```
# Install packages
!pip install pennylane pennylane-lightning pennylane-lightning[gpu] cotengra quimb --upgr
!pip install tifffile
!pip install -U "jax[cuda12_pip]" -f https://storage.googleapis.com/jax-releases/jax_cuda
# !pip install --upgrade "jax[cuda11_pip]" -f https://storage.googleapis.com/jax-releases
```

```
Collecting pennylane
  Downloading PennyLane-0.39.0-py3-none-any.whl.metadata (9.2 kB)
Collecting pennylane-lightning
  Downloading PennyLane_Lightning-0.39.0-cp310-cp310-manylinux_2_28_x86_64.whl.metada
Collecting cotengra
  Downloading cotengra-0.6.2-py3-none-any.whl.metadata (4.5 kB)
Collecting quimb
  Downloading quimb-1.9.0-py3-none-any.whl.metadata (5.5 kB)
Requirement already satisfied: numpy<2.1 in /usr/local/lib/python3.10/dist-packages (
Requirement already satisfied: scipy in /usr/local/lib/python3.10/dist-packages (from
Requirement already satisfied: networkx in /usr/local/lib/python3.10/dist-packages (f
Collecting rustworkx>=0.14.0 (from pennylane)
  Downloading rustworkx-0.15.1-cp38-abi3-manylinux_2_17_x86_64.manylinux2014_x86_64.w
Requirement already satisfied: autograd in /usr/local/lib/python3.10/dist-packages (f
Requirement already satisfied: toml in /usr/local/lib/python3.10/dist-packages (from
Collecting appdirs (from pennylane)
  Downloading appdirs-1.4.4-py2.py3-none-any.whl.metadata (9.0 kB)
Collecting autoray>=0.6.11 (from pennylane)
  Downloading autoray-0.7.0-py3-none-any.whl.metadata (5.8 kB)
Requirement already satisfied: cachetools in /usr/local/lib/python3.10/dist-packages
Requirement already satisfied: requests in /usr/local/lib/python3.10/dist-packages (f
Requirement already satisfied: typing-extensions in /usr/local/lib/python3.10/dist-pa
Requirement already satisfied: packaging in /usr/local/lib/python3.10/dist-packages (
Collecting pennylane-lightning-gpu (from pennylane-lightning[gpu])
  Downloading PennyLane_Lightning_GPU-0.39.0-cp310-cp310-manylinux_2_28_x86_64.whl.me
Collecting cytoolz>=0.8.0 (from quimb)
  Downloading cytoolz-1.0.0-cp310-cp310-manylinux_2_17_x86_64.manylinux2014_x86_64.wh
Requirement already satisfied: numba>=0.39 in /usr/local/lib/python3.10/dist-packages
Requirement already satisfied: psutil>=4.3.1 in /usr/local/lib/python3.10/dist-packag
Requirement already satisfied: tqdm>=4 in /usr/local/lib/python3.10/dist-packages (fr
Requirement already satisfied: toolz>=0.8.0 in /usr/local/lib/python3.10/dist-package
Requirement already satisfied: llvmlite<0.44,>=0.43.0dev0 in /usr/local/lib/python3.1
Requirement already satisfied: charset-normalizer<4,>=2 in /usr/local/lib/python3.10/
Requirement already satisfied: idna<4,>=2.5 in /usr/local/lib/python3.10/dist-package
Requirement already satisfied: urllib3<3,>=1.21.1 in /usr/local/lib/python3.10/dist-p
Requirement already satisfied: certifi>=2017.4.17 in /usr/local/lib/python3.10/dist-p
Downloading PennyLane-0.39.0-py3-none-any.whl (1.9 MB)
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Downloading quimb-1.9.0-py3-none-any.whl (1.7 MB)
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Downloading autoray-0.7.0-py3-none-any.whl (930 kB)
   ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 930.0/930.0 kB 48.8 MB/s eta 0:00:00
Downloading cytoolz-1.0.0-cp310-cp310-manylinux_2_17_x86_64.manylinux2014_x86_64.whl
```

```
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Downloading rustworkx-0.15.1-cp38-abi3-manylinux_2_17_x86_64.manylinux2014_x86_64.whl
━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 2.0/2.0 MB 73.3 MB/s eta 0:00:00
Downloading appdirs-1.4.4-py2.py3-none-any.whl (9.6 kB)
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━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 776.7/776.7 kB 38.7 MB/s eta 0:00:00
Installing collected packages: appdirs, rustworkx, cytoolz, autoray, cotengra, quimb,
Successfully installed appdirs-1.4.4 autoray-0.7.0 cotengra-0.6.2 cytoolz-1.0.0 penny
Requirement already satisfied: tifffile in /usr/local/lib/python3.10/dist-packages (2
Requirement already satisfied: numpy in /usr/local/lib/python3.10/dist-packages (from
Looking in links: https://storage.googleapis.com/jax-releases/jax_cuda_releases.html
```

```python
import jax
import jax.numpy as jnp

x = jnp.ones((1000, 1000))
y = jnp.dot(x, x)
print(jax.devices())
```

```
[CudaDevice(id=0)]
```

```python
# Import packages
import matplotlib as mpl
import matplotlib.pyplot as plt
import sys
import numpy as np
#np.set_printoptions(threshold=sys.maxsize)
import pandas as pd
from sklearn import datasets
import seaborn as sns
import jax
import time
from tifffile import tifffile
import functools

from typing import List, Union, Tuple, Dict, Optional, Any
from typing import Callable

jax.config.update("jax_enable_x64", True)
#jax.config.update("jax_debug_nans", True)
import jax.numpy as jnp

import optax  # optimization using jax

import torch  # https://pytorch.org
import torchvision  # https://pytorch.org
#torch.set_printoptions(profile="full")
import pennylane as qml
import pennylane.numpy as pnp
```

```
import os, cv2, itertools # cv2 -- OpenCV
import shutil
import zipfile
%matplotlib inline

from jax.lib import xla_bridge

def set_jax_platform():
    # Check if TPU is available
    try:
        tpu_backend = xla_bridge.get_backend('tpu')
        if tpu_backend and tpu_backend.device_count() > 0:
            # Set platform to TPU
            jax.config.update('jax_platform_name', 'tpu')
            print("Set platform to TPU")
            return
    except RuntimeError:
        pass  # No TPU found, move on to check for GPU

    # Check if GPU is available
    try:
      gpu_backend = xla_bridge.get_backend('gpu')
      if gpu_backend and gpu_backend.device_count() > 0:
          # Set platform to CUDA (GPU)
          jax.config.update('jax_platform_name', 'gpu')
          print("Set platform to GPU")
    except RuntimeError:
          # Set platform to CPU
          jax.config.update('jax_platform_name', 'cpu')
          print("Set platform to CPU")

# Call the function to set the platform
set_jax_platform()

sns.set()

seed = 1701
rng = np.random.default_rng(seed=seed)
prng = pnp.random.default_rng(seed=seed)
jrng_key = jax.random.PRNGKey(seed)
```

```
Set platform to GPU
<ipython-input-3-18305dd46f60>:40: DeprecationWarning: jax.lib.xla_bridge.get_backend
  tpu_backend = xla_bridge.get_backend('tpu')
<ipython-input-3-18305dd46f60>:51: DeprecationWarning: jax.lib.xla_bridge.get_backend
  gpu_backend = xla_bridge.get_backend('gpu')
```

```
from google.colab import drive
drive.mount('/content/drive')
```

```
Mounted at /content/drive
```

```
ROWS, COLS = 32, 32

# Extract data from https://www.kaggle.com/datasets/kmader/siim-medical-images
data_zip = '/content/drive/MyDrive/Datasets/CTMedicalImages/archive.zip'

# extract the file
with zipfile.ZipFile(data_zip, 'r') as zip_ref:
    zip_ref.extractall(os.getcwd())
```

```
from google.colab.patches import cv2_imshow
overview_df = pd.read_csv('overview.csv')
overview_df.columns = ['id']+list(overview_df.columns[1:])
overview_df['Contrast'] = overview_df['Contrast'].map(lambda x: 1 if x else 0) #1 for con
dummy_img = tifffile.imread(os.path.join('tiff_images', overview_df['tiff_name'][0]))
cv2_imshow(dummy_img)
```

```
resized = cv2.resize(dummy_img, (ROWS, COLS),interpolation=cv2.INTER_CUBIC)
cv2_imshow(resized)
```

```
def read_tiff_and_resize(filepath, resize_row = 32, resize_col = 32):
  img = tifffile.imread(filepath)
  return cv2.resize(img, (resize_row, resize_col),interpolation=cv2.INTER_CUBIC)
```

```
class CTImgHerm(torch.utils.data.Dataset):
    def __init__(self, overview_csv='overview.csv', tiff_img_folder='tiff_images', resize
        self.overview_df = pd.read_csv('overview.csv')
        self.overview_df.columns = ['idx']+list(overview_df.columns[1:])
        self.overview_df['Contrast'] = overview_df['Contrast'].map(lambda x: 1 if x else
        self.resize_col = resize_col
        self.resize_row = resize_row

        self.data = []

        for i in range(len(self.overview_df)):
          img = read_tiff_and_resize(os.path.join(tiff_img_folder, overview_df['tiff_name
          img = cv2.normalize(img, None, 0, 1.0,cv2.NORM_MINMAX, dtype=cv2.CV_32F)
          label = self.overview_df['Contrast'][i]
          self.data.append((img, label))

    def __len__(self):
        return len(self.data)

    def __getitem__(self, idx):
        img, label = self.data[idx]
        img = (img + np.transpose(img))/2
        return img, label

dummy_dataset = CTImgHerm()
dummy_dataset[0]
```

```
(array([[0.00067681, 0.00067681, 0.00067681, ..., 0.00067681, 0.00067681,
         0.00067681],
        [0.00067681, 0.00067681, 0.00067681, ..., 0.00858719, 0.00069783,
         0.00067681],
        [0.00067681, 0.00067681, 0.00067681, ..., 0.05319135, 0.00705113,
         0.00054893],
        ...,
        [0.00067681, 0.00858719, 0.05319135, ..., 0.07699811, 0.0740454 ,
         0.04863514],
        [0.00067681, 0.00069783, 0.00705113, ..., 0.0740454 , 0.41830966,
         0.02946547],
        [0.00067681, 0.00067681, 0.00054893, ..., 0.04863514, 0.02946547,
         0.00067681]], dtype=float32),
```

```
      1)


def img_hermitian_evolve(
    img:jnp.ndarray,
    t:float
)->jnp.ndarray:
  assert img.shape[-1]==32 and img.shape[-2] == 32, f"The shape of the image must be 32 by
  return jax.scipy.linalg.expm(img*( -0.5j*t))

print(
    img_hermitian_evolve(
        dummy_dataset[0][0],
        10
        )[16]
    )
```

```
 [ 0.03316834+0.10185403j  0.03990275+0.01448904j -0.01340654-0.11395637j
  -0.09734283-0.04602238j -0.02874839-0.10615511j  0.03417724-0.0231277j
   0.31355855-0.00824658j -0.02404667+0.14984643j -0.20052674-0.04882964j
   0.14395793+0.08981129j  0.1990049 -0.210869j   -0.27399263+0.07857375j
  -0.06780967-0.25949493j  0.26647484-0.00677687j  0.05676082-0.0082445j
  -0.06779608-0.04115491j  0.11816131-0.20410226j -0.08650953-0.03623072j
  -0.04554248+0.09426466j  0.15362452+0.02755849j  0.07865272-0.17987852j
  -0.07149893+0.21788627j  0.08791468-0.09473701j  0.01336784+0.02959449j
   0.23885594-0.01219563j  0.15551719-0.15525931j  0.0274305 -0.07152441j
   0.18937154+0.15392612j -0.07521366+0.0550395j  -0.04855048-0.09508036j
  -0.04012334-0.05201115j  0.05081468-0.17594492j]
```

```
ket = {
    '0':jnp.array([1,0]),
    '1':jnp.array([0,1]),
    '+':(jnp.array([1,0]) + jnp.array([0,1]))/jnp.sqrt(2),
    '-':(jnp.array([1,0]) - jnp.array([0,1]))/jnp.sqrt(2)
}

pauli = {
    'I':jnp.array([[1,0],[0,1]]),
    'X':jnp.array([[0,1],[1,0]]),
    'Y':jnp.array([[0, -1j],[1j, 0]]),
    'Z':jnp.array([[1,0],[0,-1]])
}

def tensor_product(*args):
  input_list = [a for a in args]
  return functools.reduce(jnp.kron, input_list)

def multi_qubit_identity(n_qubits:int)->jnp.ndarray:
  assert n_qubits>0
  if n_qubits == 1:
    return pauli['I']
```

```
  else:
    return tensor_product(*[pauli['I'] for _ in range(n_qubits)])

pauli_words_su4 = {}
for key1 in pauli.keys():
  for key2 in pauli.keys():
    if not (key1==key2 and key1=='I' and key2=='I'):
      pauli_words_su4[key1+key2] = tensor_product(pauli[key1], pauli[key2])

pauli_words_su8 = {}
for key1 in pauli.keys():
  for key2 in pauli.keys():
    for key3 in pauli.keys():
      if not key1+key2+key3 == 'III':
        pauli_words_su8[key1+key2+key3] = tensor_product(pauli[key1], pauli[key2], pauli[

pauli_words_su16 = {}
for key1 in pauli.keys():
  for key2 in pauli.keys():
    for key3 in pauli.keys():
      for key4 in pauli.keys():
        if not key1+key2+key3+key4 == 'IIII':
          pauli_words_su16[key1+key2+key3+key4] = tensor_product(
              pauli[key1],
              pauli[key2],
              pauli[key3],
              pauli[key4]
          )

pauli_words_su32 = {}
for key1 in pauli.keys():
  for key2 in pauli.keys():
    for key3 in pauli.keys():
      for key4 in pauli.keys():
        for key5 in pauli.keys():
          if not key1+key2+key3+key4+key5 == 'IIIII':
            pauli_words_su32[key1+key2+key3+key4+key5] = tensor_product(
                pauli[key1],
                pauli[key2],
                pauli[key3],
                pauli[key4],
                pauli[key5]
            )

observables_2_cls_5q = [0]*2
for i in ['0', '1']:
      idx = int(i, 2)
      basis_state =ket[i]
      single_qubit_obs = jnp.outer(basis_state, basis_state)
      observables_2_cls_5q[idx] = tensor_product(single_qubit_obs, multi_qubit_identity(4
```

```
print(observables_2_cls_5q)
```

```
[Array([[1, 0, 0, ..., 0, 0, 0],
        [0, 1, 0, ..., 0, 0, 0],
        [0, 0, 1, ..., 0, 0, 0],
        ...,
        [0, 0, 0, ..., 0, 0, 0],
        [0, 0, 0, ..., 0, 0, 0],
        [0, 0, 0, ..., 0, 0, 0]], dtype=int64), Array([[0, 0, 0, ..., 0, 0, 0],
        [0, 0, 0, ..., 0, 0, 0],
        [0, 0, 0, ..., 0, 0, 0],
        ...,
        [0, 0, 0, ..., 1, 0, 0],
        [0, 0, 0, ..., 0, 1, 0],
        [0, 0, 0, ..., 0, 0, 1]], dtype=int64)]
```

```
def su4_op(
    params:jnp.ndarray
):
  generator = jnp.einsum("i, ijk -> jk", params, jnp.asarray(list(pauli_words_su4.values(
  return jax.scipy.linalg.expm(1j*generator)

def brickwall_su4_5q_single_layer(
    params:jnp.ndarray
):
  """
  A brickwall layer of su4 gates on 5 qubits.
  Second stack is three su4 on qubit pairs (1,2), (3,4)
  First stack is two su4 on qubit pairs (2,3), (4,5)
  """
  second_stack = tensor_product(
      su4_op(params[:15]),
      su4_op(params[15:30]),
      multi_qubit_identity(1)
  )
  first_stack = tensor_product(
      multi_qubit_identity(1),
      su4_op(params[30:45]),
      su4_op(params[45:60]),
  )
  return jnp.dot(second_stack, first_stack)

def su32_op(
    params:jnp.ndarray
):
  generator = jnp.einsum("i, ijk - >jk", params, jnp.asarray(list(pauli_words_su32.values
  return jax.scipy.linalg.expm(1j*generator)

test_params = jax.random.normal(shape=[60], key=jrng_key)
print(brickwall_su4_5q_single_layer(test_params).shape)
```

```
(32, 32)
```

```
def measure_sv(
    state:jnp.ndarray,
    observable:jnp.ndarray
    ):
  """
  Measure a statevector with a Hermitian observable.
  Note: No checking Hermitianicity of the observable or whether the observable
  has all real eigenvalues or not
  """
  expectation_value = jnp.dot(jnp.conj(state.T), jnp.dot(observable, state))
  return jnp.real(expectation_value)

def measure_dm(
    rho:jnp.ndarray,
    observable:jnp.ndarray
):
  """
  Measure a density matrix with a Hermitian observable.
  Note: No checking Hermitianicity of the observable or whether the observable
  has all real eigenvalues or not.
  """
  product = jnp.dot(rho, observable)

  # Calculate the trace, which is the sum of diagonal elements
  trace = jnp.trace(product)

  # The expectation value should be real for physical observables
  return jnp.real(trace)

vmap_measure_sv = jax.vmap(measure_sv, in_axes=(None, 0), out_axes=0)
vmap_measure_dm = jax.vmap(measure_dm, in_axes=(None, 0), out_axes=0)

def bitstring_to_state(bitstring:str):
  """
  Convert a bit string, like '0101001' or '+-+-101'
  to a statevector. Each character in the bitstring must be among
  0, 1, + and -
  """
  assert len(bitstring)>0
  for c in bitstring:
    assert c in ['0', '1', '+', '-']
  single_qubit_states = [ket[c] for c in bitstring]
  return tensor_product(*single_qubit_states)


def qnn_hamevo(
    params:jnp.ndarray,
    t:jnp.ndarray,
```

```
    img:jnp.ndarray
)->jnp.ndarray:
  """
  A QNN that takes (M+M^T)/2
  as input, where M is the (rescaled) original image,
  as well as a trainable parameter t,
  and parameters for the trainable layers
  and output an array of 2 elements representing classification logits
  """
  single_op_params = 60 #4**5-1

  n_outer_layers = len(t)
  n_inner_layers = (len(params)//single_op_params)//n_outer_layers
  state = tensor_product(ket['+'], ket['+'], ket['+'], ket['+'], ket['+'])
  for i in range(n_outer_layers):
    state = jnp.dot(
      img_hermitian_evolve(img, t[i]),
      state
      )
    inner_layer_params = params[i*(single_op_params*n_inner_layers):(i+1)*(single_op_para
    for j in range(n_inner_layers):
      state = jnp.dot(
          brickwall_su4_5q_single_layer(inner_layer_params[j*single_op_params:(j+1)*singl
          state
      )
  return vmap_measure_sv(state, jnp.asarray(observables_2_cls_5q))
```

```
print(
    qnn_hamevo(
        jax.random.normal(shape=[60*15], key=jrng_key),
        jax.random.normal(shape=[15], key=jrng_key),
        dummy_dataset[0][0]
    )
)
```

```
 [0.44247169 0.55752831]
```

```
@jax.jit
def compute_out(weight,t, features, labels):
    """Computes the output of the corresponding label in the qcnn"""
    out = lambda weight,t, feature, label: qnn_hamevo(weight,t, feature)
    return jax.vmap(out, in_axes=(None,None,  0, 0), out_axes=0)(
        weight,t, features, labels
    )


def compute_accuracy(weight,t, features, labels):
    """Computes the accuracy over the provided features and labels"""
```

```
    out = compute_out(weight,t, features, labels)
    pred = jnp.argmax(out, axis = 1)
    return jnp.sum(jnp.array(pred == labels).astype(int)) / len(out)


def compute_cost(weight,t, features, labels):
    """Computes the cost over the provided features and labels"""
    logits = compute_out(weight,t, features, labels)
    return jnp.nanmean(optax.softmax_cross_entropy_with_integer_labels(logits, labels))


value_and_grad = jax.jit(jax.value_and_grad(compute_cost, argnums=[0,1]))


N_OUTER_LAYERS = 2
N_INNER_LAYERS = 1
N_LAYERS = N_OUTER_LAYERS*N_INNER_LAYERS
SINGLE_OP_PARAMS  = 60 #4**5-1

def init_weights_random():
    print("Random Init")
    return jax.random.normal(shape=[SINGLE_OP_PARAMS*N_LAYERS], key=jrng_key),jax.random.

def init_weights_beta(alpha=0.5, beta=2.0):
    # Initialize weights with a Beta distribution skewed towards 0
    weights = jax.random.beta(jrng_key, alpha, beta, shape=[SINGLE_OP_PARAMS*N_LAYERS])
    biases = jax.random.beta(jrng_key, alpha, beta, shape=[N_OUTER_LAYERS])
    return weights, biases


import pandas as pd
import os

def save_weights_to_csv(weights, biases, epoch, file_name='weights.csv'):
    """Saves the weights and biases to a CSV file."""
    # Convert weights and biases to a flat list
    weight_list = weights.flatten().tolist()
    bias_list = biases.flatten().tolist()

    # Create a dictionary to store the weights and biases with epoch
    data = {'epoch': [epoch], 'weights': [weight_list], 'biases': [bias_list]}

    # Convert to DataFrame
    df = pd.DataFrame(data)

    # Check if the file exists before appending
    if not os.path.isfile(file_name):
        # If the file does not exist, create it with a header
        df.to_csv(file_name, mode='w', header=True, index=False)
    else:
        # If the file exists, append the new data without a header
```

```
        df.to_csv(file_name, mode='a', header=False, index=False)



# def train_vqc(batchsize:int, n_epochs:int, seed:int=1701):
#   start = time.time()
#   pnp.random.seed(seed)
#   np.random.seed(seed)
#   # load data
#   full_dataset = CTImgHerm()
#   train_size = int(0.8 * len(full_dataset))
#   test_size = len(full_dataset) - train_size
#   train_dataset, test_dataset = torch.utils.data.random_split(full_dataset, [train_size
#   trainloader = torch.utils.data.DataLoader(
#     train_dataset, batch_size=batchsize, shuffle=True
#   )
#   testloader = torch.utils.data.DataLoader(
#     test_dataset, batch_size=batchsize, shuffle=True
#   )
#   # Exponential decay of the learning rate.
#   scheduler = optax.exponential_decay(
#     init_value=0.01,
#     transition_steps=n_epochs,
#     decay_rate=0.99)

#   # Combining gradient transforms using `optax.chain`.
#   gradient_transform = optax.chain(
#     optax.clip(1.0),
#     optax.scale_by_adam(),  # Use the updates from adam.
#     optax.scale_by_schedule(scheduler),  # Use the learning rate from the scheduler.
#     # Scale updates by -1 since optax.apply_updates is additive and we want to descend
#     optax.scale(-1.0)
#   )
#   # init weights and optimizer
#   weights, weights_last = init_weights()
#   opt_state = gradient_transform.init((weights, weights_last))
#   #data containers
#   train_cost_epochs, test_cost_epochs, train_acc_epochs, test_acc_epochs = [], [], [],
#   for step in range(n_epochs):
#         train_cost_batches = []
#         train_acc_batches = []
#         test_cost_batches = []
#         test_acc_batches = []
#         epoch_start = time.time()
#         print(f"Training at Epoch {step+1}/{n_epochs}, Train batches {len(trainloader)}
#         for batch, (x_train, y_train) in enumerate(trainloader):
#           batch_start = time.time()
#           # Training step with (adam) optimizer
#           x_train, y_train = jnp.asarray(x_train.numpy()), jnp.asarray(y_train.numpy())
#           train_cost, grad_circuit = value_and_grad(weights, weights_last, x_train, y_t
#           updates, opt_state = gradient_transform.update(grad_circuit, opt_state)
```

```
#           weights, weights_last = optax.apply_updates((weights, weights_last), updates)
#           train_acc = compute_accuracy(weights, weights_last, x_train, y_train)
#           train_cost_batches.append(train_cost)
#           train_acc_batches.append(train_acc)
#           if len(trainloader)<= 5 or (batch+1)%5==0:
#             print(f"Training at Epoch {step+1}/{n_epochs}, Batch {batch+1}, Cost {train


#         train_cost_epochs.append(np.mean(train_cost_batches))
#         train_acc_epochs.append(np.mean(train_acc_batches))


#         # load test data
#         for batch, (x_test, y_test) in enumerate(testloader):
#           batch_start = time.time()
#           x_test, y_test = jnp.asarray(x_test.numpy()), jnp.asarray(y_test.numpy())
#           # compute accuracy and cost on testing data
#           test_out = compute_out(weights, weights_last, x_test, y_test)
#           test_pred = jnp.argmax(test_out, axis=1)
#           test_acc = jnp.sum(jnp.array(test_pred == y_test).astype(int)) / len(test_out
#           test_cost = jnp.nanmean(optax.softmax_cross_entropy_with_integer_labels(test_
#           test_cost_batches.append(test_cost)
#           test_acc_batches.append(test_acc)
#           if len(testloader)<= 5 or (batch+1)%5==0:
#             print(f"Testing at Epoch {step+1}/{n_epochs}, Batch {batch+1}, Cost {test_c
#         test_acc_epochs.append(np.mean(test_acc_batches))
#         test_cost = np.mean(test_cost_batches)
#         test_cost_epochs.append(test_cost)
#         print("......")
#         print(f"Epoch {step+1}/{n_epochs}, Train: Cost {np.mean(train_cost_batches)}, A
#         print(f"Epoch {step+1}/{n_epochs}, Test: Cost {test_cost}, Acc {test_acc}. Time
#         print("=-="*10)

#   return dict(
#         n_train=[train_size] * n_epochs,
#         step=np.arange(1, n_epochs + 1, dtype=int).tolist(),
#         train_cost=[c.astype(float) for c in train_cost_epochs],
#         train_acc=[c.astype(float) for c in train_acc_epochs],
#         test_cost=[c.astype(float) for c in test_cost_epochs],
#         test_acc=[c.astype(float) for c in test_acc_epochs],
#     )
```

```
import numpy as np
import pandas as pd
import time
import torch
import optax
import jax.numpy as jnp
```

```
def train_vqc(batchsize: int, n_epochs: int, seed: int = 1701):
    start = time.time()
    np.random.seed(seed)

    # Load data once
    full_dataset = CTImgHerm()
    train_size = int(0.75 * len(full_dataset))
    val_size = int(0.05 * len(full_dataset))
    test_size = len(full_dataset) - train_size - val_size
    # Split dataset into train, validation, and test sets (fixed split for both initializ
    train_dataset, val_test_dataset = torch.utils.data.random_split(
        full_dataset, [train_size, val_size + test_size]
    )
    val_dataset, test_dataset = torch.utils.data.random_split(
        val_test_dataset, [val_size, test_size]
    )

    # Create data loaders
    trainloader = torch.utils.data.DataLoader(
        train_dataset, batch_size=batchsize, shuffle=True
    )
    valloader = torch.utils.data.DataLoader(
        val_dataset, batch_size=batchsize, shuffle=True
    )
    testloader = torch.utils.data.DataLoader(
        test_dataset, batch_size=batchsize, shuffle=False
    )

    # Exponential decay of the learning rate
    scheduler = optax.exponential_decay(
        init_value=0.01,
        transition_steps=n_epochs,
        decay_rate=0.99
    )
    gradient_transform = optax.chain(
        optax.clip(1.0),
        optax.scale_by_adam(),
        optax.scale_by_schedule(scheduler),
        optax.scale(-1.0)
    )

    def run_training(init_weights_func, file_name):
        """Run the training loop for a given initialization function."""
        weights, weights_last = init_weights_func()
        opt_state = gradient_transform.init((weights, weights_last))

        train_cost_epochs, val_cost_epochs = [], []
        train_acc_epochs, val_acc_epochs = [], []

        # Training and validation loop
```

```
    for step in range(n_epochs):
        train_cost_batches, train_acc_batches = [], []
        val_cost_batches, val_acc_batches = [], []

        print(f"Epoch {step+1}/{n_epochs} - Training...")


        # Training loop
        for x_train, y_train in trainloader:
            x_train, y_train = jnp.asarray(x_train.numpy()), jnp.asarray(y_train.nump
            train_cost, grad_circuit = value_and_grad(weights, weights_last, x_train,
            updates, opt_state = gradient_transform.update(grad_circuit, opt_state)
            weights, weights_last = optax.apply_updates((weights, weights_last), upda
            train_acc = compute_accuracy(weights, weights_last, x_train, y_train)
            print(f"Training at Epoch {step+1}/{n_epochs}, Cost {train_cost}, Acc {tr

            train_cost_batches.append(train_cost)
            train_acc_batches.append(train_acc)

        train_cost_epochs.append(np.mean(train_cost_batches))
        train_acc_epochs.append(np.mean(train_acc_batches))

        # Validation loop
        for x_val, y_val in valloader:
            x_val, y_val = jnp.asarray(x_val.numpy()), jnp.asarray(y_val.numpy())
            val_out = compute_out(weights, weights_last, x_val, y_val)
            val_pred = jnp.argmax(val_out, axis=1)
            val_acc = jnp.sum(val_pred == y_val) / len(val_out)
            val_cost = jnp.nanmean(optax.softmax_cross_entropy_with_integer_labels(va

            val_cost_batches.append(val_cost)
            val_acc_batches.append(val_acc)

        val_cost_epochs.append(np.mean(val_cost_batches))
        val_acc_epochs.append(np.mean(val_acc_batches))

    # Testing loop
    test_cost_batches, test_acc_batches = [], []
    for x_test, y_test in testloader:
        x_test, y_test = jnp.asarray(x_test.numpy()), jnp.asarray(y_test.numpy())
        test_out = compute_out(weights, weights_last, x_test, y_test)
        test_pred = jnp.argmax(test_out, axis=1)
        test_acc = jnp.sum(test_pred == y_test) / len(test_out)
        test_cost = jnp.nanmean(optax.softmax_cross_entropy_with_integer_labels(test_
        print('TEST ACC:', test_acc,'TEST COST:', test_cost)
        test_cost_batches.append(test_cost)
        test_acc_batches.append(test_acc)

    final_test_cost = np.mean(test_cost_batches)
    final_test_acc = np.mean(test_acc_batches)
```

```
        # Save weights to CSV
        save_weights_to_csv(weights, weights_last,n_epochs, file_name=file_name)

        return dict(
            n_train=[train_size] * n_epochs,
            n_val=[val_size] * n_epochs,
            n_test=[test_size] * n_epochs,
            step=np.arange(1, n_epochs + 1, dtype=int).tolist(),
            train_cost=[c.astype(float) for c in train_cost_epochs],
            train_acc=[c.astype(float) for c in train_acc_epochs],
            val_cost=[c.astype(float) for c in val_cost_epochs],
            val_acc=[c.astype(float) for c in val_acc_epochs],
            test_cost=final_test_cost.astype(float),
            test_acc=final_test_acc.astype(float)
        )

    # Run Loop 1: Random Initialization
    results_random = run_training(init_weights_random, file_name='random_weights.csv')

    # Run Loop 2: Beta Initialization
    results_beta = run_training(init_weights_beta, file_name='beta_weights.csv')

    # Return results for both initializations along with the testloader
    return results_random, results_beta, testloader
```

```
# n_epochs = 500
# n_reps = 20
# batch_size = 5000

# train_sizes = [int(0.8 * len(CTImgHerm()))]

# def run_iterations():
#     results_df = pd.DataFrame(
#         columns=["train_acc", "train_cost", "test_acc", "test_cost", "step", "n_train"]
#     )

#     for _ in range(n_reps):
#         results = train_vqc(n_epochs=n_epochs, batchsize=batch_size)
#         results_df = pd.concat(
#             [results_df, pd.DataFrame.from_dict(results)], axis=0, ignore_index=True
#         )

#     return results_df

# results_df = run_iterations()

n_epochs = 500
n_reps = 20
```

```
n_reps = 20
batch_size = 5000

# train_sizes = [int(0.8 * len(CTImgHerm()))]  # 80% train size

def run_iterations():
    results_random_df = pd.DataFrame(
        columns=["train_acc", "train_cost", "val_acc", "val_cost",
                 "test_acc", "test_cost", "step", "n_train"]
    )

    results_beta_df = pd.DataFrame(
        columns=["train_acc", "train_cost", "val_acc", "val_cost",
                 "test_acc", "test_cost", "step", "n_train"]
    )

    for _ in range(n_reps):
        # Train the model and get results
        # results = train_vqc_r(n_epochs=n_epochs, batchsize=batch_size)
        results_random, results_beta, testloader = train_vqc(batchsize=batch_size, n_epoc

        # Convert results to DataFrame and append to the results DataFrame
        results_random_df = pd.concat(
            [results_random_df, pd.DataFrame.from_dict(results_random)],
            axis=0, ignore_index=True
        )

        results_beta_df = pd.concat(
            [results_beta_df, pd.DataFrame.from_dict(results_beta)],
            axis=0, ignore_index=True
        )

    return results_random_df,results_beta_df,testloader

# Run the iterations and store the results
results_random_df,results_beta_df,testloader = run_iterations()
```

```
Random Init
Epoch 1/500 - Training...
/usr/local/lib/python3.10/dist-packages/jax/_src/lax/lax.py:3373: ComplexWarning: Cas
  x_bar = _convert_element_type(x_bar, x.aval.dtype, x.aval.weak_type)
Training at Epoch 1/500, Cost 0.683920462217599, Acc 0.6
Epoch 2/500 - Training...
Training at Epoch 2/500, Cost 0.6786253168212966, Acc 0.5866666666666667
Epoch 3/500 - Training...
Training at Epoch 3/500, Cost 0.6744901445674198, Acc 0.6533333333333333
Epoch 4/500 - Training...
Training at Epoch 4/500, Cost 0.6715121241331398, Acc 0.6533333333333333
Epoch 5/500 - Training...
Training at Epoch 5/500, Cost 0.6693050508313451, Acc 0.6533333333333333
Epoch 6/500 - Training...
Training at Epoch 6/500, Cost 0.6673731910754609, Acc 0.6533333333333333
Epoch 7/500 - Training...
```

```
Epoch 7/500 - Training...
Training at Epoch 7/500, Cost 0.6654160283852819, Acc 0.6666666666666666
Epoch 8/500 - Training...
Training at Epoch 8/500, Cost 0.6634034329329047, Acc 0.64
Epoch 9/500 - Training...
Training at Epoch 9/500, Cost 0.6614985112357895, Acc 0.6933333333333334
Epoch 10/500 - Training...
Training at Epoch 10/500, Cost 0.6599028823394565, Acc 0.7066666666666667
Epoch 11/500 - Training...
Training at Epoch 11/500, Cost 0.6587146879823255, Acc 0.7066666666666667
Epoch 12/500 - Training...
Training at Epoch 12/500, Cost 0.6578767070025507, Acc 0.7466666666666667
Epoch 13/500 - Training...
Training at Epoch 13/500, Cost 0.6572242904610075, Acc 0.7466666666666667
Epoch 14/500 - Training...
Training at Epoch 14/500, Cost 0.6565835840075206, Acc 0.7333333333333333
Epoch 15/500 - Training...
Training at Epoch 15/500, Cost 0.6558476461873717, Acc 0.7466666666666667
Epoch 16/500 - Training...
Training at Epoch 16/500, Cost 0.6549970181361655, Acc 0.7466666666666667
Epoch 17/500 - Training...
Training at Epoch 17/500, Cost 0.6540767420968209, Acc 0.7333333333333333
Epoch 18/500 - Training...
Training at Epoch 18/500, Cost 0.653148383947802, Acc 0.7333333333333333
Epoch 19/500 - Training...
Training at Epoch 19/500, Cost 0.6522468461643489, Acc 0.72
Epoch 20/500 - Training...
Training at Epoch 20/500, Cost 0.6513625026089405, Acc 0.7066666666666667
Epoch 21/500 - Training...
Training at Epoch 21/500, Cost 0.6504531313444936, Acc 0.6933333333333334
Epoch 22/500 - Training...
Training at Epoch 22/500, Cost 0.6494772214136342, Acc 0.6933333333333334
Epoch 23/500 - Training...
Training at Epoch 23/500, Cost 0.64842340431054, Acc 0.7066666666666667
Epoch 24/500 - Training...
Training at Epoch 24/500, Cost 0.6473160568682821, Acc 0.6933333333333334
Epoch 25/500 - Training...
Training at Epoch 25/500, Cost 0.6461974473038606, Acc 0.6933333333333334
Epoch 26/500 - Training...
Training at Epoch 26/500, Cost 0.6451008040543521, Acc 0.7066666666666667
Epoch 27/500 - Training...
Training at Epoch 27/500, Cost 0.6440309005119483, Acc 0.72
Epoch 28/500 - Training...
```

```
print('last epoch results')
print("RANDOM")
print(results_random_df.iloc[-1])
print("BETA")
print(results_beta_df.iloc[-1])
```

```
last epoch results
RANDOM
train_acc     0.826667
train_cost     0.55343
val_acc            0.8
```

```
val_cost       0.65327
test_acc          0.65
test_cost     0.613101
step               500
n_train             75
n_val              5.0
n_test            20.0
Name: 9999, dtype: object
BETA
train_acc     0.853333
train_cost    0.545025
val_acc            0.4
val_cost      0.678823
test_acc          0.75
test_cost     0.604731
step               500
n_train             75
n_val              5.0
n_test            20.0
Name: 9999, dtype: object
```

```
from google.colab import files

results_beta_df.to_csv('CT-beta-df.csv',index=False)
results_random_df.to_csv('CT-random-df.csv',index=False)

files.download('CT-beta-df.csv')
files.download('CT-random-df.csv')
```

```
import pandas as pd

def load_weights_and_biases_from_csv(file_name):
    # Load the weights DataFrame
    weights_data = pd.read_csv(file_name)

    # Get the last epoch's weights and biases
    last_epoch = weights_data.iloc[-1]  # Get the last row
    weights = np.array(eval(last_epoch['weights']))  # Convert string representation back
    biases = np.array(eval(last_epoch['biases']))  # Convert string representation back t

    return weights, biases
```

```
import pandas as pd
import ast
import jax.numpy as jnp
import torch
import optax
```

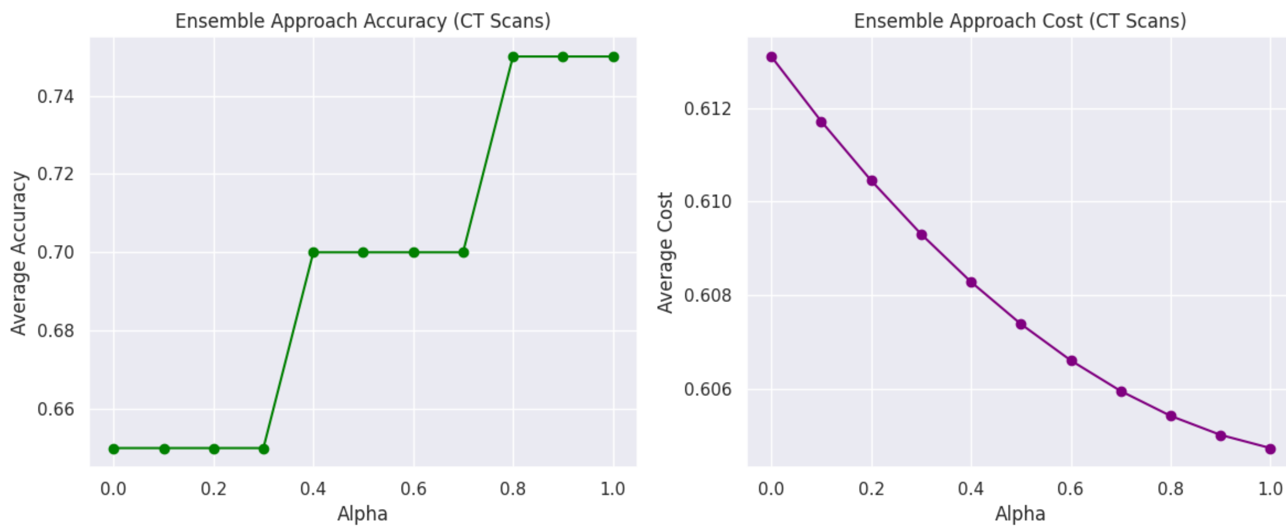
```
def ensemble_test_model(testloader, weights1, weights_last1, weights2, weights_last2, alp
    """Test the ensemble model using weights from two models and a given alpha."""
    total_cost = 0
    correct_predictions = 0
    total_samples = 0

    for x_test, y_test in testloader:
        x_test, y_test = jnp.asarray(x_test.numpy()), jnp.asarray(y_test.numpy())

        test_out1 = compute_out(weights1, weights_last1, x_test, y_test)
        test_out2 = compute_out(weights2, weights_last2, x_test, y_test)

        # Weighted ensemble output
        ensemble_out = alpha * test_out1 + (1 - alpha) * test_out2
        ensemble_pred = jnp.argmax(ensemble_out, axis=1)

        correct_predictions += jnp.sum(jnp.array(ensemble_pred == y_test).astype(int))
        total_samples += len(y_test)

        test_cost = jnp.nanmean(optax.softmax_cross_entropy_with_integer_labels(ensemble_
        total_cost += test_cost * len(y_test)

    avg_test_cost = total_cost / total_samples
    avg_test_acc = correct_predictions / total_samples

    return avg_test_cost, avg_test_acc

# Load weights for both models
weights1, weights_last1 = load_weights_and_biases_from_csv('beta_weights.csv')
weights2, weights_last2 = load_weights_and_biases_from_csv('random_weights.csv')
for x_test, y_test in testloader:
  x_test, y_test = jnp.asarray(x_test.numpy()), jnp.asarray(y_test.numpy())
  myout2 = compute_out(weights2,weights_last2,x_test,y_test)
  my2pred = jnp.argmax(myout2, axis=1)
  myacc = jnp.sum(my2pred == y_test) / len(myout2)


# Alpha values from 0.0 to 1.0 in steps of 0.1
alpha_values = [i / 10 for i in range(11)]

# Store results for each alpha
results = []

# Perform ensemble testing for each alpha (one run per alpha)
for alpha in alpha_values:
    print(f"Testing with alpha={alpha}")


    # Run the ensemble test model with the current alpha
```

```
    # Run the ensemble test model with the current alpha
    avg_cost, avg_acc = ensemble_test_model(testloader, weights1, weights_last1, weights2

    # Store the result for each alpha
    results.append([alpha, avg_cost, avg_acc])

# Create a DataFrame to display results in table format
results_df = pd.DataFrame(results, columns=['Alpha', 'Average Cost', 'Average Accuracy'])

# Display the final table of results
print("\nEnsemble Test Results by Alpha:")
print(results_df)

# Save the results to a CSV file
results_df.to_csv('ensemble_test_results_by_alpha.csv', index=False)
```

```
Testing with alpha=0.0
Testing with alpha=0.1
Testing with alpha=0.2
Testing with alpha=0.3
Testing with alpha=0.4
Testing with alpha=0.5
Testing with alpha=0.6
Testing with alpha=0.7
Testing with alpha=0.8
Testing with alpha=0.9
Testing with alpha=1.0

Ensemble Test Results by Alpha:
    Alpha        Average Cost Average Accuracy
0     0.0  0.6131014909065542             0.65
1     0.1  0.6117167292880316             0.65
2     0.2  0.6104518410481592             0.65
3     0.3  0.6093076126287471             0.65
4     0.4  0.6082847787208743              0.7
5     0.5  0.6073840203418356              0.7
6     0.6   0.606605963017278              0.7
7     0.7  0.6059511750792497              0.7
8     0.8  0.6054201660904043             0.75
9     0.9  0.6050133854040143             0.75
10    1.0  0.6047312208687539             0.75
```

```
import matplotlib.pyplot as plt

# Data from the new table
alpha_ct = results_df['Alpha']
accuracy_ct = results_df['Average Accuracy']
cost_ct = results_df['Average Cost']

# Plot Accuracy vs Alpha
plt.figure(figsize=(12, 5))
plt.subplot(1, 2, 1)
plt.plot(alpha_ct, accuracy_ct, marker='o', linestyle='-', color='green')
```

```
plt.plot(alpha_ct, accuracy_ct, marker='o', linestyle='-', color='green')
plt.title('Ensemble Approach Accuracy (CT Scans)')
plt.xlabel('Alpha')
plt.ylabel('Average Accuracy')
plt.grid(True)

# Plot Cost vs Alpha
plt.subplot(1, 2, 2)
plt.plot(alpha_ct, cost_ct, marker='o', linestyle='-', color='purple')
plt.title('Ensemble Approach Cost (CT Scans)')
plt.xlabel('Alpha')
plt.ylabel('Average Cost')
plt.grid(True)

# Show the plots
plt.tight_layout()
plt.show()
```




```
import matplotlib.pyplot as plt
import seaborn as sns

# Aggregate data for plotting (mean and std per epoch)
df_random_agg = results_random_df.groupby(["step"]).agg(["mean", "std"]).reset_index()
```

```
df_beta_agg = results_beta_df.groupby(["step"]).agg(["mean", "std"]).reset_index()

sns.set_style("whitegrid")
colors = sns.color_palette()
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(16, 8))  # 1 row, 2 columns for 2 graph
axes = axes.flatten()  # Flatten for easy indexing

# Titles and axis labels
titles = ["Loss", "Accuracy"]
ylabels = ["Loss", "Accuracy"]

# Plot metrics
for i, (df, label) in enumerate([
    (df_random_agg, "Random"),
    (df_beta_agg, "Beta"),
]):
    # Loss: Train and Validation
    train_loss = df.train_cost["mean"]
    val_loss = df.val_cost["mean"]
    # Accuracy: Train and Validation
    train_acc = df.train_acc["mean"]
    val_acc = df.val_acc["mean"]

    # Loss plot
    axes[0].plot(
        df.step, train_loss,
        label=f"{label} Train Loss",
        linestyle="--" if label == "Random" else "-",
        color=colors[2 * i],  # Unique color
        alpha=0.8
    )
    axes[0].plot(
        df.step, val_loss,
        label=f"{label} Val Loss",
        linestyle="--" if label == "Random" else "-",
        color=colors[2 * i + 1],  # Unique color
        alpha=0.8
    )

    # Accuracy plot
    axes[1].plot(
        df.step, train_acc,
        label=f"{label} Train Acc",
        linestyle="--" if label == "Random" else "-",
        color=colors[2 * i],  # Unique color
        alpha=0.8
    )
    axes[1].plot(
        df.step, val_acc,
        label=f"{label} Val Acc",
        linestyle="--" if label == "Random" else "-",
```

```
        linestyle='--' if label == 'Random' else '-',
        color=colors[2 * i + 1],  # Unique color
        alpha=0.8
    )

# Format each subplot
for idx, ax in enumerate(axes):
    ax.set_title(titles[idx], fontsize=14)
    ax.set_xlabel("Epoch")
    ax.set_ylabel(ylabels[idx])
    if idx == 1:  # Accuracy plot
        ax.set_ylim(0.05, 1.05)  # Accuracy typically ranges between 0 and 1
    ax.legend()

plt.tight_layout()
plt.show()
```