# DISCRETE ASSIGNMENT 2
21K-3153

(Q1)

(i) Undirected edges, multiple edges, ~~more than one loops~~ no loops, undirected multigraph

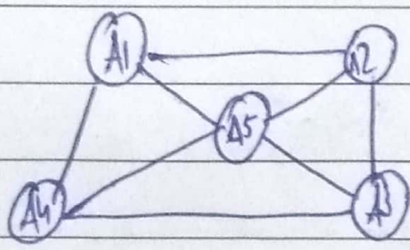
(2) undirected eges, no multiple edges, no loops, undirected simple graph

(3) Undirected edges, multiple edges, ~~more than~~ three loops, undirected pseudo graph

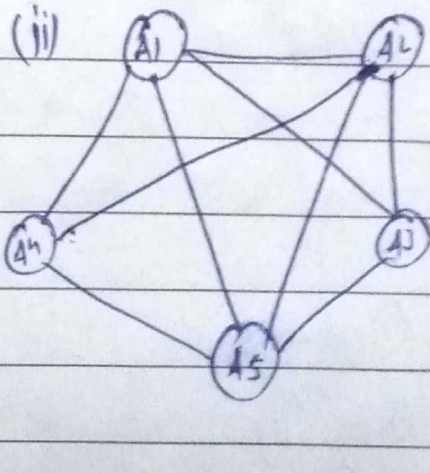
(4) directed edges, multiple edges, 2 loops, directed multi graph

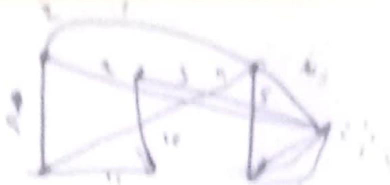
(Q2)

(i)




(ii)





(Q3)

(a)

(i) Vertices = 5

Edges = 13

deg(a) = 6 deg(b) = 6 deg(c) = 6 ~~deg~~

deg(d) = ~~4~~ 5 deg(e) = 3

(ii) Vertices = 9

Edges = 12

deg(a) = 3 deg(b) = 2 deg(c) = 4 deg(d) = 0

deg(e) = 6 deg(f) = 0 ~~deg(i)~~ 3 ~~deg(h)~~ deg(g) = 4

deg(h) = 2 deg(i) = 3

(b)

(i) Vertices = 5

Edges = 13

Indegree:

$deg^-(a) = 6$ ~~$deg^- b) = 1$~~ $deg^-(b) = 1$ $deg^-(c) = 2$

$deg^-(d) = 4$ $deg^-(e) = 0$

Out degree:

$deg^+(a) = 1$ $deg^+(b) = 5$ $deg^+(c) = 5$ $deg^+(d) = 2$

$deg^+(e) = 0$

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(ii) Vertices = ~~a~~ 4

Edges = 10

Indegree:

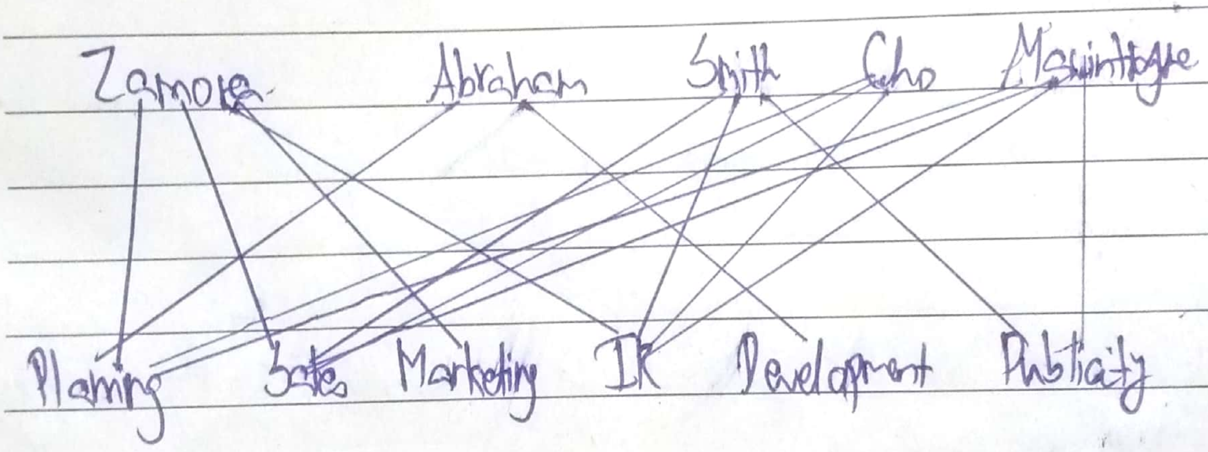
$deg^-(a) = 2$ $\quad deg^-(b) = 3$ $\quad deg^-(c) = 2$ $\quad deg^-(d) = 1$

Outdegree:

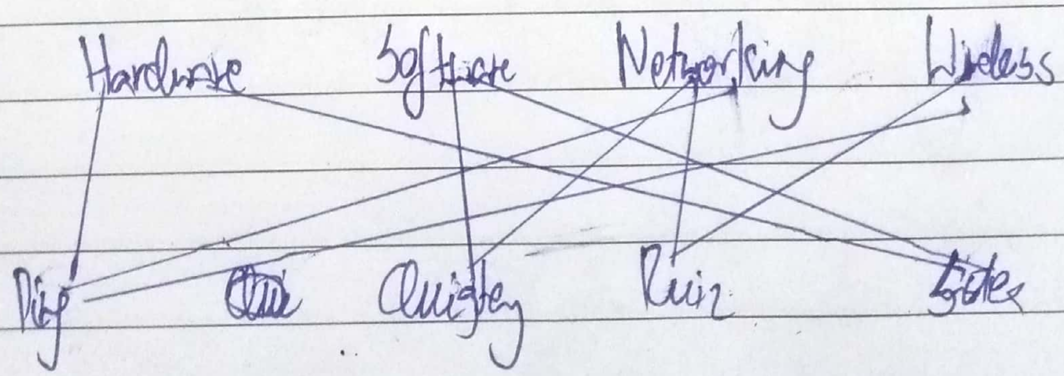
$deg^+(a) = 2$ $\quad deg^+(b) = 4$ $\quad deg^+(c) = 1$ $\quad deg^+(d) = 1$

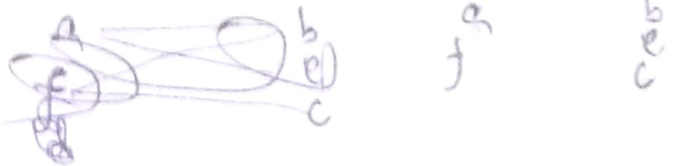
(Q4)

(a) Bipartite




(b)





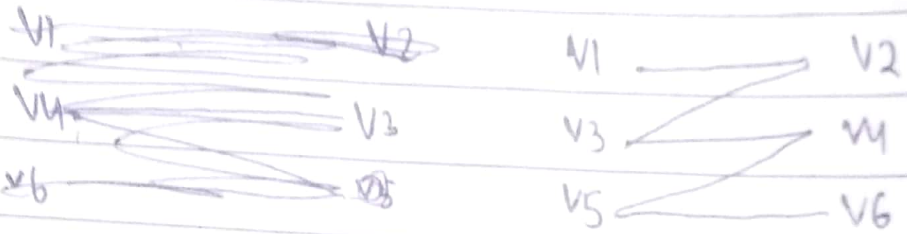

(5)

(i) Not bipartite (adjacent to b and f)

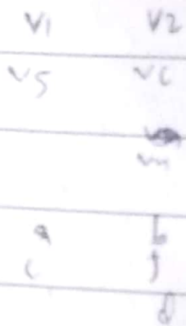
(ii)

(iii)

(ii) Bipartite




iii) Not bipartite (V4 and V5 adjacent)

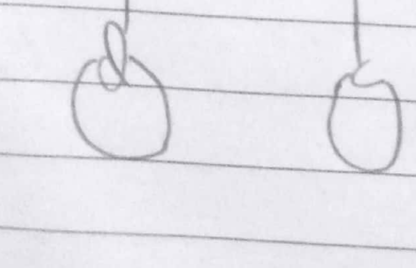


iv) Not bipartite (d and e adjacent)

Q 6) (a) No such graph exists according to Handshaking Theorem

$7 = 2m$ $\quad m \neq 3.5$

(b)



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(c) No simple graph like that exists

7(a) $15 \times 3 = 45 \rightarrow$ odd

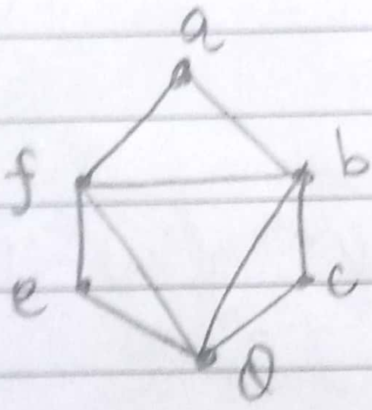
No, this cannot be possible according to Handshaking theorem as sum of all edges cannot be odd

$45 \neq 2m$

(b) Yes, as $4 \times 3 = 12$ and $12/2 = 6$

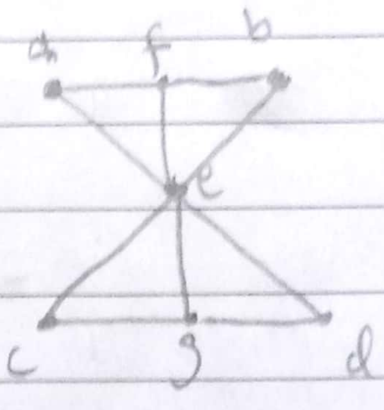
A graph is possible according to Handshaking Theorem.

8) (i)





(ii)





(b) 40 $4 \times V = 2 \times 10$

$4V = 20$

$\boxed{V = 5} \rightarrow$ 5 vertices

(3) (i)

Is isomorphic

$g(v_1) = w_2$ $g(v_2) = w_2$ $g(v_3) = w_1$

$g(v_4) = w_5$ $g(v_5) = w_4$

(ii) is ~~isompo~~ isomorphic:

$g(v_1) = u_5$ $g(v_2) = u_2$ $g(v_3) = u_4$
$g(v_4) = u_3$ $g(v_5) = u_1$ $g(v_6) = u_6$

(iii) is isomorphic

$g(v_1) = u_5$ $g(v_2) = u_4$ $g(v_3) = u_3$ $g(v_4) = u_2$
$g(v_5) = u_7$ $g(v_6) = u_1$ $g(v_7) = u_6$

(iv) Not isomorphic:
G' has a vertex of degree 4 while G does not

Q10

(i)

| N | b | c | d | e | f | g | h | i | j | k | l | m | n | o | p q | r | s | t | z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a | 2,a | 4,a | 1,a | | | | | | | | | | | | | | | | |
| ad | 2,a | 4,a | | | 6,d | 5,d | | | | | | | | | | | | | |
| adb | | 4,a | | 3,b | 6,d | 5,d | | | | | | | | | | | | | |
| adbe | | 4,a | | | 6,d | 5,d | 6,e | | | | | | | | | | | | |
| abdec | | | | | 6,c | 5,d | 6,e | | | | | | | | | | | | |
| abdecg | | | | | 6,c | | 6,e | | | 7,g | | | | | | | | | |
| abdecgf | | | | | | | 6e | 8,f | 10,f | 7,g | | | | | | | | | |
| abdecgfh | | | | | | | | 8,f | 10,f | 7,g | 9,h | | | 14,h | | | | | |
| abdecgfhk | | | | | | | | 8,f | 10,f | | 7,h | | 11,k | 14,h | | 9,k | | | |
| abdecgfhkl | | | | | | | | 8,f | 10,f | | | 10,l | 11,k | 13,l | | 9,k | | | |
| abdecgfhkli | | | | | | | | | 10,f | | | 10,l | 11k | 13l | | 9k | | | |
| abdecgfhkli | | | | | | | | | 10f | | | 10l | 10r | 13l | | 17r | | | 14r |
| abdecgfhkl rj | | | | | | | | | | | | 10l | 10r | 13l | | 17r | | | 14r |
| abdecgfhkl m | | | | | | | | | | | | | 10r | 13l | 12m | 17r | | | 14r |
| abdecgfhkl mn | | | | | | | | | | | | | | 13l | 12m | 12n | | | 14r |
| abdecgfhkl mnp | | | | | | | | | | | | | | 13l | | 12n | 14p | | 13p |
| abdecgfhkl mnpq | | | | | | | | | | | | | | 13l | | | | | |
| " pqo | | | | | | | | | | | | | | 13l | | | 14p | | 13p |
| " pqot | | | | | | | | | | | | | | | | | 14p | | |
| " pqots | | | | | | | | | | | | | | | | | 14p | | 16s z |
| " pqotsz | | | | | | | | | | | | | | | | | | | |

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adbecgfhklirmngqotsz

(ii)

| N | b | c | d | e | f | g | z |
|---|---|---|---|---|---|---|---|
| a | 4,a | 3,a | | | | | |
| ac | 4,a | | 6,c | 9,c | | | |
| acb | | | 6,c | 9,c | | | |
| acbd | | | | 7,d | 11,d | | |
| acbde | | | | | 11,d | 12,e | |
| acbdef | | | | | | 12,e | 16,f |
| acbdefg | 4,a | 3,a | 6,c | 7,d | 11,d | 12,e | 16,g |

acbdefgz

(i)

ABCDA = 125  ADBCA = 155  ACDBA = 135

ABCDA is the shortest route

(ii) ABDCA = 108  ABCDA = 97

ACBDA = 141

ABCDA = 97 is the shortest

URBANE

Date:

12(a) Yes

~~A → H → B → C → D → G → F → E~~

A → H → G → B → C → D → G → F → E

12(b) Yes, as starting and ending vertex have odd degree while other vertices have ~~an even~~ an even degree

A → G → H → I → E → F → H → E → K → J → D → C → B

or

A → G → H → F → E → I → H → E → K → J → D → C → B

both correct

0, 1, 5, 4, 7, 6, 2, 3, 0

Q13

(i) $V_0, V_1, V_2, V_6, V_5, V_4, V_7, V_3, V_0$ = Circuit

Path = $V_0, V_1, V_2, V_6, V_5, V_4, V_7, V_3$

(ii) No Hamiltonian circuit

Path: b, c, f, g, h, e, a, d

(iii) Circuit = dcbagfed

Path: dcbagfe

Date: M T W T F S S

(14a) (i) Circuit exists as all vertices have an even degree

$V_1 \to V_2 \to V_5 \to V_4 \to V_5 \to V_2 \to V_3 \to V_4 \to V_1$

(ii) All vertices do not have an even degree, thus no euler circuit

(b)

(i) Euler Path does not exist are 4 vertices have an odd degree

(ii) ~~u, v0, v1~~ u, v1, v0, v7, u, v2, v3, v4, v2, v6, v5, w, v4, v6, w

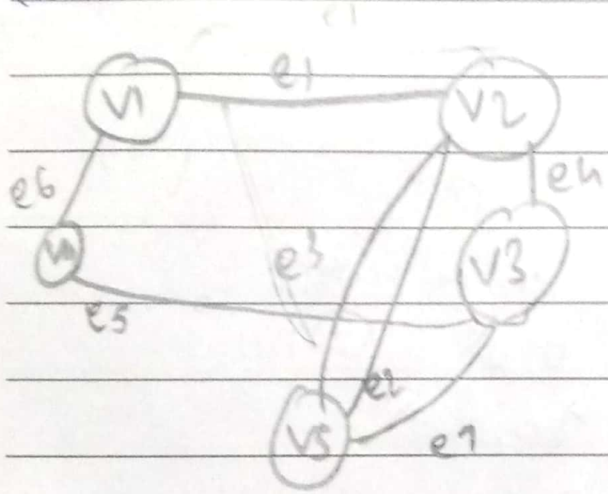
15(a)

| | | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ |
|---|---|---|---|---|---|---|---|---|
| (i) | $v_1$ | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| | $v_2$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| | $v_3$ | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| | $v_4$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| | $v_5$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| | $v_6$ | 1 | 0 | 0 | 0 | 0 | 0 | 1 |

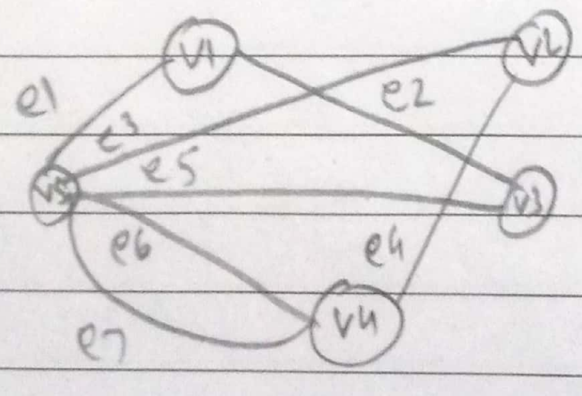
(ii)

| v/e | e1 | e2 | e3 | e4 | e5 | e6 | e7 | e8 |
|---|---|---|---|---|---|---|---|---|
| v1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| v2 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 |
| v3 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| v4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| v5 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |

(b)(i)




b(ii)



16(i)

| Initial Vertex | Terminal Vertices |
|---|---|
| a | a, b, c, d |
| b | d |
| c | a, b |
| d | b, c, d |

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 1 \\ b & 1 & 0 & 1 & 1 \\ c & 1 & 1 & 0 & 1 \\ d & 1 & 2 & 1 & 1 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(ii)

| Initial vertex | Terminal vertex |
|---|---|
| a | b, d |
| b | a, c, d, e |
| c | b, c |
| d | a, e |
| e | c, e |

$$\begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 2 & 0 & 1 & 0 \\ b & 2 & 0 & 2 & 1 & 1 \\ c & 0 & 2 & 1 & 0 & 0 \\ d & 2 & 0 & 0 & 0 & 1 \\ e & 0 & 0 & 1 & 0 & 1 \end{array}$$

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

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(17)(i) Level 3

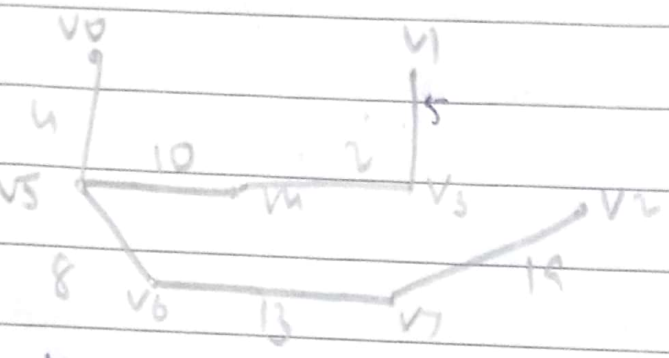
(iii) 5

(v) 8

(vii) r, s, t, x, y

(ix) v, n, h, d, a

(18i)




$V0\,V5 = 4$

$V5\,V6 = 8$

$V6\,V7 = 13$

$V7\,V2 = 19$

$V5\,V4 = 10$

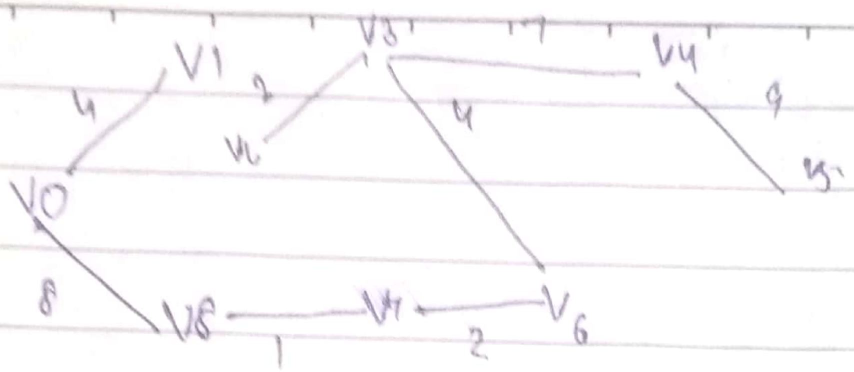
$V_4V_3 = 2$

$V_3V_1 = 5$

MST = 61

Date:

(18ii)




V0 V1 = 4

V0 V8 = 8

V8 V7 = 1

V7 V6 = 2

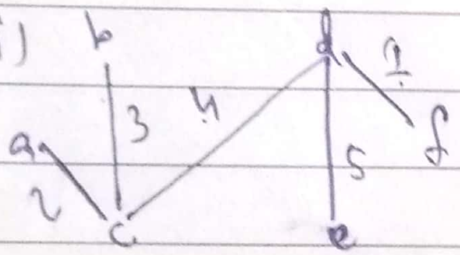
V6 V3 = 4

V3 V4 = 7

V4 V5 = 9

MST = 37

(19)(i)




MST = 15

ac = 2

cb = 3

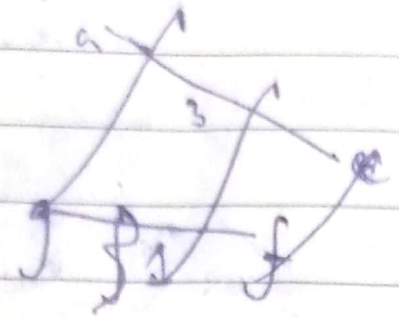
cd = 4

de = 5

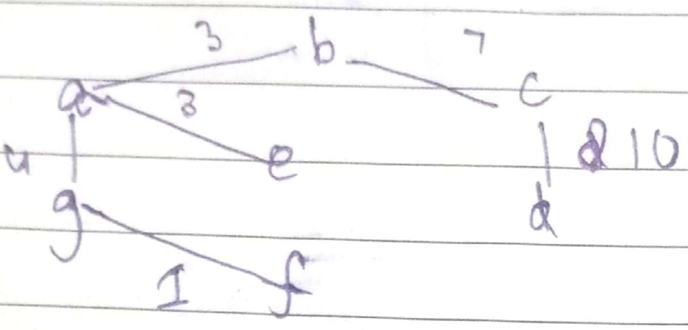
df = 1

15

(iii)



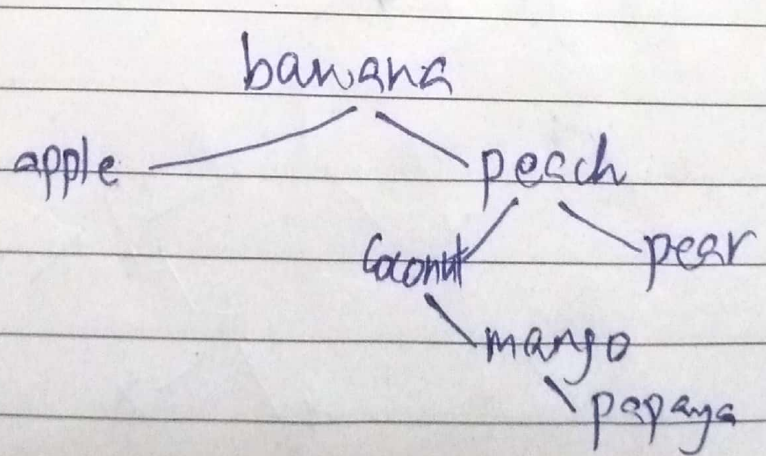

(ii)



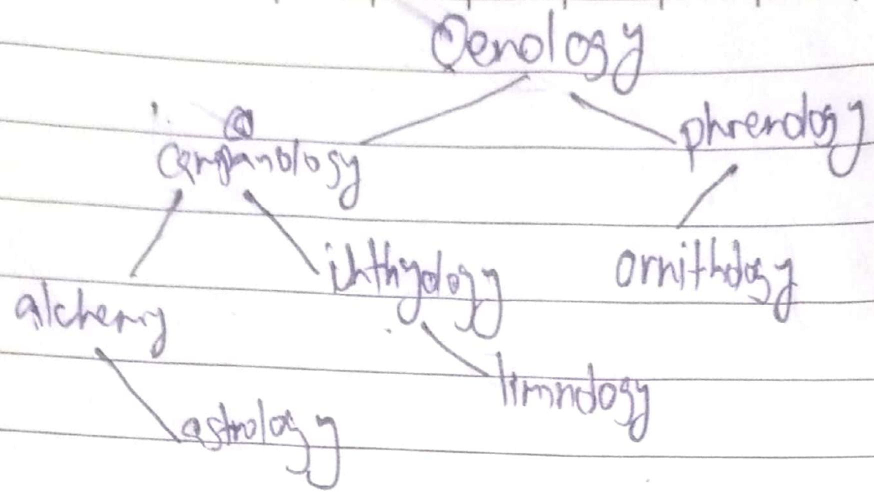

gf = 1
ae = 3
ab = 3
ag = 4
bc = 7
cd = 10

$\boxed{MST = 28}$

2019
(I)

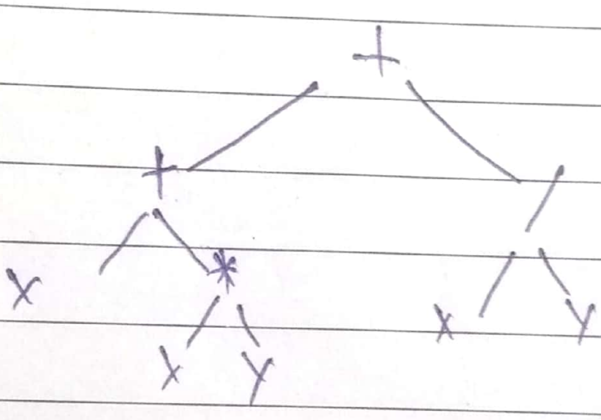


(ii)

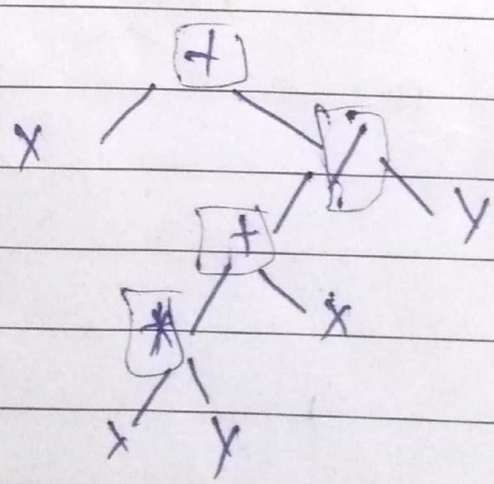



(b)

(i) $(x+xy) + (x/y)$




(ii) $x + ((xy+x)/y)$



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(2)(i)

Preorder: a b c d

ac b d

a b e j k t m f g n r s c d h o i j p q

Postorder:

k l m e f g r s n g b c o h l p q j d a

In order:

k e l m b f r n s g a c o h d i p j q

(ii) Pre order:

a b d e i j m n o c f g h k l p

Postorder:

d i m n o j e b f g k p l h c a

In order:

d b i e m j n o a f c g k h p l

2(a)

(i) $10000 - 1 = 9999$ edge

(b) $1000 * 2 = 2000$ edge

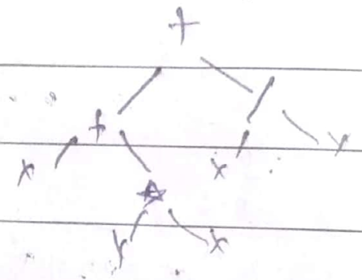
(c) $n = mi + 1$

$5 \times 100 + 1$

$\boxed{501}$ vertices

23(a)

(i) $(x + xy) + (x / y)$

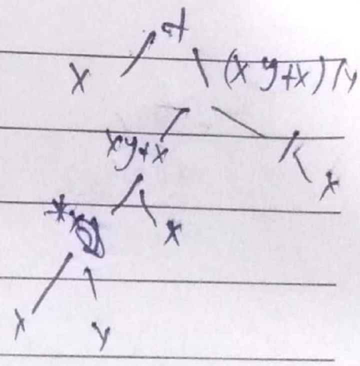



Prefix: $\boxed{+ + x * x y / x y}$

Postfix:

$\boxed{x x y * + x y / +}$

(ii) $x + ((xy + x) / y)$




~~Prefix~~ Prefix

$\boxed{+ x / + * x y x y}$

Postfix:

$\boxed{x x y * x + y / +}$

(b) (i) + - ↑ 3 2 ↑ 2 3 / 6 - 4 2

+ - ↑ 3 2 ↑ 2 3 / 6 2

+ - ↑ 3 2 ↑ 2 3 3

+ - ↑ 3 2 8 3

+ - 9 8 3

+ 1 3

$\boxed{4}$

(ii) 4 8 + 6 5 - * 3 2 - 2 2 + * /

12 6 5 - * 3 2 - 2 2 + * /

12 1 * 3 2 - 2 2 + * /

12 3 2 - 2 2 + * /

12 1 2 2 + * /

12 1 4 * /

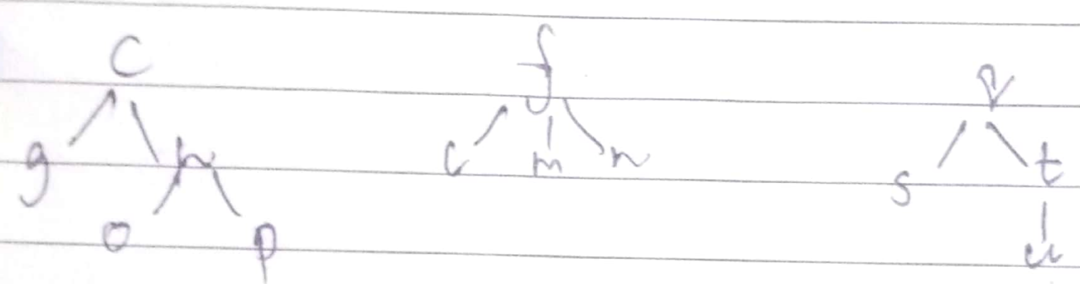
12 4 /

$\boxed{3}$

(i) Not a full m-ary tree as some have 3 children while others have 2

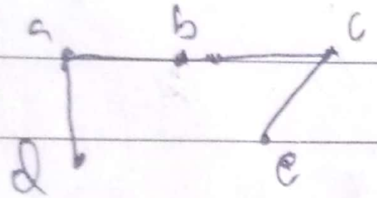
(ii) Not a balanced tree as it has leaves at level 2

(iii)



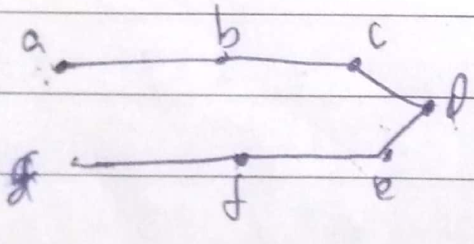

(25)

(i)



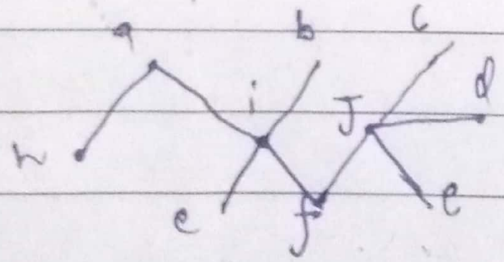

db and de removed

(ii)

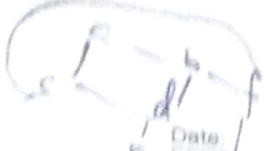



ad, gd, ge, be, bf, bj removed

(iii)

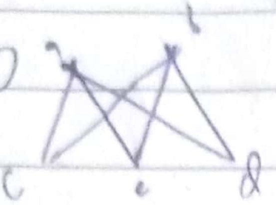



hi, ij, hg, gf, fe, ab, bc, cd removed

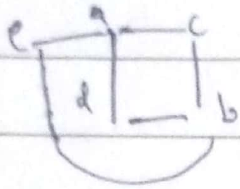


(f) $l_m$.

(2b)



(b) not planar



planar

(Q1)

$R = \{(a,a), (a,c), (a,d), (b,a), (b,b), (b,c), (b,d), (c,b), (c,c), (d,b), (d,d)\}$

(a) Reflexive : Yes as it contains $(a,a)(b,b)(c,c)(d,d)$

(b) Symmetric: not symmetric as $(a,c) \in R$ but $(c,a) \notin R$

(c) Antisymmetric: not antisymmetric and $(b,c) \in R$ and $(c,b) \in R$ but ~~b≠c~~ $b \neq c$

d) Transitive: is transitive as $(b,a) \in R$ and $(a,d) \in R$ and $(b,d) \in R$

(e) Irreflexive : not irreflexive as $(a,a)(b,b)(c,c)(d,d)$ exist

(f) Asymmetric: not asymmetric as R is not antisymmetric

(Q28) $A = \{0,1,2,3,4\}$ $B = \{0,1,2,3\}$

(a) $a = b$

$\{(0,0)\ (1,1)\ (2,2)\ (3,3)\}$

(b) $a+b=4$

$\{(1,3), (2,2), (3,1), (4,0)\}$

(c) $a > b$

$\{(4,3)\}$

~~$\{(0,1)\ (0,2)\ (0,3)\ (1,2)\ (1,2)\ (2,3)\ (4,3)\}$~~

$\{(1,0)\ (2,0)\ (2,1)\ (3,2)\ (4,3)\ (3,0)\ (3,1)$
$(3,2)\ (4,0)\ (4,1)\ (4,2)\}$

(d) $a \mid b$

$\{(1,0)\ (2,0)\ (3,0)\ (4,0)\ (1,1)\ (1,2)$
$(2,2)\ (1,3)\ (3,3)\}$

(e) $\gcd(a,b) = 1$

$\{(1,0)\ (0,1)\ (1,1)\ (1,2)\ (1,3)\ (2,1)\ (3,1)\ (4,1)\ (2,3)$
$(3,2)\ (4,3)\}$

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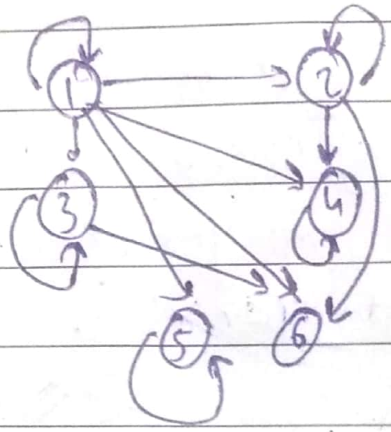
(f) lcm(a,b) = 2

{ (1,2) (2,1) (2,2) }

(Q 28) { (a,b) | a divides b } { 1, 2, 3, 4, 5, 6 }

{ (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,4) (2,6) (3,6)
(2,2) (3,3) (4,4) (5,5) (6,6) }

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 0 | 1 | 0 | 1 | 0 | 1 |
| 3 | 0 | 0 | 1 | 0 | 0 | 1 |
| 4 | 0 | 0 | 0 | 1 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 1 | 0 |
| 6 | 0 | 0 | 0 | | 0 | 1 |




{1, 2, 3, 4}

(Q30) (a) (2,2) (2,3) (2,4) (3,2) (3,3) (3,4)

not reflexive

not symmetric (2,4) ∈ R (4,2) ∉ R

not antisymmetric as 2,3 3,2 2 ≠ 3

is transitive as (2,3) (3,2) (2,2)
a b b c a c

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(b) $\{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,3)\ (4,4)\}$

is reflexive

is symmetric

is not antisymmetric as $1,2$ $2,1$ $2 \neq 1$

is transitive as $\underset{a\ b}{1,2}$ $\underset{b\ c}{2,1}$ $\underset{a\ c}{1,1}$

(c) $\{(2,4),(4,2)\}$

not reflexive, no $(1,1)$, $(3,3)$, $(2,2)$ or $(4,4)$

is symmetric

not antisymmetric $2,4$ $4,2$ $4 \neq 2$

not transitive as $2,2 \notin R$

(d) $\{(1,2)\ (2,3)\ (3,4)\}$

not reflexive

not symmetric

~~not~~ is antisymmetric as $b,a \notin R$

not transitive as $a,c$ does not exist

(e) $\{(1,1),(2,2),(3,3)\ (4,4)\}$

is reflexive

is symmetric

is antisymmetric

is transitive as for every pair $a,c \in R$

(f) {(1,3) (1,4) (2,3) (2,4) (3,1) (3,4)}

not reflexive
not symmetric
not antisymmetric $1 \neq 3$
not transitive

Q3) (a)

$a > b$ $(a,b)$

not reflexive as they cannot be the same
not symmetric as b,a cannot exist
is antisymmetric because b,a cannot exist
is asymmetric as $b,a \notin R$
is transitive as $(a_1, b_1) \in R$ & $(b_1, c_1) \in R$ then
$(a_1, c_1) \in R$

(b) a and b were born on the same day
is reflexive as they are the same
is symmetric as a,b would also mean b,a
is not antisymmetric as $b \neq a$, cannot be the same person
is transitive as if $(a,b) \to$ same day $(b,c) \to$ same day then $(a,c) \to$ same day

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(c) a has same first name as b
is reflexive as names are same
is symmetric as same first names can be flipped (b,a)
is not antisymmetric because a ≠ b, as same first names does not mean being the same person
is not asymmetric as b,a exist
is transitive as (a,b) → same name (b,c) → same name
(a,c) → same name

(d) a and b have a common grandparent

is reflexive
is symmetric
not antisymmetric as a and b are different people
not asymmetric
not irreflexive
is transitive as (a,b) → same grandparent
(b,c) → same grandparent (a,c) → same grandparent

32 (a)
both symmetric and antisymmetric
{(1,1) (2,2) (3,3) (4,4)}

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(h) neither symmetric nor antisymmetric

{(1,3) (2,4) (5,6) (3,1)}

{1,2,3}

(33) A = {1,2,3}

(a) R2∪R4

R2 = 2,1 3,1 3,2 1,1 2,2 3,3

R4 = 1,2 1,3 2,3 1,1 2,2 3,3

= (1,1) (2,2) (3,3) (2,1) (3,1) (3,2) (1,2) (1,3) (2,3)

(b) R3∪R6

R3 = 1,2 1,3 2,3

R6 = 1,2 1,3 2,1, 2,3 3,1, 3,2

= (1,2) (1,3) (2,3) (2,1) (3,1) (3,2)

(c) R3∩R6

= (1,2) (1,3) (2,3)

(d) R4∩R6

= (1,2) (1,3) (2,3)

(e) $R_3 - R_6 = \{ \}$

(f) $R_6 - R_3$

(2,1) (3,1) (3,2)

(g) $R_2 \oplus R_6$

(1,1) (2,2) (3,3) (1,2) (1,3) (2,3)

(h) $R_3 \oplus R_5$

(1,1) (2,2) (3,3) (1,2) (1,3) (2,3)

(i) $R_2 \circ R_1$

{(2,1) (3,1) (3,2)}

(j) $R_6 \circ R_6$ = {(1,1), (2,2), (3,3) (2,1) (3,1) (3,2) (1,2)
(1,3) (2,3) }

(Q34)

(i) (1,1) (1,2) (1,3)

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 1 | 0 | 0 |
| 2 | 0 | 1 | 0 |
| 3 | | | 1 |

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |

(ii) (1,2) (2,1) (2,2) (3,3)

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 0 | 1 | 0 |
| 2 | 1 | 1 | 0 |
| 3 | 0 | 0 | 1 |

(iii) (1,1) (1,2) (1,3) (2,2) (2,3) (3,3)

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 0 | 1 | 1 |
| 3 | 0 | 0 | 1 |

(b)(i) {(1,1) (1,3) (2,2) (3,1) (3,3)}

(ii) {(1,2) (2,2) (3,2) (3,3)}

(iii) (1,1) (1,2) (1,3) (2,1) (2,3) (3,1) (3,2) (3,3)

(35)

Reflexivity: $l(x) = l(x)$ xRx would be true

Symmetric : $l(b) = l(a)$ $l(a) = l(b)$ thus symmetric

Transitivity = $l(a) = l(b)$ & $l(b) = l(c)$ $l(a) = l(c)$

Thus R is an equivalent reaction

(b) $R = \{ a,b \mid a \equiv b \pmod m \}$

$a \equiv b \pmod m$ iff m divides $a-b$

Reflexive $a = a \pmod m$

$a - a = 0$ $0 = 0 \cdot m$

Symmetric = $a - b = km$ $b - a = -km$

$b \equiv a \pmod m$

transitive $a \equiv b \pmod m$ $b \equiv c \pmod m$

$a-b$ $= km$ $b-c$ $= lm$

$a - c = km + lm$

$a - c$ $(k+l)m$

thus transitive

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3G

(a)

~~(i) $2^n - 1 = 2^1, 2^1 - 1, 2^2 - 1, 2^3 - 1, 2^4 - 1, 2^5 - 1$~~

~~1, 3, 7, 15, 33~~

(i) $2^n - 1$ $\qquad n > 0$

$2-1, \quad 4-1, \quad 8-1, \quad 16-1, \quad 32-1$

$1, \quad 3, \quad 7, \quad 15, \quad 31$

(ii) $10 - \frac{3}{2}n$

$10 - \frac{3}{2}, \quad 10 - 3, \quad 10 - \frac{9}{2}, \quad 10 - 6, \quad 10 - \frac{15}{2}$

$\frac{17}{2}, \quad 7, \quad \frac{11}{2}, \quad 4, \quad \frac{5}{2}$

(iii) $\frac{(-1)^n}{n^2}$

$-1, \quad \frac{(-1)^2}{4}, \quad \frac{(-1)^3}{9}, \quad \frac{(-1)^4}{16}, \quad \frac{(-1)^5}{25}$

$-1, \quad \frac{1}{4}, \quad \frac{-1}{9}, \quad \frac{1}{16}, \quad \frac{-1}{25}$

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(iv) $\frac{3n+4}{2n-1}$

$\frac{3+4}{2-1}, \frac{6+4}{4-1}, \frac{9+4}{6-1}, \frac{12+4}{8-1}, \frac{15+4}{10-1}$

$7, \frac{10}{3}, \frac{13}{5}, \frac{16}{7}, \frac{19}{9}$

(b) (i) Arithmetic
$a+(n-1)d$

$-15+(11-1)-7$
$-15+(10)-7$
$-15-70=-85$

(ii) $a-42b, a-39b, a-36b, a-33b$
arithmetic

$(a-42b)+(15-1)3b$

$a-42b+14\times 3b$
$a-42b+42b=\boxed{a}$

(iii) $4, 3, \frac{9}{4} \ldots$ $17^{th}$

geometric

$ar^{n-1}$ $\quad r = \frac{3}{4}$

$4\left(\frac{3}{4}\right)^{17-1} = 4 \times \frac{3^{16}}{4^{16}} = \boxed{\frac{3^{16}}{4^{15}}}$

(v) $32, 16, 8 \ldots$ $9^{th}$ $\quad \frac{16}{32} = \frac{1}{2}$

$32\left(\frac{1}{2}\right)^{8} = 32 \times \frac{1}{256} = \frac{1}{8}$

(37) G.P

$T_3 = 10 \quad T_5 = 2\frac{1}{2}$

$ar^2 = 10$

$ar^4 = \frac{5}{2}$

$\frac{10}{\cancel{r^2}} r^{\cancel{4}\,2} = \frac{5}{2}$

$10r^2 = \frac{5}{2}$

$r^2 = \frac{1}{4} \quad \boxed{r = \frac{1}{2}}$

$a\left(\frac{1}{2}\right)^2 = 10$

$\frac{1}{4}a = 10$

$\boxed{a = 40}$

$40, 20, 10, 5, \frac{5}{2}$

$\frac{a}{2}$

(ii) $T_5 = 8$ $T_8 = -\frac{64}{27}$

$ar^4 = 8$

$ar^7 = -\frac{64}{27}$

$\frac{8}{\cancel{r^4}} r^{3} = -\frac{64}{27}$

$r^3 = -\frac{8}{27} \rightarrow r = -\frac{2}{3}$

$a\left(-\frac{2}{3}\right)^4 = 8 \rightarrow a = \frac{81}{2}$

$\frac{81}{2}, -27, 18, -12, 8$

38 Find AP

$T_4 = 7$ $T_{16} = 31$

$a + 3d = 7$

$a + 15d = 31$

$7 - 3d + 15d = 31$

$12d = 36$

$d = 3$

$a + 9 = 7$ $a = -2$

$1, 3, 5, 7, 9$

(ii) $T_5 = 86$ $\quad T_{10} = 146$

$a + 4d = 86$

$a + 9d = 146$

$86 - 4d + 9d = 146$

$5d = 60$

$d = 12$

$a + 48 = 86$

$a = 38$

$38, 50, 62, 74, 86$

38 (a)

$d = 7$

$\cancel{256 + (n-1)7}$ $\quad 259 + (n-1)7$

$\cancel{256 + 7n - 7}$ $\quad 259 + 7n - 7$

$\cancel{249 + 7n}$ $\quad 252 + 7n$

$784 = 259 + (n-1)\,7$

$7n - 7 = \cancel{532}\ 525$

$\boxed{\cancel{n = 76}}$

$\frac{n}{2}[2a + (n-1)d] = \frac{76}{2}[2(259) + 75 \times 7]$

$= 38[518 + 525] = \boxed{39684}$

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(b) $\frac{1}{n}$ . . . . . $\frac{n^2-n+1}{n}$

$\frac{n}{2}\left[2a+(n-1)d\right]$

$a+(n-1)d$

$\frac{1}{n}+(n-1)d=\frac{n^2-n+1}{n}$

$\frac{n}{2}\left[2\left(\frac{1}{n}\right)+(n-1)(1)\right]$

$\frac{n^2-n+1}{n}-\frac{1}{n}$

$\frac{n^2-n}{n}\times\frac{1}{n-1}$

$\frac{n}{2}\left[\frac{2}{n}+n-1\right]$

$\frac{\cancel{n}(\cancel{n-1})}{\cancel{n}}\times\frac{1}{\cancel{n-1}}$

$\frac{n}{2}\left[\frac{2+n^2-n}{\cancel{n}}\right]$

$\boxed{1}$

$\boxed{\frac{2+n^2-n}{2}}$

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(a) $\frac{1}{j}$ $j=1,2,3$

$\sum_{i=1}^{100}\frac{1}{j}$

(b) $\sum_{k=4}^{8}(-1)^k=(-1)^4+(-1)^5+(-1)^6+(-1)^7+(-1)^8$

$+1(-1)+1(-1)+1$

$\boxed{1}$

URBANE

Date:

(a) $a_n = -2a_{n-1}$ , $a_0 = -1$

~~$-2a_0$~~ , $a_1 = -2a_0 = a_1 = 2$

$a_2 = -2a_1 = -2(2) = -4$

$a_3 = -2a_3 = -2(-4) = 8$

$a_4 = -2a_3 = -2(8) = -16$
$a_5 = -2a_4 = -2(-16) = 32$
$a_6 = -2a_5 = -2(32) = -64$

$-1, 2, -4, 8, -16, 32, -64$

(b) $a_n = a_{n-1} - a_{n-2}$ $a_0 = 2$ $a_1 = -1$

~~$a_1 = a_0 - a_{-1}$~~
$a_2 = a_1 - a_0$
$-1 - 2 = \boxed{-3}$

$a_3 = a_2 - a_1$
$-3 - (-1) = \boxed{-2}$

$a_4 = a_3 - a_2$
$-2 - (-3) = \boxed{1}$

$a_5 = a_4 - a_3$
$1 - (-2) = \boxed{3}$

$2, -1, -3, -2, 1, 3$

(c) $a_n = 3a^2_{n-1}$ , $a_0 = 1$

$a_1 = 3a^2_0$

$3(1)^2 = 3$

$a_2 = 3a^2_1$

$3(3)^2 = 27$

$a_3 = 3a^2$

$3(27)^2 = 2187$

$a_4 = 3a^3$

$3(2187)^3 = 14348907$

$a_5 = 3a^4$

$3(14348907)^2 =$

$617673396283947$

1, 3, 27, 2187, 14348907, 617673396283947

(d) $a_n = na_{n-1} + a^2_{n-2}$ , $a_0 = -1$ $a_1 = 0$

$a_2 = 2a_1 + a^2_0$

$2(-1) + 1$

$-1$

$a_3 = 3a_2 + (a_1)^2$

$3(-1) + (0)^2 = \boxed{3}$

$a_4 = 4a_3 + (a_2)^2$

$4(3) + (-1)^2 = \boxed{13}$

$a_5 = 5a_4 + (a_3)^2$

$5(13) + 9$

$74$

-1, 0, -1, 3, 13, 74

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(41) Propositional Logic:

(1) Logic Circuits:

Logic gates are circuits are the building block of every computer system. They are named AND gate, OR gate, NOT gates based on the logical connectives AND, OR, NOT etc.

Building logic circuits helps make every technological hardware component there is and is the foundation of our current hardware

(2) Fuzzy Logic in AI

Fuzzy Logic means there is no absolute true or false value and is often used by AI. Propositional logic can determine a value between 1 and 0 such as 0.6. If 0 is sad and 1 is happy, fuzzy logic can output "X is slightly happy" through propositional logic.

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Predicates and Quantifiers:

(1) Computer functions

Predicate

A ~~function~~ returns a yes or no and a function in computer science an function acts like a predicate, returning a yes or no

bool ~~()~~ predicate ( parameters )

(2) ~~A~~ ~~po~~ ~~Predicate as a filter:~~ Quantifier

The biggest use of quantifiers is in spoken language and a good grasp of quantifiers can translate into writing consise and clear ~~oo~~ code for computer.

Computer Scientists use quantifiers to turn english sentences in to quantifiable ~~true and false~~ statement for a machine to understand.

Sets

(1) ~~Probability~~ Marketing:

Sets are used extensively in product placement and marketing to find out suitable demographics in specific age range.

~~For example, a baby spray company will know to exclude seni~~

(c) (b) Big data

Sets and corresponding functions are implemented on big data sets to distinguish and organize them so they can be used in economics, ~~protocol~~ finance, computer science, engineering and many other fields.

Functions:

(1) Basic economics

Functions are used in economics and finances and in many important mathematical operations.

For e.g

A weekly salary is a function of the ~~hour~~ pay rate and the number of hours worked

(2) ~~The~~ Business Mathematics:

Mathematics problems, formulas for profit and loss are all dependent on functions.

A function to calculate profit, loss, annual revenue etc.

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Relations

① Designation and Management

Relations are excellent when ~~...~~ it is needed to manage large projects with a high number of people, or when merging teams containing a large number of people.

For e.g. a company merger has taken place. Relations can help figure out who should be assigned to what.

(2) ~~...~~ ~~...~~ Organization

Relations can be used in organization and categorization of data, people, animals etc.

Can be used in major fields like marketing, finance etc.

Sequence and Series

1 Coherence in real life

Sequences are used to perceive data more easily, such as the lowest cost of an item at the supermarket at the leftmost side while the most expensive item is rightmost

2 Analysis

Companies used sequencing and series to organize large amounts of data to be ~~easily~~ analyzed made easily.

For e.g the government ~~ra~~ ranking each cities of the country by illiteracy rate to figure out where to allocate educational resources

(3) graph theory

(1) Airline Scheduling

Graph theory helps in organizing path of ~~a~~ flights so they don't ~~[illegible]~~ coincide using flow ~~[illegible]~~ ~~[illegible]~~ model.

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(2) Search Engine Algorithms

To search for a word in a database, the engine first make webgraph where edges are hyperlinks and vertices are website. Through this, a result is output

(f) Trees

(1) Programming Flowcharts

Trees can be used ~~a~~ to ~~dg~~ design flowcharts for computer programs to make them easier to understand

(2) Used in mapping algorithms ~~to~~

Trees can be used in mapping software to find the shortest path to ~~ata~~ a destination using algorithms for shortest path like Kruskal's or ~~[illegible]~~. Prim's.