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FRACTALITY AND SELF-ORGANIZED CRITICALITY OF WARS

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Abstract

This paper considers the frequency-size statistics of wars. Using several alternative measures of the intensity of a war in terms of battle deaths, we find a fractal (power-law) dependence of number on intensity. We show that the frequency-size dependence of forest fires is essentially identical to that of wars. The forest-fire model provides a basis for understanding the distribution of forest fires in terms of self-organized criticality. We extend the analogy to wars in terms of the initial ignition (outbreak of war) and its spread to a group of metastable countries.

1. INTRODUCTION

Since the fractal concept was introduced by Mandlebrot[1] it has found a broad range of applications. In this paper we consider its applicability to the statistical distribution of the intensities of wars. We will show, using several measures of the intensities of wars, that the power-law (fractal) distribution appears to be applicable. This builds upon the original statistical study of the intensities of wars carried out by Richardson.[2] We will also provide a rationale for the applicability of fractal statistics in terms of the forest-fire model and actual forest fires. We suggest that wars are an example of self-organized criticality.

One measure of fractality is a power-law dependence of the cumulative number of events with a linear dimension greater than r, N_C, on r

$$N_C \sim r^{-D} \tag{1}$$

where D is the fractal dimension. Examples include the number of earthquakes, the number of fragments and the number of lakes with a size greater than a prescribed value.[3]

2. INTENSITY OF WARS

An obvious measure of the intensity of a war I is the number of battle deaths. The frequency-size distribution of war intensities is then simply the dependence of the number of wars N on the number of battle deaths. Richardson[2] was the first to carry out this type of study using logarithmic binning. He considered 82 wars between 1820 and 1929 and found that $N = 1$ war had $\log I = 7 \pm 0.5$ (i.e. between 3 010 000 and 30 100 000 battle deaths; $N = 3$ wars with $\log I = 6 \pm 0.5$. (i.e. between 301 000 and 3 010 000 battle deaths); $N = 16$ wars with $\log I = 5 \pm 0.5$. (i.e. between 30 100 and 301 000 000 battle deaths); and $N = 62$ wars with $\log I = 4 \pm 0.5$. (i.e. between 3010 and 30 100 battle deaths). Richardson[2] pointed out that his statistical data correlated well with the relation

$$N = CI^{-D} \tag{2}$$

taking $D = 1$. Richardson[4] extended and updated his studies in his book "The Statistics of Deadly Quarrels." This study considered 105 wars between 1820 and 1949 and found that $N = 2$ with $\log I = 7 \pm 0.5$, $N = 7$ with $\log I = 6 \pm 0.5$, $N = 26$ with $\log I = 5 \pm 0.5$, and $N = 70$ with $\log I = 4 \pm 0.5$. Again there is a good correlation with Eq. (2) taking $D = 1$.

One of the major criticisms of the use of the number of battle deaths as a measure of a war's intensity is the substantial change in the global population over the period of time considered. A more logical measure would be the ratio of battle deaths to the world's population prior to the war. However, for the earlier wars, estimates of the world's population are unreliable. For this reason Levy[5] defines the intensity of a war I as the ratio of battle deaths to the population of Europe in millions at the time of the war.

Levy[5] has tabulated the intensities of 119 wars, beginning with the war of the League of Venice in 1495–1497 and ending with the Vietnam War in 1965–1973. The largest wars were the Second World War with $I = 93\,665$ and the First World War with $I = 57\,616$. When considering data of this type, one approach is to consider cumulative distributions. The number of wars with intensities greater than I, N_C, is plotted against I. However, when considering distributions that may exhibit self-organized criticality it is preferable to consider noncumulative data. One approach is to use "binned" data as described above. An alternative, but equivalent approach, is to take the derivative of the cumulative distribution with respect to intensity dN_C/dI. The derivative is obtained by taking the mean slope of a specified number of adjacent data points, in our case five. The dependence of dN_C/dI on I for the Levy[5] distribution of war intensities is given in Fig. 1. If a fractal (power-law) distribution is applicable we

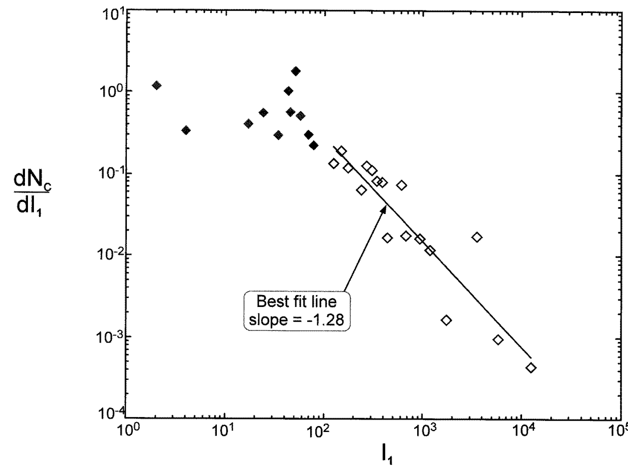

Fig. 1 Noncumulative frequency-intensity distribution of wars based on the Levy (1983) tabulation of war intensities. The noncumulative number of wars, $-dN_C/dI$, is given as a function of I. The larger wars correlate well with the fractal relation [Eq. (3)] taking $D = 1.27$.

would expect a good correlation with the relation

$$\frac{dN_C}{dI} = CI^{-D} \tag{3}$$

This correlation is illustrated in Fig. 1 taking $D = 1.27$. This is the best fit result for wars with intensities greater than 100. The fit is seen to be quite good for war intensities greater than about $I = 30$ and extends over about three orders of magnitude of data. The deviation for small wars may be real or may be due to the incompleteness of the data set.

An alternative approach to the analysis of this data is in terms of return periods T. In order to analyze this data set we order the wars from the largest (largest value of I) to the smallest (smallest value of I). The largest war is assigned a return period equal to the length of the record T_0, the second largest war has a return period $T_0/2$, the third largest has a return period $T_0/3$, and so forth. For our data set $T_0 = 1973–1495 = 478$ yrs. The dependence of the war intensity I on return period T is given in Fig. 2. One formulation of a fractal relation is to relate the war intensity I to the return period T by the relation

$$I = C_1 T^{H_a} \tag{4}$$

where H_a is the Hausdorf exponent. The straight-line correlation in Fig. 2 is with $H_a = 1.54$. The correlation with this fractal (power-law) relation is quite good for return periods between 8 and 500

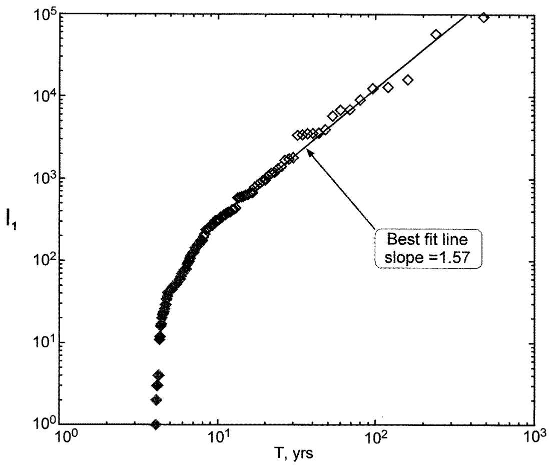

Fig. 2 Dependence of the war intensity I, as defined by Levy (1983), on the return period T. The larger wars correlate well with the fractal relation [Eq. (4)] taking $H_a = 1.54$.

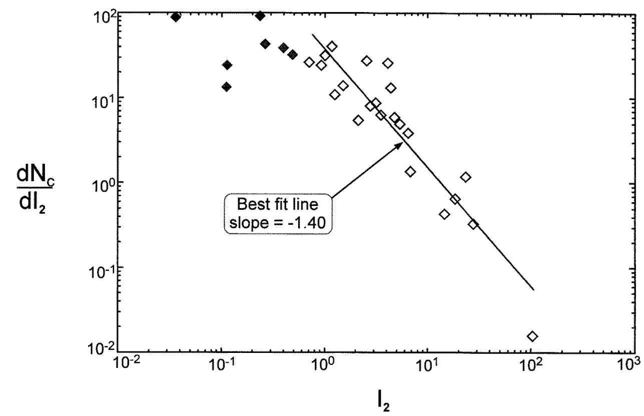

Fig. 3 Noncumulative frequency-intensity distribution of wars based on the Small and Singer (1982) tabulation of war intensities. The noncumulative number of wars, dN_C/dI_2, is given as a function of I_2. The larger wars correlate well with the fractal relation [Eq. (3)] taking $D = 1.40$.

years. Accepting the validity of the fractal relation, it is seen that the 1000-year war has an intensity of 420 000; the 100-year war an intensity of 12 000; and the ten-year war an intensity of 350.

It should be noted that the correlation given in Fig. 2 using Eq. (4) is based on cumulative statistics whereas the correlation given in Fig. 1 using Eq. (3) is based on noncumulative statistics.

An alternative definition of the intensity of a war was introduced by Small and Singer.[6] Their definition I_2 is the ratio of battle deaths to the population of the warring states in units of 10 000 people at the time of the war. These authors considered 118 wars during the period 1816–1980. Using this measure the intensity of the First World War was $I_2 = 141.5$ and the Second World War was $I_2 = 106.3$. However, the greatest value of this intensity measure was the Chaco War between Paraguay and Bolivia during 1932–1935, with $I_2 = 382.4$. It is clear that the measures of a war's intensity used by Levy[5] and by Small and Singer[6] are quite different.

We again obtain the noncumulative dependence of dN_c/dI_2 on I_2, and the result is given in Fig. 3. Using data with $I_2 > 0.7$, the best fit to the fractal relation [Eq. (3)] is found by taking $D = 1.40$. Again, good agreement is obtained for the larger wars. We also apply the alternative analysis of this data set using return periods. The result is given in Fig. 4. The straight-line correlation with the data is obtained from Eq. (4), taking $H_a = 1.39$. The results for the two definitions of war intensity are quite similar and are also reasonably close to the original result of Richardson.[2]

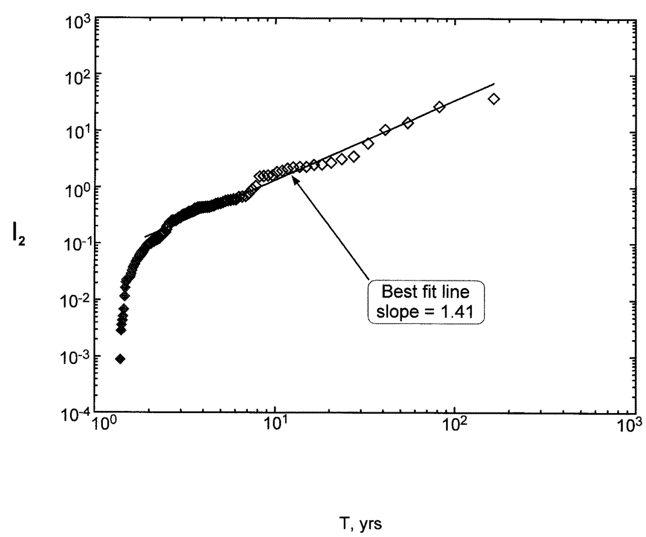

Fig. 4 Dependence of the war intensity I_2, defined by Small and Singer (1982), on the return period T. The larger wars correlate well with the fractal relation [Eq. (4)] taking $H_a = 1.39$.

Although it is certainly of interest that war intensities obey power-law (fractal) statistics, a more fundamental question is why? To address this question we consider self-organized critical behavior in general, and forest fires in particular.

3. SELF-ORGANIZED CRITICALITY

In the past ten years a variety of numerical models have been found to exhibit a universal behavior that has been called self-organized criticality.[7] In self-organized criticality the "input" to a complex system is steady; whereas the output is a series of events or "avalanches" that follow a power-law (fractal) frequency-size distribution. The concept of self-organized criticality has been primarily discussed in terms of three models; the "forest-fire" model, the "sandpile" model, and the "slider-block" model. There is evolving evidence that many natural phenomena, including such hazards as earthquakes, forest fires, and landslides, may also be examples of self-organized criticality.

Although the forest-fire model[8] was not the first model to be associated with self-organized critical behavior, it is probably the most illustrative. The forest-fire model we consider consists of a square grid of sites. At each time step, a model tree is dropped on a randomly chosen site; if the site is unoccupied, the tree is planted, or a match is dropped on a site. The sparking frequency, f_s, is the inverse number of attempted tree drops on the square grid before a model match is dropped. If $f_s = 1/100$, there have been 99 attempts to plant trees (some successful, some unsuccessful) before a match is dropped at the 100th time step. If the match is dropped on an empty site, nothing happens. If it is dropped on a tree, the tree ignites and a model fire consumes that tree and the adjacent trees.

Having specified the size of the square grid, N_g, and the sparking frequency, f_s, a simulation is run for N_S time steps and the number of fires N_F with area A_F is determined. The area, A_F, is the number of trees that burn in a fire. Examples of four typical model fires are given in Fig. 5. In these

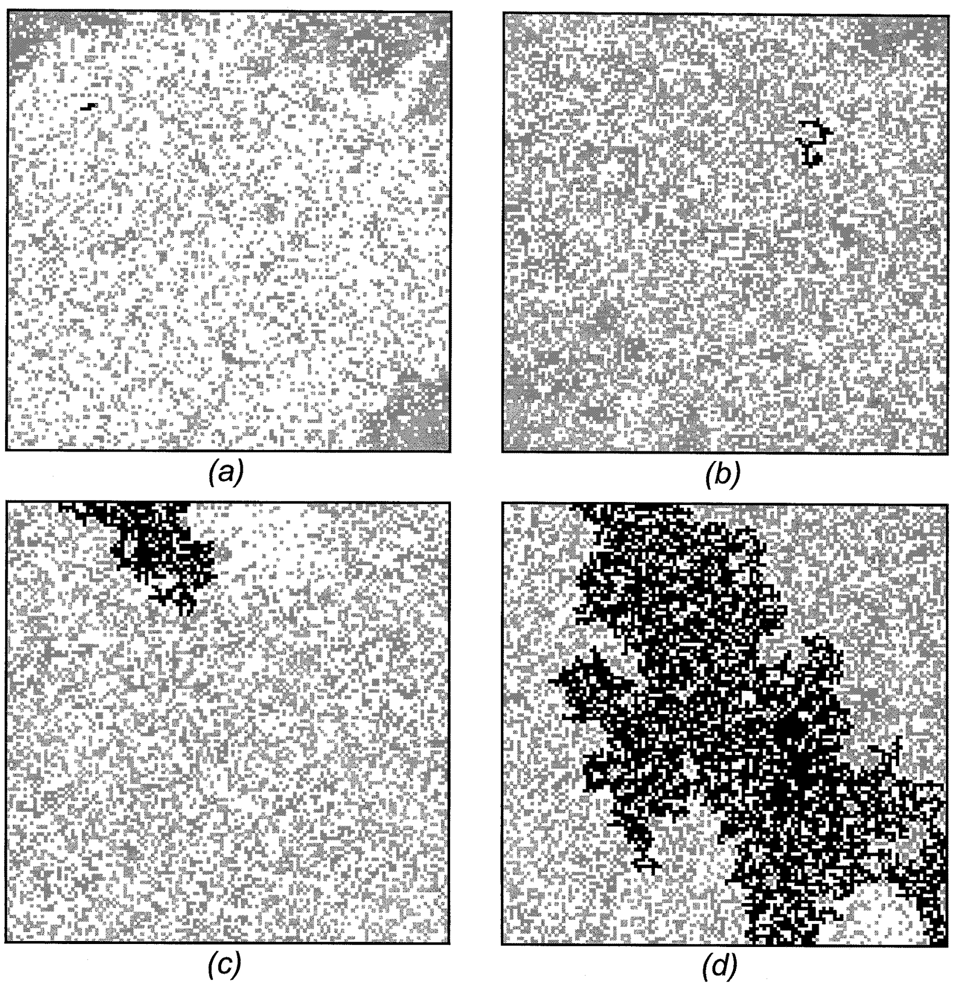

Fig. 5 Four examples of typical model forest fires are given. This run was carried out on a 128×128 grid with $f_s = 1/2000$. The heavily shaded regions are the forest fires. The lightly shaded regions are unburned forest. The white regions are unoccupied sites. The areas A_F of the four forest fires are (a) 5, (b) 51, (c) 505 and (d) 5327 trees. The largest forest fire is seen to span the entire grid.

examples, the grid size is 128×128 ($N_g = 16\,384$), $1/f_s = 2000$, and fires with $A_F = 5\,51\,506$, and 5327 trees are illustrated. Figure 5(d) is an example of a special class of forest fires which span the grid.

Noncumulative frequency-size statistics for the model forest fires are given in Fig. 6. The number of fires per time step with area A_F, N_F/N_S, is given as a function of A_F. Results are given for a grid size 128×128 and three sparking frequencies, $1/f_s = 125\,500$, and 2000. In all cases the smaller fires correlate well with the power-law (fractal) relation

$$\frac{N_F}{N_S} \sim A_F^{-\alpha} \tag{5}$$

with $\alpha \approx 1$. Since $A_F \sim r^2$, where r is the linear dimension, a comparison with Eqs. (1) and (3) yields $\alpha = D/2$.

These results clearly indicate the finite-size effect of the grid. If f_s is large, the frequency-size distribution begins to deviate significantly from a straight-line, such that there is an upper termination to the power-law distribution. In Fig. 6, the deviation begins for $1/f_s = 125$ at $A_F \approx 1000$. It is seen that large forest fires become dominant when the sparking frequency is very small. This is easily explained on physical grounds. With a large sparking frequency (for example $1/f_s = 125$), trees burn before large clusters can form. If the sparking frequency is very small (for example $1/f_s = 2000$), clusters form that span the entire grid before ignition occurs. For very small sparking frequencies there will be very few small fires. The grid will become very full before a match sparks a fire. The fires will involve a large number of trees, and in most cases the fires will span the grid.

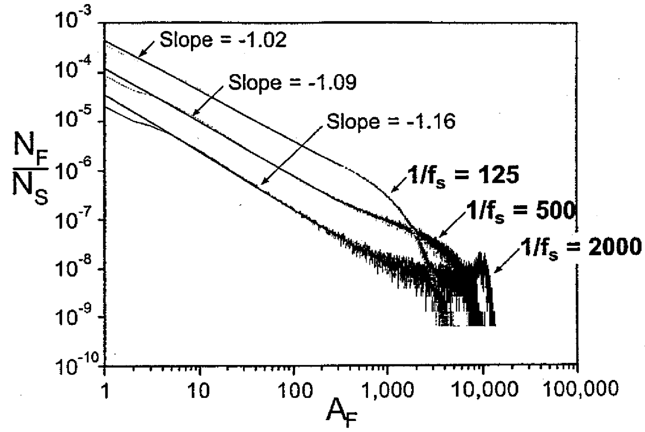

Fig. 6 Frequency-size distributions of model forest fires. The number of fires per time step with size A_F, N_F/N_S, are given as a function of A_F where A_F is the number of trees burnt in each fire. Results are given for a grid size 128×128 and three sparking frequencies, $f_s = 1/125$, 1/500, 1/2000. The small fires correlate well with the power-law relation [Eq. (4)] taking $\alpha = 1.02$ to 1.16. The finite grid-size effect can be seen at the smallest firing frequency, $f_s = 1/2000$.

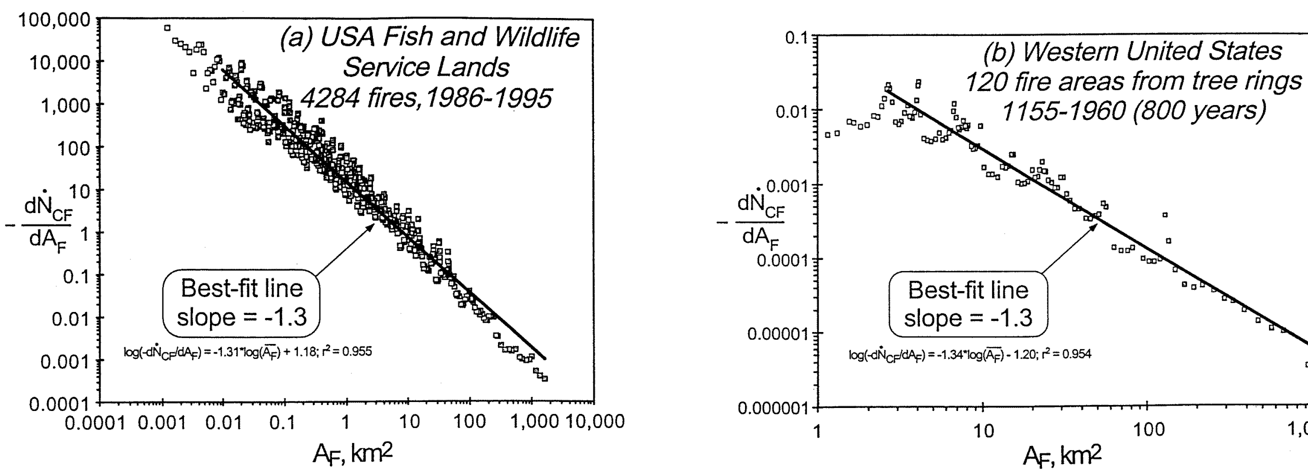

Fig. 7 Noncumulative frequency-area statistics for actual forest fires and wildfires in the United States and Australia. Four examples are given: (a) 4284 fires on US Fish and Wildlife Service Lands during 1986–1995 (National Interagency Fire Center, 1996). (b) 120 of the largest fire areas in the western United States during 1155–1960, obtained from tree ring data (Heyerdahl and Agee, 1994). (c) 164 fires in Alaskan Boreal Forests during 1990–1991 (Kasischke and French, 1995). (d) 298 fires in the Australian Capital Territory during 1926–1991 (Australian Capital Territory Bush Fire Council, 1996). The noncumulative number of fires, $-dN_{CF}/dA_F$, is given as a function of A_F. In each case, a reasonably good correlation is obtained with the power-law relation [Eq. (4)] taking $\alpha = 1.3$–1.5.

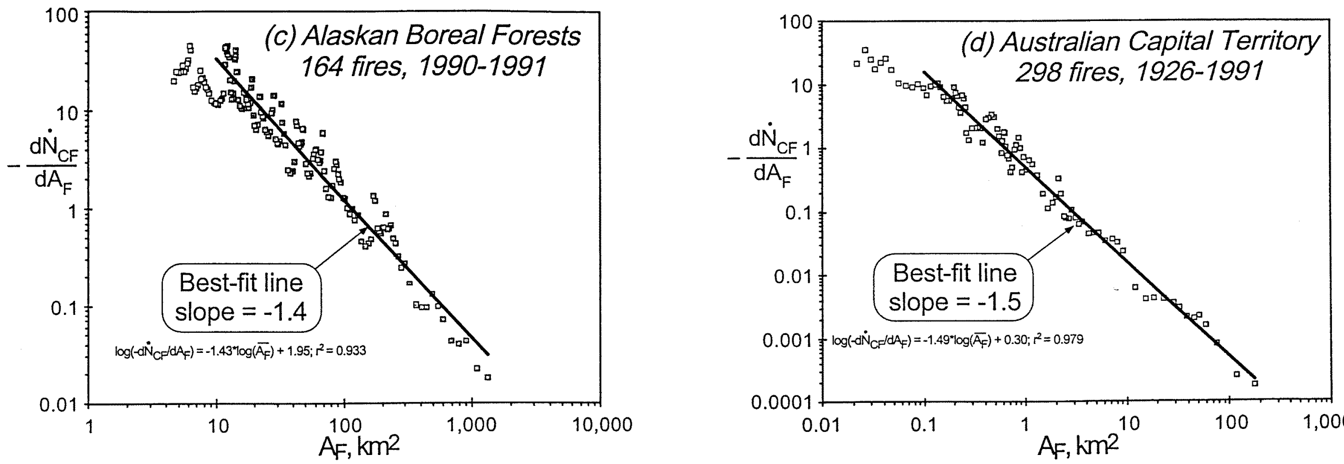

Fig. 7 (*Continued*)

We now turn our attention to the frequency-size distribution of actual forest fires and wildfires. Four forest fire and wildfire data sets from the United States and Australia are given in Fig. 7. In each case the noncumulative number of fires per year, $d\dot{N}_{CF}/dA_F$, is given as a function of A_F. We use a dot over the N to indicate that the frequency data has been divided by the length of the record to give a frequency "per year." The first data set includes 4284 fires on US Fish and Wildlife Lands during the period 1986–1995.[9] The second data set includes 120 forest fires as interpreted from tree rings for the western United States for the period 1155–1960.[10] The third data set includes 164 fires in Alaskan Boreal Forests during 1990 and 1991.[11] The fourth data set includes 298 fires in the Australian Capital Territory during 1926–1991.[12] The data sets come from a variety of geographic regions with different vegetation types and climates. The results given in Fig. 7 are in quite good agreement with the power-law relation [Eq. (5)] with $\alpha = 1.3$–1.5.

The agreement with power-law (fractal) statistics is quite good, but the slopes are somewhat higher than the model results given above. Considering the many complexities concerning the initiation and propagation of forest fires and wildfires it is remarkable that the frequency-magnitude distributions are so similar under such a wide variety of environments. The proximity of combustible material varies widely. The behavior of a particular fire depends strongly on meteorological conditions. Fire-fighting efforts extinguish many fires. Despite these complexities, the application of the statistics associated with the forest-fire model appears to be robust. We conclude that naturally-occurring forest fires are examples of self-organized critical behavior.

4. DISCUSSION

It appears reasonable to associate the number of battle deaths in a war with the number of trees that burn (the area) in a forest fire. If this is done, the frequency-intensity distributions for wars given in Figs. 1 and 3 are remarkably similar to the frequency-size distributions for forest fires given in Fig. 7. For the two definitions of war intensity we have $\alpha = 1.27$ and 1.40. For the four data sets for forest fires given in Fig. 7 we have $\alpha = 1.3$, 1.3, 1.4, 1.5. We can explain this behavior for forest fires in terms of the forest-fire model, but a key question is whether this explanation is also valid for wars.

The behavior of the forest-fire model can be explained in terms of a cascade model. If trees are randomly planted on a grid the distribution of cluster sizes is exponential (Poisonian) not power-law (fractal). The distribution of cluster sizes in the forest-fire model is power law (fractal). This is because clusters of trees continuously grow and combine to form larger clusters. Small fires sample this population of clusters but the loss of trees in fires is dominated by the largest fires. There is a self-similar cascade of trees from small to large clusters.

In terms of the forest-fire model a spark ignites a tree and the model fire consumes the entire cluster to which this tree belongs. This is also the case for real forest fires. Ignition of the forest must take place for a fire to take place, and the fire will then spread through the contiguous flammable material.

A war must begin in a manner similar to the ignition of a forest. One country may invade another country, or a prominent politician may be assassinated. The war will then spread over the contiguous region of metastable countries. Such regions of metastability could be the countries of the Middle East (Iran, Iraq, Syria, Israel, Egypt, etc.) or of the former Yugoslavia (Serbia, Bosnia, Croatia, etc.). These are then the metastable clusters. In some cases the metastable clusters could combine. Albania and Greece bridge the gap between the metastable clusters of the Middle East and the former Yugoslavia.

We now consider briefly the implications of the results given above. Saperstein[13] has discussed the relation of wars to complexity theory in a general way. One can qualitatively discuss the breakdown of order in the world in a similar manner to the "forest fires" in the forest-fire model. In the forest-fire model, sometimes a match starts a fire and sometimes it does not. Some fires are large and some are small. But the frequency-size statistics are power-law. In terms of world order there are small conflicts that may or may not grow into major wars. The stabilizing and destabilizing influences are clearly very complex. The results we have shown indicate that world order behaves as a self-organized critical system independent of the efforts made to control and stabilize interactions between people and countries.

It is easy to argue that the results given here cannot be significant. The introduction of weapons of mass destruction, particularly the atom bomb, must change global interactions and the associated wars. However, as we have shown, the frequency-area statistics of real forest fires are well approximated by power-law distributions with slopes near 1.3. Again it can be argued that attempts to extinguish fires, changing land-use practices, and other human interventions should have affected the resulting distribution of fires. But a variety of correlations show that the power-law, frequency-size distributions of these complex phenomena remain valid. We argue that this is also the case for wars.

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