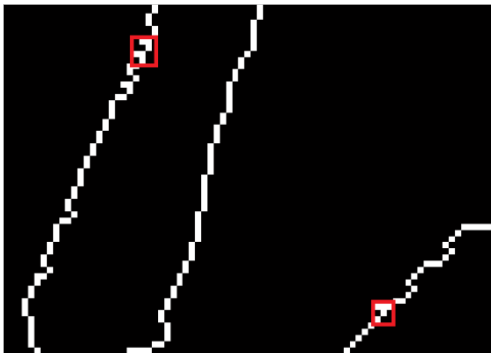
The algorithm of hand gesture recognition uses the relative position of each finger with respect to the reference point for recognition. The algorithm has 3 steps: 1). Detect the rough direction of hand and transform the contour points into *time-series curve*; 2). Locate the accurate positions of fingertips and fingerroots (two points located at the root of each finger) which are used to obtain finger directions and hand direction; 3). Recognize hand gestures using Euclidean distance metric.

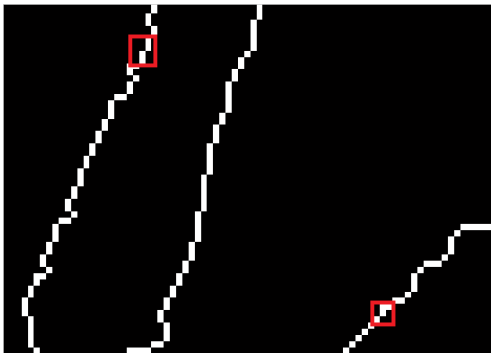
1. Hand direction detection and time-series curve

1.1. Contour points

To find the direction of the hand, the algorithm first detects the contour points. Because the contour is not always smooth after the segmentation, this may render a problem as is shown in figure 1: the points connecting algorithm can run into a dead-end when it runs into the marked points. So it's essential to eliminate the points that have two neighboring background points of the same direction in the first place.



(a). the top-left points should be eliminated



(b). result of the refined contour detection

Figure 1. Contour detection

As for the direction detection, the algorithm uses the relative position between P_i and P_{i+3} ($P_i, P_{i+3} \in P_C$, P_C is the consecutive points in the contour). Therefore there are 16 different directions and 16 direction angles ($\theta_i,\ i \in \{1,2 \dots,16\}$), see figure 2.

$$k_i = \tan(\theta_i). \quad (1)$$

The algorithm counts the number for each direction N_{d_i} ($N_{d_i} \in N_d, i \in \{1,2 \dots,16\}$), and assigns an index I_{d_i} ($I_{d_i} \in I_d,\ i \in \{1,2 \dots,16\}$) to each direction using the algorithm below.

Algorithm 1: Assign an index for each direction.

1. $N_d = N_d / \sum_{i=1}^{16} N_{d_i}$;
2. $N_{d_{I_{max}}} = max(N_d)$;
3. for $i = 1:16$
4. $\quad I_{d_i} = i - I_{max}$;
5. endfor

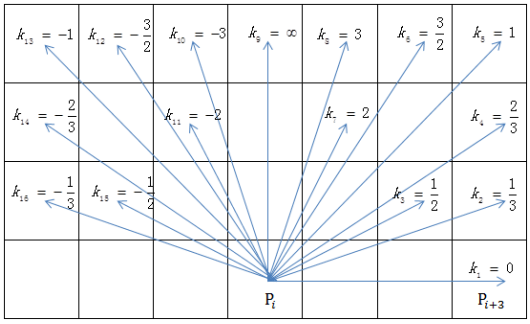


Figure 2. 16 types of direction of neighboring points (each square represents a pixel)

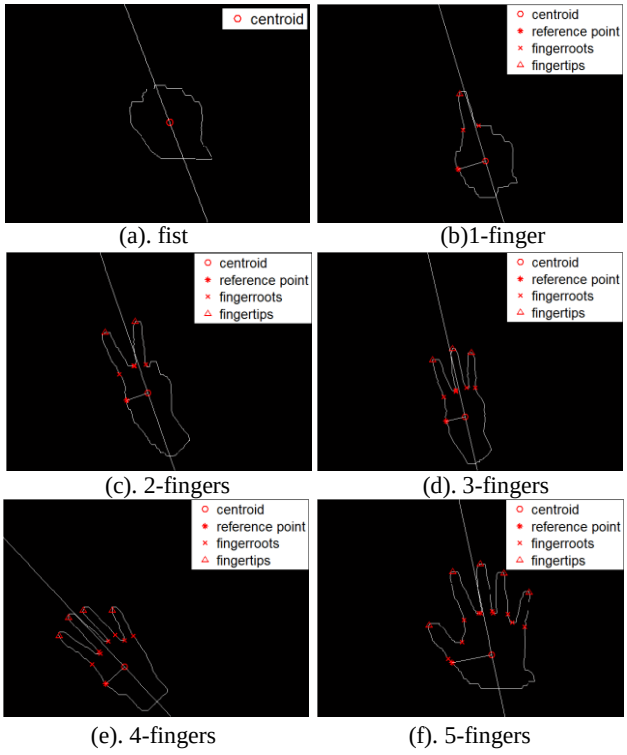
The direction index of the hand I_{hand} is obtained by equation (2):

$$I_{hand} = \sum_{i=1}^{i=16} (N_{d_i} * I_{d_i}). \quad (2)$$

I_{hand} is between two index values I_m and I_n ($m < n, m, n \in \{1,2,\dots,16\}$), which is used to calculate the hand angle θ_{hand} using (2).

$$\theta_{hand} = \theta_{I_m+I_{max}} + \left(\theta_{I_n+I_{max}} - \theta_{I_m+I_{max}}\right) * (I_{hand} - I_m) \quad (3)$$

This direction of hand is relatively rough, because not all points in the contour contribute to the direction of the hand, thus a refined way of using finger directions to represent hand direction is employed in Section 2.4 for optimal recognition. Figure 3 shows graphs with the hand contour and direction detected.



(a). fist (b)1-finger

(c). 2-fingers (d). 3-fingers

(e). 4-fingers (f). 5-fingers

Figure 3. Hand centroid and direction

1.2. Time-series curve

Once the direction is detected, the algorithm transforms the contour points into a *time-series curve* [1, 2, 3], which serves to extract hand features. Such a shape representation has been successfully used for the classification and clustering of shapes. The *time-series curve* records the relative distance between each contour point to the centroid. The centroid o is found using Distance Transform [5, 6]. The horizontal axis denotes the degree between each contour point and the reference point relative to the centroid, normalized by 360°. The vertical axis denotes the Euclidean distance between the contour points $(x_{contour}, y_{contour})$, $((x_{contour}, y_{contour}) \in P_C)$ and the centroid (x_{center}, y_{center}), normalized by the radius (R) of the maximal inscribed circle.

$$\theta_c = \frac{arctan\left(\frac{y_{contour}-y_{center}}{x_{contour}-x_{center}}\right)}{360^{\circ}}, \quad (4)$$

$$d_c = \frac{\sqrt{(x_{contour}-x_{center})^2+(y_{contour}-y_{center})^2}}{R}. \quad (5)$$

The reference point $r\,(x_r, y_r)$ is perpendicular to the hand direction and is obtained using (6):

$$\frac{y_r-y_{center}}{x_r-x_{center}} = \tan(\theta_{hand} + 90^{\circ}). \quad (6)$$

The *time-series curve* is used to detect the fingertips and fingervalleys of the hand, see figure 5, the locally highest points are detected as fingertips and locally lowest as fingervalleys using the algorithm below.

Algorithm 2: Detection of fingertips and fingervalleys.

1. for $i = 1{:}\,length(d_c)$
2. if($d_c^i < d_c^{min}$)
3. $d_c^{min} = d_c^i$;
4. $d_c^{max} = d_c^i$;
5. if($d_c^{min} < d_c^{fingervalley_{temp}}$)
6. $fingervalley_{temp} = i$;
7. endif
8. elseif($d_c^i < d_c^{max}$)
9. $fingervalley_j = fingervalley_{temp}$;
10. $firstTime = \sim firstTime$;
11. $j = j + 1$;
12. $d_c^{min} = d_c^i$;
13. $d_c^{max} = d_c^i$;
14. elseif($d_c^i > d_c^{max}$)
15. if($firstTime$)
16. $fingervalley_{temp} = i - 1$;
17. $firstTime = \sim firstTime$;
18. endif
19. $d_c^{max} = d_c^i$;
20. endif
21. endfor

Line 5-7 is used to change the index of fingervalley from fingervalley1 to fingervalley2 which makes the detection of fingervalleys more accurate, see figure 4. The initial value of the variable $firstTime$ is *true*, it's necessary because it can remain the value of $fingervalley_{temp}$ unchanged while $d_i > d_{max} = true$.

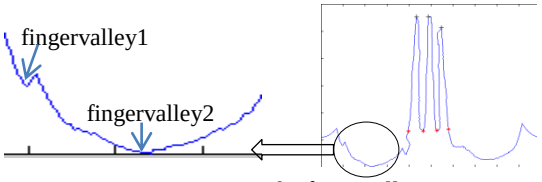
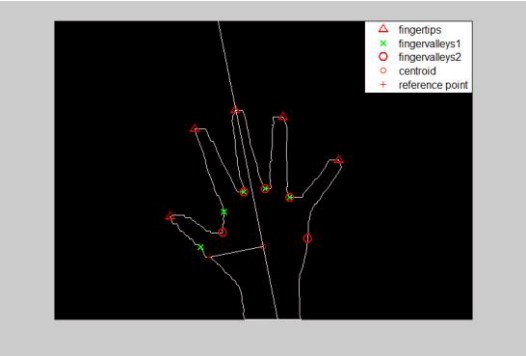
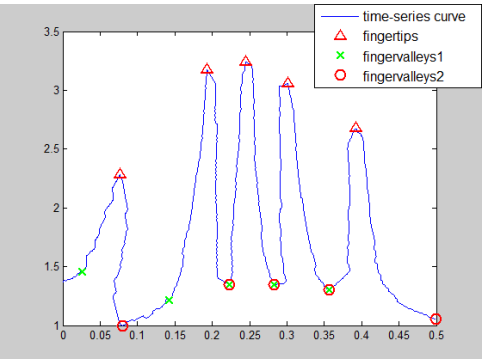


Figure 4. Move the fingervalley

The detected positions of potential fingertips and fingervalleys are shown in figure 6. The algorithm further eliminates the invalid fingertips and fingervalleys using Algorithm 3, the result of algorithm is shown in Figure 5 (b).



(a). Original hand contour with fingertips and fingervalleys detected



(b). Time-series curve with fingertips and fingervalleys detected

Figure 5. Original hand gesture and the time-series curve

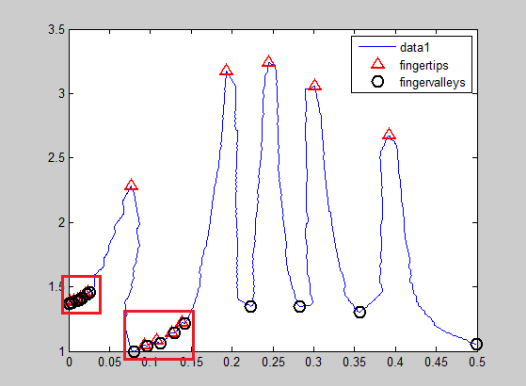


Figure 6. Fingertips & fingervalleys (Initial detection)

Algorithm 3: Eliminate invalid fingertips and fingervalleys

1. for each fingertip ft_i
2. Potential fingerroots fr_l and fr_r;
3. while(true)
4. Eliminate impossible ft_i and fr combination based on $func_{geo}(ft_i, fr_l)$ and $func_{geo}(ft_i, fr_r)$;
5. Find the most likely fr using $func_{opt}(ft_i, fr_l)$ and $func_{opt}(ft_i, fr_r)$;
6. $func_{opt}\left(ft_i, fv_{l_{opt}}\right) = max(func_{opt}(ft_i, fr_l))$;
7. $func_{opt}\left(ft_i, fv_{r_{opt}}\right) = max(func_{opt}(ft_i, fr_r))$;
8. endwhile
9. endfor

Where the initial fingerroots fr_l and fr_r are defined as the left and right fingervalleys of ft_i. Based on the hand's geometry, $func_{geo} = (\frac{d_c^{ft_i} - d_c^{fr_j}}{d_c^{fr_j}} > threshold)$, $fr_j \in \{fr_l, fr_r\}$ is used to eliminate noisy detections marked in Figure 6. As for the fingertips with more than 2 fingerroots, $func_{opt} = \frac{\left(d_c^{ft_i} - d_c^{fr_j}\right)}{d_c^{fr_j}} - w * \frac{\left|\theta_c^{ft_i} - \theta_c^{fr_j}\right|}{\theta_c^{fr_j}}$ is applied to choose the optimal ones, w is 2.5 in our code.

This is sufficient in most cases, but in others, the result may look like the table 1. It's impossible that one fingerroot belongs to several fingertips, so it's necessary to choose one between these candidate fingertips and eliminate others. The chosen fingertip should maximize Eq. (7).

$$\mathrm{d}_c^{ft} - \frac{\mathrm{d}_c^{fv1} + \mathrm{d}_c^{fv2}}{2}. \quad (7)$$

The information of fingertips and fingerroots is sufficient to discriminate the number of fingers, yet not accurate enough to recognize gestures due to the relative inaccuracy of the hand direction and reference point. Therefore, a refined way of hand direction detection is proposed next.

$Index_{ft}$	5		6		7	
$Index_{fr}$	3	8	4	8	4	8

Table 1. An example of false detection of fingertips

2. Positions of fingertips and fingerroots localization

The algorithm further detects what we call fingerroots (points located at the root of fingers, see figure 3) of each finger using the following procedure:

Algorithm 4: Refine the localization of the fingerroots.

1. while(true)
2. Detect the direction of finger θ_f^i;
3. Find the fingerroots fr^i;
4. Calculate the external rectangular area of finger A_{Rec};
5. Calculate the area of the finger A_f;
6. $R_A = A_f / A_{Rec}$;
7. if($R_A > R_{threshold}$)
8. break;
9. endif
10. endwhile

2.1. Finger direction

The finger direction θ_f^i is calculated using fingertip ft (x_{ft}, y_{ft}) and fingerroots fr_1^i (x_{fr1}^i, y_{fr1}^i) and fr_2^i (x_{fr2}^i, y_{fr2}^i) by equations (8)-(10).

$$\theta_f^i = \frac{\theta_{f1}^i + \theta_{f2}^i}{2}, \quad (8)$$

$$\theta_{f1}^i = arctan(\frac{y_{ft} - y_{fr1}^i}{x_{ft} - x_{fr1}^i}), \quad (9)$$

$$\theta_{f2}^i = arctan(\frac{y_{ft} - y_{fr3}^i}{x_{ft} - x_{fr3}^i}). \quad (10)$$

Where $\theta_{f1}^i, \theta_{f2}^i$ is respectively the direction of the fingertip and the fingerroot.

2.2. Fingerroots detection

After the detection of the finger direction, the algorithm below finds the temporary fingerroots $(x_{fr_{temp}}, y_{fr_{temp}})$ which are perpendicular to the finger direction using (11), (12).

$$\frac{y_{fr_{temp1}} - y_{fr3}^i}{x_{fr_{temp1}} - x_{fr3}^i} = \tan\left(\theta_f^i + 90^\circ\right), \quad (11)$$

$$\frac{y_{fr_{temp2}} - y_{fr1}^i}{x_{fr_{temp2}} - x_{fr1}^i} = \tan\left(\theta_f^i - 90^\circ\right). \quad (12)$$

Algorithm 5: Detection of the temporary fingerroots.

1. if $(fr_{temp1} \in \{fr_1^i : ft\})$
2. $\quad fr_1^{i+1} = fr_{temp1};$
3. $\quad \mathrm{fr}_2^{i+1} = \mathrm{fr}_2^i;$
4. elseif$(\mathrm{fr}_{temp2} \in \{ft : fr_2^i\})$
5. $\quad fr_1^{i+1} = fr_1^i;$
6. $\quad fr_1^{i+1} = fr_{temp2};$
7. else
8. $\quad fr_1^{i+1} = fr_1^i + 1;$
9. $\quad fr_1^{i+1} = fr_2^i + 1;$
10. end

Where $\{fr_1^i : ft\}$ means contour points between fr_1^i and ft. The fingerroots and fingertips are used to calculate the area of the finger (A_f) and the external rectangular area (A_{Rec}). This process will be terminated until the area ratio $(R_A = A_f / A_{Rec})$ surpasses a certain threshold.

2.3. Finger and external rectangular area

The area of finger is calculated by counting pixel numbers in the finger region.

As for the external rectangular area, the temporary finger direction is always perpendicular to the line of the fingerroots, so the height as well as the largest width is needed to calculate A_{Rec}.

$$Height = \sqrt{(x_{ft} - x_{vp})^2 + (y_{ft} - y_{vp})^2}, \quad (13)$$

$$Width = \max\left(\sqrt{(x_{contour1} - x_{vp})^2 + (y_{contour1} - y_{vp})^2}\right) + \max\left(\sqrt{(x_{contour2} - x_{vp})^2 + (y_{contour2} - y_{vp})^2}\right). \quad (14)$$

In (13), (x_{ft}, y_{ft}) is the coordinate of the fingertip, (x_{vp}, y_{vp}) is the vertical point between the fingertip and the line of the fingerroots. In (14), $(x_{contour1}, y_{contour1}) \in S_{C_1}$, S_{C_1} is the contour point set between fingerroot1 fr_1^i and fingertip ft and $(x_{contour2}, y_{contour2}) \in S_{C_2}$, S_{C_2} is the contour point set between fingertip ft and fingerroot2 fr_2^i. (x_{vp}, y_{vp}) is the vertical point between contour point and the finger direction.

2.4. Accurate Hand direction

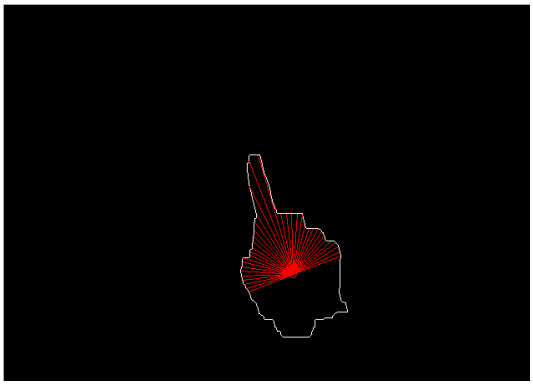
It's intuitive to assume that the directions of the fingers can represent the direction of the hand. So the refined hand direction is defined as follows:

$$\theta_{hand} = \frac{\sum_{i=1}^{i=|\theta_f|} \theta_{f_i}}{|\theta_f|}. \quad (15)$$

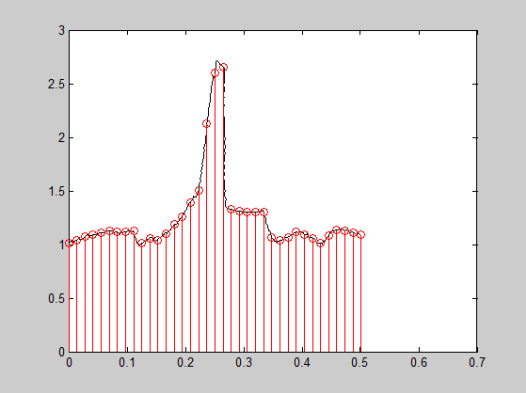
Where $|\theta_f|$ is the length of vector θ_f.

3. Recognition

For the recognition phase, the algorithm first converts the hand contour into a time-series curve with the newly detected hand direction, and then takes samples in the *time-series curve* to form a distance vector which is used for recognition. See figure 7.

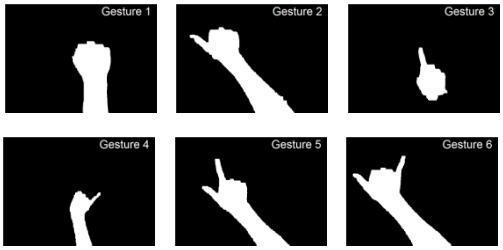


(a). Distance sample vector in hand gesture



(b). Distance sample vector in time-series curve

Figure 7. Distance sample vector



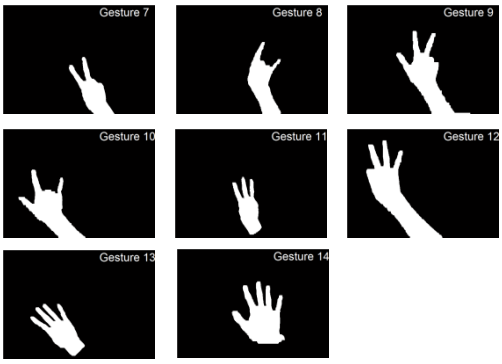


Figure 8. Predefined hand gestures

14 types of hand gestures are defined that can be achieved with no significant by ordinary human hand. See figure 8. One-versus-one multi-class Support Vector Machine is employed to train the feature vector of each hand gesture type for the ensuing recognition step.

Table 2. Results of the recognition

Gesture index	1	2	3	4	5	6	7	8	9	10	11	12	13	14
True positive (%)	100	92.1	96.1	99	90.7	100	98.6	99.5	100	100	98.3	95.6	100	100
False positive(%)	0	7.6	3.6	4.3	4.5	0	1.6	0	1.1	1.3	0	0	2.9	0

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	100%													
2		92.1%	1.4%											
3		7.8%	96.1%		0.7%		0.3%							
4				99%	5.0%									
5			1.4%	0.5%	90.7%		1.1%							
6			1.1%			100%								
7							98.6%	0.5%						
8				0.5%				99.5%						
9									100%		0.9%	2.5%		
10					3.6%					100%				
11											98.3%			
12												95.6%		
13											0.8%	1.9%	100%	
14														100%

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P.S. We are currently using Machine Learning algorithms (e. g. Logistic Regression Algorithm and Support Vector Machine) to train the data for the ensuing recognition phase. The source code is available in Github: https://github.com/imkaywu/Gesture-Recognition-SVM. The result is much more robust and the paper will be changed accordingly. Check my personal blog: http://imkaywu.com/publication.html for latest updates.