LA

ASSIGNMENT 1

21k-3153.

ASSIGNMENT 1
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1·6 (Q 13-20)

(13)
$$x_1 + 3x_2 = b_1$$
$$-2x_1 + x_2 = b_2$$

$2R_1 + R_2$

$$\begin{array}{cc|c} 1 & 3 & b_1 \\ -2 & 1 & 2b_2 \end{array} \quad = \quad \begin{array}{cc|c} 1 & 3 & b_1 \\ 0 & 7 & 2b_1 + 2b_2 \end{array}$$

$\frac{1}{7}R_2$

$$\begin{array}{cc|c} 1 & 3 & b_1 \\ 0 & 1 & \frac{2b_1}{7} + \frac{b_2}{7} \end{array} \longrightarrow \begin{array}{l} x_1 + 3x_2 = b_1 \\ x_2 = \frac{2}{7}b_1 + \frac{1}{7}b_2 \end{array} \longrightarrow$$ thus, system is consistent for all values of b_1 and b_2

(14)
$$6x_1 - 4x_2 = b_1$$
$$3x_1 - 2x_2 = b_2$$

$R_1 ↔ R_2$ $\leftarrow$
$$\begin{array}{cc|c} 6 & -4 & b_1 \\ 3 & -2 & b_2 \end{array} \rightarrow \begin{array}{cc} 6 & -4 \\ 0 & \end{array}$$

-4_x

$\frac{1}{6}R_1$ $\leftarrow$
$$\begin{array}{cc|c} 6 & -4 & b_1 \\ 3 & -2 & b_2 \end{array} \rightarrow \begin{array}{cc|c} 1 & -2/3 & b_1/6 \\ 3 & -2 & b_2 \end{array}$$

$-3R_1 + R_2$

$$\begin{array}{cc|c} 1 & -4 & \frac{1}{6}b_1 \\ 0 & 0 & -\frac{1}{2}b_1 + b_2 \end{array}$$

$x_1 - 4x_2 = \frac{1}{6}b_1 \rightarrow -\frac{1}{3}b_2$

$$-\frac{1}{2}b_1 + b_2 = 0$$
$$b_2 = \frac{1}{2}b_1 \Rightarrow 2b_2 = b_1$$

(15) $\quad x_1 - 2x_2 + 5x_3 = b_1$
$\quad\quad 4x_1 - 5x_2 + 8x_3 = b_2$
$\quad\quad -3x_1 + 3x_2 - 3x_3 = b_3$

$$
\begin{array}{c}
\\
-4R_1 + R_2 \leftarrow \\
3R_1 + R_2 \leftarrow
\end{array}
\left[\begin{array}{ccc|c}
1 & -2 & 5 & b_1 \\
4 & -5 & 8 & b_2 \\
-3 & 3 & -3 & b_3
\end{array}\right]
$$

$$
\begin{array}{c}
\\
\frac{1}{3}R_2 \\
\\
\end{array}
\left[\begin{array}{ccc|c}
1 & -2 & 5 & b_1 \\
0 & 3 & -12 & -4b_1 + b_2 \\
0 & -3 & 12 & 3b_1 + b_3
\end{array}\right]
\quad
\begin{array}{c}
2R_2 + R_3 \leftarrow \\
\longrightarrow \\
3R_2 + R_3 \leftarrow
\end{array}
\left[\begin{array}{ccc|c}
1 & -2 & 5 & b_1 \\
0 & 1 & -4 & -4/3 b_1 + b_2/3 \\
0 & -3 & 12 & 3b_1 + b_3
\end{array}\right]
$$

$$
\left[\begin{array}{ccc|c}
1 & 0 & -3 & -5/3 b_1 + 2/3 b_2 \\
0 & 1 & -4 & -4/3 b_1 + 1/3 b_2 \\
0 & 0 & 0 & -b_1 + b_2 + b_3
\end{array}\right]
$$

system is consistent for all values
$$b_1 = b_2 + b_3$$

(16)
$$
\begin{array}{c}
\\
4R_1 + R_2 \leftarrow \\
9R_2 + R_1 \leftarrow
\end{array}
\left[\begin{array}{ccc|c}
1 & -2 & 1 & b_1 \\
-4 & 5 & 2 & b_2 \\
-4 & 7 & 4 & b_3
\end{array}\right]
$$

$$
\left[\begin{array}{ccc|c}
1 & -2 & 1 & b_1 \\
0 & -3 & -2 & 4b_1 + b_2 \\
0 & -1 & 0 & 4b_1 + b_3
\end{array}\right]
\quad
\begin{array}{c}
\rightarrow R_3 b_j R_2 \\
\\
-R_2
\end{array}
\left[\begin{array}{ccc|c}
1 & -2 & 1 & b_1 \\
0 & -1 & 0 & 4b_1 + b_3 \\
0 & -3 & -2 & 4b_1 + b_2
\end{array}\right]
$$

R_2+R_1

1	-2	-1	b_1
0	1	0	$-4b_1 - b_3$
0	-3	-2	$4b_1 + b_2$

$3R_2 + R_3 \Rightarrow$

①

1	0	-1	$-8b_1 - b_3 + b_1 = -7b_1 - b_3$
0	1	0	$-4b_1 - b_3$
0	0	-2	$-8b_1 + b_2 - 3b_3$

1	0	-1	$-7b_1 - b_3$
0	1	0	$-4b_1 - b_3$
0	0	1	$-4b_1 - \frac{1}{2}b_2 + 3/2 \, b_3$

Consistant for all valve

(17)
$$\begin{cases} x_1 - x_2 + 3x_3 + 2x_4 = b_1 \\ -2x_1 + x_2 + 5x_3 + x_4 = b_2 \\ -3x_1 + 2x_2 + 2x_3 - x_4 = b_3 \\ 4x_1 - 3x_2 + x_3 + 3x_4 = b_4 \end{cases}$$

next page

$$
\left[\begin{array}{cccc|c}
1 & -1 & 3 & 2 & b_1 \\
-2 & 1 & 5 & 1 & b_2 \\
-3 & 2 & 2 & -1 & b_3 \\
4 & -3 & 1 & 3 & b_4
\end{array}\right]
\begin{array}{l}
 \\
2R_1 + R_2 \\
3R_1 + R_3 \\
-4R_1 + R_4
\end{array}
$$

$$
\begin{array}{l}
 \\
-R_2 \leftarrow
\end{array}
\left[\begin{array}{cccc|c}
1 & -1 & 3 & 2 & b_1 \\
0 & -1 & 11 & 5 & 2b_1 + b_2 \\
0 & -1 & 11 & 5 & 3b_1 + b_3 \\
0 & 1 & -11 & -5 & -4b_1 + b_4
\end{array}\right]
$$

$$
\begin{array}{l}
R_2 + R_1 \\
 \\
R_2 + R_3 \leftarrow \\
-R_2 + R_4
\end{array}
\left[\begin{array}{cccc|c}
1 & -1 & 3 & 2 & b_1 \\
0 & 1 & 11 & 5 & -2b_1 - 2b_2 \\
0 & -1 & 11 & 5 & 3b_1 + b_3 \\
0 & 1 & -11 & -5 & -4b_1 + b_4
\end{array}\right]
$$

$$
\left[\begin{array}{cccc|c}
1 & 0 & 14 & 7 & -b_1 - b_2 \\
0 & 1 & 11 & 5 & -2b_1 - b_2 \\
0 & 0 & 0 & 0 & b_1 - b_2 + b_3 \\
0 & 0 & 0 & 0 & -2b_1 + b_2 + b_4
\end{array}\right]
$$

consistent of all values

$$
b_1 - b_2 + b_3 = 0
$$
$$
-2b_1 + b_2 + b_4 = 0
$$

(iv)

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{bmatrix} \qquad Y = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad (A-I)\, x = 0$$

$$\begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ 3 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$$\begin{matrix} & 1 & 1 & 2 & | & 0 \\ -1R_1+R_2 & 2 & 1 & -2 & | & 0 \\ -3R_1+R_3 & 3 & 1 & 0 & | & 0 \end{matrix} \xrightarrow[2R_2+R_3]{-R_2} \begin{bmatrix} 1 & 1 & 2 & | & 0 \\ 0 & -1 & -6 & | & 0 \\ 0 & -2 & -6 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & | & 0 \\ 0 & 1 & 6 & | & 0 \\ 0 & 0 & 1 & | & 0 \end{bmatrix} \qquad x_1 = x_2 = x_3 = 0$$

(b) $\qquad Ax = 4x \qquad (A-4I)\, x = 0 \qquad$ $Ax - 4x = 0$

$$\begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 2 & | & 0 \\ 2 & -2 & -2 & | & 0 \\ 3 & 1 & -3 & | & 0 \end{bmatrix}$$

R_2 by R_4 $\qquad \begin{matrix} 2 & -2 & -2 & | & 0 \\ -2 & 1 & 2 & | & 0 \\ 3 & 1 & -3 & | & 0 \end{matrix}$

$$\begin{array}{c} \\ 2R_1+R_2 \\ -R_1+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{array}\right]$$

$$\begin{array}{c} R_2 \leftrightarrow \\ -R_2 \\ \\ -4R_2+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{array}{c} x_1 - x_3 = 0 \\ x_2 = 0 \\ x_3 = t \quad \boxed{x_1 = t} \end{array}$$

(19)

$$\left[\begin{array}{ccc} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{array}\right] \qquad X = \left[\begin{array}{cccc} 2 & -1 & 5 & 78 \\ 4 & 0 & -3 & 01 \\ 3 & 5 & -7 & 21 \end{array}\right] \quad Ax=b$$

$$\begin{array}{c} \\ -2R_1+R_2 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 2 & 5 & 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} R_2+R_3 \\ \\ \\ -2R_2+R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 5 & -2 & -2 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$-\frac{7}{5}R_3 + R_1$

$-\frac{3}{5}R_3 + R_2$

$-5 R_3$

$$\left[\begin{array}{ccc} 1 & 0 & 7/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & -1/5 \end{array}\right.\left|\begin{array}{ccc} 7/5 & -1/5 & 0 \\ -2/5 & -1/5 & 0 \\ 4/5 & -2/5 & 1 \end{array}\right]$$

$\frac{-13}{5}$

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right.\left|\begin{array}{ccc} 7 & -13/5 & 7 \\ 0 & 2/5 & 0 \\ 4 & -2 & -5 \end{array}\right|\left.\begin{array}{cccc} 2 & -1 & 5 & 7 & 6 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{array}\right]$$

3×5

(20)

$$\begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & -4 \end{bmatrix} X = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 6 & 7 & 8 & 9 \\ 1 & 3 & 7 & 9 \end{bmatrix}$$

$$\begin{array}{c} \textcircled{2} \\ \\ 2R_1 + R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & -4 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ -2 & 0 & 1 & 1 & 0 & 0 \end{array} \right]$$

$$\begin{array}{c} -R_2 + R_1 \\ -R_2 \\ 2R_2 + R_4 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & -4 & 0 & 0 & \textcircled{1} \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & 2 & 9 & 1 & 0 & 2 \end{array} \right]$$

$$\begin{array}{c} \\ \\ -\frac{1}{9}R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -5 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & -9 & 1 & 2 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5/9 & -1/9 & -1/9 \\ 0 & 1 & 0 & 3/9 & -7/9 & 2/9 \\ 0 & 0 & 1 & -7/9 & -2/9 & -4/9 \end{array} \right] \rightarrow \text{inverse}$$

$$\begin{bmatrix} -5/9 & -1/9 & -1/9 \\ 1/9 & -7/9 & 4/9 \\ -1/9 & -4/9 & -4/9 \end{bmatrix} \times \begin{bmatrix} 4 & 3 & 21 \\ 6 & 7 & 29 \\ 1 & 3 & 7 & 9 \end{bmatrix}$$

$-60/9 - 6/9 -1/9$	$-15/9 -7/9 -3/9$	$-60/9 - 2/9 -7/9$	$-5/9 -9/9 -3/9$
$24/9 - 28/9 -7/9$	$3/9 - 49/9 +6/9$	$2/9 -14/9 -49/9$	$1/9 -83/9 +18/9$
$-4/9 -12/9 -2/9$	$-3/9 -14/9 -6/9$	$-2/9 -4/9 +14/9$	$-11/9 -18/9 -18/9$

$$\begin{bmatrix} -3 & -21/9 & -25/9 & -13/9 \\ -4 & -40/9 & -40/9 & 8.44/9 \\ -2 & 8/9 & -3\,24/9 & -37/9 \end{bmatrix}$$

1.7

(Q) Determine whether upper, lower or or diagonal

(a) upper, invertible
(b) lower, not invertible
(c) diagonal, invertible
(d) upper triangular, not invertible
(e) lower triangular, invertible
(f) upper triangular, not invertible
(g) diagonal, invertible
(h) lower, not invertible

(Q 3) Inspection

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -4 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 4 & -1 \\ 4 & 10 \end{bmatrix}$$

(4)
$$\begin{bmatrix} 1 & 2 & -5 \\ -3 & 1 & 0 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 6 & -10 \\ -12 & 3 & 0 \end{bmatrix}$$

(5)
$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 & 4 & -4 \\ 1 & -5 & 3 & 0 & 3 \\ -6 & 2 & 2 & 2 & 2 \end{bmatrix}$$

3×3   5×5

$$\begin{bmatrix} 15 & 10 & 0 & 20 & -20 \\ 2 & -10 & 6 & 0 & 6 \\ 18 & -6 & -6 & -6 & -6 \end{bmatrix}$$

(6)
$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 4 & -1 & 3 \\ 1 & 2 & 0 \\ -5 & 1 & -2 \end{bmatrix} \begin{bmatrix} -3 & 6 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -2 & 6 \\ -1 & -2 & 0 \\ -20 & 4 & -8 \end{bmatrix} \begin{bmatrix} -3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -6 & -10 & 12 \\ 3 & -10 & 0 \\ 60 & 20 & -16 \end{bmatrix}$$

Dated:

(0) Find A^2, A^a, A^{-k}

(3) $A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$ $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1/4 \end{bmatrix}$ $A^{-2} = \begin{bmatrix} 1 & 0 \\ 0 & -1/4 \end{bmatrix}$ $A^{-k} = \begin{bmatrix} (1)^{-k} & 0 \\ 0 & -\frac{1}{(4)^k} \end{bmatrix}$

(8) $A = \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ $A^2 = \begin{bmatrix} 36 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{bmatrix}$

$A^{-2} = \begin{bmatrix} 1/36 & 0 & 0 \\ 0 & 1/9 & 0 \\ 0 & 0 & 1/25 \end{bmatrix}$ $A^{-k} = \begin{bmatrix} -\frac{1}{(6)^k} & 0 & 0 \\ 0 & \frac{1}{\frac{9}{3}^k} & 0 \\ 0 & 0 & \frac{1}{(5)^k} \end{bmatrix}$

(9) $A = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/9 & 0 \\ 0 & 0 & 1/16 \end{bmatrix}$ $A^{-2} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 16 \end{bmatrix}$

$$A^{-k} = \begin{bmatrix} (2)^k & 0 & 0 \\ 0 & (3)^k & 0 \\ 0 & 0 & (4)^k \end{bmatrix}$$

(10)
$$\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$(-4)^{-2}$

$\dfrac{1}{(-4)^2} = \dfrac{1}{4}$

$$A^2 = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \qquad A^{-2} = \begin{bmatrix} -\frac{1}{4} & 0 & 0 & 0 \\ 0 & -\frac{1}{16} & 0 & 0 \\ 0 & 0 & -\frac{1}{9} & 0 \\ 0 & 0 & 6 & \frac{1}{4} \end{bmatrix}$$

$$A^{-k} = \begin{bmatrix} -\frac{1}{(2)}^k & 0 & 0 & 0 \\ 0 & -\frac{1}{(4)}^k & 0 & 0 \\ 0 & 0 & -\frac{1}{(3)}^k & 0 \\ 0 & 0 & 0 & \frac{1}{(4)}^k \end{bmatrix}$$

Determine by inspection if invertible

$$\begin{bmatrix} 0 & 6 & -1 \\ 0 & 7 & -4 \\ 0 & 0 & -2 \end{bmatrix} \rightarrow$$ not invertible as diagonals are too should all be non-zero elements

$$\begin{bmatrix} -1 & 2 & 4 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$ → invertible as diagonal values are non-zero

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -5 & 0 & 0 \\ 4 & -3 & 4 & 0 \\ 1 & -2 & 1 & 3 \end{bmatrix}$$ → invertible as diagonals are non-zero

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ -3 & -1 & 0 & 0 \\ -4 & -6 & 0 & 0 \\ 0 & 3 & 8 & -5 \end{bmatrix}$$ → ~~invertible~~ not invertible as one diagonal value is 0

(Q) Find diagonal of AB

$$A = \begin{bmatrix} 3 & 2 & 6 \\ 0 & 1 & -2 \\ 0 & 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 2 & 7 \\ 0 & 5 & 3 \\ 0 & 0 & 6 \end{bmatrix} =$$

$$\begin{bmatrix} -3 & 6+10 & 21+6+12 \\ 0 & 5 & 3-12 \\ 0 & 0 & -6 \end{bmatrix} = \begin{bmatrix} -3 & 16 & 39 \\ 0 & 5 & -9 \\ 0 & 0 & -6 \end{bmatrix}$$

-3, 5   diagonal