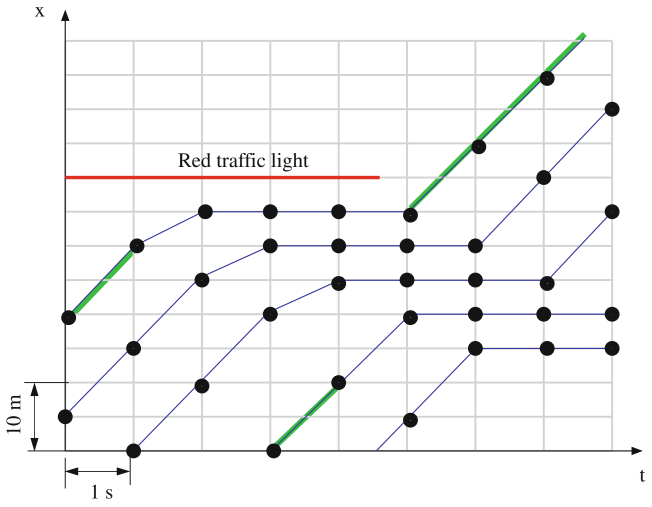


Fig. 10.8 Trajectory plot of the OVM with the triangular fundamental diagram ($l_{\text{eff}} = 5$ m, $v_0 = 10$ m/s, $T = \tau = 1$ s) with an update time $\Delta t = 1$ s, and using Eq. (10.26) for the positional update. Shown are vehicles approaching a traffic light (*red line*) turning *green* at $t = 4.5$ s (*end of the line*). The vehicle trajectories correspond to free traffic if drawn *green*, and to bound traffic, otherwise

which will be discussed in Chap. 12 in more detail. This model has several interesting properties which we now investigate further.

Relation to the Optimal Velocity Model. According to Eq. (10.11), Newell's model is mathematically equivalent to the OVM (10.19) in the car-following regime (bound traffic) if one sets $\tau = T$ and updates the OVM speed according to the explicit integration scheme (10.9) and the vehicle positions by the simple *Euler scheme*[20]

$$x_\alpha(t + \Delta t) = x_\alpha(t) + v_\alpha(t + \Delta t)\,\Delta t. \tag{10.26}$$

As a consequence, the parameter T of Newell's model has the additional meaning of a speed adaptation time τ.

Figure 10.8 shows that this equivalence only applies for the triangular fundamental diagram and only in the bound traffic regime, i.e., for gaps s satisfying $v_e(s) < v_0$ or $s < s_0 + v_0 T$. Otherwise, discretization errors are present.

Generally, the OVM is updated with time steps significantly smaller than the adaptation time. However, this does not invalidate the reasoning above, at least, qualitatively. In any case, the steady-state equilibria of the two models are equivalent.

[20] We emphasize that the usual second-order "ballistic" update scheme (10.8) may not be applied, here.

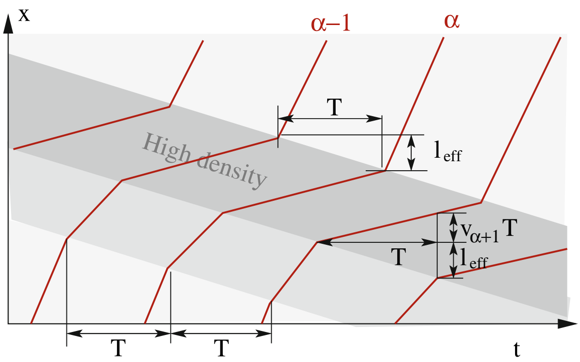


Fig. 10.9 Relation between Newell's model and the Section-Based Model: The *shaded regions* represent the evolution of the macroscopic local density. The gradient $v_e(\rho) = (1/\rho - l_{\text{eff}})/T$ corresponds to the local speed

Relation to the macroscopic Section-Based Model. When disaggregating the solutions of the Section-Based Model (8.2) with the function (8.11) by generating trajectories from the density and speed fields of congested traffic using the macro-micro relation (8.23), these trajectories are simultaneously solutions of Newell's model (as illustrated by Fig. 10.9).

Interpretation from the driver's point of view. The trajectories corresponding to the solution of Newell's model for congested traffic shown in Fig. 10.9 are given by the recursive relations

$$\begin{aligned} x_\alpha(t+T) &= x_{\alpha-1}(t) + wT = x_{\alpha-1}(t) - l_{\text{eff}}, \\ v_\alpha(t+T) &= v_{\alpha-1}(t). \end{aligned} \tag{10.27}$$

This means that the trajectory of the follower is completely determined by the trajectory of the leading vehicle.

In Newell's car-following model, the position of a vehicle following another vehicle at time $t + T$ is given by the position of the leader at time t minus the (effective) vehicle length l_{eff}. As a corollary, the speed profile of a vehicle *exactly* reproduces that of its leader with a time delay T.

The different meanings of the parameter T. From the above considerations we conclude that the parameter T of Newell's model can be interpreted in four different ways:

1. As the *reaction time* when interpreting Eq. (10.25) as a time-delay differential equation or when considering the trajectories (10.27).

2. As the *time gap* of the microscopic fundamental diagram (10.22).
3. As the *speed adaptation time* following from the equivalence between Newell's model and the OVM combined with speed update rule (10.9).
4. And as the *numerical update time* $T = \Delta t$ when interpreting Eq. (10.25) as a discrete-time model.

The interpretation in terms of a reaction time or a time gap can only be applied for congested traffic. In contrast, the interpretation as a speed adaptation time or a numerical update time is generally valid.

Relation to the macroscopic Payne's model. Besides illustrating another interesting property of Newell's model, this paragraph shows how to derive macroscopic from microscopic traffic flow models by the micro–macro relations between the vehicle speed and the local speed field and between distance and local density, respectively, and by first-order Taylor expansions.

Left-hand side of Newell's model equation. In deriving a macroscopic equivalent of $v_\alpha(t+T)$, we start with the expression (10.17) for the macroscopic local speed. For points (x,t) lying on the trajectory of vehicle α, we have

$$v_\alpha(t) = V(x_\alpha(t), t) = V(x, t). \tag{10.28}$$

Furthermore, the change in position and speed during one update time step $\Delta t = T$ is expressed in terms of local macroscopic fields by a Taylor expansion up to first order,

$$\begin{aligned} v_\alpha(t+T) &= V(x_\alpha + v_\alpha T, t+T) \\ &= V(x_\alpha, t) + \frac{\partial V(x,t)}{\partial x} v_\alpha T + \frac{\partial V(x,t)}{\partial t} t \\ &= V(x,t) + \left(V(x,t)\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} \right) T. \end{aligned} \tag{10.29}$$

Right-hand side of Newell's car-following model equation. First, we apply the micro–macro relation between the optimal velocity function and the macroscopic speed-density relation, $v_{\text{opt}}(s) = v_e(s) = V_e(\rho)$. In a second step, the local density ρ in the argument is determined such that the approximation error is minimal. Since $d_\alpha = s_\alpha + l_{\alpha-1} = 1/\rho$ denotes the distance *between* the vehicles α and $\alpha - 1$, we evaluate ρ at the intermediate location $x_\alpha + d_\alpha/2 = x + d_\alpha/2$ (Fig. 10.10) and consistently express everything up to first order by macroscopic local quantities,

$$\begin{aligned} v_{\text{opt}}(s_\alpha(t)) &= V_e\left(\rho\left(x + d_\alpha/2, t\right)\right) \\ &= V_e(\rho(x,t)) + V_e'(\rho)\frac{\partial \rho}{\partial x}\frac{d_\alpha}{2} \\ &= V_e(\rho(x,t)) + \frac{V_e'(\rho)}{2\rho}\frac{\partial \rho}{\partial x}. \end{aligned} \tag{10.30}$$

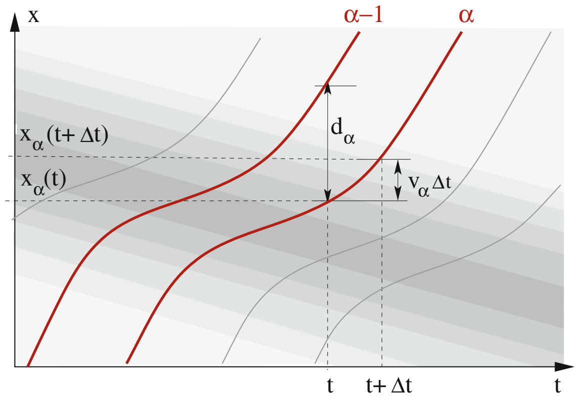


Fig. 10.10 Illustration of the derivation of Payne's model from the (*generalized*) Newell's model. Shown are the trajectories $x_\alpha(t)$ and $x_{\alpha-1}(t)$ of the subject and leading vehicles, respectively, and the associated local density (*shaded*). The error of the micro–macro transition is minimal when defining the local density at as $x_\alpha + d_\alpha/2$ as the inverse of the distance Δx_α

In the second line we have applied the first-order Taylor expansion, and the chain rule. In the third line, we have expressed $d_\alpha/2$ by $\frac{1}{2\rho(x,t)}$.[21] Equating (10.29) with Eq. (10.30) leads to

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = \frac{V_e(\rho) - V}{T} + \frac{V_e'(\rho)}{2\rho T}\frac{\partial \rho}{\partial x}. \tag{10.31}$$

When identifying $T = \tau$ this corresponds to Payne's model (9.18). We conclude that Newell's model and its extensions to other fundamental diagrams are always *approximatively* equivalent to Payne's model. Simultaneously, Newell's model is exactly equivalent to the Section-Based Model if restricting conditions apply (traffic in the car-following regime, triangular fundamental diagram).

Relation of Newell's model with anticipation to the FVDM. In order to compensate for at least part of the reaction time delay described by Newell's model, a driver would try to predict the distance gap (the only exogenous stimulus of Newell's model) by a certain time interval T_a into the future. Using the rate of change $\dot{s} = -\Delta v$ for an estimate of the gap at this time, $\hat{s}(t + T_a) = s(t) - T_a \Delta v$, this results in the *generalized Newell's model*

$$v(t+T) = v_{\text{opt}}\left(s(t) - T_a \Delta v\right) \approx v_{\text{opt}}(s(t)) - v'_{\text{opt}}(s) T_a \Delta v. \tag{10.32}$$

According to Eq. (10.11) this is equivalent to a time-continuous model given by

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{v_{\text{opt}}(s) - v}{T} - \frac{T_a v'_{\text{opt}}(s)}{T}\,\Delta v.$$

[21] The displacement d_α appears only in terms that are already of first order. Therefore, we only need the zeroth order relation $d_\alpha(t) = 1/\rho(x,t)$.

This corresponds to a Full Velocity Difference Model with a gap dependent sensitivity $\gamma(s) = T_a v'_{\text{opt}}(s)/T$. From the OV plausibility conditions (10.20) it follows that $\lim_{s\to\infty} v'_{\text{opt}}(s) = 0$, i.e., the sensitivity tends to zero when the gap becomes sufficiently large (for the triangular fundamental diagram it is exactly zero for $s > s_0 + v_0 T$). This means, the resulting FVDM-like model is complete, similarly to the "improved" FVDM presented in Sect. 10.7.

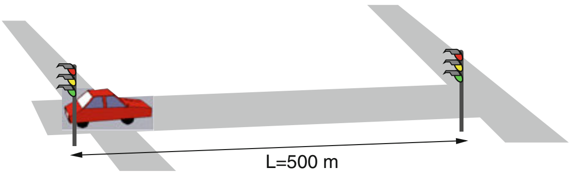
Problems

10.1 Dynamics of a single vehicle approaching a red traffic light

A single car in city-traffic conditions can be described by the following time-continuous acceleration model:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \begin{cases} \frac{v_0 - v}{\tau} & \text{if } \Delta v \le \sqrt{2b(s - s_0)}, \\ -b & \text{otherwise.} \end{cases}$$

Here, s denotes the distance to the next car or the next traffic light (whichever is nearer), and Δv is the approaching rate. A red traffic light is modeled by a virtual standing vehicle of zero dimension at the stopping line which is removed when the light turns green.



1. What is the meaning of the model parameters v_0, τ, s_0, and b? Describe the qualitative acceleration profile after the initially standing car starts moving, and the deceleration profile when approaching a red traffic light. Which essential human property is not taken care of by this model?
2. The first traffic light turns green at $t = 0$ s. Calculate the speed and the acceleration as a function of time for general model parameters assuming that the second traffic light is always green.
3. Consider now a situation where the subject car is approaching a red traffic light with cruising speed $v_0 = 50$ km/h assuming $s_0 = 2$ m and $b = 2$ m/s^2. At which distance to the traffic light does the driver initiate his or her braking maneuver? What is the braking deceleration and the final distance of the standing car to the stopping line?
4. Calculate the trajectory and the speed profile of the car during the complete start-stop cycle for a distance of 500 m between the stopping lines of the two traffic lights assuming $\tau = 5$ s and values for the other model parameters as above.