Alaskan Brown Bears and Pacific Salmon Facing the Effects of Global Warming

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Overview

Climate Change

James Hansen—1988

In 1988, James Hansen gave his testimony to Congress on the dangers of climate change. He warned the country about the disastrous weather that will come from global warming.

Polar Bears

Climate warming is causing sea ice to melt, resulting in shorter time periods for polar bears to hunt seals [11].

Koala Bears

Droughts and Heat waves cause extreme declines for the koala bear population. With future predictions of global warming, it is likely they will lose their habitats and face a significant decrease in the number of koala bears [1].

Pacific Salmon Exponential Growth

- 5 salmon make it to adulthood [13].
- 32% survive commercial harvesting each year [9].
- Salmon die shortly after spawning [10].

Consider the exponential equation

$$x(t) = x_0(0.32 * 5)^t, \tag{1}$$

where x represents the salmon population at time t, in years, x_0 is the initial population of salmon.

Assumptions:

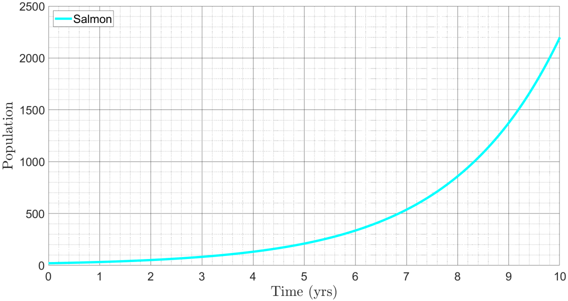
1. Salmon return each year to reproduce.
2. Each salmon that survive commercial harvesting will survive spawning migration.
3. Salmon are not affected by any other variables.

Exponential Differential Equation

The exponential salmon growth equation can be written as a first-order differential equation, as shown below,

$$\frac{dx}{dt} = \frac{d}{dt} x_0 (0.32 * 5)^t = \frac{d}{dt} x_0 e^{\ln(0.32*5)t} = r_x x, \tag{2}$$

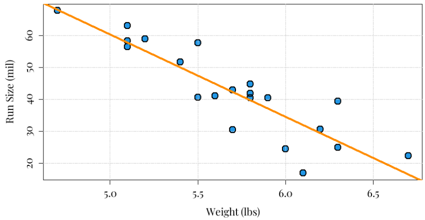
where $r_x = \ln(0.32 * 5)$ is our estimated growth rate for the salmon population. Let $x_0 = 20$ as a starting point.



Salmon Carrying Capacity

Table: Sockeye Comparison Between Weight and Run Size in Bristol Bay

Year	Weight (lbs)	Run (mil)
2001	6.7	22.3
2002	6.1	16.9
2003	6.3	24.9
2004	5.8	41.9
⋮	⋮	⋮
2017	5.5	57.6
2018	5.1	63.0
2019	5.1	56.4
2020	5.1	58.3
2021	4.7	67.7



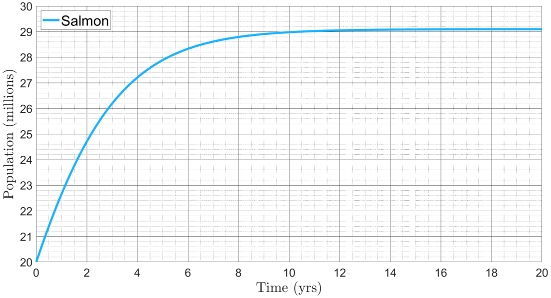
The variables, sockeye salmon run size and their average weight per run, have a strong negative linear correlation of -0.88. We use this correlation to determine a carrying capacity of 29.1 million salmon.

Logistic Growth ODE

Consider the logistic population growth equation

$$\frac{dx}{dt} = r_x x \left(1 - \frac{x}{K_x}\right), \tag{3}$$

where $r_x = \ln(0.32 * 5)$ is the growth rate from the exponential model and $K_x = 29.1 * 10^6$ represents the carrying capacity of the salmon species. Let $x_0 = 20 * 10^6$ as our new starting point.



Alaskan Brown Bears

Alaskan brown bear estimated growth rates:

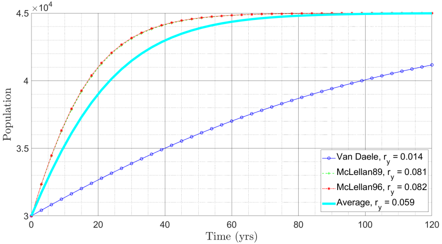
1. $r_y = 0.014$, by Victor G. Barnes and Lawerence J. Van Daele using a deterministic model [3].
2. $r_y = 0.081$, by Bruce McLellan using the Lotka equation [6].
3. $r_y = 0.082$, by Fredrick Hovey and Bruce McLellan using the Lotka equation with bootstrap [5].
4. $r_y = 0.059$, Average of the 3 growth rates above.

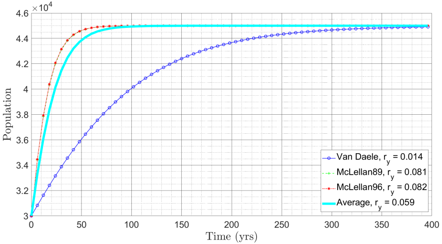
Using the logistic growth equation again for the brown bear species, we get

$$\frac{dy}{dt} = r_y y \left(1 - \frac{y}{K_y}\right), \tag{4}$$

where y represents the brown bear population at time t, in years, r_y is the growth rate, and K_y is the carrying capacity.

Compare Growth Rates





- Let the initial population be 30,000 and the carrying capacity be 45,000 [2].

Time span of reaching carrying capacity:

- Using McLellan's growth rates show a time span of approximately 80 years.
- Barnes' and Van Daele's growth rate shows about 400 years.
- The Average of the growth rates estimates a time span of around 100 years.

Base Models for Both Species

To recap, the base model for the pacific salmon is

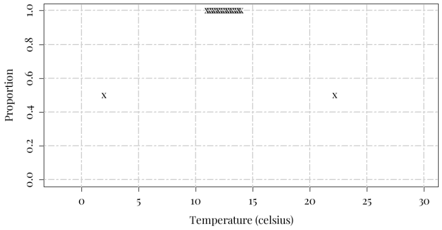
$$\frac{dx}{dt} = r_x x \left(1 - \frac{x}{K_x}\right),$$

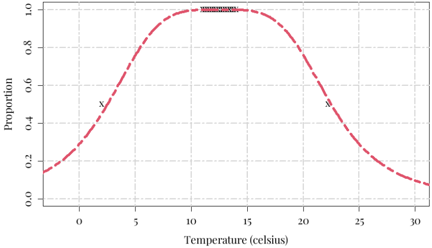
where $r_x = \ln(0.32 * 5)$ and $K_x = 29.1 * 10^6$. The base model for Alaskan brown bears is

$$\frac{dy}{dt} = r_y y \left(1 - \frac{y}{K_y}\right),$$

where $r_y = 0.059$ and $K_y = 45,000$.

Salmon Survival Depends on Temperature





- Using articles by Dr. Phyllis Weber Scannell and Katherine Carter, we estimate lethal and optimal temperature ranges for salmon during spawning migration [12, 4].
- 100% chance of survival for the optimal temperature range.
- 50% chance of survival at the beginning of the lethal temperatures.

$$P(T) = \frac{1}{1 + 10^{-4}(T - 12.5)^4}. \quad (5)$$

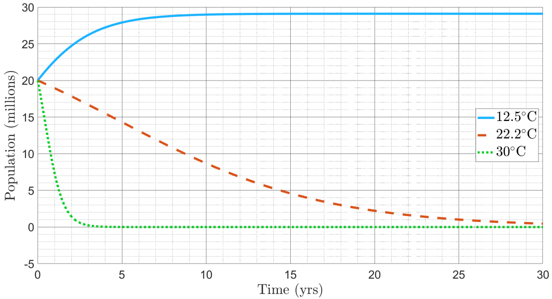
Growth Rate Depends on Temperature

Incorporating the survival proportion function with our growth rate $r_x = \ln(0.32 * 5)$, we get

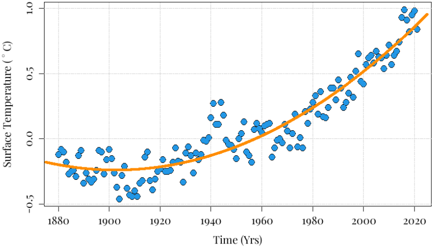
$$R(T) = \ln(0.32 * 5 * P(T)) = \ln\left(\frac{0.32 * 5}{1 + 10^{-4}(T - 12.5)^4}\right), \tag{6}$$

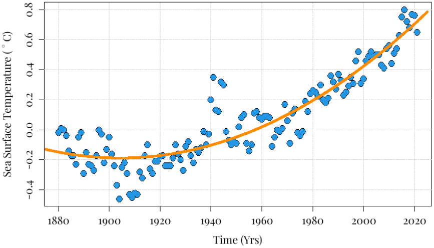
where the salmon population model is now

$$\frac{dx}{dt} = R(T)x\left(1 - \frac{x}{K_x}\right). \tag{7}$$



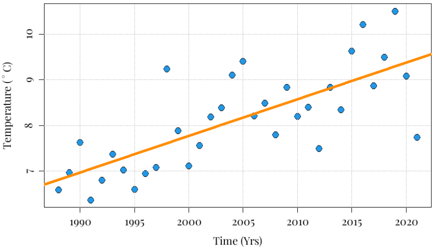
Temperature Changing





- Average annual temperatures compared to the 20th century average of 13.9° [8].
- Fitted with polynomial regression models.
- The sea surface anomalies are similar to the global surface anomalies.
- After 1970, the sea surface temperatures (SST) appear more linear than polynomial or exponential.

Linear Trend



- Sampled 5 rivers in Alaska that have consistently recorded data for at least 15 years.
- Average annual water temperatures during spawning months.
- Fit the data with a linear regression line,

$$T(t) = 0.08t + 9.54. \tag{8}$$

Replacing Parameter T with $T(t)$

As a reminder, the current growth rate function depends on temperature,

$$R(T) = \ln(0.32 * 5 * P(T)) = \ln\left(\frac{0.32 * 5}{1 + 10^{-4}(T - 12.5)^4}\right).$$

When replacing the temperature parameter with the temperature linear regression model, we get

$$\begin{aligned} R(T(t)) = \ln\left[0.32 * 5 * P(T(t))\right] &= \ln\left(\frac{0.32 * 5}{1 + 10^{-4}(T(t) - 12.5)^4}\right) \\ &= \ln\left(\frac{0.32 * 5}{1 + 10^{-4}(0.08t - 2.96)^4}\right). \end{aligned} \tag{9}$$

Growth Rate Depends on Time I

As a reminder, the stochastic growth rate, $r_x = \ln(0.32 * 5)$, was found after taking the derivative of exponential population equation, $x(t) = x_0 e^{\ln(0.32*5)t}$. So, by replacing r_x with $R(t)$ we get

$$x(t) = x_0 e^{R(t)t}.$$

Then, taking the derivative produces the following,

$$\frac{dx}{dt} = \big[R(t) + R'(t)t\big] x_0 e^{R(t)t} = \big[R(t) + R'(t)t\big] x,$$

where

$$R'(t) = \frac{d}{dt} \ln(0.32 * 5 * P(t)) = \frac{P'(t)}{P(t)}.$$

Since

$$P(t) = \frac{1}{1 + 10^{-4}(0.08t - 2.96)^4},$$

Growth Rate Depends on Time II

then

$$P'(t) = \frac{-4 * 10^{-4} * 0.08(0.08t - 2.96)^3}{\left(1 + 10^{-4}(0.08t - 2.96)^4\right)^2}.$$

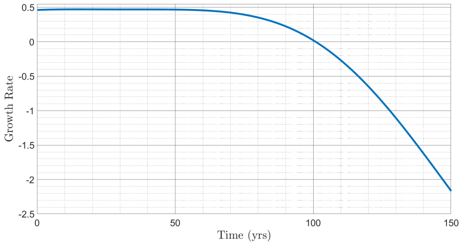
So, the growth rate function can now be written as

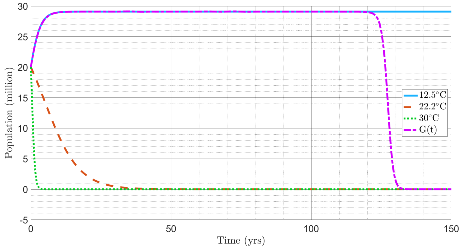
$$G(t) = \ln\left(\frac{0.32 * 5}{1 + 10^{-4}(0.08t - 2.96)^4}\right) - \frac{4*10^{-4}*0.08t(0.08t-2.96)^3}{1+10^{-4}(0.08t-2.96)^4}, \tag{10}$$

or in a more simplified form,

$$G(t) = R(t) + \frac{P'(t)t}{P(t)}. \tag{11}$$

Salmon Model with Proposed Growth Rate Function





- Incorporating the growth rate function, G(t), in the salmon model produces

$$\frac{dx}{dt} = G(t)x\left(1 - \frac{x}{K_x}\right). \quad (12)$$

- Using the **vpasolve** function in MATLAB, we found that the growth rate becomes negative after approximately 101 years, causing the salmon population to decrease.

$$T(101) = 17.62^{\circ}\text{C}$$

Models Recap

To recap, we have the base model for salmon where they are not affected by temperature,

$$\frac{dx}{dt} = r_x x \left(1 - \frac{x}{K_x}\right),$$

where $r_x = \ln(0.32 * 5)$ and $K_x = 29.1 * 10^6$. The model where salmon are affected by temperature is

$$\frac{dx}{dt} = G(t)x \left(1 - \frac{x}{K_x}\right), \tag{13}$$

where $G(t) = R(t) + \dfrac{P'(t)}{P(t)}$ and $K_x = 29.1 * 10^6$. The base model for Alaskan brown bears is

$$\frac{dy}{dt} = r_y y \left(1 - \frac{y}{K_y}\right),$$

where $r_y = 0.059$ and $K_y = 45,000$.

Introducing Interaction I

The Lotka-Volterra equations,

$$\begin{aligned} \frac{dx}{dt} &= \alpha x - \beta xy, \\ \frac{dy}{dt} &= \delta xy - \gamma y, \end{aligned} \tag{14}$$

are commonly used to describe organic systems where two species interact with each other. To implement logistic growth with interaction, Theodore Modis' developed a competitive predator-prey model,

$$\begin{aligned} \frac{dx}{dt} &= a_x x - b_x x^2 + c_{xy} xy, \\ \frac{dy}{dt} &= a_y y - b_y y^2 + c_{yx} xy, \end{aligned} \tag{15}$$

where a_x, a_y, b_x, b_y, c_{xy} and c_{yx} are real parameters that describe the interaction of the two species [7].

Introducing Interaction II

First, we use a variation of Theodore Modis' equations to introduce interaction into our model when neither species is affected by temperature,

$$\begin{aligned} \frac{dx}{dt} &= r_x x\left(1-\frac{x}{K_x}\right) - c_{xy}xy = r_x x - \frac{r_x x^2}{K_x} - c_{xy}xy, \\ \frac{dy}{dt} &= r_y y\left(1-\frac{y}{K_y}\right) + c_{yx}xy = r_y y - \frac{r_y y^2}{K_y} + c_{yx}xy, \end{aligned} \tag{16}$$

where $r_x = \ln(0.32 * 5),\ r_y = 0.059,\ K_x = 15,\ K_y = 5$, and $c_{xy},\ c_{yx} > 0$. Then, substituting $a_x = r_x$, $b_x = \frac{r_x}{K_x}$, $a_y = r_y$, and $b_y = \frac{r_y}{K_y}$ we get

$$\begin{aligned} \frac{dx}{dt} &= a_x x - b_x x^2 - c_{xy}xy, \\ \frac{dy}{dt} &= a_y y - b_y y^2 + c_{yx}xy. \end{aligned} \tag{17}$$

Critical Points

Setting $\frac{dx}{dt}$ and $\frac{dy}{dt}$ equal to zero, we get

$$\begin{aligned} 0 &= x(a_x - b_x x - c_{xy} y), \\ 0 &= y(a_y - b_y y + c_{yx} x). \end{aligned}$$

Then, we solve for x and y to get the following critical points;

$$\begin{aligned} x_1^* &= 0, & y_1^* &= 0, \\ x_2^* &= \tfrac{a_x}{b_x} = K_x, & y_2^* &= 0, \\ x_3^* &= 0, & y_3^* &= \tfrac{a_y}{b_y} = K_y, \\ x_4^* &= \frac{a_x b_y - c_{xy} a_y}{c_{xy} c_{yx} + b_x b_y}, & y_4^* &= \frac{a_x c_{yx} + b_x a_y}{c_{xy} c_{yx} + b_x b_y}. \end{aligned}$$

Characteristic Polynomial of the Jacobian I

We begin by constructing the Jacobian matrix,

$$J_{(x,y)} = \begin{pmatrix} a_x - 2b_x x - c_{xy} y & -c_{xy} x \\ c_{yx} y & a_y - 2b_y y + c_{yx} x \end{pmatrix}. \tag{18}$$

Then, we derive the characteristic polynomial,

$$\begin{aligned} \det(J_{(x,y)} - \lambda I) = \lambda^2 & \\ & - \left[(a_x - 2b_x x - c_{xy} y) + (a_y - 2b_y y + c_{yx}) \right] \lambda \\ & + \left[(a_x - 2b_x x - c_{xy} y)(a_y - 2b_y y + c_{yx}) \right. \\ & \left. + c_{xy} x c_{yx} y \right]. \end{aligned} \tag{19}$$

Note that the trace and determinant of the Jacobian matrix are

$$\begin{aligned} \mathrm{T} &= \operatorname{tr}(J_{(x,y)}) = (a_x - 2b_x x - c_{xy} y) + (a_y - 2b_y y + c_{yx}), \\ \mathrm{D} &= \det(J_{(x,y)}) = (a_x - 2b_x x - c_{xy} y)(a_y - 2b_y y + c_{yx}) + c_{xy} x c_{yx} y. \end{aligned}$$

Characteristic Polynomial of the Jacobian II

Substituting the above variables in the characteristic polynomial produces

$$\det(J_{(x,y)} - \lambda I) = \lambda^2 - \mathrm{T}\lambda + \mathrm{D}.$$

Now, solving for our eigenvalues, λ, gives the below equation,

$$\lambda = \frac{\mathrm{T} \pm \sqrt{\mathrm{T}^2 - 4\mathrm{D}}}{2}.$$

Using the equation above, we can determine the signs of the real and imaginary parts of the eigenvalues, providing insight into the stability near each critical point.

Stability Near (0, 0)

Starting with the first critical point, (0, 0), we get

$$\mathrm{T} = a_x + a_y,$$
$$\mathrm{T}^2 - 4\mathrm{D} = (a_x - a_y)^2.$$

So, substituting in the values for a_x and a_y, we get

$$\mathrm{T} = \ln(0.32 * 5) + 0.059 \approx 0.529,$$
$$\mathrm{T}^2 - 4\mathrm{D} = (\ln(0.32 * 5) - 0.059)^2 \approx 0.169.$$

Since $\mathrm{T} \approx 0.529 > 0.411 \approx \sqrt{\mathrm{T}^2 - 4\mathrm{D}}$, both eigenvalues are positive real values, which implies that the first critical point is an unstable node.

Stability Near $\left(\frac{a_x}{b_x},\ 0\right)$

For the second critical point, $\left(\frac{a_x}{b_x},\ 0\right)$, we get

$$\mathrm{T} = a_x\left(\frac{c_{yx}}{b_x} - 1\right) + a_y,$$
$$\mathrm{T}^2 - 4\mathrm{D} = \frac{(a_x(b_x + c_{yx}) + a_y b_x)^2}{b_x^2},$$

where the stability is dependent on c_{yx}. Substituting each parameter with its value allows us to develop the following criterion;

$$\mathrm{T} + \sqrt{\mathrm{T}^2 - 4\mathrm{D}} > 0,$$
$$\mathrm{T} - \sqrt{\mathrm{T}^2 - 4\mathrm{D}} < 0.$$

This results in the second critical point being a saddle point.

Stability Near $\left(0,\ \frac{a_y}{b_y}\right)$

Looking at the third critical point, $\left(0,\ \frac{a_y}{b_y}\right)$, the trace and discriminant are;

$$\mathrm{T} = a_x - \frac{a_y(b_y + c_{xy})}{b_y},$$
$$\mathrm{T}^2 - 4\mathrm{D} = \frac{(a_x b_y + a_y(b_y - c_{xy}))^2}{b_y^2},$$

where the stability is now dependent on c_{xy}. Substituting each parameter with its value allows us to develop the following criterion;

$$\mathrm{T} + \sqrt{\mathrm{T}^2 - 4\mathrm{D}} > 0 \quad \text{when} \quad c_{xy} < 0.094,$$
$$\mathrm{T} - \sqrt{\mathrm{T}^2 - 4\mathrm{D}} < 0.$$

From the signs of the eigenvalues, we determine that $\left(0,\ \frac{a_y}{b_y}\right)$ is a saddle point when $c_{xy} < 0.094$, and a stable node when $c_{xy} > 0.094$.

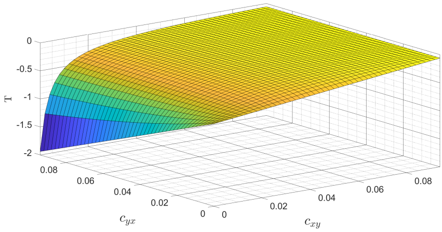
Stability Near $\left(\frac{a_x b_y - c_{xy} a_y}{c_{xy} c_{yx} + b_x b_y}, \frac{a_x c_{yx} + b_x a_y}{c_{xy} c_{yx} + b_x b_y}\right)$ I

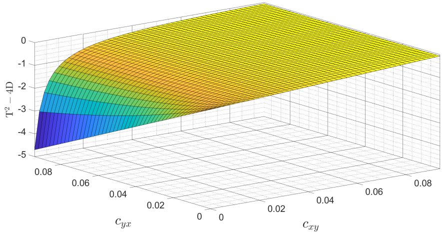
The last critical point has the following trace and discriminant;

$$\begin{aligned}
\mathrm{T} &= \frac{a_y b_x (c_{xy} - b_y) - a_x b_y (b_x + c_{yx})}{b_x b_y + c_{xy} c_{yx}}, \\
\mathrm{T}^2 - 4\mathrm{D} &= \frac{(a_y b_x (c_{xy} - b_y) - a_x b_y (b_x + c_{yx}))^2}{(b_x b_y + c_{xy} c_{yx})^2} \\
&\quad - \frac{4(a_x c_{yx} + a_y b_x)(a_x b_y - a_y c_{xy})(b_x b_y + c_{xy} c_{yx})}{(b_x b_y + c_{xy} c_{yx})^2},
\end{aligned}$$

where the stability is dependent on parameters, c_{xy} and c_{yx}. We construct the following constraints for the critical point;

$$x_4^* = \frac{a_x b_y - c_{xy} a_y}{c_{xy} c_{yx} + b_x b_y} > 0, \quad \text{and} \quad y_4^* = \frac{a_x c_{yx} + b_x a_y}{c_{xy} c_{yx} + b_x b_y} > 0.$$





We create the below criterion for c_{xy} and c_{yx};

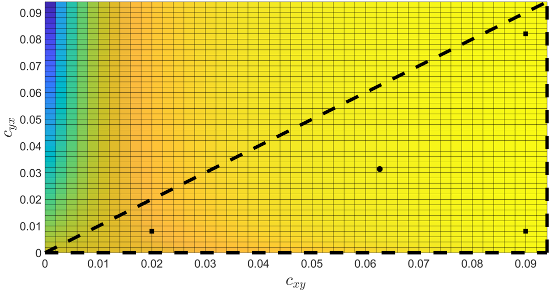
$$0 < c_{xy} < \frac{a_x b_y}{a_y} \approx 0.094,$$
$$c_{yx} > 0.$$

The brown bear population should have a higher effect on the salmon population. Therefore, the constraints for the parameters c_{xy} and c_{yx} are

$$0 < c_{xy} < 0.094,$$
$$0 < c_{yx} < c_{xy}.$$

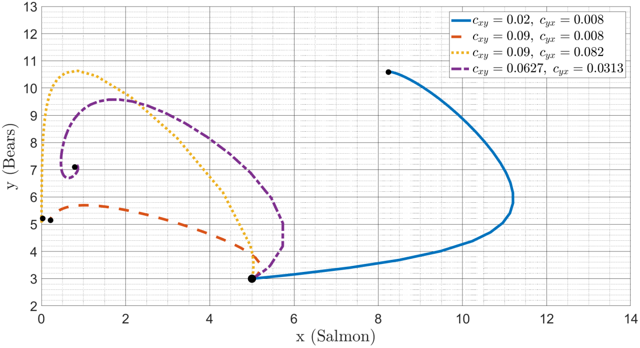
Stability Near $\left(\frac{a_x b_y - c_{xy} a_y}{c_{xy} c_{yx} + b_x b_y}, \frac{a_x c_{yx} + b_x a_y}{c_{xy} c_{yx} + b_x b_y}\right)$ III

This criterion forces the trace and discriminant to be negative, resulting in the last critical point being a stable spiral point.



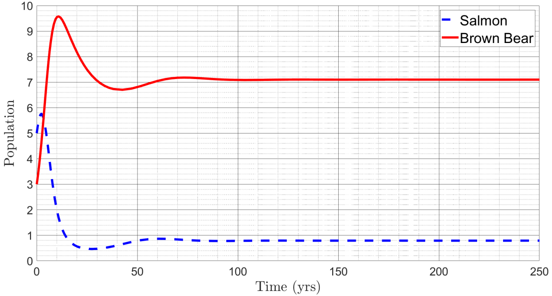
- The triangle outlines the region for viable interaction parameters.
- The values chosen for $(c_{xy},\ c_{yx})$ are $(0.02,\ 0.008)$, $(0.09,\ 0.008)$, $(0.09,\ 0.082)$, and $(0.0627,\ 0.0313)$.

Testing Different c_{xy} and c_{yx} Values



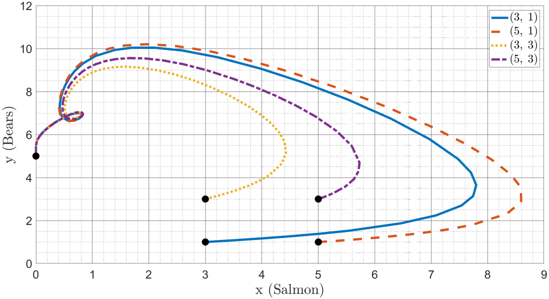
- Each of these parameters affects the location of the critical point, (x_4^*, y_4^*), as well as the oscillations of the populations.
- When the pair of parameters is equal to $(0.0627,\ 0.0313)$, the population oscillates and converges to its critical point $(0.79, 7.1)$.

Autonomous Model



- Solutions to the model with respect to time with $c_{xy} = 0.0627$ and $c_{yx} = 0.0313$.
- Both populations oscillate towards their equilibrium point, $(0.79, 7.1)$.

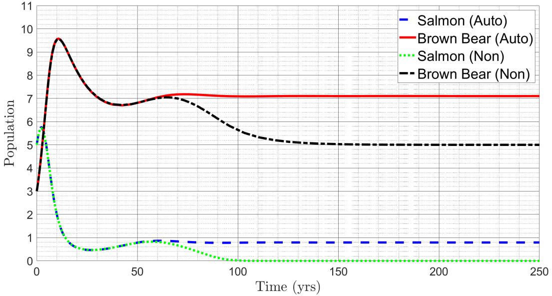
Non-Autonomous Model I



Now, including the effects of temperature, we get the following non-autonomous system of equations;

$$\begin{aligned} \frac{dx}{dt} &= G(t)x\left(1 - \frac{x}{K_x}\right) - c_{xy}xy, \\ \frac{dy}{dt} &= r_y y\left(1 - \frac{y}{K_y}\right) + c_{yx}xy. \end{aligned} \tag{20}$$

Non-Autonomous Model II



- According to our temperature function the projected Alaskan river temperature in 60 years is $T(60) \approx 14.34^{\circ}$C.
- Soon after the river temperature leaves the optimal range, the difference in the outcomes of the species' populations becomes prominent.

Concluding Remarks and Future Work

- We conclude that global warming could eventually cause the salmon population to go regionally extinct and the brown bear population to decrease in size to accommodate for the lack of a resource, assuming the brown bear population is not affected in any other way.
- Exploring different models to represent the species separately.
- More research in determining survival rates for migrating salmon at different temperatures.
- including a term to represent the change in harvest rates would significantly improve the accuracy of our model.
- We hope our model is used in further research to protect salmon and brown bears from the extreme weather that global warming will bring.

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