

Fourier级数逼近

Fourier级数展开

➤ 给定周期函数 $f(x)$, $x \in [-L, L]$, $T = 2L$

➤ 满足 $f(x) = f(x + kT)$

➤ 一般 (a, b) 区间, 变量替换 $L = \frac{b - a}{2}$, $x_1 = x + L + a$

➤ Fourier级数展开

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L}x + b_n \sin \frac{n\pi}{L}x \right)$$

➤ 系数
$$\begin{cases} a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx, & n = 0, 1, 2, \dots \\ b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx, & n = 1, 2, 3, \dots \end{cases}$$

Fourier 级数展开函数编写

➤求解 $[F, A, B] = \text{fseries}(f, x, p, a, b)$

➤求Fourier级数展开的MATLAB代码

```
function [F,A,B]=fseries(f,x,varargin)
[p,a,b]=default_vals({6,-pi,pi},varargin{:});
L=(b-a)/2; if a+b, f=subs(f,x,x+L+a); end
A=int(f,x,-L,L)/L; B=[]; F=A/2;
for n=1:p
    an=int(f*cos(n*pi*x/L),x,-L,L)/L; A=[A, an];
    bn=int(f*sin(n*pi*x/L),x,-L,L)/L; B=[B,bn];
    F=F+an*cos(n*pi*x/L)+bn*sin(n*pi*x/L);
end
if a+b, F=subs(F,x,x-L-a); end
```

支持函数的编写

➤ 给函数参数分派默认值

```
function varargout=default_vals(vals,varargin)
if nargin~=length(vals),
    error('number of arguments mismatch');
else, nn=length(varargin)+1;
    varargout=varargin;
    for i=nn:nargout, varargout{i}=vals{i};
end, end, end
```

➤ 其他函数也可能使用

➤ 后面将用到这个公用函数

➤数学描述

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L}x + b_n \sin \frac{n\pi}{L}x \right)$$

➤核心代码

$$\begin{cases} a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx, & n = 0, 1, 2, \dots \\ b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx, & n = 1, 2, 3, \dots \end{cases}$$


```
A=int(f,x,-L,L)/L; B=[]; F=A/2;  
for n=1:p  
    an=int(f*cos(n*pi*x/L),x,-L,L)/L; A=[A, an];  
    bn=int(f*sin(n*pi*x/L),x,-L,L)/L; B=[B,bn];  
    F=F+an*cos(n*pi*x/L)+bn*sin(n*pi*x/L);  
end
```

例3-33 函数的Fourier近似


➤ 函数 $y = x(x - \pi)(x - 2\pi)$

➤ 其中 $x \in (0, 2\pi)$

➤ 求它的Fourier级数展开

```
 >> syms x; f=x*(x-pi)*(x-2*pi);  
      [F,A,B]=fseries(f,x,12,0,2*pi);F
```

➤ 结果比较

```
 >> ezplot(f,[0,2*pi]),  
      hold on, ezplot(F,[0,2*pi]), hold off
```

展开的结果和效果

➤ 更大的区域 $x \in (-\pi, 3\pi)$



```
>> ezplot(f, [-pi, 3*pi]),  
hold on, ezplot(F, [-pi, 3*pi]), hold off
```

➤ 数学描述

$$\begin{aligned} f(x) = & 12 \sin x + \frac{3 \sin 2x}{2} + \frac{4 \sin 3x}{9} + \frac{3 \sin 4x}{16} + \\ & \frac{12 \sin 5x}{125} + \frac{\sin 6x}{18} + \frac{12 \sin 7x}{343} + \frac{3 \sin 8x}{128} + \\ & \frac{4 \sin 9x}{243} + \frac{3 \sin 10x}{250} + \frac{12 \sin 11x}{1331} + \frac{\sin 12x}{144} \end{aligned}$$

➤ 一般形式 $f(x) = \sum_{n=1}^{\infty} \frac{12}{n^3} \sin nx$

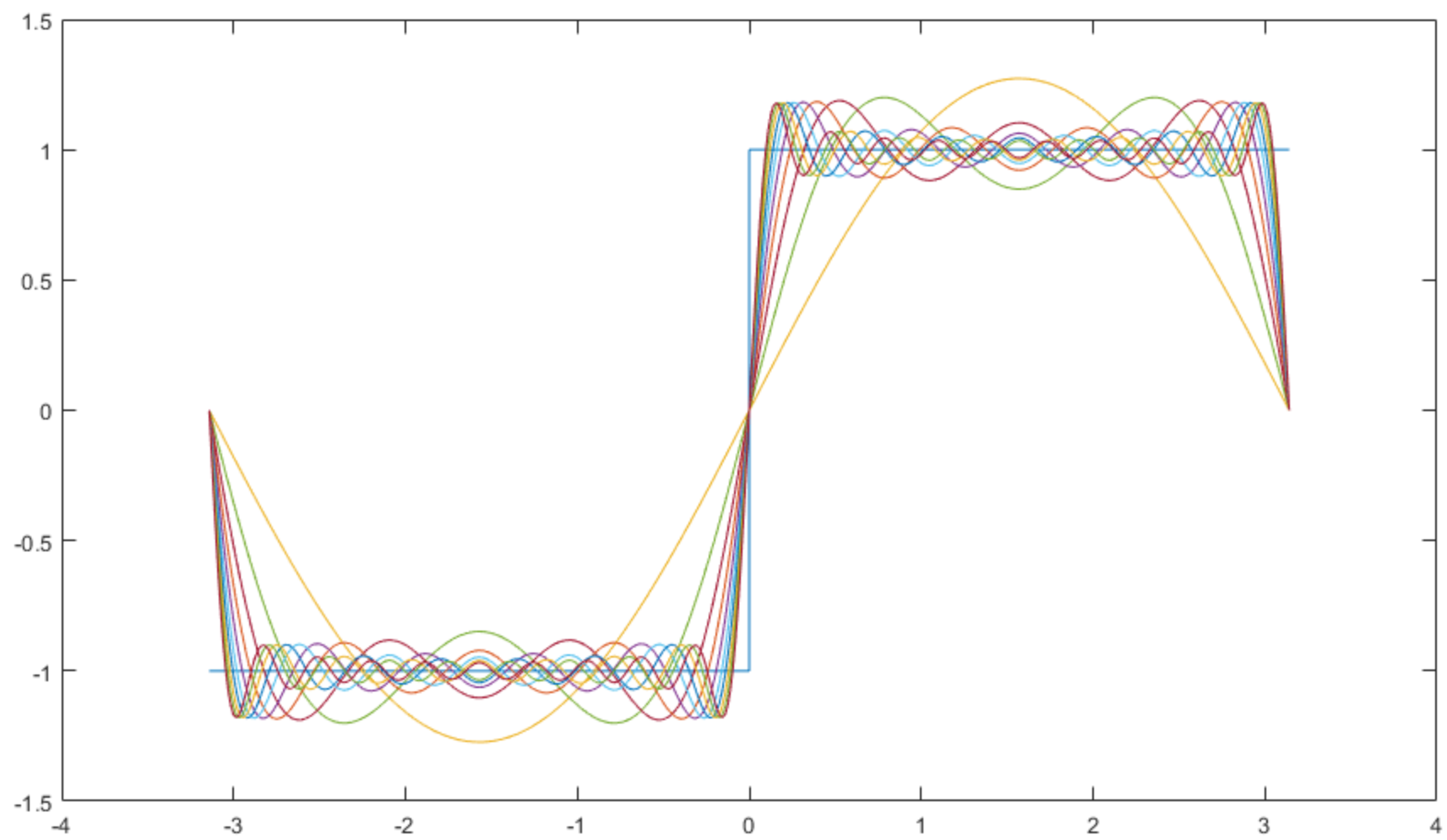
例3-34 分段函数的近似

➤ 给定函数 $y = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases} \quad x \in (-\pi, \pi)$

➤ 原函数可以表示成 $f(x) = \frac{|x|}{x}$



```
>> syms x; f=abs(x)/x; xx=[-pi:pi/200:pi];  
xx=xx(xx~=0); xx=sort([xx,-eps,eps]);  
yy=subs(f,x,xx); plot(xx,yy), hold on  
for n=1:20  
    [f1,a,b]=fseries(f,x,n);  
    y1=subs(f1,x,xx); plot(xx,y1)  
end
```

展开效果与函数

➤ 前14项的Fourier级数展开



```
>> [f1,a,b]=fseries(f,x,14); f1
```

➤ 数学形式

$$f(x) \approx 4 \frac{\sin x}{\pi} + \frac{4 \sin 3x}{3\pi} + \frac{4 \sin 5x}{5\pi} + \frac{4 \sin 7x}{7\pi} \\ + \frac{4 \sin 9x}{9\pi} + \frac{4 \sin 11x}{11\pi} + \frac{4 \sin 13x}{13\pi}$$

➤ 一般形式 $f(x) = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\sin(2k-1)x}{2k-1}$

周期函数的假设

➤ 在区间 $[-2\pi, 2\pi]$ 进行拟合效果比较



```
>> xx=[-2*pi:pi/200:2*pi];  
xx=xx(xx~=0);  
xx=sort([xx,-eps,eps]);  
yy=subs(f,x,xx);  
plot(xx,yy), hold on  
y1=subs(f1,x,xx); plot(xx,y1), hold off
```

➤ 回忆一下开始的假设 $f(t) = f(kT + t)$

