This means that the model possesses a unique steady-state flow-density relation, i.e., a fundamental diagram.[1]

These conditions are necessary but not sufficient. For example, when in the car-following regime (steady-state congested traffic), the time gap to the leader has to remain within reasonable bounds (say, between 0.5 and 3 s). Furthermore, the acceleration has to be constrained to a "comfortable" range (e.g., $\pm 2\ \mathrm{m/s^2}$), or at least, to physically possible values. Particularly, when approaching the leading vehicle, the quadratic relation between braking distance and speed has to be taken into account. Finally, any car-following model should allow instabilities and thus the emergence of "stop-and-go" traffic waves, but should not produce accidents, i.e., negative bumper-to-bumper gaps $s < 0$.[2]

Which of the car-following models introduced in Chap. 10 satisfy the conditions (11.1)–(11.5)?

11.2 Gipps' Model

Gipps' model presented here is a modified version of the one described in his original publication. It is simplified, but conceptually unchanged. Although it produces an unrealistic acceleration profile, this model is probably the simplest complete and accident-free model that leads to accelerations within a realistic range.

11.2.1 Safe Speed

Accidents are prevented in the model by introducing a "safe speed" $v_{\text{safe}}(s, v_l)$, which depends on the distance to and speed of the leading vehicle. It is based on the following assumptions:

1. Braking maneuvers are always executed with constant deceleration b. There is no distinction between comfortable and (physically possible) maximum deceleration.
2. There is a constant "reaction time" Δt.

[1] If one were to weaken condition (11.1) to $\partial a_{\text{mic}}/\partial v \leq 0$, it is possible to formulate models that do *not* have a fundamental diagram. Such models are proposed in the context of B. Kerner's *three-phase theory*.

[2] Traffic-flow models are meant to describe *normal* conditions, while accidents are almost always caused by *exceptional* driving mistakes that are not part of normal driving behavior and thus not part of the intended scope of the model.

3. Even if the leading vehicle suddenly decelerates to a complete stop (worst case scenario), the distance gap to the leading vehicle should not become smaller than a minimum gap s_0.[3]

Condition 1 implies that the *braking distance* that the leading vehicle needs to come to a complete stop is given by

$$\Delta x_l = \frac{v_l^2}{2b}.$$

From condition 2 it follows that, in order to come to a complete stop, the driver of the considered vehicle needs not only his or her braking distance $v^2/(2b)$, but also an additional *reaction distance* $v\,\Delta t$ travelled during the reaction time.[4] Consequently, the *stopping distance* is given by

$$\Delta x = v\Delta t + \frac{v^2}{2b}. \tag{11.8}$$

Finally, condition 3 is satisfied if the gap s exceeds the required minimum final value s_0 by the difference $\Delta x - \Delta x_l$ between the stopping distance of the considered vehicle and the breaking distance of the leader:

$$s \geq s_0 + v\Delta t + \frac{v^2}{2b} - \frac{v_l^2}{2b}. \tag{11.9}$$

The speed v for which the equal sign holds (the highest possible speed) defines the "safe speed"

$$v_{\text{safe}}(s, v_l) = -b\Delta t + \sqrt{b^2\Delta t^2 + v_l^2 + 2b(s - s_0)}. \tag{11.10}$$

11.2.2 Model Equation

The simplified Gipps' model is defined as an iterated map with the "safe speed" (11.10) as its main component:

$$\boxed{v(t + \Delta t) = \min\left[v + a\Delta t, v_0, v_{\text{safe}}(s, v_l)\right] \quad \text{Gipps' model.}} \tag{11.11}$$

This model equation reflects the following properties:

- The simulation update time step is equal to the reaction time Δt.

[3] This condition is not present in the original paper, but is necessary to ensure an accident-free model in the presence of numerical errors arising from discretization.

[4] In contrast to the original publication, we assume the speed to be constant within the reaction time.

- If the current speed is greater than $v_{\text{safe}} - a\Delta t$ or $v_0 - a\Delta t$, the vehicle will reach the minimum of v_0 and v_{safe} during the next time step.[5]
- Otherwise the vehicle accelerates with constant acceleration a until either the safe speed or the desired speed is reached.

11.2.3 Steady-State Equilibrium

The homogeneous steady state implies $v(t + \Delta t) = v_l = v$, thus

$$v = \min(v_0, v_{\text{safe}}) = \min\left(v_0, -b\Delta t + \sqrt{b^2\Delta t^2 + v^2 + 2b(s - s_0)}\right),$$

which yields the steady-state speed-gap relation

$$v_e(s) = \max\left[0, \min\left(v_0, \frac{s - s_0}{\Delta t}\right)\right] \tag{11.12}$$

and, assuming constant vehicle lengths l, the familiar "triangular" fundamental diagram

$$Q_e(\rho) = \min\left(v_0\rho, \frac{1 - \rho l_{\text{eff}}}{\Delta t}\right), \tag{11.13}$$

where $l_{\text{eff}} = (l + s_0)$. As in the Newell model, the parameter Δt can be interpreted in four different ways: (i) As the reaction time introduced in the derivation of v_{safe}, (ii) as the numerical update time step of the actual model equation (11.11), (iii) as a speed adaption time in Eq. (11.11) (at least, if $v(t + \Delta t)$ is restricted by v_{safe} or v_0), or (iv) as the "safety time gap" $(s - s_0)/v_e$ in congested traffic as deduced from the fundamental diagram (11.12).

11.2.4 Model Characteristics

Unlike the minimal models described in the previous chapter, the Gipps' model is transparently derived from a few basic assumptions and uses parameters that are easy to interpret and assign realistic values (Table 11.1). Furthermore, Gipps' model is—again, in contrast to the minimal models—*robust* in the sense that meaningful results can be produced from a comparatively wide range of parameter values.
Highway traffic. The simulation of the highway scenario (Fig. 11.1, left) produces more realistic results than the OVM or the Newell model: The speed field in panel

[5] Strictly speaking, this means that deceleration $(v - v_{\text{safe}})/\Delta t$ is not restricted to b. In multi-lane simulations, it can be greater if another vehicle "cuts in" in front of the considered vehicle.

Table 11.1 Parameters of the simplified Gipps' model and typical values in different scenarios

Parameter	Typical value Highway	Typical value City traffic
Desired speed v_0	120 km/h	54 km/h
Adaption/reaction time Δt	1.1 s	1.1 s
Acceleration a	1.5 m/s^2	1.5 m/s^2
Deceleration b	1.0 m/s^2	1.0 m/s^2
Minimum distance s_0	3 m	2 m

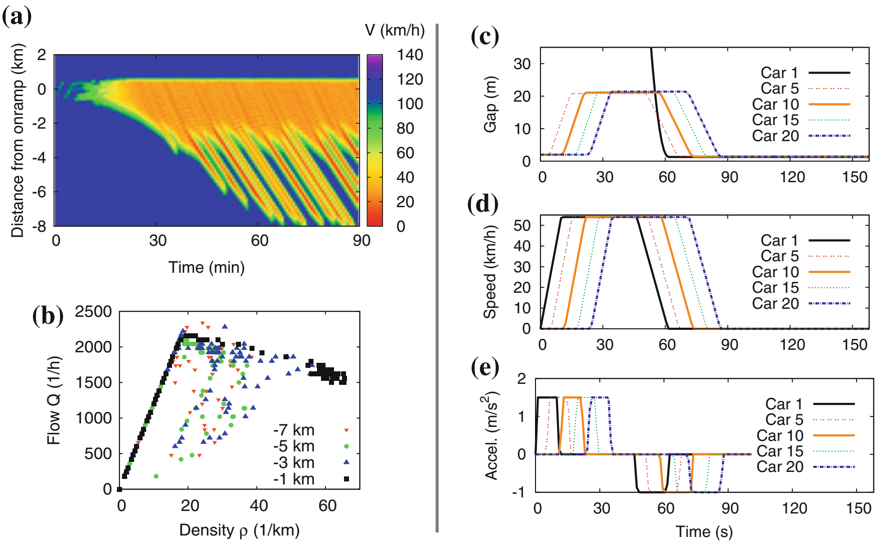

Fig. 11.1 Fact sheet of Gipps' model (11.11), (11.10). Simulation of the two standard scenarios "highway" (*left*) and "city traffic" (*right*) with the parameter values listed in Table 11.1. See Chap. 10.5 for a detailed description of the scenarios

(a) exhibits small perturbations which are caused by vehicles merging from the on-ramp and grow into stop-and-go waves while propagating upstream. The propagation velocity $c_{\text{cong}} = -l_{\text{eff}}/\Delta t$ is constant and of the order of the empirical value (≈ -15 km/h). Furthermore, the wave length (of the order of 1–1.5 km) is not too far away from the empirical values (1.5–3 km).

The flow-density diagram in Fig. 11.1b, obtained from virtual detectors, shows a strongly scattered cloud of data points in the region of congested traffic, i.e., everywhere to the right of the straight line indicating free traffic. Such a wide scattering is in agreement with empirical data (cf. Figs. 4.11 and 4.12). By looking at scatter plots of individual detectors, one observes that detectors that are closer to the bottleneck produce data points that are shifted towards greater densities and closer to the fundamental diagram of steady-state traffic. Moreover, the data points of virtual detectors positioned inside the region of stationary traffic immediately upstream of the bottleneck (solid black squares) lie on the fundamental diagram itself. This apparent density increase near the outflow region of a congestion, also known as *pinch effect*, can be observed empirically. However, the systematic density underestimation, which conspicuously increases with the degree of the scattering of the data points, suggests that the *real* density increase is smaller, or even nonexistent. This means that the pinch effect is essentially a result of data misinterpretation, or, more specifically, by estimating the density with the time mean speed instead of the space mean speed (cf. Sect. 3.3.1). This interpretation is confirmed by simulation as will be shown in Fig. 11.5b. We draw an important conclusion that is not restricted to Gipps' model (and not even to traffic flow models):

> When using empirical data to assert the accuracy and predictive power of models, one has to simulate both the actual traffic dynamics *and* the process of data capture and analysis.

City traffic. Compared to the simple models of the previous chapter, the city-traffic scenario (Fig. 11.1, right column) is closer to reality as well. However, the acceleration time-series is unrealistic. By definition, there are only three values for the acceleration: Zero, a, and $-b$ (cf. Panel (e)). The resulting driving behavior is excessively "robotic" and the abrupt transitions are unrealistic.

Moreover, Gipps' model does not differentiate between comfortable and maximum deceleration: Assuming that b in Eq. (11.10) denotes the maximum deceleration, the model is accident-free but every braking maneuver is performed *very* uncomfortably with full brakes. On the other hand, when interpreting b as the comfortable deceleration and allowing for heterogeneous and/or multi-lane traffic the model possibly produces accidents if leading vehicles (which might be simulated using different parameters or even different models) brake harder than b.

In summary, Gipps' model produces good results in view of its simplicity. Modified versions of this model are used in several commercial traffic simulators. One example of such a modification is *Krauss' model* which essentially is a stochastic version of the Gipps model.

11.3 Intelligent Driver Model

The time-continuous *Intelligent Driver Model* (IDM) is probably the simplest complete and accident-free model producing realistic acceleration profiles and a plausible behavior in essentially all single-lane traffic situations.