Bundesliga Pythagorean Prediction*

Estimating league-specific coefficients (2010/11–2023/24) and prospective PiT accuracy in 2024/25

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We study a four-parameter Pythagorean points model for the German Bundesliga. Match-level CSVs are cleaned and standardized, season tables are constructed, and end-of-season (EoS) team totals are pooled across 2010/11–2023/24. We estimate league-specific coefficients (a, b, c, d) by minimizing mean absolute error (MAE) using a multi-start Nelder–Mead procedure. Generalization is evaluated with leave-one-season-out (LOSO) validation, summarizing accuracy by the median MAE and Pearson correlation between predicted and realized EoS points. Using the fitted coefficients, we generate "points-in-table" (PiT) forecasts for 2024/25 at 9, 18, and 27 rounds by combining realized points to date with Pythagorean-based projections for remaining fixtures. We report accuracy at each checkpoint, visualize predicted vs. actual points, and highlight the largest over- and under-performers. All code is reproducible from the project scripts, and intermediate/summary artifacts are written to the `output/` directory.

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*Code and data: https://github.com/Clearsky21z/Bundesliga_Pythagorean_Prediction

1 Introduction

Forecasting league tables from scoring profiles is a long-standing idea in sports analytics. This paper tailors a **four-parameter Pythagorean model** to the Bundesliga, fits the parameters on seasons **2010/11–2023/24**, and evaluates *prospective* accuracy on **2024/25** using "points-in-table" (PiT) projections at Round 9, 18, and 27.

We proceed as follows. Section (**ref?**)(sec-methods) formalizes the model and the reproducible pipeline used to clean raw CSVs, construct season tables, estimate coefficients, and perform validation. Section (**ref?**)(sec-results) reports fitted coefficients, in-sample diagnostics (Table Table 1), LOSO validation (Table Table 3), and prospective PiT performance for 2024/25 (Table Table 5; Figures Figure 1–Figure 3). Section (**ref?**)(sec-discussion) interprets the findings, limitations, and extensions. All artifacts are read from the `output/` directory.

2 Methodology

2.1 Model

For team–season observation i, with goals for GF_i, goals against GA_i, and matches played PLD_i, we use

$$\text{frac}_i = \frac{GF_i^b}{GF_i^c + GA_i^d}, \qquad \widehat{PTS}_i = a \cdot \text{frac}_i \cdot PLD_i.$$

Here $a > 0$ scales the expected fraction to the 3-points-per-win system, and $b, c, d > 0$ control curvature and the relative impact of scoring vs conceding.

Estimation. Let y_i be realized EoS points and $\hat{y}_i(a, b, c, d)$ the model prediction. We choose (a, b, c, d) to minimize mean absolute error (MAE) across the pooled team–season set (2010/11–2023/24) using **Nelder–Mead** with multiple starts.

Validation. We perform **leave-one-season-out (LOSO)** validation: for each season s, refit on all other seasons and evaluate on s. We summarize performance by median MAE and median Pearson correlation r across held-out seasons.

2.2 Data and Measurement

Raw match CSVs for each season (e.g., `D1_2018_2019.csv`) are cleaned to standardize columns, repair result codes from the recorded goals, and parse date/time. For each season we construct an EoS table (played, goals for/against, goal difference, points); these are also stacked into a pooled dataset used for estimation. Variables are all game-level primitives or simple team-level aggregates. The full pipeline lives in `script/` and writes all analysis-ready artifacts to `output/`.

3 Results

3.1 Fitted coefficients (2010/11–2023/24)

The pooled Bundesliga coefficients are **a=2.4177, b=1.2318, c=1.1785, d=1.2174** (see `output/bundesliga_coefs_pooled.csv`).

3.1.1 In-sample diagnostics

- **MAE**: 2.98
- **Correlation (r)**: 0.968

Table 1: In-sample metrics by season (2010/11–2023/24). Lower MAE and higher r are better.

Table 2: In-sample metrics by season (2010/11–2023/24).

Season	MAE	r
D1_2010_2011.csv	3.67	0.931
D1_2011_2012.csv	2.98	0.976
D1_2012_2013.csv	2.94	0.971
D1_2013_2014.csv	3.58	0.969
D1_2014_2015.csv	2.89	0.973
D1_2015_2016.csv	1.64	0.992
D1_2016_2017.csv	3.66	0.950
D1_2017_2018.csv	2.95	0.971
D1_2018_2019.csv	2.80	0.975
D1_2019_2020.csv	2.62	0.977
D1_2020_2021.csv	2.84	0.973
D1_2021_2022.csv	2.94	0.966
D1_2022_2023.csv	3.62	0.952
D1_2023_2024.csv	2.61	0.986

3.1.2 Leave-one-season-out (LOSO)

Overall across held-out seasons:

- **Median MAE**: 3.00
- **Median r**: 0.972

Table 3: LOSO per-season holdout performance (train on all-but-one, test on the held-out season).

Table 4: LOSO per-season holdout performance.

Season	a	b	c	d	MAE	r
D1_2010_2011.csv	2.401115	1.227495	1.170509	1.212918	3.70	0.931
D1_2011_2012.csv	2.490733	1.232563	1.186197	1.226450	3.00	0.975
D1_2012_2013.csv	2.133682	1.162348	1.071932	1.120948	3.20	0.969
D1_2013_2014.csv	2.225726	1.201309	1.126899	1.165948	3.72	0.968
D1_2014_2015.csv	2.751209	1.240201	1.216807	1.260475	3.07	0.973
D1_2015_2016.csv	2.286555	1.226001	1.157699	1.198718	1.69	0.992
D1_2016_2017.csv	2.403808	1.227016	1.170236	1.212843	3.70	0.951
D1_2017_2018.csv	2.402950	1.233615	1.178997	1.217211	2.95	0.971
D1_2018_2019.csv	2.471014	1.226372	1.177819	1.218519	2.82	0.975
D1_2019_2020.csv	2.175335	1.230578	1.151851	1.189216	2.68	0.976
D1_2020_2021.csv	2.492685	1.274063	1.230646	1.263104	2.99	0.972
D1_2021_2022.csv	2.487198	1.222908	1.175689	1.217528	2.99	0.966
D1_2022_2023.csv	2.296429	1.254117	1.190680	1.222013	3.70	0.951
D1_2023_2024.csv	2.403005	1.233461	1.178837	1.217079	2.62	0.986

3.2 Prospective 2024/25 PiT

PiT projects the final table using realized points to date plus Pythagorean projections for remaining fixtures under the fitted coefficients.

Table 5: Bundesliga 2024/25 PiT summary after 9, 18, and 27 rounds.

Table 6: Bundesliga 2024/25 PiT summary after 9, 18, and 27 rounds.

Rounds	r	MAE
9	0.833	6.91
18	0.942	4.13
27	0.960	3.32

3.2.1 Visual comparisons (predicted vs actual)

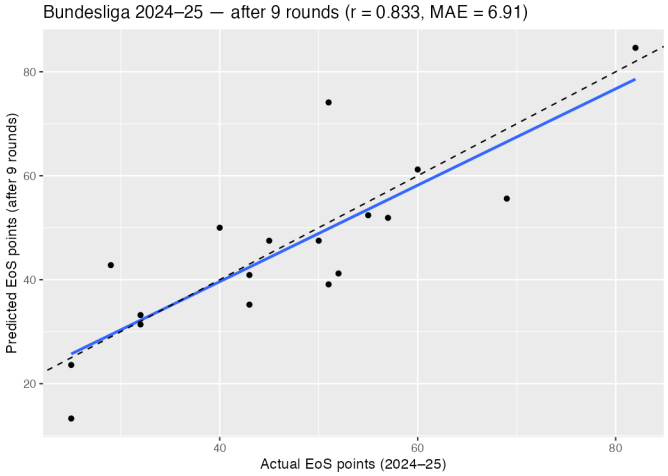

Figure 1: Predicted vs actual EoS points using PiT after Round 9 (2024/25).

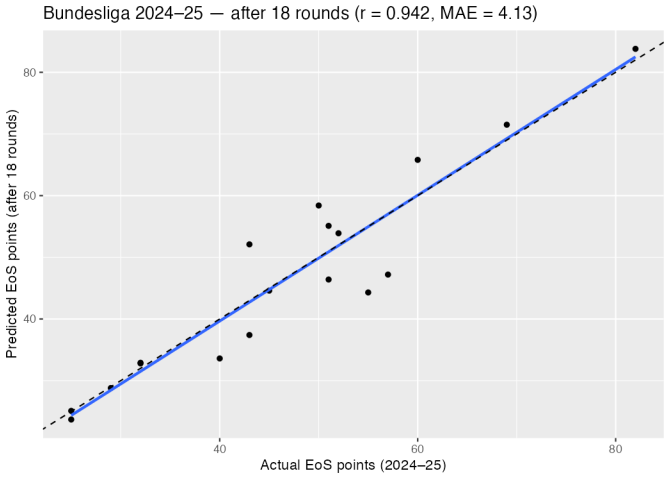

Figure 2: Predicted vs actual EoS points using PiT after Round 18 (2024/25).

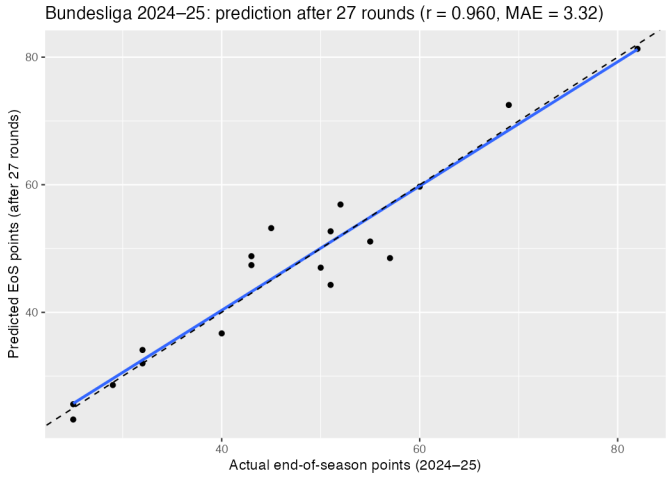

Figure 3: Predicted vs actual EoS points using PiT after Round 27 (2024/25).

3.2.2 Biggest over/under at Round 27

Table 7: Top-5 absolute deviations between predicted total (after Round 27) and final EoS points, 2024/25.

Table 8: Top-5 absolute deviations at Round 27.

Team	Predicted_Total	Actual_PTS	Error	Abs_Error	Direction
Dortmund	48.5	57	-8.5	8.5	Under (pred<actual)
M'gladbach	53.2	45	8.2	8.2	Over (pred>actual)
Werder Bremen	44.3	51	-6.7	6.7	Under (pred<actual)
Wolfsburg	48.8	43	5.8	5.8	Over (pred>actual)
Mainz	56.9	52	4.9	4.9	Over (pred>actual)

4 Discussion

Interpretation. The exponents b, c, d govern curvature and the asymmetric impact of scoring vs conceding; a maps expected fractions to points. The fitted values track Bundesliga totals closely across the modern era.

Generalization. LOSO medians (MAE 3.00, r 0.972) indicate stable transportability within 2010/11–2023/24.

Prospective accuracy. On 2024/25, error shrinks with more rounds observed (Table Table 5; Figures Figure 1–Figure 3). By Round 27, correlation is $ $0.96 with MAE $ $3.3 points—useful for table-level forecasting.

Limitations & extensions. PiT assumes future match expectation depends only on current GF/GA, not schedule strength, injuries, or tactical change. Extensions include opponent-adjusted projections, hierarchical priors, alternative robust losses, and uncertainty bands.