3/10/21

Calculus Assignment 2

Q65 Show that

$$f(x) = \begin{cases} x^2+x+1, & x \le 1 \\ 3x, & x > 1 \end{cases}$$

is continuos at $x=1$. Find if differentiable at $x=1$. find derivative there. Sketch graph.

① $f(1) = 3$

3	$3x$
1	

② $\lim_{\to 3^-} 3 = \lim_{\to 3^+} 3(1)$

$\boxed{3=3}$

③ $f(1) = f(x)$

$\boxed{3=3}$ → continuous

④ $\dfrac{f(x_0+h) + f(x_0)}{h}$

x^2+x+1	$3x$
1	

$x_0 = 1 \rightarrow f(x_0) = 3$

$f(x_0+h) = (1+h)^2 + (1+h)+1$

$1 + 2h + h^2 + 1 + h + 1$

$h^2 + 3h + 3$

↓

$\dfrac{h^2+3h+3-3}{h} \rightarrow \dfrac{h^2 \cancel{+3h+6}}{h}$

$$\frac{h^2 + 3h}{h} \rightarrow \frac{h(h+3)}{h} \rightarrow h+3$$

$h \to 0$

$$\lim_{h \to 0^-} = \boxed{3}$$

RHS: $f(x_0) = 3$

$f(x_0 + h) = 3(1+h) \rightarrow 3 + 3h$

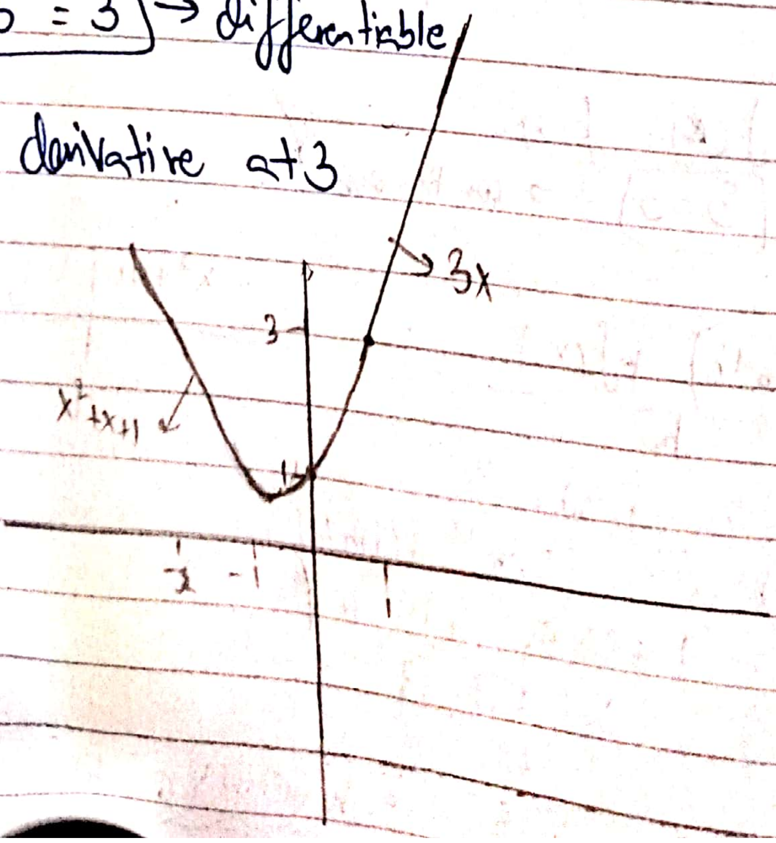
$$\frac{3 + 3h - 3}{h}$$

$$\frac{3h}{h} = \boxed{3}$$

LHS = RHS

$\boxed{3 = 3} \rightarrow$ differentiable

derivative at 3



Derivative is 3

(6b) $f(x) = \begin{cases} x^2 - 16x, & x \leq 9 \\ \sqrt{x}, & x \geq 9 \end{cases}$

Continous at $x=9$, differentiable at $x=9$?
find derivative

$\begin{array}{c|c} x^2-16x & \sqrt{x} \end{array}$

① $f(9) = \pm 3$

② $\lim_{x \to 9^-} f(x) = \lim_{x \to 9^+} f(x)$

$x^2 - 16x = \cancel{\sqrt{9}}\ \sqrt{x}$

$81 - 144 = \cancel{3}\ \sqrt{9}$

$-63 = 3 \Rightarrow \times$

not continuous thus ~~the~~ not differentiable.

(67) $f(x) = \begin{cases} x^2 , & x \leq 1 \\ \sqrt{x} , & x > 1 \end{cases}$

differentiable at $x=1$! find derivative

(1) $f(1) = 1$

x^2	$\sqrt{x}$

(2) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

$x^2 = \sqrt{x}$

$1 = 1$

(3) $\boxed{1=1} \Rightarrow$ continuous

(4) $f(\alpha\, x_0) = 1$

$f(x_0+h) = (1+h)^2 = 1+2h+h^2$

LHS:

$$\frac{1+2h+h^2-1}{h} = \frac{h^2+2h}{h} = \frac{\cancel{h}(h+2)}{\cancel{h}}$$

$$\lim_{h \to 0} \boxed{h+2} = \boxed{2}$$

RHS :

$f(x_0) = 1$

$(x_0 + h) : \sqrt{1+h}$

$$\frac{\sqrt{1+h} - 1}{h}$$

$$\frac{\sqrt{1+h} - 1}{h} \times \frac{\sqrt{1+h} + 1}{\sqrt{1+h} + 1}$$

$$\frac{(\sqrt{1+h})^2 - (1)^2}{h\sqrt{1+h} + h} = \frac{1+h-1}{h\sqrt{1+h} + h}$$

$$\frac{\cancel{h}}{\cancel{h}(\sqrt{1+h} + 1)} = \frac{1}{\sqrt{1+h} + 1}$$

$$\lim_{h \to 0} = \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$

LHS = RHS

$2 \neq \frac{1}{2}$ → not differentiable

2.4

Q1) Q. Find $\frac{dy}{dx}$ at $x=1$

$y = \frac{2x-1}{x+3}$ $\qquad \frac{vu' - uv'}{v^2}$

$u' = 2 \qquad v' = 1$

$\frac{dy}{dx} = \frac{(x+3)(2) - ((2x-1)(1))}{(x+3)^2}$

$\frac{(1+3)(2) - ((2-1)(1))}{(4)^2}$

$\frac{8-1}{16} = \boxed{\frac{7}{16}}$

Q2) $y = \frac{4x+1}{x^2-5}$ $\qquad x=1$ $\qquad u' = 4$, $v' = 2x$

$\frac{dy}{dx} = \frac{(x^2-5)(4) - ((4x+1)(2x))}{(x^2-5)^2}$

$\frac{(-4)(4) - ((5)(2))}{(-4)^2} = \frac{-16-10}{-16}$

$\frac{-26}{-16} = \frac{-\cancel{26}^{13}}{\cancel{16}_8} = \boxed{-\frac{13}{8}}$

Q23 $y = \left(\frac{3x+2}{x}\right)\left(x^{-5}+1\right)$

$\cancel{3x^{-4}+2x^{-5}}$

$3x+2 \left(x^{-6} + \frac{1}{x}\right.$

$3x+2 \times \left(\frac{1}{x^6}+\frac{1}{x}\right)$

$3x+2 \times \left(\frac{1+x^5}{x^6}\right)$

$\frac{(3x+2)(1+x^5)}{x^6}$

$\frac{3x+3x^6+2+2x^5}{x^6} \rightarrow \frac{3x^6+2x^5+3x+2}{x^6}$

$u' = 18x^5+10x^4+3$

$v' = 6x^5$

$\frac{dy}{dx}$ $\frac{x^6(18x^5+10x^4+3)-((3x^6+2x^5+3x+2)(6x^5))}{(x^6)^2}$

$\frac{(1)(18+10+3)-((3+2+3+2)(6))}{1}$

$$\frac{31-60}{1} = \boxed{-29} \to \frac{dy}{dx}$$

(2h) $y = (2x^7 - x^2)\frac{(x-1)}{(x+1)}$

$$\frac{2x^8 - x^3 - 2x^7 + x^2}{x+1}$$

$u' = \underline{16x^7 - 3x^2 - 14x^6 + 2x}$

$v' = 1$

$$\frac{dy}{dx} = \frac{(x+1)(16x^7 - 3x^2 - 14x^6 + 2x) - ((2x^8 - x^3 - 2x^7 + x^2)(1))}{(x+1)^2}$$

$$\frac{(1+1)(16-3-14+2) - ((2-1-2+1)(1))}{(2)^2}$$

$$\frac{2 - 0}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

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Ex 3.1

(Q25) $x^4 + y^4 = 16$ $(1, \sqrt[4]{15})$

Ex 2.5

(Q15) find $f'(x)$

$f(x)$ 15

(15) $f(x) = \sin^2 x + \cos^2 x$

$\frac{dy}{dx} = \boxed{2 \sin x \cos x + -2 \cos x \sin x}$

$\sin^2 x + \cos^2 x = 1$

so $f'(1) = \boxed{0}$

$\boxed{\frac{dy}{dx} = 0}$

$\sec^2 = \sec x \tan x$

(16) $f(x)$ $\sec^2 x - \tan^2 x$

2

~~2 s~~

$2 \sec x \, (\sec x \tan x)$

$2 \sec x \tan x \sec x - 2 \tan x \, (\sec^2 x)$

$2 \sec x \tan x \sec x - 2 \tan x \sec^2 x$

$2 \times \frac{1}{\cos x} \times \frac{\sin x}{\cos x} \times \frac{1}{\cos x} - 2 \frac{\sin x}{\cos x} \times \frac{1}{\cos^2 x}$

$$\boxed{\frac{2 \sin x}{\cos^3 x} - \frac{2 \sin x}{\cos^3 x} = 0}$$

(17) $f(x) = \dfrac{\sin x \sec x}{1 + x\tan x}$

~~$u' = \sin x \sec x \tan x + \sec x \cos x$~~

$u' = \sin x \dfrac{1}{\cos x} \tan x$

$\tan^2 x \quad \cancel{+} \quad \dfrac{1}{\cos x} \cancel{\cos x}$

$\boxed{\cancel{u' = \tan^2 x}}$

$\sin x \dfrac{1}{\cos x} = \boxed{\tan x}$

$\dfrac{\tan x}{1 + x\tan x}$

$u' = \sec^2 x$

$v' = x(\sec^2 x) + \tan x(1)$

$v' = x\sec^2 x + \tan x$

$\dfrac{(1 + x\tan x)(\sec^2 x) - ((\tan x(x\sec^2 x + \tan x))}{(1 + x\tan x)^2}$

$\dfrac{\sec^2 x + x\sec^2 x\tan x - ((x\sec^2 x\tan x + \tan^2 x)}{(1 + x\tan x)^2}$

$\dfrac{\sec^2 x - \tan^2 x}{(1 + x\tan x)^2} = \boxed{\dfrac{1}{(1 + x\tan x)^2}}$

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(ii) $f(x) = \dfrac{(x^2+1)(\cot x)}{3 - \cos x \operatorname{cosec} x}$

$$\frac{x^2+1 \; (\cot x)}{3 - \cos x \times \frac{1}{\sin x}}$$

$$\frac{(x^2+1)\{\cot x\}}{(3 - \cot x)}$$

~~$u' = 2x \cot x$~~

~~$u' = 2(x^2+1)(\operatorname{cosec} x$~~

$u' = (x^2+1)(-\operatorname{cosec}^2 x) + \cot x (2x)$

$-x^2 \operatorname{cosec}^2 x - \operatorname{cosec}^2 x \quad + 2x \cot x$

$\operatorname{cosec}^2 x(-x^2 - 1)$

$v' = \operatorname{cosec}^2 x$

$$\frac{dy}{dx} = \frac{(3-\cot x)(2x\cot x - \operatorname{cosec}^2 x(x^2+1)) - (x^2+1)\cot x \operatorname{cosec}^2 x)}{(3-\cos x)^2}$$

$6x\cot x - 2x\cot^2 x - 3x^2 + 3\operatorname{cosec}^2 x + \cancel{\cot x \operatorname{cosec}^2 x} -$ x^2+1

$x^2 + 1 \cot x \operatorname{cosec}^2 x$

$$\boxed{\frac{6x\cot x - 2x\cot^2 x - 3(x^2+1)\operatorname{cosec}^2 x}{(3-\cos x)^2}}$$

~~sin2x~~

$3\sin x \cancel{\cos x}$ ~~SEC~~

Ex 2.6 find $\frac{dy}{dx}$

(31) $y = \cos(\cos x)$

$-\sin x(\cos x) - \sin x = \sin x(\cos x)\sin x$

~~sin x (cos x) (sin x)~~

$\boxed{\sin^2 x(\cos x)}$

(34) $y = \sin(\tan 3x)$

$\cos(\tan 3x) \times (\sec^2 3x\,(3))$

$\cos(\tan 3x) \times 3\sec^2 3x$

~~$3\sec^2 x \cos$~~

$3\sec^2 3x \cos(\tan 3x)$

(33) $y = \cos^3(\sin 2x)$

$3\cos^2(\sin 2x) \times (\cos 2x\,(2))$

$2\cos 2x)$

$\boxed{\cancel{6\cos 2x \cos^2(\sin 2x)}}$

$3\cos^2 x(\sin 2x)$ $(\ldots)(-\sin x(\sin 2x))$

$(2\sin 2x)$

$\boxed{-6(\sin x)(\sin 2x)\cos(\sin 2x)\cos^2(\sin 2x)}$

(34) $y = \dfrac{1 + \operatorname{cosec}(x^2)}{1 - \cot(x^2)}$

$u' = -\cot x \operatorname{cosec} x\,(x^2)\,(2x)$

$-2x \cot x \operatorname{cosec} x\,(x^2)$

$v' = \operatorname{cosec}^2(x^2)\,(2x)$

$2x \operatorname{cosec}^2(x^2)$

$\cancel{\dfrac{dy}{dx} = 1 - \cot(x^2)\left(-2x\cot x \operatorname{cosec} x(x^2)\right)}$

$\dfrac{dy}{dx}$:

$$\frac{(1-\cot(x^2))(-2x\cot x \operatorname{cosec} x(x^2)) - (1+\operatorname{cosec}(x^2))(2x\operatorname{cosec}^2(x^2))}{(1-\cot(x^2))^2}$$

(35)

$$y = (5x+8)^7 (1-\sqrt{x})^6$$

$$u' = 7(5x+8)^6 (5)$$

$$\boxed{35(5x+8)^6}$$

$$v' = \cancel{6}^{3}(1-\sqrt{x})^5 \left(\frac{-1}{\cancel{2}\sqrt{x}}\right)$$

$$\boxed{\frac{-3(1-\sqrt{x})^5}{\sqrt{x}}}$$

$$(5x+8)^7 \left(\frac{-3(1-\sqrt{x})^5}{\sqrt{x}}\right) + (1-\sqrt{x})^6 \left(35(5x+8)^6\right)$$

$$\cancel{\frac{-3}{\sqrt{x}} (5x+8)^7 (1-\sqrt{x})^5 + 35(1-\sqrt{x})}$$

$$\cancel{\frac{3}{\sqrt{x}} (5x+8)^7}$$

$$\boxed{\frac{-3}{\sqrt{x}} (5x+8)^7 (1-\sqrt{x})^5 + 35(1-\sqrt{x})^6 (5x+8)^6}$$

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Q (36) $y = (x^2+x)^5 \sin^8 x$

$u' = 5(x^2+x)^4 (2x+1)$

$(10x+5)(x^2+x)^4$

$v' = 8 \sin^7 x (\cos x)$

$8 \cos x \sin^7 x$

$(x^2+x)^5 (8 \cos x \sin^7 x) + \sin^8 x ((10x+5)(x^2+x)^4)$

$(x^2+x)^5 (8\cos x \sin^7 x) + (10x+5)(x^2+x)^4 \sin^8 x$

$$\boxed{8 (x^2+x)^5 (\cos x \sin^7 x) + 5 (\sin^8 x)(x^2+x)^4 (2x+1}$$

Ex 3.1

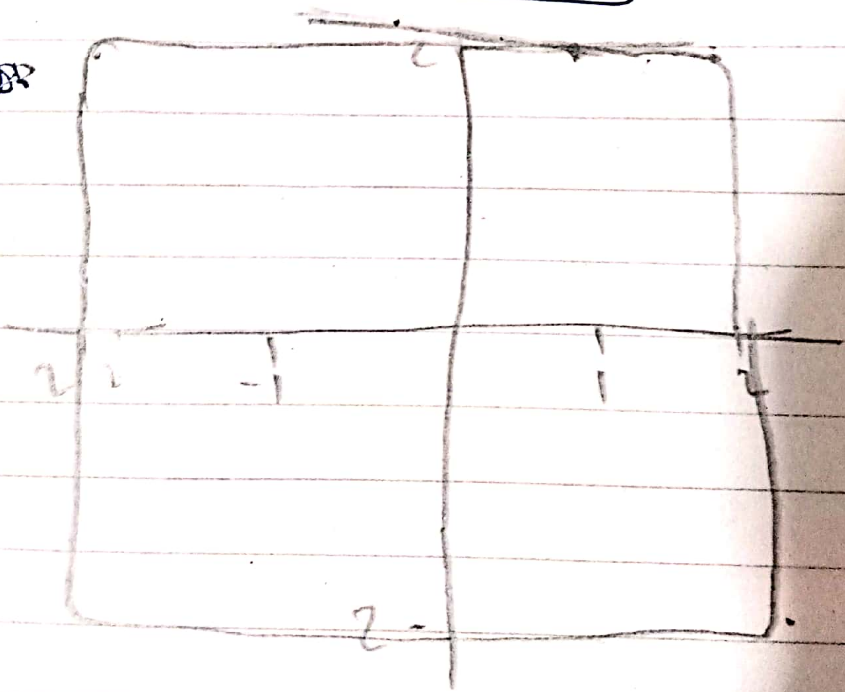
Q25 $x^4 + y^4 = 16$ $(1, \sqrt[4]{15})$

$4x^3 + 4y^3\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-\cancel{4}x^3}{\cancel{4}y^3}$ $\frac{dy}{dx} = -\frac{x^3}{y^3}$

$\frac{-(1)^3}{(\sqrt[4]{15})^3} = \frac{-1}{(15)^{\frac{3}{4}}}$

$= \boxed{-0.1312}$



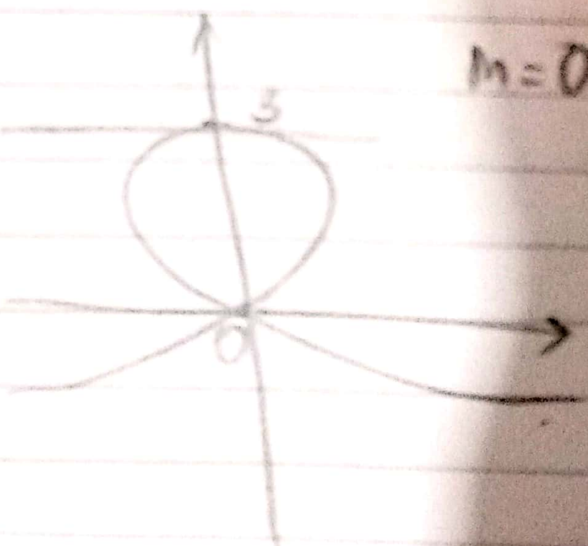
(2b) $y^3 + yx^2 + x^2 - 3y^2 = 0$ $(0, 3)$

$2y^2 \frac{dy}{dx} + \frac{dy}{dx} 2x + 2x - 6y \frac{dy}{dx} = 0$

$\cancel{\frac{dy}{dx} (2y^2 + 2x - 6y) + 2x = 0}$

$\frac{dy}{dx} (2y^2 + 2x - 6y) + 2x = 0$

$\frac{dy}{dx} = \frac{-2x}{2y^2 + 2x - 6y} = \frac{0}{\cancel{18} + 0 - \cancel{18}} = \boxed{\frac{0}{0}}$



$m = 0$

(27) $2(x^2+y^2)^2 = 25(x^2-y^2)$; $(3,1)$

$$4(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = 25\left(2x-2y\frac{dy}{dx}\right)$$

$$(4x^2+4y^2)\left(2x+2y\frac{dy}{dx}\right) = 50x - 50y\frac{dy}{dx}$$

$$8x^3+8x^2y\frac{dy}{dx}+8xy^2+8y^3\frac{dy}{dx} \quad -50x+50y\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(8x^2y+8y^3+50y\right)+8x^3+8xy^2-50x$$

$$\frac{dy}{dx} = \frac{50x-8x^3-8xy^2}{8x^2y+8y^3+50y}$$

$$\frac{50(3)-8(3)^3-8(3)(1)^2}{8(3)^2(1)+8(1)^3+50(1)} = \boxed{-\frac{9}{13}}$$

(8) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ $\quad (-1, 3\sqrt{3})$

$$\cancel{\frac{2}{3\sqrt[3]{x}}} + \cancel{\frac{2}{3y}}$$

$$\frac{2}{3\sqrt[3]{x}} + \frac{2}{3\sqrt[3]{y}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\cancel{2}}{3\sqrt[3]{x}} \times \frac{3\sqrt[3]{y}}{\cancel{2}}$$

$$\frac{3\sqrt[3]{y}}{3\sqrt[3]{x}} = -\frac{\cancel{3}\sqrt[3]{3\sqrt{3}}}{\cancel{3}\sqrt[3]{-1}} = -$$

$$\boxed{1.73} = \boxed{\sqrt{3}}$$