Taxation Notes

UMA Project

December 17, 2019

1 Non-technical Summary

The UMA Data Verification Machine (DVM) is responsible for producing "honest" prices that can be used to evaluate financial smart contracts on the blockchain. The DVM relies on votes by individuals who own UMA vote tokens to accurately report prices from the fiat world. In order to ensure that the DVM cannot be manipulated for financial gain, we establish conditions under which it is not economically profitable to corrupt the system by purchasing enough tokens to corrupt the DVM by voting dishonestly[1].

The tool we use to incentivize honest behavior is to conduct buybacks of the UMA vote token from token holders. We view these buybacks as a way to ensure that there are positive returns paid for holding vote tokens. We prevent corruption by ensuring that the expected present discounted value of all future buybacks exceeds the amount that an individual could gain by corrupting the system. These buybacks are financed through a mixture of current taxes on the margin in the system and a "rainy day fund" which is savings held from unspent margin taxes from the past. We build a mathematical model that describes these interactions and lay out the conditions necessary to ensure that there is no economically profitable corruption opportunities.

We find that

1. The executed buybacks at high levels of margin are typically higher than the buybacks at lower levels of margin.
2. The expectation of growth in the total margin secured by the DVM coupled with the fact that there are higher levels of buybacks at higher levels of margin results in the system being secure with low (or zero) buybacks in its early stages. Depending on beliefs about margin growth and realized margin growth, this results in the system being secure for a non-trivial amount of time (10+ years) without the need to implement buybacks.
3. When the system's "rainy day fund" is empty, taxes must be weakly larger than the buybacks which occur for each level of margin. Additionally, for certain levels of margin that have positive buybacks, the system collects more in taxes than needed so that it can use those funds to finance future buybacks in states where there are higher required buybacks.

These findings motivate our proposed taxation structure which levies low taxes while the system is growing and then uses a pro-cyclical tax structure with higher taxes when margin is high relative to its persistent level.

[1] There are other attacks that involve bribing voters to take a particular action, but we address these concerns in a different research document

2 Technical Details

We now move on to describe the technical details of our findings.

The core argument will revolve around what we call the "Profit from Corruption (PfC) less than Cost of Corruption (CoC)" inequality ($PfC < CoC$). Preserving this inequality ensures that no individual can profitably corrupt the DVM.

Throughout this work, we assume that the PfC is proportional to the amount of margin that is held in the system, PfC $= \gamma M_t$, and that the CoC is equal to buying sufficient vote tokens to corrupt the system[2], CoC $= \chi p_t S_t$ where p_t denotes the dollar price of the UMA vote tokens, S_t denotes the outstanding volume of tokens, and χ tells us what fraction of the tokens are needed to corrupt the system. We will also rely on the assumption that the market price for tokens are accurate — Namely that if X_t are the dollar amounts of buybacks in each period t and r is the risk-free interest rate, then the market cap of the token would be $p_t S_t = E_t\left[\sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s X_t\right]$. Our baseline assumption will be that the proportion of margin that can be stolen is $\frac{1}{2}$ and the fraction of tokens required to corrupt is $\frac{1}{2}$.

In each of the following subsections, we will use the notation presented below

- B_t denotes the number of tokens bought back prior to vote
- D_t denotes the dollar value of funds held in the "rainy day fund"
- M_t denotes total dollar value of margin in system
- τ_t denotes the tax rate on the system
- p_t denotes the price (in dollars) of a single vote token
- S_t denotes the number of vote tokens outstanding
- $F_t \equiv \tau_t M_t$ denotes total fee levied on system
- $X_t \equiv B_t p_t$ denotes the dollar value of expenditure on buybacks

Additionally the following are parameters

- χ is the fraction of voters required to corrupt the system
- η is percent of people who do not vote
- γ the fraction of margin seized if DVM is corrupted
- π is the inflation rate used to reward voters
- r is the risk-free return on other investments

[2]There are other types of attacks in which individuals can offer bribes to individual voters in order to incentivize them to corrupt the system — We have forthcoming work in which we can show that the cheapest attack on our system involves a 51% attack

We can expand and rearrange the $PfC < CoC$ inequality to express the price needed to secure the system.

$$
\begin{aligned}
PfC_t &\leq CoC_t \\
&\Rightarrow \gamma M_t \leq \frac{1-\eta}{2} p_t S_t \\
&\Rightarrow p_t \geq \frac{2}{1-\eta} \frac{\gamma M_t}{S_t}
\end{aligned}
\qquad \Rightarrow p_t \geq \frac{2}{1-\eta} \frac{\gamma M_t}{S_t}
$$

Thus as long as the price of a token is higher than $\frac{2}{1-\eta} \frac{\gamma M_t}{S_t}$, then it will not be profitable to corrupt the system.

The remaining analysis is done in pieces. We start with the simplest possible model and slowly add additional features to deepen our insights. We briefly describe these sections:

- Section 2.1 asks the question, "In a deterministic world in which the system margin has reached a steady level of margin, what is the lowest tax rate we can implement that ensure incorruptibility?" We find that the lowest tax rate achievable is the same as the return that we would like the tokens to offer.
- Section 2.2 asks the question, "If we knew exactly how margin would grow in the future, what is the minimum amount of taxes that we could impose at each period while maintaining the $PfC < CoC$ guarantee?" We find that the rate of change in the tax rates is similar to the rate of change of the margin held in the system. We find that, under a particular parameterization, we can achieve a zero tax rate for almost 20 years before being required to increase taxes to steady state amounts.
- Section 2.3 analyzes a model in which margin follows a stochastic process and thinks about the right way to smooth taxes while maintaining the ability to make payments that ensure the system is "incorruptible."

2.1 Deterministic Steady State

In this version of the model, we will consider the system margin being constant over time, i.e. $M_t = \bar{M}\ \forall t$. However, it is important to note that not all of the variables will be constant in the steady state because there is potentially non-zero inflation in the number of outstanding tokens.

The total number of tokens follows

$$S_{t+1} = (1+\pi)(S_t - B_t)$$

Also, recall that the price that ensures $PfC < CoC$ is given by $p_t \geq \frac{2}{1-\eta}\frac{PfC_t}{S_t}$.

The token market cap, $p_t S_t$, will be a constant since

$$\begin{aligned}
p_t S_t &= \frac{2}{1-\eta}\frac{PfC_t}{S_t}S_t \\
&= \frac{2}{1-\eta}PfC_t \\
&= \frac{2}{1-\eta}\gamma M_t \\
&= \frac{2}{1-\eta}\gamma \bar{M}
\end{aligned}$$

Assume that we'd like to achieve an aggregate period-by-period return of r to token holders.

$$\begin{aligned}
(1+r) &= \frac{p_{t+1}(S_{t+1}+B_{t+1})}{p_t S_t} \\
(1+r) &= \frac{p_{t+1}S_{t+1}+X_{t+1}}{p_t S_t} \\
(1+r) &= \frac{\frac{2}{1-\eta}\frac{PfC_{t+1}}{S_{t+1}}S_{t+1}+X_{t+1}}{\frac{2}{1-\eta}\frac{PfC_t}{S_t}S_t} \\
(1+r)PfC_t &= PfC_{t+1}+\frac{1-\eta}{2}X_{t+1}
\end{aligned}$$

In the SS this means that

$$\begin{aligned}
(1+r)P\bar{f}C &= P\bar{f}C+\frac{1-\eta}{2}\bar{X} \\
\rightarrow \bar{X} &= \frac{2P\bar{f}C}{1-\eta}r
\end{aligned}$$

If we assume that $P\bar{f}C = \frac{1}{2}\bar{M}$ and that there is full participation then this reduces to $\bar{X} = r\bar{M}$ which means that the fee rate is given by,

$$\bar{\tau} \equiv \frac{\bar{X}}{\bar{M}} = r$$

The question of how high fees should be in the steady state depend on what one believes about the return that token holders are willing to accept on their tokens.

If the following three things were true,

1. Blockchains become a widely accepted and used technology
2. UMA was core to decentralized finance
3. People were confident that UMA would continue to be a core component of decentralized finance

then it is feasible that token holders would accept interest rates near the risk-free rate. Under current market conditions, this could translate into a required annualized fee rate of between 2% and 5%, but, even in more "investor friendly times," this should not exceed 5%-10%.

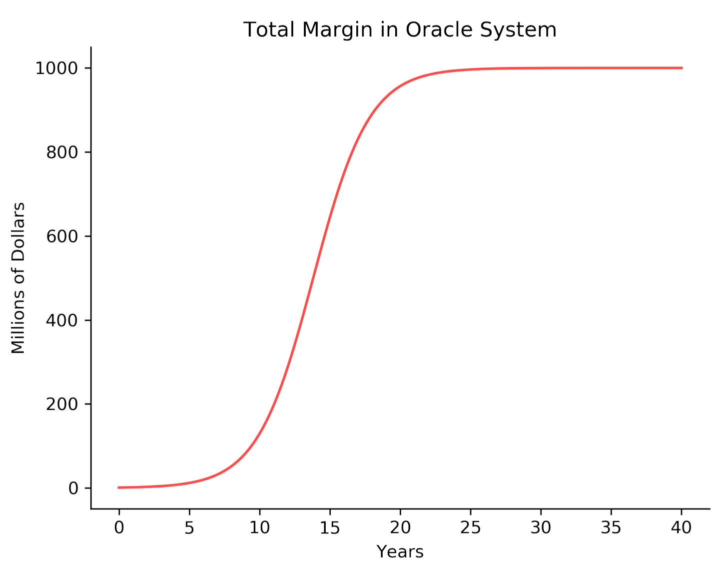
2.2 Determisitic Growth

We now consider how to implement fees in a world where the system margin grows over time according to a deterministic process. One of the reasons that it is important to model growth in this system is that, if people believe that there will be higher buybacks in the future, then we might be able to sustain lower buybacks today.

For simplicity, we assume that margin follows

$$M_{t+1} = M_t + gM_t \left(1 + \frac{M_t}{\bar{M}}\right)$$

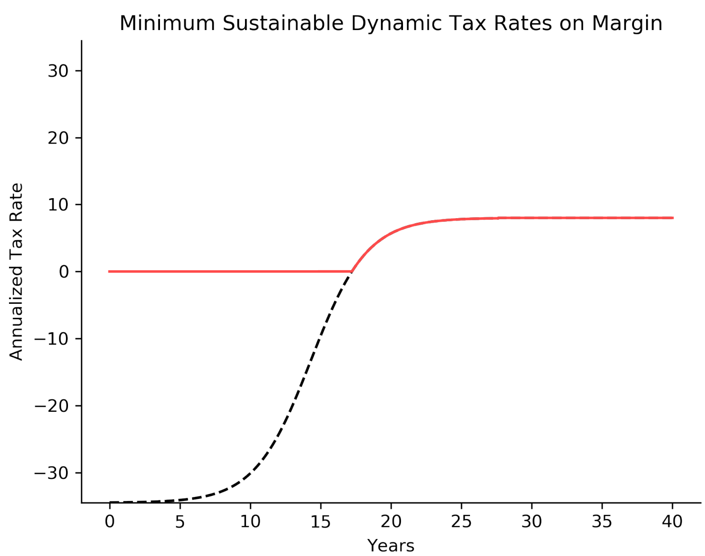
This process, known as logistic growth, generates "S-shaped" growth. We can see the implications that this process has for the total system margin in Figure 2.2.



We assume that we would like to charge the minimum amount of fees while maintaining the system's incorruptibility. This produces the following mathematical program:

$$
\begin{aligned}
\min_{F_t} E &\left[\sum_{t=0} \left(\frac{1}{1+r} \right)^t F_t \right] \\
&\text{subject to} \\
PfC_t &\leq \frac{1}{2} P_t S_t = \frac{1}{2} E \left[\sum_{s=0} \left(\frac{1}{1+r} \right)^s X_{t+s} \right] \\
X_t &= F_t \\
0 &\leq F_t \\
F_t &\leq \bar{\tau} \bar{M} \\
M_{t+1} &= M_t + g M_t \left(1 + \frac{M_t}{\bar{M}} \right)
\end{aligned}
$$

We can solve this program indirectly by choosing an s such that $M_s \approx \bar{M}$. At this point, we have arrived in the steady state and the fee rate implemented will be $F_t = \bar{\tau} M_t$. We can then step back by one period to $t = s - 1$ and compute what the required X_t in that period would be. We can proceed to step this back until we reach our initial condition of M_0 which traces out a path of fee collections. This process generates a sequence of fees that look like Figure 2.2.



We plot the solution to two different mathematical programs. The first, plotted as a solid line, corresponds to the program that we described above. The second, plotted as a dashed line, corresponds to what fee rates would be if we did not impose the non-negativity constraint. Notice that in the case in which there is no non-negativity constraint, the fees go negative in order to meet the $PfC < CoC$ constraint with equality.

The most interesting observation we make from this chart is that fees can start low and stay low for a prolonged period of time before rising to the steady state levels. The reason that this satisfies the $PfC < CoC$ inequality is that individuals understand that there will be growth in the system — The future promise of increased buybacks in the future is enough to secure the system while it is new (and small).

We can be see this by looking at the equation that determines the CoC:

$$CoC_t = \chi p_t S_t = \chi E \left[\sum_{s=0}^{\infty} \left(\frac{1}{1+r} \right)^s X_{t+s} \right]$$

High values of X_{t+s} increase the cost of corruption in period t. This insight is somewhat general, but may be potentially too strong in this model due to the fact that there is certainty about the future growth. This concern motivates our next section in which we introduce uncertainty about the growth of the system.

2.3 Stochastic Markov

In the previous two models discussed there was no uncertainty about what future margin would be. Obviously in practice, there is significant uncertainty about how margin will evolve and this uncertainty will have implications for token prices and other model outcomes. We add uncertainty to this model by assuming that margin evolves according to a stochastic Markov process.

One additional difference we introduce in this section is a separation between the amount of fees collected, F_t, and the size of the buyback, X_t. In the deterministic model, there were no unexpected fluctuations and margin was either constant or always growing. These two features made it unnecessary to store a portion of the fees collected today to fund future buybacks.

We assume that margin follows a Markov process, that is, that tomorrow's margin only depends on what margin was today.

$$M_{t+1} = f(M_t, \varepsilon_{t+1})$$

A first order Markov process can be quite general and the margin processes from Section 2.1 and Section 2.2 are special cases of the above process. The only difference is that now, margin will begin low and then will (potentially) grow to higher levels with some uncertainty about the path it takes to get there.

Our objective will be to choose $\{X_t(M^t), F_t(M^t)\}_{t=0}^{\infty}$ to optimize certain goals. In this document, we will focus on minimizing the present discounted value of payouts to token holders and keeping fee rates at a low and consistent level[3].

Two Part Solution

We will approach this problem by breaking it into two parts:

1. Solve for the minimum cost policy function $X^*(M^t)$
2. Find the fee function $F^*(M^t)$ that meets our objective of low and non-volatile fees such that we can fund any sequence of buybacks, $\{X(M^t)\}$, without debt

Buyback Policy

In the discussion that follows, we will focus on the case in which there are no negative buybacks. There are a few interesting features associated with the unconstrained case, but we relegate their discussion to Appendix A. The constrained program can be written as

$$\begin{aligned}
&\min_{\{X_t\}} E\left[\sum_t \left(\frac{1}{1+r}\right) X_t\right] \\
&\text{subject to} \\
&2PfC_t \leq E\left[\sum_{s=0}^{\infty} \left(\frac{1}{1+r}\right)^s X_{t+s}\right] \quad (\lambda_t) \\
&X_t \geq 0 \quad (\mu_t)
\end{aligned}$$

If we add the restriction that the policy is Markov, $X^*(M^t) = X^*(M_t)$, then we know

$$\begin{aligned}
2PfC_t &\leq E\left[\sum_{s=0}^{\infty} \left(\frac{1}{1+r}\right)^s X^*(M^s)\right] \\
&\leq \sum_{s=0}^{\infty} \left(\frac{1}{1+r}\right)^s E\left[X^*(M_s)|M_t\right]
\end{aligned}$$

Finally, if we assume that the margin process, $\{M_t\}$, follows a discrete Markov chain with N states, then the objective function can be simplified to

[3]We do this by using a quadratic punishment function which punishes any deviation of the fee rate from 0 quadratically. The quadratic structure ensures that large fees are penalized more heavily than smaller fees — Under certain assumptions, such a structure often produces a solution in which fees tomorrow are the same as the fees today in expectation

$$E\left[\sum_t \left(\frac{1}{1+r}\right) X_i(t)\right]$$
$$\Rightarrow \pi_0(I - \frac{1}{1+r}P)^{-1}\vec{X}$$

where π_0 is a vector that denotes the initial distribution across the states of margin and $\vec{X} \equiv \{X_1, \ldots, X_i, \ldots, X_N\}$ denotes a buyback for each of the margin states. Our program can then be written as a linear program:

$$\begin{aligned} &\min_{\vec{X}} \pi_0(I - \frac{1}{1+r}P)^{-1}\vec{X} \qquad \text{subject to} \\ &\quad 2PfC_i \leq (I - \frac{1}{1+r}P)^{-1}X_i \quad (\forall i) \\ &\quad X_i \geq 0 \quad (\forall i) \end{aligned}$$

Given the model parameters, we can solve this program with any standard linear program solver.

Tax Policy

We can then formalize the second step with

$$\begin{gathered} \min_{\{F_t\}} E\left[\sum_t \left(\frac{1}{1+r}\right)^t (F_t - \hat{\tau})^2\right] \\ \text{subject to} \\ F_t + (1+r)D_t \geq D_{t+1} + X_t^* \quad (\mu_t) \\ D_t \geq 0 \end{gathered}$$

where D_t denotes the amount that is currently stored in the rainy day fund and $\hat{\tau}$ denotes a target level for our fee rate. We can write this recursively as

$$\begin{aligned} V(D_t, M_t) = \max_{F_t} \; &(F_t - \hat{\tau})^2 + \frac{1}{1+r}E[V(D_{t+1}, M_{t+1})] \\ &\text{Subject to} \\ &D_{t+1} + X_t \leq (1+r)D_t + F_t \\ &D_t \geq 0 \end{aligned}$$

Although most quadratic problems have near analytical solutions, we cannot exploit them in this case due to no borrowing inequality constraint on the rainy day fund because it adds a non-linearity to the problem. We can still compute a solution to this recursive program using a "brute-force" style method called value function iteration.

The output of such a solution will be a rule, $T^*(D_t, M_t)$, which expresses the fee that should be imposed as a function of the amount currently stored in the rainy day fund and the current margin in the system. Additionally, this "policy function" will imply a law of motion for the size of the rainy day fund

Numerical Example

In this subsection, we describe a single numerical example[4]. This will help us in the following subsection when we discuss the insights that come out of this model.

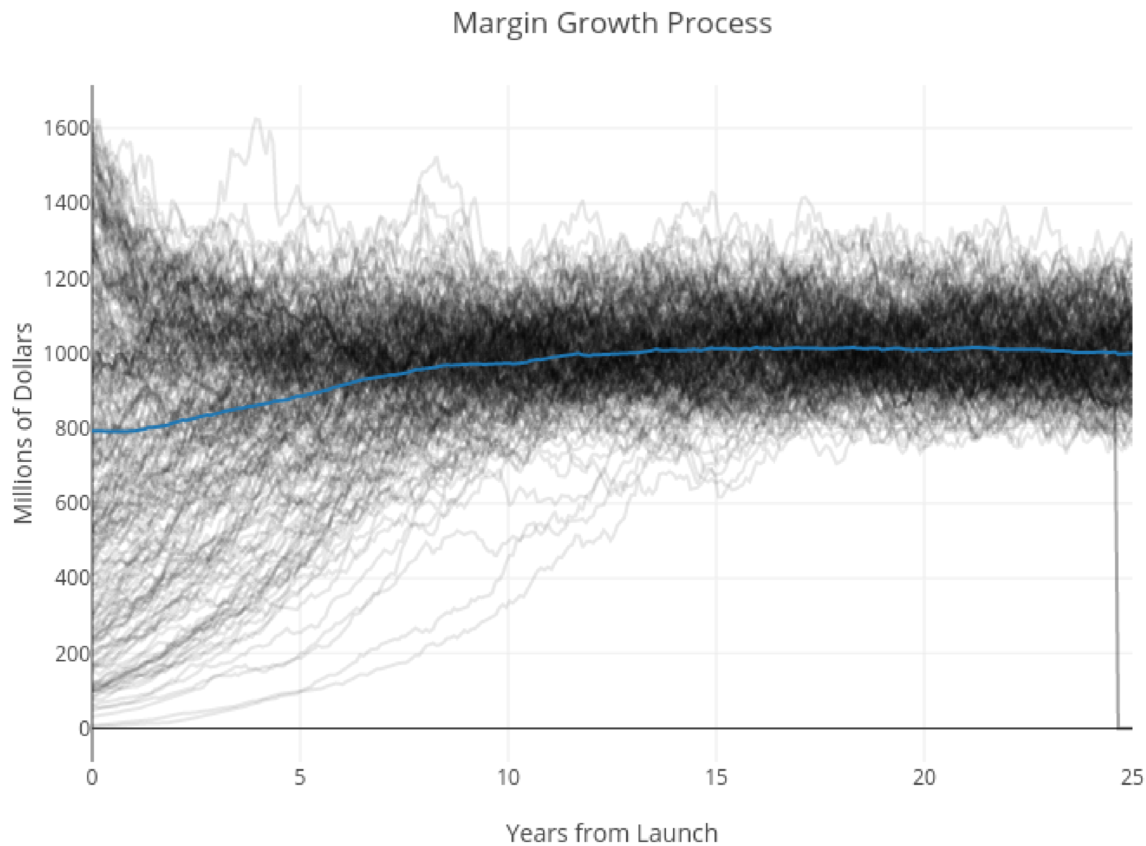
There are some issues that arise when we set too fine of a time-scale, so for now we focus on a monthly scale. We set most of the parameters to our "baseline" assumptions:

- $\chi = \frac{1}{2}$: Corrupted with half of the votes
- $\eta = 0$: Full participation
- $\gamma = \frac{1}{2}$: Half of the margin is vulnerable to being stolen
- $r = 0.0021$: Monthly interest risk-free rate which annualizes to about 2.5%
- $\hat{\tau} = 0.000165$: A target fee-rate of 0.2% of margin

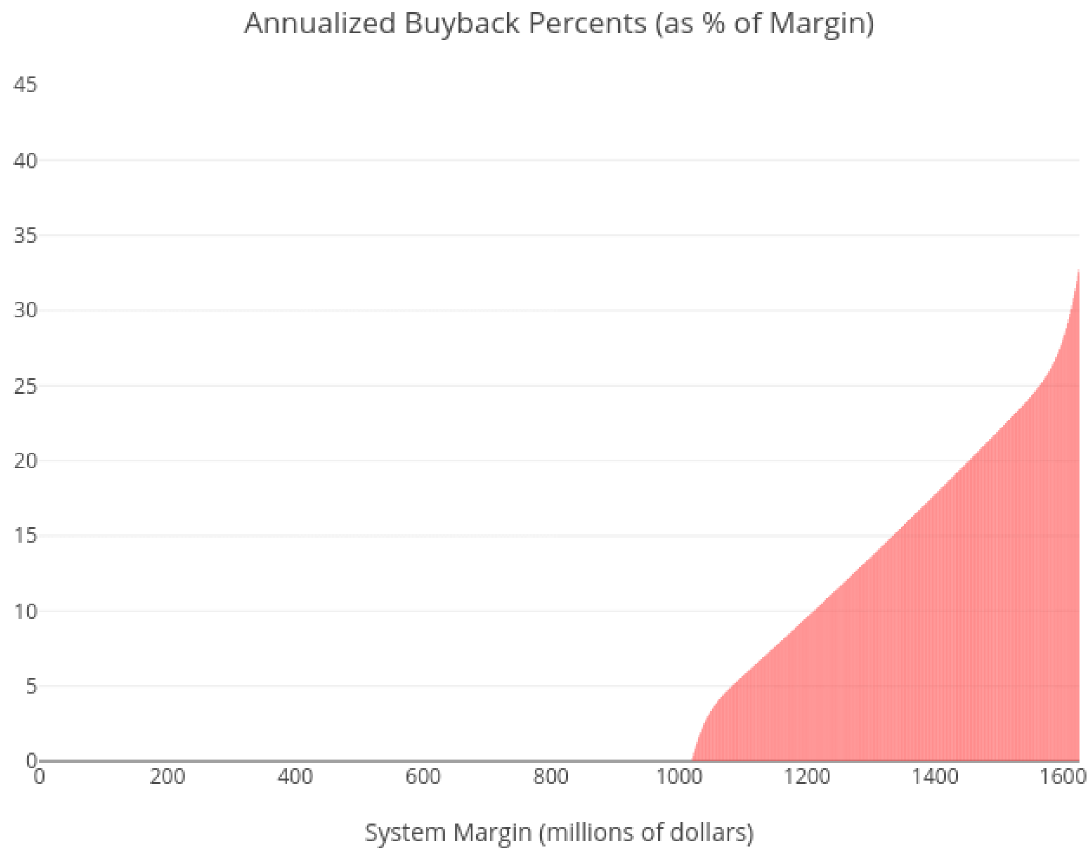
The only remaining question is how to pick our Markov process for M_t. We use a discrete Markov approximation of the Logistic Growth process with some added disaster risk[5]. We describe how we generate this approximation in Appendix B and leave it to the interested reader to investigate further. We plot many possible histories of this process below to help with the visualization of the implied outcomes associated with this process.

[4]For additional parameterizations, we refer you to Appendix C in which we display similar outcomes for a variety of other parameterizations

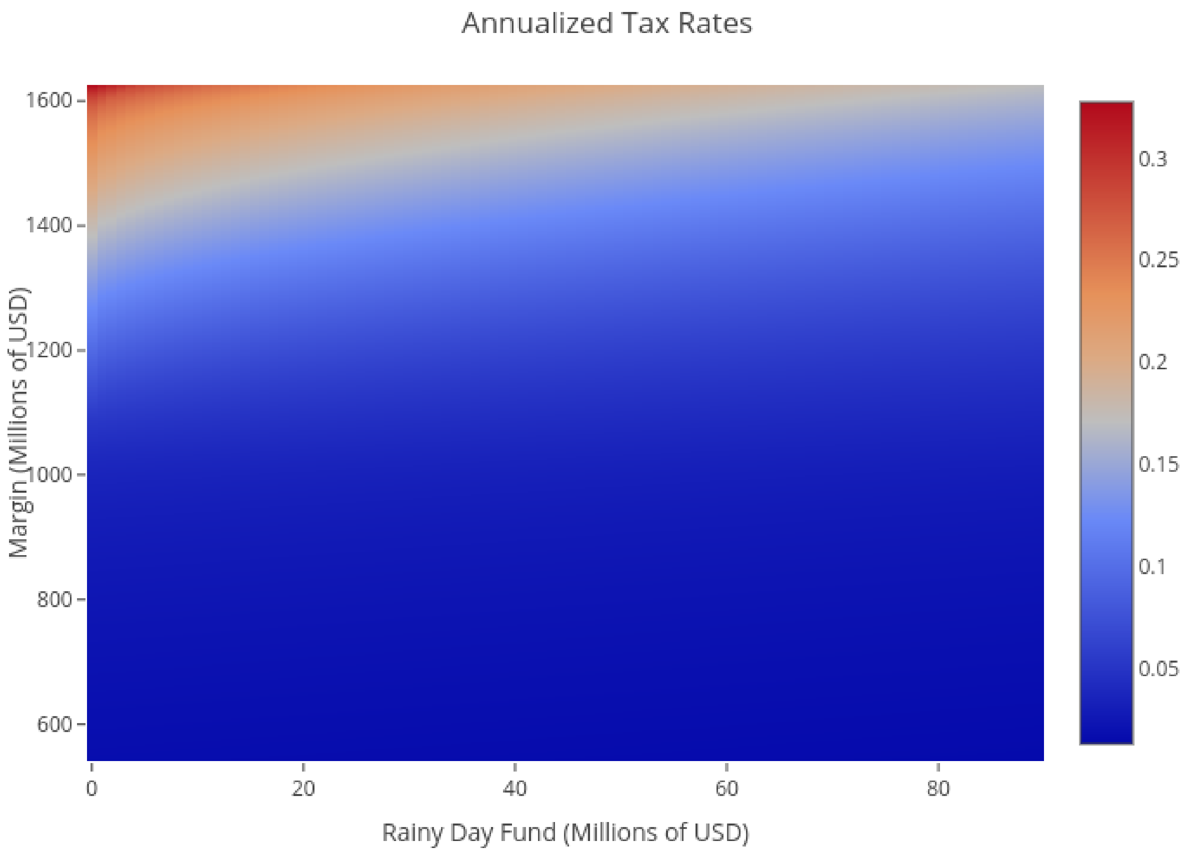
[5]When we say disaster risk here, we mean that there's an absorbing state with margin at 0 and there's positive probability that the process reaches that state from any other state



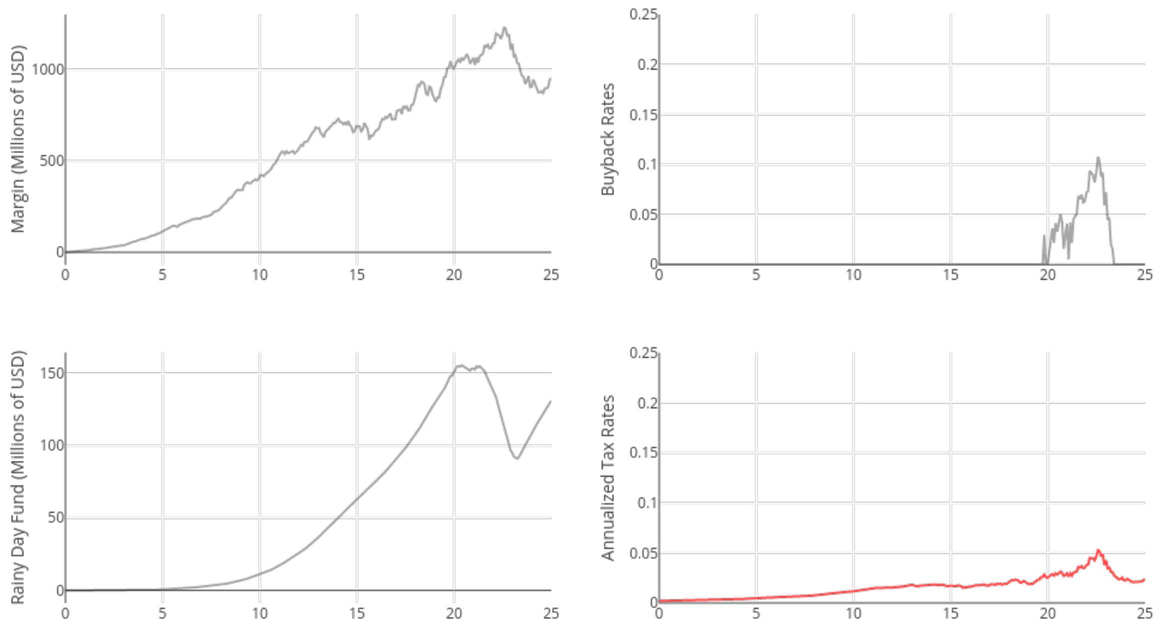
We first illustrate how the buybacks vary by the current level of margin. Note that this is very similar to what we observed in the deterministic case — At low levels of margin, we can support a policy of zero buybacks because there is positive probability that the margin will grow to a point where there will be relatively large buybacks. In the image below, we annualize the monthly buybacks as a percent of margin



We now demonstrate how fees vary by the status of the rainy day fund and the current margin using a heatmap. Current margin increases along the y-axis, the rainy day fund increases along the x-axis, and the color of the image at a point demonstrates how high the annualized fee rates are at that point.



Finally, we simluated the entire system one time for 25 years. We plot each of the series of interest below. We note that the fee rates are quite smooth relative to the buyback rates or the amount held in the rainy day fund — We view this as an indicator that the solution to our mathematical program was a success.



Tax Insights

In this subsection, we discuss some of the findings associated with our numerical examples:

If there are no rainy day funds, fees must reflect at least the full buyback amount

This point follows almost immediately from the fact that we have chosen to disallow any borrowing by the DVM. If the rainy day fund, D_t, is currently at 0 then

$$\begin{aligned} D_{t+1} + X_t &\leq (1 + r) * D_t + F_t \\ D_{t+1} + X_t &\leq F_t \end{aligned}$$

The question is whether we will raise extra fees to not experience the same state tomorrow. It turns out that if the margin is relatively low then we impose a slightly higher fee than needed to provide additional stability for times in which we will have higher buybacks.

Rainy day funds are saved until periods when the buybacks are high

This point is aimed at exploring $\frac{\partial X^*(M_t) - T^*(D_t, M_t)}{\partial M_t}$ and $\frac{\partial X^*(M_t) - T^*(D_t, M_t)}{\partial D_t}$. It says, conditional on positive buybacks, that the change in per-period savings is decreasing in both (1) how much savings there already is and (2) the margin in the system.

$$\frac{\partial(X^*(M_t) - T^*(D_t, M_t))|X^*(M_t) > 0}{\partial D_t} < 0$$
$$\frac{\partial(X^*(M_t) - T^*(D_t, M_t))|X^*(M_t) > 0}{\partial M_t} < 0$$

Imposing low, but positive, fee rates while system is growing allows relatively low fee rates once margin is stable

If we look at Figure 2.3, we can note that the annualized feerates are approximately 0.2% for the first 15 years and that during that time there are no buybacks. This allows the system to build up its rainy day fund to almost 150 million USD. Shortly thereafter, the system hits its maximum possible buyback levels which are approximately 10% of the margin — Rather than impose extra high fee, it only raises the fee rates by a few percentage points and uses the rainy day fund to fund the difference.

3 Conclusion

This document explores potential policies that allow UMA's DVM to support honest prices.

It explores three (related) mathematical models which explore the necessary conditions for DVM corruption to be unprofitable for attackers. We demonstrate how UMA could secure the DVM using a mixture of fees and token buybacks and propose a loose structure for how to implement these policies.

A Negative Buybacks

Theory

In this appendix section, we return to the problem of finding a minimum cost buyback policy. However, we will no longer impose a non-negativity constraint. The modified mathematical program becomes

$$\begin{aligned}
\min_{\{X_t\}} \; & E\left[\sum_t \left(\frac{1}{1+r}\right) X_t\right] \\
& \text{subject to} \\
& 2PfC_t \leq E\left[\sum_{s=0}^{\infty} \left(\frac{1}{1+r}\right)^s X_{t+s}\right] \quad (\lambda_t)
\end{aligned}$$

We will continue to focus on Markov policies, $X^*(M^t) = X^*(M_t)$, so, as before, we know:

$$\begin{aligned}
2\gamma M_t &\leq E\left[\sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s X^*(M^s)\right] \\
&\leq \sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s E\left[X^*(M_s)|M_t\right]
\end{aligned}$$

In the discrete Markov chain case (where P denotes the Markov transition matrix), then this reduces to:

$$\begin{aligned}
2\gamma M_t &= \sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s E\left[X^*(M_s)|M_t\right] \\
2\gamma \vec{M} &= \sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s P^s \vec{X} \\
&\dots \\
\vec{X} &= 2\gamma\left(I - \frac{1}{1+r}P\right)\vec{M}
\end{aligned}$$

Interpretation

For certain parameterizations, it is optimal to have periods of negative buybacks ($X_t < 0$). We discuss how this should be interpreted and propose a potential mechanism to implement negative buybacks.

Consider the process that determines the number of tokens at a given time:

$$\begin{aligned}
S_{t+1} &= \pi(S_t - B_t) \\
p_t S_{t+1} &= \pi(p_t S_t - X_t) \\
X_t &= p_t \frac{\pi S_t - S_{t+1}}{\pi}
\end{aligned}$$

Thus if $X_t < 0$ then $S_{t+1} > \pi S_t$

$$\begin{aligned}
S_{t+1} &> piS_t \\
\pi(S_t - B_t) &> \pi S_t \\
&\rightarrow B_t < 0
\end{aligned}$$

The negative buybacks mean that we need to be increase the supply of tokens by more than the typical inflation rate. One generic mechanism that would allow for positive and negative buybacks is simply a period-by-period auction.

- If $B_t > 0$ then the DVM performs an auction and allows token holders to name a price at which they would be willing to sell a certain number of their tokens. It then uses X_t dollars from the rainy day fund to purchase the tokens from these individuals.
- If $B_t < 0$ then the DVM performs an auction to sell off newly minted tokens. It allows individuals to mark a price at which they would be willing to purchase tokens and collects X_t dollars for the rainy day fund by selling to the individuals who value the tokens the most.

An additional benefit of this process is that it helps ensure that tokens stay in the hands of those who value them most which provides additional protections for the DVM.

B Discrete Markov Approximation to Logistic Growth

We use a stochastic version of the logistic growth process:

$$M_{t+1} = M_t \left(1 + g\left(1 - \frac{M_t}{\bar{M}}\right)\right) + \sigma(M_t)\varepsilon_{t+1}$$

State values

We create create a vector of evenly spaced points between some initial value, M_0, and the deterministic steady state, $\bar{M}$. These will serve as the state values that margin can take. The more points we place between these two numbers, the more accurate our approximation will become.

Transition matrix

To determine the probability of transitioning between two points M_i and M_j, we take the difference in value of the conditional cumulative distributions. Thus, the probability of transitioning from M_i to M_j is given by

$$p_{ij} = F\left(M_j - M_i\left(1 + g\left(1 - \frac{M_i}{\bar{M}}\right)\right)\right)$$

where F is a Normal distribution with mean 0 and standard deviation $\sigma(M_i)$.

Additionally, we add a small positive probability (inversely proportional to the size of the system) of transitioning from any state to a "disaster" state which is absorbing[6].

[6] This actually results in the guaranteed (eventual) demise of the system.

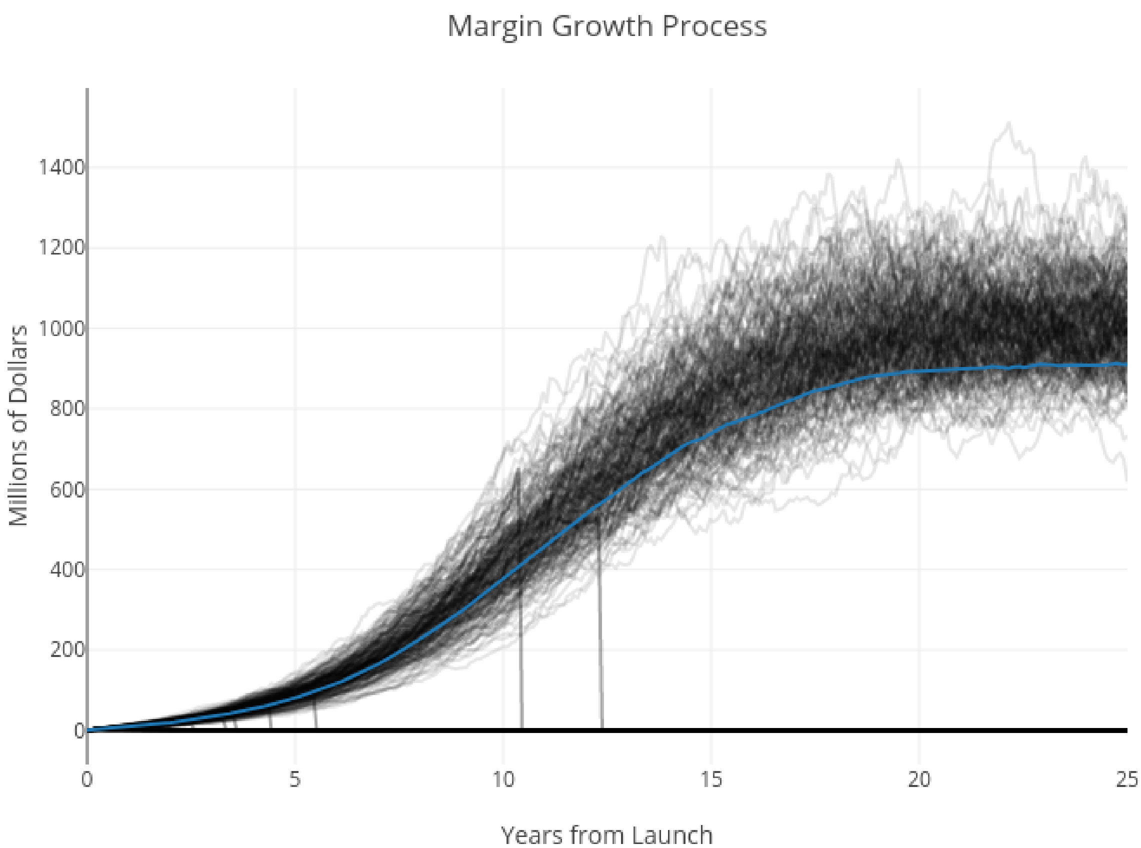
C Parameterization Robustness Checks

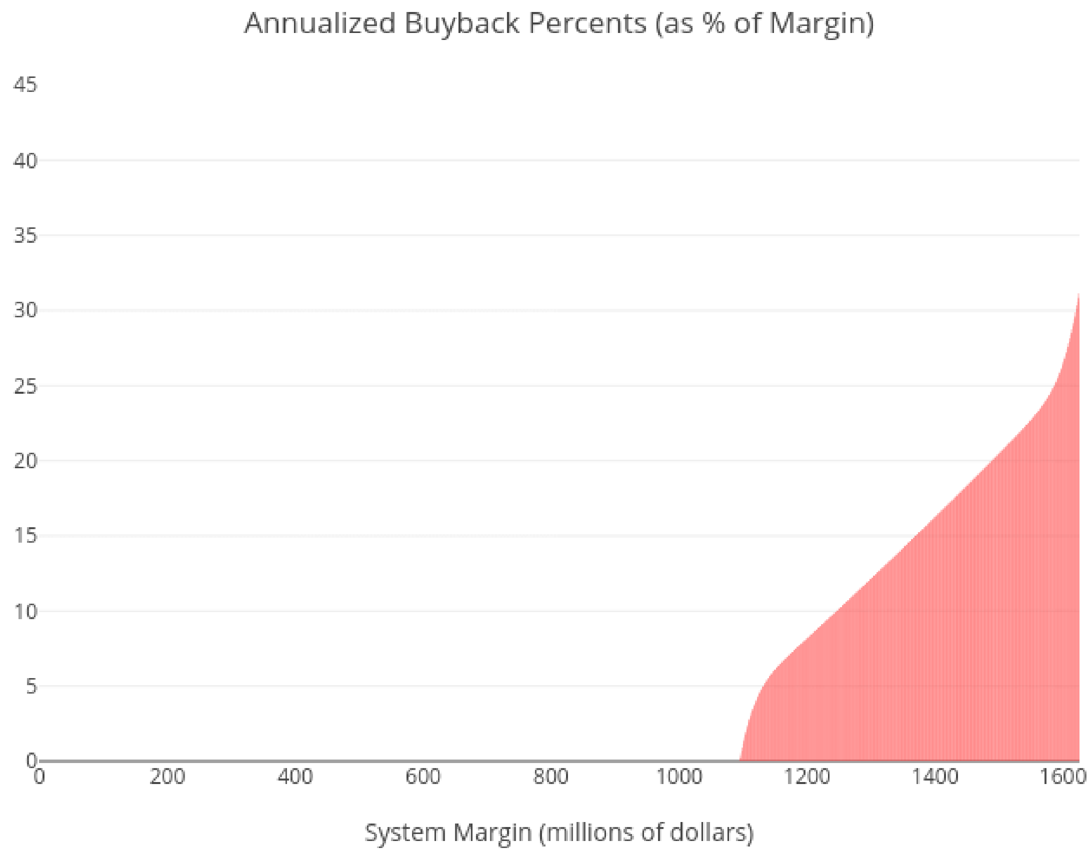
In this appendix, we plot all of the graphs from the numerical section for alternative parameterizations.

C.1 Fluctuating r

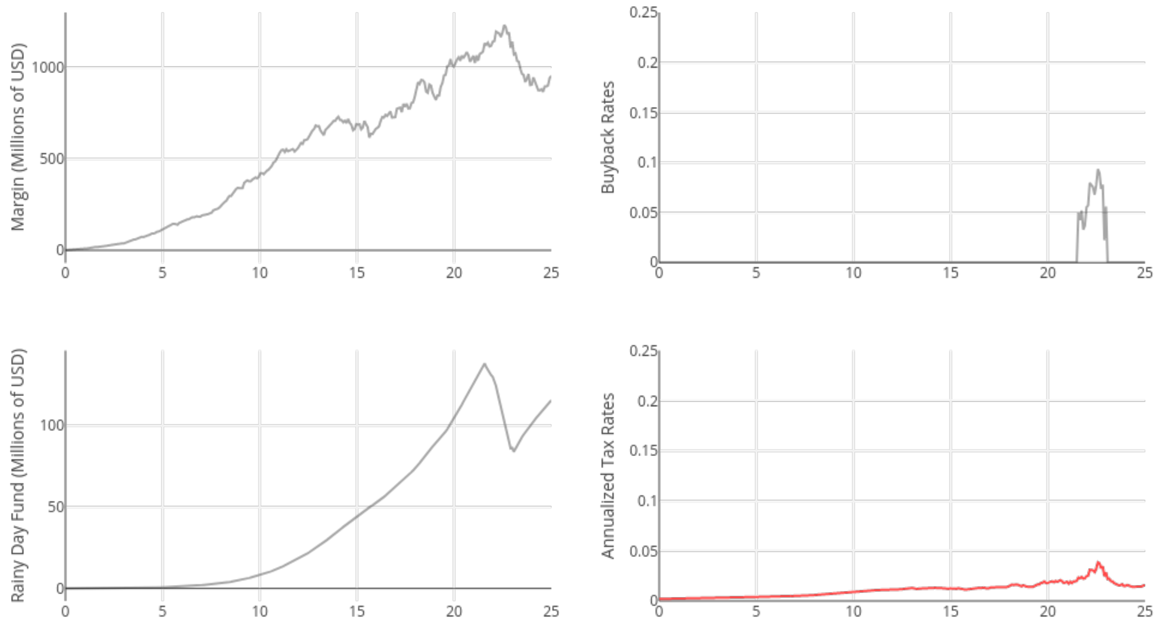
In this section, we consider three alternative values for the risk-free rate

C.1.1 Graphs with $r = 0.01$ (annualized)

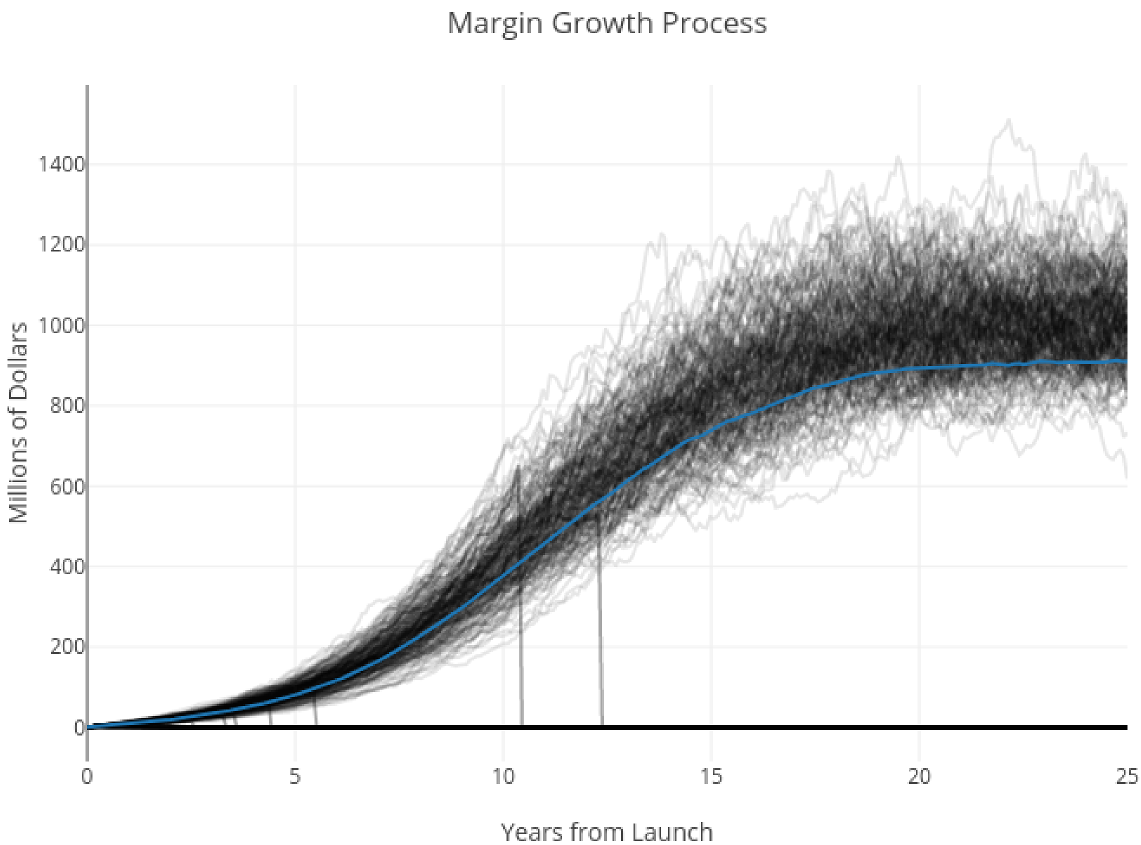


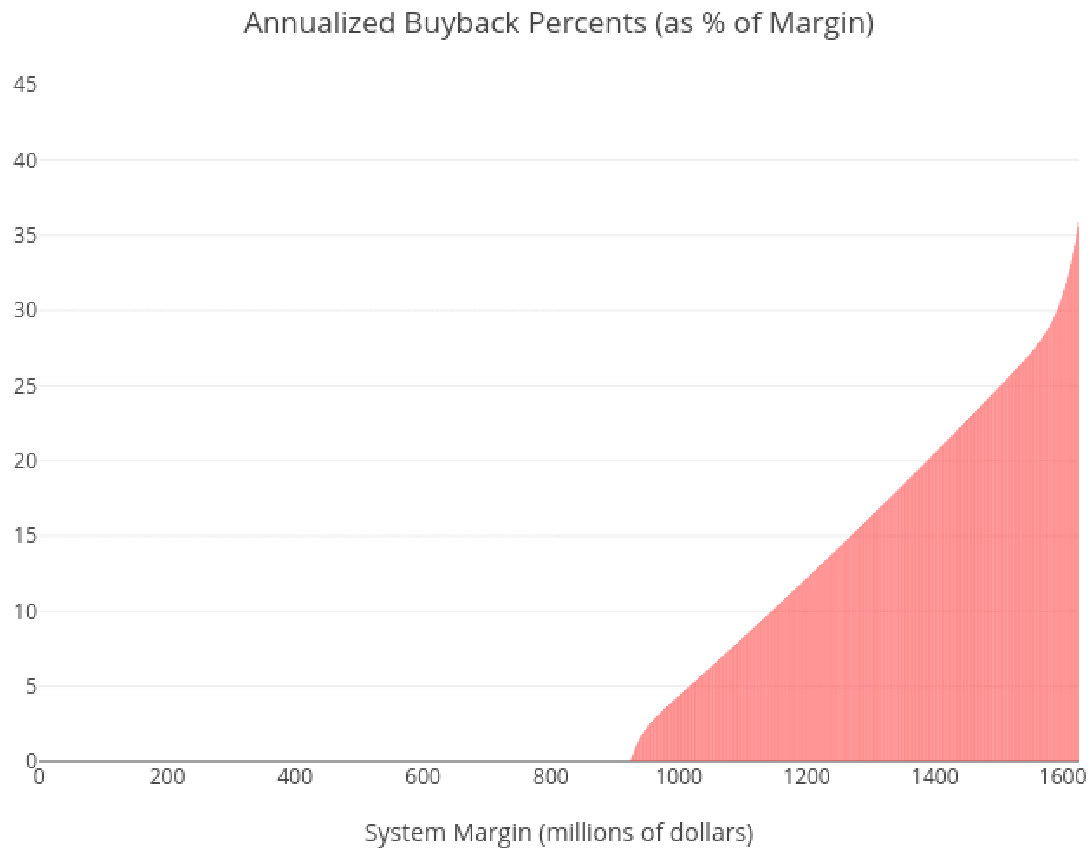

Annualized Buyback Percents (as % of Margin)
45
40
35
30
25
20
15
10
5
0
0
200
400
600
800
1000
1200
1400
1600
System Margin (millions of dollars)

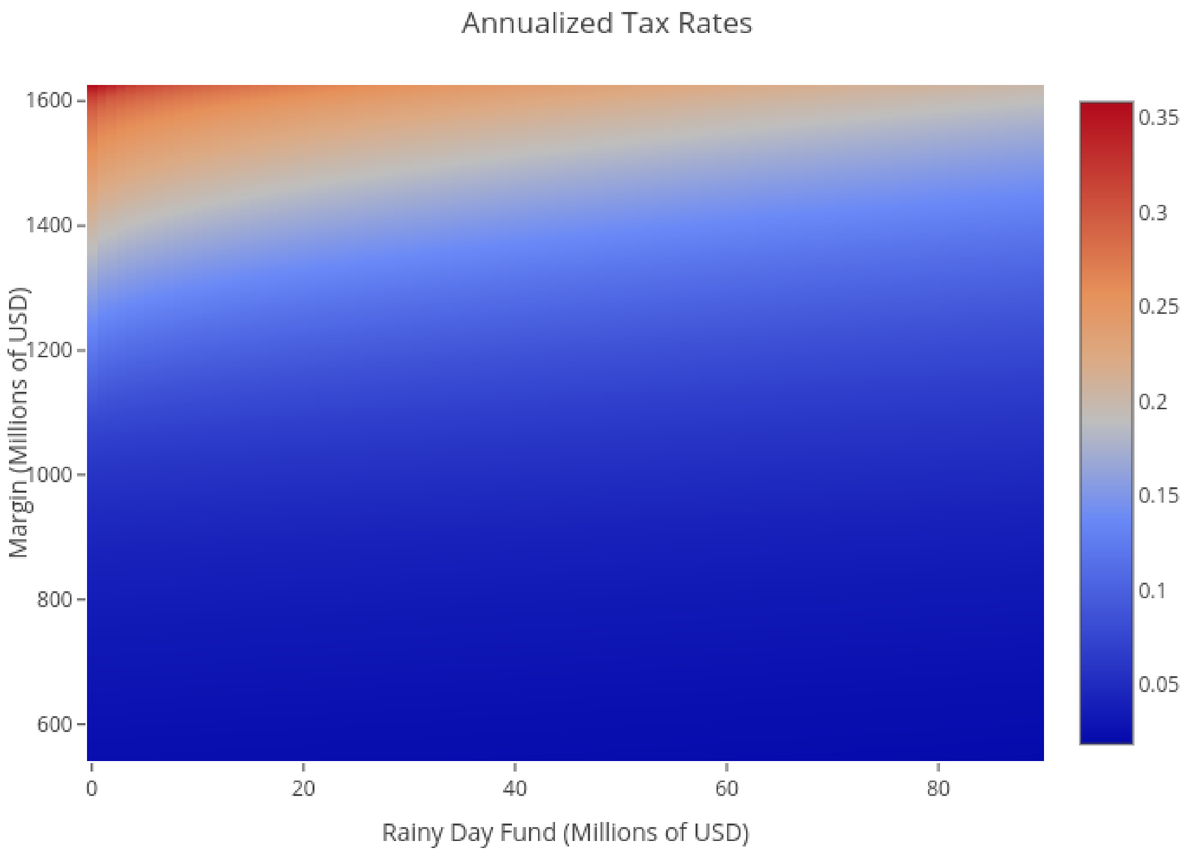

Annualized Tax Rates
Margin (Millions of USD)
1600
1400
1200
1000
800
600
0
20
40
60
80
Rainy Day Fund (Millions of USD)
0.3
0.25
0.2
0.15
0.1
0.05

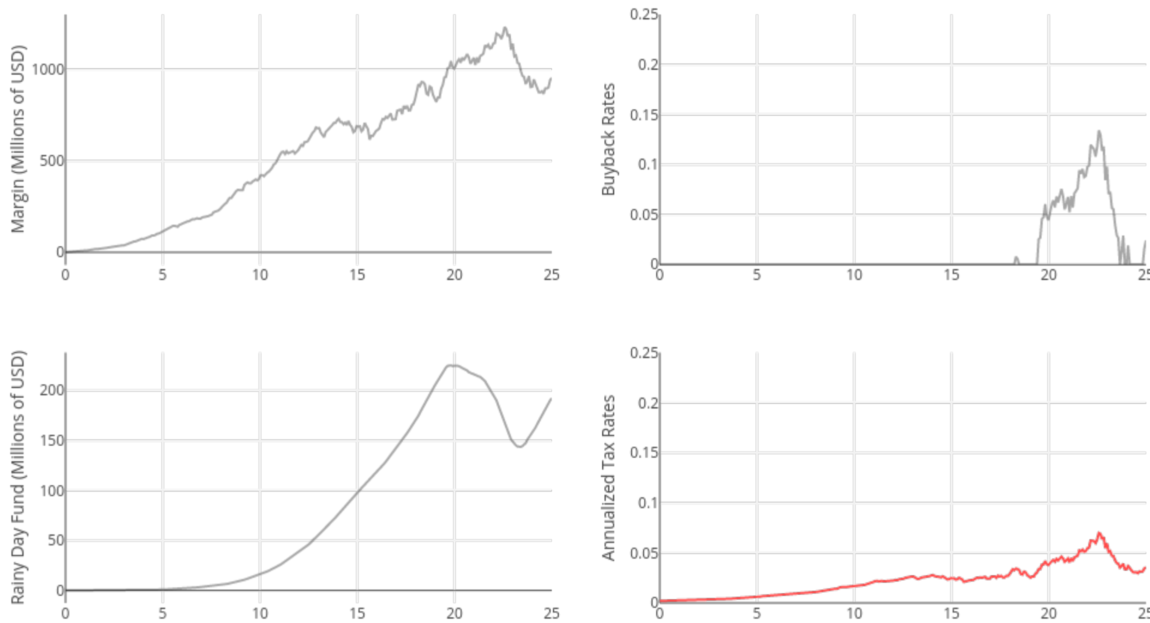

Margin (Millions of USD)
Buyback Rates
Rainy Day Fund (Millions of USD)
Annualized Tax Rates
0
500
1000
0.05
0.1
0.15
0.2
0.25
50
100
5
10
15
20
25

C.1.2 Graphs with $r = 0.05$ (annualized)

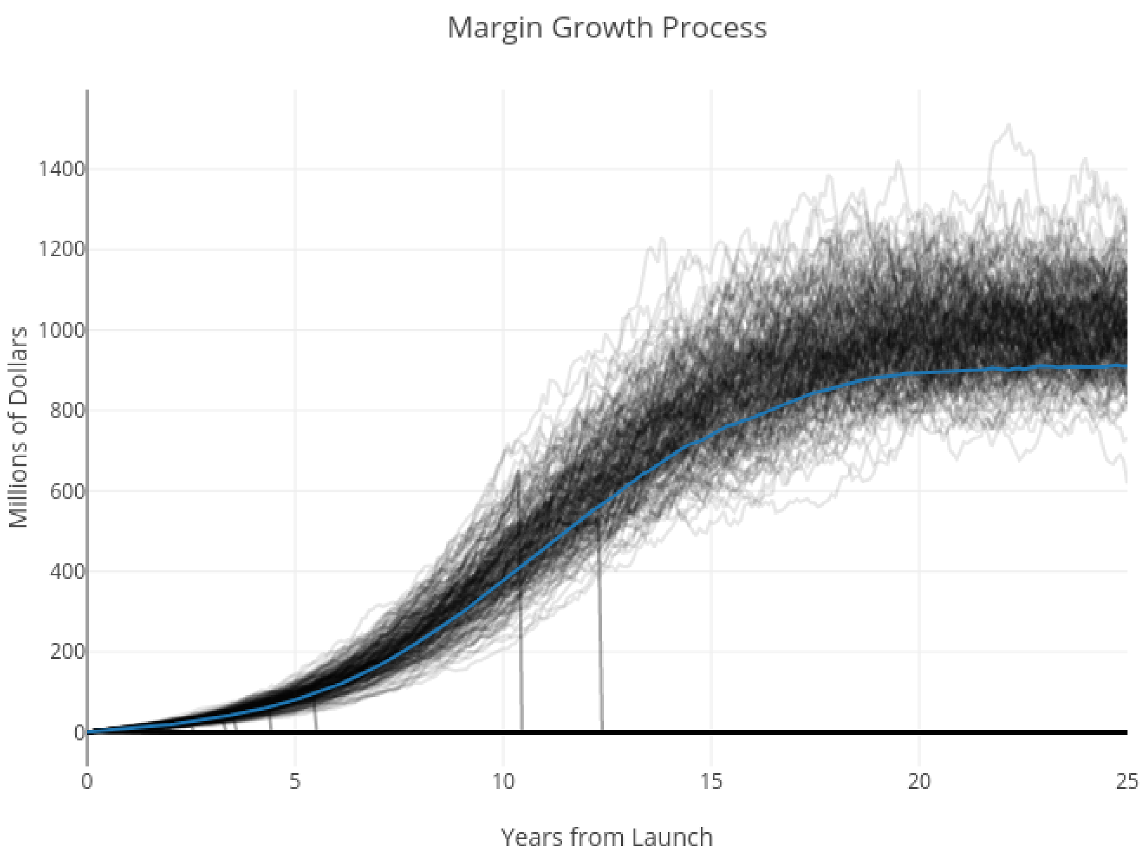


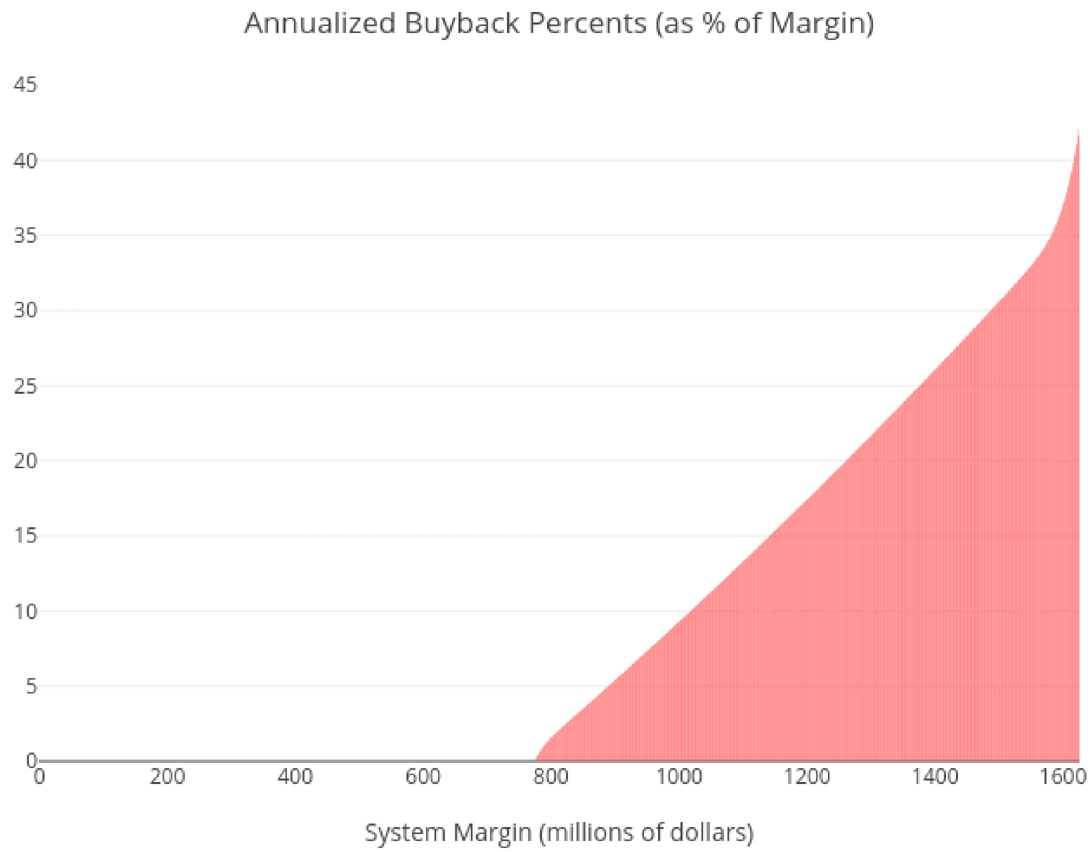

Annualized Buyback Percents (as % of Margin)
45
40
35
30
25
20
15
10
5
0
0
200
400
600
800
1000
1200
1400
1600
System Margin (millions of dollars)

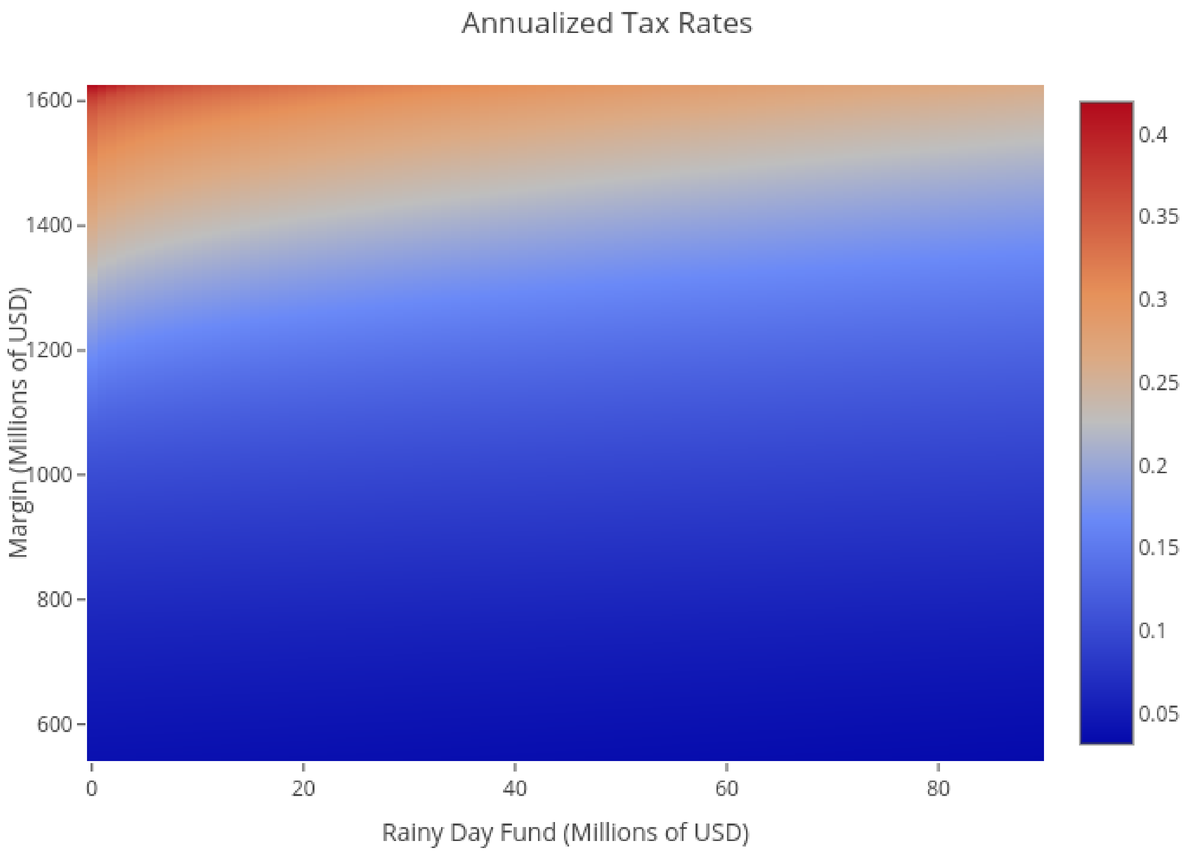

Annualized Tax Rates
Margin (Millions of USD)
1600
1400
1200
1000
800
600
0
20
40
60
80
Rainy Day Fund (Millions of USD)
0.35
0.3
0.25
0.2
0.15
0.1
0.05

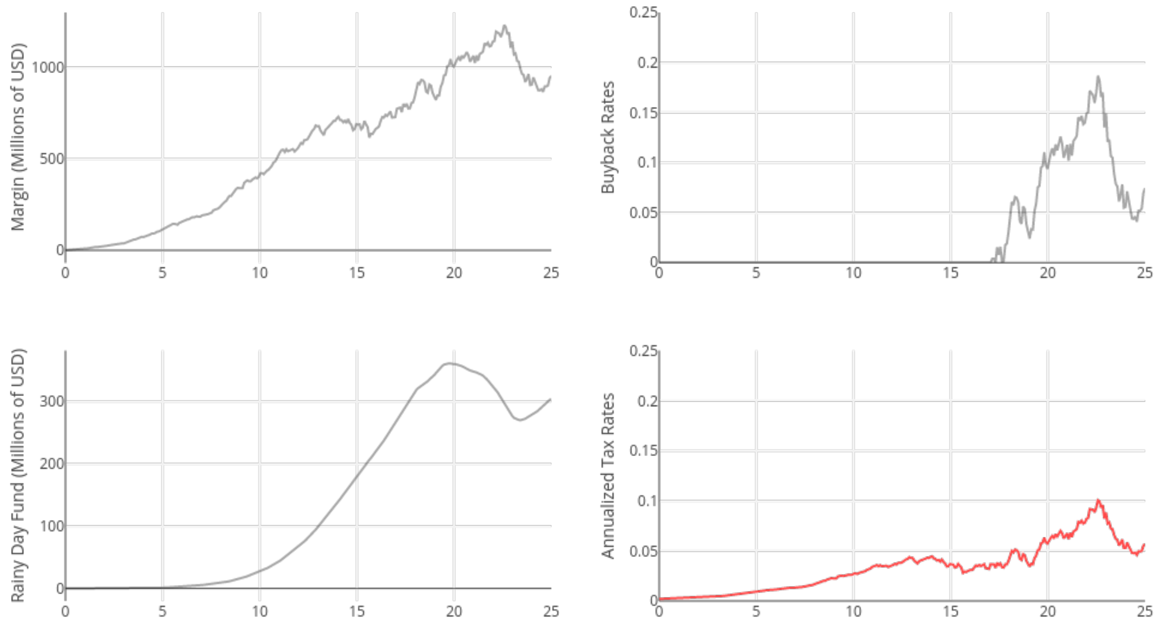

Margin (Millions of USD)
Buyback Rates
Rainy Day Fund (Millions of USD)
Annualized Tax Rates

C.1.3 Graphs with $r = 0.10$ (annualized)




Annualized Buyback Percents (as % of Margin)
45
40
35
30
25
20
15
10
5
0
0
200
400
600
800
1000
1200
1400
1600
System Margin (millions of dollars)

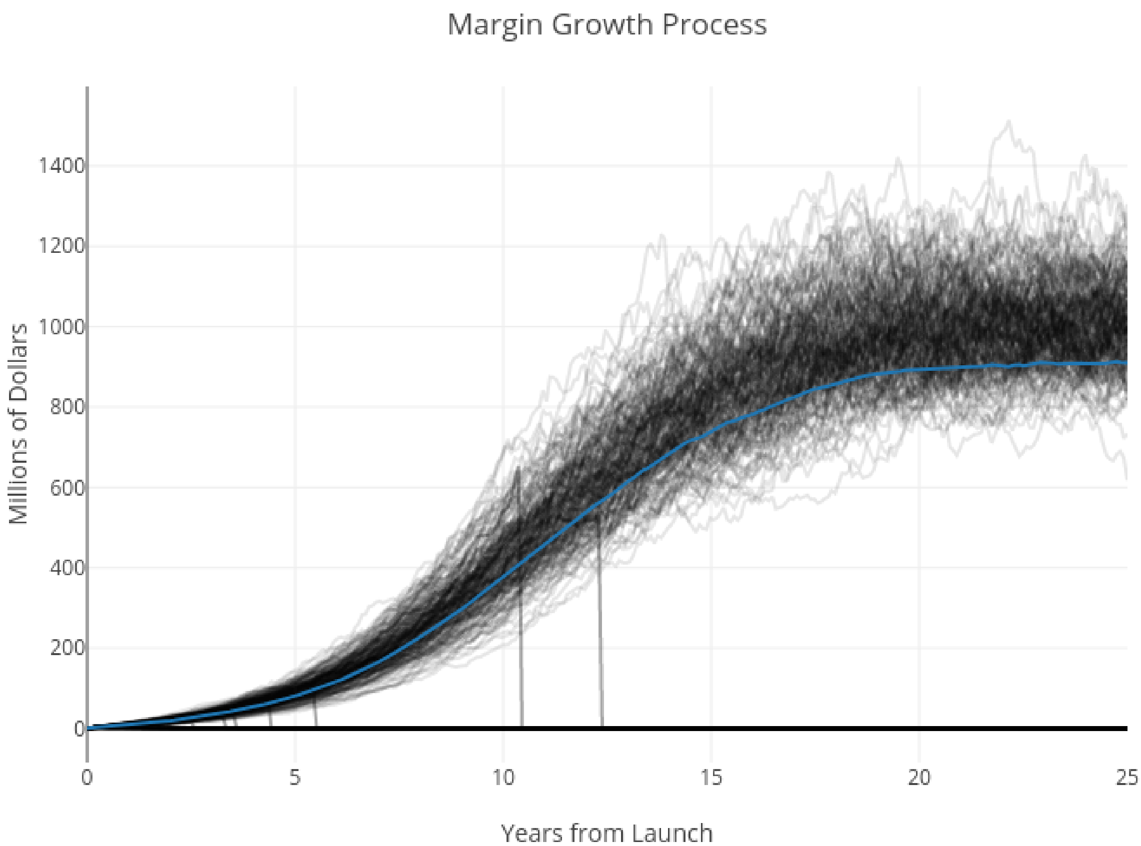

Annualized Tax Rates
Margin (Millions of USD)
1600
1400
1200
1000
800
600
0
20
40
60
80
Rainy Day Fund (Millions of USD)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05

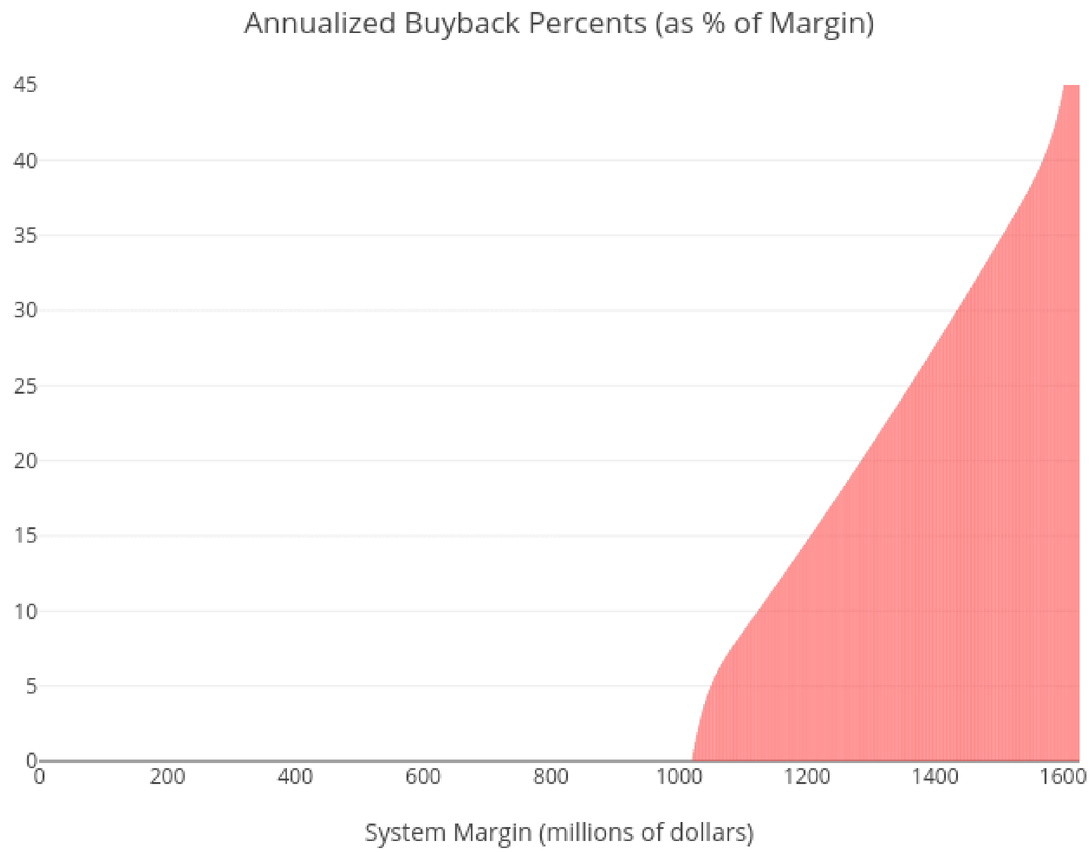

Margin (Millions of USD)
Buyback Rates
Rainy Day Fund (Millions of USD)
Annualized Tax Rates

C.2 Fluctuating γ

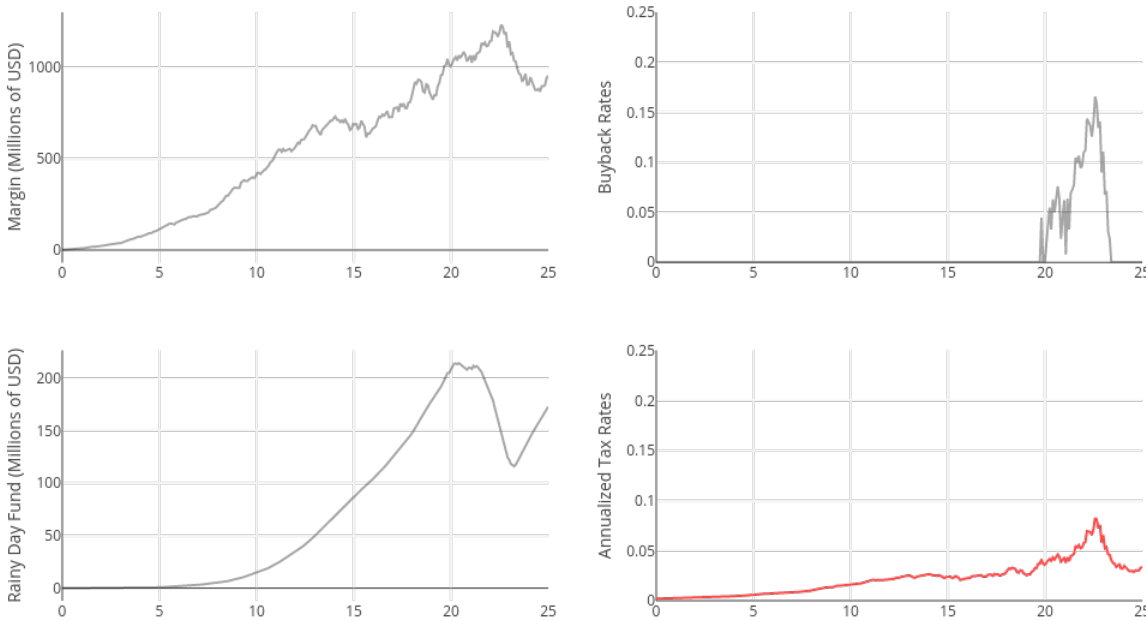
In this section, we consider two alternative values for the percent of margin that can be seized.

C.2.1 Graphs with $\gamma = 0.75$

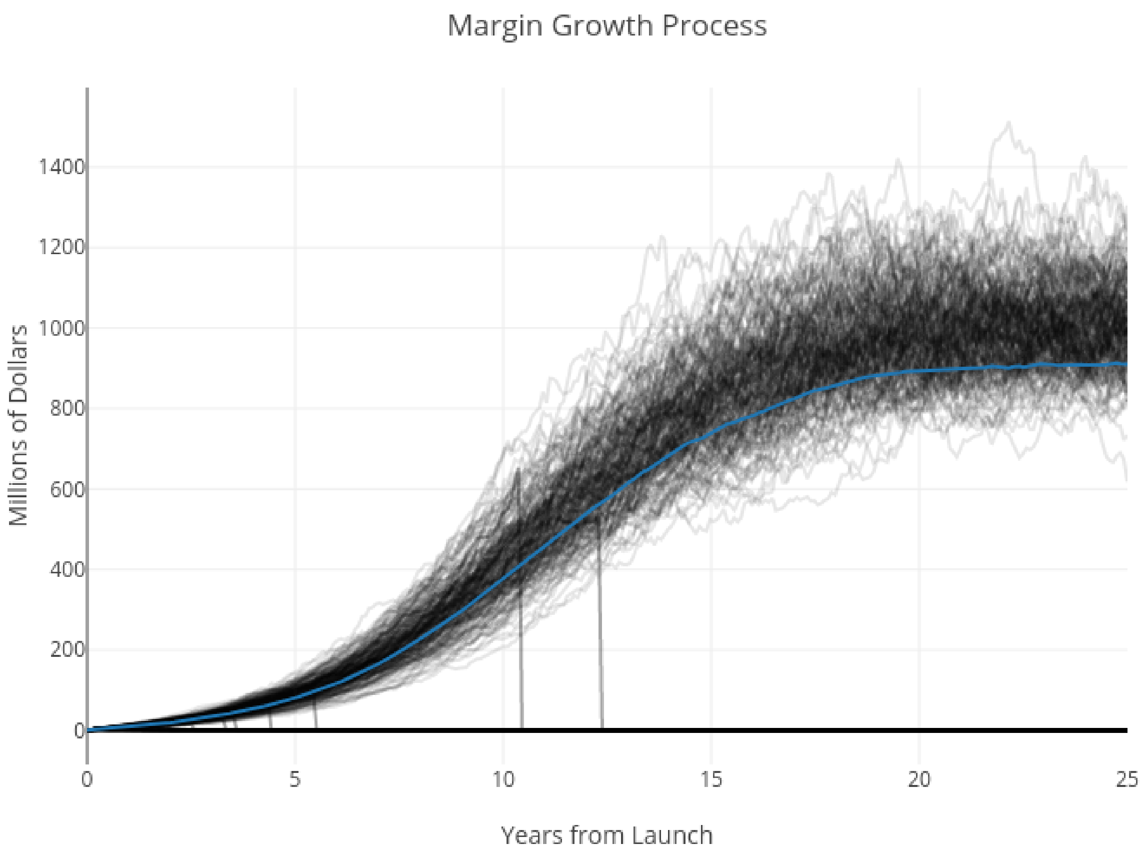


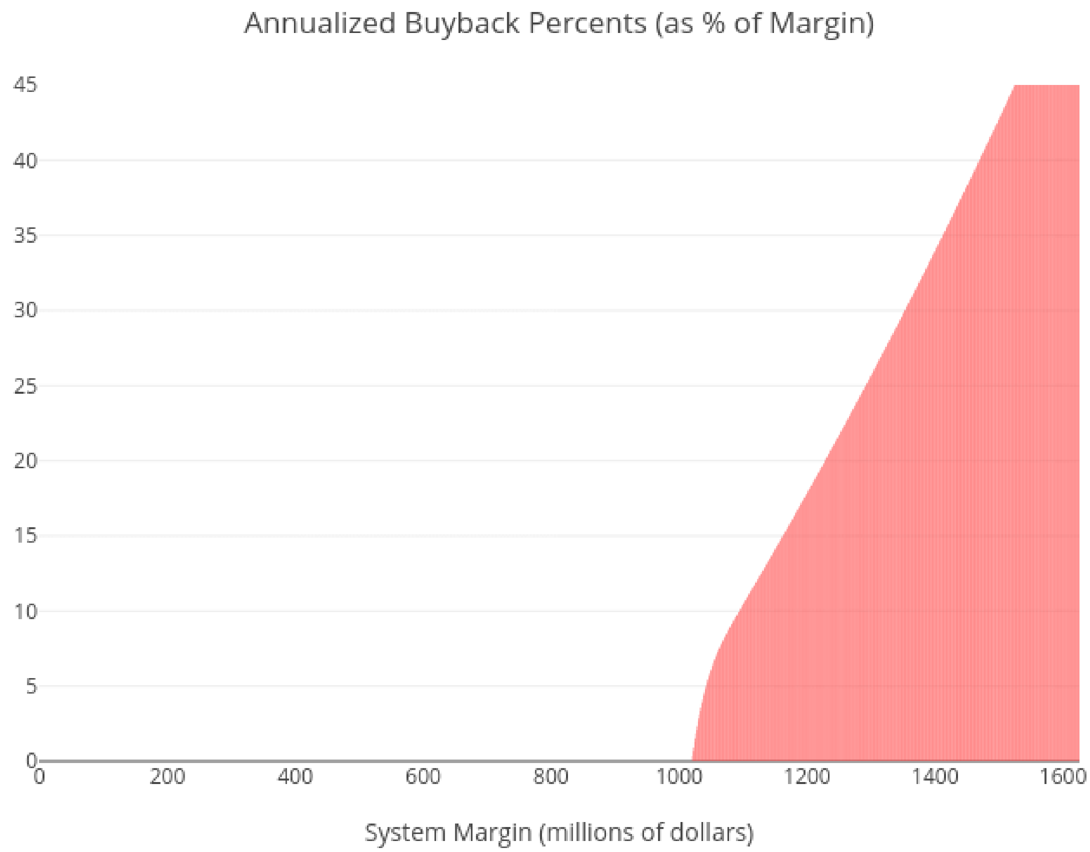

Annualized Buyback Percents (as % of Margin)
45
40
35
30
25
20
15
10
5
0
0
200
400
600
800
1000
1200
1400
1600
System Margin (millions of dollars)


Annualized Tax Rates
1600
1400
1200
1000
800
600
Margin (Millions of USD)
0
20
40
60
80
Rainy Day Fund (Millions of USD)
0.5
0.4
0.3
0.2
0.1

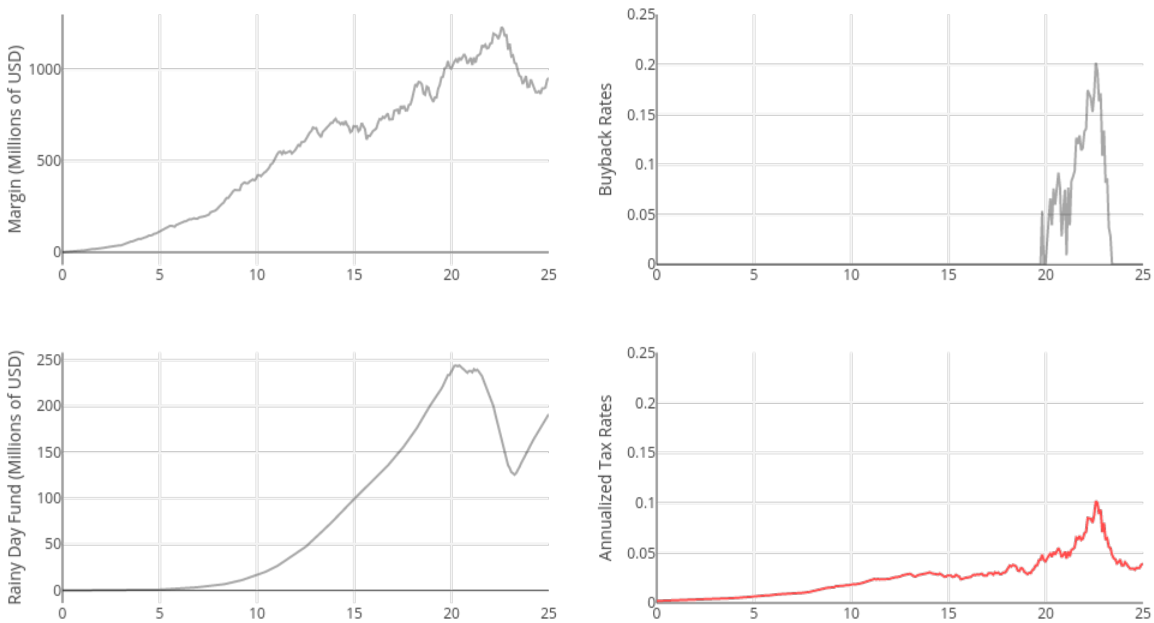

Margin (Millions of USD)
1000
500
0
0
5
10
15
20
25
Buyback Rates
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
25
Rainy Day Fund (Millions of USD)
200
150
100
50
0
0
5
10
15
20
25
Annualized Tax Rates
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
25

C.2.2 Graphs with $\gamma = 0.90$




Annualized Buyback Percents (as % of Margin)
45
40
35
30
25
20
15
10
5
0
0
200
400
600
800
1000
1200
1400
1600
System Margin (millions of dollars)


Annualized Tax Rates
1600
1400
1200
1000
800
600
Margin (Millions of USD)
0
20
40
60
80
Rainy Day Fund (Millions of USD)
0.6
0.5
0.4
0.3
0.2
0.1

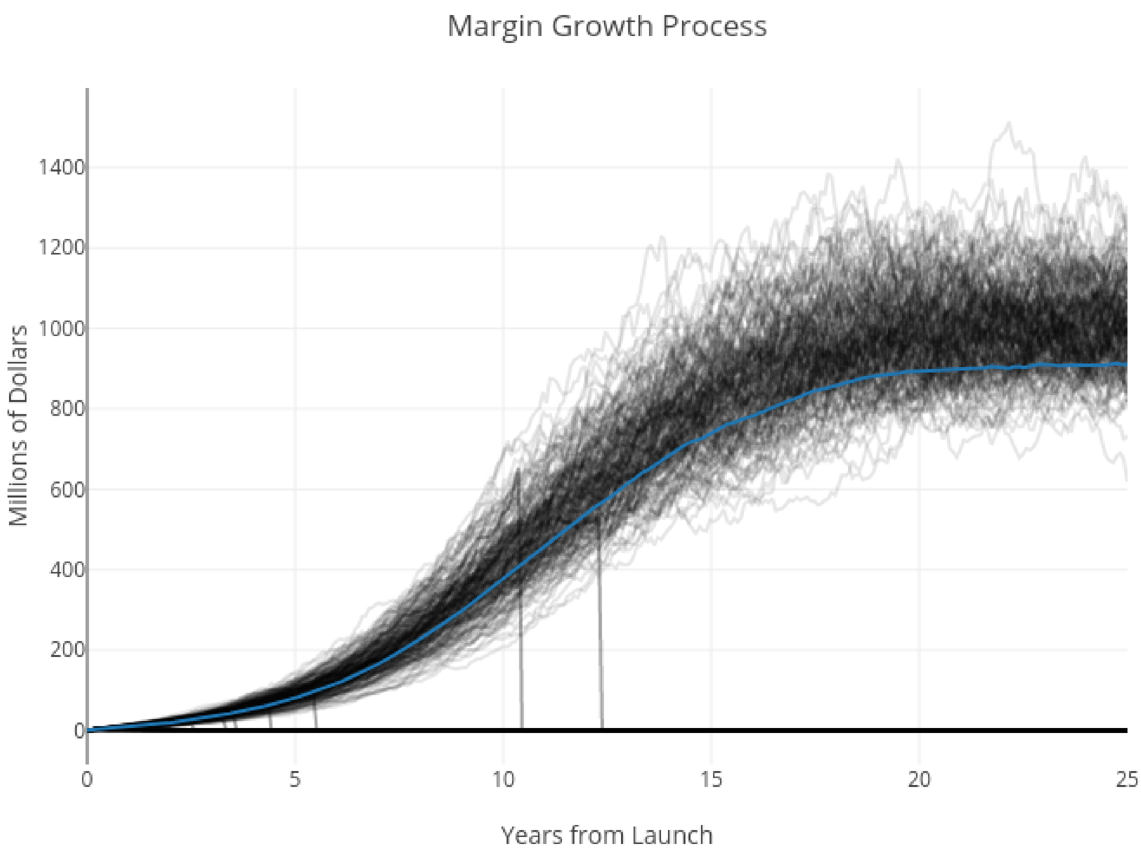


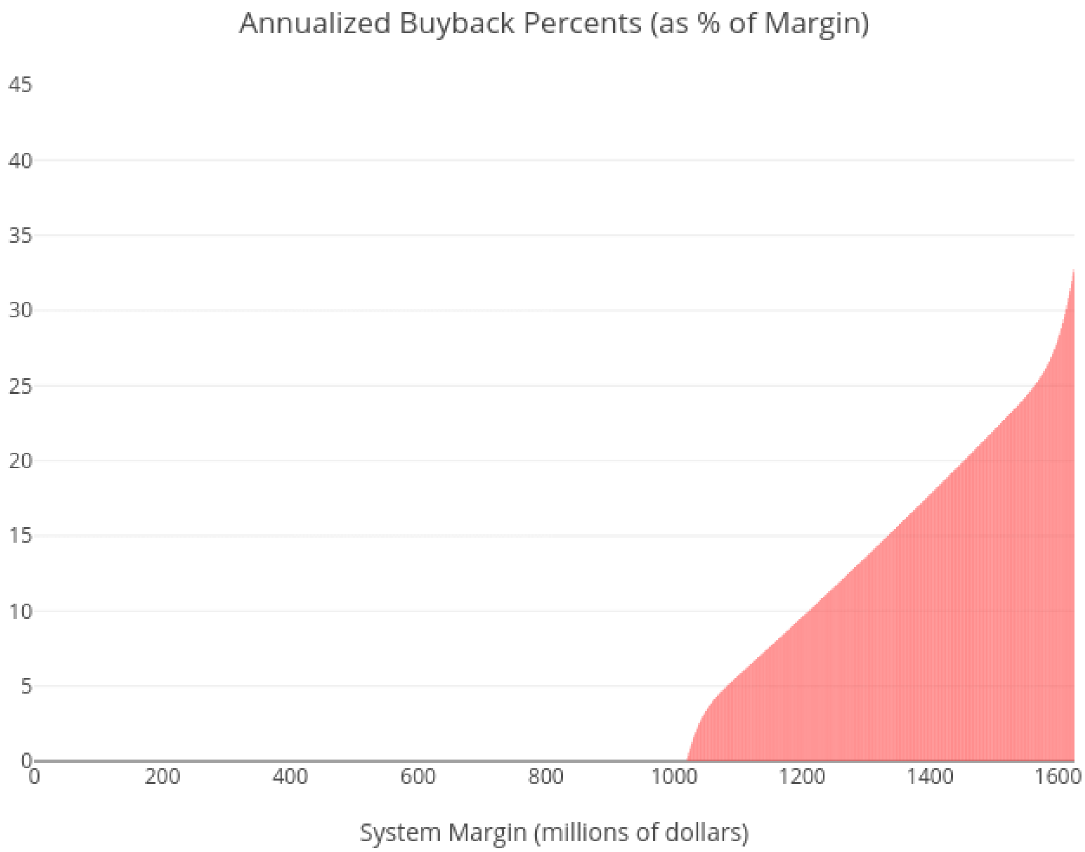
Margin (Millions of USD)
0
500
1000
Buyback Rates
0
0.05
0.1
0.15
0.2
0.25
Rainy Day Fund (Millions of USD)
0
50
100
150
200
250
Annualized Tax Rates
0
5
10
15
20
25

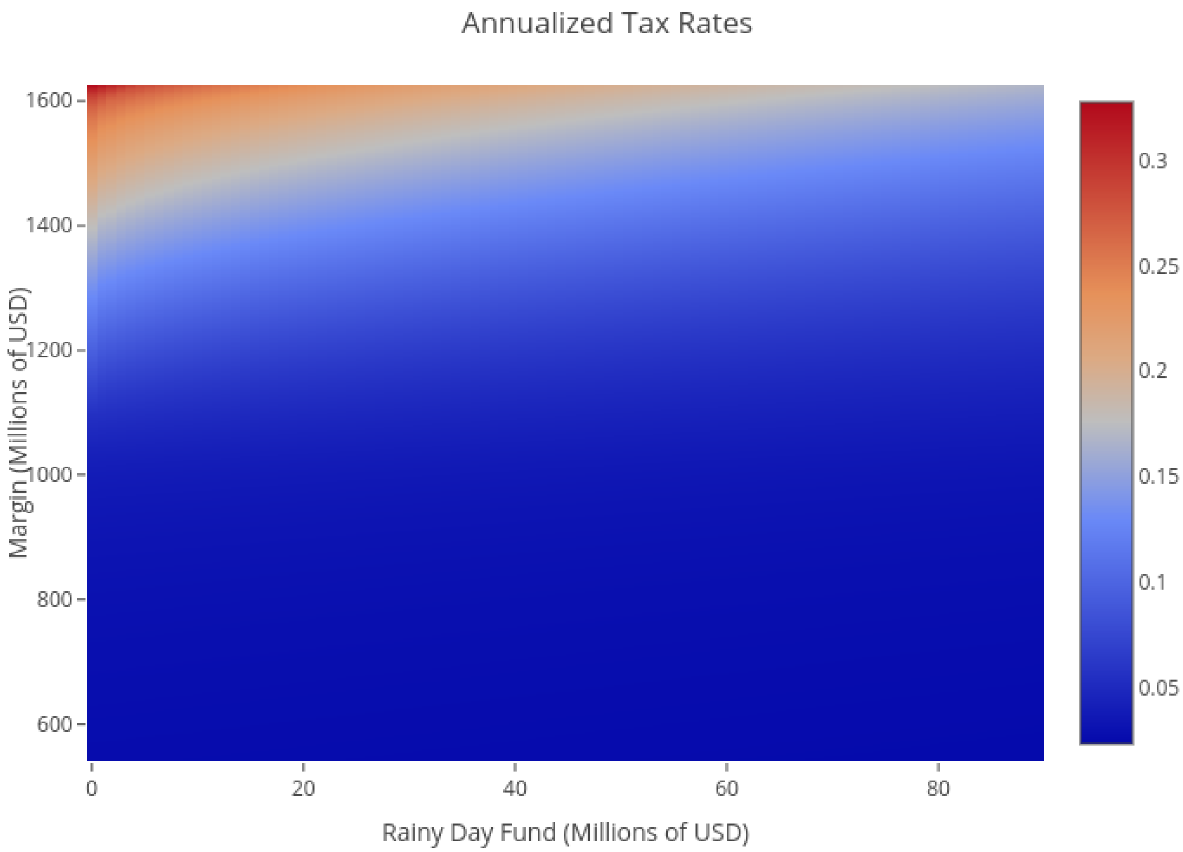
C.3 Fluctuating $\hat{\tau}$

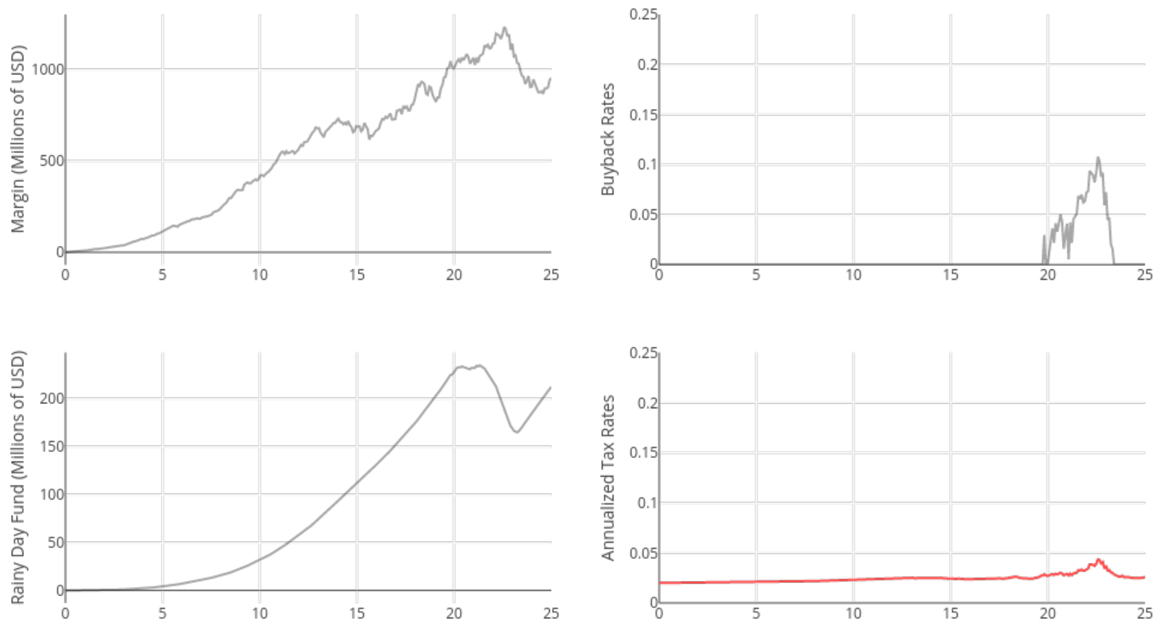
In this section, we consider three alternative values for the baseline fee amount

C.3.1 Graphs with $\hat{\tau} \approx 0.02$ (annualized)

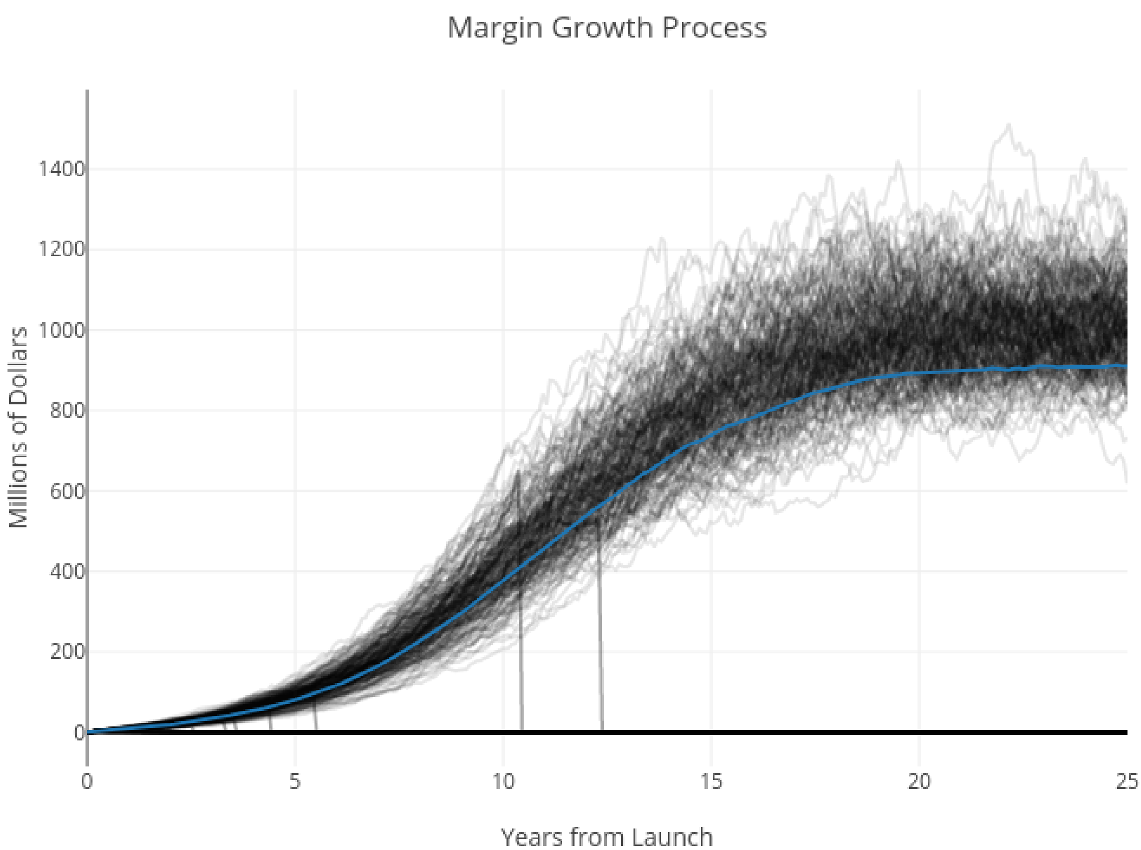


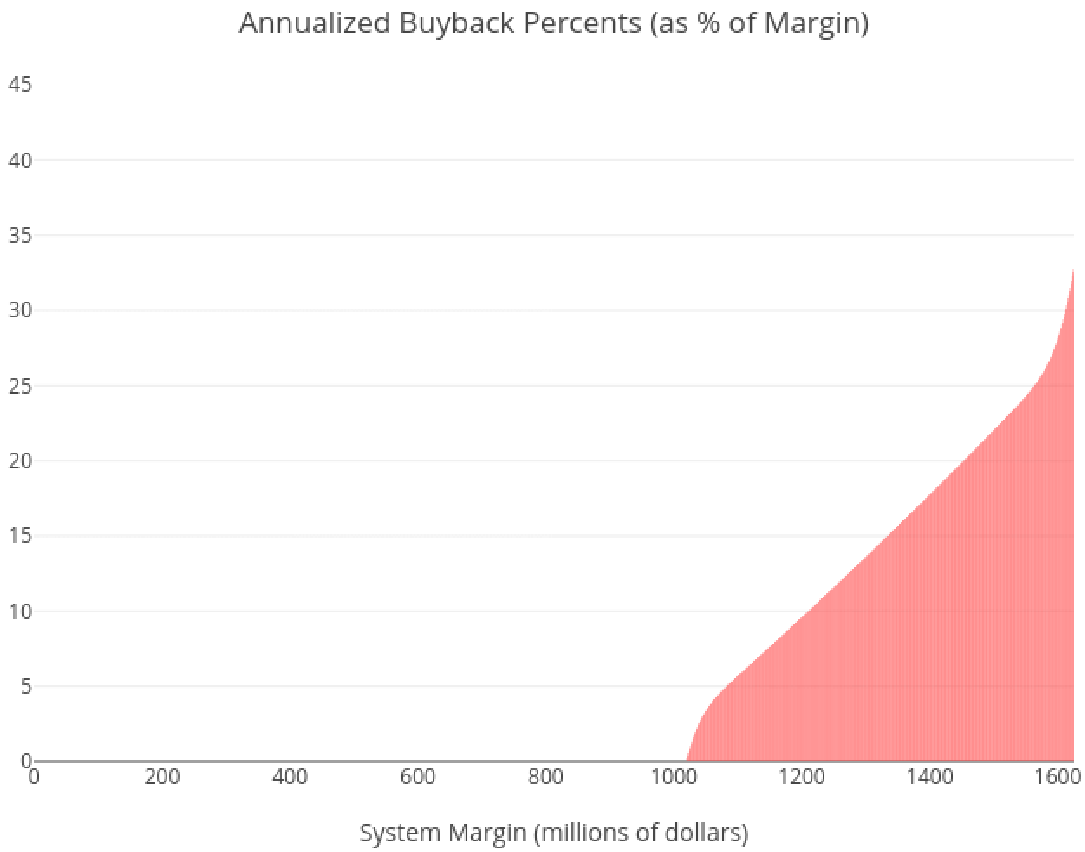

Annualized Buyback Percents (as % of Margin)
45
40
35
30
25
20
15
10
5
0
0
200
400
600
800
1000
1200
1400
1600
System Margin (millions of dollars)

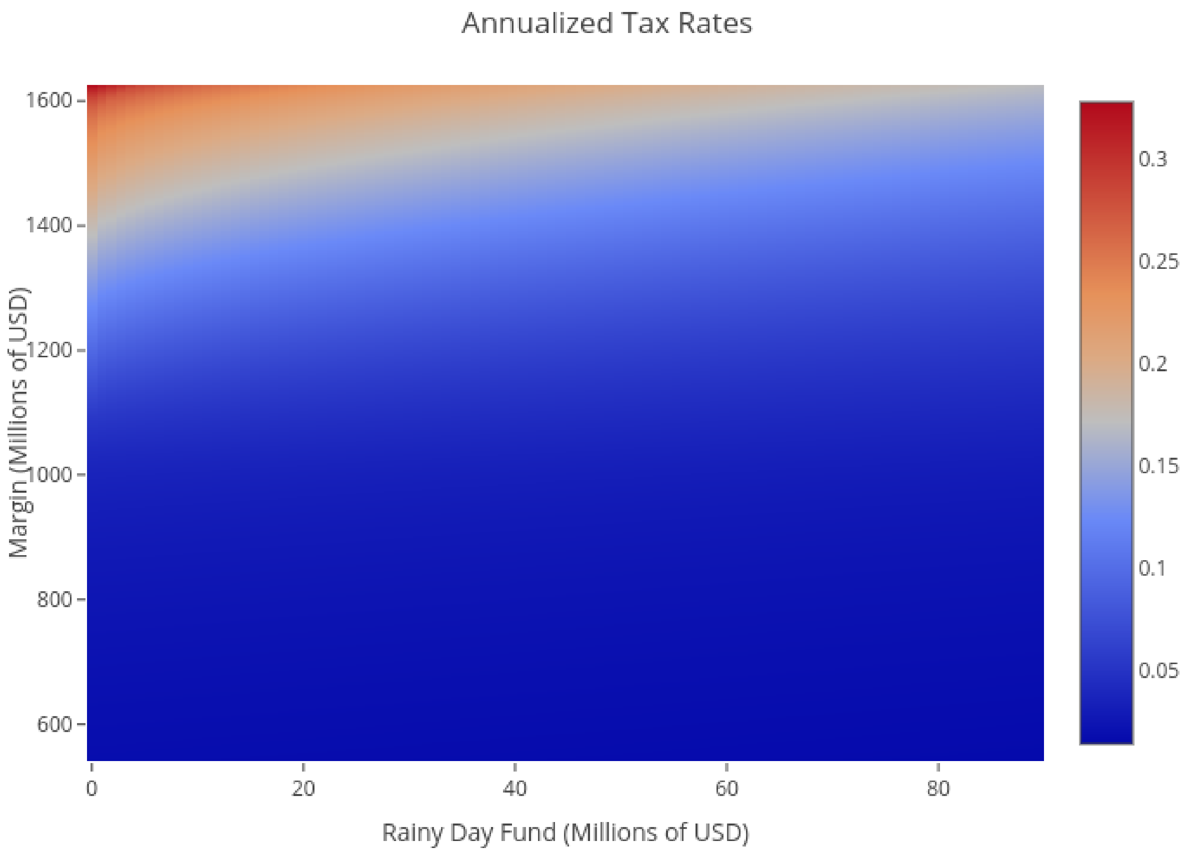

Annualized Tax Rates
Margin (Millions of USD)
1600
1400
1200
1000
800
600
Rainy Day Fund (Millions of USD)
0
20
40
60
80
0.3
0.25
0.2
0.15
0.1
0.05

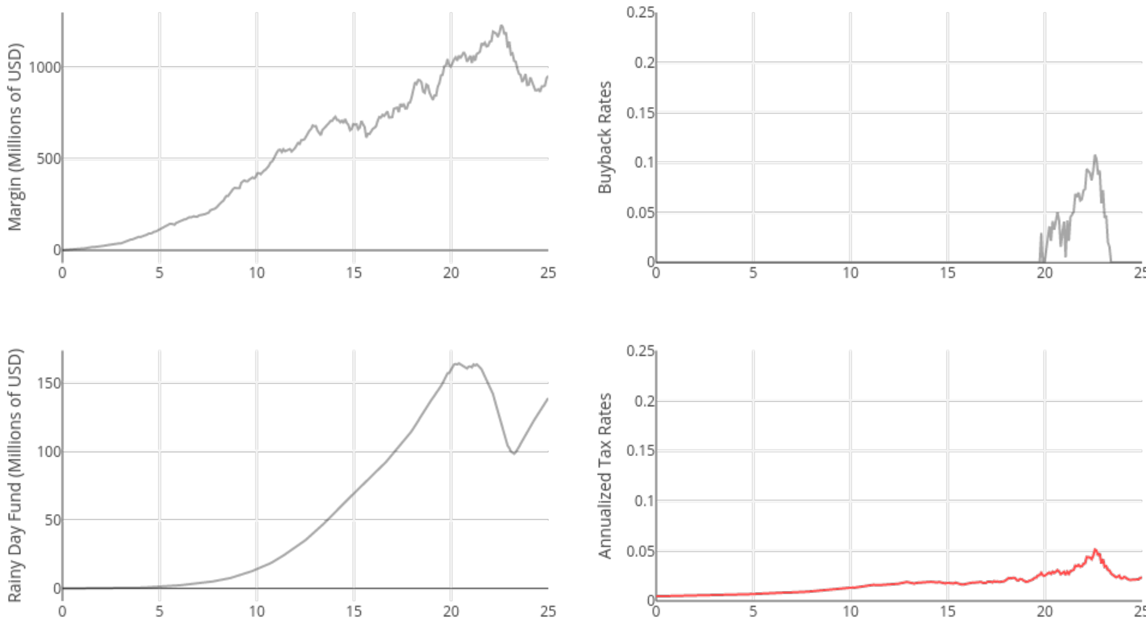

Margin (Millions of USD)
Buyback Rates
Rainy Day Fund (Millions of USD)
Annualized Tax Rates

C.3.2 Graphs with $\hat{\tau} \approx 0.004$ (annualized)

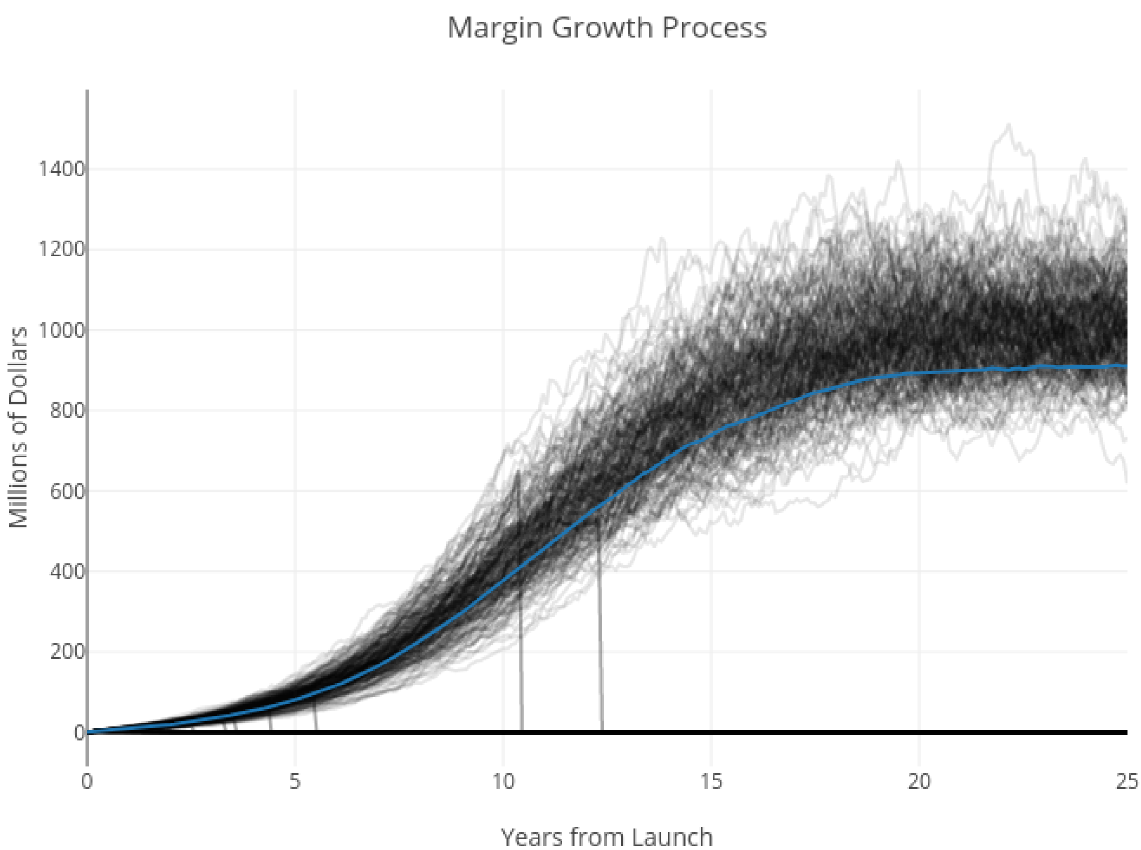


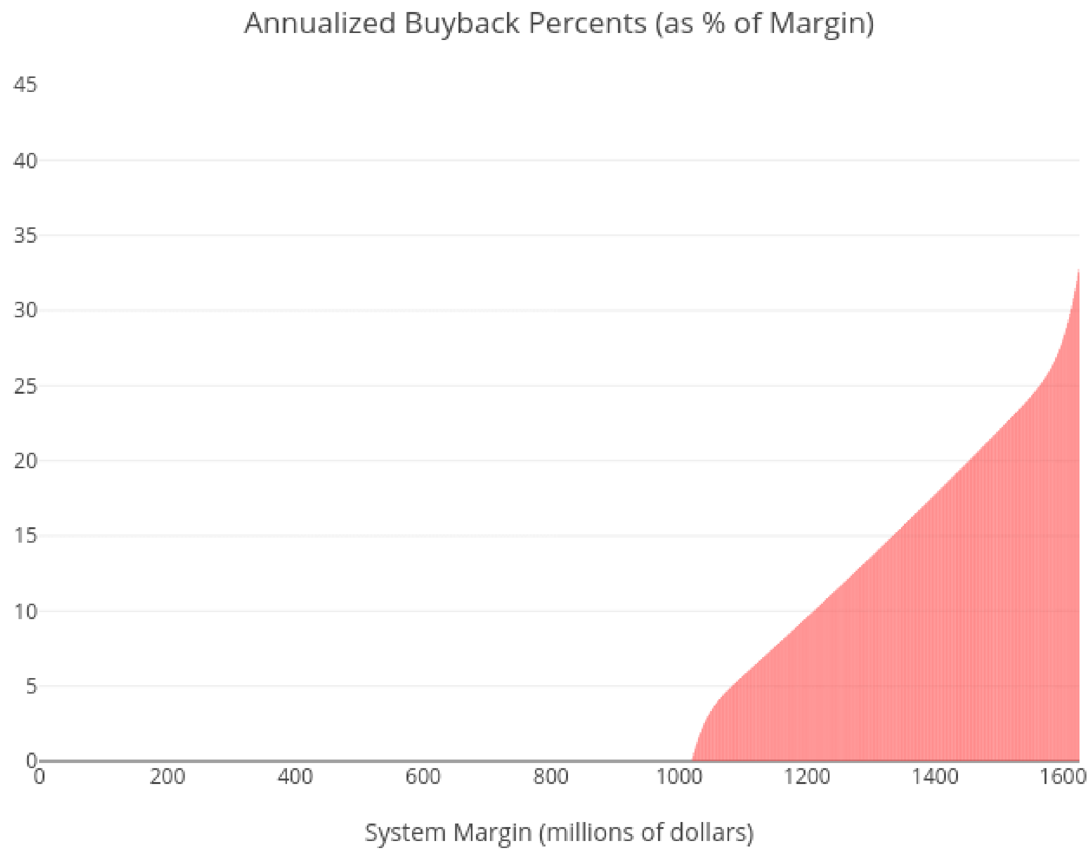

Annualized Buyback Percents (as % of Margin)
45
40
35
30
25
20
15
10
5
0
0
200
400
600
800
1000
1200
1400
1600
System Margin (millions of dollars)


Annualized Tax Rates
Margin (Millions of USD)
1600
1400
1200
1000
800
600
0
20
40
60
80
Rainy Day Fund (Millions of USD)
0.3
0.25
0.2
0.15
0.1
0.05

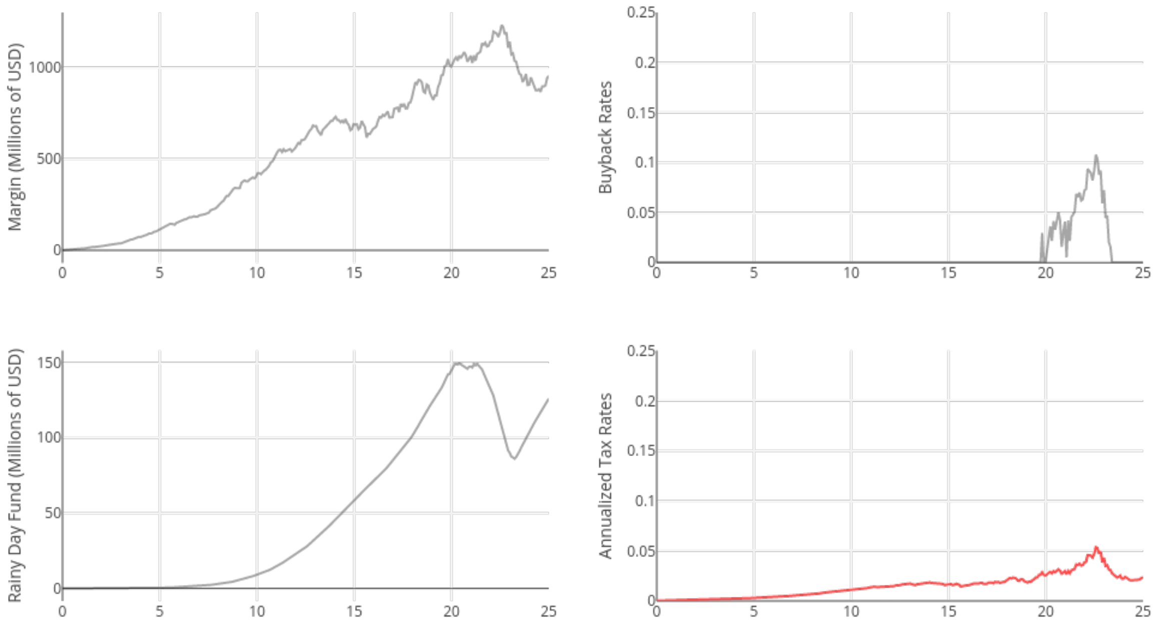

Margin (Millions of USD)
Buyback Rates
Rainy Day Fund (Millions of USD)
Annualized Tax Rates

C.3.3 Graphs with $\hat{\tau} = 0.0001$ (annualized)




Annualized Buyback Percents (as % of Margin)
45
40
35
30
25
20
15
10
5
0
0
200
400
600
800
1000
1200
1400
1600
System Margin (millions of dollars)


Annualized Tax Rates
Margin (Millions of USD)
1600
1400
1200
1000
800
600
0
20
40
60
80
Rainy Day Fund (Millions of USD)
0.3
0.25
0.2
0.15
0.1
0.05


Margin (Millions of USD)
Buyback Rates
Rainy Day Fund (Millions of USD)
Annualized Tax Rates
0
500
1000
50
100
150
0.05
0.1
0.15
0.2
0.25
5
10
15
20
25