D. E Assignment1

Problem 1

(a) Partial diff eq, 3rd order, 1st degree, non linear

(b) ordinary diff eq, 2nd order, $\frac{3}{2}$ degree, non linear

Problem 2

(a) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0$ $\quad y(0) = -2,\ y'(0) = 6$

$y = c_1 e^{4x} + c_2 e^{-3x}$

$-2 = c_1 e^{4(0)} + c_2 e^{-3(0)}$

$\boxed{-2 = c_1 + c_2}$

$y' = 4c_1 e^{4x} - 3c_2 e^{-3x}$

$y'(0) = 6$

$\boxed{6 = 4c_1 - 3c_2}$

$c_1 = 0$

$c_2 = -2$

$y = -2e^{-3x}$

$y' = 6e^{-3x}$

$y'' = -18e^{-3x}$

$-18e^{-3x} - 6e^{-3x} - 12(-2e^{-3x}) = 0$

$-18e^{-3x} - 6e^{-3x} + 24e^{-3x} = 0$

$-24e^{-3x} + 24e^{-3x} = \boxed{0}$ proven

(b) $x^3\frac{d^3y}{dx^3} - 3x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} - 6y = 0$

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$y(2) = 2 \qquad y'(2) = 2 \qquad y''(2) = 6$

$y = c_1x + c_2x^2 + c_3x^3$

$\boxed{2 = 2c_1 + 4c_2 + 8c_3}$

$y' = c_1 + 2c_2x + 3c_3x^2$

$\boxed{2 = c_1 + 4c_2 + 12c_3}$

$y'' = 2c_2 + 6c_3x$

$6 = 2c_2 + 6c_3(2)$

$\boxed{6 = 2c_2 + 12c_3}$

$6 = 2(c_2 + 6c_3)$

$3 = c_2 + 6c_3$

$\boxed{3 - 6c_3 = c_2}$

$2 = c_1 + 4(3-6c_3) + 12c_3$

$2 = c_1 + 12 - 24c_3 + 12c_3$

$2 = c_1 + 12 - 12c_3$

$-10 = c_1 - 12c_3$

$12c_3 - 10 = c_1$

$2 = 2(12c_3 - 10) + 4(3-6c_3) + 8c_3$

$2 = \cancel{24}c_3 - 20 + 12 - \cancel{24}c_3 + 8c_3$

$\frac{10}{8} = c_3$

$2 = c_1 + 4\left(-\frac{9}{2}\right) + 12\left(\frac{5}{4}\right)$

$2 = c_1 - 18 + 15$

$2 = c_1 - 3$

$\boxed{5 = c_1}$

$3 - 6\left(\frac{5}{4}\right) = c_2$

$3 - \frac{15}{2} = c_2$

$\boxed{-\frac{9}{2} = c_2}$

$c_1 = 5$

$c_2 = -\frac{9}{2}$

$c_3 = \frac{5}{4}$

(c)

(b) $y = 5x - \frac{9}{2}x^2 + \frac{5}{4}x^3$

$y' = 5 - 9x + \frac{15}{4}x^2$

$y'' = -9 + \frac{15}{2}x$

$y''' = \frac{15}{2}$

$x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0$

$\frac{15}{2}x^4 - 3x^2\left(-9 + \frac{15}{2}x\right) + 6x\left(5 - 9x + \frac{15}{4}x^2\right) - 6y = 0$

$\cancel{\frac{15}{2}x^3} + \cancel{27x^2} - \frac{45}{2}x^3 + 30x - \cancel{54x^2} + \frac{45}{2}x^3 - 6($

$5x - \frac{9}{2}x^2 + \frac{5}{4}x^3)$

$-\cancel{30x} + 27x^2 - \frac{15}{2}x^3$

$\boxed{0}$

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(c) $\frac{d^2y}{dx^2} + y = 0 \qquad y(0) = 1, \; y'(\frac{\pi}{2}) = -1$

$y = c_1 \sin x + c_2 \cos x$

$(0) = 1$

$1 = c_2$

$y' = c_1 \cos x - c_2 \sin x$

$(\frac{\pi}{2}) = -1$

$-1 = c_1 (0) - c_2 (1)$

$\boxed{c_2 = 1}$

no solution due to c1 not possible

~~Problem 3~~

(a) $\cancel{x^3 + y^3 = 3cxy}$

$\cancel{\frac{dy}{dx}\left(3x^2 + 3y^2 \frac{dy}{dx}\right) = \frac{d}{dx}(3cxy)}$

$\cancel{u = 3cx} \qquad \cancel{v = y}$

$\cancel{u' = 3c} \qquad v = \frac{dy}{dx}$

$\cancel{3cx \frac{dy}{dx} + 3cy}$

Problem

$$x^3+y^3 = 3cxy$$

$$\frac{x^3+y^3}{3xy} = c$$

$\frac{dy}{dx}(x^3)(3x)$

$\frac{dy}{dx}$: $\frac{d}{dx}(3cxy)$

$$3x^2+3y^2\frac{dy}{dx} = 3cx\frac{dy}{dx}+3cy$$

$$\cancel{3x^2+3y\frac{dy}{dx} = 3\left(\frac{x^3+y^3}{3xy}\right)x\frac{dy}{dx} + 3\left(\frac{x^3+y^3}{3xy}\right)y = 0}$$

$$\left(3y^2-3cx\right)\frac{dy}{dx} = 3cy-3x^2$$

$$\frac{dy}{dx} = \frac{3cy-3x^2}{3y^2-3cx} = \frac{3\left\{\frac{x^3+y^3}{3xy}\right\}y-3x^2}{3y^2-3\left\{\frac{x^3+y^3}{3xy}\right\}x}$$

$$\frac{\left[\frac{x^3+y^3}{xy}\right]y-3x^2}{3y^2-\left\{\frac{x^3+y^3}{y}\right\}} = \frac{\frac{x^3+y^3}{x}-3x^2}{3y^2-\frac{x^3-y^3}{y}}$$

$$\frac{\frac{x^3+y^3-3x^3}{x}}{\frac{3y^3-x^3-y^3}{y}} = \frac{\frac{-2x^3+y^3}{x}}{\frac{2y^3-x^3}{y}}$$

$$\boxed{\frac{\left(-2x^3+y^3\right)y}{\left(2y^3-x^3\right)x}}$$

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(b)

$$\cancel{3y = \frac{4x^3}{x^2+1}}$$

$$3y = \frac{4x^3}{x^2+1} + 3c$$

$$3y = \frac{4x^3+3c}{x^2+1}$$

$$\frac{dy}{dx}(3y) = \frac{dy}{dx}\left(\frac{4x^3+3c}{x^2+1}\right)$$

$$3y(x^2+1)$$

$$3x^2y + 3y = 4x^3 + 3c$$

$$\cancel{6x\frac{dy}{dx} + 3\frac{dy}{dx} = 12x^2}$$

$$\cancel{\frac{dy}{dx}(6x+3) = 12x}$$

$u = 3x^2 \quad v = y$

$u' = 6x \quad v' = \frac{dy}{dx}$

$$3x^2\frac{dy}{dx} + 6xy + 3\frac{dy}{dx} = 12x^2$$

$$\frac{dy}{dx}(3x^2+3) + 6xy = 12x^2$$

$$\boxed{\frac{dy}{dx} = \frac{12x^2-6xy}{3x^2+3}}$$

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Q1(a) ~~ln(y+2~~

$$(xy + 2x + y + 2)\frac{dx}{dx} + (x^2 + 2x)\frac{dy}{dx} = 0$$

$$y(x+1) + 2(x+1) + (x^2+2x)\frac{dy}{dx} = 0$$

$$(x+1)(y+2) + (x^2+2x)\frac{dy}{dx} = 0$$

$$(x^2+2x)\frac{dy}{dx} = -(x+1)(y+2)$$

$$\int \frac{dy}{y+2} = \int -\frac{(x+1)}{x^2+2x}$$

$$\ln(y+2) = -\int \frac{x+1}{x(x+2)}$$

$$-\frac{1}{2}\int \frac{2x+1}{x^2+2x}\,dx$$

$$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x) + c$$

$$\cancel{c = \ln(y+2) - \ln(x^2+2x)^{-\frac{1}{2}}}$$

$$y+2 = x^2+2x^{-\frac{1}{2}}\,c$$

$$y = \frac{c}{(x^2+2x)^{\frac{1}{2}}} - 2$$

$$\boxed{y = \frac{c}{\sqrt{x^2+2x}} - 2}$$

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Q4 (b) Solve the linear D.E

(h) $\frac{dy}{dx} + \frac{y}{x \ln(x)} = \frac{3x^2}{\ln(x)}$

$\frac{dy}{dx} + P(x) y = f(x)$

$\frac{dy}{dx} + \frac{1}{x\ln(x)} y = \frac{3x^2}{\ln x}$

$I.F = e^{\int Px} \to e^{\int \frac{1}{x\ln(x)}}$

let $\ln(x) = u$ $\qquad \frac{du}{dx} = \frac{1}{x}$

$\int \frac{1}{\cancel{x} u} \cancel{x} \, du$ $\qquad du = \frac{1}{x} dx$

$\int \frac{1}{u} du$ $\qquad dx = x\,du$

$\ln u$

$\ln(\ln(x))$

$I.F \; e^{\ln(\ln x)} \to I.F = \boxed{\ln(x)}$

$\int \frac{d}{dx} (\ln(x) y) = \int \frac{3x^2}{\ln x} \times \cancel{\ln(x)} \to dx$

$\ln(x) y = 3 \frac{x^3}{3}$

$\ln(x) y = x^3 + c$

$y = \frac{x^3 + c}{\ln(x)}$

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Q4(c) Solve exact D.E

$$e^x\{y-3(e^x+1)^2\}dx+(e^x+1)dy=0$$

$$y(0)=4$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

$$\cancel{e^0\{4-3(e^0+1)^2\}}$$

$$e^x=e^x$$

$$\frac{\partial f}{\partial x}=M$$

$$\frac{\partial f}{\partial x}=e^x\{y-3(e^x+1)^2\}$$

$$\int\frac{\partial f}{\partial y}=\int e^x+1\,\partial y$$

$$f=(e^x+1)y+g(x)$$

$$\frac{\partial f}{\partial x}=e^xy+g'(x)$$

$$e^x\{y-3(e^x+1)^2\}=e^xy+g'(x)$$

$$\cancel{e^xy}-3e^x(e^x+1)^2=\cancel{e^xy}+g'(x)$$

$$g'(x)=-3e^x(e^x+1)^2$$

(e^x)

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$g = \int g^{\prime} = \int -3e^x(e^x+1)^2$

$-3e^x(e^{2x} + 2e^x + 1)$

$\int -3e^{3x} - 6e^{2x} - 3e^x$

$-3\int \frac{e^{3x}}{3} - 6\int e^{2x} - 3\int e^x$

$g(x) = e^{3x} - 3e^{2x} - 3e^x$

$f = e^x y + y + e^x - e^{3x} - 3e^{2x} - 3e^x$

$\cancel{f = e^x y + y + e^{3x}} - 3e^{2x} - 3e^x$

$\boxed{f = e^x y + y - 2e^x - e^{3x} - 3e^{2x}}$