Traffic Modeling Based on FARIMA Models

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Abstract

We provide in this paper a procedure to fit a FARIMA(*p,d,q*) (fractional autoregressive integrated moving average) model to the actual traffic trace, as well as a method to generate a FARIMA process with given parameters. We showed how to model traffic by fitting FARIMA models to four measured traces. Our experiments illustrated that FARIMA model is a good traffic model and is capable of capturing the property of real traffic with long-range and short-range dependent behavior. Unlike previous work on FARIMA models, we deduce some guidelines to reduce the complexity of fitting the FARIMA model which would allow us to reduce the computational time of fitting.

Key words: FARIMA, long-range dependence, traffic modeling, self-similarity.

1. INTRODUCTION

Time series modeling holds a great promise as a tool for studying network traffic. However, traditional models can only capture short-range dependence; for examples, Poisson process, Markov processes, AR, MA, ARMA and ARIMA processes [1].

The studies of high quality traffic measurements have revealed that traffic in high-speed networks exhibits self-similarity, i.e. long-range dependence [2] that can't be captured by previous models. Hence, self-similar models, such as FGN (fractional Gaussian noise) model [3] and FDN (fractional differencing noise) model [4] have been developed. Unfortunately, these models can't be used to describe the short-range dependence. Recent real traffic measurements found the co-existence of both long-range and short-range dependence in traffic traces [5]. Therefore, models are required to describe both long-range and short-range dependence simultaneously. We consider FARIMA(*p,d,q*) (fractional autoregressive integrated moving average) model as one of good models with this capability.

This paper studied the FARIMA models in its implementation detail. We provide a procedure to fit a FARIMA model to the actual traffic trace, as well as a method to generate a FARIMA process with given parameters. We use the techniques of backward-prediction, fractal de-filter (fractional differencing), and a combination of rough estimation and accurate estimation to provide guidelines for simplifying the FARIMA model fitting procedure, in order for us to reduce the time of traffic modeling.

This paper is organized as follows. Section 2 summarizes the basics mathematics of the FARIMA models. Section 3 provides the method to generate a FARIMA(*p,d,q*) process. Section 4 shows how to build a FARIMA(*p,d,q*) model to describe a traffic trace. Section 5 studies the feasibility of FARIMA(*p,d,q*) models. Section 6 is the concluding remarks.

2. THE MODEL

FARIMA processes are the natural generalizations of standard ARIMA (*p,d,q*) processes defined in [1] when the degree of differencing d is allowed to take nonintegeral values [4]. A FARIMA*(p,d,q)* process $\{X_t$: $t=\ldots,-1, 0, 1,\ldots\}$ is defined to be

$$\Phi(B)\Delta^d X_t = \Theta(B)a_t, \qquad (2\text{-}1)$$

where $\{a_t\}$ is a white noise and $d \in(-0.5, 0.5)$,

$$\Phi(B) = 1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p,$$
$$\Theta(B) = 1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q,$$

$\Theta(B)$, $\Phi(B)$ have no common zeroes, and also no zeroes in $|B|\leq 1$ while p and q are non-negative integers. B is the backward-shift operator, i.e. $BX_t = X_{t-1}$. $\Delta = (1-B)$ is the differencing operator and Δ^d denotes the fractional differencing operator,

$$\Delta^d = (1-B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k , \qquad (2\text{-}2)$$

$$\binom{d}{k} = \Gamma(d+1)/[\Gamma(k+1)\Gamma(d-k+1)],$$

Γ denotes the Gamma function.

Clearly, if $d = 0$, FARIMA(p,d,q) processes are the usual ARMA(p,q) processes. If $d \in (0, 0.5)$, then long-range dependence or persistence occurs in the FARIMA processes. FARIMA($0,d,0$) process, i.e. FDN, is the simplest and most fundamental form of the FARIMA processes. The property of FARIMA($0,d,0$) process is similar to FGN process which can only describe long-range dependence. The parameter d in FARIMA($0,d,0$) process is the indicator for the strength of long-range dependence, just like the Hurst parameter H in FGN process. In fact, $H = d + 0.5$. Both processes have autocorrelation functions which behave asymptotically as k^{2d-1} with different constants of proportionality [2].

For $d \in (0, 0.5)$, $p \neq 0$ and $q \neq 0$, a FARIMA(p,d,q) process can be regarded as an ARMA(p,q) process driven by FDN. From (2-1), we obtain

$$X_t = \Phi^{-1}(B)\Theta(B)Y_t , \qquad (2\text{-}3)$$

where

$$Y_t = \Delta^{-d} a_t . \qquad (2\text{-}4)$$

Here, Y_t is a FDN. Consequently, compared with ARMA and FGN processes, the FARIMA(p,d,q) processes are flexible and parsimonious [6] with regard to the simultaneous modeling of the long-range and short-range dependent behavior of a time series.

3. METHOD TO GENERATE A FARIMA*(p,d,q)* PROCESS

From (2-3) and (2-4), one sees that the FARIMA(p,d,q) process X_t can be regarded as an ARMA(p,q) process driven by a FDN Y_t. So, we propose a two-step procedure to generate a FARIMA(p,d,q) process as follows:

Step1: Generating a FDN Y_t using (2-4).

Step2: Generating a FARIMA(p,d,q) process X_t using (2-3).

In step 1, note that according to the definition of fractional differencing operator (2-2) and the equation (2-4), a FDN process can be obtained for a given parameter d. Here, we make an important assumption that all the X_t values are equal to zero for negative time index. Under this assumption, we can get a finite time series defined by

$$X_n = \sum_{k=0}^{n} \pi_k X_{n-k} + a_n , \qquad (3\text{-}1)$$

where

$$\pi_0 = 1, \pi_1 = d, \pi_k = \frac{k-1-d}{k}\pi_{k-1}, k = 2,3,\ldots,\infty.$$

We call this method the direct definition method.

Another way of generating a FARIMA($0,d,0$) process was introduced by Hosking [4]. The basic equations for the Hosking's algorithm are as follows. The process X_t has Guassian margins with zero mean and variance v_0 and fractional differencing parameter $d = H - 1/2$. The autocorrelation function has an asymptotically hyperbolic shape, and is determined from d as

$$\rho_k = \frac{d(1+d)\cdots(k-1+d)}{(1-d)(2-d)\cdots(k-d)}.$$

X_0 is chosen from the Normal distribution N($0,v_0$). Set $N_0 = 0$ and $D_0 = 1$. Then generate n points by iterating the following steps for $k = 1, ..., n$:

$$N_k = \rho_k - \sum_{j=1}^{k-1} \phi_{k-1,j}\rho_{k-j} ,$$

$$D_k = D_{k-1} - N_{k-1}^2 / D_{k-1} ,$$

$$\phi_{kk} = N_k / D_k ,$$

$$\phi_{kj} = \phi_{k-1,j} - \phi_{kk}\phi_{k-1,k-j} , j = 1,\ldots,k-1,$$

$$m_k = \sum_{j=1}^{k} \phi_{kj} X_{k-j} ,$$

$$v_k = (1-\phi_{kk}^2)v_{k-1} .$$

Choose each X_k from N(m_k,v_k).

In step2, we use the known ARMA process generating method to generate the FARIMA(p,d,q) process X_t by replacing a_t (white noise) with Y_t (FDN).

3.1 Verification Experiments

Since the generation of a fractionally differenced noise FARIMA($0,d,0$) process is the key step, we did the following experiments to verify it. We used the above two ways (the direct definition method and the Hosking's algorithm) to generate FARIMA($0,d,0$) processes for five parameters (d = 0.1, 0.2, 0.3, 0.4, 0.45). For each generated process, we got 20,000 samples. Then, we estimated the Hurst parameter H ($d = H - 0.5$) using periodogram-based analysis for each generated process. Table 1 shows that the results of two generating ways are very close. The accuracy of estimated parameter d of generated processes with respect to the objective (given) parameter d is +5 to +15 %. So both of the above two ways are good for generating FARIMA(*0,d,0*) processes.

4. CONSTRUCTING A FARIMA*(p,d,q)* MODEL TO FIT AN ACTUAL TRACE

In order to build an appropriate model for a given traffic trace, the following three basic steps can be used iteratively until a successful model is obtained:

1). Model identification:

In this step, one wants to determine the likely values of the order of the model, i.e., the p, d and q parameters of the FARIMA model. Normally, there will be several plausible models to be examined. The order to determine the likely values of the parameters is:

- d, the level of differencing,
- p, the autoregression, and finally
- q, the moving average.

2). Parameter estimation:

Once a set of possible models has been selected, parameter values are determined for each model.

3). Diagnostic checking:

This involves checking how well the fitted model conforms to the trace, and it also suggests how the model should be changed when a good-fit is not obtained.

The problem remains open on how to fit an appropriate FARIMA model to the given trace, and this topic is also being studied in statistics field because it is more difficult to fit a FARIMA model than an ARMA model. As discussed above, FARIMA models come from ARMA models. Since there are several known ways for fitting ARMA models, we can take advantage of this by transferring the FARIMA problem to an ARMA problem. For a given time series X_t, we can obtain from (2-1)

$$W_t = \Delta^d X_t , \tag{4-1}$$

where

$$W_t = \Phi^{-1}(B)\Theta(B)a_t . \tag{4-2}$$

4.1 Steps of Fitting Traffic

We propose a way to solve (4-1) and to obtain W_t. That is, we use known methods to estimate the Hurst parameter H [7], and the level of differencing $d = H - 0.5$, then we do the fractional differencing as detailed in next paragraph. Using the known ways for fitting ARMA model [1], we can solve (4-2) and obtain model parameters $\phi_1, \phi_2, \dots, \phi_p, \theta_1, \theta_2, \dots, \theta_q$.

Here, we propose the following procedure to fit a FARIMA model to the traffic trace. Explanation is provided where necessary.

step 1: Pre-processing the measured traffic trace to get a zero-mean time series $X_t : t = 1, 2,\dots$

step 2: Obtaining an approximate value of d according to the relationship $d = H - 0.5$. The Hurst parameter H can be obtained using the known Hurst parameter estimates methods such as Variance-time plots, R/S analysis and periodogram-based method [7]. This is the rough estimation of d.

step 3: Doing fractional differencing on X_t. From (2-1) we can get the precise expression

$$W_t = \Delta^d X_t = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k X_t = \sum_{j=0}^{\infty} \pi_j X_{t-j} , \tag{4-3}$$

where

$$\pi_j = \frac{(-1)^j \Gamma(1+d)}{\Gamma(1+j)\Gamma(1+d-j)} .$$

In practice, from (4-3), we need the values of samples X_0, $X_{-1}, \dots , X_{-M}$ in back of the first sample (where M must be sufficiently large). Using EX_t instead of them may cause larger error. We realized its approximation form using auxiliary AR model based "backward-prediction". After fractional differencing on X_t, we obtain W_t [7].

step 4: Determining p and q using known ways for fitting ARMA models. We begin with several sets of p and q of models, and the value of p and q should be 0, 1, or 2 (p and q should not be 0 simultaneously in one set) because we found that p and q of fitted traffic FARIMA models are usually small. Then, we determine the best (p, q) combination according to the model identification and diagnostic checking. If no suitable combination is found, we will increase p and q then repeat above steps.

Step 5: Estimating parameters. After we determine the order p and q of the fitted model, we can get all parameters $\phi_1, \phi_2, \dots, \phi_p, \theta_1, \theta_2, \dots, \theta_q$, and σ^2 by using parameter estimation [8]. This step is called the accurate estimation of parameters.

Finally, we get the fitted FARIMA*(p,d,q)* model as form (2-1).

4.2 Verification Experiments

The key step of the above procedure for fitting a FARIMA model is the fractional differencing. So we did some experiments to verify if the fractional differencing step can remove long-range dependence, i.e., reduce d to close to zero.

In the first experiment, we generated the FARIMA($0,d,0$) traces for five parameters (d = 0.1, 0.2, 0.3, 0.4, 0.45) using the direct definition method as introduced in Section 3. After fractional differencing on these traces, we estimated their Hurst parameters using the periodgram-based analysis. The results are shown in Table 1. We found that after fractional differencing, the Hurst parameters are near 0.5 (0.51 - 0.54), and the parameters d are near zero (0.01 - 0.04). There is almost no long-range dependence (self-similarity) after fractional differencing.

We also studied a Bellcore trace pAug.TL to verify that the fractional differencing can remove long-range dependence. We got a data set which has 2000 samples, and each sample represents the arrival packet number during 0.01 second interval. We did Hurst parameter estimates for the trace pAug.TL and got $H = 0.8$, $d = 0.3$. Figure 1 and Figure 2 showed the autocorrelation function and spectral density of pAug.TL before and after fractional differencing respectively. After the fractional differencing, the parameter d decreased to 0.03 (near zero). We found that the autocorralation function decayed faster with the lag increase after the fractional differencing. In self-similar processes the low-frequency part is characteristic for a power-law behavior of the spectral density around zero, but the above behavior disappear after the fractional differencing. Therefore, the experiments verified that the fractional differencing significantly took off long-range dependence as we expected [9].

5. FEASIBILITY STUDY

In order to study the feasibility of FARIMA model based modeling, we build the FARIMA models for four real traffic traces, use some experiments to verify the fitted FARIMA model and compare the FARIMA model with other models. The possibility to simplify the FARIMA model based modeling is discussed here also.

5.1 Constructing FARIMA Models for Actual Traffic

In order to study the FARIMA-based traffic modeling, we have carried out some study on the measured traffic traces obtained from inside and outside China. We have two traces (C1003 and C1008) from CERNET (the Chinese Education and Research Network) center. The two traces were conducted on an Ethernet cable at the CERNET center, which carried all traffic to and from the Internet of China. We also have two traces (pAug.TL and pOct.TL) from Bellcore Lab [2].

In the experiment, we preprocessed the measured traces to get four data sets. Each data set has 2000 samples, and each sample represents the arrival packet number during 0.01 second interval. We used the procedure proposed in Section 4 to fit FARIMA models to the four data sets. The four fitted FARIMA models are given in Table 2. The FARIMA models of the four traffic traces showed that these traffic traces exhibit both long-range and short-range dependence because their $d \neq 0$, and $p, q \neq 0$ simultaneously.

5.2 Experiments to Verify the Derived FARIMA Models

Here, we use the traces to verify that the fitted FARIMA model can describe the dependence structure of the actual traffic trace. For example, Figure 3 shows the autocorrelation functions of the trace pAug.TL and its fitted FARIMA model. We found that the two curves are very close in both long range and short range. Therefore, the experiment verified that FARIMA models can simultaneously model the long-range and short-range dependent behavior of actual traffic.

5.3 Experiments to Compare with Other Models

We also did some experiments for comparing FARIMA model with other models such as ARIMA, AR and FGN. Here, we fitted the FARIMA, ARIMA, AR and FGN models to the trace pAug.TL. The four fitted models for pAug.TL are in Table 3.

Figures 3 to 6 show the autocorrelation functions of the fitted FARIMA, ARIMA, AR and FGN models of the trace pAug.TL respectively. As can be seen, the FGN model can't have a good fit of the autocorraltion function with actual trace at the short range, and AR model can't have a good fit at the long range. For both short and long ranges, the FARIMA and ARIMA models can have a good fit, but in comparison, the ARIMA model would need more parameters.

Table 4 shows the MSE (Mean-Squared Errors) [7] of the fitted models of the trace pAug.TL. Based on the results in Table 4, we observe that the FARIMA model has the lowest (best) MSE value, followed in order by the ARIMA, AR and FGN models.

5.4 Simplification Methods

Although FARIMA models have several advantages, one drawback appears to be the high computational complexity and the long procedure involved. In our experiments, we found that the order (p,q) of the fitted FARIMA model for an actual measured trace usually remains fixed for some time. This interesting finding suggests a method to simplify the modeling procedure, and this will make it more easy to implement the modeling procedure.

As an example, we studied the measured trace pAug.TL. The length of the trace is 100 seconds, and it has 10,000 samples, each representing packet arrivals during 0.01 second. Then we divide the trace to five data sets, Aug1 to Aug5, each with 2000 samples. We fit a FARIMA model to each data set. The five fitted FARIMA models are given in Table 5. Observe that the order of the five FARIMA models are constant at p = 1 and q = 1. This phenomenon implies that it is not necessary to do model identification and diagnostic checking for the trace pAug.TL in 100 seconds. We have the similar observations from other experiments that the (p,q) values may not change during long periods. So it is possible to simplify the FARIMA model fitting procedure and reduce the time for the FARIMA model based traffic modeling.

6. CONCLUSIONS

In this paper, we have studied how to fit FARIMA models to measured traffic traces. Our experiments showed that the FARIMA model is a good traffic model and is capable of capturing the properties of real traffic; this is because the FARIMA processes are much more flexible and capable of simultaneously modeling both the long-range and the short-range dependent behavior of a time series. Compared with other short-range dependent processes such as ARMA models, less parameters are required by the FARIMA models. Compared with other long-range dependent processes such as FGN models and FARIMA(0,d,0) models, FARIMA(p,d,q) models are more superior in their capability.

We also show that it is possible to simplify the fitting procedure. Consequently, one can reduce the time to build traffic models and do real-time modeling.

Future work includes applying FARIMA models in network design, management, traffic prediction and measurement-based network control.

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Table 1: *H* and *d* of generated FARIMA(*0,d,0*) and after fractional differencing

target parameter d	FARIMA(0,d,0) Definition algorithm		FARIMA(0,d,0) Hosking algorithm		after fractional differencing	
	$\hat{H}$	$\hat{d}$	$\hat{H}$	$\hat{d}$	$\hat{H}$	$\hat{d}$
0.1	0.614	0.114	0.613	0.113	0.521	0.021
0.2	0.721	0.221	0.714	0.214	0.540	0.040
0.3	0.819	0.319	0.821	0.321	0.537	0.037
0.4	0.924	0.424	0.911	0.411	0.544	0.044
0.45	0.961	0.461	0.966	0.466	0.543	0.043

Table 2: Fitted FARIMA models of CERNET and Bellcore traces

trace	FARIMA(p,d,q) model parameters				
	p	ϕ_1	d	q	θ_1
PAug.TL	1	-0.1714523	0.2949911	1	-0.3738780
pOct.TL	1	-0.1511854	0.2520007	1	-0.2518229
C1003	1	0.3040983	0.1978055	0	
C1008	1	-0.0439937	0.1834420	0	

Table 3: Fitted FARIMA, ARIMA, AR, FGN of pAug.TL

model	parameters
FARIMA(1,d,1)	ϕ_1=-0.1714523, d = 0.2940911, θ_1=-0.373878
ARIMA(10,1,0)	ϕ_1=-0.4736030, ϕ_2=-0.4879495, ϕ_3=-0.3929721 ϕ_4=-0.2892086, ϕ_5=-0.2043307, ϕ_6=-0.2398785 ϕ_7=-0.1701698, ϕ_8=-0.1083781, ϕ_9=-0.0693743 ϕ_{10}=-0.0394189
AR(8)	ϕ_1=0.4840741, ϕ_2= -0.0335644, ϕ_3=0.0692946 ϕ_4=0.0813773, ϕ_5=0.0663615, ϕ_6=-0.0509957 ϕ_7=0.0478156, ϕ_8=0.0471420
FGN	H=0.8, Mean=3.8995, Variance=7.357078

Table 4: MSE values

model	MSE value
FARIMA(1,d,1)	0.0006031
ARIMA(10,1,0)	0.0011448
AR(8)	0.0026151
FGN	0.0398209

Table 5: Fitted FARIMA models of five data sets of trace pAug.TL

Data set	FARIMA(p,d,q) model parameters				
	p	ϕ_1	d	q	θ_1
Aug1	1	-0.1714523	0.2949911	1	-0.373878
Aug2	1	-0.1143173	0.2783244	1	-0.305762
Aug3	1	-0.1853628	0.3515382	1	-0.329502
Aug4	1	-0.1763141	0.2813467	1	-0.306262
Aug5	1	-0.1599587	0.2995875	1	-0.280055

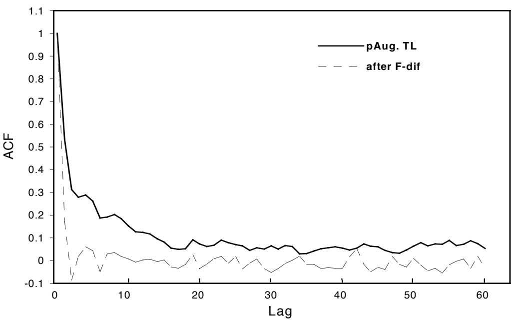

Figure 1: ACF of pAug.TL before and after fractional differencing

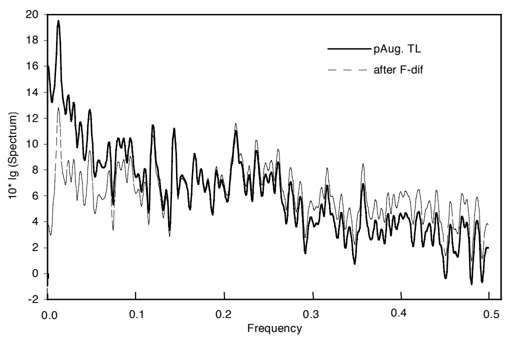

Figure 2: Spectrum of pAug.TL before and after fractional differencing

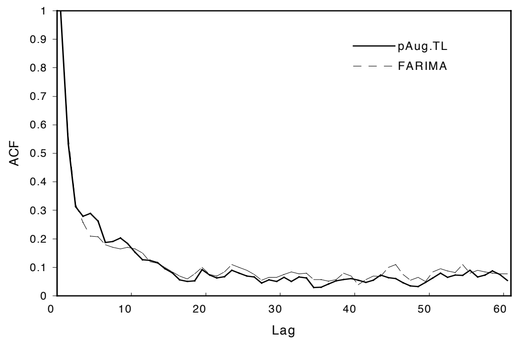

Figure 3: ACF of pAug.TL and its FARIMA model

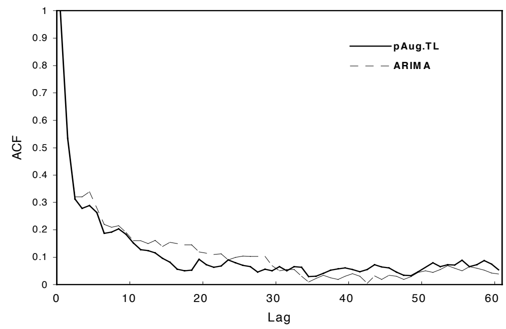

Figure 4: ACF of pAug.TL and its ARIMA model

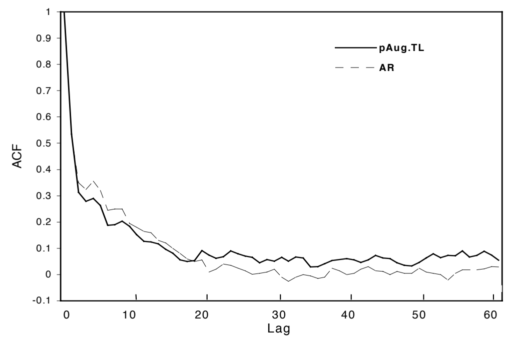

Figure 5: ACF of pAug.TL and its AR model

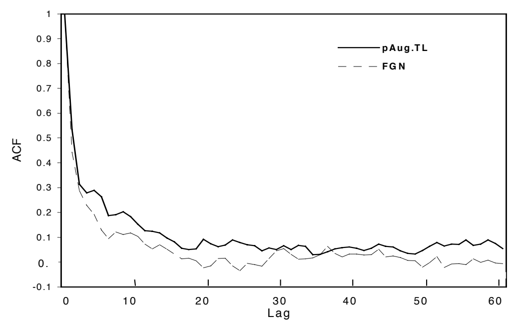

Figure 6: ACF of pAug.TL and its FGN model