Lab2

2024-03-05

```
rm(list = ls())
# Load required libraries
#fix using tidyverse
library(openxlsx)
```

```
## Warning: package 'openxlsx' was built under R version 4.3.3
```

```
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
library(lubridate)
```

```
## Warning: package 'lubridate' was built under R version 4.3.3
```

```
##
## Attaching package: 'lubridate'
```

```
## The following objects are masked from 'package:base':
##
##     date, intersect, setdiff, union
```

```
library(ggplot2)
library(data.table)
```

```
## Warning: package 'data.table' was built under R version 4.3.3
```

```
##
## Attaching package: 'data.table'
```

```
## The following objects are masked from 'package:lubridate':
##
##     hour, isoweek, mday, minute, month, quarter, second, wday, week,
##     yday, year

## The following objects are masked from 'package:dplyr':
##
##     between, first, last
```

```
library(tidyverse)
```

```
## -- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
## v forcats 1.0.0     v stringr 1.5.1
## v purrr   1.0.2     v tibble  3.2.1
## v readr   2.1.5     v tidyr   1.3.0

## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x data.table::between() masks dplyr::between()
## x dplyr::filter()       masks stats::filter()
## x data.table::first()   masks dplyr::first()
## x data.table::hour()    masks lubridate::hour()
## x data.table::isoweek() masks lubridate::isoweek()
## x dplyr::lag()          masks stats::lag()
## x data.table::last()    masks dplyr::last()
## x data.table::mday()    masks lubridate::mday()
## x data.table::minute()  masks lubridate::minute()
## x data.table::month()   masks lubridate::month()
## x data.table::quarter() masks lubridate::quarter()
## x data.table::second()  masks lubridate::second()
## x purrr::transpose()    masks data.table::transpose()
## x data.table::wday()    masks lubridate::wday()
## x data.table::week()    masks lubridate::week()
## x data.table::yday()    masks lubridate::yday()
## x data.table::year()    masks lubridate::year()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library(readxl)

df <- read_excel("C:/Users/Clayton-George Reid/Downloads/FTSM.xlsx",
                   sheet = "linear_reg") %>%
  mutate( x = row_number(),
    y = value
  )

# Display the first few rows of the dataframe
head(df)
```

```
## # A tibble: 6 x 4
##   DateTime            value     x     y
##   <dttm>              <dbl> <int> <dbl>
## 1 2000-01-03 00:00:00 -2.73     1 -2.73
## 2 2000-01-04 00:00:00 -3.53     2 -3.53
```

```
## 3 2000-01-05 00:00:00 -3.84     3 -3.84
## 4 2000-01-07 00:00:00 -1.85     4 -1.85
## 5 2000-01-10 00:00:00 -1.86     5 -1.86
## 6 2000-01-11 00:00:00 -2.72     6 -2.72
```

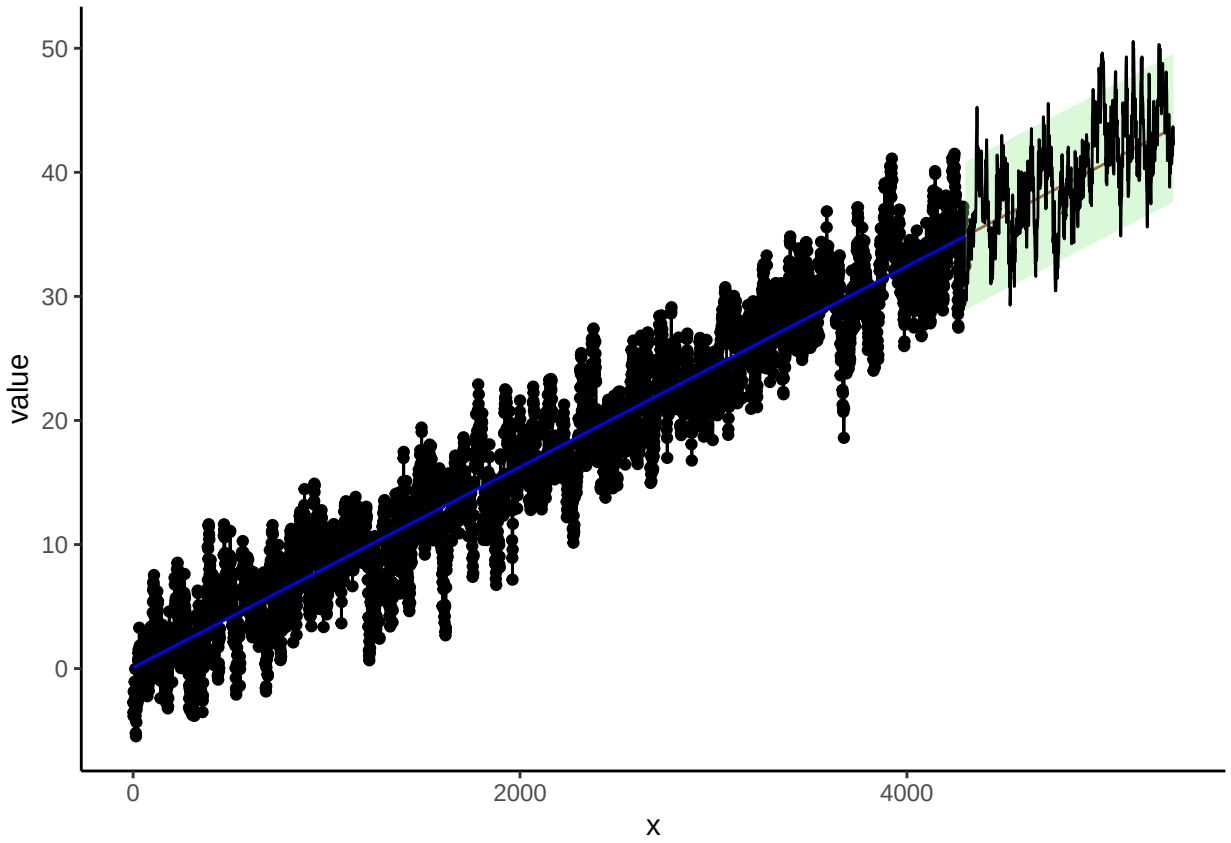
```
# Split data into training and testing sets
train <- df %>%
  slice(1:round(0.8 * n()))

test <- anti_join(df, train, by = 'x')

# Fit linear regression model
tslm <- lm(y ~ x, data = train)
summary(tslm)
```

```
##
## Call:
## lm(formula = y ~ x, data = train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.1911  -1.9687  -0.0005   2.0555   9.3079
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 7.780e-02  9.259e-02    0.84    0.401
## x           8.088e-03  3.727e-05  217.01   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.036 on 4300 degrees of freedom
## Multiple R-squared:  0.9163, Adjusted R-squared:  0.9163
## F-statistic: 4.71e+04 on 1 and 4300 DF,  p-value: < 2.2e-16
```

```
# Calculate residuals and fitted values
train <- train %>%
  mutate(e = tslm$residuals,
         yhat = tslm$fitted.values)

# Prepare data for prediction
new_data <- test %>%
  arrange(x) %>%
  select(x)
```

```
# Make predictions
predicted_y <- predict(tslm, newdata = new_data, interval = "prediction") %>%
  data.frame() %>%
  mutate(x = new_data %>% pull(x))

# Merge predicted values with test data
predicted_y <- merge(predicted_y, test, by = "x")

train %>%
```

```
  ggplot(mapping = aes(x = x, y = value))+
  geom_point()+
  geom_line()+
  theme_classic()+
  geom_line(mapping = aes(x = x, y = yhat), color= "blue")+
  geom_line(data = predicted_y, mapping = aes(x = x, y = fit), color = "brown")+
  geom_ribbon(data = predicted_y, mapping = aes(ymin = lwr, ymax = upr), fill = "light green", alpha = 
  geom_line(data = predicted_y, mapping = aes(x = x, y = value))
```



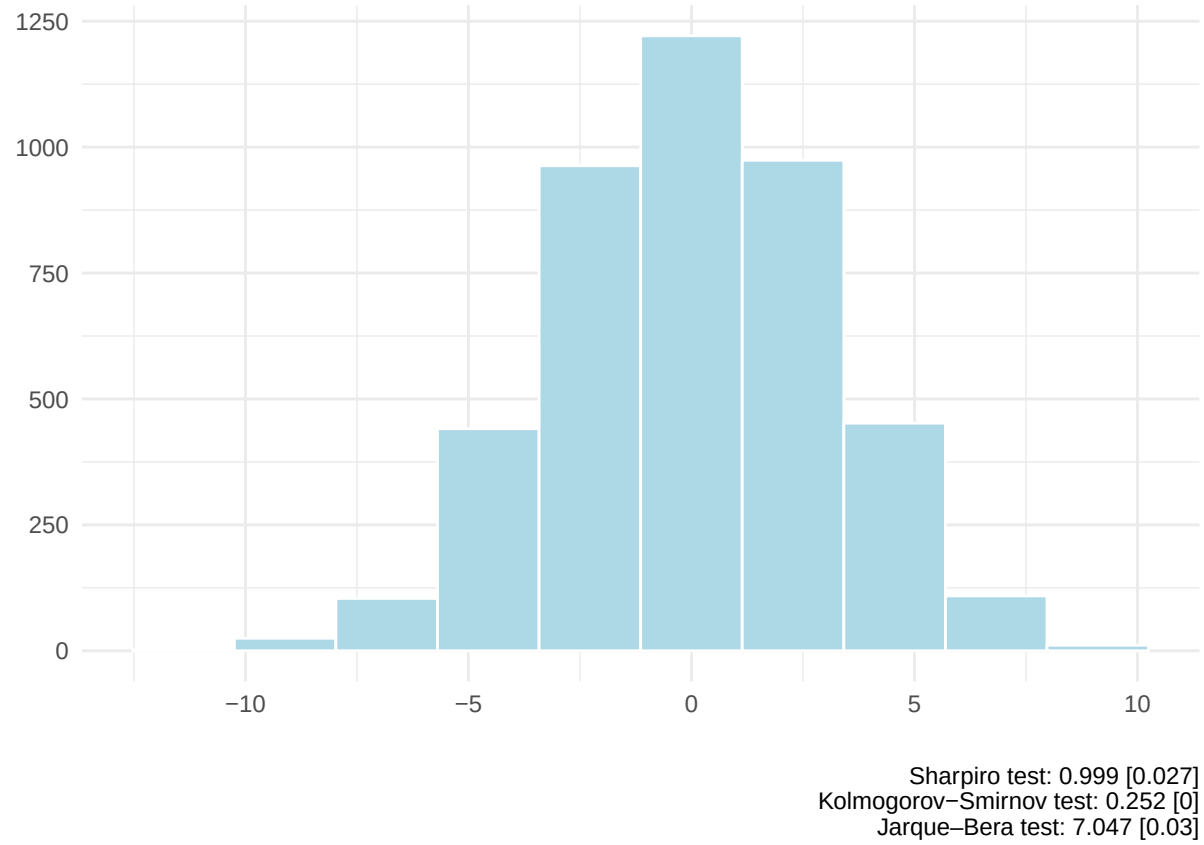
the prediction graph provides a comprehensive overview of observed trends and predicted trajectories, along with associated uncertainty. It enables viewers to assess the reliability of the predictions and make informed decisions based on the projected outcomes.

```
sw <- train %>% pull(e) %>% stats::shapiro.test()
ks <- train %>% pull(e) %>% stats::ks.test(y = "pnorm")
jb <- train %>% pull(e) %>% tseries::jarque.bera.test()
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
histogram_error <- ggplot(data = train, mapping = aes(x = e)) +
geom_histogram(bins = 10,
color = "white",
fill = "lightblue") +
```

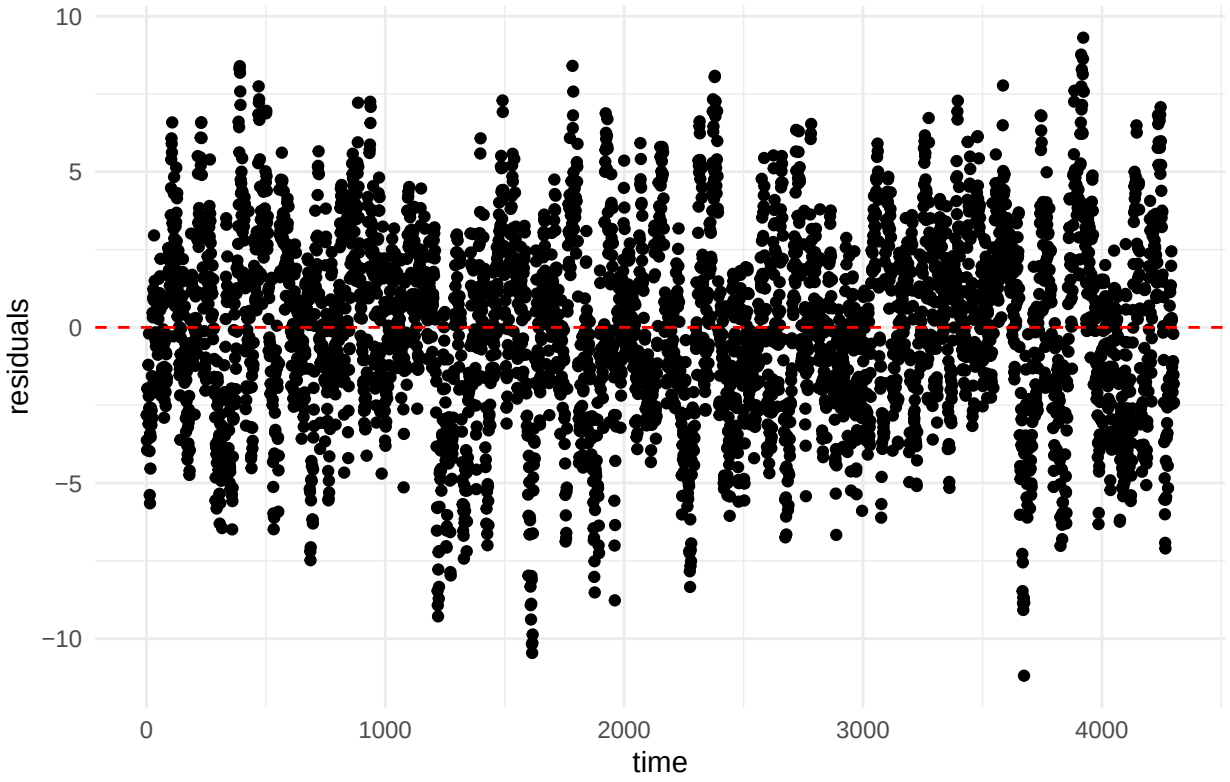
```
theme_minimal() +
labs(x = "", y = "", caption = paste0(
paste0("Sharpiro test: ", sw$statistic %>% round(3),
" [", sw$p.value %>% round(3), "]"), "\n",
paste0("Kolmogorov-Smirnov test: ", ks$statistic %>% round(3),
" [", ks$p.value %>% round(3), "]"), "\n",
paste0("Jarque-Bera test: ", jb$statistic %>% round(3),
" [", jb$p.value %>% round(3), "]")
))

print(histogram_error)
```



I conducts the Shapiro-Wilk test , Kolmogorov-Smirnov test, and Jarque-Bera test on the residuals (e). These test assess the normality of the residuals,empirical cumulative distribution function of the residuals with the theoretical normal cumulative distribution function, and skewness and kurtosis of the residuals.

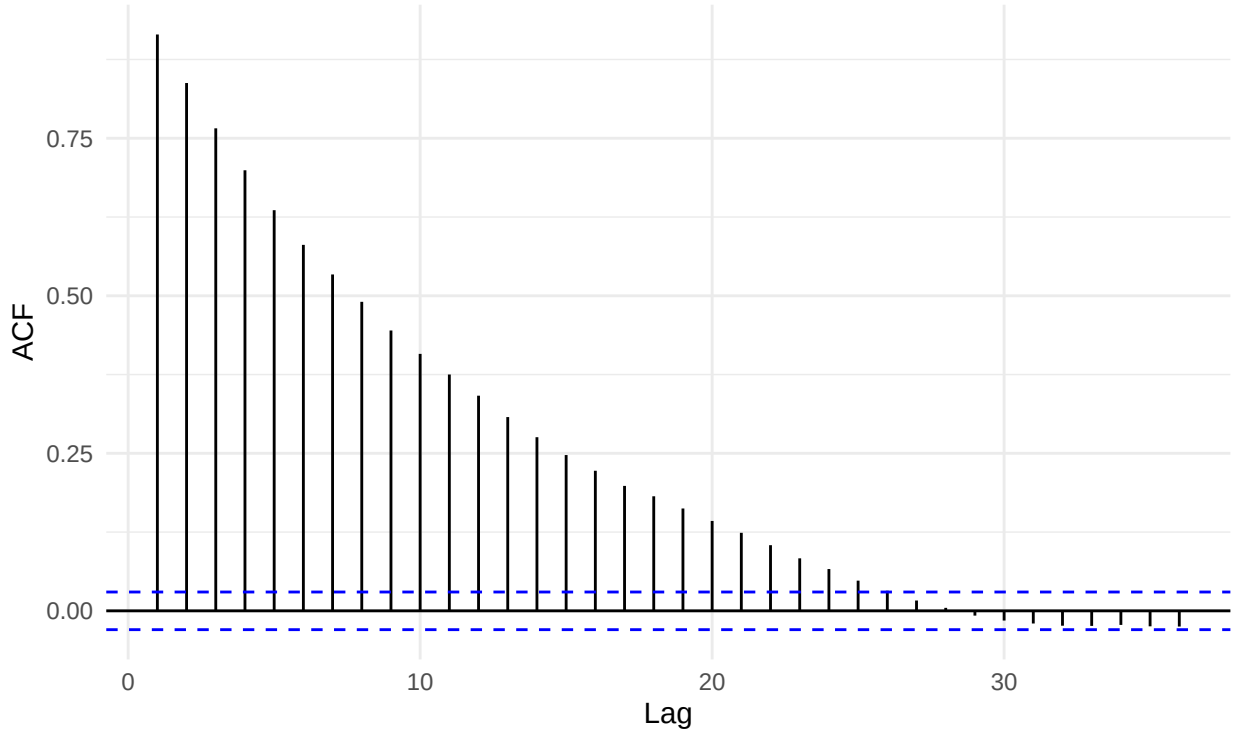
```
BP <- tslm %>% lmtest::bptest()
GQ <- tslm %>% lmtest::gqtest()
homoskedasticity_plot <- ggplot(data = train, mapping = aes(x = x, y = e)) +
geom_point() +
geom_hline(yintercept = 0,
color = "red",
linetype = "dashed")+
theme_minimal() +
labs(x = "time", y = "residuals",
caption = paste0(
```

```
paste0("Breusch-Pagan test: ", BP$statistic %>% round(3),
" [", BP$p.value %>% round(3), "]"), "\n",
paste0("Goldfeld-Quandt test: ", GQ$statistic %>% round(3),
" [", GQ$p.value %>% round(3), "]")))

print(homoskedasticity_plot)
```



Breusch−Pagan test: 3.915 [0.048]
Goldfeld−Quandt test: 1.011 [0.404]

```
BO <- train %>% pull(e) %>% stats::Box.test(lag = 10, type = "Box-Pierce")
LB <- train %>% pull(e) %>% stats::Box.test(lag = 10, type = "Ljung-Box")
acf_plot <- train %>% pull(e) %>% forecast::ggAcf()+
theme_minimal()+
labs(title = "", caption = paste0(
paste0("Box-Pierce test: ", BO$statistic %>% round(3),
" [", BO$p.value %>% round(3), "]"), "\n",
paste0("Ljung-Box test: ", LB$statistic %>% round(3),
" [", LB$p.value %>% round(3), "]")))

print(acf_plot)
```



Box−Pierce test: 18261.197 [0]
Ljung−Box test: 18287.081 [0]

```
# Take testing data set for the new value of x for prediction
new_data <- test %>% arrange(x) %>% select(x)
# Predict the corresponding y values along
# with a 95% prediction interval
predicted_y <- predict(tslm, newdata = new_data, interval = "prediction") %>%
data.frame() %>% mutate(x = new_data %>% pull(x), .before = fit)
predicted_y
```

```
##         x      fit      lwr      upr
## 1    4303 34.88165 28.92703 40.83627
## 2    4304 34.88974 28.93512 40.84436
## 3    4305 34.89783 28.94321 40.85245
## 4    4306 34.90592 28.95129 40.86054
## 5    4307 34.91400 28.95938 40.86863
## 6    4308 34.92209 28.96747 40.87672
## 7    4309 34.93018 28.97555 40.88481
## 8    4310 34.93827 28.98364 40.89290
## 9    4311 34.94636 28.99173 40.90099
## 10   4312 34.95445 28.99981 40.90908
## 11   4313 34.96253 29.00790 40.91717
## 12   4314 34.97062 29.01598 40.92526
## 13   4315 34.97871 29.02407 40.93335
## 14   4316 34.98680 29.03216 40.94144
## 15   4317 34.99489 29.04024 40.94953
## 16   4318 35.00297 29.04833 40.95762
## 17   4319 35.01106 29.05642 40.96571
```

```
## 18   4320 35.01915 29.06450 40.97380
## 19   4321 35.02724 29.07259 40.98189
## 20   4322 35.03533 29.08068 40.98998
## 21   4323 35.04342 29.08876 40.99807
## 22   4324 35.05150 29.09685 41.00616
## 23   4325 35.05959 29.10493 41.01425
## 24   4326 35.06768 29.11302 41.02234
## 25   4327 35.07577 29.12111 41.03043
## 26   4328 35.08386 29.12919 41.03852
## 27   4329 35.09195 29.13728 41.04661
## 28   4330 35.10003 29.14537 41.05470
## 29   4331 35.10812 29.15345 41.06279
## 30   4332 35.11621 29.16154 41.07088
## 31   4333 35.12430 29.16962 41.07897
## 32   4334 35.13239 29.17771 41.08706
## 33   4335 35.14048 29.18580 41.09515
## 34   4336 35.14856 29.19388 41.10324
## 35   4337 35.15665 29.20197 41.11133
## 36   4338 35.16474 29.21006 41.11942
## 37   4339 35.17283 29.21814 41.12751
## 38   4340 35.18092 29.22623 41.13560
## 39   4341 35.18900 29.23432 41.14369
## 40   4342 35.19709 29.24240 41.15179
## 41   4343 35.20518 29.25049 41.15988
## 42   4344 35.21327 29.25857 41.16797
## 43   4345 35.22136 29.26666 41.17606
## 44   4346 35.22945 29.27475 41.18415
## 45   4347 35.23753 29.28283 41.19224
## 46   4348 35.24562 29.29092 41.20033
## 47   4349 35.25371 29.29901 41.20842
## 48   4350 35.26180 29.30709 41.21651
## 49   4351 35.26989 29.31518 41.22460
## 50   4352 35.27798 29.32326 41.23269
## 51   4353 35.28606 29.33135 41.24078
## 52   4354 35.29415 29.33944 41.24887
## 53   4355 35.30224 29.34752 41.25696
## 54   4356 35.31033 29.35561 41.26505
## 55   4357 35.31842 29.36370 41.27314
## 56   4358 35.32651 29.37178 41.28123
## 57   4359 35.33459 29.37987 41.28932
## 58   4360 35.34268 29.38795 41.29741
## 59   4361 35.35077 29.39604 41.30550
## 60   4362 35.35886 29.40413 41.31359
## 61   4363 35.36695 29.41221 41.32168
## 62   4364 35.37504 29.42030 41.32977
## 63   4365 35.38312 29.42839 41.33786
## 64   4366 35.39121 29.43647 41.34595
## 65   4367 35.39930 29.44456 41.35404
## 66   4368 35.40739 29.45265 41.36213
## 67   4369 35.41548 29.46073 41.37022
## 68   4370 35.42356 29.46882 41.37831
## 69   4371 35.43165 29.47690 41.38640
## 70   4372 35.43974 29.48499 41.39449
## 71   4373 35.44783 29.49308 41.40258
```

```
## 72   4374 35.45592 29.50116 41.41067
## 73   4375 35.46401 29.50925 41.41876
## 74   4376 35.47209 29.51734 41.42685
## 75   4377 35.48018 29.52542 41.43494
## 76   4378 35.48827 29.53351 41.44303
## 77   4379 35.49636 29.54159 41.45112
## 78   4380 35.50445 29.54968 41.45921
## 79   4381 35.51254 29.55777 41.46731
## 80   4382 35.52062 29.56585 41.47540
## 81   4383 35.52871 29.57394 41.48349
## 82   4384 35.53680 29.58203 41.49158
## 83   4385 35.54489 29.59011 41.49967
## 84   4386 35.55298 29.59820 41.50776
## 85   4387 35.56107 29.60628 41.51585
## 86   4388 35.56915 29.61437 41.52394
## 87   4389 35.57724 29.62246 41.53203
## 88   4390 35.58533 29.63054 41.54012
## 89   4391 35.59342 29.63863 41.54821
## 90   4392 35.60151 29.64672 41.55630
## 91   4393 35.60960 29.65480 41.56439
## 92   4394 35.61768 29.66289 41.57248
## 93   4395 35.62577 29.67097 41.58057
## 94   4396 35.63386 29.67906 41.58866
## 95   4397 35.64195 29.68715 41.59675
## 96   4398 35.65004 29.69523 41.60484
## 97   4399 35.65812 29.70332 41.61293
## 98   4400 35.66621 29.71141 41.62102
## 99   4401 35.67430 29.71949 41.62911
## 100  4402 35.68239 29.72758 41.63720
## 101  4403 35.69048 29.73566 41.64529
## 102  4404 35.69857 29.74375 41.65338
## 103  4405 35.70665 29.75184 41.66147
## 104  4406 35.71474 29.75992 41.66956
## 105  4407 35.72283 29.76801 41.67765
## 106  4408 35.73092 29.77610 41.68574
## 107  4409 35.73901 29.78418 41.69383
## 108  4410 35.74710 29.79227 41.70192
## 109  4411 35.75518 29.80035 41.71001
## 110  4412 35.76327 29.80844 41.71810
## 111  4413 35.77136 29.81653 41.72619
## 112  4414 35.77945 29.82461 41.73428
## 113  4415 35.78754 29.83270 41.74237
## 114  4416 35.79563 29.84079 41.75047
## 115  4417 35.80371 29.84887 41.75856
## 116  4418 35.81180 29.85696 41.76665
## 117  4419 35.81989 29.86504 41.77474
## 118  4420 35.82798 29.87313 41.78283
## 119  4421 35.83607 29.88122 41.79092
## 120  4422 35.84416 29.88930 41.79901
## 121  4423 35.85224 29.89739 41.80710
## 122  4424 35.86033 29.90548 41.81519
## 123  4425 35.86842 29.91356 41.82328
## 124  4426 35.87651 29.92165 41.83137
## 125  4427 35.88460 29.92973 41.83946
```

```
## 126  4428 35.89268 29.93782 41.84755
## 127  4429 35.90077 29.94591 41.85564
## 128  4430 35.90886 29.95399 41.86373
## 129  4431 35.91695 29.96208 41.87182
## 130  4432 35.92504 29.97017 41.87991
## 131  4433 35.93313 29.97825 41.88800
## 132  4434 35.94121 29.98634 41.89609
## 133  4435 35.94930 29.99442 41.90418
## 134  4436 35.95739 30.00251 41.91227
## 135  4437 35.96548 30.01060 41.92036
## 136  4438 35.97357 30.01868 41.92845
## 137  4439 35.98166 30.02677 41.93654
## 138  4440 35.98974 30.03486 41.94463
## 139  4441 35.99783 30.04294 41.95272
## 140  4442 36.00592 30.05103 41.96081
## 141  4443 36.01401 30.05911 41.96890
## 142  4444 36.02210 30.06720 41.97699
## 143  4445 36.03019 30.07529 41.98508
## 144  4446 36.03827 30.08337 41.99317
## 145  4447 36.04636 30.09146 42.00127
## 146  4448 36.05445 30.09955 42.00936
## 147  4449 36.06254 30.10763 42.01745
## 148  4450 36.07063 30.11572 42.02554
## 149  4451 36.07872 30.12380 42.03363
## 150  4452 36.08680 30.13189 42.04172
## 151  4453 36.09489 30.13998 42.04981
## 152  4454 36.10298 30.14806 42.05790
## 153  4455 36.11107 30.15615 42.06599
## 154  4456 36.11916 30.16423 42.07408
## 155  4457 36.12724 30.17232 42.08217
## 156  4458 36.13533 30.18041 42.09026
## 157  4459 36.14342 30.18849 42.09835
## 158  4460 36.15151 30.19658 42.10644
## 159  4461 36.15960 30.20467 42.11453
## 160  4462 36.16769 30.21275 42.12262
## 161  4463 36.17577 30.22084 42.13071
## 162  4464 36.18386 30.22892 42.13880
## 163  4465 36.19195 30.23701 42.14689
## 164  4466 36.20004 30.24510 42.15498
## 165  4467 36.20813 30.25318 42.16307
## 166  4468 36.21622 30.26127 42.17116
## 167  4469 36.22430 30.26936 42.17925
## 168  4470 36.23239 30.27744 42.18734
## 169  4471 36.24048 30.28553 42.19543
## 170  4472 36.24857 30.29361 42.20352
## 171  4473 36.25666 30.30170 42.21161
## 172  4474 36.26475 30.30979 42.21970
## 173  4475 36.27283 30.31787 42.22779
## 174  4476 36.28092 30.32596 42.23589
## 175  4477 36.28901 30.33405 42.24398
## 176  4478 36.29710 30.34213 42.25207
## 177  4479 36.30519 30.35022 42.26016
## 178  4480 36.31328 30.35830 42.26825
## 179  4481 36.32136 30.36639 42.27634
```

```
## 180  4482 36.32945 30.37448 42.28443
## 181  4483 36.33754 30.38256 42.29252
## 182  4484 36.34563 30.39065 42.30061
## 183  4485 36.35372 30.39873 42.30870
## 184  4486 36.36180 30.40682 42.31679
## 185  4487 36.36989 30.41491 42.32488
## 186  4488 36.37798 30.42299 42.33297
## 187  4489 36.38607 30.43108 42.34106
## 188  4490 36.39416 30.43917 42.34915
## 189  4491 36.40225 30.44725 42.35724
## 190  4492 36.41033 30.45534 42.36533
## 191  4493 36.41842 30.46342 42.37342
## 192  4494 36.42651 30.47151 42.38151
## 193  4495 36.43460 30.47960 42.38960
## 194  4496 36.44269 30.48768 42.39769
## 195  4497 36.45078 30.49577 42.40578
## 196  4498 36.45886 30.50385 42.41387
## 197  4499 36.46695 30.51194 42.42196
## 198  4500 36.47504 30.52003 42.43005
## 199  4501 36.48313 30.52811 42.43814
## 200  4502 36.49122 30.53620 42.44623
## 201  4503 36.49931 30.54429 42.45433
## 202  4504 36.50739 30.55237 42.46242
## 203  4505 36.51548 30.56046 42.47051
## 204  4506 36.52357 30.56854 42.47860
## 205  4507 36.53166 30.57663 42.48669
## 206  4508 36.53975 30.58472 42.49478
## 207  4509 36.54784 30.59280 42.50287
## 208  4510 36.55592 30.60089 42.51096
## 209  4511 36.56401 30.60898 42.51905
## 210  4512 36.57210 30.61706 42.52714
## 211  4513 36.58019 30.62515 42.53523
## 212  4514 36.58828 30.63323 42.54332
## 213  4515 36.59636 30.64132 42.55141
## 214  4516 36.60445 30.64941 42.55950
## 215  4517 36.61254 30.65749 42.56759
## 216  4518 36.62063 30.66558 42.57568
## 217  4519 36.62872 30.67366 42.58377
## 218  4520 36.63681 30.68175 42.59186
## 219  4521 36.64489 30.68984 42.59995
## 220  4522 36.65298 30.69792 42.60804
## 221  4523 36.66107 30.70601 42.61613
## 222  4524 36.66916 30.71410 42.62422
## 223  4525 36.67725 30.72218 42.63231
## 224  4526 36.68534 30.73027 42.64040
## 225  4527 36.69342 30.73835 42.64849
## 226  4528 36.70151 30.74644 42.65659
## 227  4529 36.70960 30.75453 42.66468
## 228  4530 36.71769 30.76261 42.67277
## 229  4531 36.72578 30.77070 42.68086
## 230  4532 36.73387 30.77878 42.68895
## 231  4533 36.74195 30.78687 42.69704
## 232  4534 36.75004 30.79496 42.70513
## 233  4535 36.75813 30.80304 42.71322
```

```
## 234  4536 36.76622 30.81113 42.72131
## 235  4537 36.77431 30.81921 42.72940
## 236  4538 36.78240 30.82730 42.73749
## 237  4539 36.79048 30.83539 42.74558
## 238  4540 36.79857 30.84347 42.75367
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## 1061 5363 43.45522 37.49806 49.41239
## 1062 5364 43.46331 37.50614 49.42048
## 1063 5365 43.47140 37.51423 49.42857
## 1064 5366 43.47949 37.52231 49.43666
## 1065 5367 43.48757 37.53040 49.44475
## 1066 5368 43.49566 37.53848 49.45284
## 1067 5369 43.50375 37.54657 49.46093
## 1068 5370 43.51184 37.55466 49.46902
## 1069 5371 43.51993 37.56274 49.47711
## 1070 5372 43.52802 37.57083 49.48521
## 1071 5373 43.53610 37.57891 49.49330
## 1072 5374 43.54419 37.58700 49.50139
## 1073 5375 43.55228 37.59508 49.50948
## 1074 5376 43.56037 37.60317 49.51757
## 1075 5377 43.56846 37.61125 49.52566
```
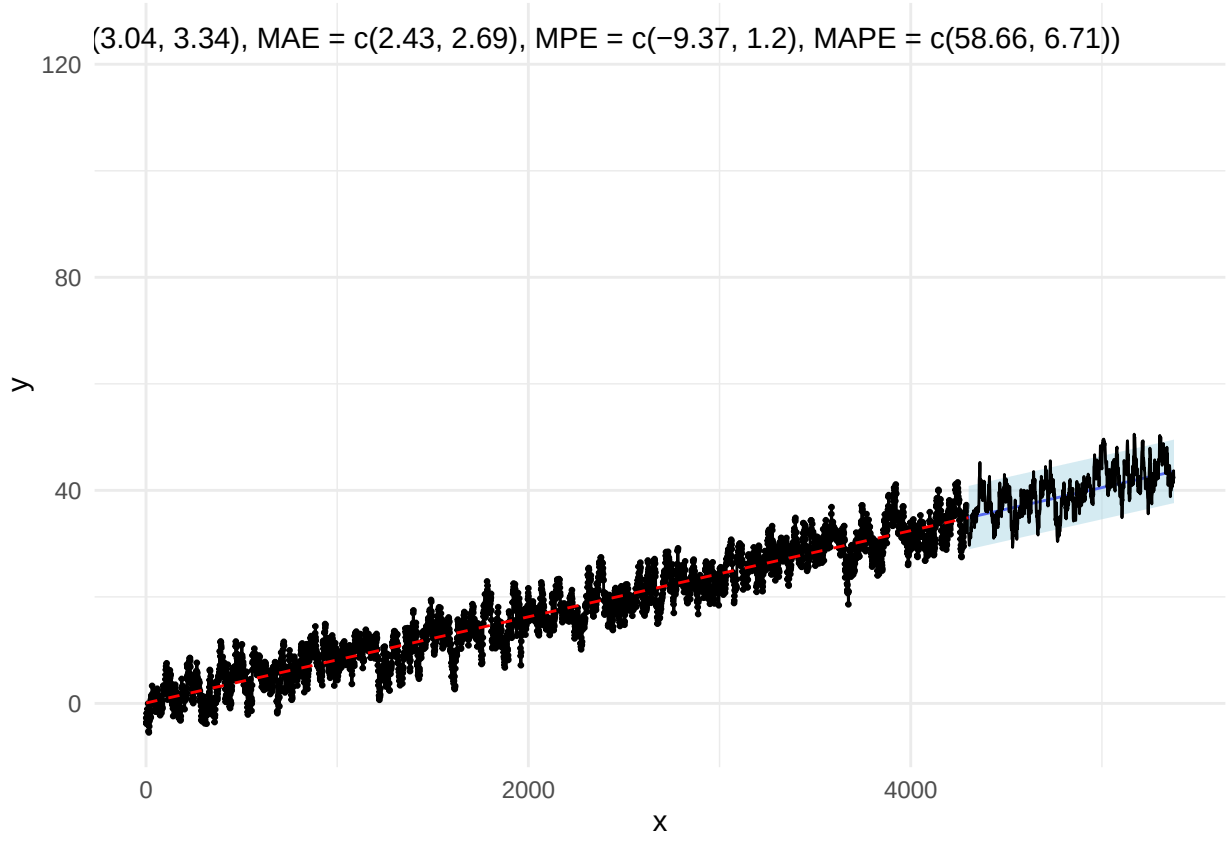
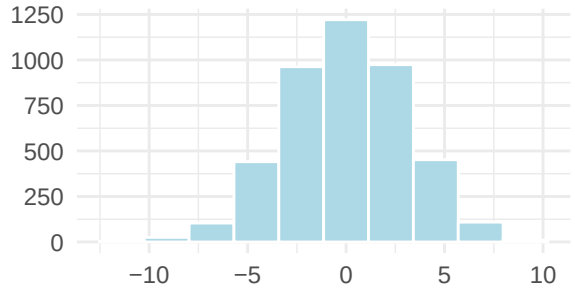

```
 invsout <- list(
 Insample = train %>% pull(yhat) %>%
 forecast::accuracy(train %>% pull(y)) %>%
 data.frame() %>% round(2),
 Outsample = predicted_y %>% pull(fit) %>%
 forecast::accuracy(test %>% pull(y)) %>%
data.frame() %>% round(2)
 ) %>%
 rbindlist(use.names = T,
 idcol = "Accuracy",
fill = T)
final_plot <- ggplot(data = train, aes(x = x, y = y)) +
geom_point(size = 0.5) +
geom_line() +
theme_minimal() +
geom_line(mapping = aes(x = x, y = yhat), linetype = "dashed", color = "red") +
geom_line(data = data.frame(x = new_data$x, y = predicted_y$fit), color = "blue") +
geom_ribbon(data = data.frame(x = new_data$x, y = predicted_y$fit,
                              ymin = predicted_y$lwr, ymax = predicted_y$upr),
              aes(ymin = predicted_y$lwr, ymax = predicted_y$upr),
             fill = "lightblue",
```

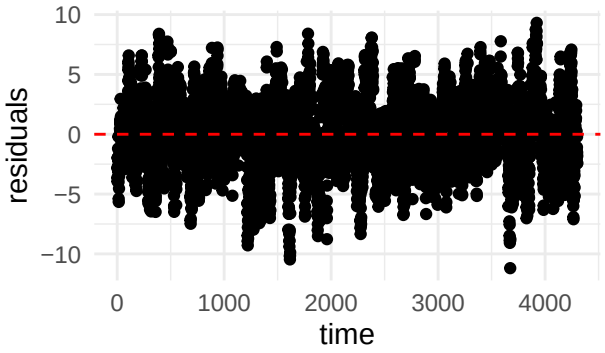
```
            alpha = 0.5) +
 geom_line(data = data.frame(x = test$x, y = test$y), color = "black") +
  annotate(geom = "text", x = 0, y = 125, label = list(invsout))

print(final_plot)
```



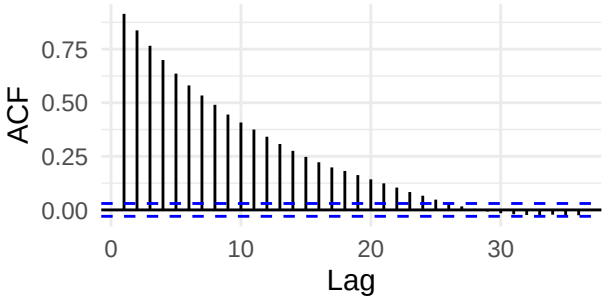

```
ggpubr::ggarrange(
plotlist = list(histogram_error,
homoskedasticity_plot,
acf_plot,final_plot ),
nrow = 2,
ncol = 2
)
```



Sharpiro test: 0.999 [0.027]
Kolmogorov−Smirnov test: 0.252 [0]
Jarque–Bera test: 7.047 [0.03]



Breusch−Pagan test: 3.915 [0.048]
Goldfeld−Quandt test: 1.011 [0.404]



Box−Pierce test: 18261.197 [0]
Ljung−Box test: 18287.081 [0]