

Dated:

LA Assignment 2

21K-3153

Dated:

~~Assignment~~ Assignment 2 LA

Example 1:

$$0 + 0 = 0$$

$$k0 = 0$$

① $\overset{u}{0} \in V \quad \overset{v}{0} \in V \quad \overset{u}{0} + \overset{v}{0} = 0 \in V$
closed under addition

② $\underset{0}{0} + \underset{0}{0} = \underset{0}{0} + \underset{0}{0} \quad \checkmark$

③ $\underset{0}{0} + (\underset{0}{0} + \underset{0}{0}) = (\underset{0}{0} + \underset{0}{0}) + \underset{0}{0} \quad \checkmark$

④ $\overset{\rightarrow}{0} + \underset{0}{0} = \underset{0}{0} + \overset{\rightarrow}{0} \quad \checkmark$

⑤ $\underset{0}{0} + (-\underset{0}{0}) = -\underset{0}{0} + \underset{0}{0} \quad \checkmark$

⑥ $0 \in V \quad k(0) = \{0\} = 0 \in V \quad (\text{closed under multiplication})$

⑦ $\underset{0}{k(0+0)} = \underset{0}{k0 + k0} \quad \checkmark$

⑧ $\underset{0}{(k+m)0} = \underset{0}{k0 + m0} \quad \checkmark$

Dated:

$$(9) \quad k(\mathbf{0}) = \mathbf{0} \quad \checkmark$$

$$(10) \quad \begin{aligned} 1U &= U \\ 1\mathbf{0} &= \mathbf{0} \\ \mathbf{0} &= \mathbf{0} \end{aligned} \quad \checkmark$$

Example 2:

$$\begin{aligned} U + V &= (u_1, u_2, \dots, u_n) + (v_1, v_2, \dots, v_n) = (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n) \\ kU &= (ku_1 + kv_1, \dots, ku_n + kv_n) \end{aligned}$$

$$(1) \quad (u_1, u_2, \dots, u_n) \in V \quad (v_1, v_2, \dots, v_n) \in V$$

$$(u_1 + v_1, u_2 + v_2, \dots, u_n + v_n) \in V \quad \checkmark$$

closed under additions

$$(2) \quad \begin{aligned} (u_1, u_2) + (v_1, v_2) &= (v_1, v_2) + (u_1, u_2) \quad \checkmark \\ (u_1 + v_1, u_2 + v_2) &= (u_1 + v_1, u_2 + v_2) \end{aligned}$$

$$(3) \quad (u_1, \dots, u_n) + ((v_1, \dots, v_n) + (w_1, \dots, w_n)) = (u_1, \dots, u_n) + (v_1 + w_1, \dots, v_n + w_n)$$

$$(u_1, \dots, u_n) + (v_1 + w_1, \dots, v_n + w_n) = (u_1 + v_1 + w_1, \dots, u_n + v_n + w_n)$$

$$(u_1 + v_1 + w_1, \dots, u_n + v_n + w_n) = (u_1 + v_1 + w_1, \dots, u_n + v_n + w_n)$$

Dated:

④ ~~0 +~~
 $0 + (u_1, u_2, \dots, u_n) = (u_1, u_2, \dots, u_n) + 0 = (u_1, u_2, \dots, u_n)$

✓

⑤ $(u_1, u_2, \dots, u_n) + (-u_1, -u_2, \dots, -u_n) = (0, 0, \dots, 0)$

✓

⑥ $kU_0 = (ku_1, ku_2, \dots, ku_n) \in V$
closed under multiplication

⑦ $k(u+v) = ku + kv$
 $k(u_1 + v_1, \dots, u_n + v_n) = (ku_1, ku_2, \dots, ku_n) + (kv_1, kv_2, \dots, kv_n)$
 $(k(u_1 + v_1), \dots, k(u_n + v_n)) =$

$(ku_1 + kv_1, \dots, ku_n + kv_n) = \text{~~ku_1, kv_1, \dots, ku_n, kv_n~~}$

$(ku_1 + kv_1, ku_2 + kv_2, \dots, ku_n + kv_n) = (ku_1 + kv_1, ku_2 + kv_2, \dots, ku_n + kv_n)$

⑧ $(km)u = ku + mu$

$(k+m)(u_1, u_2, \dots, u_n) = k(u_1, u_2, \dots, u_n) + m(u_1, u_2, \dots, u_n)$
 $(ku_1 + mu_1, ku_2 + mu_2, \dots, ku_n + mu_n) = (ku_1 + mu_1, ku_2 + mu_2, \dots, ku_n + mu_n)$

Dated:

$$⑨ \quad K(mu) = (ku)u$$

$$K(mu_1, mu_2, \dots, mu_n) = ku \\ (ku_1, ku_2, \dots, ku_n) = (ku_1, ku_2, \dots, ku_n)$$

⑩

$$Iu = u$$

$$I(u_1, u_2, \dots, u_n) = (u_1, u_2, \dots, u_n)$$

$$(u_1, u_2, \dots, u_n) = (u_1, u_2, \dots, u_n)$$

Example 3

$$u = (u_1, u_2, \dots, u_n, \dots)$$

$$⑦ ② \quad (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n, \dots) = (v_1 + u_1, v_2 + u_2, \dots, v_n + u_n, \dots)$$

~~closed under addition~~

⑥

$$⑧ ① \quad (u_1, u_2, \dots, u_n, \dots) \in V \quad (v_1, v_2, \dots, v_n, \dots) \in V$$

$$(u_1 + v_1, u_2 + v_2, \dots, u_n + v_n, \dots) \in V$$

closed under addition

④

$$③ \quad u_1, u_2, \dots$$

→

Dated:

$$\textcircled{3} (u_1, u_2, \dots, u_n, \dots) + ((u_1, v_2, \dots, v_n, \dots) + (w_1, w_2, \dots, w_n, \dots))$$

$$= (u_1 + v_1 + w_1, u_2 + v_2 + w_2, \dots, u_n + v_n + w_n, \dots)$$

$$(u_1 + v_1 + w_1, u_2 + v_2 + w_2, \dots, u_n + v_n + w_n, \dots)$$

=

$$((u_1, u_2, \dots, u_n, \dots) + (v_1, v_2, \dots, v_n, \dots)) + (w_1, w_2, \dots, w_n, \dots)$$

$$(u_1 + v_1, u_2 + v_2, \dots, u_n + v_n, \dots) + (w_1, w_2, \dots, w_n, \dots)$$

$$(u_1 + v_1 + w_1, u_2 + v_2 + w_2, \dots, u_n + v_n + w_n, \dots) \quad \checkmark$$

$$\textcircled{4} \textcircled{5} (0, 0, \dots, 0, \dots) + (u_1, u_2, \dots, u_n, \dots) = (u_1, u_2, \dots, u_n, \dots)$$
$$(u_1, u_2, \dots, u_n, \dots) \quad \checkmark$$

$$\textcircled{5} (u_1, u_2, \dots, u_n, \dots) + (-u_1, -u_2, \dots, -u_n, \dots) = 0$$
$$(0, 0, \dots, 0, \dots) \quad \checkmark$$

$$\textcircled{6} ku = (ku_1, ku_2, \dots, ku_n, \dots)$$

Dated:

$$\textcircled{a} \quad k(u+v) = ku + kv = (ku_1, ku_2, \dots, ku_n) + (kv_1, kv_2, \dots, kv_n) \\ = k(u_1 + v_1, u_2 + v_2, \dots, u_n + v_n) \\ = (ku_1 + kv_1, ku_2 + kv_2, \dots, ku_n + kv_n) = (k(u_1 + v_1), k(u_2 + v_2), \dots, k(u_n + v_n))$$

$$\textcircled{b} \quad (k+m)u = ku + mu \\ = (ku_1 + mu_1, ku_2 + mu_2, \dots, ku_n + mu_n) \\ = (ku_1, ku_2, \dots, ku_n) + (mu_1, mu_2, \dots, mu_n) \\ = (ku_1 + mu_1, ku_2 + mu_2, \dots, ku_n + mu_n)$$

$$\textcircled{c} \quad k(mu) = (km)u \\ = (km u_1, km u_2, \dots, km u_n) \\ = (km u_1, km u_2, \dots, km u_n) = (km u_1, km u_2, \dots, km u_n)$$

$$\textcircled{d} \quad 1u = u \\ 1(u_1, u_2, \dots, u_n) = (u_1, u_2, \dots, u_n) \\ (u_1, u_2, \dots, u_n) = (u_1, u_2, \dots, u_n)$$

Dated:

Example 4:

$$u = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$$

$$v = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$$

$$ku = \begin{bmatrix} ku_{11} & ku_{12} \\ ku_{21} & ku_{22} \end{bmatrix}$$

① $u+v \in V$

$$\begin{bmatrix} u_{11}+v_{11} & u_{12}+v_{12} \\ u_{21}+v_{21} & u_{22}+v_{22} \end{bmatrix} \in V \quad \text{closed under addition}$$

② $\begin{bmatrix} u_{11}+v_{11} & u_{12}+v_{12} \\ u_{21}+v_{21} & u_{22}+v_{22} \end{bmatrix} = \begin{bmatrix} v_{11}+u_{11} & v_{12}+u_{12} \\ v_{21}+u_{21} & v_{22}+u_{22} \end{bmatrix}$ ✓

③

$$\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} + \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} = \left[\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} \right] + \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

$$\begin{bmatrix} u_{11}+v_{11}+w_{11} & u_{12}+v_{12}+w_{12} \\ u_{21}+v_{21}+w_{21} & u_{22}+v_{22}+w_{22} \end{bmatrix} = \begin{bmatrix} u_{11}+v_{11}+w_{11} & u_{12}+v_{12}+w_{12} \\ u_{21}+v_{21}+w_{21} & u_{22}+v_{22}+w_{22} \end{bmatrix}$$

Dated:

$$\textcircled{4} \quad 0 + u = u$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$$

$$\textcircled{5} \quad \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} + \begin{bmatrix} -u_{11} & -u_{12} \\ -u_{21} & -u_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\textcircled{6} \quad k u \cdot \varepsilon V$$

$$\begin{bmatrix} k u_{11} & k u_{12} \\ k u_{21} & k u_{22} \end{bmatrix} \varepsilon V$$

$$\textcircled{7} \quad k \begin{bmatrix} u_{11} + v_{11} & u_{12} + v_{12} \\ u_{21} + v_{21} & u_{22} + v_{22} \end{bmatrix} = \begin{bmatrix} k u_{11} & k u_{12} \\ k u_{21} & k u_{22} \end{bmatrix} + \begin{bmatrix} k v_{11} & k v_{12} \\ k v_{21} & k v_{22} \end{bmatrix}$$

$$\begin{bmatrix} k u_{11} + k v_{11} & k u_{12} + k v_{12} \\ k u_{21} + k v_{21} & k u_{22} + k v_{22} \end{bmatrix} = \begin{bmatrix} k u_{11} + k v_{11} & k u_{12} + k v_{12} \\ k u_{21} + k v_{21} & k u_{22} + k v_{22} \end{bmatrix}$$

Dated:

$$(8) (k+u)u = (ku + mu)$$

$$k+u \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} = \begin{bmatrix} ku_{11} & ku_{12} \\ ku_{21} & ku_{22} \end{bmatrix} + \begin{bmatrix} mu_{11} & mu_{12} \\ mu_{21} & mu_{22} \end{bmatrix}$$

$$\begin{bmatrix} ku_{11} + mu_{11} & ku_{12} + mu_{12} \\ ku_{21} + mu_{21} & ku_{22} + mu_{22} \end{bmatrix} = \begin{bmatrix} ku_{11} + mu_{11} & ku_{12} + mu_{12} \\ ku_{21} + mu_{21} & ku_{22} + mu_{22} \end{bmatrix}$$

$$(9) k(mu) = (km)u$$

$$k \begin{bmatrix} mu_{11} & mu_{12} \\ mu_{21} & mu_{22} \end{bmatrix} = \begin{bmatrix} kmu_{11} & kmu_{12} \\ kmu_{21} & kmu_{22} \end{bmatrix}$$

$$\begin{bmatrix} kmu_{11} & kmu_{12} \\ kmu_{21} & kmu_{22} \end{bmatrix} \quad \checkmark$$

$$(10) 1u = u$$

$$1 \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$$

$$\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$$

Dated:

Example 5

$$u = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \\ \vdots & \vdots \\ u_{m1} & u_{m2} \end{bmatrix}$$

$$u = \begin{bmatrix} u_{11} & u_{12} & \dots & u_{1n} \\ u_{21} & u_{22} & \dots & u_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ u_{m1} & u_{m2} & \dots & u_{mn} \end{bmatrix}$$

① $u + v \in V$

$$\begin{bmatrix} u_{11} + v_{11} & u_{12} + v_{12} & \dots & u_{1n} + v_{1n} \\ u_{21} + v_{21} & u_{22} + v_{22} & \dots & u_{2n} + v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ u_{m1} + v_{m1} & u_{m2} + v_{m2} & \dots & u_{mn} + v_{mn} \end{bmatrix} \in V$$

$$\begin{bmatrix} u_{11} + v_{11} & u_{12} + v_{12} & \dots & u_{1n} + v_{1n} \\ u_{21} + v_{21} & u_{22} + v_{22} & \dots & u_{2n} + v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ u_{m1} + v_{m1} & u_{m2} + v_{m2} & \dots & u_{mn} + v_{mn} \end{bmatrix} \in V$$

$$\begin{bmatrix} v_{11} + u_{11} & v_{12} + u_{12} & \dots & v_{1n} + u_{1n} \\ v_{21} + u_{21} & v_{22} + u_{22} & \dots & v_{2n} + u_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{m1} + u_{m1} & v_{m2} + u_{m2} & \dots & v_{mn} + u_{mn} \end{bmatrix}$$

Dated:

$$\textcircled{3} \quad u + (v + w) = (u + v) + w$$

~~Let~~ u, v, w

$$\begin{array}{ccc|ccc} u_{11} & u_{12} & \dots & u_{1n} & v_{11}+v_{11} & v_{12}+v_{12} & \dots & v_{1n}+v_{1n} \\ u_{21} & u_{22} & \dots & u_{2n} & v_{21}+v_{21} & v_{22}+v_{22} & \dots & v_{2n}+v_{2n} \\ 1 & 1 & & & 1 & 1 & & 1 \\ u_{m1} & u_{m2} & \dots & u_{mn} & v_{m1}+v_{m1} & v_{m2}+v_{m2} & \dots & v_{mn}+v_{mn} \end{array}$$

$$\begin{array}{ccc|ccc} u_{11}+v_{11}+w_{11} & u_{12}+v_{12}+w_{12} & \dots & u_{1n}+v_{1n}+w_{1n} \\ u_{21}+v_{21}+w_{21} & u_{22}+v_{22}+w_{22} & \dots & u_{2n}+v_{2n}+w_{2n} \\ 1 & 1 & & 1 \\ u_{m1}+v_{m1}+w_{m1} & u_{m2}+v_{m2}+w_{m2} & \dots & u_{mn}+v_{mn}+w_{mn} \end{array}$$

$$\begin{array}{ccc|ccc} u_{11}+v_{11} & u_{12}+v_{12} & \dots & u_{1n}+v_{1n} \\ u_{21}+v_{21} & u_{22}+v_{22} & \dots & u_{2n}+v_{2n} \\ 1 & 1 & & 1 \\ u_{m1}+v_{m1} & u_{m2}+v_{m2} & \dots & u_{mn}+v_{mn} \end{array} + \begin{array}{ccc|ccc} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{2n} \\ 1 & 1 & & 1 \\ w_{m1} & w_{m2} & \dots & w_{mn} \end{array}$$

Dated:

(4) $0 + U = U$

$$\begin{pmatrix} 0 & 0 & -0 \\ 0 & 0 & -0 \\ 1 & 1 & 1 \\ 0 & 0 & -0 \end{pmatrix} + \begin{pmatrix} u_{11} & u_{12} & -u_{1n} \\ u_{21} & u_{22} & -u_{2n} \\ 1 & 1 & 1 \\ u_{n1} & u_{n2} & -u_{nn} \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} & -u_{1n} \\ u_{21} & u_{22} & -u_{2n} \\ 1 & 1 & 1 \\ u_{n1} & u_{n2} & -u_{nn} \end{pmatrix}$$

(5) $u + (-u) = 0$

$$\begin{pmatrix} u_{11} & u_{12} & -u_{1n} \\ u_{21} & u_{22} & -u_{2n} \\ 1 & 1 & 1 \\ u_{n1} & u_{n2} & -u_{nn} \end{pmatrix} + \begin{pmatrix} -u_{11} & -u_{12} & -(-u_{1n}) \\ -u_{21} & -u_{22} & -(-u_{2n}) \\ 1 & 1 & 1 \\ -u_{n1} & -u_{n2} & -(-u_{nn}) \end{pmatrix} =$$

$$\begin{pmatrix} 0 & 0 & -0 \\ 0 & 0 & -0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Dated:

(6) $KU \in V$

$$\begin{bmatrix} ku_1 & ku_2 & -ku_n \\ ku_2 & ku_2 & -ku_n \\ | & | & | \\ ku_1 & ku_2 & ku_n \end{bmatrix} \in V$$

(7) $K(u+v) = Ku + Kv$

$$K \begin{bmatrix} u_1+v_1 & u_2+v_2 & -u_n+v_n \\ u_2+v_2 & u_2+v_2 & -u_n+v_n \\ | & | & | \\ u_1+v_1 & u_2+v_2 & u_n+v_n \end{bmatrix} = \begin{bmatrix} ku_1+kv_1 & ku_2+kv_2 & -ku_n+kv_n \\ ku_2+kv_2 & ku_2+kv_2 & -ku_n+kv_n \\ | & | & | \\ ku_1+kv_1 & ku_2+kv_2 & ku_n+kv_n \end{bmatrix}$$

$$\begin{bmatrix} ku_1+kv_1 & ku_2+kv_2 & -ku_n+kv_n \\ ku_2+kv_2 & ku_2+kv_2 & -ku_n+kv_n \\ | & | & | \\ ku_1+kv_1 & ku_2+kv_2 & ku_n+kv_n \end{bmatrix}$$

Dated:

$$(8) (k+m)u = k+mu$$

$$\left(\begin{array}{ccc} k_{u1} + m_{u1} & k_{u2} + m_{u2} & - k_{u1} + m_{u1} \\ k_{u2} + m_{u2} & & - k_{u2} + m_{u2} \\ k_{um1} + m_{um1} & k_{um2} + m_{um2} & k_{um1} + k_{um2} \end{array} \right)$$

$$\left(\begin{array}{ccc} k_{u1} + m_{u1} & k_{u2} + m_{u2} & - k_{u1} + m_{u1} \\ k_{u2} + m_{u2} & & - k_{u2} + m_{u2} \\ k_{um1} + m_{um1} & k_{um2} + m_{um2} & - k_{um1} + m_{um2} \end{array} \right)$$

$$(9) k(mu) = (k+mu)$$

$$k \left(\begin{array}{ccc} m_{u1} & m_{u2} & - m_{u1} \\ m_{u2} & & - m_{u2} \\ m_{um1} & m_{um2} & - m_{um1} \end{array} \right) = \left(\begin{array}{ccc} k_{m11} & k_{m12} & - k_{m11} \\ k_{m21} & k_{m22} & - k_{m21} \\ k_{mum1} & k_{mum2} & k_{mum1} \end{array} \right)$$

$$\left(\begin{array}{ccc} k_{m11} & k_{m12} & - k_{m11} \\ k_{m21} & k_{m22} & - k_{m21} \\ k_{mum1} & k_{mum2} & - k_{mum1} \end{array} \right)$$

Dated:

⑩ $U = U$

$$I \begin{bmatrix} u_{11} & u_{12} & -u_{1n} \\ u_{21} & u_{22} & -u_{2n} \\ 1 & 1 & 1 \\ u_{m1} & u_{m2} & u_{mn} \end{bmatrix} = \begin{matrix} \text{---} \\ \\ \\ \end{matrix} U$$

$$\begin{bmatrix} u_{11} & u_{12} & -u_{1n} \\ u_{21} & u_{22} & -u_{2n} \\ 1 & 1 & 1 \\ u_{m1} & u_{m2} & u_{mn} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & -u_{1n} \\ u_{21} & u_{22} & -u_{2n} \\ 1 & 1 & 1 \\ u_{m1} & u_{m2} & u_{mn} \end{bmatrix}$$