Portfolio Choice Problem

UMA Project

January 14, 2019

1 Baseline Heterogeneous Agent Model

Prior to discussing the details of the model, it is helpful to have a broad sense of what the inputs to the model will be:

1. There is an Oracle who is responsible for reporting some stochastic state, but is unable to determine this information on its own. In order to get the information from other individuals, the Oracle will issue voting tokens and reward individuals for reporting information, but must ensure that it is in people's interests to tell the truth.
2. There are individuals who like consumption and participate in financial markets and can help (by reporting the truth), or hinder (by reporting the wrong state), the Oracle in its attempts to determine the stochastic state.
3. Though not modeled directly, there is a financial market that depends on the information being provided by the Oracle. If the Oracle reports the wrong stochastic state then some of the collateral from this market can be stolen.

These components give rise to various tradeoffs and we will look for policies that can sensibly ensure that individuals find it optimal to report the truth and guarantee that the collateral from the market is safe from being stolen.

We now proceed to describe these components in more detail

1.1 Oracle

The Oracle issues total supply 1 of the vote token and desires to report some state $S_t \in \{0, 1\}$ correctly. The Oracle does not know the true value of S_t but believes that $S_t = 1$ with probability $p_0 \geq 0.5$. The Oracle then uses votes by individuals as signals about the true state and will report 0 if enough voters report $s = 0$[1].

[1]Enough can eventually be determined by treating the Oracle as a Bayesian decision maker who has a loss function over reporting a lie, postponing reporting, and reporting the truth.

The Oracle's policy is to punish those who vote against its reported state by taking a fraction γ of their vote tokens and by rewarding those who voted with majority by giving them the seized tokens. The majority receives $(1-\gamma)\frac{\tilde{\pi}}{1-\tilde{\pi}}$ where $\tilde{\pi}$ is the percent of individuals who vote against the majority.

The Oracle is responsible for protecting μ_t margin in some system and generates revenue $\Pi_t = \tau_t(\mu_t)\mu_t$. μ_t follow some stochastic process. The Oracle pays the revenue out as dividends to vote token holders.

1.2 Individuals

Individuals are allowed access to two financial assets:

1. Risk-free bonds which pay return r
2. Vote tokens issued by the Oracle which will pay an amount which depends on the individual's vote, s_t, and the Oracle's reported state, $\hat{S}_t$, according to $(1+\gamma(s_t, \hat{S}_t))(d_t + p_t)$ where d_t is period ts dividend and p_{t+1} is the value at which the token could be resold

Voters who choose to vote dishonestly may successfully corrupt the Oracle. In order to successfully corrupt the Oracle, they must own enough votes to convince the Oracle to report the wrong state. If the dishonest voters successfully corrupt the Oracle then they are able to seize a fraction κ of the margin in the system, but, afterwards, everyone will enter a new regime where they only have access to the risk-free bond, and, not the vote tokens.

Individuals receive utility from consumption and maximize their consumption by choosing a portfolio of assets and to report honestly or dishonestly. Additionally, when voting, individuals who would like to report the truth forget to vote with probability η and make a mistake with their vote with probability π. We assume that dishonest voters are more deliberate in the sense that they neither forget nor make mistakes in their votes.

Agents experience wealth shocks (forced consumption shocks) throughout their life. If an agent has wealth w_t going into period t then, when they start making their decisions, they have wealth $w_t + \varepsilon_t$ where $\varepsilon \sim N(0, \sigma^2)$.

An agent with wealth w_t, with the system uncorrupted, and margin at μ_t faces the following problem:

$$V(w_t, 0) = \max_{b_{t+1}, x_{t+1}, s_t} u(c_t) + \beta E_x \left[V(w_{t+1}, \text{Corrupt}_t) \right] \tag{1}$$

$$w_{t+1} = (1+r)b_{t+1} + x_{t+1}(1 + \gamma(s_{t+1}, \hat{S}_{t+1}))(d_{t+1}(\mu_{t+1}) + p_{t+1}(\mu_{t+1}, \hat{S}_t)) \tag{2}$$

$$c_t = w_t + \varepsilon_t - b_{t+1} - p_t x_{t+1} \tag{3}$$

and agent with wealth w_t with a corrupted system faces

$$
\begin{aligned}
V(w_t, 1) &= \max_{b_{t+1}} u(c_t) + \beta V(w_{t+1}, 1) \\
w_{t+1} &= (1+r)b_{t+1} \\
c_t &= w_t + \varepsilon_t - b_{t+1}
\end{aligned}
$$

When computing expectations, agents believe that others will report the truth (besides for non-participation and trembling hand mistakes). Thus they will only attack if they can do so alone profitably.

1.3 Truthful Oracle

A *Truthful Oracle* is an Oracle policy $(\gamma, \tau_t(\mu_t)$ such that $\hat{S}_t = 1$. This is done by ensuring that the truth-tellers value the token more than the liars do

A *Truthful Equilibrium* is

1. Value functions $V : \mathcal{R}^+ \times \mathcal{R}^+ \to \mathcal{R}$
2. Portfolio decision functions $b^*, x^* : \mathcal{R}^+ \times \mathcal{R}^+ \to \mathcal{R}^+$
3. Vote decision function $s^* : \mathcal{R}^+ \times \mathcal{R}^+ \to \{0, 1\}$
4. Price $p \in \mathcal{R}^+$
5. Oracle redistribution policy: $\gamma \in (0, 1)$

such that

- Value functions and policy functions are optimal given prices
- The Oracle policy produces a truthful oracle
- Token price, p, clears token market