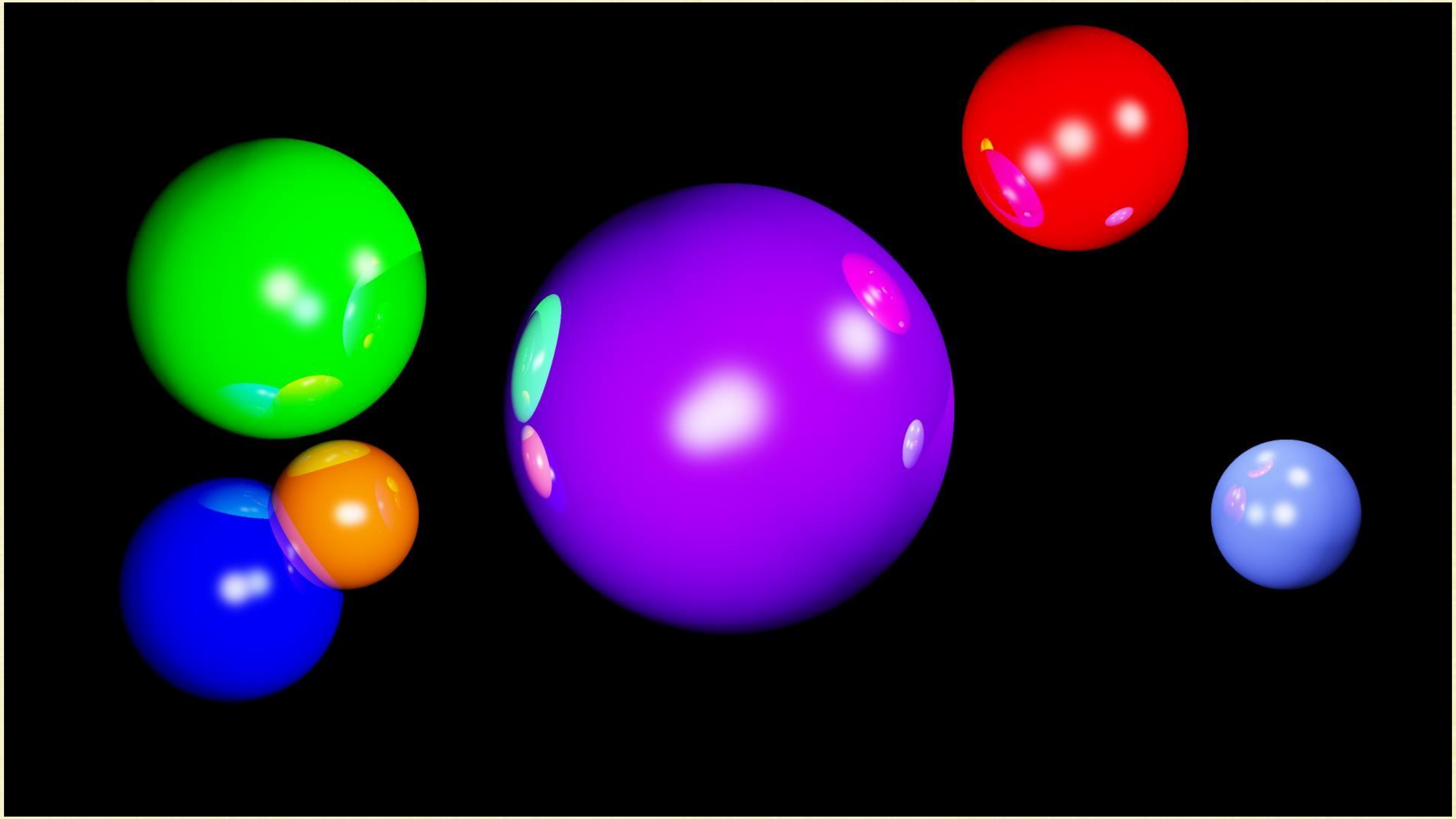
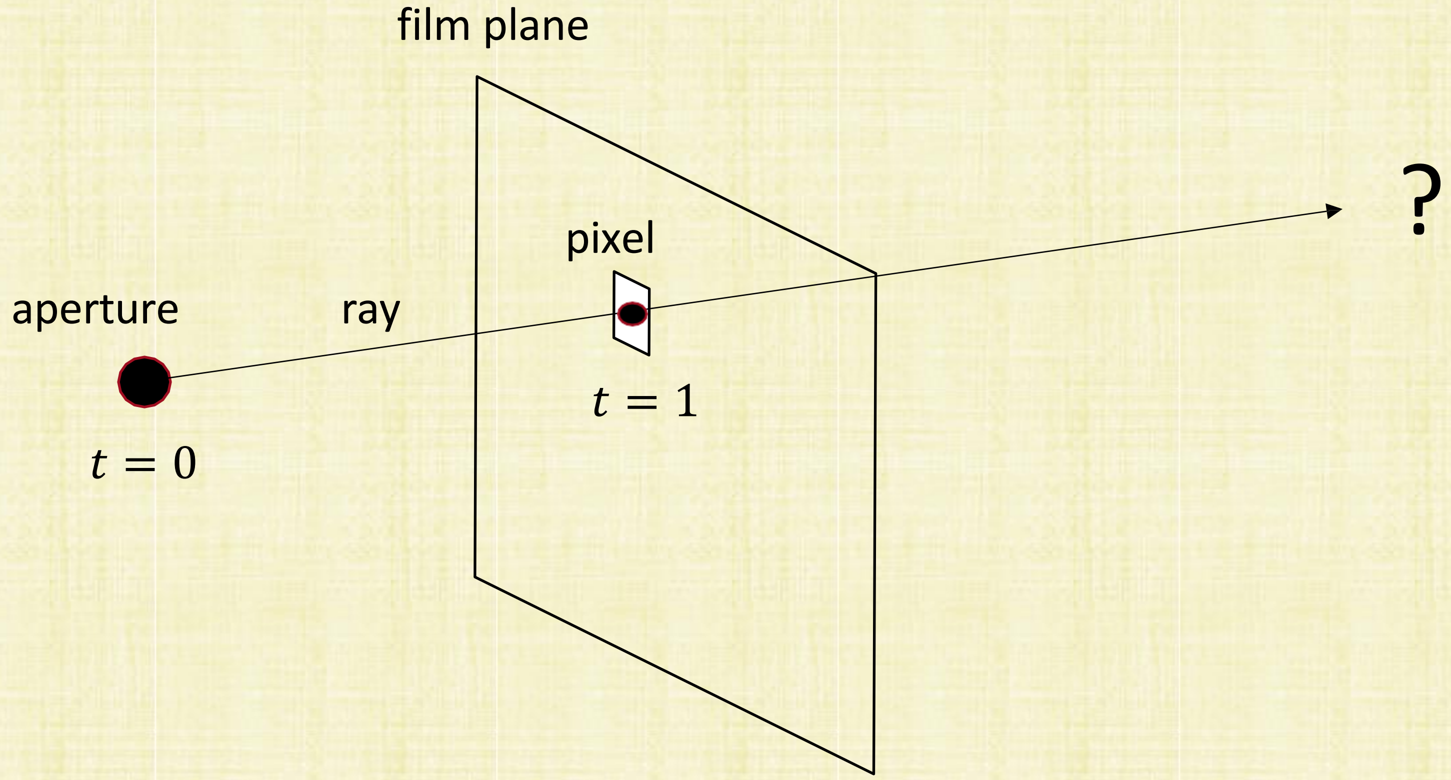
Ray Tracing



Constructing Rays

- For each pixel, create a ray and intersect it with objects in the scene
  - The first intersection is used to determine a color for the pixel
- The ray is $R(t) = A + (P - A)t$ where A is the aperture and P is the pixel location
- The ray is defined by $t \in [0, \infty)$, although only $t \in [1, t_{far}]$ is inside the viewing frustum
  - We only care about the first intersection with $t \geq 1$



Ray-Triangle Intersection

- Given the enormous number of triangles, many approaches have been implemented and tested in various software/hardware settings:

- Option 1: Triangles are contained in planes, so consider the Ray-Plane intersection first
  - A Ray-Plane intersection yields a point, and a subsequent test determines if that point lies inside the triangle
  - Option 1A: Both the triangle and the point can be projected into 2D; then, the 2D triangle rasterization test can be used to determine "inside"
    - Can project into the xy, xz, yz plane by just dropping the z, y, x coordinate (respectively) from the triangle vertices and the point
    - Most robust to drop the coordinate with the largest component in the triangle's normal (so that the projected triangle has maximal area)
  - Option 1B: There is a fully 3D version of the 2D rasterization

- Option 2: Skip the Ray-Plane intersection and consider the Ray-Triangle intersection directly
  - This is similar to how ray tracing works for non-triangle geometry (ray tracers handle non-triangle geometry better than scanline rendering does)

Ray-Plane Intersection

- A plane is defined by a point p_o (that lies on it) and a normal direction N
- A point p is on the plane if $(p - p_o) \cdot N = 0$
- A ray $R(t) = A + (P - A)t$ intersects the plane when $(R(t) - p_o) \cdot N = 0$ for some $t \geq 0$
- That is, $(A + (P - A)t - p_o) \cdot N = 0$ or $(A - p_o) \cdot N + (P - A) \cdot Nt = 0$
- So, $t = \frac{(p_o - A) \cdot N}{(P - A) \cdot N}$

- As always, if $t \notin \left[1, t_{far}\right]$ or another intersection has a smaller t value, then this intersection is ignored

Notes:

- The length of N cancels (so it need not be unit length)
- A (non-unit length) triangle normal can be computed by taking the cross product of any two edges (as long as the triangle does not have zero area)
- Any triangle vertex can be used as a point on the plane

Direct Ray-Triangle Intersection (Option 2)

- Triangle Basis Vectors: $p = p_2 + \beta_0 u + \beta_1 v$ with $\beta_0, \beta_1 \in [0,1]$ and $\beta_0 + \beta_1 \leq 1$
- Points on the ray have $R(t) = A + (P - A)t$
- An intersection point has $A + (P - A)t = p_2 + \beta_0 u + \beta_1 v$
- Or $\begin{pmatrix} u & v & A - P \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ t \end{pmatrix} = A - p_2$ where $\begin{pmatrix} u & v & A - P \end{pmatrix}$ is a 3x3 matrix and $A - p_2$ is a 3x1 vector (3 equations with 3 unknowns)
- The 3x3 coefficient is degenerate when its columns are not full rank
  - That happens when the triangle has zero area or the ray direction, $P - A$, is perpendicular to the plane's normal
- Otherwise, there is a unique solution

- $R(t_{int})$ is inside the triangle, when that unique solution has: $\beta_0, \beta_1 \in [0,1]$ and $\beta_0 + \beta_1 \leq 1$

- As always, if $t \notin [1, t_{far}]$ or another intersection has a smaller t value, then this intersection is ignored

Ray-Object Intersections

- As long as a Ray-Geometry intersection routine can be written, ray tracing can be applied to any representation of object geometry
  - In contrast to scanline rendering, where objects need to be turned into triangles
- Besides triangle meshes, ray tracers use: analytic descriptions of geometry, implicitly defined surfaces, parametric surfaces, etc.

- The surfaces of many objects can be written as <u>implicit functions</u>
- That is, $f(p) = 0$ if and only if p is on the surface (e.g. the equation for a plane)
- Sometimes there are additional constraints (such as on the barycentric weights for triangles)

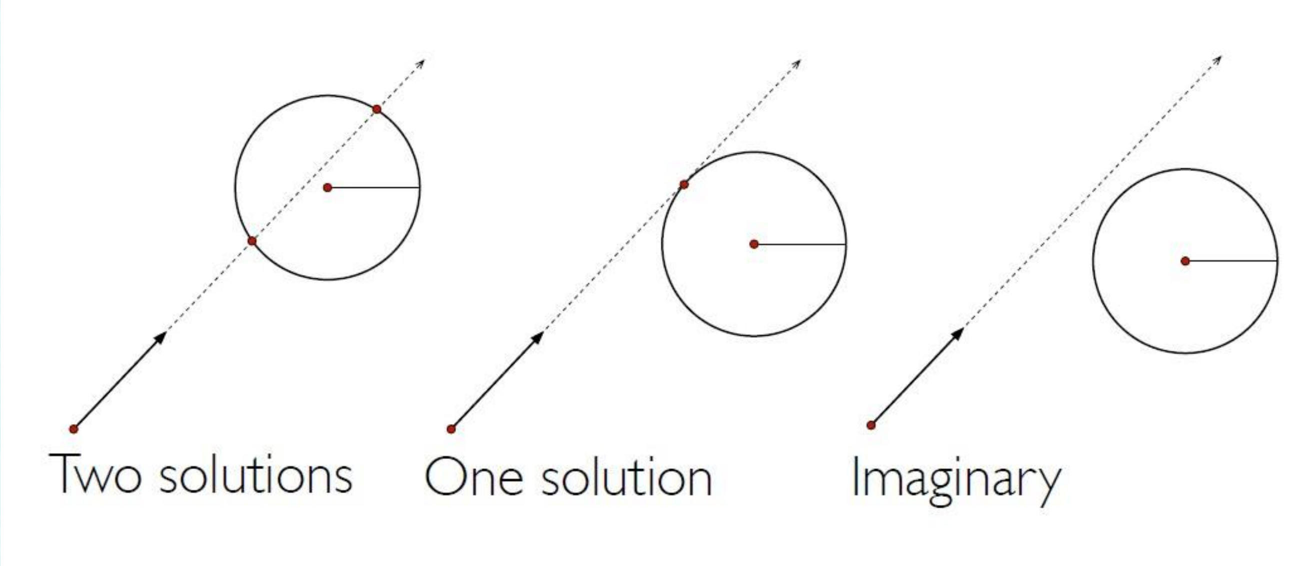
- Ray-Object intersection routines often proceed as follows:
  - substitute the ray equation in for the point, i.e. $f(R(t)) = 0$
  - solve for t
  - check the solution against any additional constraints

Ray-Sphere Intersections

- A point p is on a sphere with center C and radius r when $\|p - C\|_2 = r$
  - Squaring both sides: $(p - C) \cdot (p - C) = r^2$
- Substitute $R(t) = A + (P - A)t$ in for p to get a quadratic equation in t:

$$(P - A) \cdot (P - A)t^2 + 2(P - A) \cdot (A - C)t + (A - C) \cdot (A - C) - r^2 = 0$$

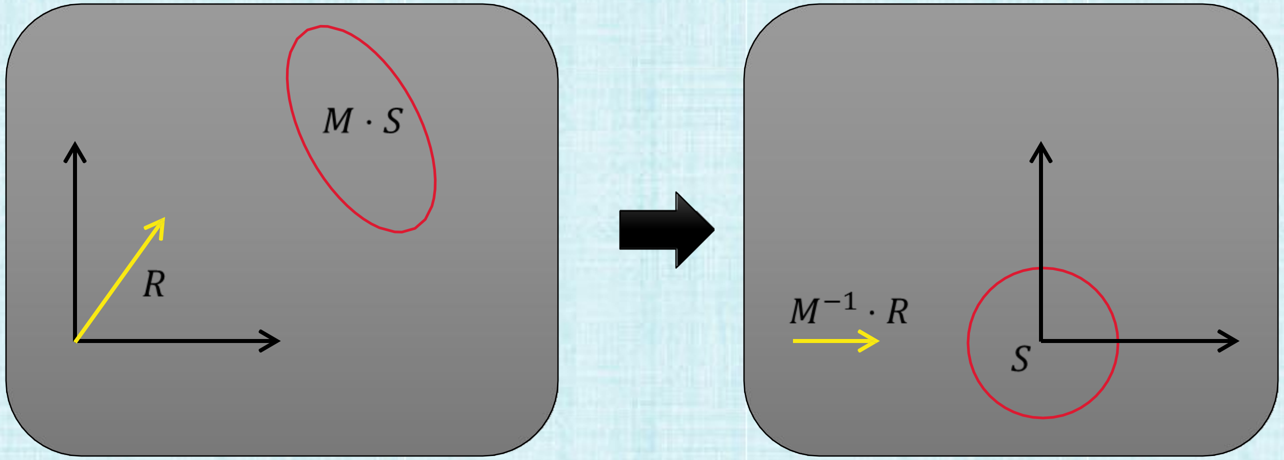
- When the discriminant of this quadratic equation is positive, there are two solutions (choose the one the ray hits first)
- When the discriminant is zero, there is one solution (the ray tangentially grazes the sphere)
- When the discriminant is negative, there are no solutions



Ray Tracing Transformed Objects

- Geometry is typically kept in a convenient object space
- The object space representation is often simpler to deal with
  - E.g., spheres can be centered at the origin, objects are not sheared, coordinates may be non-dimensionalized for numerical robustness, there may be (auxiliary) geometric acceleration structures, more convenient color and texture information, etc.

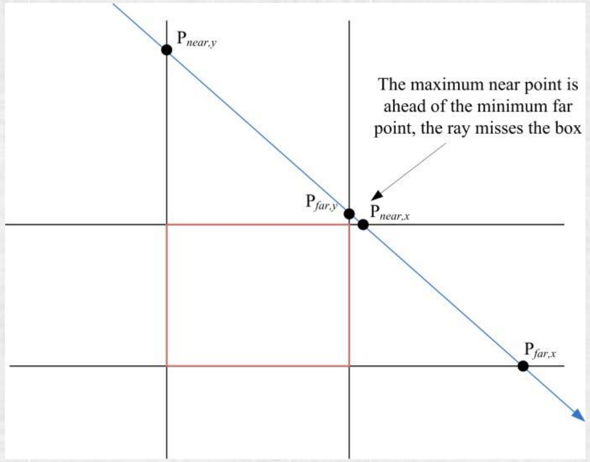
- It is typically preferable to ray trace in object space, rather than world space
- Transform the ray into object space and find the ray-object intersection, then transform the relevant information back to world space

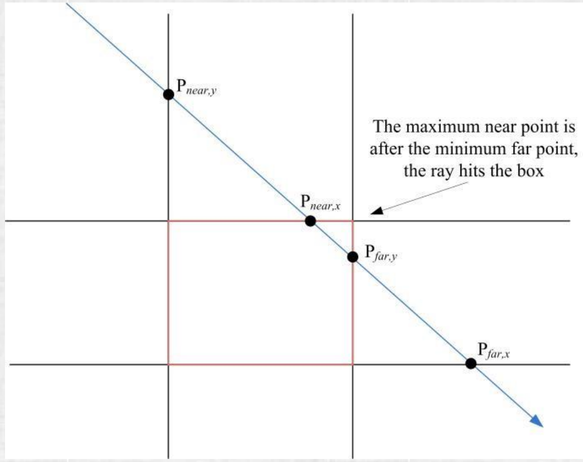


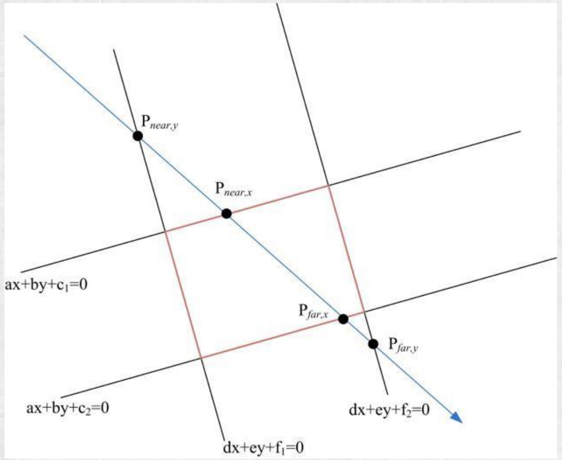
$$\boldsymbol{M \cdot R_{object}(t_{int})}$$

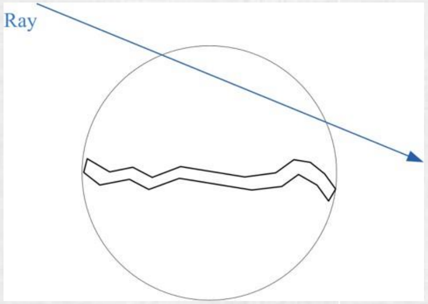
Aside: Code Acceleration

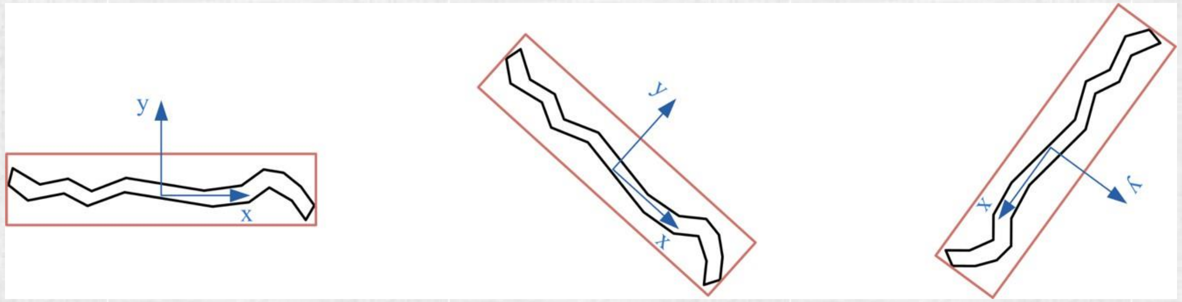
- Ray-Object intersections can be expensive
- So, put complex objects inside simpler objects, and first test for intersections against the simpler object (potentially skipping tests against the complex object)
- Simple <u>bounding volumes</u>: spheres, axis-aligned bounding boxes (AABB), or oriented bounding boxes (OBB)







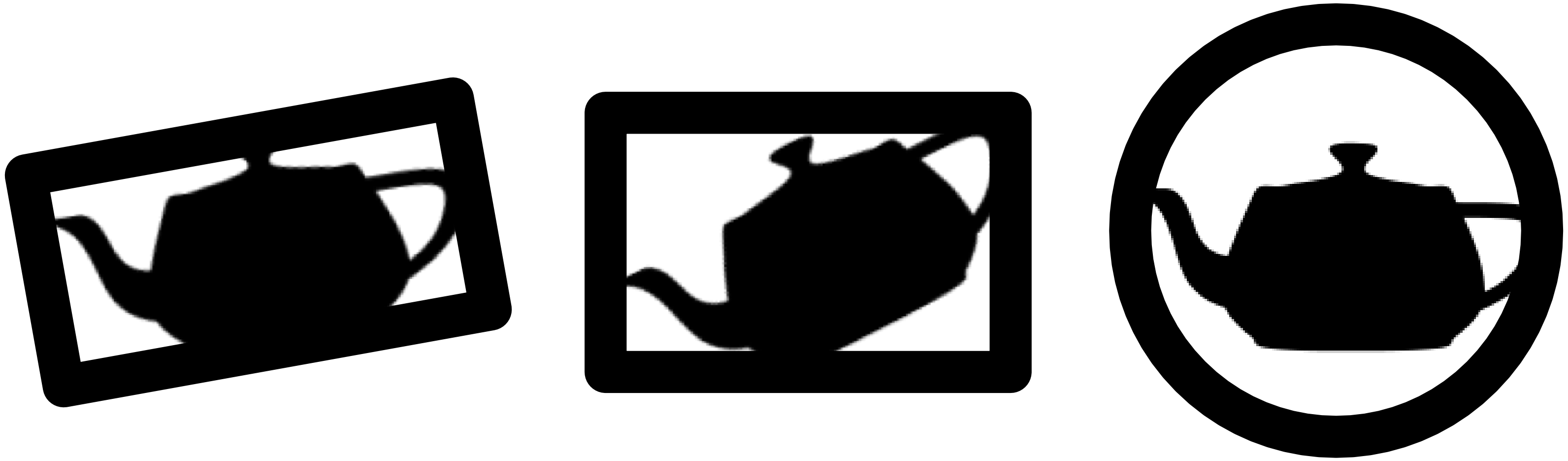




Bounding Volumes

Quick way to avoid intersections: bound complex object with a simple volume

- Object is fully contained in the volume
- If it doesn't hit the volume, it doesn't hit the object
- So test bvol first, then test object if it hits

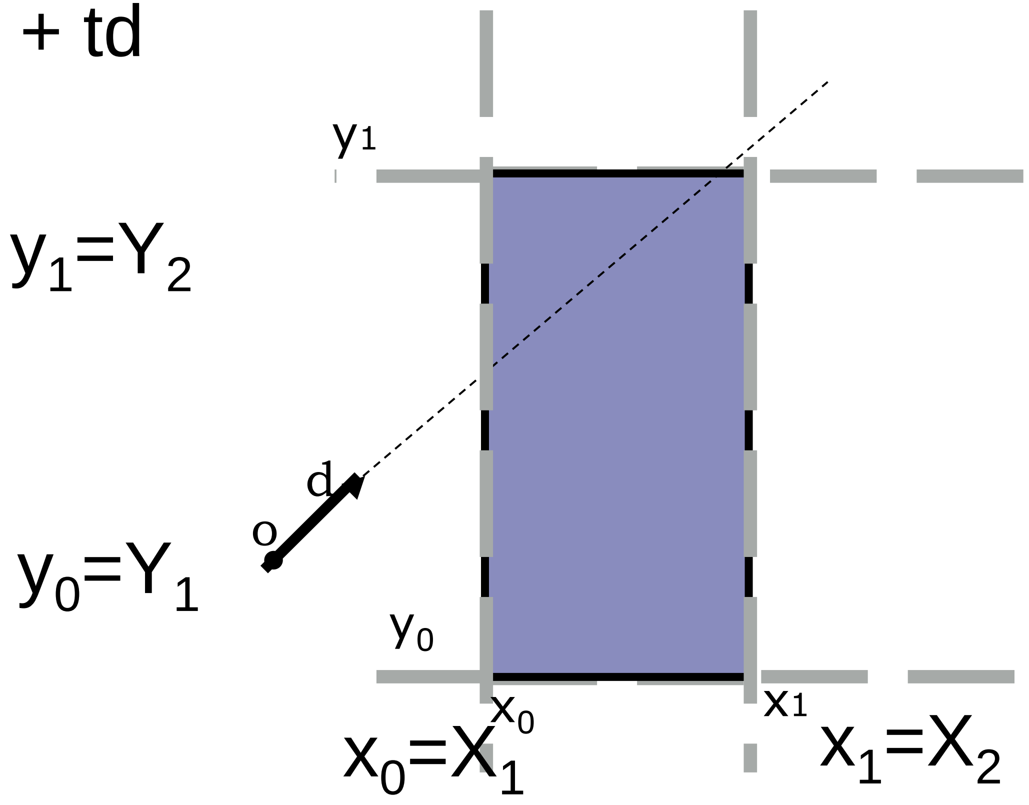


Axis aligned Box

Axis- aligned box

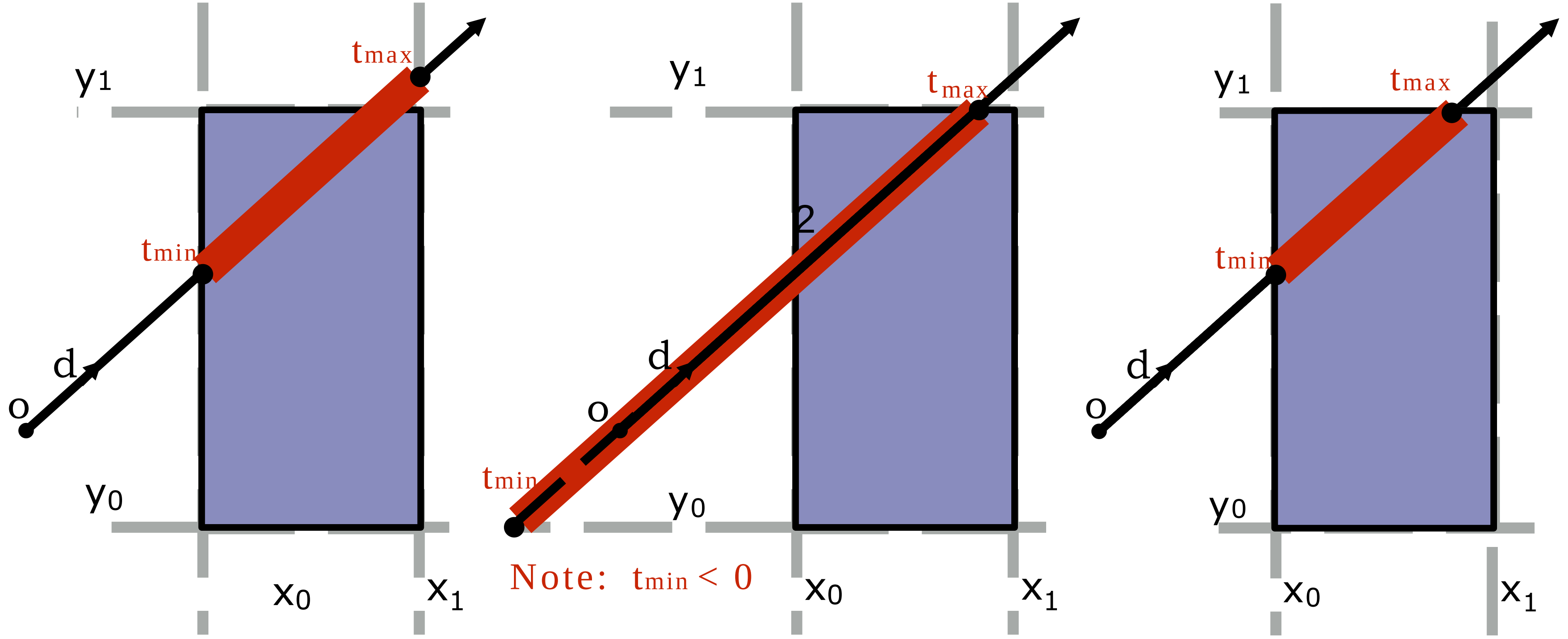
Box: $(X_1,Y_1,Z_1)\text{->}(X_2,Y_2,Z_2)$

Ray: $P(t)=o + td$



Ray Intersection with Axis-Aligned Box

2D example; 3D is the same! Compute intersections with slabs and take intersection of t_{min}/t_{max} intervals

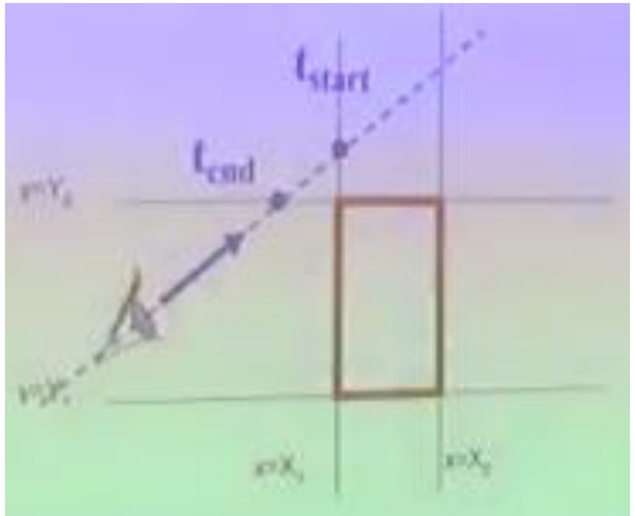


Intersections with *x* planes **Intersections with *y* planes** **intersection result**

Is there an intersection

1) If $t_{start} > t_{end}$ -> box is missed.
2) If $t_{end} < t_{min}$ -> box is behind.
3) If $t_{start} > t_{min}$ -> closest intersection at t_{start}

   Else -> closest intersection at t_{end}



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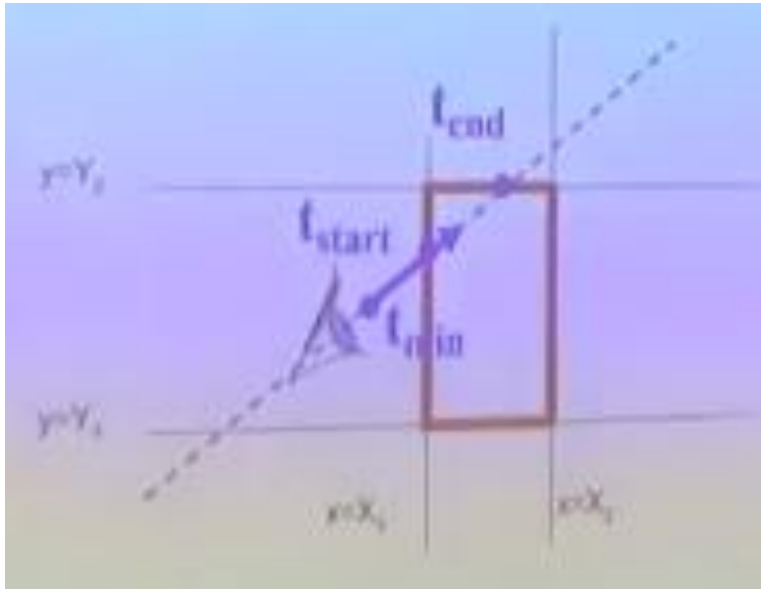

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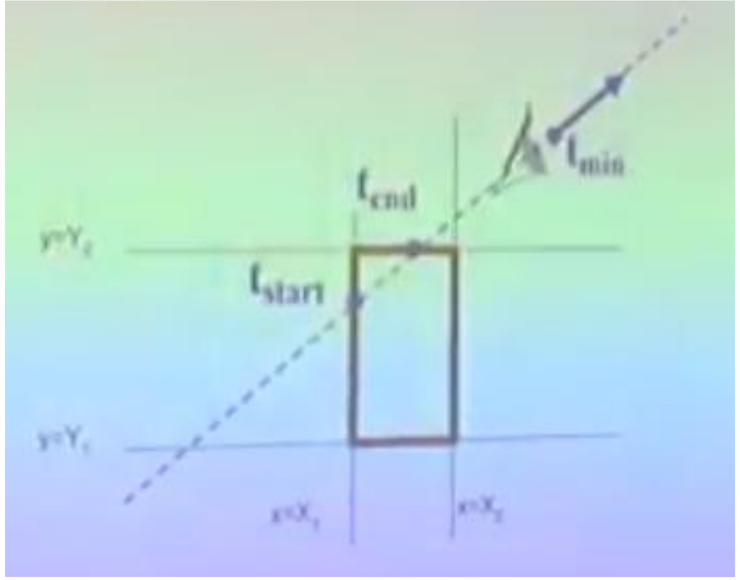
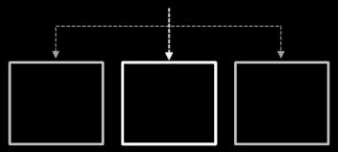
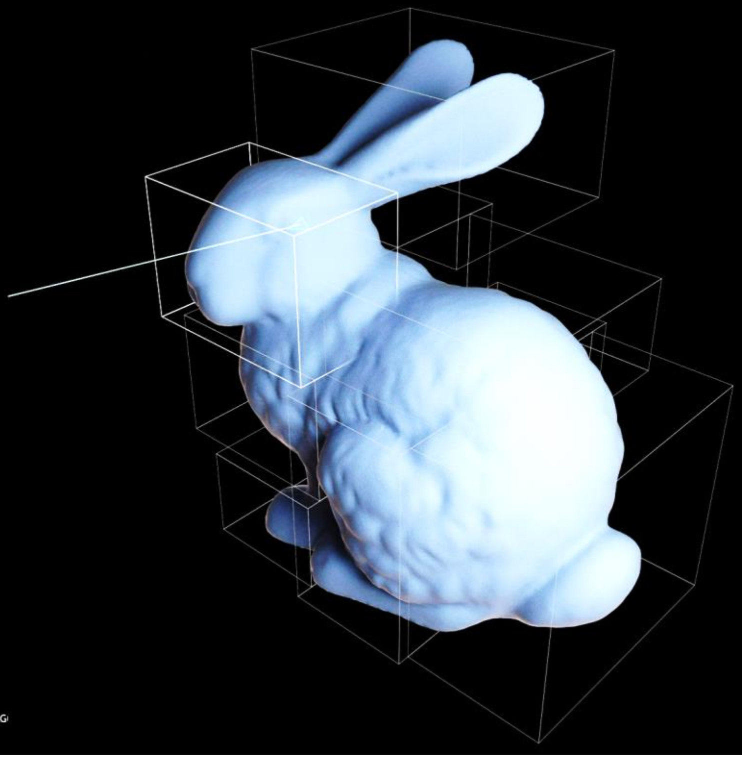


2

BVH ALGORITHM

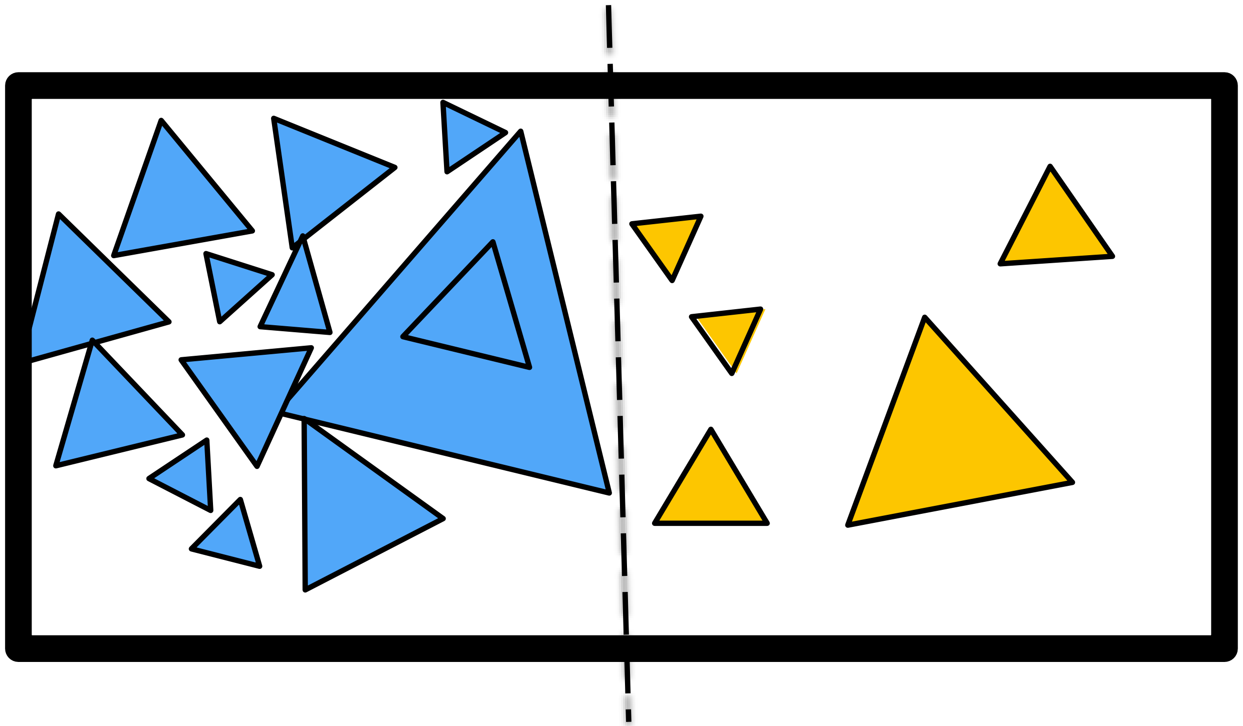
BVH = “Bounding Volume Hierarchy”





Bounding Volume Hierarchy (BVH)

- Find the bounding box of objects primitives.
- Split objects/primitives into two child BV's
- Recurse, build a binary tree.



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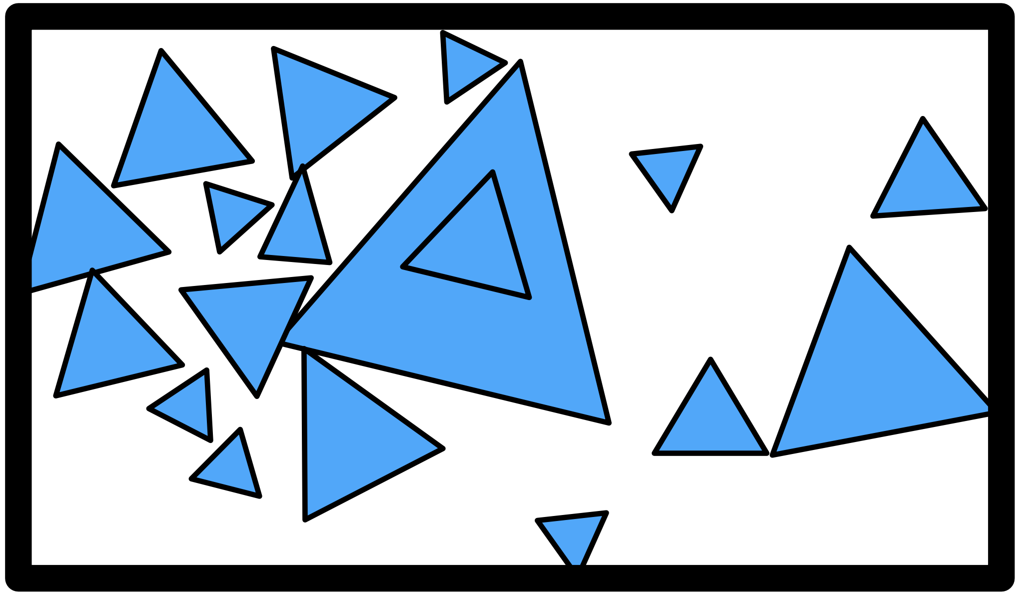
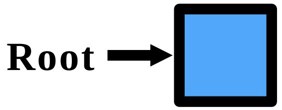
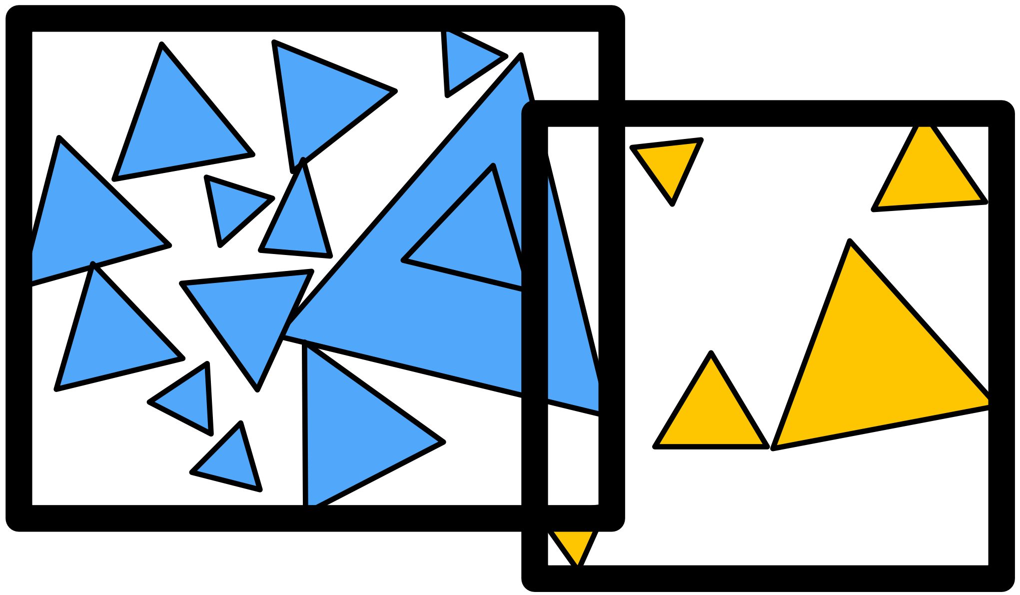
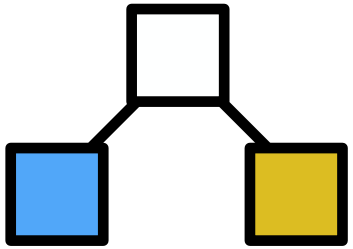




Bounding Volume Hierarchy (BVH)





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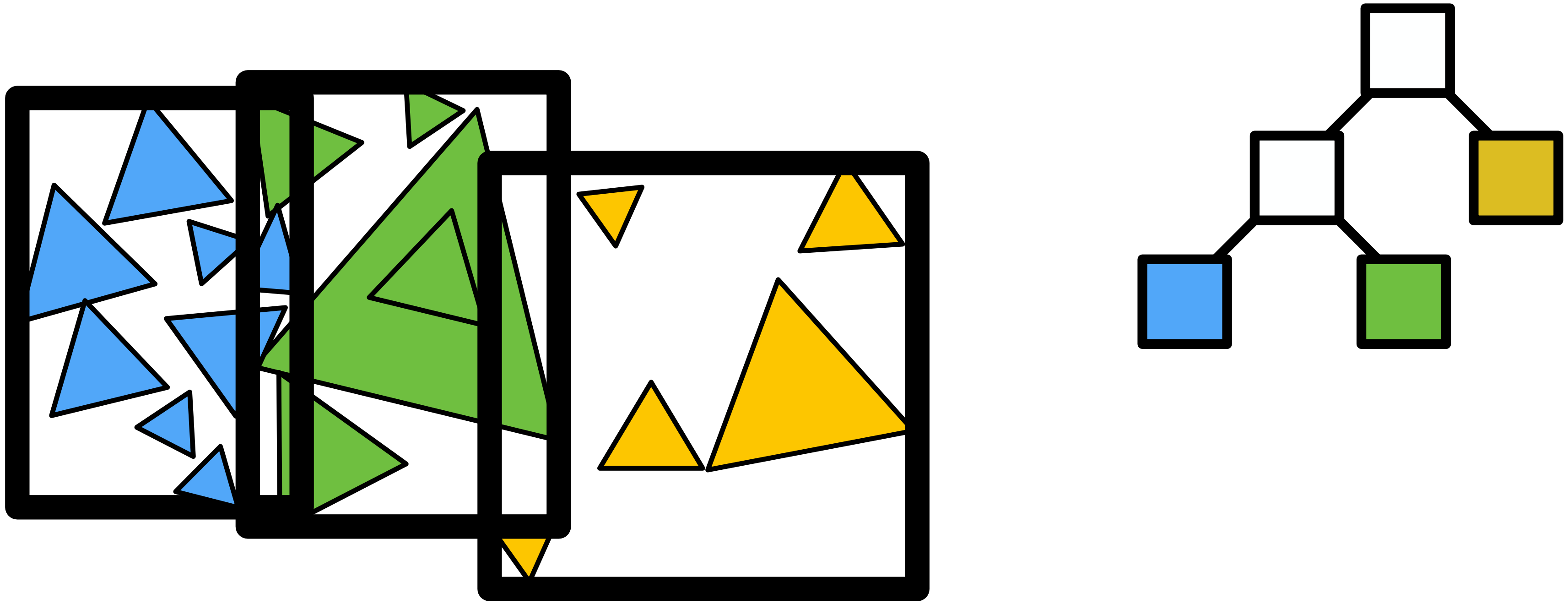
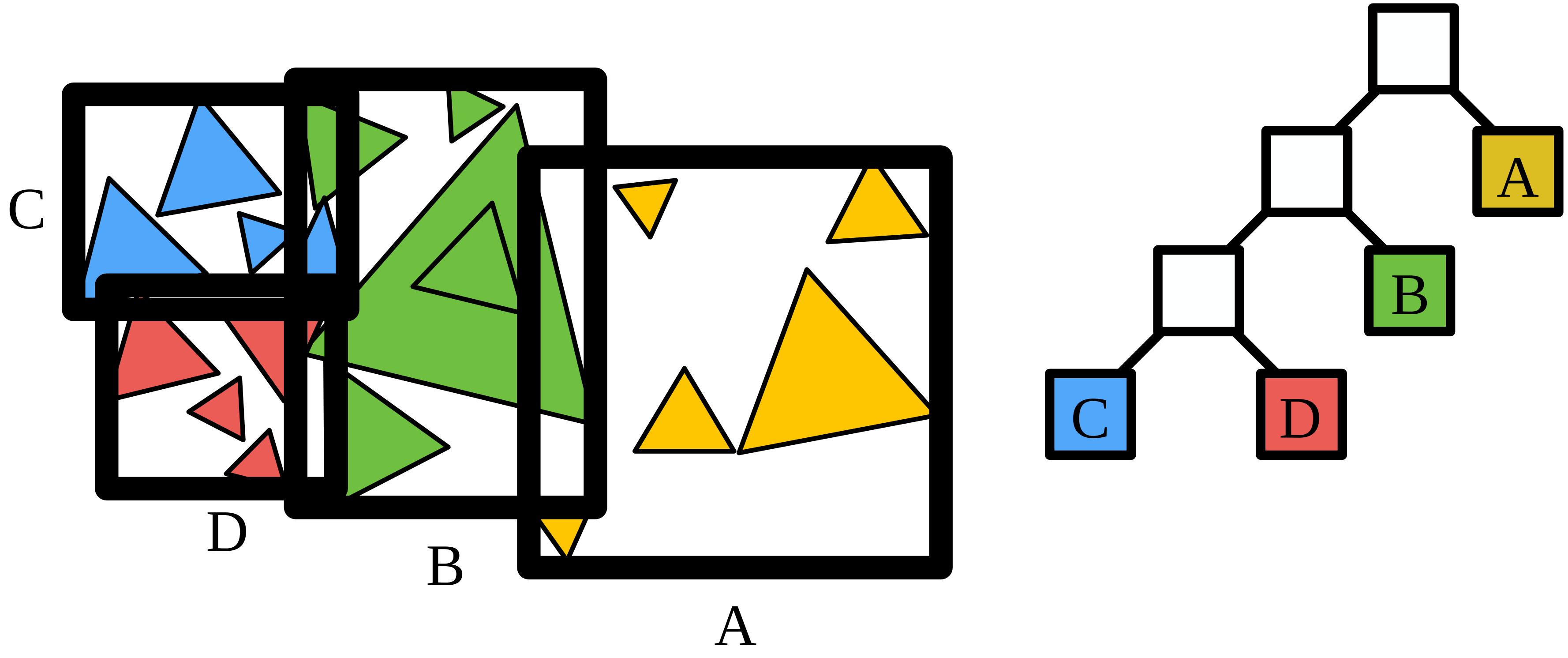
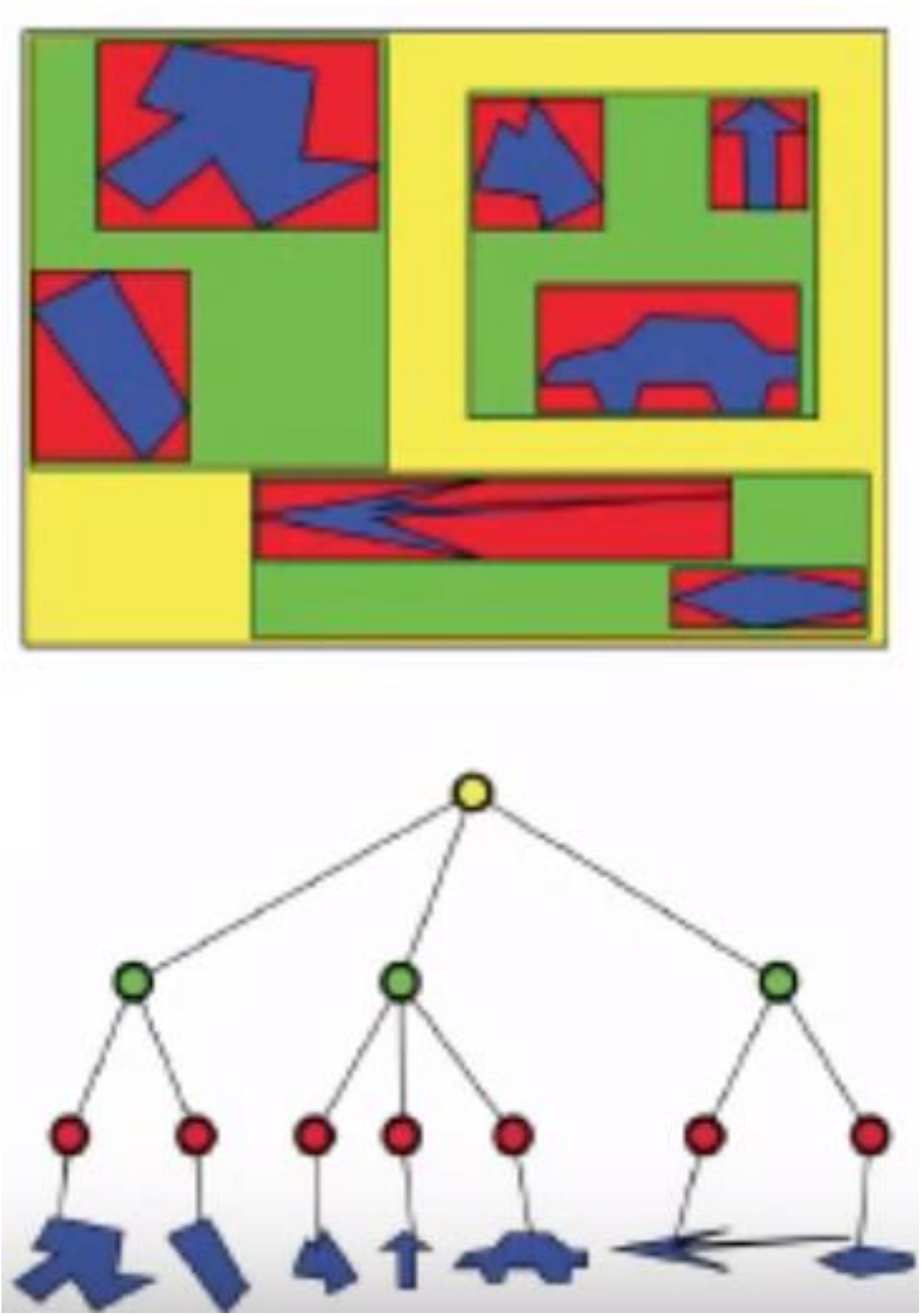


Bounding Volume Hierarchy (BVH)





Bounding Volume Hierarchy (BVH)

Internal nodes store

- Bounding box
- Children: reference to child nodes

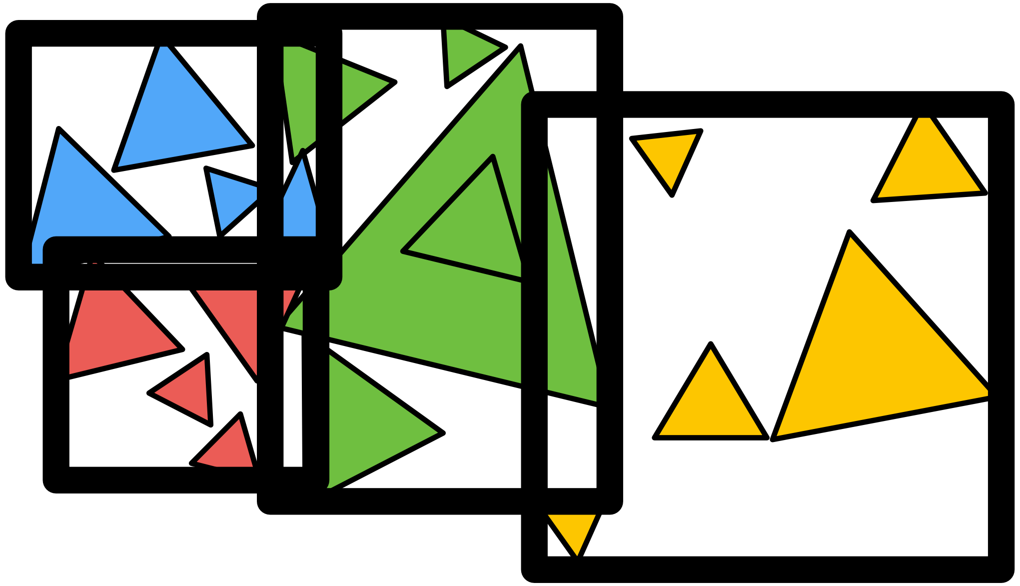
Leaf nodes store

- Bounding box
- List of objects

Nodes represent subset of primitives in scene

- All objects in subtree

BVH Pre-Processing



- Find bounding box
- Recursively split set of objects in two subsets
- Stop when there are just a few objects in each set
- Store obj reference(s) in each leaf node

BVH Pre-Processing

Choosing the set partition

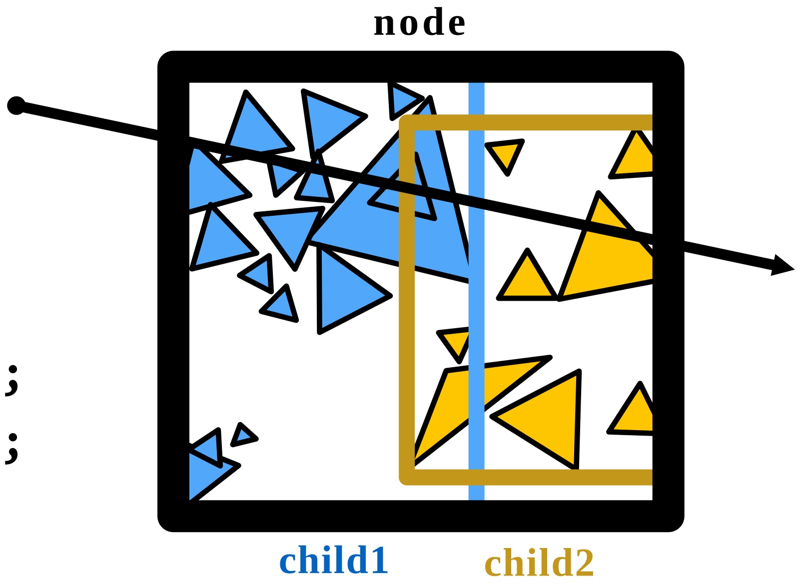
- Choose a dimension to split or optimize over x,y,z
- Simple #1: Split objects around midpoint
- Simple #2: Split at location of median object
- Ideal: split to minimize expected cost of ray intersection

Termination criteria?

- Simple: stop when node contains few elements (e.g. 5)
- Ideal: stop when splitting does not reduce expected cost of ray intersection

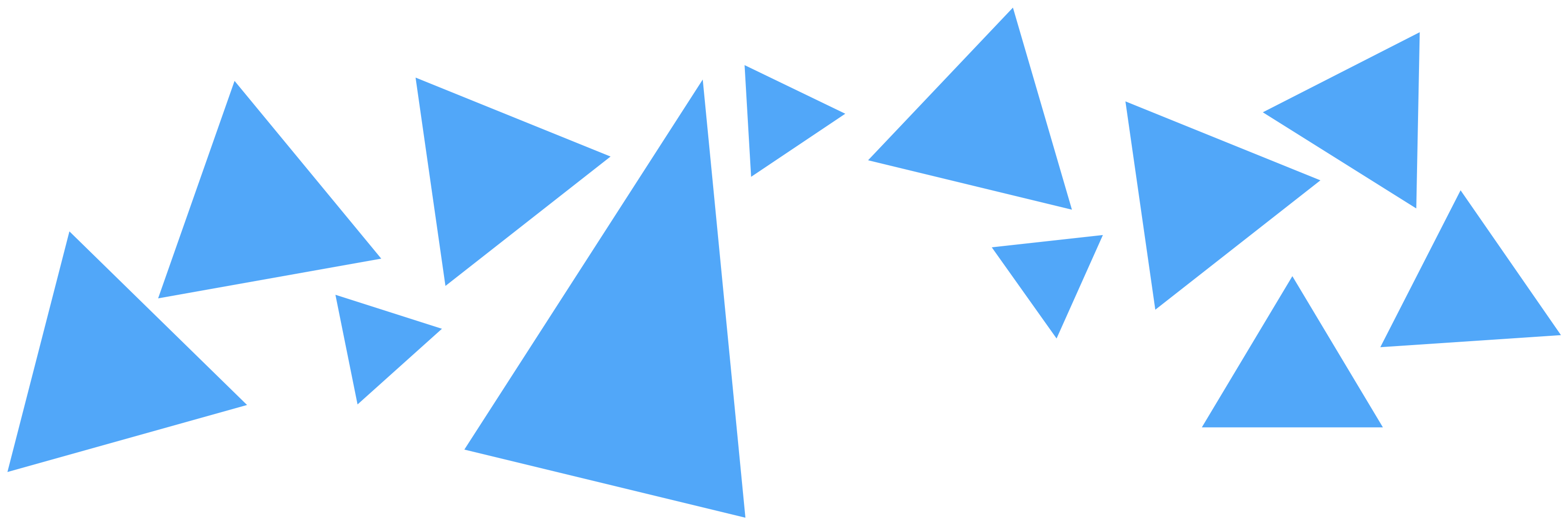
BVH Recursive Traversal

```
Intersect (Ray ray, BVH node)
  if (ray misses node.bbox) return;
  if (node is a leaf node)
     test intersection with all objs;
     return closest intersection;
  hit1 = Intersect (ray, node.child1);
  hit2 = Intersect (ray, node.child2);
  return closer of hit1, hit2;
```

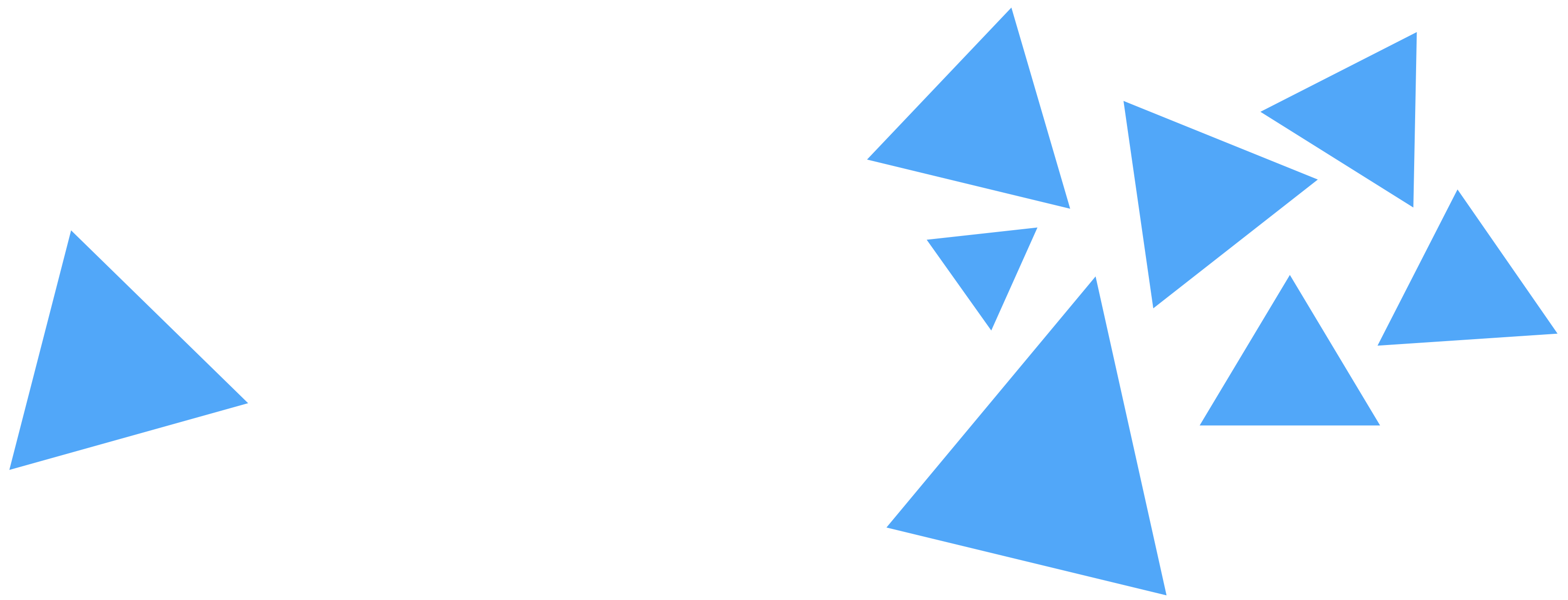


Optimizing Hierarchical Partitions
(How to Split?)

How to Split into Two Sets? (BVH)



How to Split into Two Sets? (BVH)



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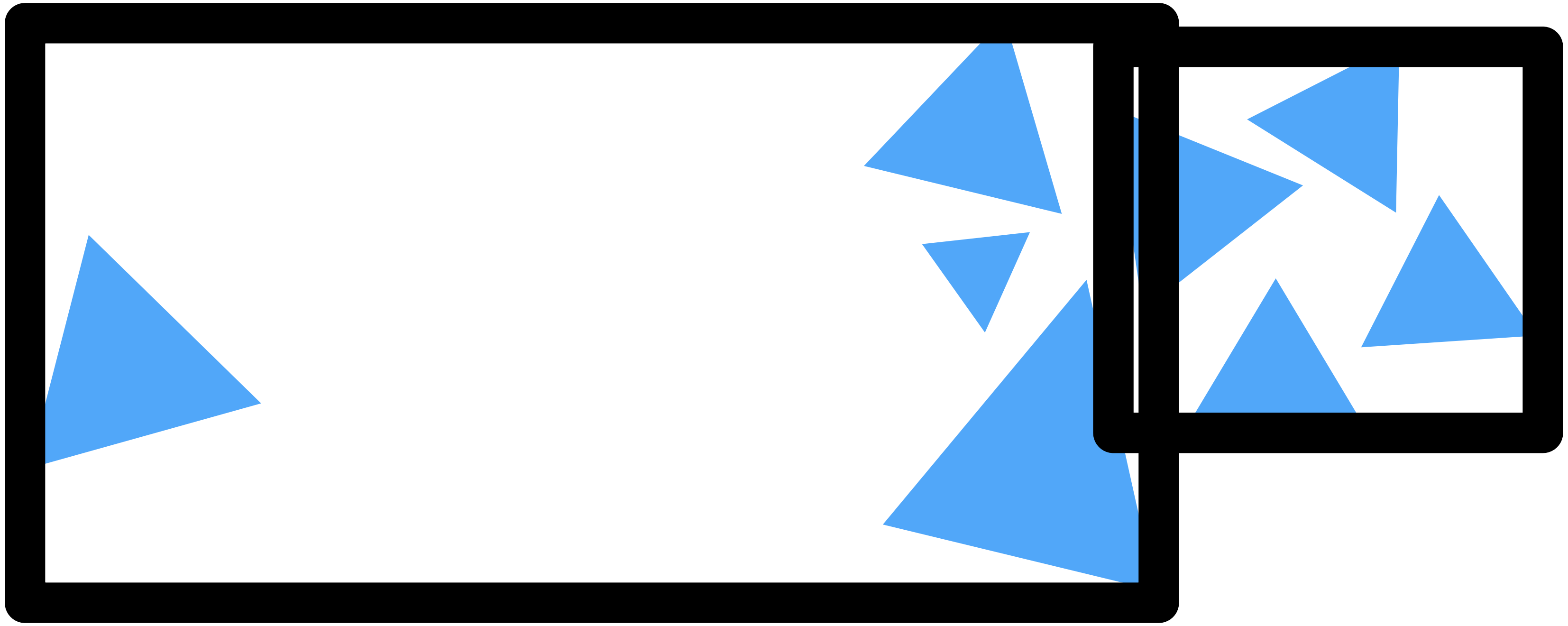
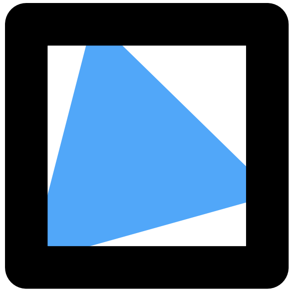
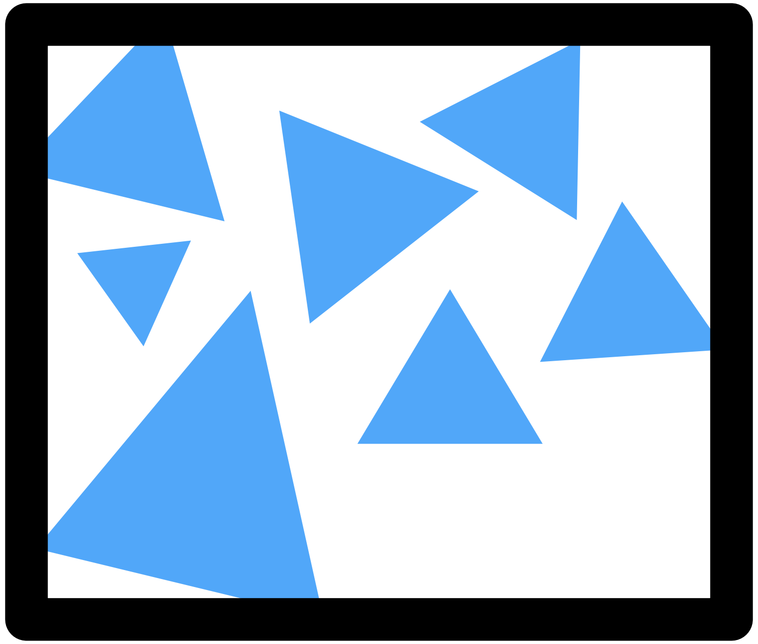


Split at median element

Child nodes have equal numbers of elements

How to Split into Two Sets? (BVH)



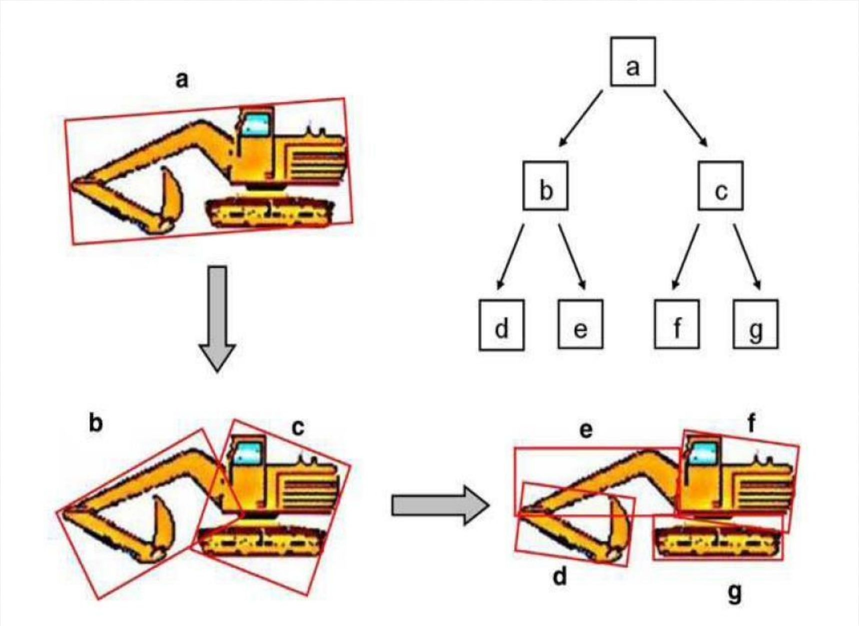


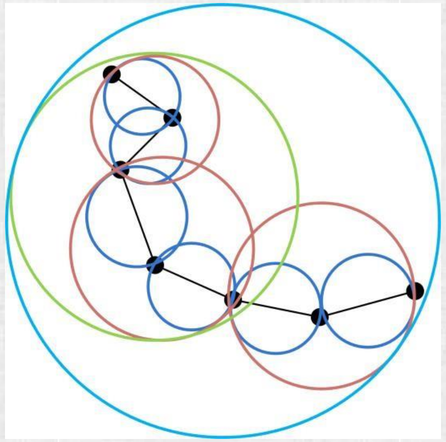
A better split?

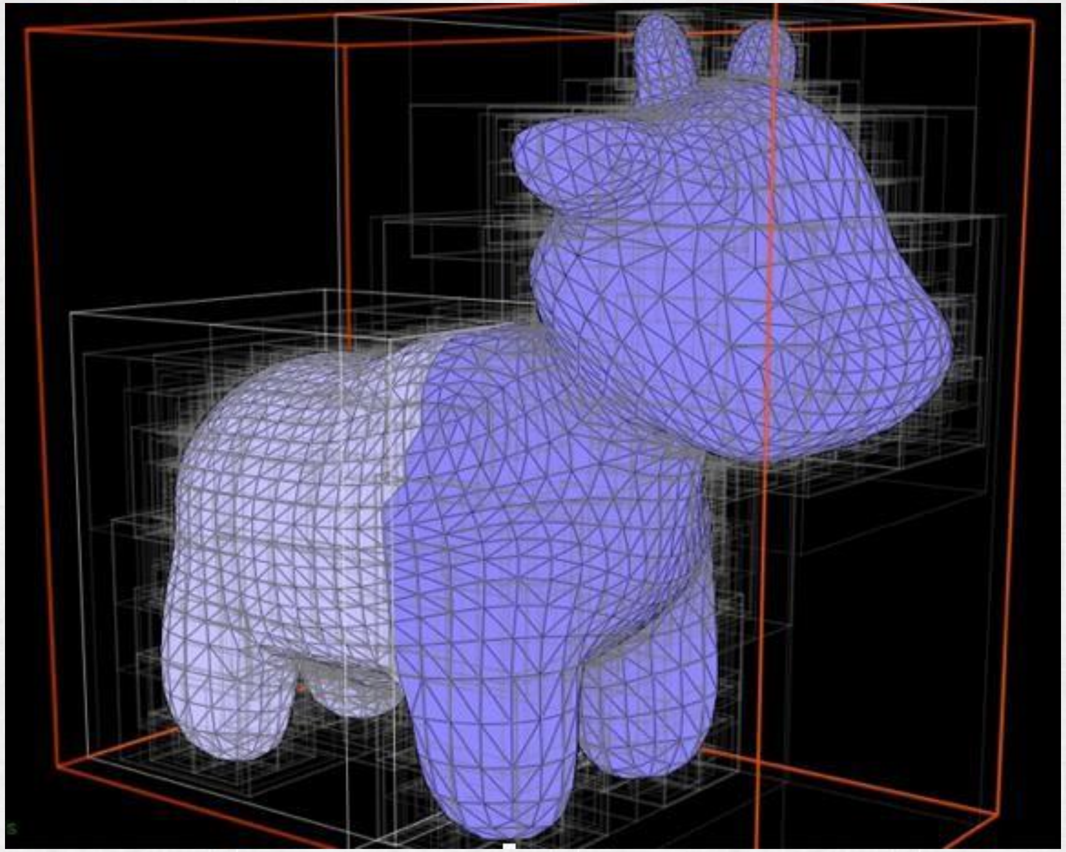
Smaller bounding boxes, avoid overlap and empty space

Aside: Code Acceleration

- For complex objects, build a <u>hierarchical tree structure</u> in object space
- The lowest levels of the tree contain the primitives used for intersections (and have simple geometry bounding them); then, these are combined hierarchically into a $\log n$ height tree
- Starting at the top of a Bounding Volume Hierarchy (BVH), one can prune out many nonessential (missed) ray-object collision checks

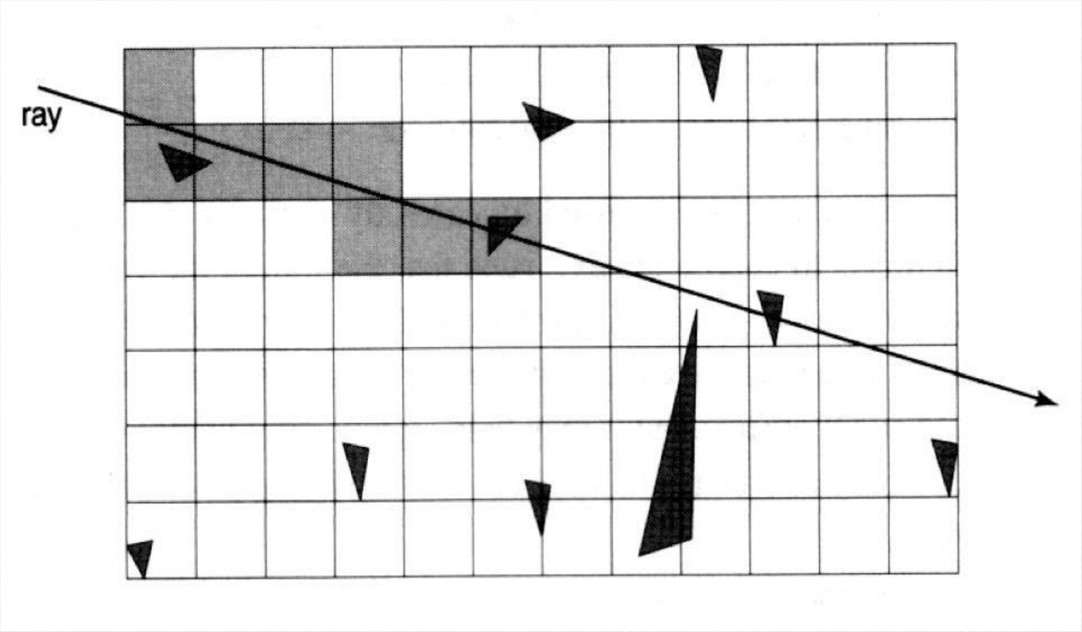


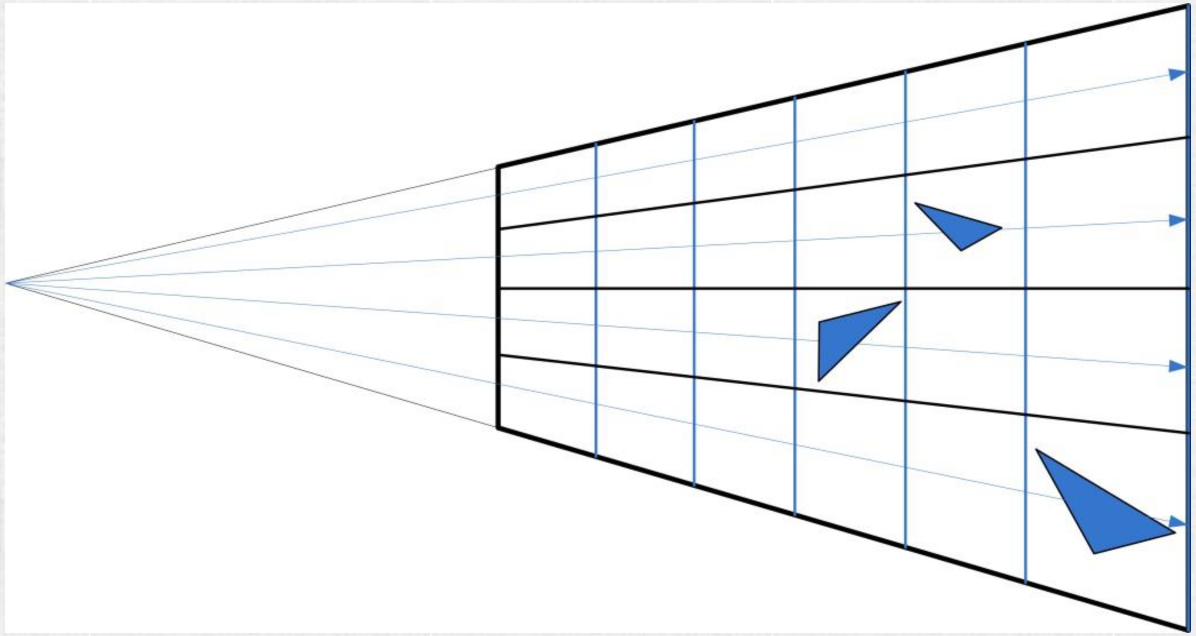




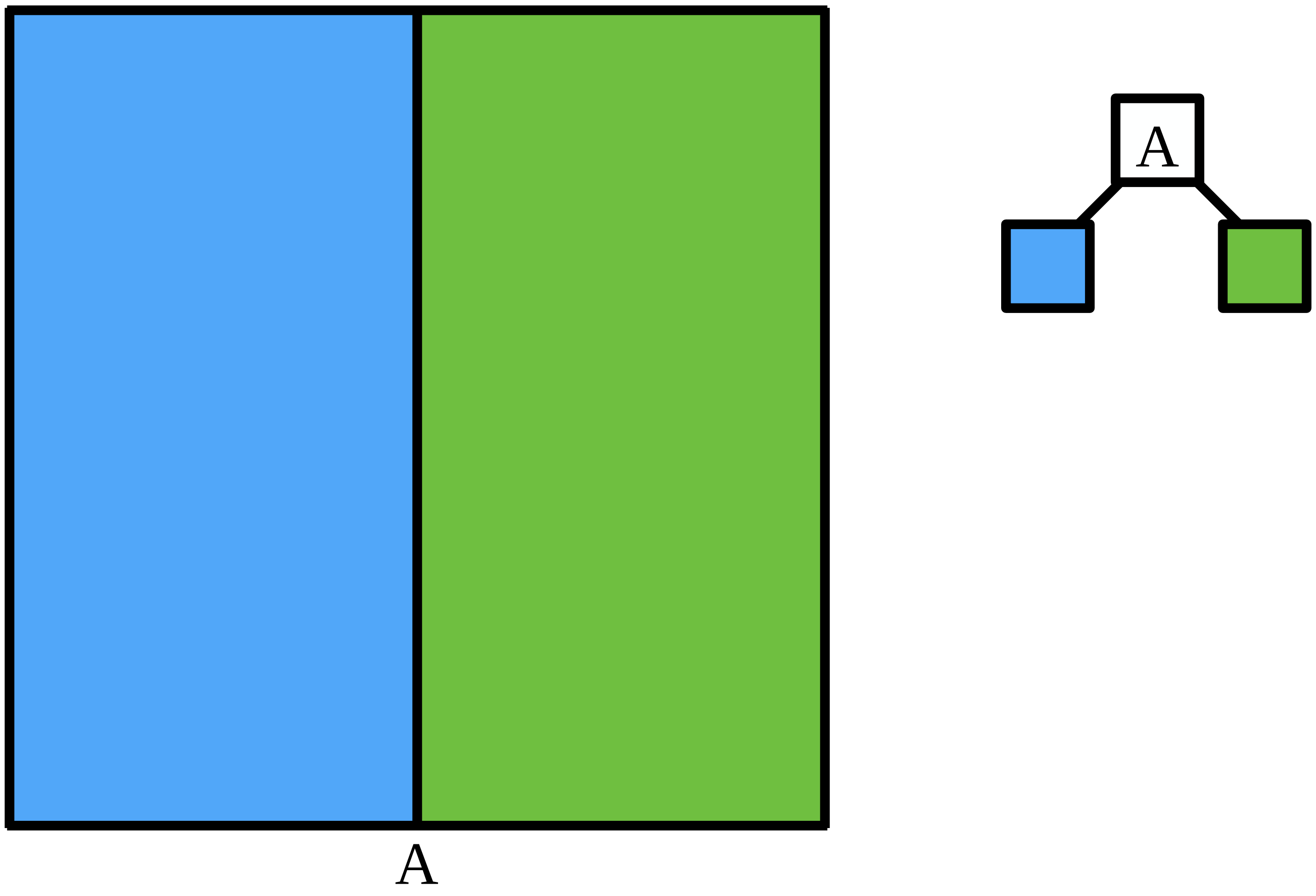
Aside: Code Acceleration

- When there are many objects in the scene, checking rays against all of their top level simple bounding volumes can become expensive
- Thus, world space bounding volume hierarchies, octrees, and K-D trees are used
- Also useful (but flat instead of hierarchical) are uniform spatial partitions (uniform grids) and viewing frustum partitions

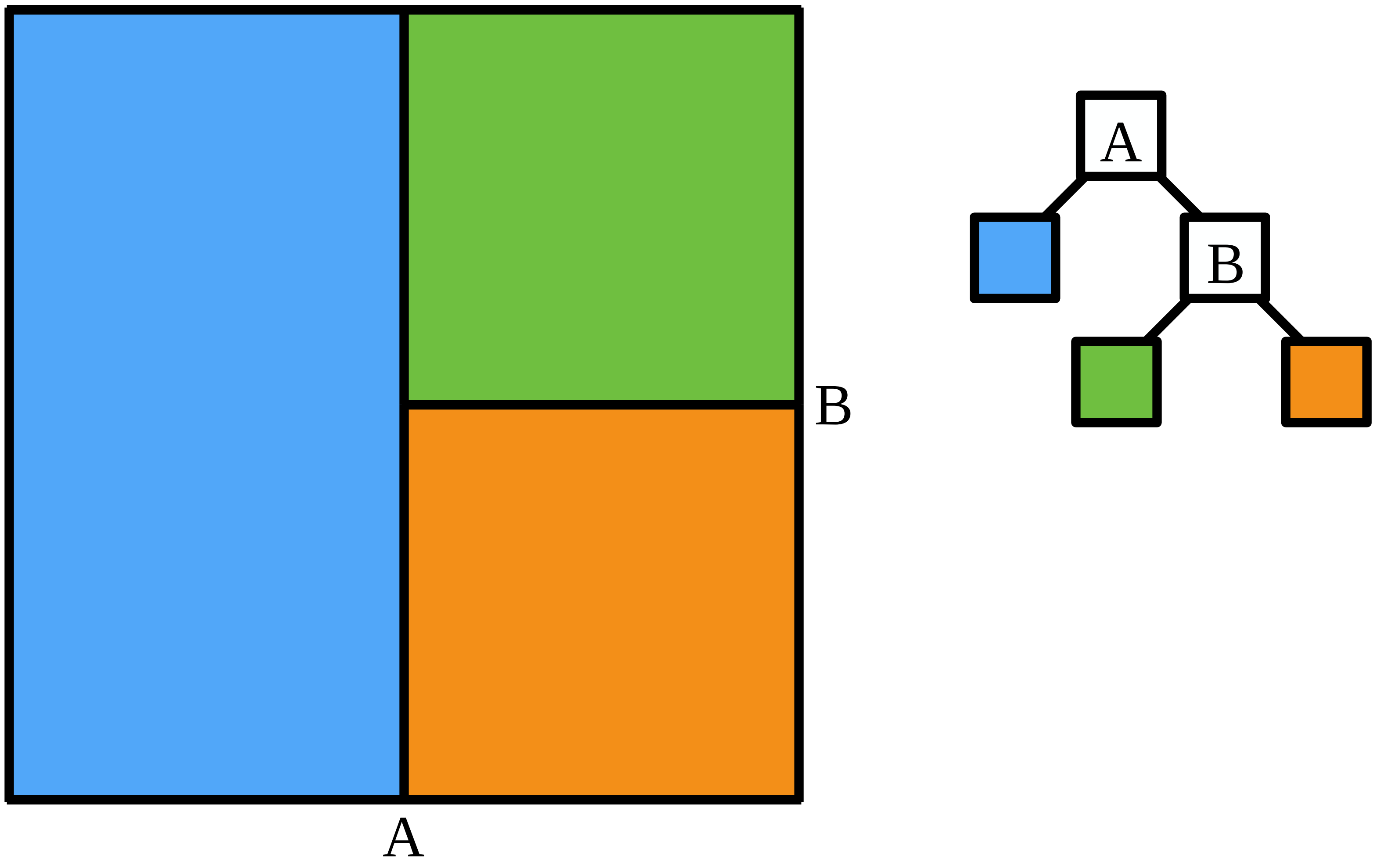




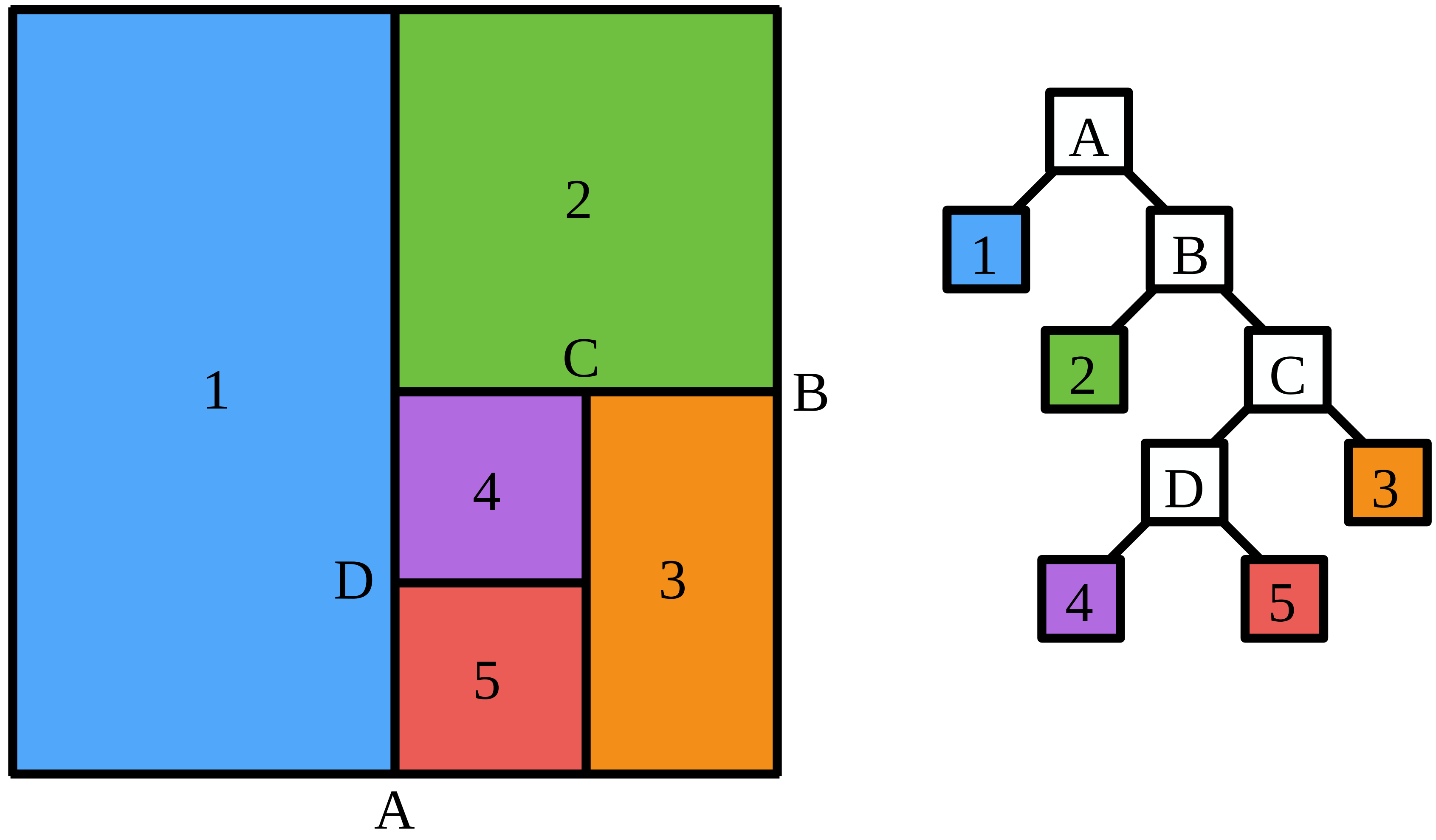
Spatial Hierarchies



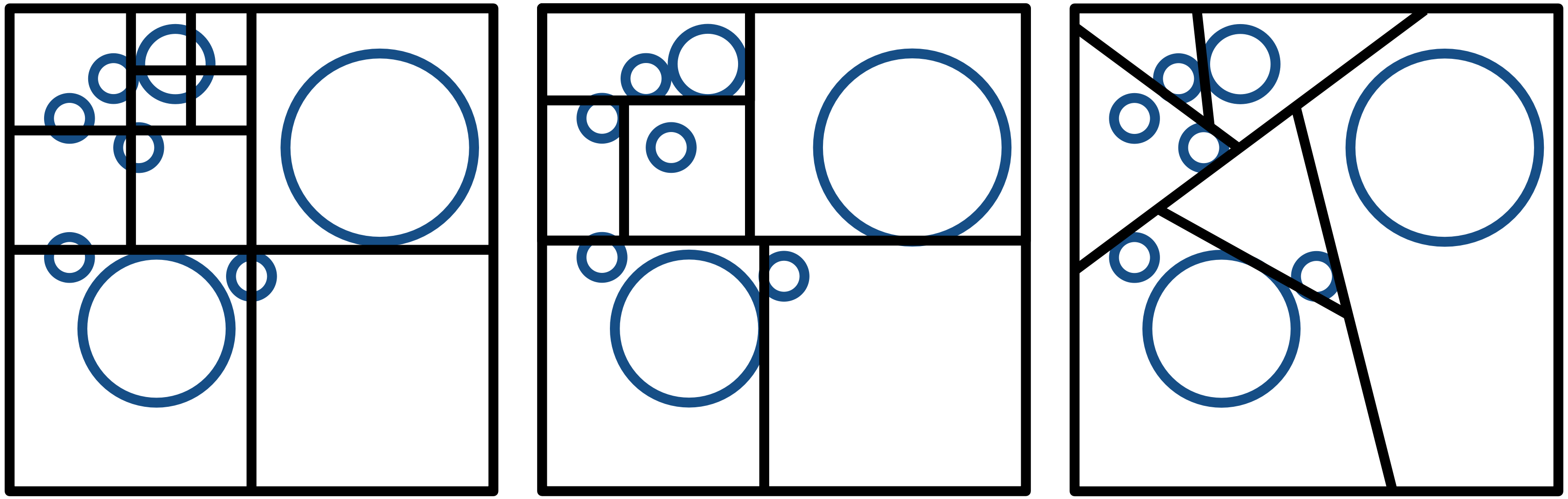
Spatial Hierarchies



Spatial Hierarchies



Spatial Partitioning Variants



Oct-Tree KD-Tree BSP-Tree

KD-Trees

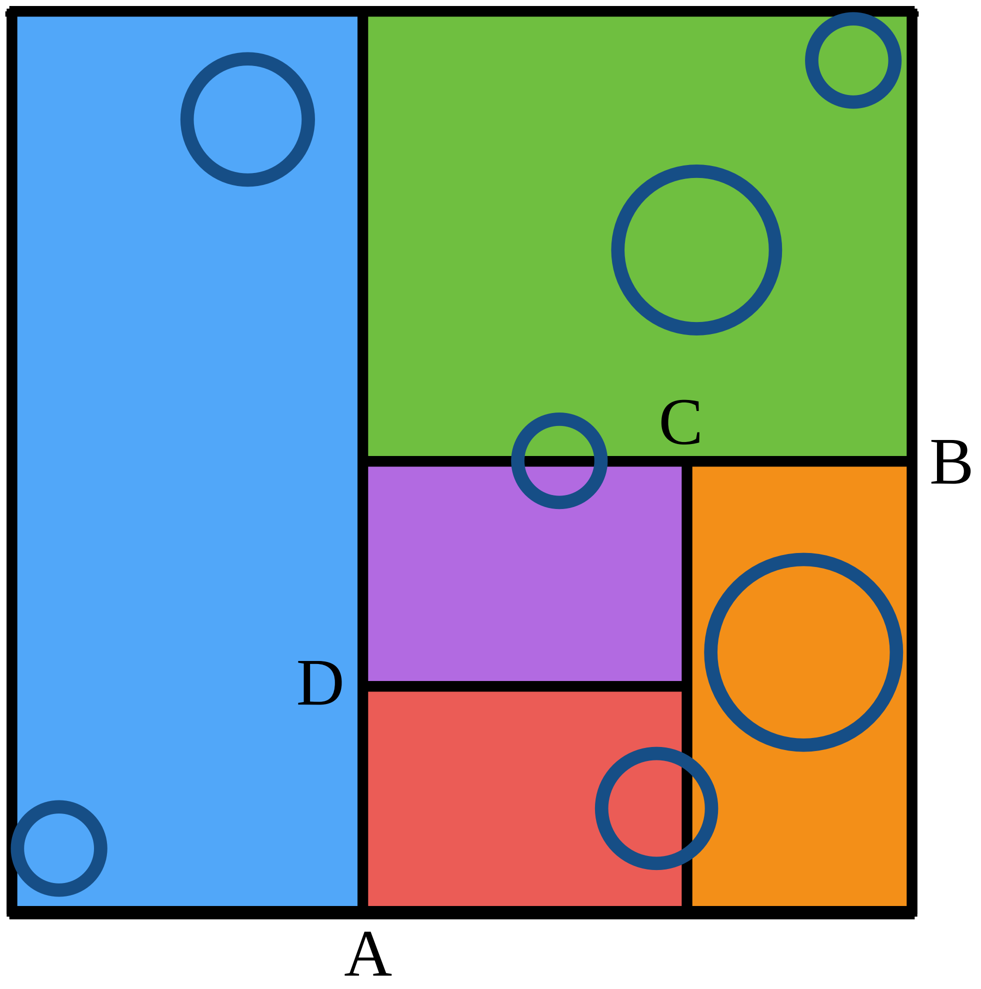
Internal nodes store

- split axis: x-, y-, or z-axis
- split position: coordinate of split plane along axis
- children: reference to child nodes

Leaf nodes store

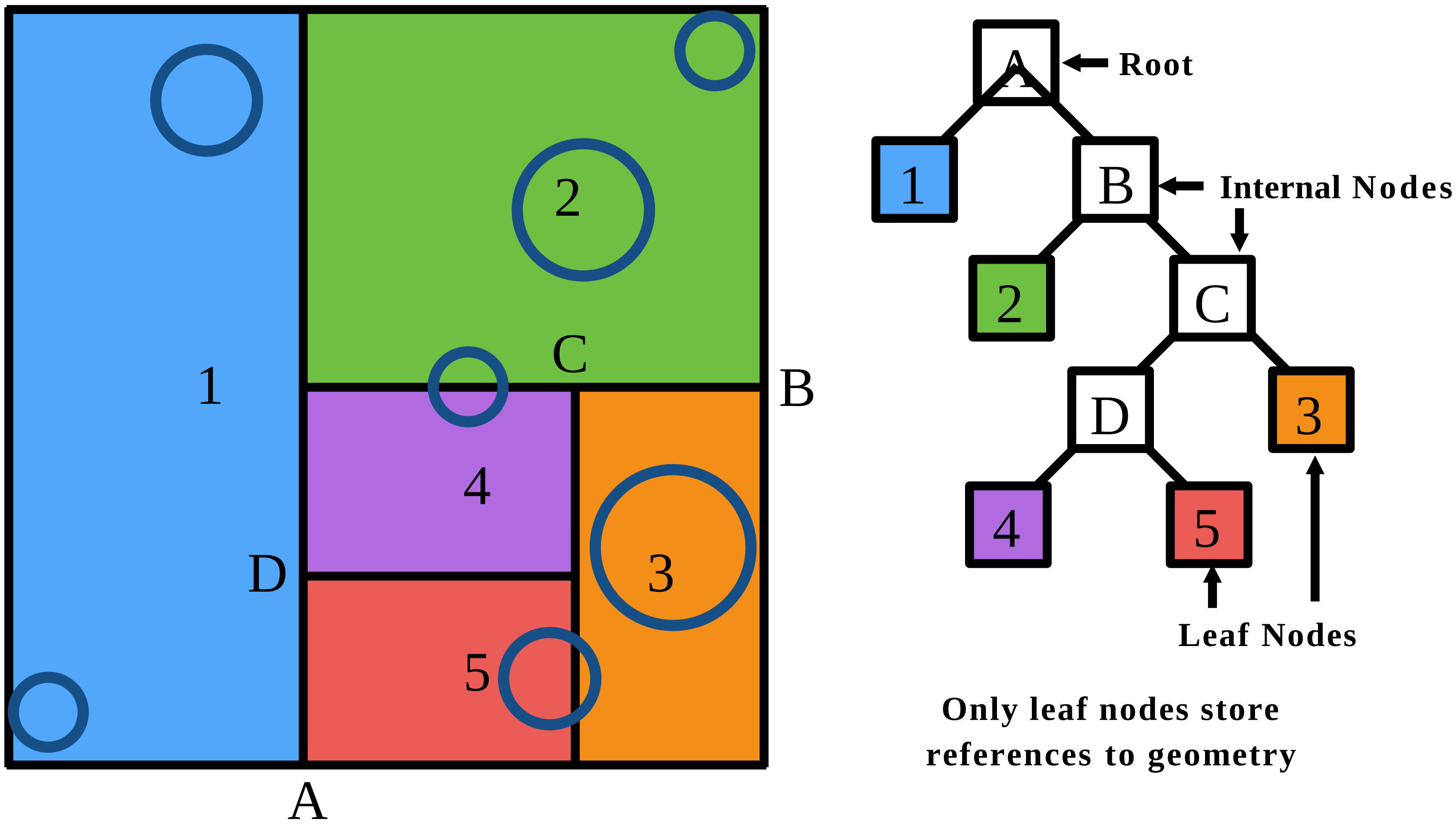
- list of objects
- mailbox information

KD-Tree Pre-Processing



- Find bounding box
- Recursively split cells, axis-aligned planes
- Until termination criteria met (e.g. max #splits or min #objs)
- Store obj references with each leaf node

KD-Tree Pre-Processing



KD-Tree Pre-Processing

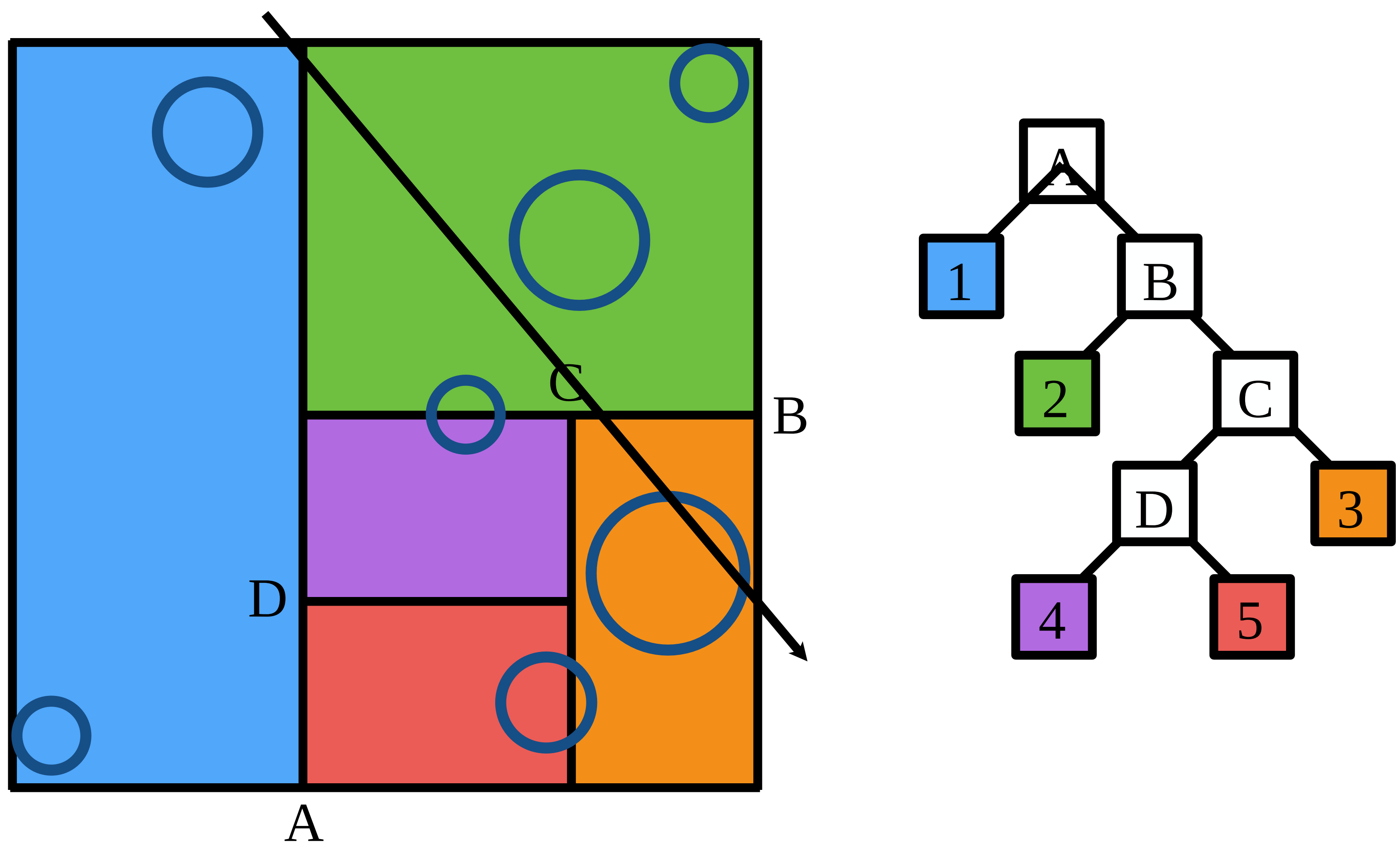
Choosing the split plane

- Simple: midpoint, median split
- Ideal: split to minimize expected cost of ray intersection

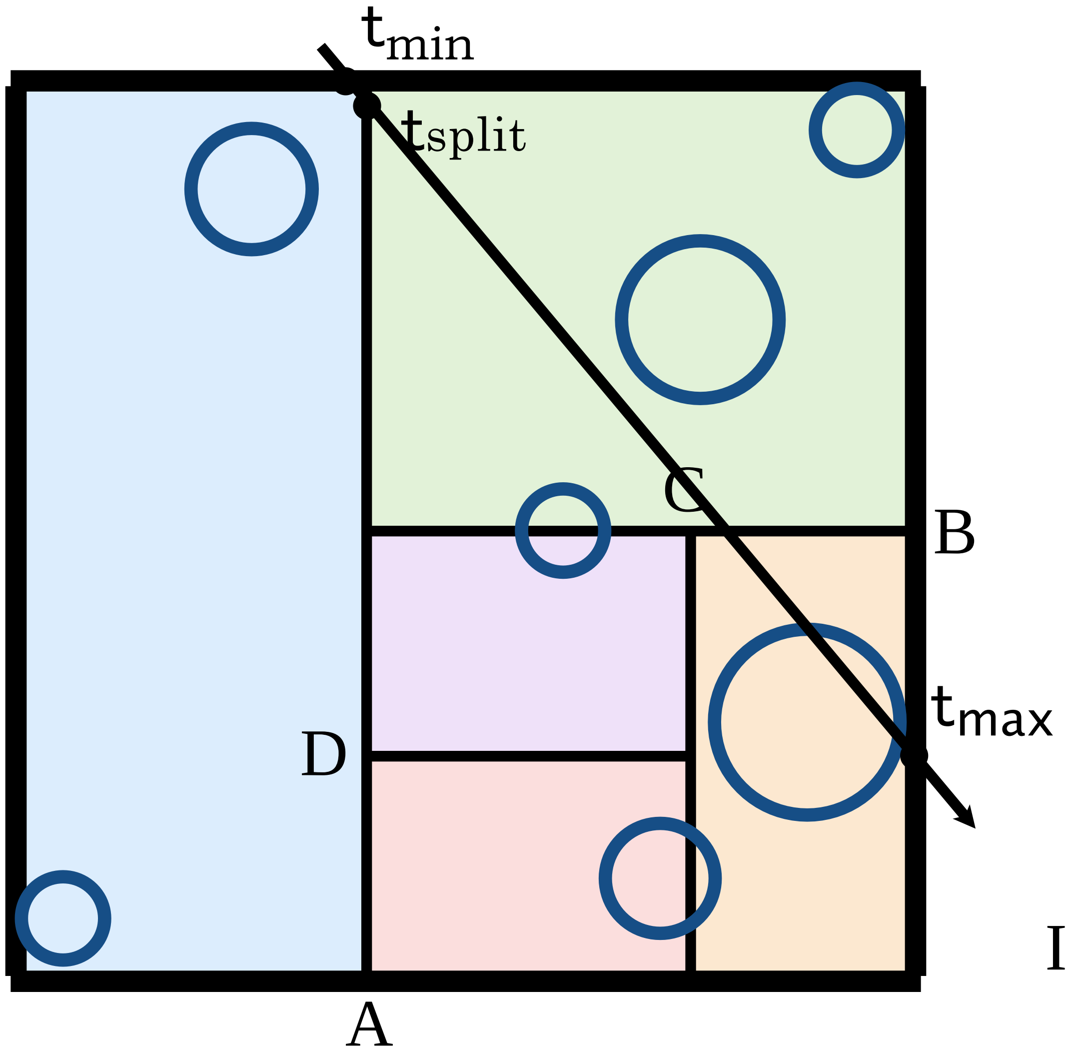
Termination criteria?

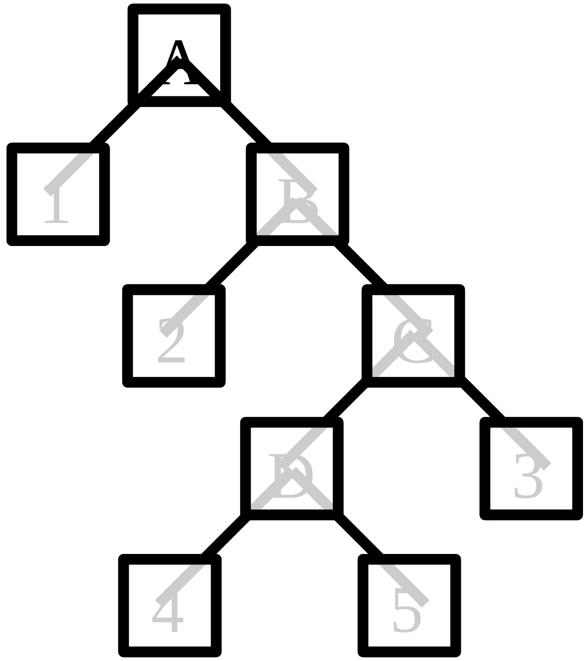
- Simple: common to prescribe maximum tree depth
- Ideal: stop when splitting does not reduce expected cost of ray intersection

Top-Down Recursive In-Order Traversal



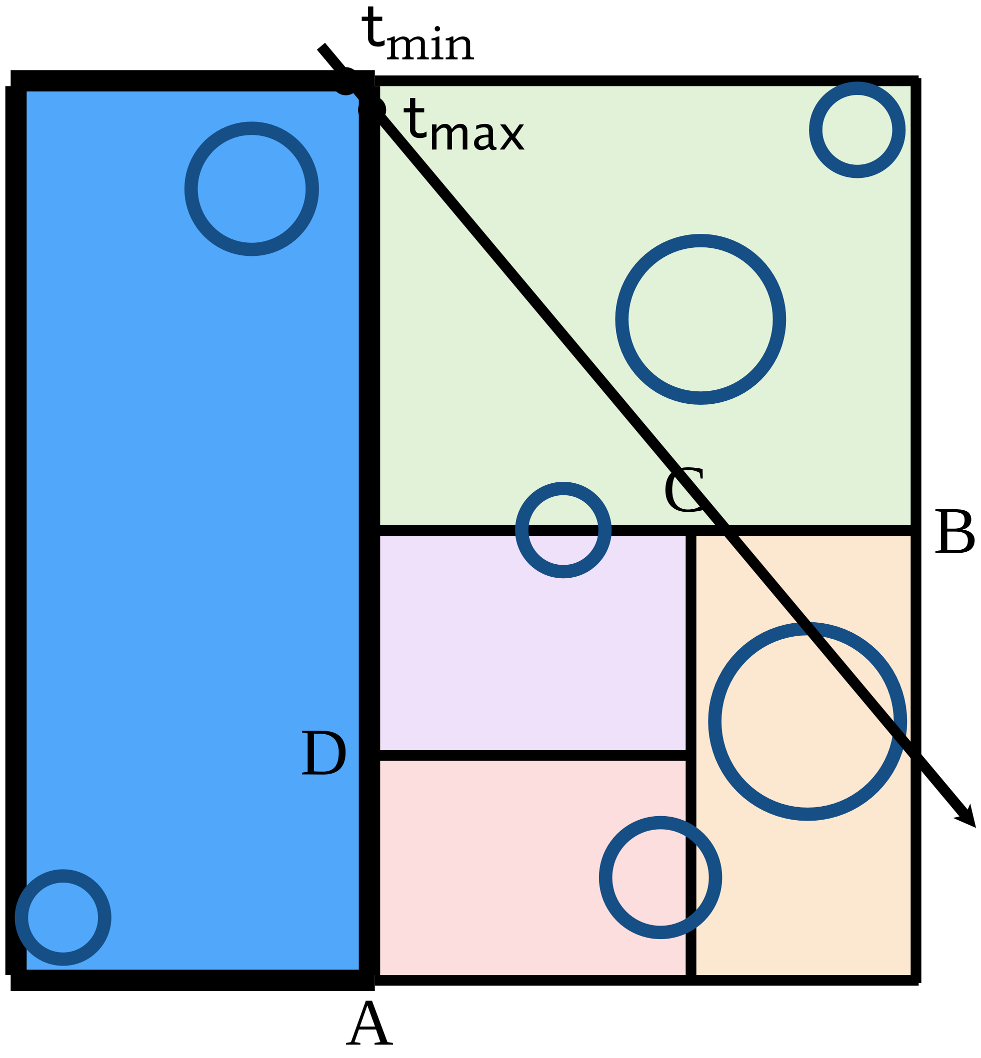
Top-Down Recursive In-Order Traversal

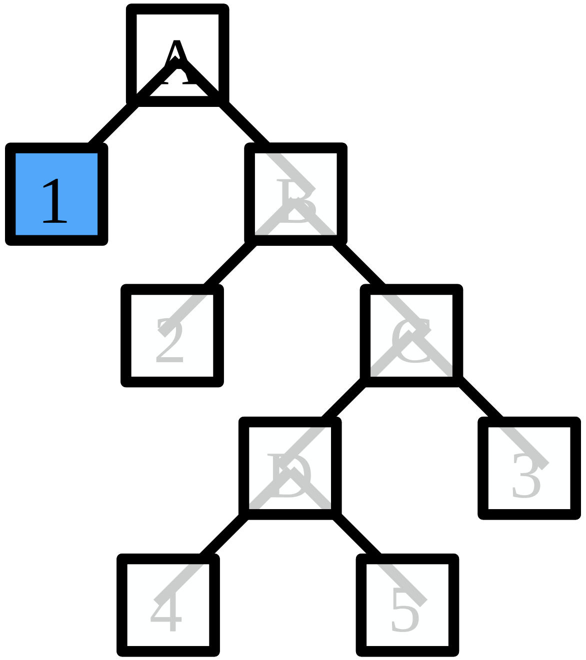




Internal node: split

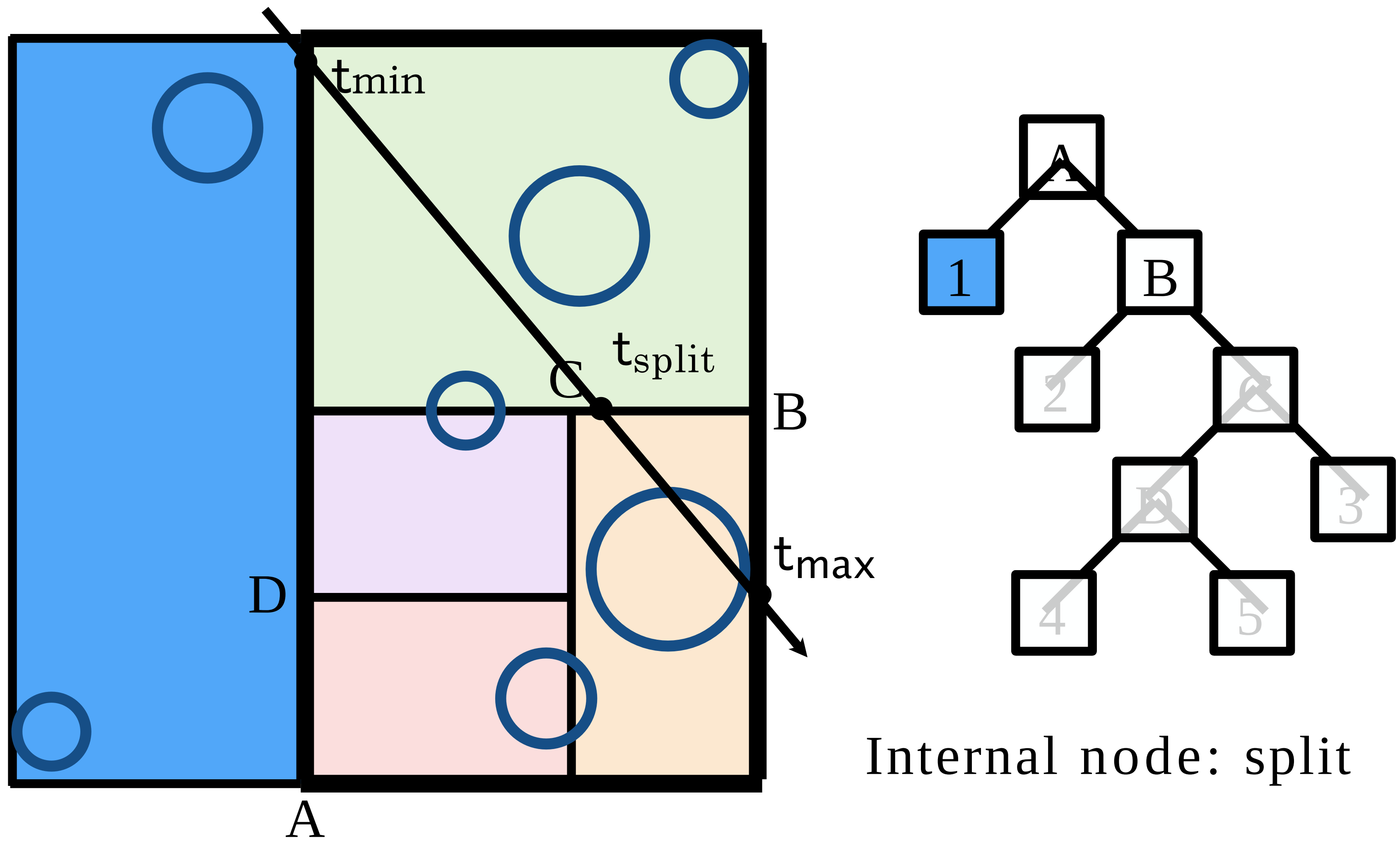
Top-Down Recursive In-Order Traversal





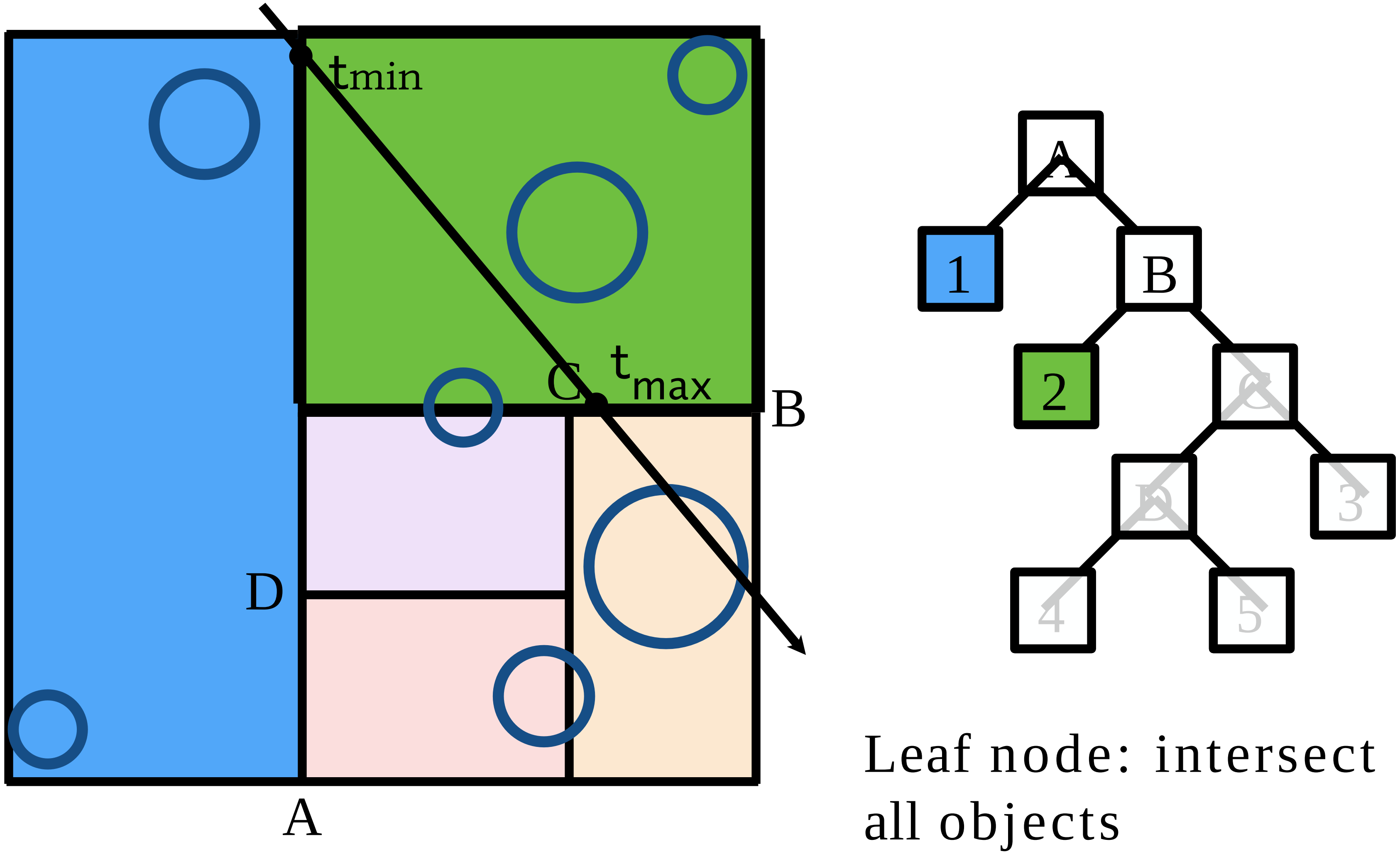
Leaf node: intersect all objects

Top-Down Recursive In-Order Traversal



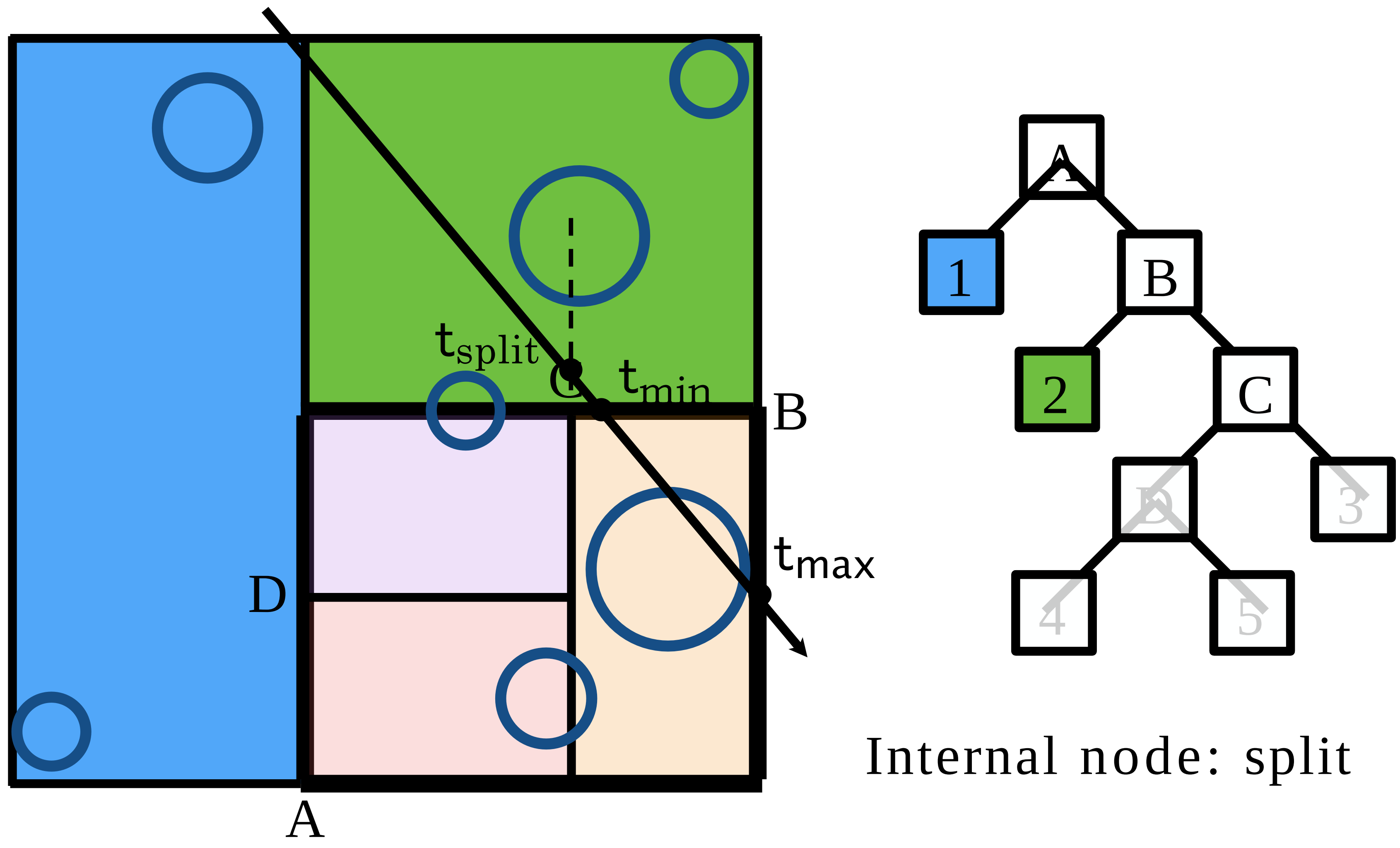
Internal node: split

Top-Down Recursive In-Order Traversal



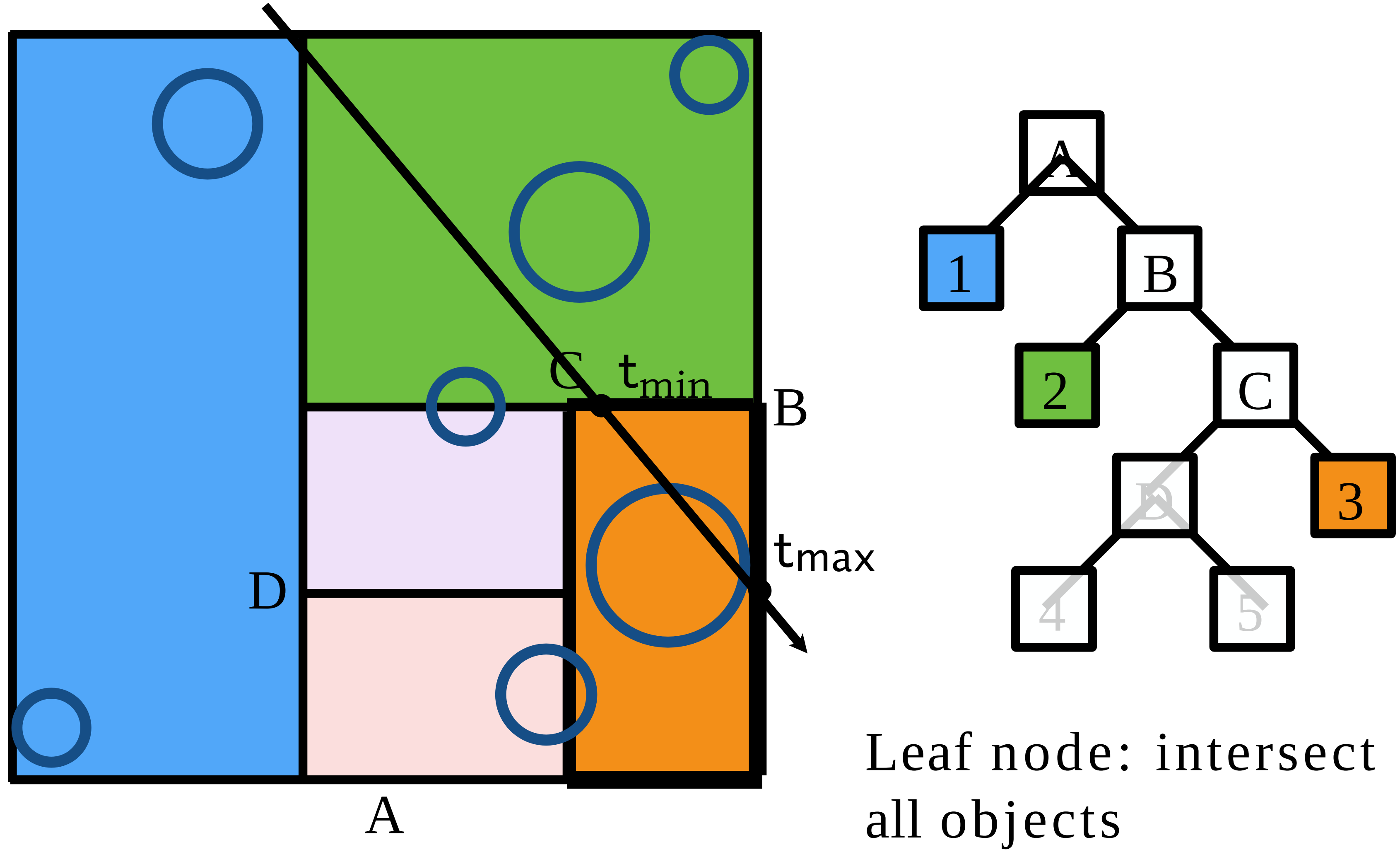
Leaf node: intersect all objects

Top-Down Recursive In-Order Traversal



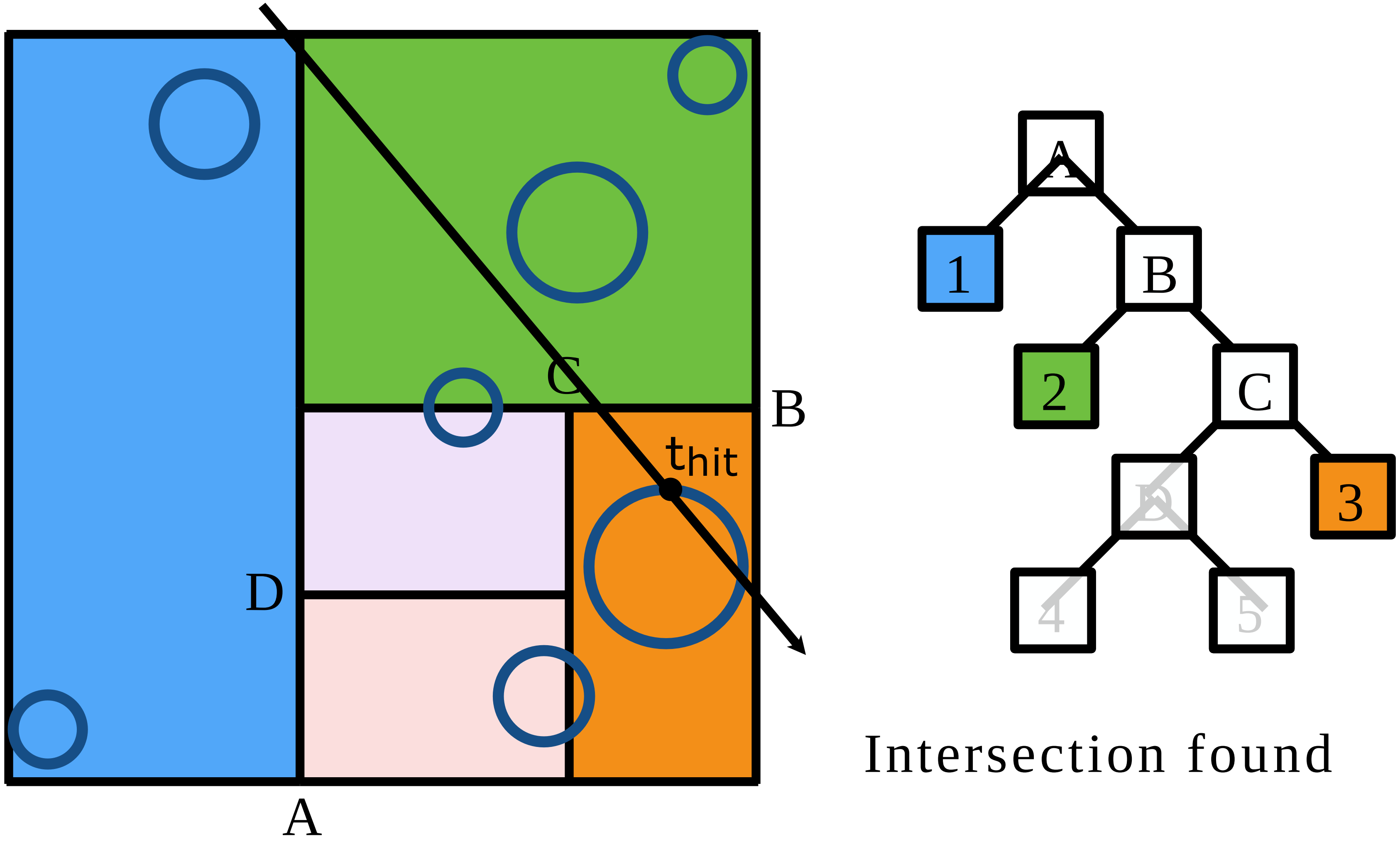
Internal node: split

Top-Down Recursive In-Order Traversal

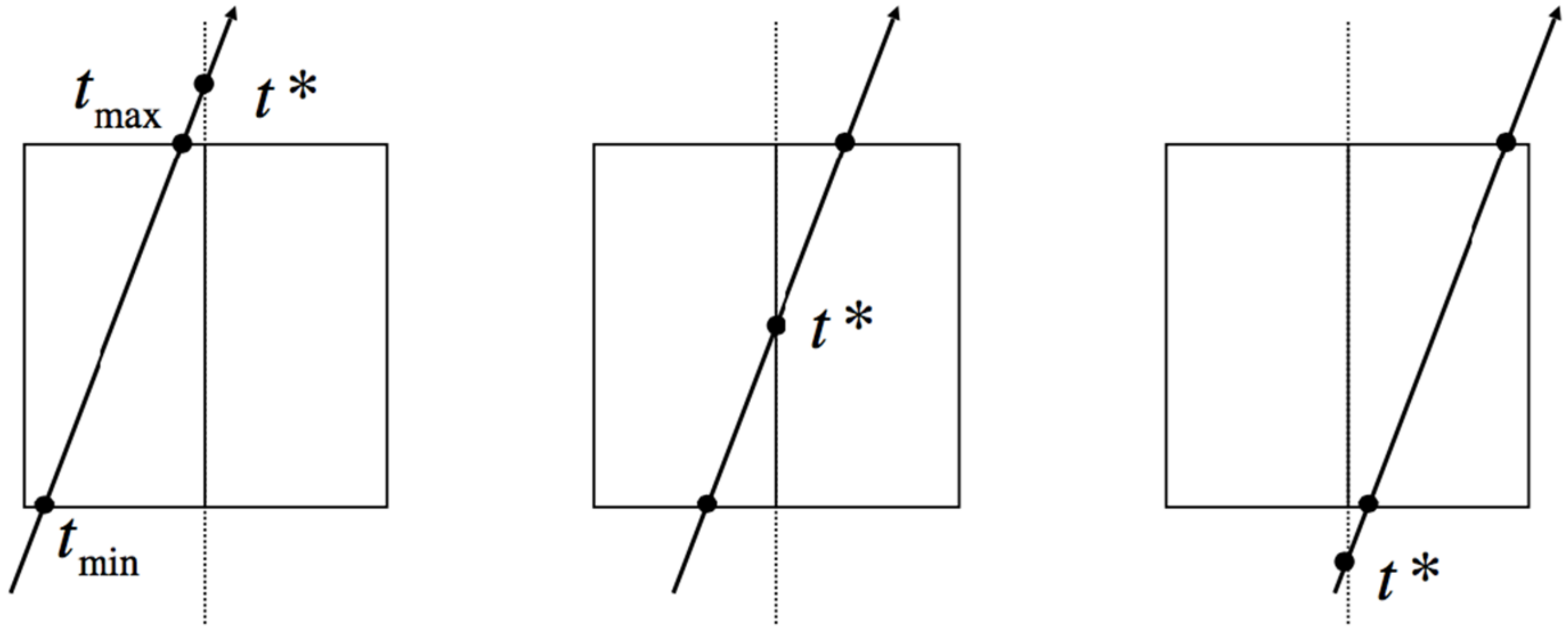


Leaf node: intersect all objects

Top-Down Recursive In-Order Traversal



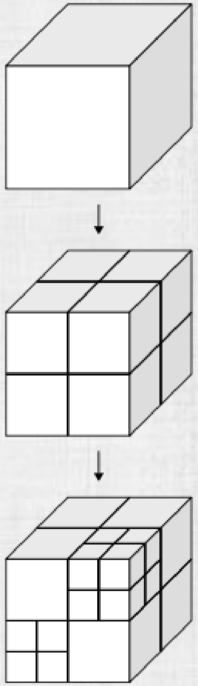
KD-Trees Traversal – Recursive Step

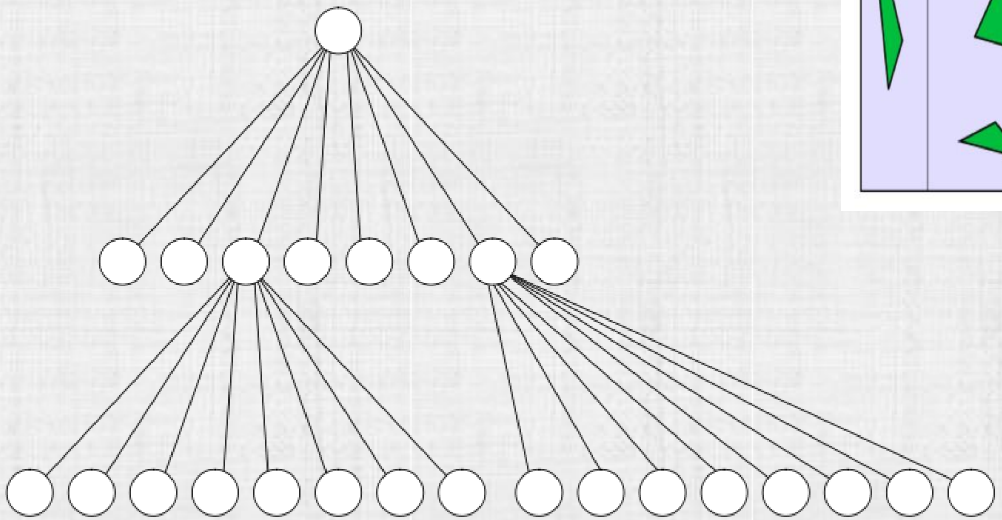


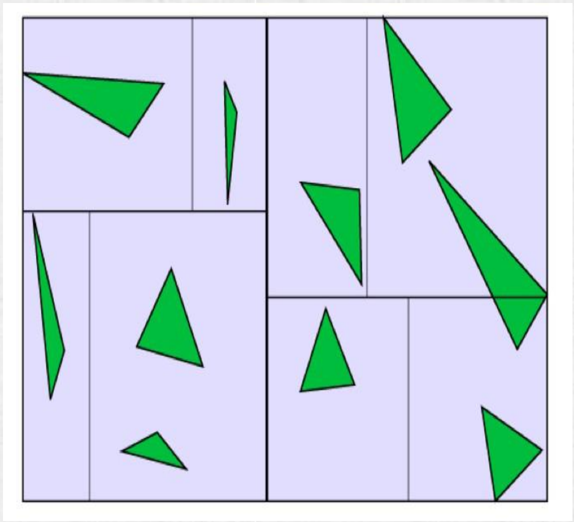
$t_{max} < t^*$	$t_{min} < t^* < t_{max}$	$t^* < t_{min}$
`Intersect(L,tmin,tmax)`	`Intersect(L,tmin,t*)` `Intersect(R,t*,tmax)`	`Intersect(R,tmin,tmax)`

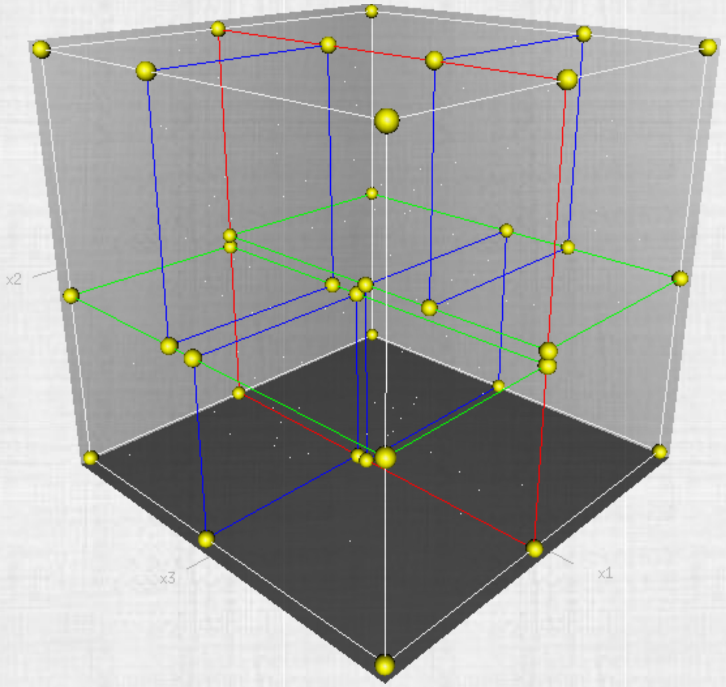
Aside: Code Acceleration

- Instead of a bottom-up bounding volume hierarchy approach, <u>octrees</u> and <u>K-D trees</u> take a top-down approach to hierarchically partitioning objects (and space)





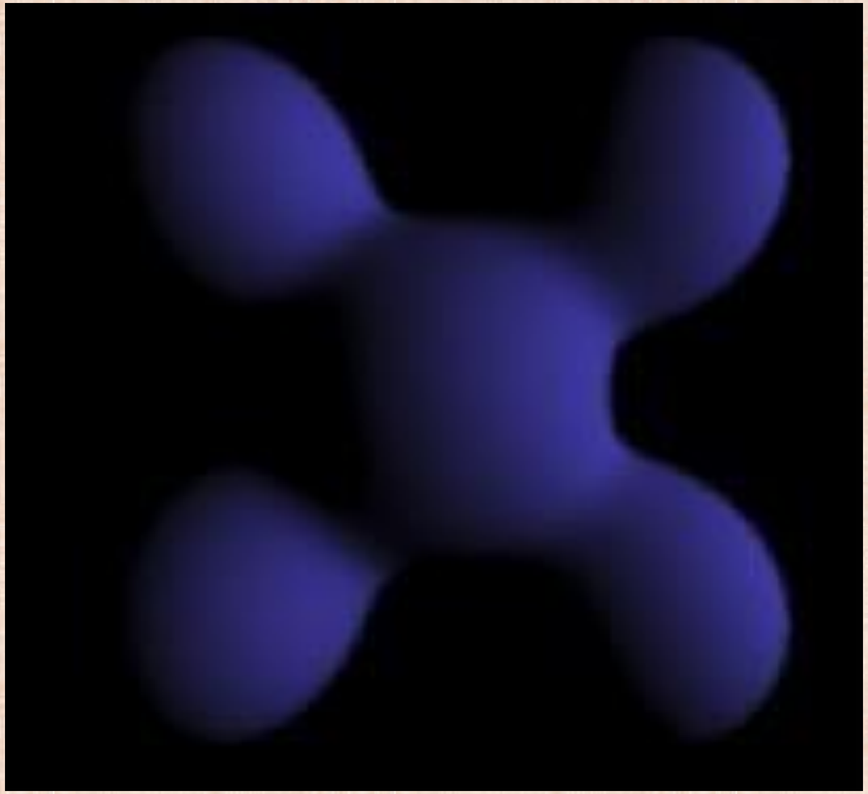




Ambient vs. Diffuse Shading

- Ambient shading colors a pixel when its ray intersects the object
- Diffuse shading attenuates object color based on how far the unit normal is tilted away from the incoming light (note how your eyes/brain imagine a 3D shape)

Ambient



Diffuse