**Vanderbilt University + College of Engineering**

**CS 3250 Algorithms, Spring 2024**

**Homework #1B**

Sections 01/02/03

**Honor Statement:**

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| **HWS are 60% of your final grade. HW1 is worth 8%** | | | |
| **Component** | **Due date** | **Details** | **Percent of final grade** |
| HW1A: Brightspace Calibration Quiz | Wednesday, 1/17, 9:00 AM | Complete the calibration quiz in Brightspace.  *No late homework is accepted.* | ~1% |
| **HW1B** | | | |
| HW1B: Brightspace Asymptotic Analysis Quiz | Wednesday, 1/24, 9:00 AM | Take the quiz in Brightspace.  *No late homework is accepted.* | ~2% |
| HW1B: Gradescope Written assignment | Wednesday, 1/24, 9:00 AM | Upload to Gradescope  *Late homework rules apply* | ~5% |

1. **A Stunning Result From Nadha Skolar? (5 points)** Nadha Skolar read online at “Geeks for Geeks” that Ω(nlogn) is the supposed lower bound for all comparison-based sorting algorithms in the worst case. Skeptical about this claim, Nadha asks his little five-year-old sister, Stellar, to tackle the problem (unlike her brother, Stellar is naturally gifted in CS). Using an unusual technique, Stellar devises a comparison-based sorting algorithm with a worst-case running time of O (𝑛 log √𝑛). Based on this result, Nadha quickly writes up a research paper claiming the two Skolars have just vanquished the lower-bound of nlogn for comparison based-sorting. Nadha believes these results are stunning enough that his paper should get published in an upcoming issue of the Journal of the ACM, ensuring him of his long-awaited fame.

Is it possible that Nadha and Stellar shattered the glass floor on comparison-based sorting algorithms? If so, briefly explain why and how their algorithm can be asymptotically faster than every known comparison-based sorting algorithm. If not, assuming Stellar is correct, explain why Nadha’s enthusiasm for Stellar’s results is misguided. Be sure to state your reasoning clearly and concisely in no more than a single short paragraph.

Their algorithm is still O(nlogn). Rewrite It is easy to see that this is still O(nlogn).

1. **Prove It! (10 points – 5 each).** In asymptotic analysis n refers to the input size for an algorithm, so you may always assume n is an integer in this context.
2. **(5 points)** Prove 2n is O(n!) for n > 3.

Via induction, I will show that is O(n!) for any n > 3.

Base case: n = 4. In this case, ­. Since n! is greater than here, the upper bound holds.

Inductive hypothesis: Assume that k! for some integer k. We want to show that

Inductive step: . Rewrite this inequality as needing to show that . By the inductive hypothesis, this simplifies to needing to show . But since the domain of k is restricted to k > 3, this must hold. For a function to be O(n!), n! must always sit above the graph of for some choice of k and n0. Since it was just shown that this holds for k = 1, n0 = 4, 2n is O(n!). QED.

1. **(5 points)** Write a well-structured proof demonstrating that 7log3n is Θ(log2n).

Recall that log3(n) is equal to . Rewrite 7log3(n) as . Now we must prove that this function is O(log2n) and Ω(log2(n)).

Proving O(log2n): We must find some

1. **Snack Attack (15 points).** Professor Arena is planning to reward his algorithm students for surviving the first exam in CS3250. One idea he has is to distribute snack bags (e.g., Lays Potato Chips, Pretzels, Doritos, Cheetos, etc.) at the lecture following the exam. Unbeknownst to many, Prof. Arena is a bit underpaid and underfunded. Consequently, he must devise a reward system so that he is using his money wisely. After consulting his college-age daughter, Mia, for some advice, he arrives at the following gameplan:

* All students who took the exam must receive at least one snack bag, even if they performed poorly.
* Every student should receive more snack bags than any student to their immediate left or right if they scored higher than that student. Similarly, if a student scored lower than a student to their immediate left or right, they must receive fewer snack bags than their neighbor, who scored higher.

You are provided as input a paired list of exam scores and student names in an array corresponding to the order in which students are seated (you can imagine this as one big, long row of students). You may further assume that all scores are unique (i.e., no two students received the same score on the exam).

1. **(10 points)** Write an efficient algorithm to determine the minimum number of snack bags Prof. Arena must purchase so he can distribute them to his class according to the criteria listed above. Your algorithm should print out each student’s name and the number of snack bags they will receive in addition to the total number of snack bags Prof. Arena must purchase in advance.

**An Example:**

Scores: [80, 90, 20, 75]

Names: [Abby, Lucy, Donald, Cesar]

Abby: 1

Lucy: 2

Donald: 1

Cesar: 2

Total snack bags needed: 6

1. **(5 points)** Provide a worst-case run-time analysis with a tight bound (i.e., a Θ bound or tight big-O bound).