Derivation of the Magnetic Field Inside a Finite Solenoid

1 Introduction

We derive the magnetic field $B(z)$ inside a solenoid of finite length L, inner radius r, and outer radius R using the Biot-Savart law.

2 Magnetic Field Due to a Thin Solenoid

For a single-radius solenoid of radius a, the field along the axis is given by:

$$B_a(z) = \frac{\mu_0 n I}{2} \left[\frac{L/2 + z}{\sqrt{a^2 + (L/2 + z)^2}} + \frac{L/2 - z}{\sqrt{a^2 + (L/2 - z)^2}} \right]. \tag{1}$$

3 Averaging Over the Radial Thickness

For a solenoid with thickness from r to R, we integrate over all current loops:

$$B(z) = \frac{1}{R - r} \int_r^R B_a(z)\, da. \tag{2}$$

Using the integral identity:

$$\int \frac{da}{\sqrt{a^2 + C^2}} = \ln\left(a + \sqrt{a^2 + C^2}\right), \tag{3}$$

we obtain:

$$\int_r^R \frac{da}{\sqrt{a^2 + C^2}} = \ln\left(\frac{R + \sqrt{R^2 + C^2}}{r + \sqrt{r^2 + C^2}} \right). \tag{4}$$

Applying this result to both terms in $B_a(z)$, we derive the final expression:

$$B(z) = \frac{\mu_0 n I}{R - r} \ln\left(\frac{(R + \sqrt{R^2 + (L/2 + z)^2})(R + \sqrt{R^2 + (L/2 - z)^2})}{(r + \sqrt{r^2 + (L/2 + z)^2})(r + \sqrt{r^2 + (L/2 - z)^2})} \right). \tag{5}$$