FORECASTING TRAFFIC FLOW CONDITIONS IN AN URBAN NETWORK: COMPARISON OF MULTIVARIATE AND UNIVARIATE APPROACHES

YIANNIS KAMARIANAKIS*, Associate Researcher, Regional Analysis Division, Institute of Applied and Computational Mathematics, Foundation for Research and Technology, Vasilika Vouton, GR 71110, Heraklion Crete, Greece, Tel: +30 81 391771, Fax: +30 81 391761, E-mail: kamarian@iacm.forth.gr

POULICOS PRASTACOS, Director of Research, Regional Analysis Division, Institute of Applied and Computational Mathematics, Foundation for Research and Technology, Vasilika Vouton, GR 71110, Heraklion Crete, Greece, Tel: +30 81 391767, Fax: +30 81 391761, E-mail: poulicos@iacm.forth.gr

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* Corresponding Author

Abstract. Several univariate and multivariate models have been proposed for performing short term forecasting of traffic flow. In this paper two different univariate (historical average and ARIMA) and two multivariate (VARMA and STARIMA) models are presented and discussed. A comparison of the forecasting performance of these four models is undertaken using datasets from 25 loop detectors located in major arterials in the city of Athens, Greece. The variable under study is the relative velocity that is the traffic volume divided by the road occupancy. Although the specification of the network's neighborhood structure for the STARIMA model was relative simple and can be further refined, the results obtained indicate a comparable forecasting performance for the ARIMA, VARMA and STARIMA models. The historical average model could not cope with the variability of the data sets at hand.

INTRODUCTION

Short term forecasting of traffic flow conditions has been for several years a hot topic of research. Several forecasting techniques have been proposed which attempt to develop a mathematical framework based on theoretical or assumed empirical relationships. In this paper, we concentrate on statistical methods that aim to forecast the traffic flow conditions in an urban network at future time instants, based on real time data. When the forecasting horizon is relatively long, various alternative-to-purely-statistical methodologies have been proposed. A brief categorization according to Boyce *(1)* is the following:

- *Individual travel choice behavior* based on consumer choice theory as found in micro-economics and psychology; this approach is largely from the perspective of the trips made by each individual, taking into account constraints imposed by others.
- *Daily activities incorporating time-space constraints,* as found in human geography; this approach emphasizes more the activities performed by the household as a decision unit, and the interactions of its members.
- *Equilibria of aggregate travel and location choices,* as found in operations research; this approach is from the transportation and spatial systems perspective, especially as it pertains to the joint effects of individual choices such as congestion.

The short-term forecasting models proposed can be categorized as univariate or multivariate. In univariate models, separate models are estimated for each loop detector, whereas in the multivariate case a single model is fitted to the whole dataset for the urban network under study. Univariate models are flexible and can adapt to the specific characteristics of a distinct time series; on the other hand, multivariate models are expected to gain in accuracy through the incorporation of information taken from sites closely located or with similar traffic flow patterns to the one being considered each time.

The vast majority of the presented methods are univariate in nature; historical traffic flow data from a given location are used for modeling and predicting future behavior in the same location. Specifications have ranged from Kalman filtering *(2),* non-parametric regression *(3),* regression with time varying coefficients *(4),* neural networks *(5)* and ARIMA models *(6, 7)*. Williams *(8)* used separate ARIMAX models for a set of loop detectors that incorporate information from upstream measurement locations.

Multivariate models that simultaneously describe traffic flow in various locations were applied through the state-space formulation by Ben-Akiva et al. *(9),* Whittaker et al *(10)* and Stathopoulos and Karlaftis (*11)* using a relatively small number of measurement locations.

Recently, Kamarianakis and Prastacos *(12)* described the spatiotemporal evolution of traffic flow in an urban network by a single space-time ARIMA (STARIMA) model. This formulation permits to quantify the effects of the flow shocks occurring at some location to its neighbors.

The STARIMA model class was developed at the early eighties in a series of papers by Pfeifer and Deutsch (*13, 14, 15).* Since then, it has been used for the analysis of spatial time series data coming from various disciplines such as environmetrics *(14, 16, 17),* epidemiology *(13)* and spatial econometrics (*18, 19).* This model class expresses each observation at time *t* and location *i* as a weighted linear combination of previous observations and innovations lagged both in space and time. The basic mechanism for this representation is the hierarchical ordering of the neighbors of each site and a corresponding sequence of weighting matrices, the specification of which, is a matter left to the model builder to capture the physical properties that are being considered endogenous to the particular spatial system being analyzed. STARIMA models can be viewed as special cases of the Vector Autoregressive Moving Average (VARMA) models that describe a set of time series by using $N \times N$ autoregressive and moving average parameter matrices to represent all autocorrelations and cross-correlations within and among the *N* time series under study *(19).* In fact, STARIMA models are constrained VARMA models where the constraints are made in a way to reflect the topology of a spatial network and result in a drastic reduction in the number of parameters that must be estimated.

In this paper, a comparison of the forecasting performance of several alternative models is attempted. Two univariate methods, namely --historical average and ARIMA-- and two multivariate --STARIMA and VARMA-- have been selected. Similar comparison tests for several univariate methods can be found in Smith et al. *(20).* For the evaluation of the models a dataset that includes traffic flow data collected at 25 loop detectors located on major arterials leading to the center of the city of Athens, Greece was used. The datasets are for two separate months and data were collected at each point every 90 seconds.

The next section of this paper includes a brief exposition of the methodological issues involved, while the next two sections contain a detailed discussion of the application and the results obtained.

RELATIVE VELOCITY FORECASTING MODELS

Historical Average

The historical average model simply uses an average of past observations to forecast future ones. As such, it relies upon the cyclical nature of traffic conditions. However, the model has no way to react to dynamic changes, such as incidents, in the transportation system. Forms of the historical average model have been applied to the urban traffic control system *(21),* as well as, to various traveler information systems in Europe such as AUTOGUIDE *(22)* and LISB *(23).*

The present model was simply formulated by finding the average volume for each interval at each site. The average was computed using the development database only and the model works as follows: at time *t*, the relative velocity at site s $V_s(t+h)$ is estimated as $V_{hist}(t+h)$.

ARIMA Models

Univariate time-series models, such as the autoregressive integrated moving average (ARIMA) model, attempt to develop a mathematical model explaining the past behavior of the series and then apply it to forecast future behavior. ARIMA models have been applied to the UTCS *(24)* and to freeway volume forecasting *(5).* Such models have been extensively analyzed and applied in the transportation literature.

Vector Autoregressive Moving Average Models (VARMA)

The VARMA models can be employed to estimate the dynamic interactions among multiple time series. The estimation refers to the detection, quantification and prediction of these dynamic interactions. VARMA models can be viewed as a subclass of the State-Space models which are widely used in physics and engineering. The latter models bring together multiple input and multiple output systems and have recently been applied for short term traffic flow forecasts to datasets coming from successive loop detectors placed in a single road *(10*). VARMA models can be applied to traffic flow data from multiple detectors of a road network in both unrestricted and restricted forms. In unrestricted form, when the objective is to identify similarities in traffic flow patterns among various roads occurring at different time lags. In restricted form -in that case for each detector the upstream neighbors are used as inputs - when the objective is to determine the output's dependence to each input at different time lags.

VARMA processes are distinguished by their autoregressive and moving average orders. The p-th autoregressive order, q-th moving average order process is written as

$$y_t = \delta + \Phi_1 y_{t-1} + ... + \Phi_p y_{t-p} + \varepsilon_t - \Theta_1 \varepsilon_{t-1} - ... - \Theta_q \varepsilon_{t-q} \tag{1}$$

where $y_t = (y_{1t},..., y_{kt})', t = 0,1,...$ denotes a k-dimensional time series vector which in our case contains the observations of the k loop detectors under study at time t, ε_t is a vector white noise process with $\varepsilon_t = (\varepsilon_{1t},..., \varepsilon_{kt})'$ such that $E(\varepsilon_t) = 0, E(\varepsilon_t \varepsilon_s') = 0$ for $t \neq s$ and $\Phi_1,..., \Phi_{t-p}$, $\Theta_1,..., \Theta_{t-q}$ represent matrices containing unknown coefficients that have to be estimated. Analyzing and modeling a set of series jointly, enables one to understand the dynamic relationships over time among the series and to improve the accuracy of forecasts for individual series by utilizing the additional information available from the related series and their forecasts.

When a general VARMA model is applied to the spatial time series taken from a set of a set of detectors, at each measurement location, each observation is modeled as a linear combination of its own past, plus a linear combination of past observations of all other measurement locations plus a linear combination of past errors. Even in its simplest VAR(1) form, this model class contains a large amount of parameters that have to be estimated. This form is helpful when low-dimensional systems are modeled (e.g. as in macroeconomics where these models are very popular) but when a road network with a large number of measurement locations is under study, the vast majority of the coefficients is not expected to differ significantly from zero either because some variables are not Granger-causal*, for the remaining ones or because the data provided information is not rich enough to provide sufficiently precise estimates with confidence intervals that do not contain zero.

* Granger *(25)* has defined a concept of causality, which under suitable conditions is easy to deal with in the context of VAR models. The idea is that the cause cannot come after the effect, thus if a variable x affects a variable z, the former should help improving the predictions of the latter variable.

In order to obtain a meaningful model with reasonable standard errors for the estimates, the model builder should restrict a respectable amount of the parameters that lie in the autoregressive and moving average parameter matrices to be zero. These restrictions can be based either in considerations that relate the model's causal relations with the network's neighbourhood structure (i.e. acting a-priori one may let unrestricted only the coefficients that correspond to the upstream neighbours of a downstream measurement site, thus, if we want to quantify the dependence of detector *b* to some upstream ones that give information that serve as input we can restrict the *b*-th row of the matrices $\Phi_1,...,\Phi_{t-p}$, $\Theta_1,...,\Theta_{t-q}$ to contain zero elements at columns that do not correspond to input detectors.) or in inductive ones that declare that for each endogenous variable of the system the set of predictors is comprised by the subset of the variables that best satisfies the Granger causality argument (i.e. acting a-posteriori one may let unrestricted only the coefficients that are proved to be statistically significant irrespective of the position of the measurement location they correspond to).

In practice, one first estimates the appropriate order of the model through a penalized likelihood information criterion like Akaike's information criterion or Schwarz Bayesian criterion and afterwards proceeds to coefficient estimation through least squares or maximum likelihood. Finally, the residuals have to be checked on the satisfaction of the model hypotheses. VARMA models can easily be implemented through commercial software such as SAS or MATLAB. For VAR models, parameter estimation usually takes only some seconds even when large systems with many observation locations are analyzed; when moving average terms are present however, the coefficient estimation may take several hours if the datasets are large.

SPACE-TIME ARIMA Models

A multivariate time series model can be further refined if the process under study exhibits systematic dependence between the observations at each location and the observations at neighboring locations. This phenomenon is called "spatial correlation". Models that explicitly attempt to explain these dependencies across space are referred to as space-time models. Various formulations have appeared in the literature depending on the form of data and the aim of the application. In this study, we follow the Space Time Autoregressive Integrated Moving Average (STARIMA) approach firstly illustrated by Pfeifer and Deutsch *(12, 13)* that can be easily implemented through relatively simple modifications of vector autoregressive models *(19)*. A special class of the STARIMA models are the STARMA models which refer to spatial time series with non-seasonal fluctuations. Seasonalities can be removed by appropriate differencing operations and a STARIMA model may be seen as a STARMA model for the differenced data. In this section, the STARMA models will be presented since their formulation is easier for the inexperienced reader.

STARMA models are characterized by linear lagged spatiotemporal dependence. Suppose the observations $z_i(t)$ of the random variable $Z_i(t)$, which in our case reflects traffic conditions, are available for *N* spatial locations *(i=1,..,N)* and *T* time periods. The autoregressive structure of the STARMA model expresses the observations at time *t* taken from the *i-th* spatial location as a linear combination of past observations taken from location *i* and its neighboring measurement locations. If the same relationship holds for every site in the system, the process is said to exhibit spatial stationarity and is thus amenable to these forms of space-time models.

Let $L^{(l)}$, the spatial lag operator of spatial order l, be such that

$$L^{(0)} z_i(t) = z_i(t)$$

$$L^{(l)} z_i(t) = \sum_{j=1}^{N} w_{ij}^{(l)} z_j(t)$$

where $w_{ij}^{(l)}$ are a set of weights with

$$\sum_{j=1}^{N} w_{ij}^{(l)} = 1$$

for all i and $w_{ij}^{(l)}$ nonzero only if sites i and j are l^{th} order neighbors. The matrix representation of the set of weights $w_{ij}^{(l)}$ is $W^{(l)}$, an $N \times N$ square matrix with each row summing to one. For the $N \times 1$ column vector $z(t)$ that contains the observations $z_i(t), i = 1,..., N,$ the following relationships hold

$$L^{(0)} z(t) = W^{(0)} z(t) = \mathrm{I}_N z(t)$$

and

$$L^{(l)} z(t) = W^{(l)} z(t) \qquad \text{for } l>0.$$

The specification of the form of weights $w_{ij}^{(l)}$ should reflect the configuration of the road network. Through $w_{ij}^{(l)}$ one may incorporate the distances of the measurement locations i and j, natural barriers, or even the ease in accessibility of site i to site j. First order neighbors are those "closest" to the site of interest. Second order neighbors should be "farther" away than first order neighbors, but "closer" than third order neighbors (see scheme below).

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

First and second order neighbours for site 0.

As in univariate time series, $z_i(t)$ is expressed as a linear combination of past observations and errors. Here however, instead of allowing dependence of $z_i(t)$ only with past observations and errors at site i, dependence is allowed with neighboring sites of various spatial order. In particular

$$z_i(t) = \sum_{k=1}^{p} \sum_{l=0}^{\lambda_k} \phi_{kl} L^{(l)} z_i(t-k) - \sum_{k=1}^{q} \sum_{l=0}^{m_k} \theta_{kl} L^{(l)} \varepsilon_i(t-k) + \varepsilon_i(t) \qquad (2)$$

where p is the autoregressive order, q is the moving average order, λ_k is the spatial order of the k^{th} autoregressive term, m_k is the spatial order of the k^{th} moving average term, ϕ_{kl} θ_{kl} are parameters, and the $\varepsilon_i(t)$ are random normal errors with

$$E[\varepsilon_i(t)] = 0$$

$$E[\varepsilon_i(t)\varepsilon_j(t+s)] = \sigma^2 \text{ for } i=j,\ s=0 \text{ and } 0 \text{ otherwise.}$$

The same model in vector form is

$$z(t)=\sum_{k=1}^{p}\sum_{l=0}^{\lambda_k}\phi_{kl}W^{(l)}z_i(t-k)-\sum_{k=1}^{q}\sum_{l=0}^{m_k}\theta_{kl}W^{(l)}\varepsilon(t-k)+\varepsilon(t) \tag{3}$$

where $\varepsilon(t)$ are normal with mean zero and

$E[\varepsilon(t)\varepsilon(t+s)]=\sigma^2 \mathrm{I}_N$ when s=0 and 0 otherwise.

In order for the STARMA model to represent a stationary process, one in which the covariance structure does not change with time, certain conditions must be met. These conditions are called stationarity conditions and require that every x_u that solves

$$\det\left[x_u^p \mathrm{I}-\sum_{k=1}^{p}\sum_{l=0}^{\lambda_k}\phi_{kl}W^{(l)}x_u^{p-k}\right]=0$$

lies inside the unit circle $(|x_u|<1)$. Effectively this requirement serves to determine a region of possible ϕ_{kl} values that will result in a stationary process. For the spatial and temporal order identification, analogously to univariate time series, the sample properties of two functions, the spatial temporal autocorrelation function and the spatial temporal partial autocorrelation function, are examined.

THE APPLICATION

The study area and the data analyzed

The urban area of Athens, the capital of Greece, has an area of 60 km^2 and a population of approximately four million people. Total daily demand for travel is about 5.5 million trips with about 1 million occurring during the 2-hour peak period *(11).* A set of 88 loop detectors (figure 1) has been installed by the Ministry of Environment and Public Works at major roads of the Athens network to measure traffic volume and road occupancy. The measurements take place every 90 seconds and are immediately transmitted to the Urban Traffic Control Center where they are used by the Siemens MIGRA traffic control system to adjust street lights timing, stored in databases for further analysis and displayed on a website (http://test.AthensTraffic.gr) that shows real time traffic conditions in Athens *(26).* An indicator of data quality ranging from 1 to 3 is transmitted as well since often electronic or system failures result in measurements that might not be accurate.

In this study, the dataset consists of the available information, notably traffic volume and road occupancy, in major arterials leading to the center of the city. Nine out of the 34 loop detectors that matched our criteria (leading to the center of the city) provided measurements of questionable quality according to the data quality indicator so they were discarded from further analysis. The 25 remaining ones are highlighted in figure 1 and table 1.

The variable under study was the relative velocity, which was defined to be equal to the traffic volume divided by the road occupancy. This is a variable less stable than the other two, but it reflects traffic conditions in a clear way. When multiplied by a constant related to the average vehicle length it can provide a proxy for the exact speed *(4).* Averages over five consecutive time intervals were taken in order to ease the implementation and smooth out the noise, so each loop provided 192 measurements per day. Data measurements for weekends were discarded since traffic conditions during these two days differ significantly from the other weekdays. The observations on five out of the twenty-five loop detectors for both periods of the study are highlighted at figure 2 and the basic statistical measures of the data are depicted at table 2. One firstly observes a sinusoidal pattern in relative velocities with a lower part that corresponds to morning until afternoon when congestion occurs and an upper part that corresponds to the night hours. There is also a sign of dependence between the level of relative velocities and their variation.

Separate models were fitted for two periods. The first one was from 31 July 2001 to 27 August 2001 and contained 3,727 observations (almost 20 days); the second one was from 11 February 2002 to March 10, 2002 and contained 4,242 observations corresponding to 22 weekdays. August is a month of atypical traffic flow characteristics since most of the Athens' population takes their vacation at that time, whereas the second period is considered a typical one. As expected, figure 2 indicates lower relative velocities for the second period of the study. Previous studies have shown that traffic conditions are more variable when there are more vehicles in the network and that is revealed in the increased variations of the relative velocities for the second period of the study one observes at table 2. The different traffic conditions in the two periods of the study are evident in figure 3 where the occupancy-volume scatterplots along with a loess-smoothing curve are presented for the raw data of three loops.

The ARIMA, VARMA and STARIMA models should be employed to stationary* data. Both the augmented Dickey Fuller and the Bos Fetherston tests were performed (see *(12)* for details) and as expected, they indicated nonstationarity for the raw data. The daily periodicity was removed by differencing of order 192 and although apart from the sinusoidal evolution of the relative velocities through the day one also observes a possible temporally dependent variation, this fact did not affect the aforementioned tests that did not reject the null hypothesis of stationarity on the differenced data. The same strategy as far as differencing is concerned is followed by Smith et al. *(27)*. The transformation-to-stationarity practically implies that the variable finally modeled was the daily increment in relative velocity. Finally, mean standardization also took place so that the STARIMA models can easily be applied according to the approach of *(14-16)*.

Model fitting

Concerning univariate modeling, 50 (25 loop detectors, two time periods) ARIMA models were fitted to the time series of the two datasets of the application. The three stages of the ARIMA model building procedure indicated that models of factored form provided parsimonious representation and increased forecasting performance as well when compared to the classical ones. Factored models represent ARIMA models with a large number of parameters as a product of simpler ARIMA models; the interested reader will find a thorough discussion on these concepts in the aforementioned textbooks. The patterns of the fitted (by SAS/ETS PROC ARIMA) models (table 2) were quite similar for the series within the datasets that correspond to each of the two periods examined. The vast majority of the models that correspond to the first period under study contain one autoregressive term corresponding to the instant past and two moving average ones, the first corresponding to the model errors at lag one and the second represents the prediction error one day ago (lag 192). The stability of

* That is why in econometric/statistical analyses of that kind, one often sees the models to be applied to the transformed-to-stationarity data (e.g. differenced or log transformed).

the parameters of the corresponding lags is surprising and it is due to the homogeneous and stable traffic conditions in Athens during August. In the second period under study however, traffic conditions are a lot more volatile due to significantly heavier volumes and that resulted in many different model specifications for the 25 loops. One may observe a relative stability in the estimates though (check for example the autoregressive parameter at lag 1).

For the application of the multivariate models, a hierarchical system of neighbors (table 4) was defined. Concerning the two (one for each dataset) VARMA models the neighborhood structure and the dependence of the downstream loops to the upstream ones, was represented by restricting a priori the coefficients that correspond to non-neighbors for all lags to be zero. For the STARMA models, the neighborhood structure was represented by two matrices where all first and second order neighbors of each measurement site are equally weighted. This specification was done a priori and allowed the W_l matrices to be treated as exogenous constants rather than model parameters. There are considerable gains in simplicity and ease of model identification and estimation relative to unrestricted multiple time series modeling that are achieved through this mechanism. The aforementioned definitions limit the STARMA model family used in this study to models with maximum spatial order of two.

The two VARMA models that correspond to the two datasets under study were performed by the SAS/ETS PROC VARMAX (X stands for possible exogenous variables that may be used as inputs to the VARMA model). For the tentative model order selection process the minimum Akaike's information criterion was used and indicated an autoregressive model of order 7 for the first dataset and of order 12 for the second; its surprising that for both models moving average terms were not found to be of statistical significance. The a-priori restrictions that were posed to reflect the neighborhood structure resulted in a more than 80% reduction of the parameters that would have been estimated by the full model (a full model specification would result in 4375 parameters for the first and 7500 parameters to be estimated for the second dataset). Even after the restrictions, a respectable amount of the estimated parameters (slightly above 30%) was not statistically significant.

After examination of the space-time autocorrelations and partial autocorrelations (again the reader is referred to *(12)* for details concerning the STARMA model fitting procedure on the datasets of this study) the candidate models for the two periods under study are of the form

$$Z_t = \phi_{10} Z_{t-1} + \phi_{20} Z_{t-2} + \phi_{30} Z_{t-3} + \phi_{11} W_1 Z_{t-1} + \phi_{12} W_2 Z_{t-1} - \theta_{20} a_{t-192} - \theta_{10} a_{t-1} + a_t \tag{4}$$

Thus, each measurement taken at a specific site at time *t* is modeled as a linear combination of eight terms; namely the

- Three previous measurements at this site,
- Weighted average of the measurements taken from its first order neighbors at time *t*-1
- Weighted average of the measurements taken from its second order neighbors at time *t*-1,
- Prediction error that was made the day before at the same time,
- Previous prediction error,
- Random error.

The non-linear least squares estimates of the parameters are depicted in Table 5. The estimation was performed through a run of the SAS/ETS procedure PROC MODEL (if spatially weighted moving average terms were present in (4) at least two recursive runs should have been performed). A surprising result is that the parameters that correspond to the second order neighbors appear to be more significant than the ones that correspond to first order neighbors probably implying that the temporal intervals between observations were relatively long.

The forecast test

To assess the forecasting performance of the four models the following procedure was adopted. For each dataset, the observations that correspond to the first ten days were used just for model calibration; no forecasting was performed during that period. Then, ten different points in time were randomly selected, parameter estimation took place until these points and forecasts were estimated with the calibrated models for five steps after the selected points. Thus, for each one of the fifty time series that corresponds to the observations taken from a loop detector in one of the two periods of the study, there were fifty pairs of observations-predicted values for each model. Root mean square errors (figure 4) and relative errors (table 6) were then used to compare observed and forecasted relative velocities.

As depicted at figure 4 and table 6 the historical average model, was by far the worst and this was due to its inability to catch short-term dependencies in traffic conditions. The ARIMA models slightly outperformed the VARMA and STARIMA models that provided similar results. The distribution of the relative error indicates that about forty five percent of the observed-predicted pairs for the ARIMA, VARMA and STARIMA models have an absolute relative error of more than twenty five percent. For the historical average, more than the eighty percent of the observed-predicted pairs lies in that region.

The road network had a large contribution on the performance of the multivariate models. The loops are sparsely placed and a respectable number of them is not related to any neighbors; for measurement sites of that kind the univariate ARIMA formulation clearly has an advantage since the VARMA and STARIMA methodologies base their accuracy on loop-dependence relations. It should be underlined that the neighborhood structure imposed to the multivariate models was a rather naïve one and can be further improved with obvious implications on the forecasting results.

CONCLUSIONS

This study demonstrated the application of univariate and multivariate techniques in an urban network using the same data sets originating from a set of loop detectors. The results obtained demonstrate the following

- Univariate techniques, namely ARIMA models, can be effectively used for modeling the flow speed. However, in a network where there may be hundreds of loop detectors computational problems might arise and excessive computer time might be needed. This may render this technique difficult to be applied in a real world urban network.
- Multivariate techniques that permit the modeling of traffic speed while accounting for the spatial characteristics of the network as well as the temporal evolution of traffic in the different locations in the network, provide almost equal good performance. With this approach, the characteristics of the flow and the way it propagates through the network are accounted for and are considered in the models estimates. The forecasting accuracy of these models is directly tied to the number of loop detectors. When the number of loop detectors is relatively small and located in many disperse locations, then the space dependencies cannot be accounted for and in effect, these models reduce to univariate models. However, when the number of loop detectors increases, the performance of these models can improve

significantly since the relationship between the measurements occurring in different locations and time are captured.

For forecasting purposes the issues are to not only be able to forecast traffic speeds when there are stable patterns but also to be able to estimate traffic speeds across the network when a traffic flow shock is introduced at some point of the network. With univariate models, this is not possible since they are based on equations that fail to account for any interdependencies between loop detectors. On the contrary, this is possible with the space-time models presented in this paper because speeds in the various locations are modeled in a way that permits to account for what is happening in the adjacent and other locations in the network.

The proposed multivariate methodologies can be refined by introducing time varying coefficients for temporal periods of the day with different traffic flow characteristics or variable causal relationships among loop detectors according to the state of traffic. In case of free flow the downstream measurement locations may depend on the upstream ones but in case of congestion the relation may be inverted.

REFERENCES

1. Boyce, D. E. A Practitioner's Guide to Urban Travel Forecasting Models. *12th International EMME/2 Users' Conference,* 1997.

2. Whittaker, J., S. Garside, and K. Lindveld. Tracking and predicting a network traffic process. *International Journal of Forecasting,* Vol. 13*,* 1997.

3. Davis, G.A., and N.L. Nihan. Nonparametric regression and short-term freeway traffic forecasting. *ASCE Journal of Transportation Engineering,* Vol. 117(2), 1991.

4. Rice, J., and E. van Zwet. A simple and effective method for predicting travel times on freeways. *IEEE Intelligent Transportation Systems Proceedings*, 2001.

5. Van Lint, J.W.C., and S.P. Hoogendoorn. Freeway travel time prediction with state-space neural networks. *81th Annual Transportation Research Board Meeting*, 2002.

6. Williams, B.M., P.K. Durvasula, and D.E. Brown. Urban freeway travel prediction: application of seasonal ARIMA and Exponential Smoothing Models. *77th Annual Transportation Research Board Meeting*, 1997.

7. Lee, S., and D.B. Fambro. Application of subset autoregressive integrated moving average model for short-term freeway traffic volume forecasting. *Transportation Research Record 1678*, 1999.

8. Williams, B.M. Multivariate vehicular traffic flow prediction: an evaluation of ARIMAX modeling. *80th Annual Transportation Research Board Meeting,* 2001.

9. Ben-Akiva, M., E. Cascetta, H. Gunn, D. Inaudi, and J. Whittaker. In dynamic traffic prediction for motorway networks. *Advanced Methods In Transportation Analysis*, edited by L. Bianco and P. Toth. Springer-Verlag. 1996.

10. Whittaker, J., S. Garside, and K. Lindveld. Tracking and predicting a network traffic process. *International Journal of Forecasting 13,* 1997.

11. Stathopoulos, A., and G.M. Karlaftis. A multivariate state-space approach for urban traffic flow modeling and prediction. *81th Annual Transportation Research Board Meeting*, 2002.

12. Kamarianakis, I., and P. Prastacos. Space-time modeling of traffic flow. *Methods of spatial analysis – spatial time series analysis, ERSA Proceedings*, 2002.

13. Pfeifer, P.E., and S.J. Deutsch. A three-stage iterative procedure for space-time modeling. *Technometrics,* Vol. 22(1), 1980.

14. Pfeifer, P.E., and S.J. Deutsch. Identification and Interpretation of First-Order Space-Time ARMA Models. *Technometrics,* Vol. 22 (3), 1980.

15. Pfeifer, P.E., and S.J. Deutsch. Variance of the Sample-Time Autocorrelation Function of Contemporaneously Correlated Variables. *SIAM Journal of Applied Mathematics, Series A,* Vol. 40(1), 1981.

16. Pfeifer, P.E., and S.J. Deutsch. Seasonal Space-Time ARIMA modeling. *Geographical Analysis,* Vol. 13 (2), 1981.

17. Deutsch, S.J., and J.A Ramos. Space-Time modeling of vector hydrologic sequences. *Water Resources Bulletin,* Vol. 22(6), 1986.

18. Elhorst, J. P. Dynamic Models in Space and Time, manuscript. University of Groningen, Netherlands, 2000.

19. Giacomini, R., and C.W.J. Granger. Aggregation of space-time processes, manuscript. Department of Economics, University of California, San Diego, 2001.

20. Smith, B.L., and M.J. Demetsky. Traffic flow forecasting: Comparison of modeling approaches. *Journal of Transportation Engineering, ASCE.* 1997.

21. Stephanides, Y.J., P.G. Michalopoulos, and R.A. Plum. Improved estimation of traffic flow for real-time control. *Transportation Research Record 795*, Transportation Research Board, Washington, D.C. 28-39. 1981.

22. Jeffrey, D.J., K. Russam, and D.I. Robertson. Electronic route guidance by AUTOGUIDE: the research background. *Traffic Engineering and Control,* 1987.

23. Kaysi, I., M. Ben-Akiva, and H. Koutsopoulos. An integrated approach to vehicle routing and congestion prediction for real-time driver guidance. *Transportation Research Record 1408,* Transportation Research Board, Washington, D.C., 1993.

24. Okutani, I, and Y.J. Stephanides. Dynamic prediction of traffic volume through Kalman filtering theory. *Transportation Research Part B*, Vol 17, 1984.

25. Granger, C.W.J. Investigating causal relations by econometric models and cross-spectral methods. *Econometrica*, Vol. 37, 1969.

26. Kotzinos, D. Advanced Traveler's Information Systems. *Unpublished Ph.D. dissertation,* Technical University of Crete, Chania, Greece, 2001.

27. Smith, B.L., B.M. Williams, and R.K. Oswald. Comparison of parametric and nonparametric models for traffic flow forecasting. *Transportation Research Part C*, Vol 10, 2002.

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LABEL	*ROAD*	*INTERSECT. STR.*
M1	Mesogion	ERT
M2	Mesogion	Ipirou
M3	Mesogion	Paritsi
K1	Kifisias	Karella
K2	Kifisias	28 Oktovriou
K3	Kifisias	Ethn. Antistas.
K4	Kifisias	25 Martiou
KAP	Kapodistriou	El. Venizelou
TRAL	Veikou	Tralleon
GAL	Galatsiou	Veikou
P1	Patision	Kiprou
P2	Patision	Derigni
P3	Patision	Ipirou
SEP	Tr. Septemvr.	Marni
L1	Liosion	Sepolion
L2	Liosion	Pl. Vathis
AL	Alexandras	Panormou
BM1	Vas. Sofias	Mesogion
BM2	Vas. Sofias	Mesogion
BM3	Vas. Sofias	Alexandras
BM4	Vas. Sofias	Alexandras
KAT	Katehaki	Alimou
MIX	Mihalakopoul	Sinopis
A	Amalias	Vas. Sofias
S	Sigrou	Frantzi

TABLE 1 Location of the loop detectors under study

SITE	MEAN (AUGUST)	VARIANCE (AUGUST)	SKEWNESS (AUGUST)	MEAN (FEB-MAR)	VARIANCE (FEB-MAR)	SKEWNESS (FEB-MAR)
M1	-7.299171e-015	1.910245e+004	4.193507e-001	2.859715e-015	8.283703e+004	6.965089e-004
M2	1.716657e-015	1.261564e+003	2.419429e-001	5.087837e-016	1.367126e+004	7.938480e-002
M3	4.454296e-014	4.754314e+003	5.047244e-001	-1.235116e-015	2.834469e+004	3.501394e-002
KAT	9.286167e-015	7.986365e+002	1.684261e-001	3.684296e-016	6.592629e+003	1.491311e-002
K1	-4.031440e-014	8.874021e+003	4.570294e-001	-1.198273e-015	4.625433e+004	1.056192e-002
K2	1.245365e-014	8.171604e+003	4.382056e-001	3.059720e-015	3.915146e+004	3.471764e-002
K3	-4.149714e-014	4.702235e+003	6.957575e-001	-2.006187e-015	2.421069e+004	6.542651e-002
K4	-1.617983e-014	2.859344e+003	4.204714e-001	-1.256169e-015	1.409399e+004	8.135375e-002
KAP	-7.686658e-015	1.559097e+003	2.831333e-001	6.175581e-016	1.113532e+004	5.877742e-002
TRAL	6.042091e-015	1.837857e+003	3.887191e-001	7.228237e-016	2.581437e+004	8.899183e-003
GAL	5.303163e-015	2.442650e+003	2.028006e+000	-3.052702e-016	7.847806e+003	4.795165e-003
L2	-3.204651e-016	1.168043e+002	2.486567e-002	8.631778e-016	1.504720e+003	1.312467e-002
SEP	-2.297887e-016	4.170068e+002	1.417909e-001	-1.868464e-015	6.547218e+003	5.023091e-002
P1	-1.659210e-015	4.507253e+002	9.567459e-002	-8.298437e-016	5.394936e+003	3.062323e-002
P2	-2.127798e-015	6.417203e+002	4.058264e-002	-9.123018e-017	4.368673e+003	4.674180e-002
P3	1.397037e-015	1.949005e+002	3.043126e-002	1.228099e-016	1.842022e+003	3.901963e-002
S	-3.304902e-015	9.603511e+002	1.129470e-001	1.550913e-015	7.045201e+003	1.598506e-001
A	-4.573246e-015	1.665012e+003	5.719351e-002	-2.303562e-015	1.998369e+004	5.164928e-002
BM1	-3.288006e-015	2.239783e+002	1.115568e-001	-8.574760e-017	2.864741e+003	6.020362e-003
BM2	8.958380e-015	4.345446e+002	1.160451e-001	3.008841e-016	4.073353e+003	1.137899e-002
BM3	2.532181e-015	2.000895e+002	6.066701e-001	-3.223759e-016	9.563090e+002	1.190688e-001
BM4	1.242887e-014	6.284769e+002	1.481245e-001	1.614072e-016	5.065504e+003	4.021932e-002
MIX	-1.194451e-014	3.056875e+002	1.774820e-001	-5.614165e-017	9.923221e+003	3.288007e-002
L1	-4.618302e-016	2.463585e+002	7.130476e-002	-7.894919e-017	1.274060e+003	1.912110e-002
AL	-5.226567e-016	3.025340e+002	6.209760e-002	-5.438722e-017	2.985394e+003	5.475049e-002

TABLE 2 Sample moments of the seasonally differenced, mean corrected data.

LOOP	*August 2001*			*February-March 2002*		
M1 AR Op.	1-0.97 B(1)			1-0.87 B(1)	1-0.94 B(2)	1+0.56 B(192)
MA Op.	1-0.81 B(1)	1-0.44 B(192)		1-0.32 B(1)	1-0.91 B(2)	1-0.7 B(192)
M2 AR Op.	1- 0.8 B(1)			1+0.55 B(1)	1-0.65 B(2) 1- 0.86 B(3)	1+0.54 B(192) 1+0.88 B(193)
MA Op.	1-0.87 B(1)	1- 0.5 B(192)		1+0.87 B(1)	1-0.3 B(2) 1-0.8 B(3)	1-0.67 B(192) 1+0.9 B(193)
M3 AR Op.	1-0.97 B(1)			1-0.84 B(1)		1+0.52 B(192)
MA Op.	1-0.76 B(1)	1-0.47 B(192)		1–0.48 B(1)		1-0.71 B(192)
KAT AR Op.	1-0.97 B(1)			1-0.44 B(1)	1+0.98 B(3)	1+0.57 B(192) 1- B(193)
MA Op.	1-0.84 B(1)	1-0.43 B(192)		1-0.22 B(1)	1+0.99 B(3)	1-0.7 B(192) 1-0.99 B(193)
K1 AR Op.	1-0.91 B(1)			1-0.89 B(1)	1-0.73 B(3)	1+0.58 B(192) 1- B(193)
MA Op.	1-0.67 B(1)	1-0.43 B(192)		1-0.55 B(1)	1-0.74 B(3)	1-0.74 B(192) 1-0.99 B(193)
K2 AR Op.	1-0.95 B(1)			1-0.93 B(1)	1+0.95 B(2) 1+ B(3)	1+0.54 B(192) 1+ B(193)
MA Op.	1-0.61 B(1)	1-0.47 B(192)		1-0.6 B(1)	1+0.94 B(2) 1+ B(3)	1-0.72 B(192) 1+B(193)
K3 AR Op.	1-0.96 B(1)			1-0.9 B(1)	1-0.98 B(2)	1+0.58 B(192)
MA Op.	1-0.69 B(1)	1-0.47 B(192)		1-0.61 B(1)	1-0.99 B(2)	1-0.72 B(192)
K4 AR Op.	1-0.97 B(1)			1-0.87 B(1)		1+0.58 B(192)
MA Op.	1-0.73 B(1)	1-0.45 B(192)		1-0.64 B(1)		1-0.7 B(192)
KAP AR Op.	1-0.98 B(1)			1-0.93 B(1)	1-0.54 B(2)	1+0.57 B(192)
MA Op.	1-0.84 B(1)	1-0.45 B(192)		1-0.78 B(1)	1-0.42 B(2)	1-0.72 B(192)
TRA AR Op.	1-0.98 B(1)			1-0.9 B(1)	1+0.72 B(3)	1+0.55 B(192) 1+0.66 B(193)
MA Op.	1-0.86 B(1)	1-0.44 B(192)		1-0.49 B(1)	1+0.61 B(3)	1-0.7 B(192) 1+0.63 B(193)
GAL AR Op.	1-0.8 B(1)	1-0.18 B(192)		1-0.86 B(1)	1-0.56 B(2) 1- B(3)	1+0.57 B(192)
MA Op.	1- 0.4 B(1)	1-0.48 B(192)		1-0.6 B(1)	1-0.63 B(2) 1-0.99 B(3)	1-0.74 B(192)
L2 AR Op.	1-0.97 B(1)			1-0.9 B(1)		1+0.54 B(192)
MA Op.	1-0.89 B(1)	1-0.45 B(192)		1-0.65 B(1)		1-0.69 B(192)
SEP AR Op.	1-0.92 B(1)			1-0.94 B(1)	1-0.98 B(2) 1-0.62 B(3)	1+0.57 B(192)
MA Op.	1-0.71 B(1)	1-0.49 B(192)		1-0.51 B(1)	1-0.97 B(2) 1-0.7 B(3)	1-0.72 B(192)
P1 AR Op.	1-0.97 B(1)			1-0.79 B(1)	1-0.87 B(3)	1+0.55 B(192)
MA Op.	1-0.88 B(1)	1-0.45 B(192)		1-0.35 B(1)	1-0.85 B(3)	1-0.72 B(192)
P2 AR Op.	1- 0.9 B(1)				1-0.8 B(2) 1-0.75 B(3)	1+0.55 B(192) 1-0.4 B(193)
MA Op.	1-0.8 B(1)	1- 0.46 B(192)			1-0.68 B(2) 1-0.72 B(3)	1-0.71 B(192) 1-0.44 B(193)
P3 AR Op.	1-0.9 B(1)	1- 0.04 B(192)		1-0.93 B(1)	1-0.45 B(2) 1+0.56 B(192)	1-0.52 B(193)
MA Op.	1-0.7 B(1)	1-0.52 B(192)		1-0.65 B(1)	1-0.58 B(2)	1-0.65 B(192) 1-0.56 B(193)
S AR Op.	1+0.46 B(1)	1-0.17 B(2)		1-0.78 B(1)	1-0.96 B(2) 1-0.93 B(3)	1+0.56 B(192)
MA Op.	1+0.62 B(1)	1-0.45 B(192)		1-0.53 B(1)	1-0.91 B(2) 1-0.92 B(3)	1-0.68 B(192)
A AR Op.	1-0.1 B(1)-0.84 B(2)			1+0.39 B(1)	1-0.64 B(2) 1-0.98 B(3)	1+0.55 B(192)
MA Op.	1+0.16 B(1) 1-0.71 B(2)	1-0.5 B(192)		1+0.78 B(1)	1-0.21 B(2) 1-0.98 B(3)	1-0.74 B(192)
BM1 AR Op.	1-1.13 B(1) +0.17 B(2)			1-0.8 B(1)	1-0.98 B(2) 1+0.88 B(3)	1+0.56 B(192) 1-0.84 B(193)
MA Op.	1-0.83 B(1)	1-0.48 B(192)		1-0.44 B(1)	1-0.98 B(2) 1+ 0.9 B(3)	1-0.7 B(192) 1-0.85 B(193)
BM2 AR Op.	1-0.98 B(1)			1-0.73 B(1)	1-0.98 B(2)	1+0.55 B(192)

MA Op.	1-0.89 B(1)	1-0.47 B(192)		1-0.59 B(1)	1-0.98 B(3) 1-0.95 B(2) 1-0.99 B(3)	1-0.73 B(192)
BM3 AR Op.	1-1.06 B(1) +0.09 B(2)			1+0.46 B(1)	1-0.78 B(2) 1-0.56 B(3)	1+0.55 B(192) 1+0.64 B(193)
MA Op.	1-0.74 B(1)	1-0.46 B(192)		1+0.86 B(1)	1-0.3 B(2) 1-0.5 B(3)	1-0.69 B(192) 1+0.68 B(193)
BM4 AR Op.	1-1.3 B(1) +0.32 B(2)			1-0.79 B(1)	1+0.69 B(2)	1+0.57 B(192)
MA Op.	1-0.88 B(1)	1-0.47 B(192)		1-0.3 B(1)	1+0.61 B(2)	1-0.75 B(192)
MIX AR Op.	1-1.03 B(1) +0.04 B(2)			1+0.11 B(1)	1-0.28 B(2) 1-0.89 B(3)	1+0.56 B(192)
MA Op.	1-0.89 B(1)	1-0.47 B(192)		1+0.6 B(1)	1-0.85 B(3)	1-0.75 B(192)
L1 AR Op.	1-0.93 B(1) - 0.05 B(2)			1+0.78 B(1)	1-0.83 B(2) 1+0.68 B(3)	1+0.56 B(192) 1+0.75 B(193)
MA Op.	1-0.87 B(1)	1-0.46 B(192)		1+0.92 B(1)	1-0.62 B(2) 1+0.74 B(3)	1-0.68 B(192) 1+0.77 B(193)
AL AR Op.	1-0.97 B(1)			1-0.88 B(1)	1+0.56 B(192)	1+ B(193)
MA Op.	1-0.82 B(1)	1-0.51 B(192)		1-0.6 B(1)	1-0.68 B(192)	1+0.99 B(193)

TABLE 3 ARIMA model formulation and coefficient estimates for the fifty time series of the study.

LOOP DETECTOR	*FIRST ORDER NEIGHBORS*	*SECOND ORDER NEIGHBORS*
M1	-	K1, K2, K3
M2	M1	K1, K2, K3, K4
M3	M1, M2	K1, K2, K3, K4
KAT	-	M2, M3
K1	-	M1, M2
K2	K1, KAP	M1, M2, M3
K3	K1, K2, KAP	M1, M2, M3
K4	K1, K2, K3	M1, M2, M3
KAP	-	K1
TRAL	-	KAP, K1, K2, K3
GAL	TRAL	K3, K4
L2	L1, SEP	P1, P2, P3
SEP	P1, P2, P3	L1
P1	-	GAL
P2	P1	GAL
P3	P1, P2	GAL
S	-	-
A	S	-
BM1	BM2, BM3	BM4, AL
BM2	BM1, BM3	BM4, AL
BM3	BM4	KAT, M3
BM4	BM3	KAT, M3
MIX	BM1, BM2	KAT, M3
L1	-	P1, P2
AL	BM4	K4

TABLE 4 First and second order neighbors of each measurement location.

	ϕ_{10} (T-VALUE)	ϕ_{20} (T-VALUE)	ϕ_{30} (T-VALUE)	ϕ_{11} (T-VALUE)	ϕ_{12} (T-VALUE)	θ_{20} (T-VALUE)	θ_{10} (T-VALUE)	**RMSE**
Model 1 (Aug.)	0.241 (68.75)	0.2264 (64.23)	0.1615 (45.89)	0.0572 (14.43)	0.1 (27.05)	0.0074 (2.11)	0.0133 (3.09)	42.1168
Model 2 (Feb.-Mar.)	0.3834 (122.79)	0.165 (49.92)	0.1188 (38.13)	0.0185 (5.29)	0.0354 (9.96)	0.03 (9.68)	-0.0113 (-3.13)	100.8

TABLE 5 Parameter estimation (after diagnostic checks) and root mean square error for the two STARMA models.

	PERCENTAGE	*WITHIN*	*RELATIVE*	*ERROR*	*BAND*		
	<-25	-25 to -15	-15 to –5	-5 to 5	5 to 15	15 to 25	>25
HIST.AV	45%	4.5%	2%	1%	3.5%	7%	38%
ARIMA	21%	12%	9%	18%	7%	9%	24%
VARMA	17%	14%	10%	12%	6%	13%	28%
STARIMA	26%	11%	6%	9%	11%	17%	20%

TABLE 6 Distribution of the forecasting errors

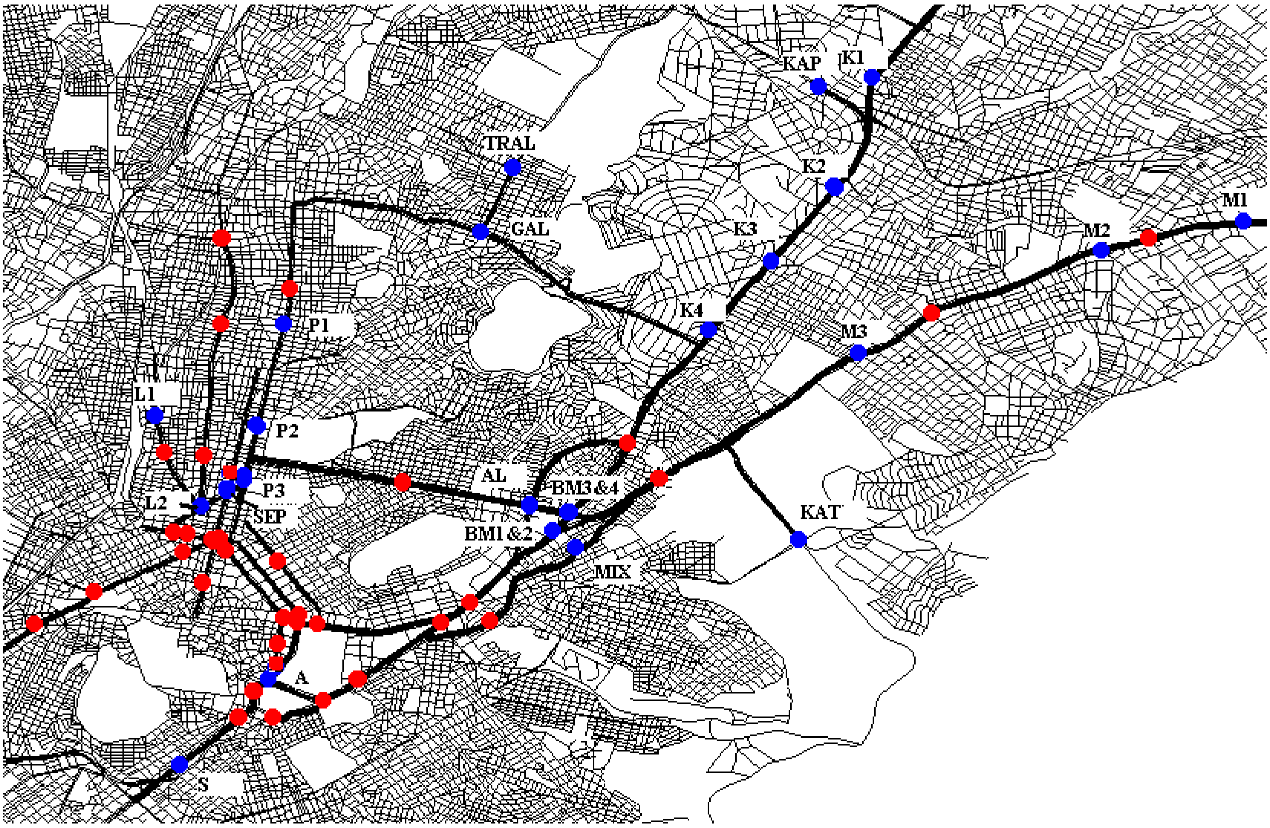

FIGURE 1 Loop detectors at the Athens road network. The ones used in this study are highlighted with different color and a label.

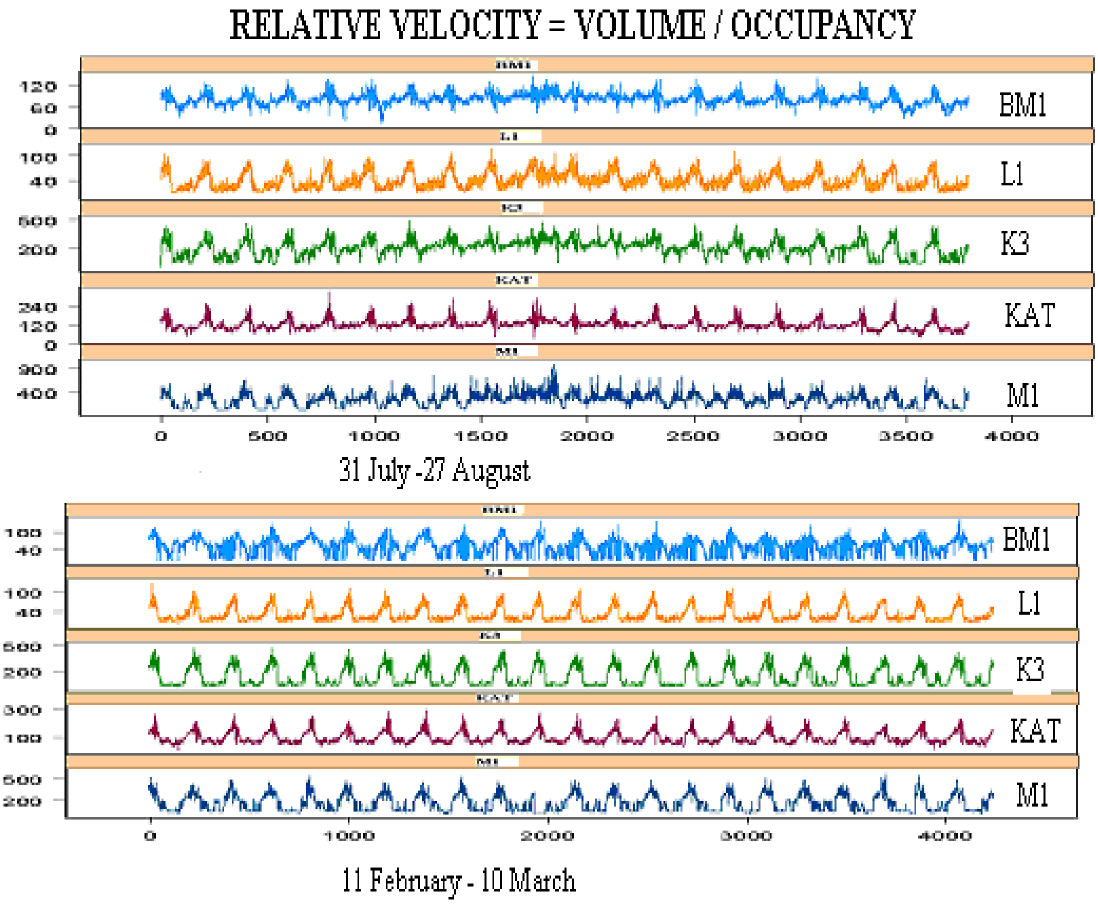

FIGURE 2 Evolution of relative velocity through time for the two periods under study for 5 of the 25 loop detectors.

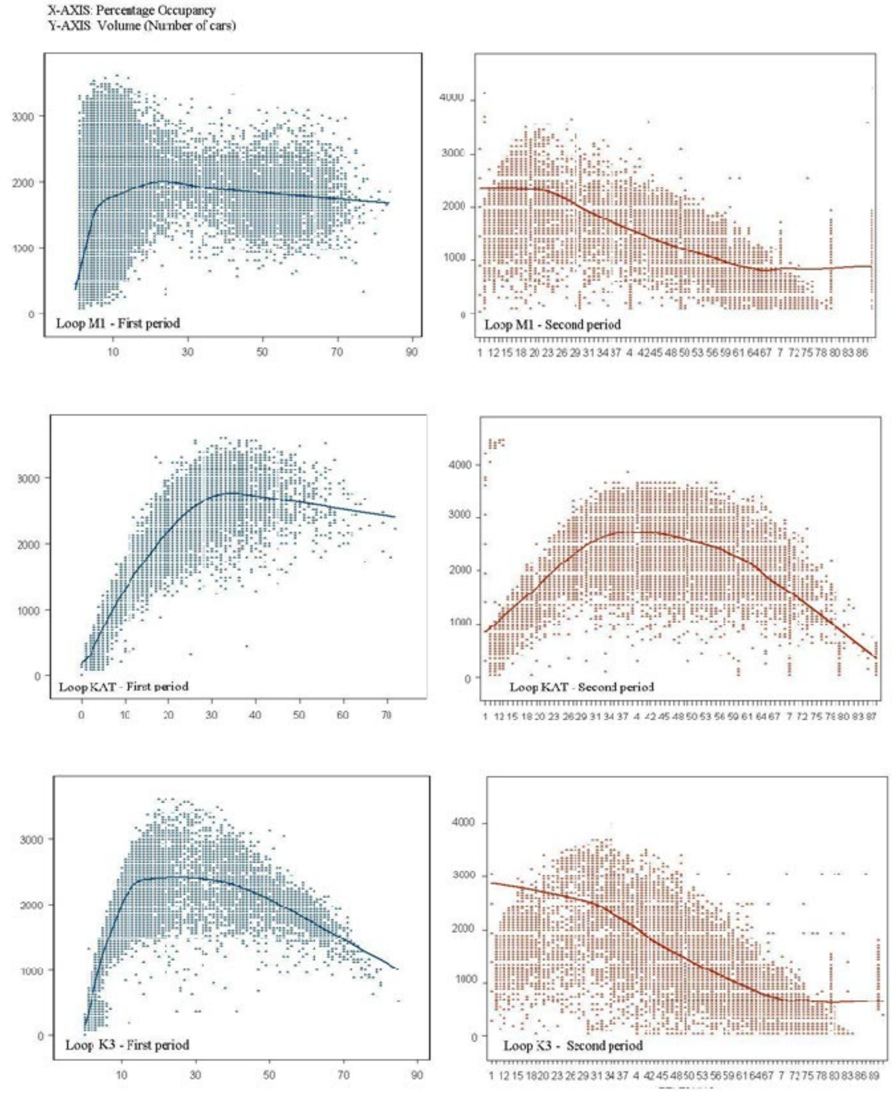

FIGURE 3 Occupancy-Volume scatterplots and loess smoothing curves for three loops of the study. The left column corresponds to the first period under study and the right one to the second period.

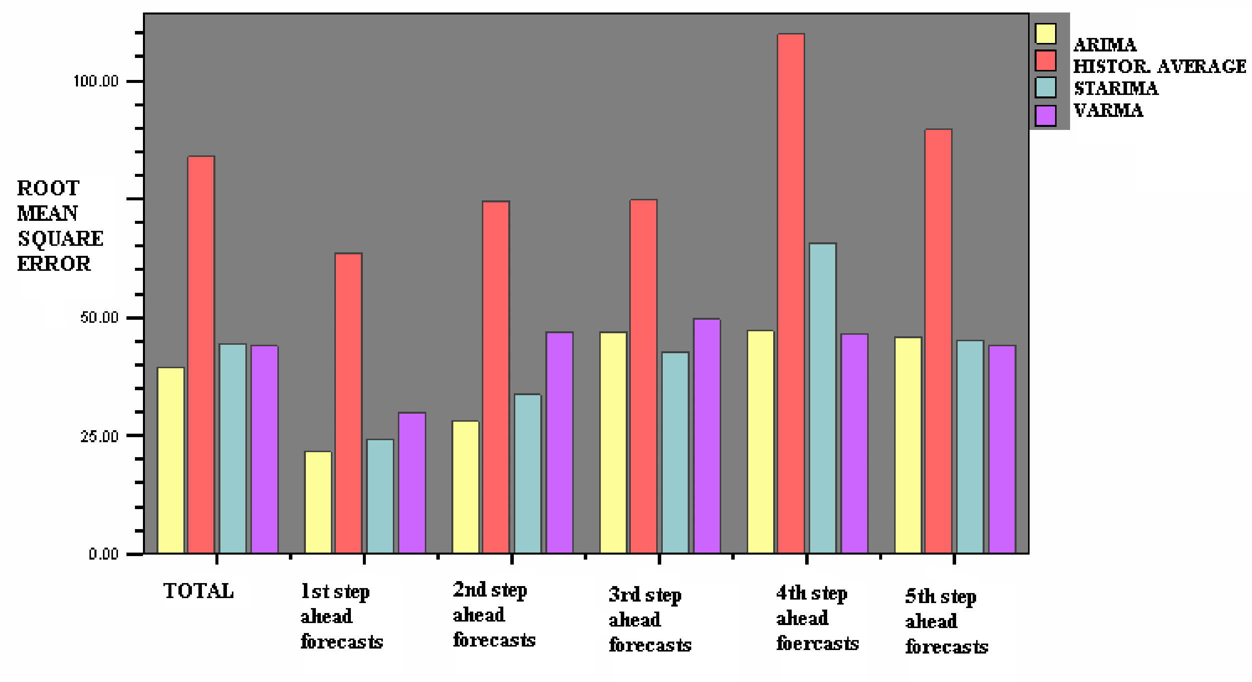

FIGURE 4 Root mean square error performance of the proposed methods.