Traffic Flow Forecasting: Comparison of Modeling Approaches

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Abstract: The capability to forecast traffic volume in an operational setting has been identified as a critical need for intelligent transportation systems (ITS). In particular, traffic volume forecasts will support proactive, dynamic traffic control. However, previous attempts to develop traffic volume forecasting models have met with limited success. This research effort focused on developing traffic volume forecasting models for two sites on Northern Virginia's Capital Beltway. Four models were developed and tested for the freeway traffic flow forecasting problem, which is defined as estimating traffic flow 15 min into the future. They were the historical average, time-series, neural network, and nonparametric regression models. The nonparametric regression model significantly outperformed the other models. A Wilcoxon signed-rank test revealed that the nonparametric regression model experienced significantly lower errors than the other models. In addition, the nonparametric regression model was easy to implement, and proved to be portable, performing well at two distinct sites. Based on its success, research is ongoing to refine the nonparametric regression model and to extend it to produce multiple interval forecasts.

INTRODUCTION

The primary purpose of advanced traffic management and information systems (ATMIS) is to operate the surface transportation system as efficiently as possible. To accomplish this, ATMIS rely upon surveillance technology to collect data describing the status of the system, as well as decision support software to assist in developing optimal traffic control strategies. The key challenge to developing ATMIS is to effectively use the surveillance data to derive control strategies.

An ATMIS cannot successfully operate in a reactive mode. In fact, one could argue that real-time traffic control is even an example of a reactive system. A real-time system collects data and (almost) immediately modifies the control strategy. However, such a system cannot anticipate system changes and modify the control strategy accordingly. To operate in a proactive manner, an ATMIS must incorporate an anticipatory or forecasting capability. In fact, an early report on intelligent transportation systems (ITS) architecture concluded that "the ability to make and continuously update predictions of traffic flows and link times for several minutes into the future using real-time data is a major requirement for providing dynamic traffic control" (Cheslow et al. 1992).

Unfortunately, the development of an effective traffic flow forecasting model has proven to be difficult. As Davis and Nihan (1991) conclude, "the short-term forecasting of traffic conditions has had an active but somewhat unsatisfying research history." In this paper we discuss the application of two advanced modeling techniques–neural networks and nonparametric regression—to freeway traffic flow forecasting. In addition, we compare the performance of these models to two traditional models—historical average and time series—at a challenging test location using data collected by an operational ATMIS.

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PROBLEM DEFINITION

It is important to formulate the freeway traffic flow prediction problem in such a way that it can be incorporated in existing and future ATMIS. Therefore, one must first consider what information will be readily available to serve as model inputs. Basically, one can only expect to have access to data describing current and previous traffic volume. Sensors will collect flow data, creating an extensive database of past conditions. An effective traffic flow prediction model must exploit the information contained in such a database.

Furthermore, as field controllers become more sophisticated, it is likely that ATMIS will use highly distributed architectures. This means that much of the computational load, such as performing calculations to estimate future traffic volumes, will fall upon the controllers. To support such an architecture, and to minimize the communications requirements, the assumption was made that only data local to the controller would be available to serve as model inputs. While other data, such as volumes from upstream detectors, may likely provide useful information for traffic flow forecasting, the availability of such data in a distributed architecture is doubtful.

Therefore, it is assumed here that at the time of prediction, t, a model will have access to the site's current volume, $V(t)$, and the volume at the previous prediction period (in this case we assume that volume predictions will occur every 15 min). In addition, it is assumed that the database of past conditions can be used to calculate historical average volumes, V_{hist}, at the site. A formal statement of the problem is as follows:

Predict: $V(t + 15)$
Given: $V(t)$
$V(t - 15)$
$V_{hist}(t)$
$V_{hist}(t + 15)$

The prediction interval is defined as 15 min in this research effort because the variation in flow rates for shorter periods is unstable (McShane and Roess 1990). In fact, the *Highway Capacity Manual* (1994) suggests using 15-min flow rates for operational analyses.

TEST LOCATION

The four traffic flow forecasting models were developed and tested using data collected at sites monitored by the Northern

Virginia traffic management system (TMS). The TMS is an early generation ATMIS, using loop detectors in each lane to collect traffic volume data. The two sites chosen within the TMS are both located on the Capitial Beltway, the most congested interstate facility in the Commonwealth of Virginia. One site is southbound near the Telegraph Road interchange, while the other site is northbound at the Woodrow Wilson Bridge.

Two independent sets of data were collected for model development and model evaluation. The development database consists of traffic volumes collected from June through August 1993, while the evaluation database consists of traffic volumes collected from September through October 1993. Seasonal differences may exist between these databases. This allows the researcher to investigate whether seasonal effects significantly deteriorate the performance of a traffic flow forecasting model.

FREEWAY TRAFFIC FLOW FORECASTING MODELS

Each of the four freeway forecasting models developed in this research effort are described as follows.

Historical Average

The historical average model simply uses an average of past traffic volumes to forecast future traffic volume. As such, it relies upon the cyclical nature of traffic flow. However, the model has no way to react to dynamic changes, such as incidents, in the transportation system. Forms of the historical average model have been applied to the urban traffic control system (UTCS) (Stephanedes et al. 1981), as well as to various traveler information systems in Europe such as AUTOGUIDE (Jeffrey et al. 1987) and LISB (Kaysi et al. 1993).

The present model was simply formulated by finding the average volume for each time interval at each site. The average was computed using the development database only. The model does not need to use the full set of independent variables included in the definition of $x(t)$. The model works as follows: at time t, $V(t + D)$ is estimated as $V_{\mathrm{hist}}(t + D)$.

Time Series

Statistical time-series models, such as the autoregressive integrated moving average (ARIMA) model, attempt to develop a mathematical model explaining the past behavior of a series and then apply it to forecast future behavior. ARIMA models have been applied to the UTCS (Okutani and Stephanedes 1984) and to freeway volume forecasting (Kim and Hobeika 1993).

ARIMA models rely on an uninterrupted series of data. Unfortunately, systems such as the Northern Virginia TMS operate in a harsh environment that often results in missing data. Whether it be faulty loop detectors, noise in the communications network, or software problems, one must expect that missing data will be problematic for some time to come in ATMIS. While data filling techniques can address this problem, research has shown that they should "be used with caution as the complexity of the situation increases" (Terry et al. 1986).

Based on this conclusion, it appears that time-series modeling is not well suited for wide-scale field application to freeway traffic flow forecasting. For this study, a simplified ARIMA model was developed based on average volumes for each time interval on each week day, computed from the Telegraph Road development database. The model was developed with five consecutive days of data, with no missing values. Using a statistical software package, an ARIMA(2,1,0) process was identified and then the parameters were estimated.

Back-Propagation Neural Network

Neural networks, a mathematical modeling approach developed in the field of artificial intelligence, have recently gained significant attention from the transportation research community. They have been applied to areas such as traffic flow modeling, traffic signal control, and transportation planning.

While a number of neural network paradigms exist, the most widely applied paradigm to date is back-propagation. Back-propagation was selected for this application based on its well-developed theory, and its ability to model relationships between continuously valued variables. Back-propagation networks approximate a mathematical function to perform a mapping. In Fig. 1 a back-propagation network accepts inputs at the input layer, passes the weighted input values for summation at the hidden layer, and then weights the sums and passes them to the output layer. Back-propagation networks "learn" connection weight values (the parameters of the model) based on a set of training input/output data. Back-propagation networks do not require a modeler to define the functional form of the model a priori. As seen in Fig. 1, the mapping between the inputs and output is distributed through the network structure. While this distributed representation of the model allows for flexibility, it also makes determination of the relationships between the variables difficult.

Back-propagation neural networks have proven to be effective at representing complex nonlinear relationships. However, it is possible to overtrain the network, resulting in a memorization of the training data rather than a generalization of the relationship. To produce an effective model, a modeler must have access to a large training database and must exercise judgment on when to quit training.

The neural network model for this effort was trained with the development database collected at the Telegraph Road location. The model contained 10 hidden processing elements on one hidden layer. The model was not retrained at the Wilson Bridge location. This decision was made based on the belief that it is highly unlikely that personnel with sufficient skill and experience will be available to train neural networks at different sites in the field. To be considered for wide-scale use, the neural network model must be usable at multiple sites without retraining.

Nonparametric Regression

Nonparametric regression can be thought of as a dynamic clustering model. This approach attempts to identify groups of past cases whose input values, or states, are similar to the state of the system at prediction time. It is considered dynamic in that it defines a group of similar past cases (or the neighborhood) around the current input state, instead of defining a number of groupings prior to the time of prediction. Nonparametric regression has been developing over the last 20 years,

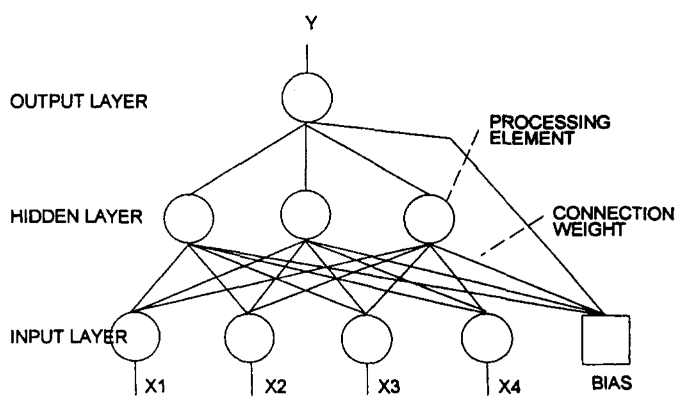

FIG. 1. Backpropagation Neural Network

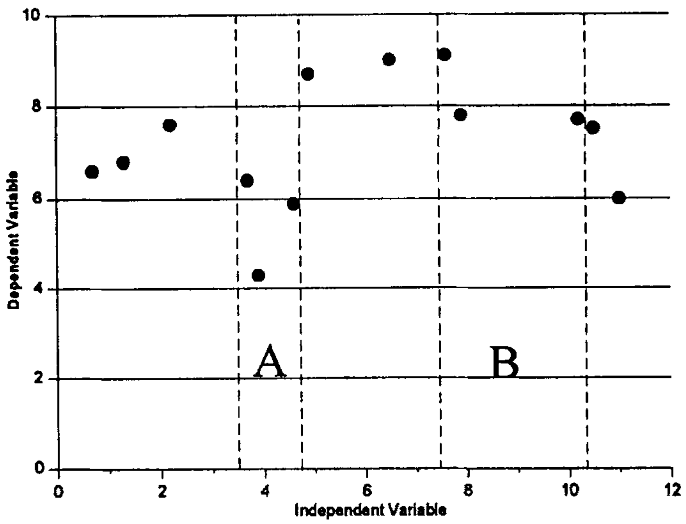

FIG. 2. Nearest Neighbor Example

At time t, given the state of the system, $X(t)$:

0. Initialize list of nearest neighbors, NB, to contain cases 1, 2, ..., k of the development database.

1. For each element c of the development database:
   a. Calculate $DISTANCE(X(t),X_c)$
   b. IF $DISTANCE(X(t),X_c) < MaxDISTANCE$(NB), where $MaxDISTANCE$(NB) is the largest distance between a member of NB, case LONGEST, and $X(t)$.
      i. Remove LONGEST from NB
      ii. Add c to NB

2. Estimate $V(t+D)$ as:

$$\hat{V}(t+D)=\frac{\sum_{q\in NB} Vf_q}{k}$$

FIG. 3. Nonparametric Regression Algorithm

and has seen limited application to areas such as traffic flow modeling (Davis and Nihan 1991) and hydrology (Karlsson and Yakowitz 1987).

The nearest neighbor formulation of nonparametric regression is used in this research effort. This form of nonparametric regression defines neighborhoods as those k cases with the least distance to the input state (Altman 1992). Euclidean distance is used to determine distance between the input state and cases in the database. Once the neighborhood is defined, it is easy to calculate the estimate. The estimate is an average of observed output values for cases that fall within the neighborhood.

A simple example of the nearest neighbor formulation of nonparametric regression is presented as follows. Consider a bivariate relationship, $f(x) = y$, for which estimates of y are desired for $x_A = 4$ and $x_B = 9$. A database of 13 previously observed x, y pairs is available, and is shown graphically in Fig. 2.

Assuming a neighborhood size of $k = 3$, neighborhoods A and B are identified; they include the three cases with x-values closest to x_A and x_B, respectively. Estimates for y are then computed by averaging the y-values of the cases in the neighborhood. For x_A, y is estimated as 5.6, the average of the observed y values of 6.5, 4.3, and 5.9. For x_B, y is estimated as 8.2.

In this research effort an algorithm was coded to automate the neighborhood definition and forecast generation processes. This algorithm is generic in the sense that it can be applied at a site by simple linking to the site's historical database. Therefore, here the algorithm was linked to the development database at the Telegraph Road and Woodrow Wilson Bridge sites, respectively. The development database is made up of state, future volume pairs. Using c to index the database, the state, future volume pair may be described as X_c, Vf_c. In this case X_c is defined by the values of the given variables: $V(t)$, $V(t - 15)$, $V_{hist}(t)$, $V_{hist}(t + 15)$. The complete algorithm is presented in Fig. 3.

TABLE 1. Comparison of Models

Model (1)	Strengths (2)	Weaknesses (3)
Historical average	Ease of implementation Speed of execution	Response to unexpected events
ARIMA	Time-series applications Established technique	Handling of missing values
Neural network	Suited to complex, nonlinear relationships	Black box Complex training procedure
Nonparametric regression	Requires no assumption of underlying relationship Pattern recognition applications	Complexity of search in identifying "neighbors"

Comparison Summary

Table 1 presents a summary of the strengths and weaknesses of the models considered for application to freeway traffic flow forecasting.

RESULTS

To assess and compare the effectiveness of the four freeway volume prediction models, the models were applied to the evaluation database collected at the Northern Virginia TMS. The use of this independent data set allows for an objective analysis of each model's performance.

Index of Performance

To fully evaluate the performance and potential for field implementation of the four modeling techniques, an index of performance was established. The index of performance for this research effort consists of four components: absolute error, distribution of error, ease of model implementation, and model portability.

The absolute error component of the index serves as the primary measure of model accuracy. The measure simply describes how far one would expect a predicted volume to deviate from (either above or below) the actual volume. To assess the statistical significance of differences between absolute average errors of the models, the Wilcoxon signed-rank test was used. This nonparametric test was used in place of a more traditional analysis of variance approach because the distribution of the absolute errors is not normal. The Wilcoxon test examines paired sample cases. In this application, the pairs may be defined as the absolute error experienced by two models at a given prediction time.

The null hypothesis of the Wilcoxon test states that the mean of the two populations, in this case defined as the absolute errors for two models, are equivalent. This can be represented as $\mu_1 - \mu_2 = 0$. This hypothesis is tested by computing the difference for each of the paired observations, and then ranking the differences without regard to sign. The positive difference ranks and negative difference ranks are then summed up. In general, the null hypothesis is accepted if the sum of positive difference ranks and that of negative difference ranks are roughly equivalent. If the sum of the positive difference ranks is substantially greater than that of negative difference ranks, the alternative hypothesis of $\mu_1 - \mu_2 > 0$ can be accepted. In this application, this would indicate that the average absolute error for model 1 is significantly greater than the average absolute error for model 2. A Z-statistic can be calculated to determine the level of significance.

When considering a particular model's distribution of error, it is especially important to evaluate its tendency to grossly overestimate or underestimate future volume. Such "bad misses" are particularly detrimental in ATMIS applications. Therefore, this component of the index of performance measures the number of extreme errors one may expect from the application of a model. Two thresholds are defined: errors greater than 10% and errors greater than 20%.

The ease of implementing a traffic flow forecasting model will have a direct impact on the deployment of such models in ITS. Simply put, there will not be a large number of highly skilled, technical individuals available to develop models for different geographic regions. For example, if the effective implementation of a model requires personnel to conduct a lengthy parameter definition process, it is likely that the model will never be fielded or that it will be deployed with inadequate parameter definitions. This component of the performance index is evaluated based on experience gained in developing the models.

Finally, the consistent performance of a model at multiple sites, or model portability, is a desired characteristic. Once a model is developed for a region, it should perform comparably at the numerous sites where it is deployed. This component of the performance index will be measured by comparing the difference in the errors experienced by each model at the Telegraph Road and Woodrow Wilson Bridge site.

Telegraph Road Site

The measures of error for the four models at the Telegraph Road site are displayed in Table 2. Since the time-series model cannot function in an environment with missing data, the evaluation database would only allow for testing of the model on two consecutive days. Table 3 shows the results of the Wilcoxon signed-rank tests. Again, since this test requires paired samples, the ARIMA model could not be included.

Table 4 describes the distribution of errors for each model. Of particular note is the distribution of underestimates and overestimates.

On examining Tables 2–4, it is clear that the nearest neighbor model outperforms the other models at the Telegraph Road site. This is particularly true in that the site's average absolute error is shown to be significantly less than that experienced by the other models, as proved in Table 3. The model's effectiveness is further illustrated by the fact that it estimates future traffic volume within 10% error in more than 75% of the test cases. Finally, the Telegraph Road site presents the most "level" comparison between the models, in that each model was developed with data from the Telegraph Road site.

The historical average model experienced the highest level of error among the four models, averaging 9.57% error per estimate. While this aggregate measure of error was highest, the model did not demonstrate a tendency to grossly overestimate or underestimate future traffic volumes when compared to the other models. For example, the historical average model experienced errors of 20% or more on 10.9% of the evaluation cases, as compared to 11.8% of the cases for the neural network model and 10.4% of the cases for the ARIMA model. These results indicate that while the model cannot be expected to produce the most accurate forecasts, it should at least be within a reasonable range of the true value.

However, Table 4 illustrates that the historical average model has a problem with frequently overestimating future traffic flow. For example, the model overestimated by more than 20% in 6.95% of the cases, while it underestimated by more than 20% in 3.95% of the cases. This can most likely

TABLE 2. Error Measures—Telegraph Road Site

Model (1)	Average absolute error (vehicles/h) (2)	Average error (%) (3)
Historical average (HA)	214.6	9.57
Neural network (NNw)	182.5	8.93
Nearest neighbor (NNb)	167.3	7.54
ARIMA[a]	195.0	9.03

Note: Number of samples (n) = 2,404.
[a]Includes only two days of evaluation data.

TABLE 3. Wilcoxon Signed-Rank Tests—Telegraph Road Site

Null hypothesis (1)	Alternative hypothesis (2)	Z (3)	Significant at 0.01 level? (4)	Preferred model (5)
$\mu_{HA} - \mu_{NNb} = 0$	$\mu_{HA} - \mu_{NNb} > 0$	11.92	Yes	Nearest neighbor
$\mu_{HA} - \mu_{NNw} = 0$	$\mu_{HA} - \mu_{NNw} > 0$	6.32	Yes	Neural network
$\mu_{NNw} - \mu_{NNb} = 0$	$\mu_{NNw} - \mu_{NNb} > 0$	5.68	Yes	Nearest neighbor

TABLE 4. Error Distribution—Telegraph Road Site

Model (1)	Percent Cases Over 10% Error		Percent Cases Over 20% Error	
	Underestimate (2)	Overestimate (3)	Underestimate (4)	Overestimate (5)
Historical average	14.02	19.30	3.95	6.95
Neural network	24.08	7.28	10.02	1.75
Nearest neighbor	13.31	10.98	1.87	4.08
ARIMA[a]	16.67	16.15	5.21	5.21

[a]Includes only two days of evaluation data.

be attributed to the fact that the historical average model has no way of reacting to external changes such as incidents in the system. It is likely that incidents occurred, reducing roadway capacity and effectively metering traffic. In such cases, the model was forecasting on the basis of "normal," nonmetered conditions.

As seen in Tables 2 and 4, the ARIMA model performed slightly better than the historical average model. It experienced roughly the same number of bad misses as the other models (with the exception of the nearest neighbor model, which experienced significantly fewer misses), but the distribution of the misses were attractive. For both the 10 and 20% levels, the ARIMA model was equally likely to overpredict or underpredict the future volume.

However, the fact that the ARIMA model could only be applied to two days of the evaluation database, which spans a period of two months, is evidence that the model has poor potential for effective field application. Regardless of its performance under ideal conditions, the ARIMA model is clearly not well suited for application to the traffic flow prediction problem. For this reason, the ARIMA model was dropped from consideration and not evaluated at the Woodrow Wilson Bridge site.

As seen in Table 2, the neural network model was the second-most effective model among the four, experiencing an average error of 8.93% per estimate. Furthermore, Table 3 shows that the neural network's average absolute error is significantly less than that experienced by the historical average model. The most troubling aspect of the model's performance was in the distribution of error, as shown in Table 4. For example, in nearly a quarter of the cases in the evaluation database the neural network underestimated future traffic flow by at least

10%, while there were overestimates of 10% or more in only 7.3% of the cases. In other words, for some reason the neural network model consistently produces significant underestimates of future traffic volume.

This problem is most likely due to the network training process. It is likely that cases in the training data that describe incident conditions resulted in overly extreme modifications of connection weights. In other words, a few incident conditions in the development database may have caused the network to reduce flow estimates across the board. This illustrates how important it is to select training data carefully. In addition, it illustrates another significant challenge to effectively develop a neural network model in a field application.

Woodrow Wilson Bridge Site

The measures of error for the models at the Woodrow Wilson Bridge site are displayed in Table 5. As described earlier, the time-series model was dropped from consideration, and therefore was not evaluated at this site. Table 6 shows the results of the Wilcoxon signed-rank tests.

Table 7 describes the distribution of error at the Wilson Bridge site. Of particular note for many of the models is the distribution of underestimates and overestimates.

The nearest neighbor model was the most effective model at the Woodrow Wilson Bridge site, significantly outperforming the other models as shown in the Wilcoxon signed-rank test results in Table 6. In addition, its performance is comparable to that observed at Telegraph Road. The model experienced an average of 8.0% error per estimate at the Wilson Bridge site, as compared to 7.5% at Telegraph Road. Furthermore, only 6.2% of the cases in the evaluation data set experienced errors of more than 20%.

On examining Tables 2–7, it is clear that the historical average model exhibited comparable performance at both the Telegraph Road and the Wilson Bridge sites. For example, the model experienced an average error per estimate of 9.86% at the Wilson Bridge site and an average error of 9.57% at Telegraph Road. Similarly, 10.9% of the cases at the Wilson Bridge site experienced errors greater than 20%, as compared to 11.0% of the cases at Telegraph Road. In addition, the errors were evenly distributed at both sites.

These results illustrate that the historical average model is portable. However, it has serious drawbacks. As illustrated at the Telegraph Road site, and reflected in the relatively high error measures at Wilson Bridge, the model is unable to respond to current conditions such as incidents. Therefore, the model will result in a high percentage of bad misses.

As shown in Tables 5–7, the neural network model experiences significantly higher error at the Woodrow Wilson Bridge site. The average error per estimate at the Wilson Bridge is 11.0% as compared to 8.9% at Telegraph Road. The most problematic indication of the model's poor performance is the fact that nearly half (45.0%) of the cases in the evaluation data set experience errors of 10% or more, and 20.1% of the cases experienced errors of more than 20%. In addition, the model, as also seen in the Telegraph Road results, is much more likely to underestimate than overestimate future volume.

The results at the Wilson Bridge site clearly show that the neural network model is not portable. It was developed at the Telegraph Road site, where it did a reasonable job of estimating future volume. However, the model does not capture a "universal" underlying relationship between the system's current status and its future volume. Rather, it is clear that for the model to be effective, it must be trained with data collected at each site where it will be used.

TABLE 5. Error Measures—Woodrow Wilson Bridge Site

Model (1)	Average absolute error (vehicles/h) (2)	Average error (%) (3)
Historical average (HA)	300.4	9.86
Neural network (NNw)	450.3	11.00
Nearest neighbor (NNb)	229.3	8.07

Note: Number of samples (n) = 2,544.

TABLE 6. Wilcoxon Signed-Rank Tests—Woodrow Wilson Bridge Site

Null hypothesis (1)	Alternative hypothesis (2)	Z (3)	Significant at 0.01 level? (4)	Preferred model (5)
$\mu_{HA} - \mu_{NNb} = 0$	$\mu_{HA} - \mu_{NNb} > 0$	12.82	Yes	Nearest neighbor
$\mu_{HA} - \mu_{NNw} = 0$	$\mu_{HA} - \mu_{NNw} > 0$	7.60	Yes	Historical average
$\mu_{NNw} - \mu_{NNb} = 0$	$\mu_{NNw} - \mu_{NNb} > 0$	16.43	Yes	Nearest neighbor

TABLE 7. Error Distribution—Woodrow Wilson Bridge Site

Model	Percent Cases Over 10% Error		Percent Cases Over 20% Error	
	Underestimate (2)	Overestimate (3)	Underestimate (4)	Overestimate (5)
(1)				
Historical average	13.99	19.85	4.60	6.33
Neural network	32.86	12.11	14.47	5.58
Nearest neighbor	11.01	14.58	1.53	4.68

CONCLUSIONS

The nearest neighbor formulation of nonparametric regression holds considerable promise for application to traffic flow forecasting. The strengths of this model have been demonstrated using actual data collected by an early ATMIS and discussed by considering the index of performance.

First, as seen in the results of the Wilcoxon signed-rank tests, the nearest neighbor model experienced significantly less error than the other three models at both sites. Furthermore, the error was well distributed, with very few extreme overestimates or underestimates. The model is easy to implement; all that is required is the coupling of a generic algorithm with a site-specific database. Finally, such ease of implementation allows the model to be successfully applied at multiple sites. This is demonstrated by the comparable error characteristics the model demonstrated at the Telegraph Road and Woodrow Wilson Bridge sites.

Furthermore, the theoretical foundation of nonparametric regression is ideal for traffic flow forecasting application. By selectively choosing past experiences, the model can exploit information contained within a large set of data. Given the large variations in traffic flow, such an approach is likely to react to unexpected changes, such as incidents, more effectively than a model that attempts to develop a single mapping function.

Further work is ongoing to extend the nonparametric model to forecast multiple intervals into the future. Such an extended forecast range will allow for the development of more sophisticated traffic control strategies under ATMIS. Finally, work is ongoing to develop a software architecture that will integrate the functionality of the forecasting model with other ATMIS components.

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