

$$\frac{1.5}{2}$$

NC Assignment 1 21K-3153

(1) bisection , $f(x) = \sqrt{x} - \cos(x)$
absolute error < 0.01

$$[0, 1]$$

$$f(0) = -1 \quad f(1) = 1.5 \times 10^{-4}$$

$$\text{iterations } n = \log_2 \left(\frac{|1-0|}{0.01} \right) = 6.6437$$

n	a	b	c	f(c)
1	0	1	0.5	-0.2928551419
2	0.5	1	0.75	-0.1343565344
3	0.5	0.75	0.625	-0.02039370446
4	0.625	0.75	0.6875	0.05632125144
5	0.625	0.6875	0.65625	0.0178061262
6	0.625	0.65625	0.640625	-1.33184449 $\times 10^{-3}$
7	0.640625	0.65625	0.6484375	8.227740279 $\times 10^{-3}$

$$0.6484375$$

(Q2) False position

$$\sqrt{x} - \cos x \quad \text{error} < 0.01$$

$$\begin{matrix} a & b \\ [0, 1] \end{matrix}$$

$$c = a - \frac{f(a)(b-a)}{f(b)-f(a)}$$

$$\begin{aligned} f(0) &= - \\ f(1) &= + \end{aligned}$$

n	a	b	c	f(c)	f(c)
1	0	1	0.6850733573		0.0533894965
2	0	0.6850733573	0.6503949801		0.01062599405
3	0	0.6503949801	0.643556552		2.253032581 × 10 ⁻²

error

0.5

0.05

0.006 → error < 0.01

0.006

0.006

③ $x = 2e^{-x} - x$ bisection and regular fab $[0, 1]$
percent relative true error $< 1\%$

$$f(0) = 2 \quad f(1) = 0 \quad 1\% = 0.01$$

bisection :

$$n = \log_2 \left(\frac{1}{0.01} \right) = \text{Iterations}$$

n	a	b	c	f(c)
1	0	1	0.5	1.213061319
2	0.5	1	0.75	0.1947331055
3	0.75	1	0.875	-0.04127546064
4	0.75	0.875	0.8125	0.07499462016
5	0.8125	0.875	0.84375	0.01643928128
6	0.84375	0.875	0.859375	-0.01252171743
7	0.84375	0.859375	0.8515625	

$$0.8515625 \approx 0.85156$$

$$x = 2e^{-x}$$

$$2e^{-x} - x = 0$$

False position				
n	a	b	c	f(c)
1	0	1	0.8832981542	-0.0564638995
2	0	0.8832981542	0.8590456379	-0.0119138801
3	0	0.8590456379	0.8539588662	-2.5 × 10 ⁻³

root

$$= 0.8539588662$$

$$0.85396$$

error

$$0.11 \dots$$

$$6.02 \dots$$

$$5 \times 10^{-3}$$

$$0.005 \rightarrow \text{stop}$$

QUESTION 5

$$f(x) = x^4 - x - 10$$

$$x^4 + 10 = x^4$$

$$x = \sqrt[4]{x+10}$$

$$x_0 = 4$$

$$\text{error} < 0.00001$$

testing over $[1, 2]$

$$f(1) = 1.8 \quad f(2) = 1.8 \quad f(1.5) = 1.8 \quad \checkmark$$

$$0.0002 \times 10^{-5}$$

$$0.00001 \times 10^{-4}$$

on	x_i	$g(x_i)$	error $ x_{i+1} - x_i $
1	4	1.93433642	
2	1.93433642	1.858658358	0.07567806174
3	1.858658358	1.855704793	0.00295356451
4	1.855704793	1.855589234	0.000115559
5	1.855589234	1.855584713	0.00004...

$$\boxed{1.855584713} \rightarrow \text{root}$$

QUESTION 9

$$f(x) = x^{\frac{1}{2}} + x^{-\frac{1}{2}} - 3$$

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} - \frac{1}{2} x^{-\frac{3}{2}}$$

$$0.000324 \times 10^{-7}$$

$$x_0 = 2$$

$$\text{error} < 0.0001$$

n	x_i	error
1	2	4.970562748
2	6.910562748	0.46785618
3	6.85377713	0.000324
4	6.854101964	
5	6.854101966	

$$2.1 \times 10^{-9}$$

stop

$$6.854101966$$

$$\approx 6.85410$$



QUESTIONS

$$P(t) = t \cos(t) + \sin(t)$$

$$t_0 = 2 \quad t_1 = 3$$

$$\text{error} < 0.00001$$

Q

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i+1} - x_i)}{f(x_{i+1}) - f(x_i)}$$

$$0.002 \times 10^{-1}$$

$$0.0001 \times 10^{-4}$$

n	t ₀	t ₁	t _{next}	error
1	2	3	2.02649946	0.9755000
2	3	2.02849946	2.02859488	0.002045
3	2.02859488	2.0		
3	2.02649946	2.02859488	2.028757961	0.000163
4	2.02859488	2.028757961	2.028757961	0.00000014

2.028757961