\exists y\, (P(y) \wedge \neg Q(y))$
(c) $\forall x\, (P(x) \vee Q(x))$
(d) $\neg \exists x\, (P(x) \vee Q(x))$

(16) $Q(x, y)$ = x has sent an email message to y

(a) Some student has sent an email message to some student

(b) Some student has sent an email to all students in the class.

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(c) All students have sent an email to atleast one student in the class

(d) There is a student who has been sent an email message by everyone in the class

(e) All students in the class have been sent an email message from atleast one student in the class

(f) Every student in the class has sent an email to every student in the class

(17) $P(x,y)$ = Student x has taken class y
x = students y = computer science course

(a) $\exists x \exists y P(x,y)$: ~~Some student has tak~~
At least one student has taken atleast one computer science course.

(b) $\exists x \forall y P(x,y)$:
At least one student has taken all the computer science course.

(c) $\forall x \exists y P(x,y)$

All students have taken atleast one computer science course

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(d) $\exists y \forall x \; P(x,y)$

At least one computer science course has been taken by all students.

(e) $\forall y \exists x P(x,y)$

All computer science courses have been taken by at least one student.

(f) $\forall x \forall y \; P(x,y)$

Every student has taken every computer science course

Q(18) (a) $\frac{p}{p \vee q}$ Addition

(b) $\frac{p \wedge q}{p}$ Simplification

(c) $r \to p$, r Modus ponens

(d) Modus tollens

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(e) Hypothetical syllogism

~~(18)~~ (a)

~~T → true~~

~~F → T6~~

~~T V ES~~

(19) (a)

T = tuesday MT = Math Test ET = Economics Test ES = Econ prof sick

① $T \wedge ES$ ② T simplification

③ $T \to (MT \vee ET)$

④ $MT \vee ET$ modus ponens

⑤ ES

⑥ $ES \to \neg ET$

⑦ $\neg ET$ modus ponens

⑧ $MT \vee ET$

⑨ MT = mathematics test elimination

(b) AL = Ali is a lawyer AM = ali is ambitious ER = early riser
DC = does not like chocolate

① $AL \to AM$

② $ER \to DC$

③ $AM \to ER$ ~~hypothetical syllogism~~

④ $AL \to ER$ H.S of 1 and 3

⑤ $ER \to DC$ $\boxed{⑥ AL \to DC}$ H.S of 4 and 5

H.S

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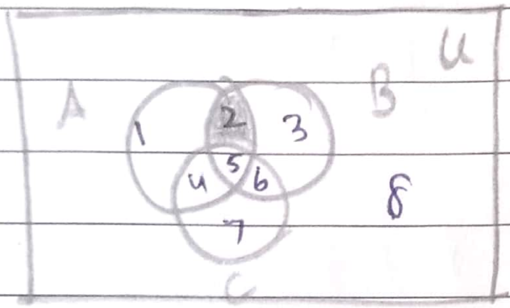
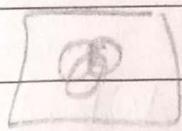
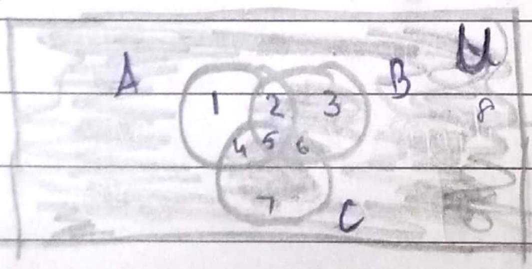
Q) $U = \{1,2,3,4,5,6,7,8\}$
$A = \{1,2,4,5\}$
$B = \{2,3,5,6\}$
$C = \{4,5,6,7\}$

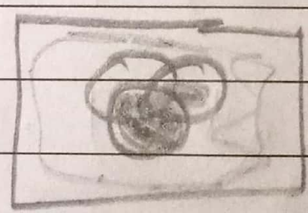
(a) $(A \cap B) \cap \bar{C}$





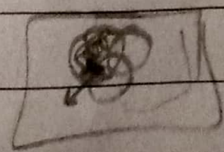
(b) $\bar{A} \cup (B \cup C)$

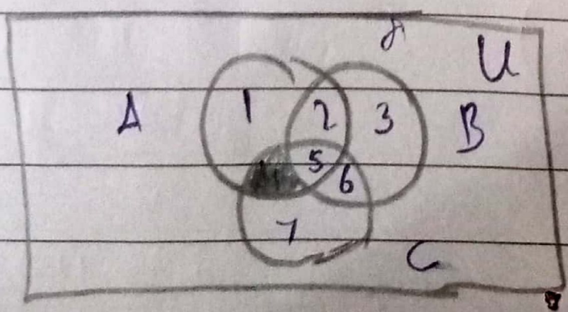




(c) $(A - B) \cap C$
$(A \cap \bar{B}) \cap C$

$A - B = A \cap \bar{B}$





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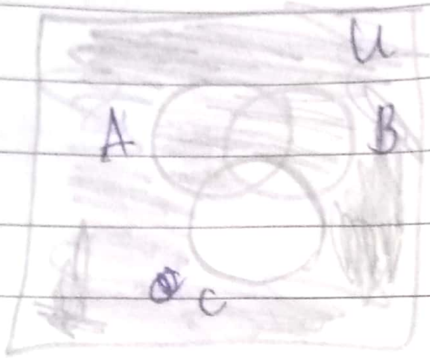
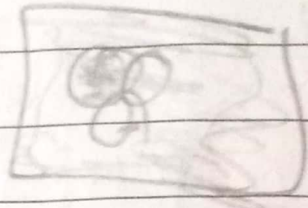
(d) $(A \cap \bar{B}) \cup \bar{C}$





(2) (i) $(A - (A \cap B)) \cap (B - (A \cap B)) = \phi$

$A - B = A \cap \bar{B}$

$A \cap (\overline{A \cap B}) \quad \cap (B \cap (\bar{A} \cup \bar{B}))$

$(A \cap (\bar{A} \cup \bar{B})) \quad \cap (B \cap (\bar{A} \cup \bar{B})$

ϕ

$(A \cap \bar{B}) \quad \cap (\bar{A} \cap B)$

$(A \cap B) \quad \cap (\overline{A \cap B}) \quad = \phi$

(b) $(A - B) \cup (A \cap B) = A$

$(A \cap \bar{B}) \quad \cup (A \cap B)$

$A \cap (\bar{B} \cup B)$

$A \cap U$

$\boxed{A}$

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(c) $(A-B) - C = (A-C) - B$

$(A \cap \bar{B}) - C$ $\quad$ $A \cap \bar{B}$

$A \cap \bar{C} \cap \bar{B}$

$a \cap \bar{C}$

$(A-B) \cap \bar{C}$

$(A \cap \bar{B}) \cap \bar{C}$

$A \cap \bar{B} \cap \bar{C} \equiv A \cap \bar{C} \cap \bar{B}$

(d) $\overline{(\bar{B} \cup (\bar{B}-A))} = B$

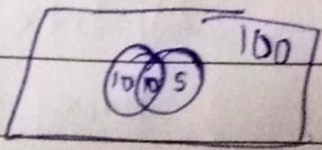
$\bar{\bar{B}} \cap \overline{(\bar{B}-A)}$

$\overline{(\bar{B} \cap \bar{A})}$

$B \cap (B \cup A)$

$B \cap (B \cup A) = \boxed{B}$

(21) (a)



(c) 100 - 25 = 75 apples

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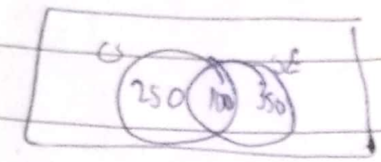
(b)



1000 - 700 = 300 like neither

250 like CS . 350 like SC

n(CS) = 250

n(SE) = 350

n(CS ∧ SE) = 100

n(all) - n(CS ∪ SE) = 300

(c) ~~n(I) = 32~~

n(mixed) = 78

n(Irish) = 32

n(trinidian) = 57

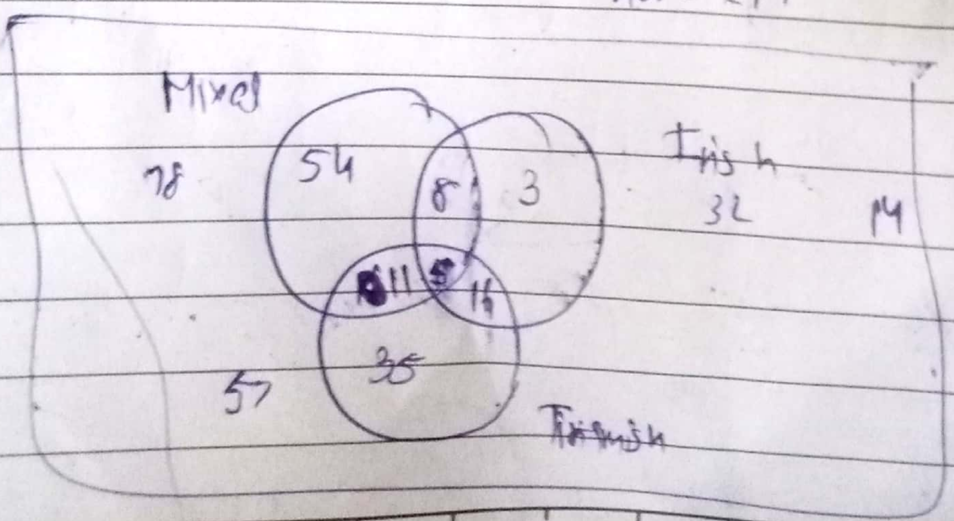
n(mixed ∧ Irish) = 13

n(Irish ∧ trini) = 21

n(mb ∧ tris) = 16

all three = 5

none = 14



54+8+3+11+5+16+35

14 = $\boxed{136}$

students

were surveyed

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(d) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$\{(x,y) : x \in A \wedge (y \in B \cap C)\}$

$(x \in A) \wedge (y \in B) \wedge (y \in C)$

$(x \in A) \wedge (x \in A) \wedge (y \in B) \wedge (y \in C)$

$((x \in A) \wedge (y \in B)) \cap \{(x \in A) \wedge y \in C)$

$(A \times B) \cap (A \times C)$

(Q13) $A = \{a,b,c,d\}$ $B = \{a,b,c,d\}$

(i) (a) $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$

domain: $\{a,b,c,d\}$ range: $\{a,b,c,d\}$ co-domain: $\{a,b,c,d\}$

(b) $f(a) = b$, $f(b) = b$, $f(c) = d$, $f(d) = c$

domain: $\{a,b,c,d\}$ range: $\{b,c,d\}$ co-domain: $\{a,b,c,d\}$

(c) $f(a) = d$, $f(b) = b$, $f(c) = c$, $f(d) = d$

domain: $\{a,b,c,d\}$ range: $\{b,c,d\}$ codomain: $\{a,b,c,d\}$

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(d) $f(a)=c$, $f(b)=a$, $f(c)=b$, $f(d)=d$

domain: $\{a,b,c,d\}$ range: $\{a,b,c,d\}$ codomain: $\{a,b,c,d\}$

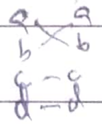
(ii) (a) Injective and Bijective

(b) Neither

(c) Neither

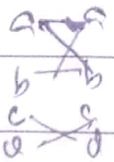
(d) Injective and Bijective

(iii) (a)



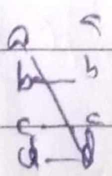
$f^{-1}(b)=a$, $f^{-1}(a)=b$ $f^{-1}(c)=d$
$f^{-1}(d)=d$

(b)



no inverse

(c)



$f^{-1}(c)=a$, $f^{-1}(a)=b$, $f^{-1}(b)=c$, $f^{-1}(d)=d$

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(24)

(a) $f(x) = \left\lfloor \frac{x^2}{3} \right\rfloor$ $f(S) = ?$

(i) $S = \{-2, -1, 0, 1, 2, 3\}$

$f(S) = \{\frac{4}{3}, \frac{1}{3}, 0, \frac{1}{3}, \frac{4}{3}, 3\}$ floor $\boxed{f(S) = \{1, 0, 0, 0, 1, 3\}}$

(ii) $S = \{0, 1, 2, 3, 4, 5\}$

$f(S) = \{0, 1/3, 4/3, 9, 16/3, 25/3\}$

$\boxed{\text{floor} = f(S) = \{0, 0, 1, 3, 5, 8\}}$

(iii) $S = \{1, 5, 7, 11\}$

$f(S) = \{1/3, \frac{25}{3}, 49/3, 121/3\}$

$f(S) = \{0, 8, 16, 40\}$

iv $S = \{2, 6, 10, 14\}$

$f(S) = \{4/3, 12, 100/3, \frac{196}{3}\}$

$f(S) = \{1, 12, 33, 65\}$

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(b) (i) $\lceil \frac{3}{4} \rceil = 1$

(ii) $\lfloor \frac{7}{8} \rfloor = 0$

(iii) $\lceil \frac{-3}{4} \rceil = 0$

(iv) $\lfloor \frac{-7}{8} \rfloor = -1$

(v) $\lceil 3 \rceil = 3$

(vi) $\lfloor -1 \rfloor = -1$

(vii) $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor$

$\frac{1}{2} + 2$

$\lfloor \frac{5}{2} \rfloor = 2$

(viii) $\lfloor \frac{1}{2} \cdot \lfloor \frac{5}{2} \rfloor \rfloor = \boxed{1}$

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(c) $\lfloor -x \rfloor = -\lceil x \rceil$ and $\lceil -x \rceil = -\lfloor x \rfloor$

$\lfloor -\frac{5}{2} \rfloor = \boxed{-3}$ | $-\lceil \frac{5}{2} \rceil = \boxed{-3}$ proved

$\lceil -\frac{5}{2} \rceil = -\lfloor \frac{5}{2} \rfloor$

$\boxed{-2 = -2} \rightarrow$ ~~proved~~ Proof

(25) $f(a) = 2a+3$ $g(a) = 3a+2$

(a) $(f \circ g)(a) = f(g(a)) = 2(3a+2)+3$
$6a+4+3$
$\boxed{6a+7}$

$g \circ f = g(f(a)) = 3(2a+3)+2$
$6a+9+2 = \boxed{6a+11}$

(b) Neither subjective, bijective or injective

(c) Not invertible