- The congested flow-density data shows wide scattering.[13]
- In many cases, real flow-density data are distributed according to an inverse-λ shape corresponding to a capacity drop of typically 10–20 % and associated hysteresis effects.

10.5.2 City Scenario

The right column of the fact sheet 10.3 shows a typical inner-city situation with intersections and traffic lights. The simulation is initialized by a queue of 20 vehicles waiting behind a red traffic light which turns green at $t = 0$. During the simulation, the vehicle queue moves in single file to the next red traffic light 740 m further downstream. Red traffic lights are simulated by a standing virtual vehicle of zero length positioned at the stopping line which is removed when the light turns green. Following driving characteristics can be compared with reality:

- Starting phase: The acceleration ranges between 1 and 2.5 m/s^2 and smoothly decreases to zero as cruising speed is approached. The first vehicle takes 3–4 s to pass the stopping line of the traffic light. Afterwards, the cars pass at a rate of one vehicle every 1.5–2 s.
- Cruising phase: In this phase, all vehicles should travel at close to the desired speed. The vehicles should move as a platoon with time gaps of the order of 1–2 s corresponding to distance gaps of about 15–30 m.
- Approaching phase: All vehicles should decelerate smoothly such that the values for the *jerk*

$$J = \left| \frac{\mathrm{d}\dot{v}}{\mathrm{d}t} \right| = \left| \frac{\mathrm{d}^2 v}{\mathrm{d}t^2} \right| \tag{10.18}$$

  remain below 2 m/s^3. The braking decelerations themselves generally do not exceed 2 m/s^2 and the braking becomes less pronounced for vehicles more at the back of the platoon.

10.6 Optimal Velocity Model

The *Optimal Velocity Model* (OVM) is a time-continuous model whose acceleration function is of the form $a_{\text{mic}}(s, v)$, i.e., the speed difference exogenous variable is missing. The acceleration equation is given by

[13] We point to the fact that this difference is partly caused by the way of presentation. When plotting flow-density, speed-density or speed-flow diagrams from the same data, only the flow-density data show a distinct discrepancy of the amount of scattering on the free and congested branches, see Fig. 4.12.

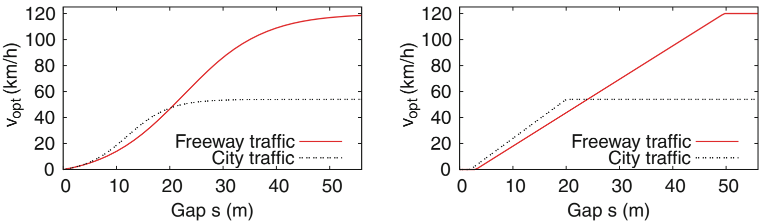

Fig. 10.4 Optimal velocity functions (10.21) (*left*) and (10.22) (*right*) for the parameter values of Table 10.1

$$\boxed{\dot{v} = \frac{v_{\mathrm{opt}}(s) - v}{\tau} \quad \text{Optimal Velocity Model.}} \tag{10.19}$$

This equation describes the adaption of the actual speed $v = v_\alpha$ to the *optimal velocity* $v_{\mathrm{opt}}(s)$ on a time scale given by the *adaptation time* τ. Comparing the acceleration equation (10.19) with the steady-state condition (10.12) it becomes evident that the optimal velocity (OV) function[14] $v_{\mathrm{opt}}(s)$ is equivalent to the microscopic fundamental diagram $v_e(s)$. It should obey the plausibility conditions

$$v'_{\mathrm{opt}}(s) \geq 0, \quad v_{\mathrm{opt}}(0) = 0, \quad \lim_{s\to\infty} v_{\mathrm{opt}}(s) = v_0, \tag{10.20}$$

but is arbitrary, otherwise. Thus, the acceleration equation (10.19) defines a whole class of models whose members are distinguished by their respective optimal velocity functions. The OV function originally proposed by Bando et al.,

$$v_{\mathrm{opt}}(s) = v_0 \frac{\tanh\left(\frac{s}{\Delta s} - \beta\right) + \tanh\beta}{1 + \tanh\beta}, \tag{10.21}$$

uses a hyperbolic tangent.[15] Besides the parameter τ which is relevant for all optimal velocity models,[16] the OVM of Bando et al. has three additional parameters, the desired speed v_0, the transition width Δs, and the form factor β (see Fig. 10.4 and Table 10.1).

A more intuitive OV function can be derived by characterizing free traffic by the desired speed v_0, congested traffic by the time gap T in car-following mode under stationary conditions, and standing traffic by the minimum gap s_0. In analogy to the Section-Based Model of Sect. 8.5, we obtain

[14] Strictly speaking, this is an “optimal speed function”. In order to be consistent with the literature (and the model name), we will nevertheless stick to velocity instead of speed, in this context.

[15] We have adapted the notation with respect to the original publication such that $v_0 = v_{\mathrm{opt}}(0)$ strictly has the meaning of the desired speed if no other vehicles are present.

[16] Sometimes, τ is replaced by the sensitivity parameter $a = 1/\tau$.

Table 10.1 Parameter of two variants of the Optimal Velocity Model (OVM)

Parameter	Typical value highway	Typical value city traffic
Adaptation time τ	0.65 s	0.65 s
Desired speed v_0	120 km/h	54 km/h
Transition width Δs [v_{opt} according to Eq. (10.21)]	15 m	8 m
Form factor β [v_{opt} according to Eq. (10.21)]	1.5	1.5
Time gap T [v_{opt} according to Eq. (10.22)]	1.4 s	1.2 s
Minimum distance gap s_0 [v_{opt} according to Eq. (10.22)]	3 m	2 m

$$v_{opt}(s) = \max\left[0,\ \min\left(v_0, \frac{s - s_0}{T}\right)\right]. \tag{10.22}$$

This relation is the microscopic equivalent to the triangular fundamental diagrams of the macroscopic Section-Based and Cell-Transmission Models (cf. Fig. 10.4). The simulation results are similar to that of the hyperbolic tangent OV function. Again, typical parameter values are given in Table 10.1.

Model properties. For the simulations of the model fact sheet in Fig. 10.3, we have assumed a speed adaptation time $\tau = 0.65$ s. Obviously this is an unrealistically low value since typical time scales for reaching a desired speed are of the order of 10 s. As a consequence, the periods of the stop-and-go waves (about 1–2 min) are too low. Furthermore, in the simulation of the city scenario, the accelerations and decelerations of the vehicle platoon (up to 22 m/s^2 and down to -10 m/s^2, respectively) differ from real accelerations by a factor of about ten.[17] However, when increasing the adaptation time τ by only 5 %, the simulations eventually lead to negative distance gaps s_α corresponding to *accidents*. On the other hand, when decreasing τ by 5 %, we obtain absolute string stability even for congested freeway traffic. While this agrees with the theoretical stability limits (see Chap. 15) it is at variance with the observations. We conclude that the simulation outcome is qualitatively correct. However:

- On a quantitative level, the OVM results are unrealistic.
- On a qualitative level, the simulation outcome has a strong dependency on the fine tuning of the model parameters, i.e., the OVM is not *robust*.

These deficiencies are mainly due to the fact that the OVM acceleration function does not contain the speed difference as exogenous variable, i.e., the simulated driver reaction depends only on the gap but is the same whether the leading vehicle is slower or faster than the subject vehicle. This corresponds to an extremely short-sighted driving style.

[17] In reality, accelerations above 4 m/s^2 (corresponding to 7 s for accelerating from zero to 100 km/h) and below -9 m/s^2 (corresponding to an emergency braking maneuver on a dry road) are physically impossible. Furthermore, everyday accelerations (which the simulations should reproduce) generally are only a fraction of the accelerations at these limits.

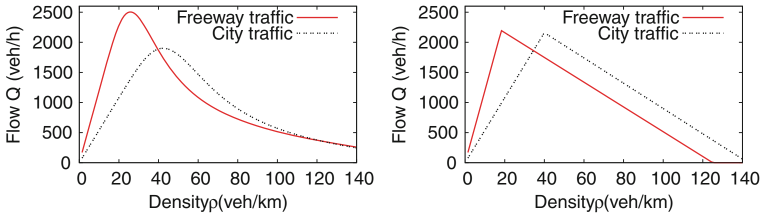

Fig. 10.5 Fundamental diagrams of the OVM for the OV functions (10.21) (*left*) and (10.22) (*right*) for the parameter values of Table 10.1

Steady-state equilibrium. As already mentioned, the steady-state condition (10.12) for the OVM leads to $v_e(s) = v_{\text{opt}}(s)$, i.e., the microscopic fundamental diagram Fig. 10.4 is given by the OV function. The macroscopic fundamental diagram (Fig. 10.5) is obtained from the microscopic one using Eq. (10.16). In particular, the OV function (10.22) results in the triangular macroscopic fundamental diagram

$$Q_e(\rho) = \min\left(v_0\rho, \frac{1-\rho(l+s_0)}{T}\right).$$

10.7 Full Velocity Difference Model

By extending the OVM with an additional linear stimulus for the speed difference, one obtains the *Full Velocity Difference Model* (FVDM):

$$\boxed{\dot{v} = \frac{v_{\text{opt}}(s) - v}{\tau} - \gamma\,\Delta v \quad \text{Full Velocity Difference Model.}} \tag{10.23}$$

As in the OVM, the steady-state equilibrium is directly given by the optimal velocity function v_{opt}. When assuming suitable values for the speed difference sensitivity γ of the order of $0.6\,\text{s}^{-1}$, the FVDM remains accident-free for speed adaptation times of the order of several seconds. Furthermore, the fact sheet (Fig. 10.6) shows that the waves in the congested region of the freeway scenario are more realistic than that of the OVM, although the wavelengths remain too short. Furthermore, the accelerations remain in a realistic range.

However, in contrast to the OVM, the Full Velocity Difference Model is not *complete* in the sense defined at the beginning of Sect. 10.1, i.e., it is not able to describe all traffic situations. The reason is that the term $\gamma\,\Delta v$ describing the sensitivity to speed difference in Eq. (10.23) does not depend on the gap. Consequently, a slow vehicle (or a red traffic light corresponding to a standing virtual vehicle) leads to a significant decelerating contribution even if it is miles away. Thus, simulated vehicles do not reach their desired speed even on a long road with no other vehicles.