National University of Computer & Emerging Sciences, Karachi

Graph Theory Assignment - 01 (CLO1)

Instructor. Dr. Nazish Kanwal, Mr. Abdul Basit & Mr. Usama Antuley

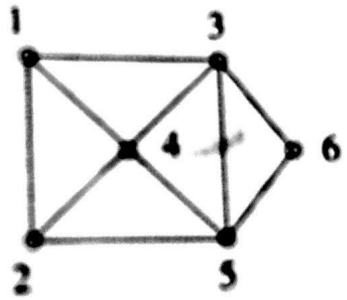
Submission Date - September 22nd, 2023 Total Marks: 30

Question 1.[04 marks]

(a) The friendship graph F_n has $2n + 1$ vertices $x, y_1, \cdots, y_n$, and $z_1, \cdots, z_n$. The vertex x is adjacent to all other vertices; also, vertices y_i and z_i are adjacent for $i = 1, \cdots, n$. There are no other edges. Draw a diagram of the friendship graph F_5.

(b) Draw a graph with six vertices at most three of which are odd and at least two of which are even.

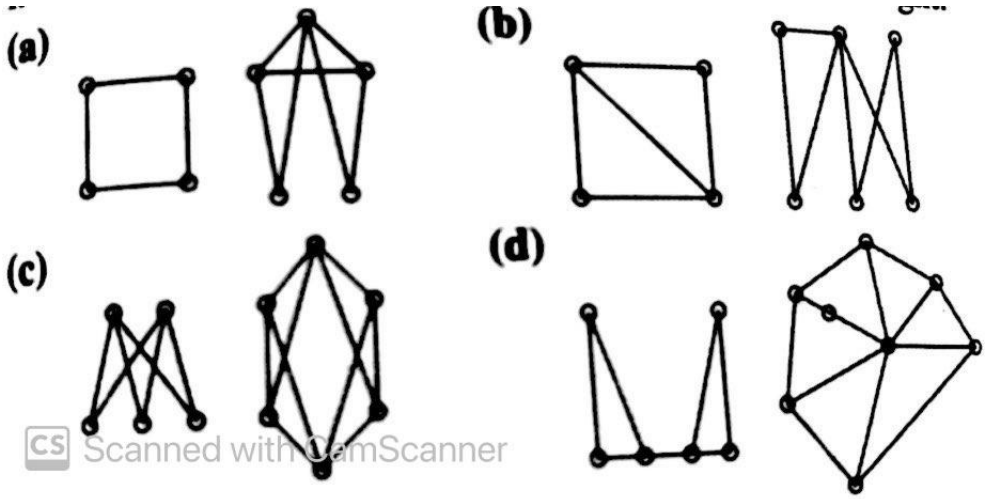
(c) In the graph, the vertices represents the rooms of a one-story house, and an edge between vertices means that the corresponding rooms have a wall in common. Draw a possible floor plan for this house.



(d) Using rectangular block whose entries are all equal, write down an adjacency matrix for $\mathsf{K}_{m,n}$.

Question 2.[05 marks]

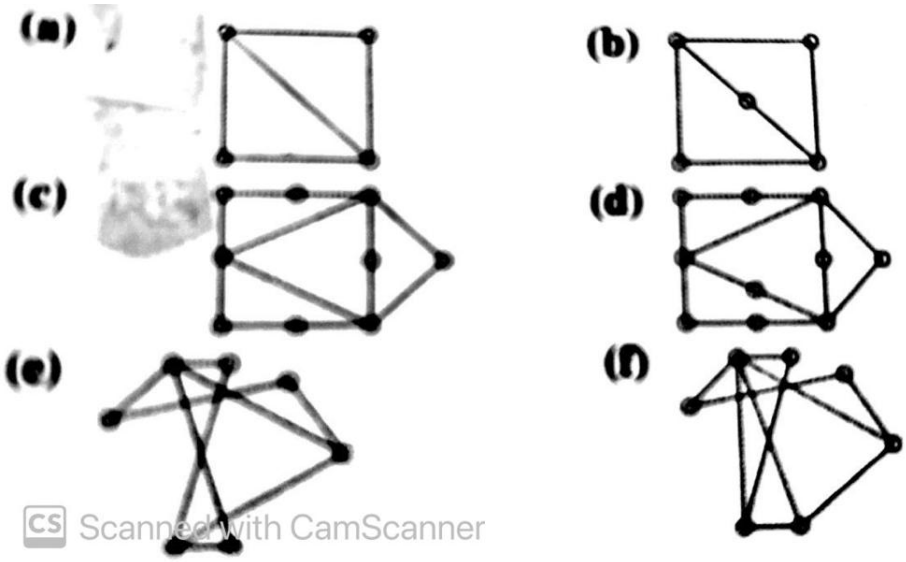
(a) For each pair of the graphs given below, discover whether the graph on left is a subgraph of the one on the right.



(i) If it is not, explain why not. (ii) If it is, label the vertices of the subgraphs, then use the same symbols to label the corresponding vertices of the graphs.

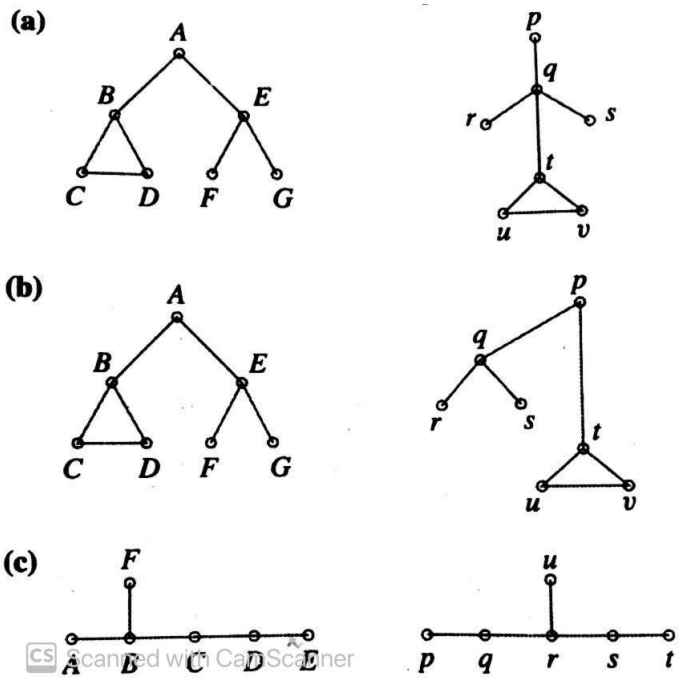
(b) Does there exist a graph G with 28 edges and 12 vertices each of degree
(i) 3 or 4 ? (ii) 3 or 6?

(c) Determine whether each of the following graphs are bipartite. In each case, give the bipartition sets or explain why the graph is not bipartite.

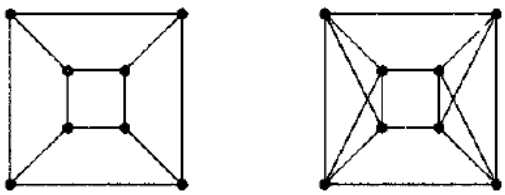


Question 3.[04 marks]

(a) (i) What is the degree sequence of each graph?
(ii)If the graph are isomorphic, exhibit an isomorphism. If not isomorphic, explain why ?

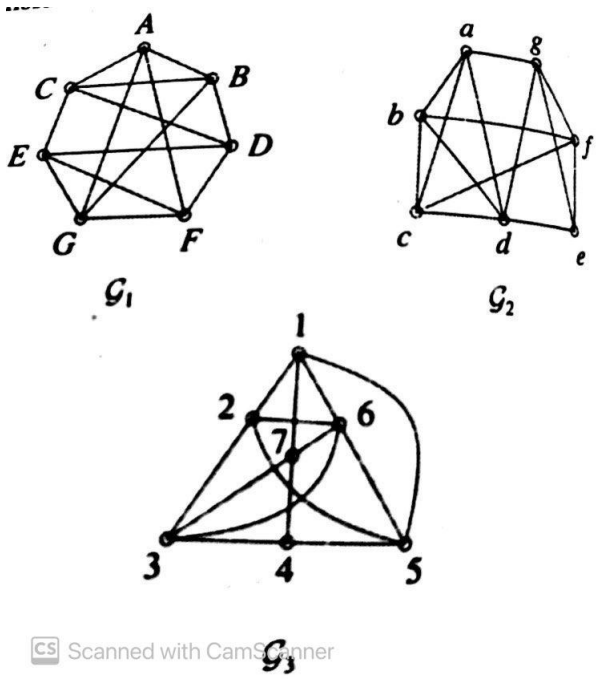


(b) Prove that the graph on the right is isomorphic to the complement of the graph on left.



Question 4.[02 marks]

(a) Consider the following three graphs.

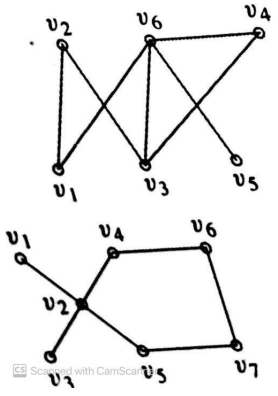


(i) What is the degree sequence of each graphs?
(ii) How many triangles in each graphs? Draw them.

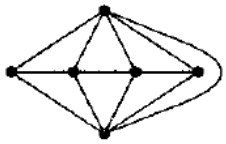
(b) A 20-vertex graph has 62-edges. Every vertex has degree 3 or 7. How many vertices have degree 7?

Question 5.[05 marks]

(a) Suppose the vertices of a graph represent cities in a certian region, and an edge joining two vertices indicates that there is a direct(nonstop) flight between thsoe two cities. Geographers define the *beta index* of a graph as the ratio of the number of edges to the number of vertices and view this number as a measure of connectivity of the region. Highly developed countries have high beta indexes; poorly developed countries has low beta indexes. Find the beta index of each of the following graph.



(b) Determine whether or not the each sequence is graphical. If it is, draw a graph with that degree sequence.
(i) $6, 5, 4, 4, 3, 2$ (ii) $6, 6, 4, 2, 2, 2, 1, 1$ (iii) $3, 3, 3, 3, 3, 3$.



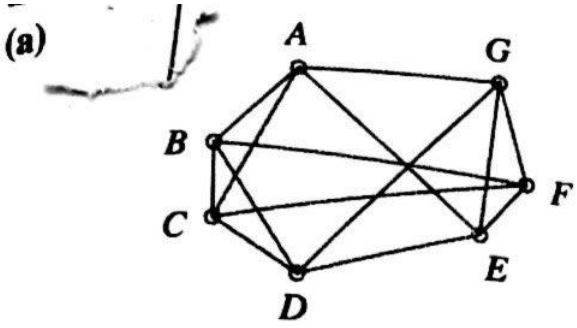
(c) Determine the size of the clique and maximum size of an independent set in the graph below.

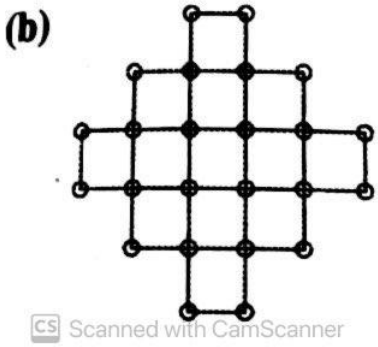
(d) Draw the graph correspond to the adjacency matrix A. (The vertices are in the order a, b, c, d)

$$A = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

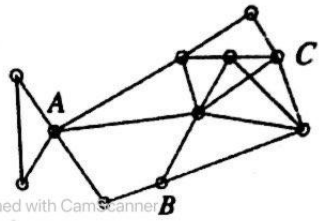
Question 6.[04 marks]

(a) For the following graphs, explain why graph is *Eulerian* and find an Eulerian circuit.



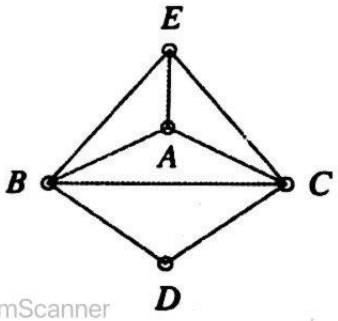


(b) Is there an *Eulerian trail* from A to B in the graph below?If yes, find one; if not , explain why not.



Question 7.[04 marks]

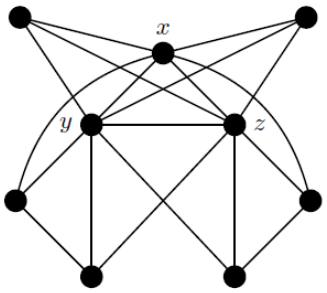
Consider the graph shown.



(i) Is it *Hamiltonian*? (ii) Is there a Hamiltonian path? (iii) Is it Eulerian? (iv) Is there an Eulerian trail?

Question 8.[02 marks]

Consider the following graph G and let $S = \{x, y, z\}$.



(i) Draw the graph $G - S$.
(ii) Find $|S|$ and $c(G - S)$.
(iii) Is $|S| < c(G - S)$?
(iv) Is G Hamiltonian?

The End