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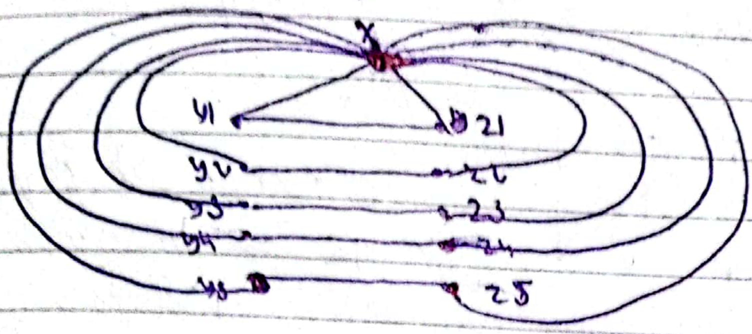
Graph Theory Assignment 1
21K-3153

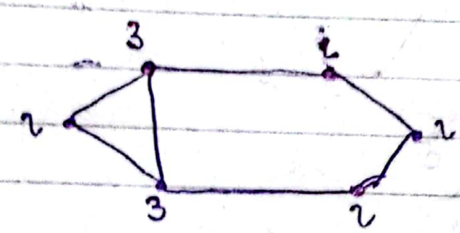
Q1

(a) $F_n = 2n+1$

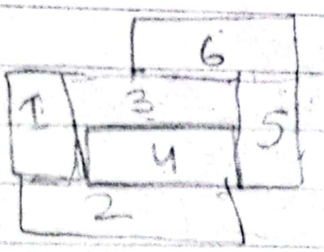
$F_5 = 10+1 = 11$ $x, y_1, y_2, y_3, y_4, y_5, z_1, z_2, z_3, z_4, z_5$

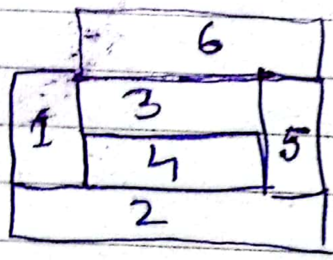


(b) 6 vertices
at most 3 odd
at least 2 even



(c)





(d)

	a	b	c	d	- - h
a	0	1	1	1	
b	1	0	1	1	
c	1	1	0	1	
d	1	1	1	0	
|	1				0
|					0
m					0

I7

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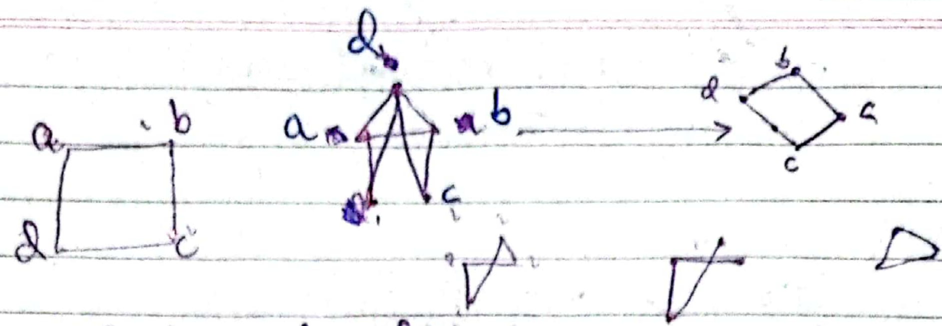
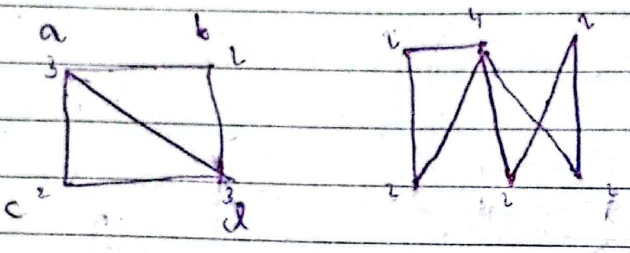
Q2

(a)



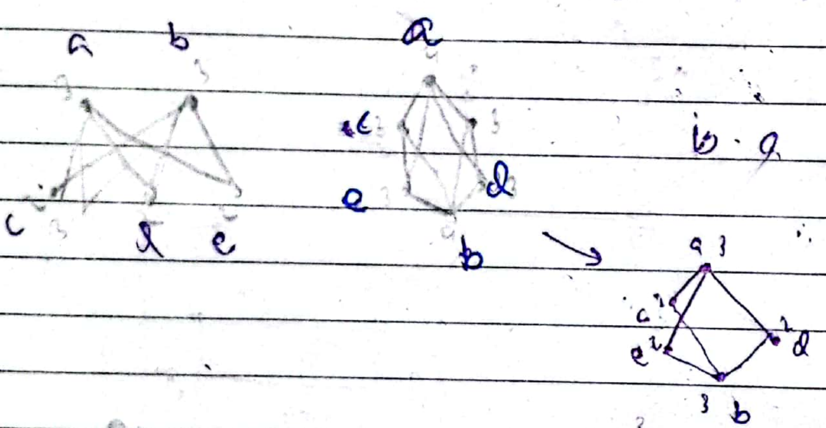
(ii) graph on the left is a subgraph

(b)



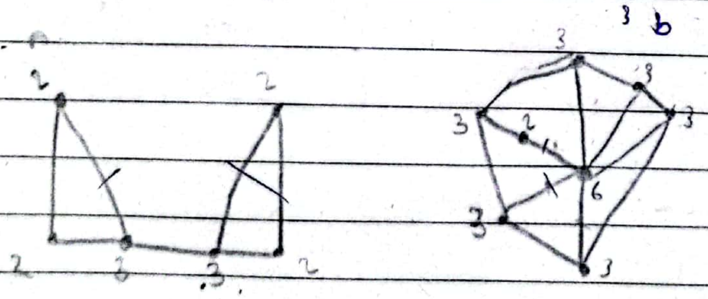
not a subgraph
as G2 does not have 2 vertices
with 3 edges
or more

(c)



is a subgraph

(d)



not a subgraph
as some edge cannot
be connected back
to same vertices

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(b) Graph → 28 edges & 12 vertices of degree
(i) 3 or 4
Using H.S.L
3 = ~~2e = Σd~~ $12 \times 3 = 36$ → sum of degrees
$2(28) = 36$ → $56 \neq 36$ → cannot be possible

4 : $4 \times 12 = 48$
$56 \neq 48$ → not possible

3 or 4 = consider 10 vertices of degree 3 & 2v of degree 4
$(30) + (8) = 38$
Using H SL : $2e = 56$ $56 \neq 38$ → not possible

11 vert of degree 3 & 1 vir of degree 4
$33 + 4 = 37$
$56 \neq 37$ → not possible

(ii) 3 or 6
3 = not possible as shown above

6 = Using H.S.L
$6 \times 12 = 72$
$2(28) = 72$ → $56 \neq 72$ → not possible

3 or 6 :
max degree sum :
$(6 \times 11) + (3 \times 1) = 69$
lower
$(9 \times 6) + (3 \times 3) = 63$

$(5 \times 4) + (7 \times 3) = 51$
↓
$(6 \times 6) + (6 \times 3) = 54$
↓
$(7 \times 4) + (5 \times 3) = 57$

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(b) Graph ~~29~~ → 28 edges ~~&~~ & 12 vertices of degree

(i) 3 or ~~&~~ 4

Using H.S.L

3 = ~~2e = ...~~ $12 \times 3 = 36$ → sum of degrees

$2(28) = 36$ → $56 \neq 36$ → cannot be possible

4 = $4 \times 12 = 48$

$56 \neq 48$ → not possible

~~... 3 or 4~~

3 or 4 = consider 10 vertices of degree 3 & 2v of degree 4

$(30) + (8) = 38$

Using H.S.L = $2e = 56$ $56 \neq 38$ → not possible

~~...~~ 11 vert of degree 3 & 1 vert of degree 4

$33 + 4 = 37$

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(ii) 3 or 6

3 = not possible as shown above

6 = Using H.S.L

$6 \times 12 = 72$

$2(28) = 72$ → $56 \neq 72$ → not possible

3 or 6:

max degree sum:

$(6 \times 11) + (3 \times 1) = 69$

lower

$(9 \times 6) + (3 \times 3) = 63$

$(5 \times 6) + (7 \times 3) = 51$

↓

$(6 \times 6) + (6 \times 3) = 54$

↓

$(7 \times 6) + (5 \times 3) = 57$

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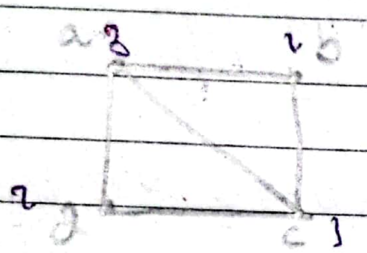
(b) 6 vert of degree 6 & 6 vert of degree 3 yields sum of 54 → thus not possible since ≠ 56

7 vert of deg(6) & 5 vertices of deg(3) = 57
$57 \neq 56$

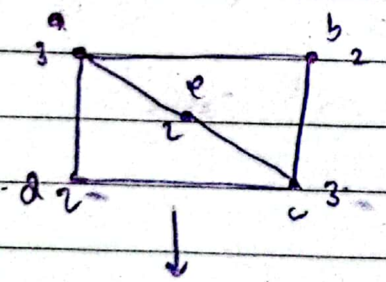
thus not possible with 3 or 6 degree

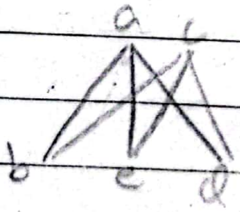
(c) (a)

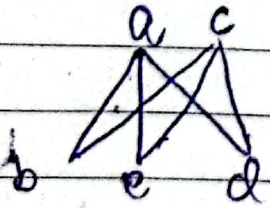


not bipartite.
a & c have 3 edges but only 2 other vertices to connect to. thus the extra edge is connected between them, making them adjacent

(b)

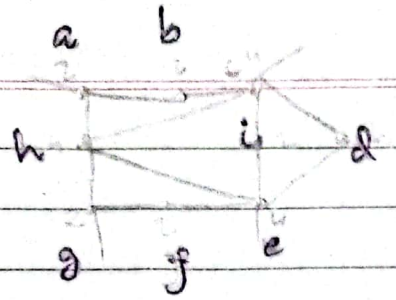




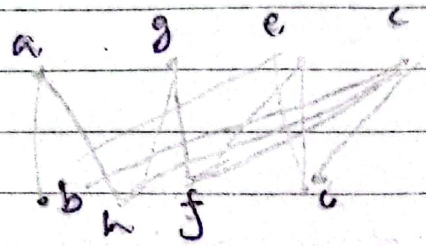


is bipartite
{a,c} {b,e,d}

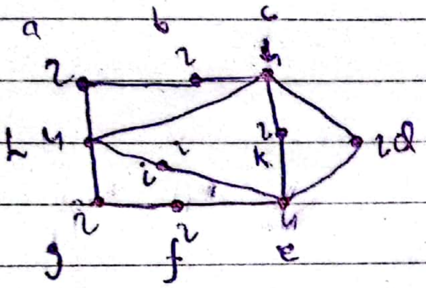
(c)

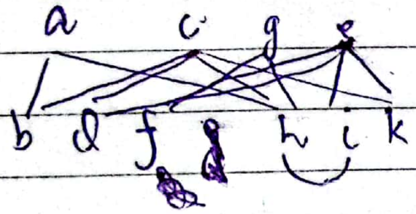


is bipartite



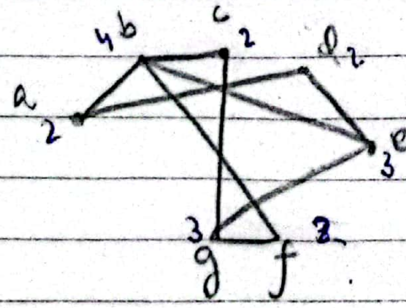
(d) ~~Same as~~

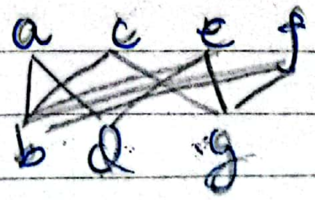




not bipartite as h is adjacent to i

(e)





is bipartite

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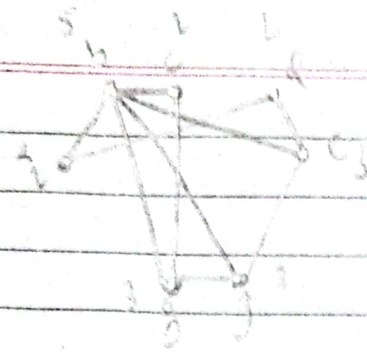
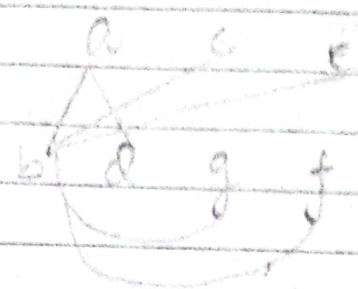
(f)

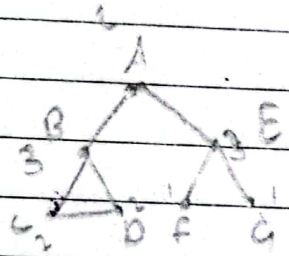


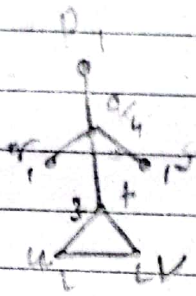


not bipartite as vertices cannot be split into two non adjacent sets

Q3

(a)

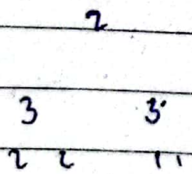


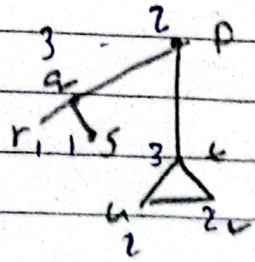


3,3,2,2,2,1,1   4,3,2,2,1,1,1

Not isomorphic, right graph has a vertice of deg(4), left graph does not

(b)





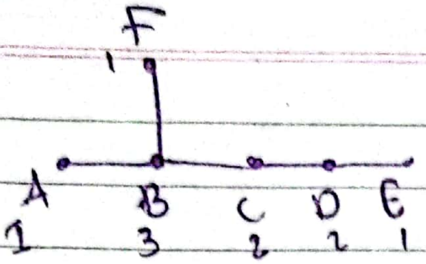
3,3,2,2,2,1,1

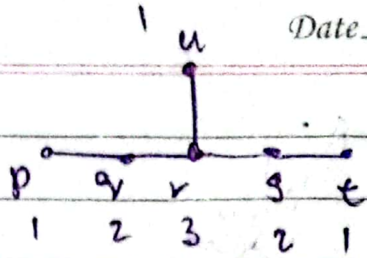
3,3,2,2,2,1,1

isomorphic

E→q | B→t | f→r | G→s | C→u | D→v | A→p |

(c)





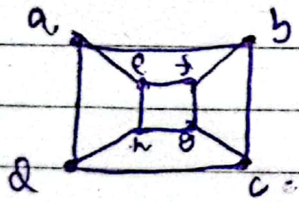
3, 2, 2, 1, 1, 1   3, 2, 2, 1, 1, 1

not isomorphic

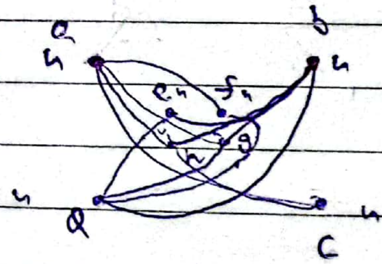
~~F→u~~
~~F→u | B→r | A→P | E→t | C→~~

left ~~right~~ graph has a vertice of degree (3) that connects to vertice of degree (1). The right graph does not.

(b) left graph G:

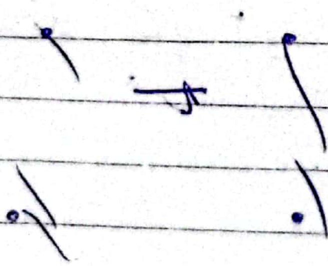


$\bar{G}$:



4, 4, 4, 4, 4, 4, 4, 4 (Regular graph)

$\bar{G}$ =



deg seq graph on right:
4, 4, 4, 4, 4, 4, 4, 4.
(~~4 reg~~) (regular graph)

thus ~~(b)~~ both graphs are isomorphic

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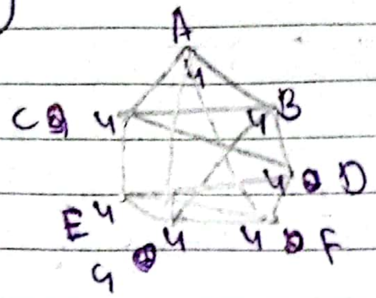
Q4

(a)



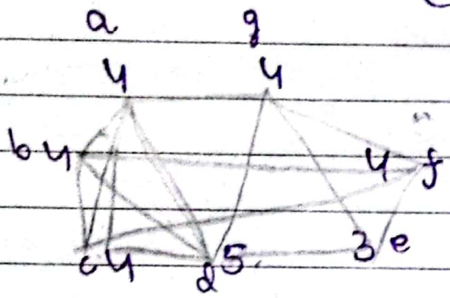
degree sequence : 4,4,4,4,4,4,4

~~AGB, AFC, AED, AGC~~

AGF, ACB, DEC, FED, ABG, BDC, CG (6)

EFG (7)

(b)



5, 4, 4, 4, 4, 4, 3

triangles: AGD, ~~AEG~~, GED, FBC, FGE, ABD, ABC, BCD

ACD (8)

(c)

3 4

4 4 5

4

4 3

5, 4, 4, 4, 4, 4, 3

Δs = 126, 176, 743 (3), 625, 236 (5), 165 (6)

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(b) 20 vert, 62 edges

$2e =$ sum of degrees
$124 =$ degrees sum

$3x + 7y = 124$
$x + y = 20$

$3(20 - y) + 7y = 124$
$60 - 3y + 7y = 124$
$4y + 60 = 124$
$4y = 64$
$\boxed{y = \cancel{8}\ 16}$

16 vertices of degree 7

Q5

(a)

(i) $\frac{7}{6}$ $= 1.16$ is the beta index

(ii) $\frac{7}{7}$ $= 1$ is the beta index

3, 3, 2, 1, 1, 0
2, 1, 0, 1, 0
1, 1 1 0 0
0 0 0 0

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(b)

(i) 6, 5, 4, 4, 3, 2

not graphical as vertice I is connected to 6 other vertices, but graph only has 6 vertices total

(ii) 6, 6, 4, 2, 2, 2, 1, 1 $\rightarrow$ 5, 3, 1, 1, 1, 0, 1

~~5, 3, 1, 1, 1, 1, 0, 1~~

~~5, 3, 1, 1, 1, 1, 0~~

5, 3, 1, 1, 1, 1, 0

2, 0, 0, 0, 0, 0

-1, -1, 0, 0, 0, 0

not graphical

2, ~~0, 0, 0, 0, 0~~

(iii) 3, 3, 3, 3, 3, 3

2, 2, 2, 3, 3

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(iii) 3, 3, 3, 3, 3, 3

2, 2, 2, 3, 3

3, 3, 2, 2, 2

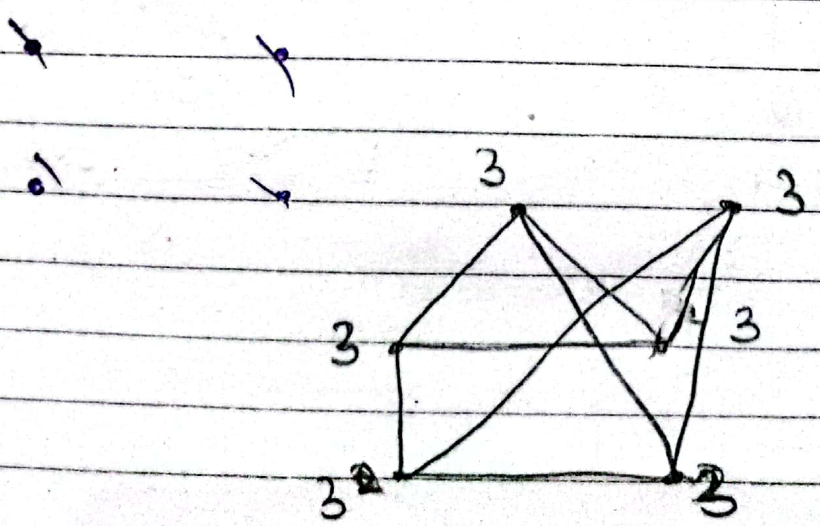
2, 1, 1, 2

2, 2, 1, 1

1, 0, 1

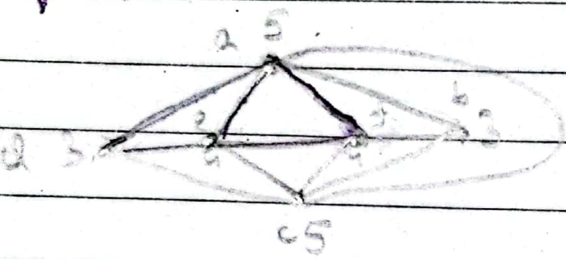
1, 1, 0

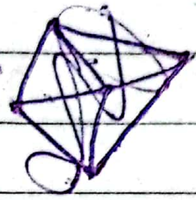
0, 0 → graphical

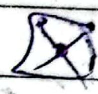


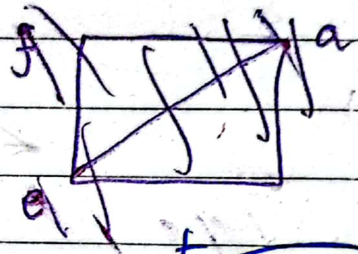
(c)

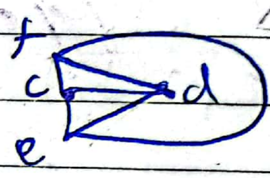
~~clique size =~~
clique size = ~~3~~ 4

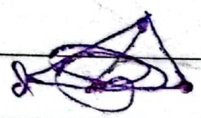


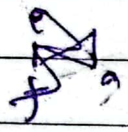


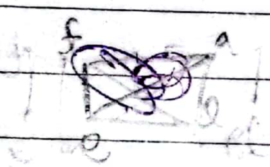








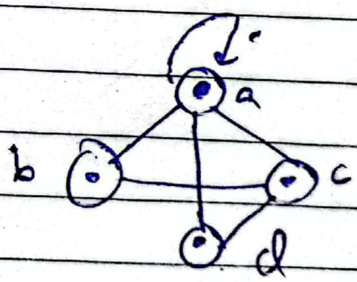




max independent size = 3

(d)

	a	b	c	d
a	a	1	1	1
b	1	0	1	0
c	1	1	0	1
d	1	0	1	0



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Q6

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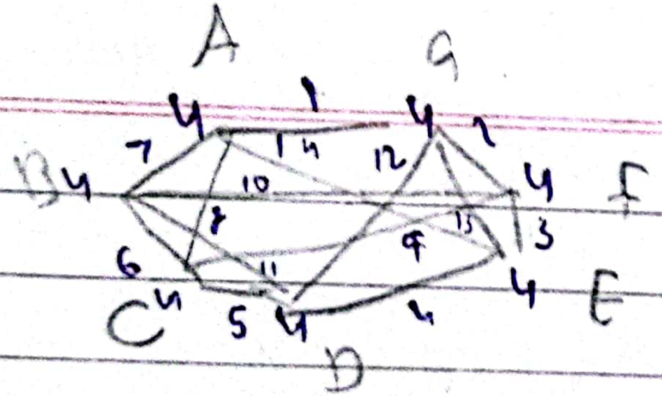
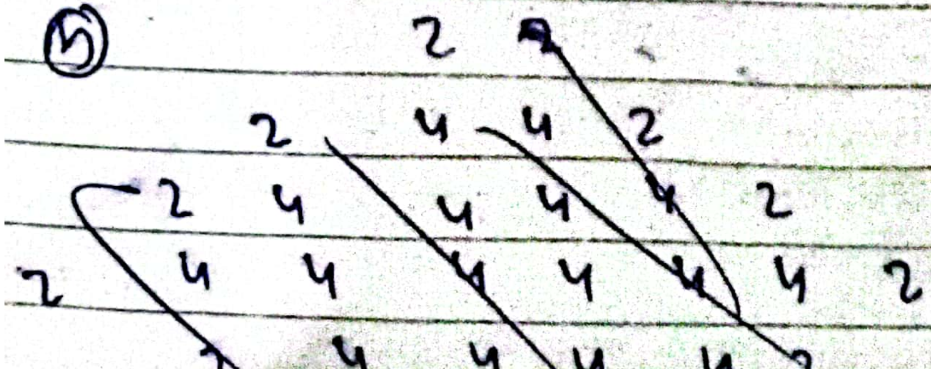
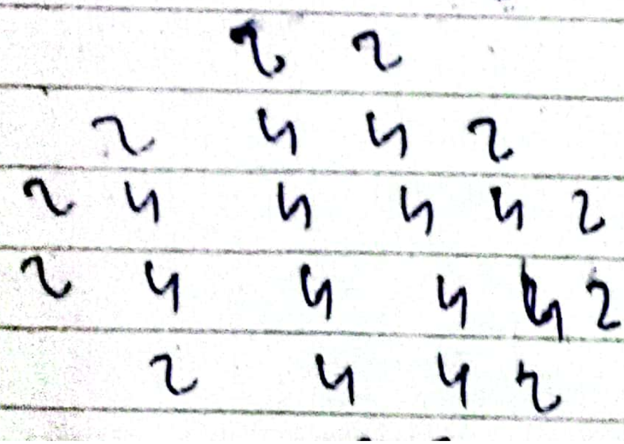


every vertices has an even degree thus eulerian

A → G → F → E → D → C → B → A → C → F → B → D → G → E → A

(b)





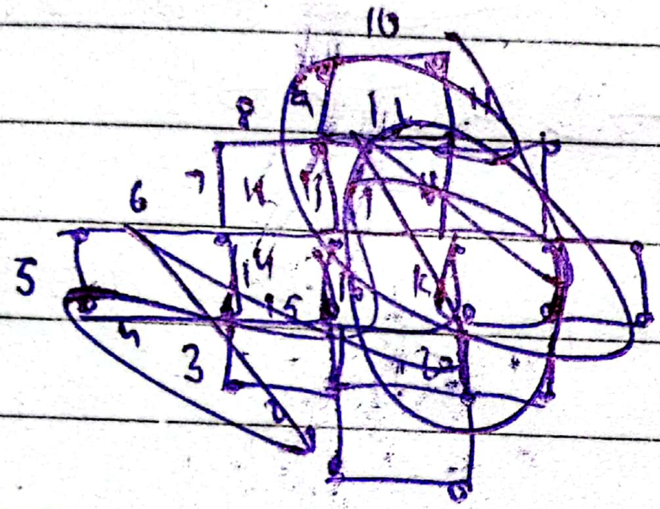
2 4 4 2   2 4

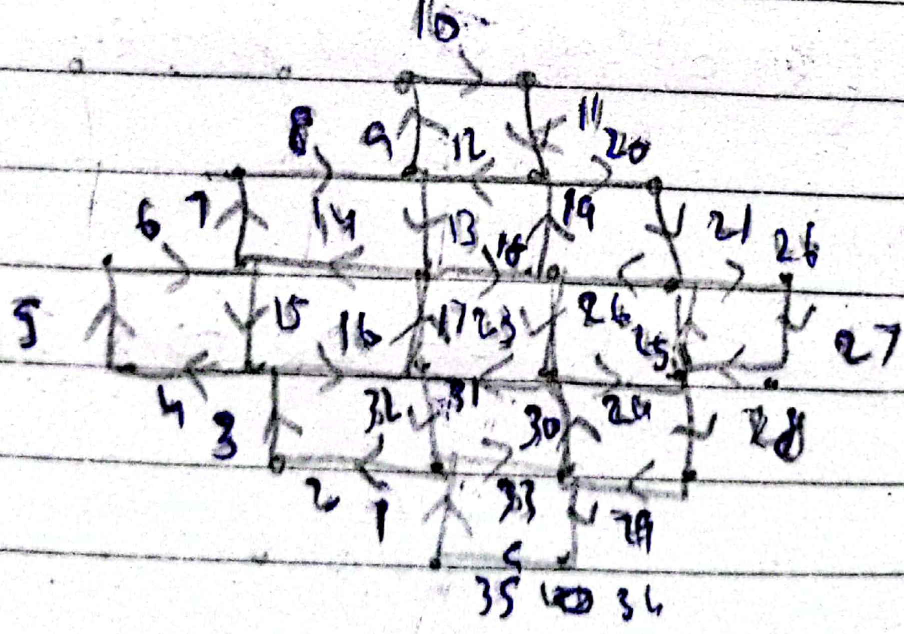
2 4 4 4 4 2   2 4

4 4 4 4 4 4 2   2

2 4 4 4 4 2

Every vertex has a degree of 4





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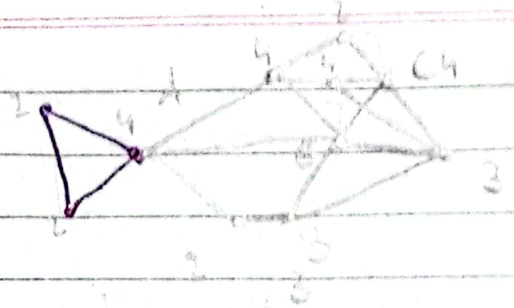
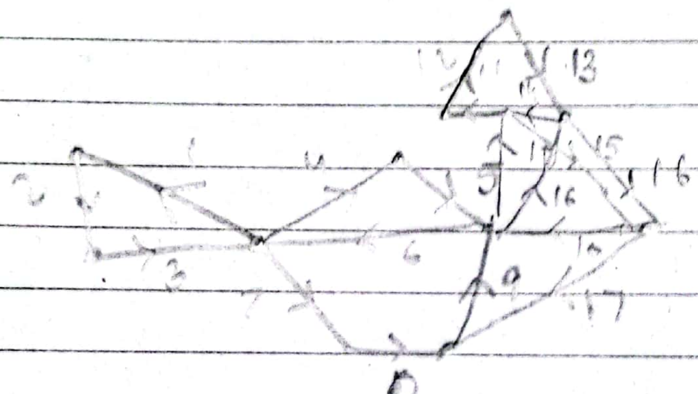
(b)



a trail exits.



Q7

① it is hamiltonian: ~~A → B → E → C → D~~

~~hamiltonian~~ E → B → D → C → A → E

② Hamiltonian path: A → B → E → C → D

③ Not eulerian (all vertices dont have even degrees)

④ trail: A → B → D → C → B → E → C → A → E

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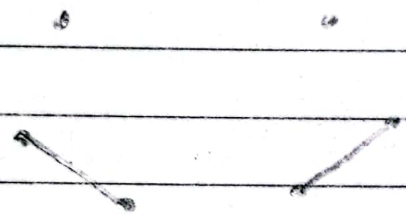
Q8 $X = \{x, y, z\}$

$G - S =$



(ii) $|S| = 3$

$c(G-S) = 4$

(iii) $|S| = 3 < 4$ = yes

(iv) $c(G-S) > |S|$ → thus not hamiltonian