

MFx – Macroeconomic Forecasting

IMFx



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Cointegration and Vector Error Correction Models

Introduction



Main Objectives of the Module

- Introduce the concept of cointegration
- Study the dynamics of cointegrated variables
- Explain the different methods used to test for cointegration

Main Objectives of the Module

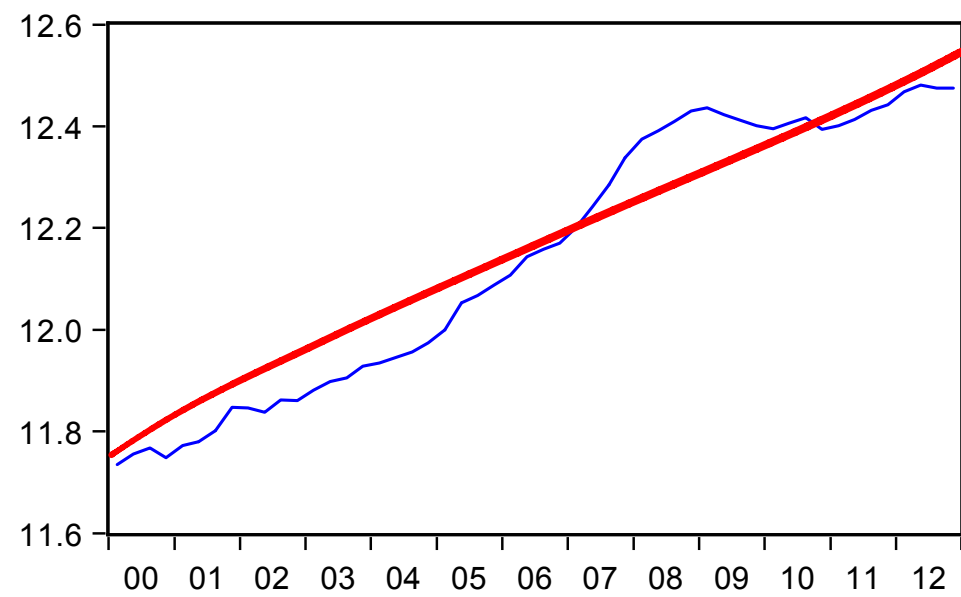
- Illustrate how to properly estimate a system of cointegrated variables
- Perform tests on the cointegration relationship

Money, Prices and GDP in Austria

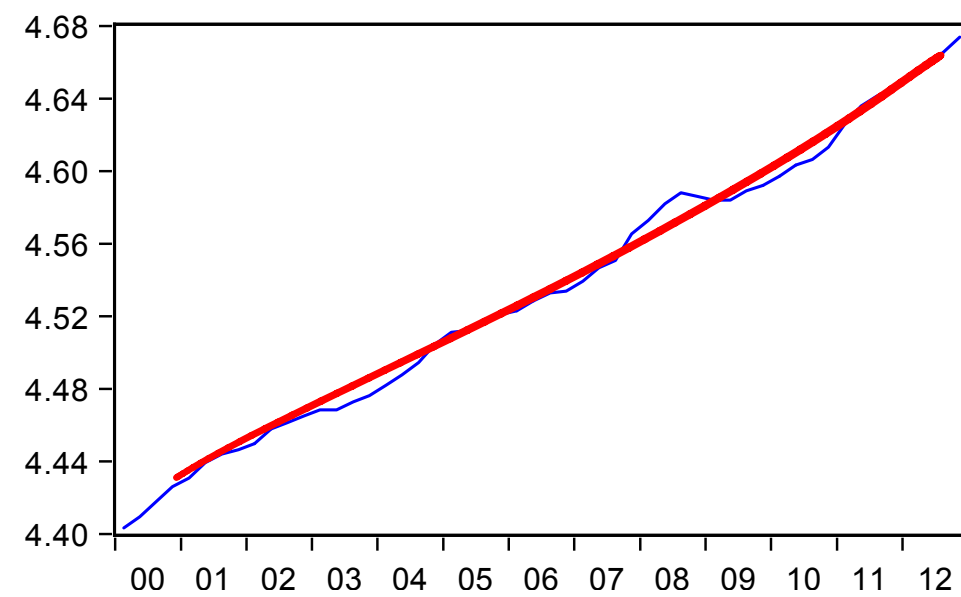
- In the data file you are given quarterly series of M2, CPI and real GDP of Austria between 1997Q3 and 2014Q4
- All series are seasonally adjusted
- Our goal is to forecast these variables over a period of 8 quarters based on a VAR

Let Us Look at the Data

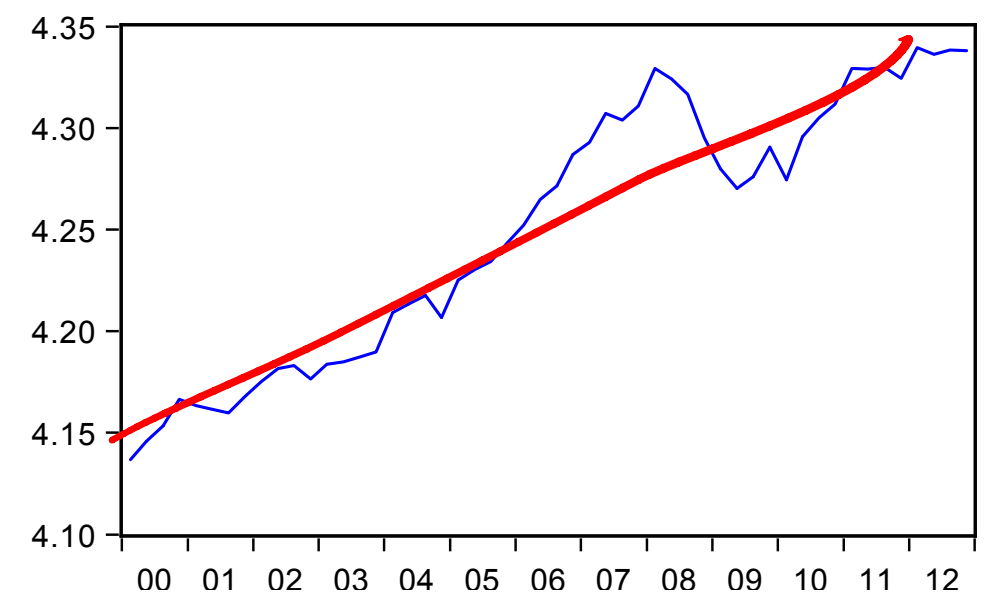
LOG(M)



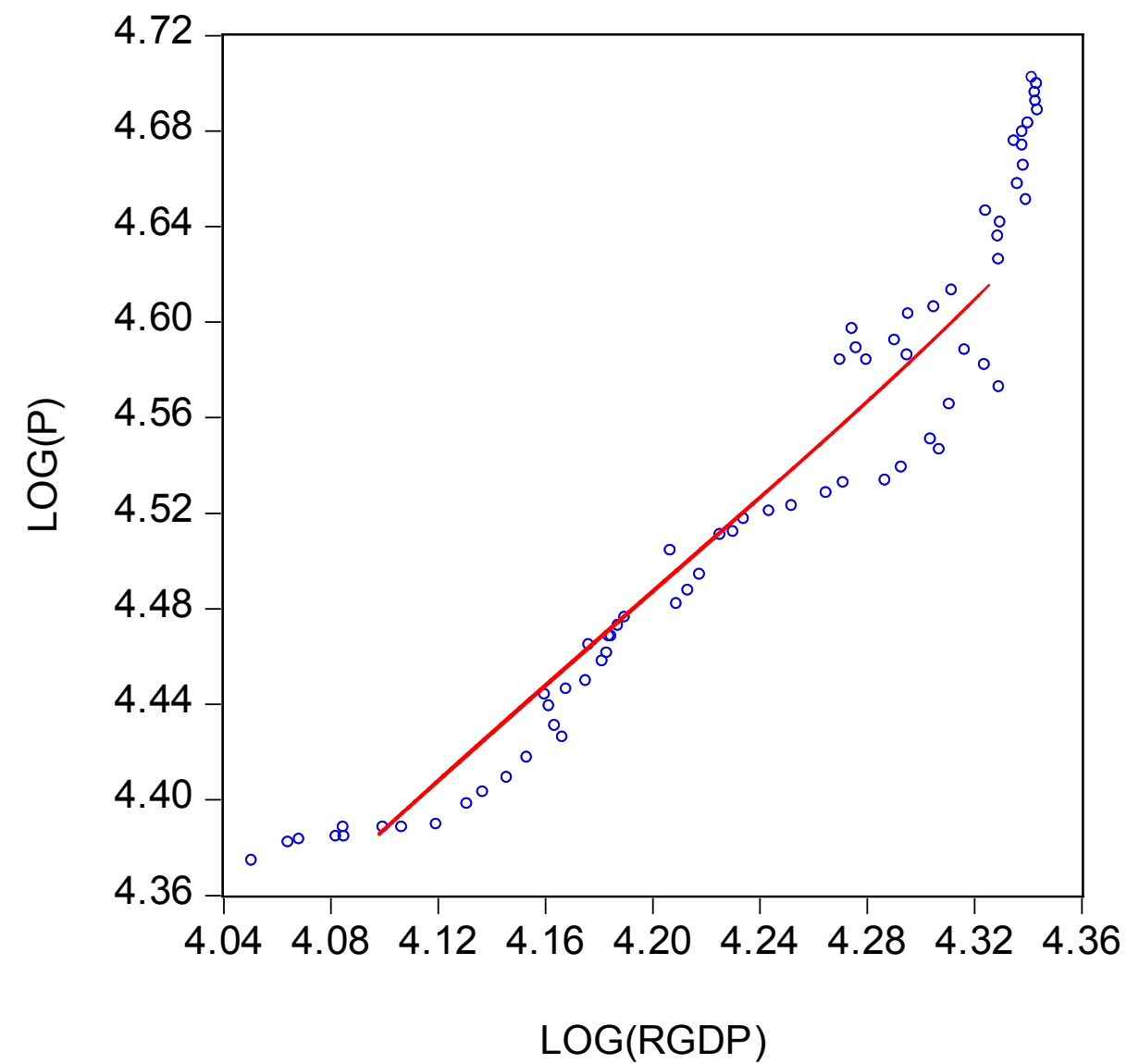
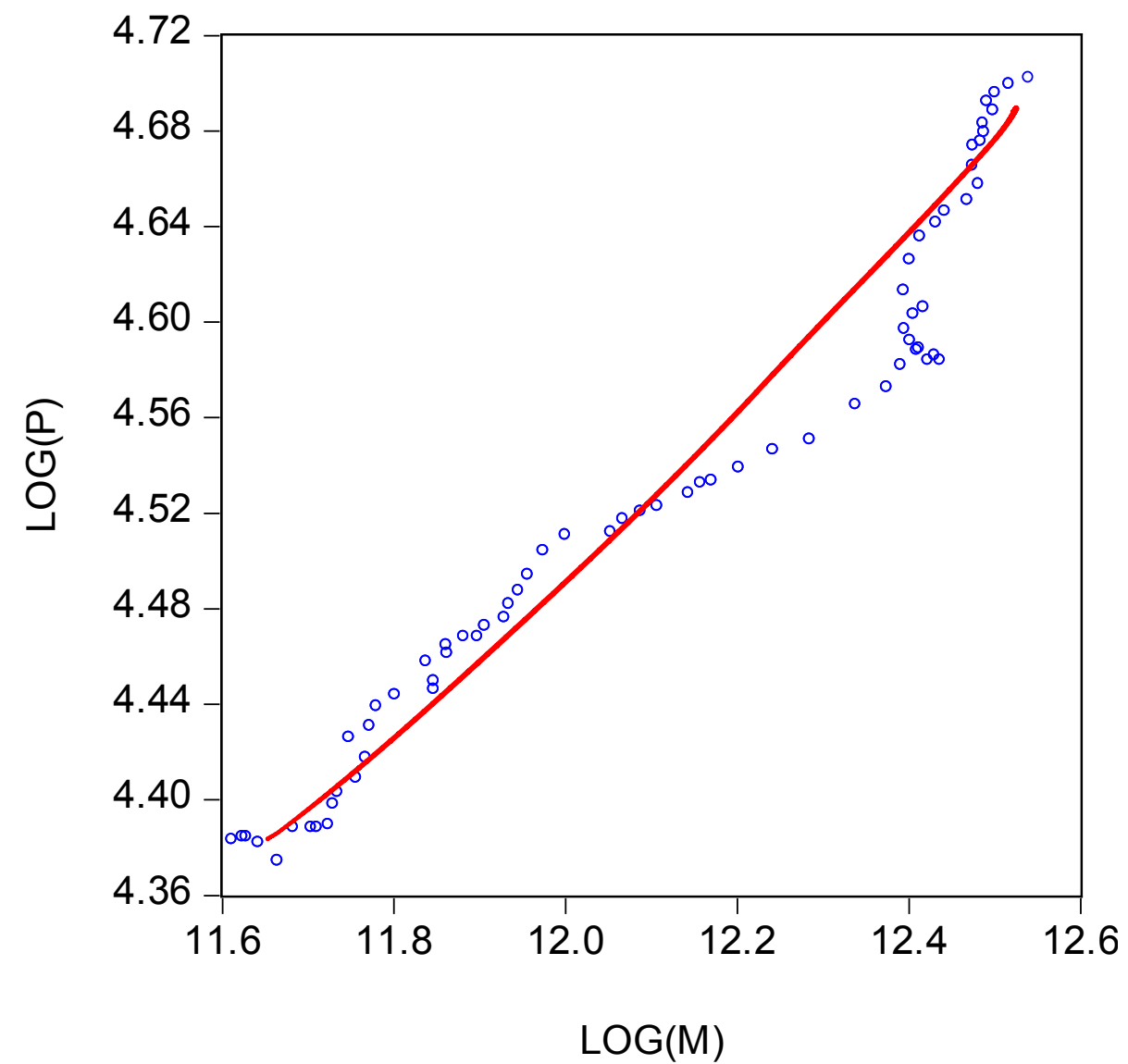
LOG(P)



LOG(RGDP)



Let Us Look at the Data More Closely



Linear Combinations of $I(1)$ series

- In general a linear combination of $I(1)$ series is also $I(1)$.
- However, in some special cases there could be a linear combination which is $I(0)$.
- So something very special has to happen for a linear combination to become stationary.

Intuition based on Theory

- Economic theories could give the intuition as to why some linear combinations of $I(1)$ series could be $I(0)$.
- For example: Uncovered Interest Parity, Permanent Income Hypothesis, Unbiased Forward Rate Hypothesis, Commodity Market Arbitrage and Purchasing Power Parity.
- Theory guides us but we can also test it.

What is Cointegration?

- $I(1)$ series are cointegrated if there exists at least one linear combination of these variables that is stationary
- In our case it means that there exist b_1 , b_2 and b_3 such that:
 $b_1m + b_2p + b_3y$ is stationary or $I(0)$ NOT $I(1)$

What Kind of VAR?

- We could estimate a VAR in levels...
- ...but as it contains $I(1)$ variables this could be problematic *less lag ← VAR in differences*
- How about a VAR in first differences? *in $I(1)$ variables*
- As we will see, this too could be highly problematic without proper testing *parameters to estimate*

What's Next?

- Why does it matter if M , P and $RGDP$ are cointegrated?
- If it makes the VAR in differences “inappropriate,” what should we do instead?
- How do we test for cointegration in the first place?

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Cointegration and Vector Error Correction Models

Error Correction



From Session 1: What is Cointegration?

- A number of $I(1)$ series are cointegrated if there exists a linear combination that would be stationary
- In our case it means that there exist b_1 , b_2 and b_3 such that:
$$b_1m + b_2p + b_3y \text{ is } I(0)$$

Does Theory Indicate Cointegration?

- The money market equilibrium could be:

$$m^s_t = m^d_t = \beta_0 + \beta_1 p_t + \beta_2 y_t + \beta_3 r_t + \underline{\underline{e_t}}$$

- However, what we observe includes an error...
- ...which should not be very persistent and its variance should not rise over time

Long-Run Relationships

- If $(b_1m + b_2p + b_3y)$ is stationary...
- ...although each series is non-stationary...
- ...there must be some adjustment made by m , p and y such that they move together
- such that deviations from $b_1m + b_2p + b_3y = 0$ remains bounded

Money and Price

- Suppose that in the long run:

$$m_t = \beta p_t + e_t, \text{ where } \beta > 0$$

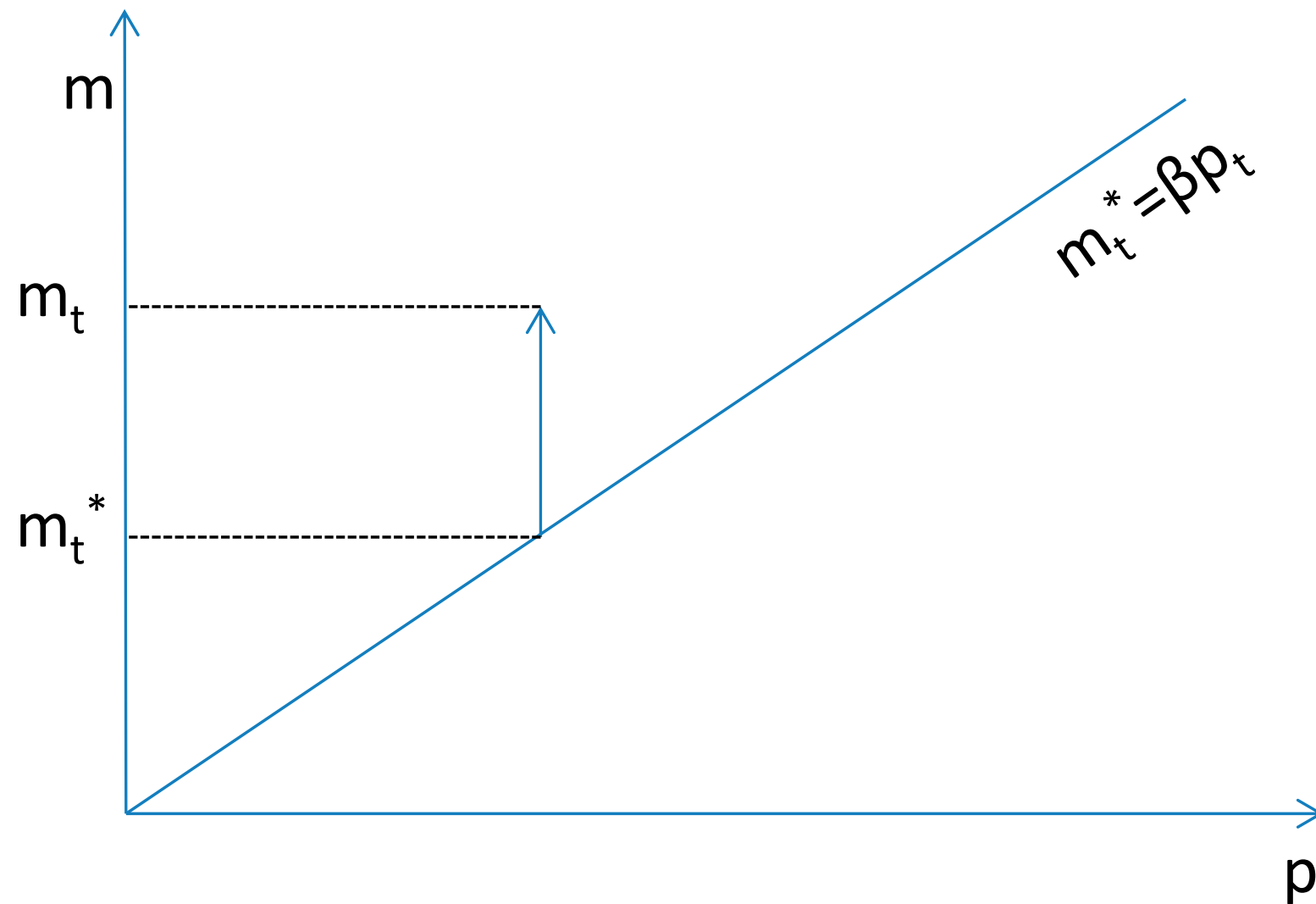
- That is, m and p are cointegrated. Note that Money Neutrality would imply $\beta = 1$
- Suppose $m \uparrow$ such that $m_t - \beta p_t > 0$
- What would be the dynamics?

$I(0)$

$$m_t - \beta p_t = \overset{0}{=} \cancel{e_t}$$

\downarrow
 \uparrow
 (β)

Deviation and Now What?



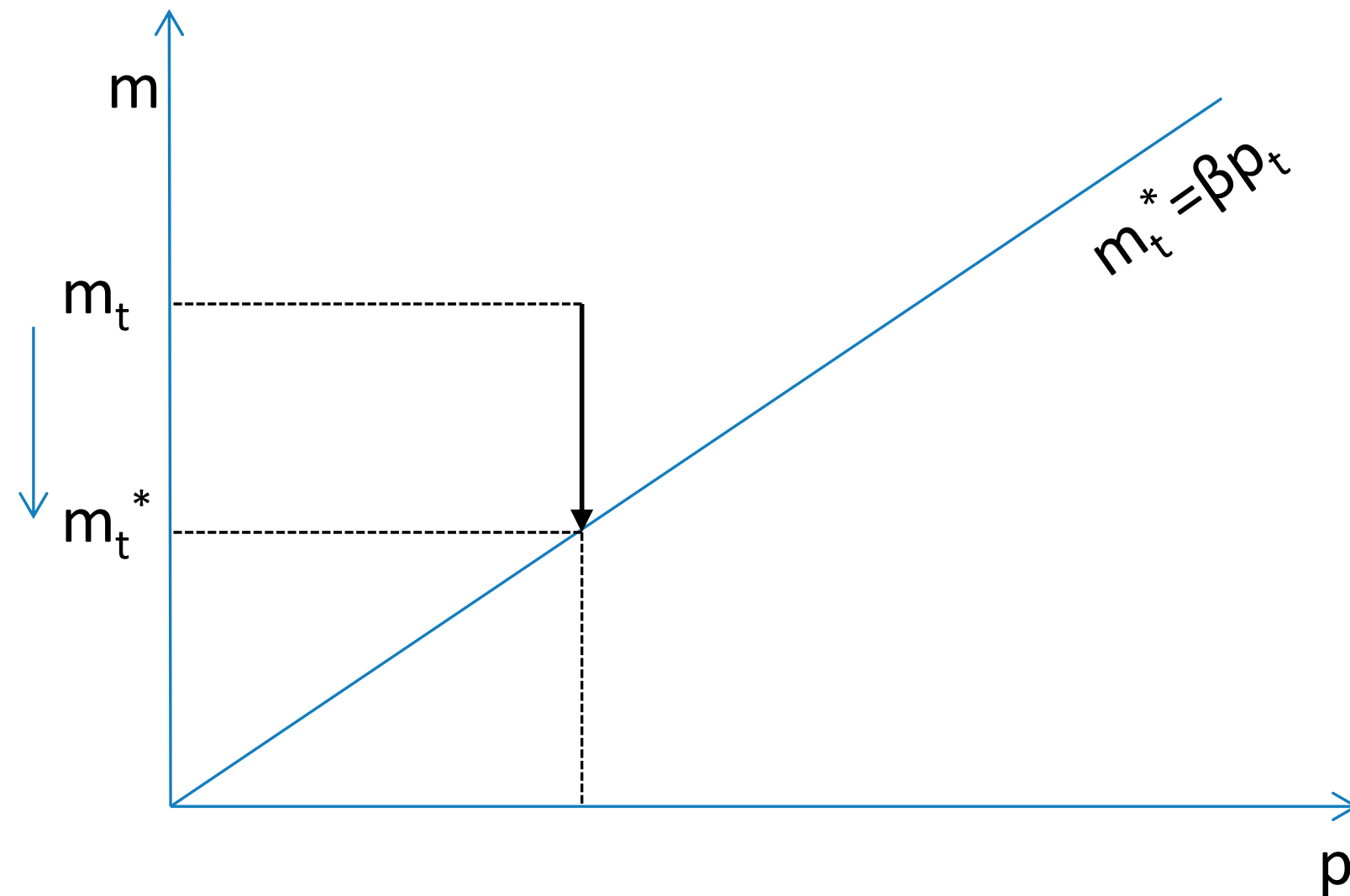
m_t is doing all the adjustment

- m_t^* is unchanged and $m_t \downarrow$

$$\Delta m_t = \alpha_m (m_{t-1} - m_{t-1}^*), \text{ where } \alpha_m < 0$$

- Short run change in m_t is a linear function of the deviation from the long run equilibrium

m_t is doing all the adjustment



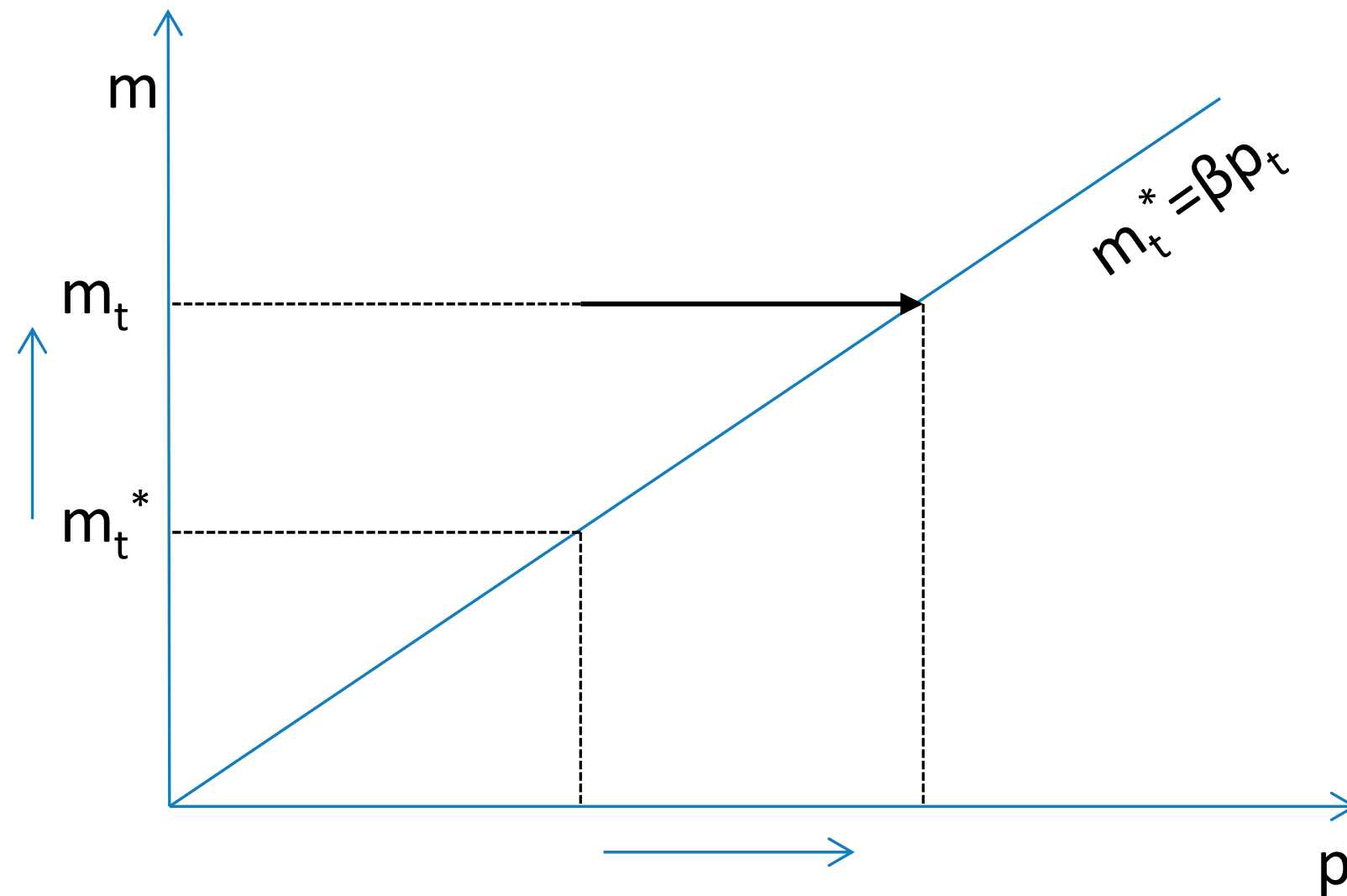
m_t^* is doing all the adjustment

- m_t is unchanged, and p_t and $m_t^* \uparrow$

$$\Delta p_t = \alpha_p (m_{t-1} - m_{t-1}^*), \text{ where } \alpha_p > 0$$

- Short run change in p_t is a linear function of the deviation from money neutrality

m_t^* is doing all the adjustment



Both m_t and p_t are Adjusting

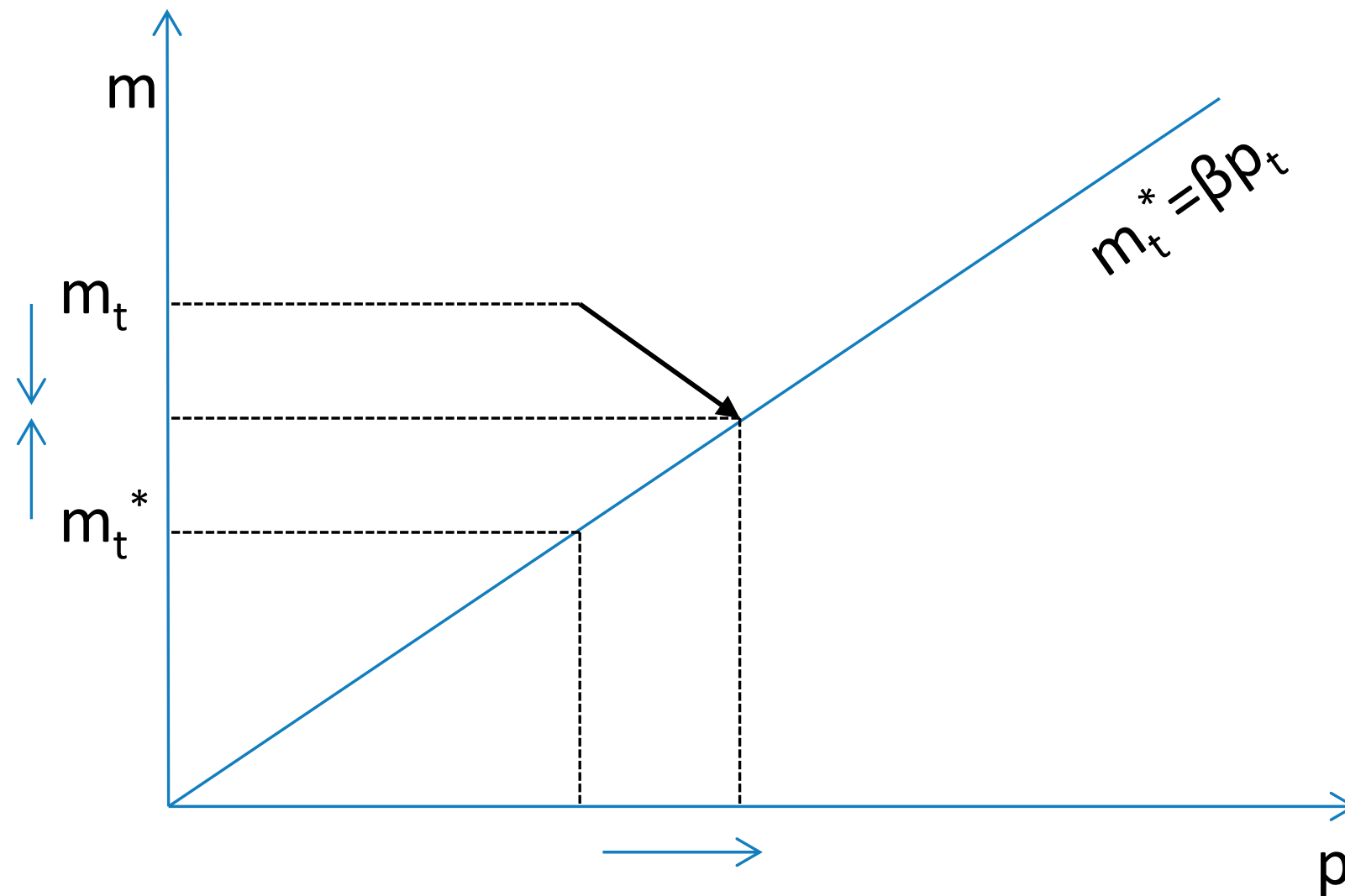
- m_t and p_t are adjusting simultaneously:

$$\Delta m_t = \alpha_m (m_{t-1} - m_{t-1}^*), \text{ where } \alpha_m < 0$$

$$\Delta p_t = \alpha_p (m_{t-1} - m_{t-1}^*), \text{ where } \alpha_p > 0$$

which is a basic error correction model

Simultaneous Adjustment



Taking Stock and What's Next?

- Error correction is a mechanism ensuring cointegration between m and p
- How about m , p and y ...or more variables?
- ...and we still don't know why it is important to take into account cointegration (and how!)

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Cointegration and Vector Error Correction Models

VECM Specification



From Session 2: Simple ECM

- m_t and p_t are cointegrated/follow the ECM:

$$\Delta m_t = \alpha_m(m_{t-1} - \beta p_{t-1}) + v_t$$

$$\Delta p_t = \alpha_p(m_{t-1} - \beta p_{t-1}) + u_t$$

- A VAR in differences would be misspecified

ECM as a Special VAR

- The previous ECM could be rewritten:

$$\begin{aligned}m_t &= (1+\alpha_m)m_{t-1}-\alpha_m\beta p_{t-1}+v_t \\p_t &= \alpha_p m_{t-1}+(1-\alpha_p\beta)p_{t-1}+u_t\end{aligned}$$

- This is clearly a VAR in levels with non-linear constraints on its coefficients

Back to Our Problem

- We believe that m_t , p_t and y_t are cointegrated
- In the long run: $E[b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4] = 0$
- Let us write a VECM and estimate it at the same time

VECM of Lag Order 0

- VECM of m , p and y with no lags of Δm_t , Δp_t , Δy_t

$$\Delta m_t = \alpha_m (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + v_t$$

$$\Delta p_t = \alpha_p (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + u_t$$

$$\Delta y_t = \alpha_y (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \eta_t$$

VAR Specification

Basics Cointegration VEC Restrictions

VAR Type

☐ Unrestricted VAR

☒ Vector Error Correction

☐ Bayesian VAR

Endogenous Variables

log(m) log(p) log(rgdp)

Estimation Sample

2000q1 2012q4

Lag Intervals for D(Endogenous):

0 0

Exogenous Variables

Do NOT include C or Trend in VEC's

OK Cancel

No lags of Δm , Δp
and Δy which could
be changed into 1 1
or 1 2

Endogenous
variables: m,p
and y

VECM of Order 1

$$\Delta m_t = \alpha_M (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{MM} \Delta m_{t-1} + \lambda_{MP} \Delta p_{t-1} + \lambda_{MY} \Delta y_{t-1} + v_t$$

$$\Delta p_t = \alpha_P (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{PM} \Delta m_{t-1} + \lambda_{PP} \Delta p_{t-1} + \lambda_{PY} \Delta y_{t-1} + u_t$$

$$\Delta y_t = \alpha_Y (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{YM} \Delta m_{t-1} + \lambda_{YP} \Delta p_{t-1} + \lambda_{YY} \Delta y_{t-1} + \eta_t$$

- How to decide the number of lags?

VAR Specification

Basics Cointegration VEC Restrictions

Rank

Number of cointegrating 1

Deterministic Trend Specification

No trend in data

☐ 1) No intercept or trend in CE or VAR

☐ 2) Intercept (no trend) in CE - no intercept in VAR

Linear trend in data

☒ 3) Intercept (no trend) in CE and VAR

☐ 4) Intercept and trend in CE - no trend in VAR

Quadratic trend in data

☐ 5) Intercept and trend in CE - linear trend in VAR

OK Cancel

Specification of
deterministic
components

The default is one
cointegration
relationship but
there could be
more...

Deterministic Components

Case 1:

$$\Delta m_t = \alpha_M (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1}) + \lambda_{MM} \Delta m_{t-1} + \lambda_{MP} \Delta p_{t-1} + \lambda_{MY} \Delta y_{t-1} + v_t$$

Case 2:

$$\Delta m_t = \alpha_M (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{MM} \Delta m_{t-1} + \lambda_{MP} \Delta p_{t-1} + \lambda_{MY} \Delta y_{t-1} + v_t$$

Case 3:

$$\Delta m_t = \mu_M + \alpha_M (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{MM} \Delta m_{t-1} + \lambda_{MP} \Delta p_{t-1} + \lambda_{MY} \Delta y_{t-1} + v_t$$

VECM of Order 1 with Constants

$$\Delta m_t = \mu_M + \alpha_M (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{MM} \Delta m_{t-1} + \lambda_{MP} \Delta p_{t-1} + \lambda_{MY} \Delta y_{t-1} + v_t$$

$$\Delta p_t = \mu_P + \alpha_P (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{PM} \Delta m_{t-1} + \lambda_{PP} \Delta p_{t-1} + \lambda_{PY} \Delta y_{t-1} + u_t$$

$$\Delta y_t = \mu_Y + \alpha_Y (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{YM} \Delta m_{t-1} + \lambda_{YP} \Delta p_{t-1} + \lambda_{YY} \Delta y_{t-1} + \eta_t$$

- Under option 3

Vector Error Correction Estimates
Date: 03/14/15 Time: 15:15
Sample (adjusted): 1998Q1 2014Q4
Included observations: 68 after adjustments
Standard errors in () & t-statistics in []

Cointegrating Eq:	CointEq1		
LOG(M(-1))	1.000000		
LOG(P(-1))	1.422433 (0.65845) [2.16027]		
LOG(RGDP(-1))	-5.586962 (0.76354) [-7.31718]		
C	5.122412		
Error Correction:	D(LOG(M))	D(LOG(P))	D(LOG(RGD...
CointEq1	-0.062230 (0.02113) [-2.94539]	-0.011734 (0.00397) [-2.95576]	0.022170 (0.01208) [1.83515]
D(LOG(M(-1)))	0.208293 (0.11837) [1.75971]	-0.026039 (0.02224) [-1.17075]	0.067627 (0.06768) [0.99917]
D(LOG(P(-1)))	-0.122835 (0.65459) [-0.18765]	0.214317 (0.12300) [1.74244]	0.416759 (0.37429) [1.11345]

Once the specification
is set EViews will use
Maximum Likelihood to
estimate the
parameters and
produce the output:

Taking Stock and What's Next?

- We now know why it is important to take into account cointegration if it is there...
- ...but is there cointegration?
- ...and if so how many cointegrating vectors are there?
- ...and what are those cointegrating vectors?

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Cointegration and Vector Error Correction Models

Cointegration Test



From Session 3: VECM

$$\Delta m_t = \mu_m + \alpha_m (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_3) + \lambda_{mm} \Delta m_{t-1} + \lambda_{mp} \Delta p_{t-1} + \lambda_{my} \Delta y_{t-1} + v_t$$

$$\Delta p_t = \mu_p + \alpha_p (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_3) + \lambda_{pm} \Delta m_{t-1} + \lambda_{pp} \Delta p_{t-1} + \lambda_{py} \Delta y_{t-1} + u_t$$

$$\Delta y_t = \mu_y + \alpha_y (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_3) + \lambda_{ym} \Delta m_{t-1} + \lambda_{yp} \Delta p_{t-1} + \lambda_{yy} \Delta y_{t-1} + \eta_t$$

- We have only one lag for illustration

VECM in Matrix Form

- We can stack the equations and variables into a vector X_t to obtain:

$$\Delta X_t = C + \Pi X_{t-1} + \Lambda \Delta X_{t-1} + e_t$$

- ΠX_{t-1} represents the error correction terms

Johansen's Methodology

- Johansen's methodology rests on estimating the rank of Π
- The rank of Π is the maximum number of independent vectors it contains
- **It cannot exceed n , the number of variables we have in the system (in our case 3)**

The Rank Informs Us about Cointegration

- If $\text{rank}(\Pi)=0$: no cointegration
- If $\text{rank}(\Pi)=n$ (full rank): all variables are $I(0)$
- If $0<\text{rank}(\Pi)=r<n$ (less than full rank): there are r independent cointegration relationships

Cointegration Test

Johansen Cointegration Test

Cointegration Test Specification **VEC Restrictions**

Deterministic trend assumption of test

Assume no deterministic trend in data:

☐ 1) No intercept or trend in CE or test VAR

☐ 2) Intercept (no trend) in CE - no intercept in VAR

Allow for linear deterministic trend in data:

☒ 3) Intercept (no trend) in CE and test VAR

☐ 4) Intercept and trend in CE - no intercept in VAR

Allow for quadratic deterministic trend in data:

☐ 5) Intercept and trend in CE - intercept in VAR

Summary:

☐ 6) Summarize all 5 sets of assumptions

* Critical values may not be valid with exogenous variables; do not include C or Trend.

Exog variables*

Lag intervals

1 2

Lag spec for differenced endogenous

Critical Values

☒ **M**HM

Size 0.05

☐ Osterwald-Lenum

OK Cancel

Johansen's Statistics

- **Trace statistic for r :** the null hypothesis is that the rank is at most r vs. the rank is strictly greater
- **Maximum eigenvalue statistic for r :** the null hypothesis is that the rank is r vs. the rank is $r+1$

Sample: 2000Q1 2012Q4
Included observations: 52
Trend assumption: Linear deterministic trend
Series: LOG(M) LOG(P) LOG(RGDP)
Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.443819	35.02620	29.79707	0.0114
At most 1	0.067420	4.519818	15.49471	0.8575
At most 2	0.016974	0.890212	3.841466	0.3454

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.443819	30.50639	21.13162	0.0018
At most 1	0.067420	3.629605	14.26460	0.8962
At most 2	0.016974	0.890212	3.841466	0.3454

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Taking Stock and What's Next?

- We now know how to test for cointegration...
-we determined that there exists a single cointegrating relationship between m , p and y
- So we can go back and estimate our VECM.
But how do we test whether y adjusts to long-run relationship?

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Cointegration and Vector Error Correction Models

Restricted VECM



From Session 4

- We established that there is one cointegration relationship...
- ...we can estimate the VECM specified in Session 3
- But we would like to know whether y adjusts to deviations from the long-run relationship

From Session 3: VECM

$$\Delta m_t = \mu_m + \alpha_m (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{mm} \Delta m_{t-1} + \lambda_{mp} \Delta p_{t-1} + \lambda_{my} \Delta y_{t-1} + v_t$$

$$\Delta p_t = \mu_p + \alpha_p (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{pm} \Delta m_{t-1} + \lambda_{pp} \Delta p_{t-1} + \lambda_{py} \Delta y_{t-1} + u_t$$

$$\Delta y_t = \mu_y + \alpha_y (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{ym} \Delta m_{t-1} + \lambda_{yp} \Delta p_{t-1} + \lambda_{yy} \Delta y_{t-1} + \eta_t$$

- We would like to test whether $\alpha_y = 0$. This is a test of weak exogeneity of RGDP

Let's First Look at the Output of Our Unrestricted VECM More Closely

Estimation proc:

=====

EC(C,1) 1 2 LOG(M) LOG(p) LOG(RGDp) @ C

VAR Model:

=====

$D(\text{LOG}(M)) = A(1,1) * (B(1,1) * \text{LOG}(M(-1)) + B(1,2) * \text{LOG}(p(-1)) + B(1,3) * \text{LOG}(\text{RGDp}(-1)) + B(1,4)) + C(1,1) * D(\text{LOG}(M(-1))) + C(1,2) * D(\text{LOG}(M(-2))) + C(1,3) * D(\text{LOG}(p(-1))) + C(1,4) * D(\text{LOG}(p(-2))) + C(1,5) * D(\text{LOG}(\text{RGDp}(-1))) + C(1,6) * D(\text{LOG}(\text{RGDp}(-2))) + C(1,7)$

$D(\text{LOG}(p)) = A(2,1) * (B(1,1) * \text{LOG}(M(-1)) + B(1,2) * \text{LOG}(p(-1)) + B(1,3) * \text{LOG}(\text{RGDp}(-1)) + B(1,4)) + C(2,1) * D(\text{LOG}(M(-1))) + C(2,2) * D(\text{LOG}(M(-2))) + C(2,3) * D(\text{LOG}(p(-1))) + C(2,4) * D(\text{LOG}(p(-2))) + C(2,5) * D(\text{LOG}(\text{RGDp}(-1))) + C(2,6) * D(\text{LOG}(\text{RGDp}(-2))) + C(2,7)$

$D(\text{LOG}(\text{RGDp})) = \mathbf{A(3,1)} * (B(1,1) * \text{LOG}(M(-1)) + B(1,2) * \text{LOG}(p(-1)) + B(1,3) * \text{LOG}(\text{RGDp}(-1)) + B(1,4)) + C(3,1) * D(\text{LOG}(M(-1))) + C(3,2) * D(\text{LOG}(M(-2))) + C(3,3) * D(\text{LOG}(p(-1))) + C(3,4) * D(\text{LOG}(p(-2))) + C(3,5) * D(\text{LOG}(\text{RGDp}(-1))) + C(3,6) * D(\text{LOG}(\text{RGDp}(-2))) + C(3,7)$

Restricted VECM Estimation

Why also impose
 $B(1,1)=1$?

The screenshot shows the 'VAR Specification' dialog box with the 'VEC Restrictions' tab selected. The dialog has three tabs: 'Basics', 'Cointegration', and 'VEC Restrictions'. The 'VEC Restrictions' tab contains a text area for restrictions, a 'VEC Coefficient Restrictions' section with a checked 'Impose Restrictions' checkbox and a text input field, and an 'Optimization' section with 'Max Iterations' and 'Convergence' input fields. The text area contains the formula $B(r,1)*LOG(M) + B(r,2)*LOG(P) + B(r,3)*LOG(RGDP)$. The 'VEC Coefficient Restrictions' section has the text 'Enter restriction: (Example: B(1,1)=1, A(2,1)=0)' and the input field contains $B(1,1)=1, A(3,1)=0$. The 'Optimization' section has 'Max Iterations' set to 500 and 'Convergence' set to 0.0001. 'OK' and 'Cancel' buttons are at the bottom right.

VAR Specification

Basics Cointegration VEC Restrictions

Restrictions may be placed on the coefficients $B(r,k)$ of the r -th cointegrating relation:

$B(r,1)*LOG(M) + B(r,2)*LOG(P) + B(r,3)*LOG(RGDP)$

VEC Coefficient Restrictions

☒ Impose Restrictions

Enter restriction: (Example: $B(1,1)=1, A(2,1)=0$)

$B(1,1)=1, A(3,1)=0$

Optimization

Max Iterations: 500

Convergence: 0.0001

OK Cancel

Weak Exogeneity Test of RGDP

Sample: 2000Q1 2012Q4
Included observations: 52
Standard errors in () & t-statistics in []

Cointegration Restrictions:
B(1,1)=1,A(3,1)=0
Convergence achieved after 11 iterations.
Restrictions identify all cointegrating vectors
LR test for binding restrictions (rank = 1):
Chi-square(1) 3.503393
Probability 0.061243

Cointegrating Eq:	CointEq1
LOG(M(-1))	1.000000
LOG(P(-1))	1.489042 (0.66084) [2.25326]
LOG(RGDP(-1))	-5.591641 (0.75494) [-7.40675]
C	4.873434

Multiple Restrictions

The screenshot shows the 'VEC Restrictions' tab of the 'VAR Specification' dialog box. At the top, there are three tabs: 'Basics', 'Cointegration', and 'VEC Restrictions'. Below the tabs, a text box explains that restrictions can be placed on the coefficients $B(r,k)$ of the r -th cointegrating relation. A scrollable area contains the formula $B(r,1)*LOG(M) + B(r,2)*LOG(P) + B(r,3)*LOG(RGDP)$. Below this, the 'VEC Coefficient Restrictions' section has a checked 'Impose Restrictions' checkbox and a text box for entering restrictions, with an example $B(1,1)=1, A(2,1)=0$ and the entered text $B(1,1)=1, B(1,2)=-1, B(1,3)=0$. To the right, the 'Optimization' section shows 'Max Iterations' set to 500 and 'Convergence' set to 0.0001. At the bottom are 'OK' and 'Cancel' buttons.

One could test instead
 $B(1,1)=1, B(1,2)=-1, B(1,3)=0$
which means in the LR:
 $b_1 m_{t-1} + b_4 = p_{t-1}$

Sample: 2000Q1 2012Q4
Included observations: 52
Standard errors in () & t-statistics in []

Cointegration Restrictions:
 $B(1,1)=1, B(1,2)=-1, B(1,3)=0$
Convergence achieved after 1 iterations.
Restrictions identify all cointegrating vectors
LR test for binding restrictions (rank = 1):
Chi-square(2) 26.87158
Probability 0.000001

Restricted VECM Then What?

- If the test of weak exogeneity indicates that we can't reject it...
- ...then we can estimate the restricted VECM (it is already done as a byproduct of the test!)
- **The restricted model will produce different forecasts, tests and impulse responses**

Taking Stock and What's Next?

- We now know how to test restrictions...
- ...we determined that RGDP is weakly exogenous
- Is the restricted VECM the best for forecasting? How does it compare to VAR in differences?

MFx – Macroeconomic Forecasting

IMFx



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Cointegration and Vector Error Correction Models

Forecasting with VECM



From Session 5

- We determined that RGDP was weakly exogenous and we estimated the model...
- ...but we were not sure which model is best for forecasting
- We will build different forecasts and compare them

Forecasting with Three Models

- VECM
- Constrained VECM where RGDP is weakly exogenous
- VAR in differences

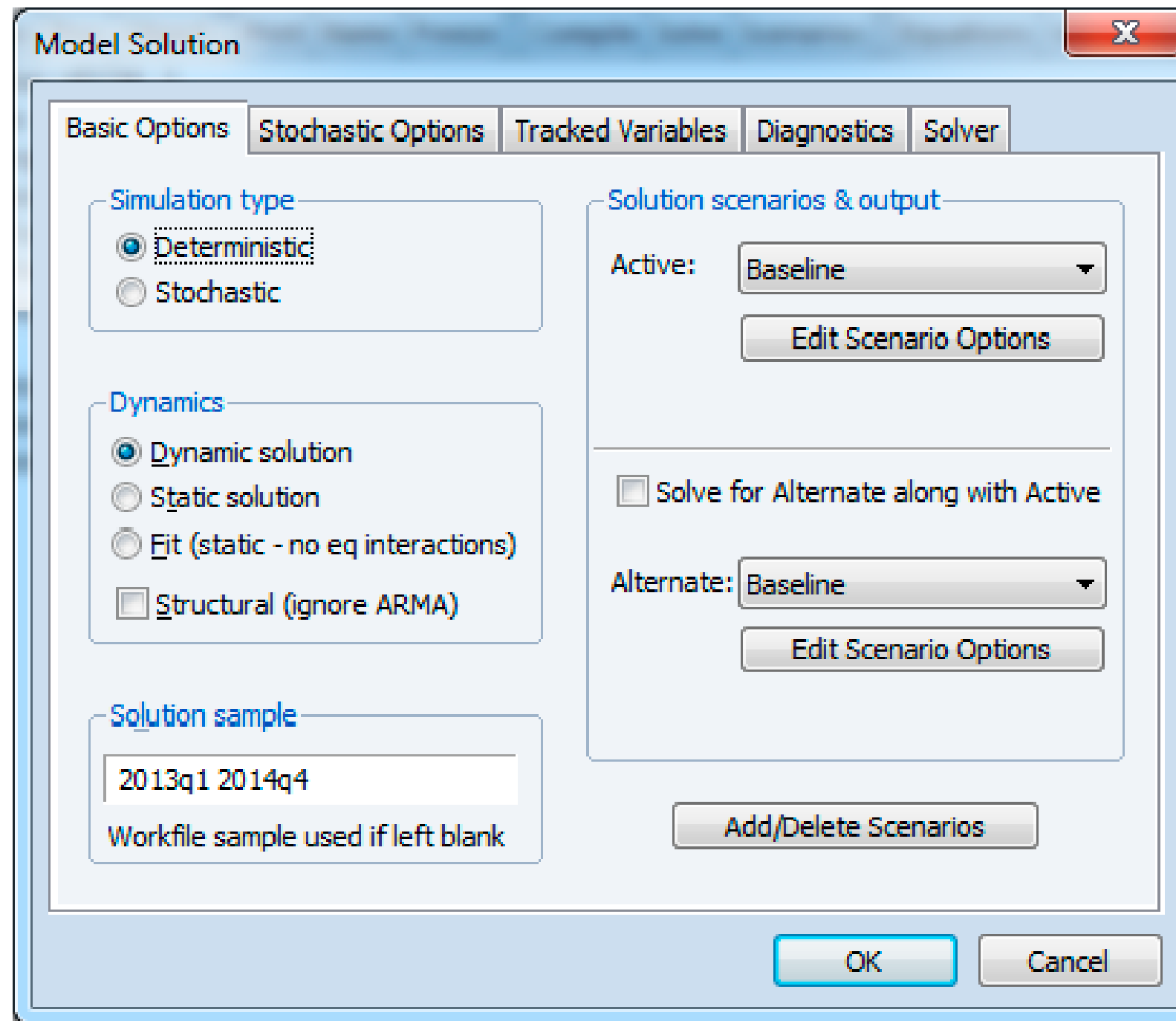
Forecasting: Steps for Each Model

- Estimate over 2000Q1-2012Q4
- Dynamic forecast over 2013Q1-2014Q4
- Compare to outcomes over the forecasting period and calculate RMSE's

Specification of the VECM: Summary

- Lags: 2
- Deterministic components: Option 3
- Number of cointegration relationships: 1
- Sample: 2000Q1-2012Q4
- Once estimated: **Proc/Make Model**

Solve the Model: Dynamic Forecast



The image shows a 'Model Solution' dialog box with a title bar and a close button. It contains five tabs: 'Basic Options', 'Stochastic Options', 'Tracked Variables', 'Diagnostics', and 'Solver'. The 'Basic Options' tab is active and contains three sections: 'Simulation type' with radio buttons for 'Deterministic' (selected) and 'Stochastic'; 'Dynamics' with radio buttons for 'Dynamic solution' (selected), 'Static solution', 'Fit (static - no eq interactions)', and a checkbox for 'Structural (ignore ARMA)'; and 'Solution sample' with a text box containing '2013q1 2014q4' and a note 'Workfile sample used if left blank'. The 'Solution scenarios & output' section on the right includes a dropdown for 'Active' (Baseline), an 'Edit Scenario Options' button, a checkbox for 'Solve for Alternate along with Active', a dropdown for 'Alternate' (Baseline), another 'Edit Scenario Options' button, and an 'Add/Delete Scenarios' button. 'OK' and 'Cancel' buttons are at the bottom right.

Model Solution

Basic Options Stochastic Options Tracked Variables Diagnostics Solver

Simulation type

☒ Deterministic

☐ Stochastic

Dynamics

☒ Dynamic solution

☐ Static solution

☐ Fit (static - no eq interactions)

☐ Structural (ignore ARMA)

Solution sample

2013q1 2014q4

Workfile sample used if left blank

Solution scenarios & output

Active: Baseline

Edit Scenario Options

☐ Solve for Alternate along with Active

Alternate: Baseline

Edit Scenario Options

Add/Delete Scenarios

OK Cancel

Proc/Make Graph

Or select
“Endogenous
variables” which is
the same in our
case.

Make Graph

Model variables

Select: All variable types

From: ☒ All model variables
☐ Listed variables

Graph series

Solution series: Deterministic Solutions

☒ Actuals

☒ Active: Baseline

☐ Compare: Baseline

☐ Deviations: Active from Compare

☐ % Deviation: Active from Compare

Transform: Level

Series grouping

☐ Each series in its own graph

☒ Group by Model Variable

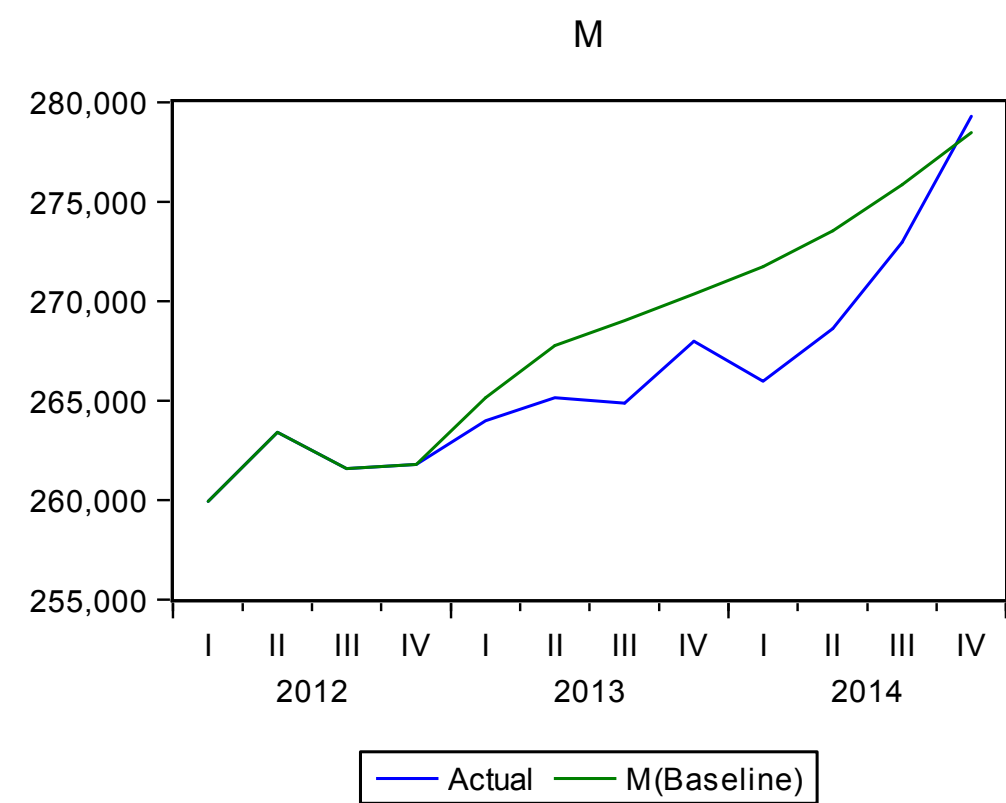
☐ Group by Scenario/Actuals/Deviations/etc.

Sample for graph

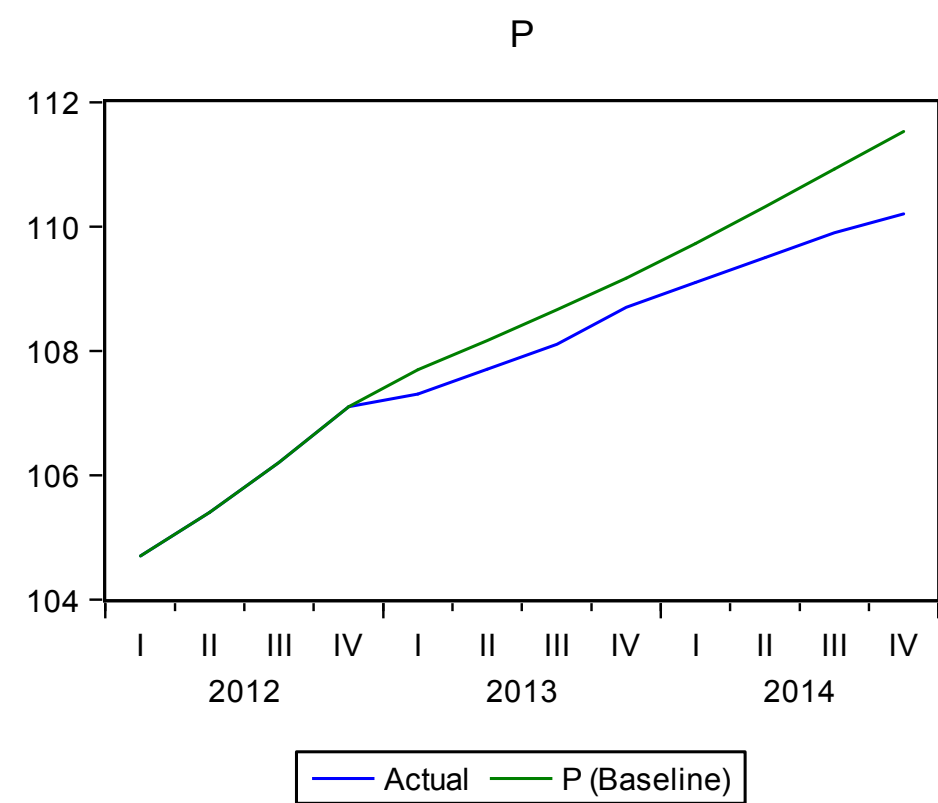
2012Q1 2014Q4

OK Cancel

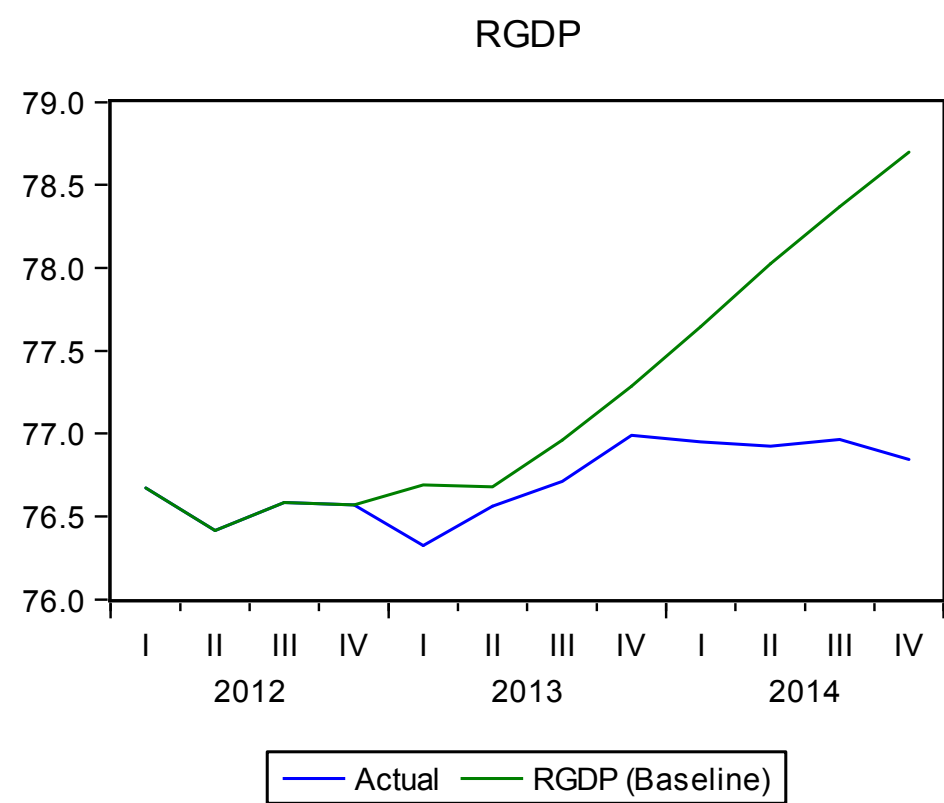
Forecasting Performance



RMSE=2851



RMSE=0.63



RMSE=0.78

Summary: Comparison of RMSEs

	VECM	Constrained VECM	VAR in Differences
P	0.6	0.3	0.5
M	2851.4	5446.8	8609.8
RGDP	0.8	0.7	0.5

Taking Stock and What's Next?

- Taking into account the long term relationship yielded superior forecasts of M and P.
- ...imposing restrictions (pre-tested!) does not necessarily improve the forecast
- What should we do if there are more than one cointegration relationships?

MFx – Macroeconomic Forecasting

IMFx



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Cointegration and Vector Error Correction Models

Multiple Cointegrations

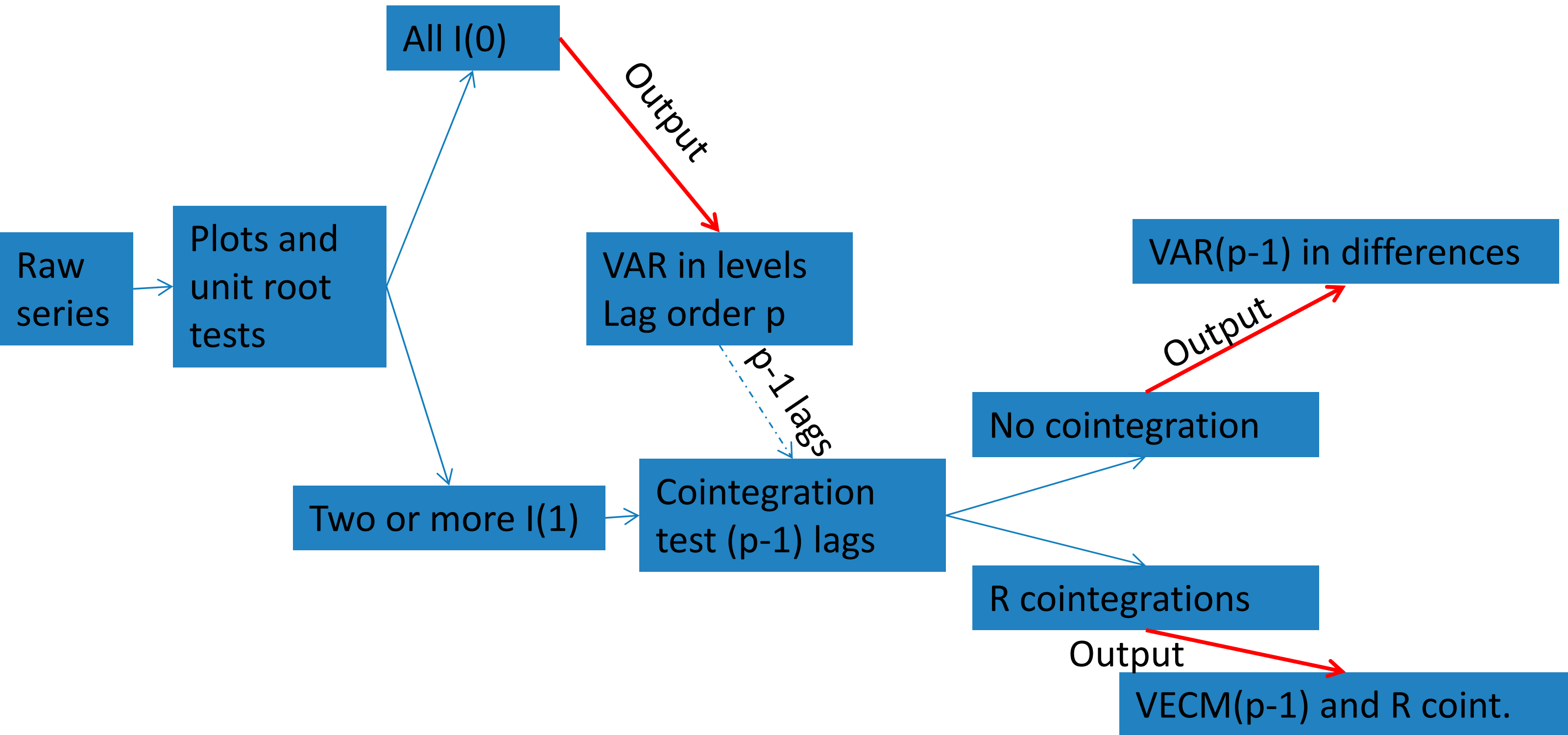


From Session 6

- We saw from the forecasts that cointegration should be taken into account...
- ...but we found that there was only one cointegration relationship
- How to deal with multiple relationships?

Let Us Study a Similar Problem

- We use Germany instead of Austria
- We have M , P , $RGDP$ and i (interest rate)
- Data are over the same period



Sample: 2000Q1 2012Q4
Included observations: 52
Trend assumption: Linear deterministic trend
Series: LOG(M) LOG(P) LOG(RGDP) I
Lags interval (in first differences): No lags

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.1 Critical Value	Prob.**
None *	0.466975	67.17082	44.49359	0.0003
At most 1 *	0.380054	34.45313	27.06695	0.0135
At most 2	0.167375	9.590746	13.42878	0.3135
At most 3	0.001265	0.065822	2.705545	0.7975

Trace test indicates 2 cointegrating eqn(s) at the 0.1 level
* denotes rejection of the hypothesis at the 0.1 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

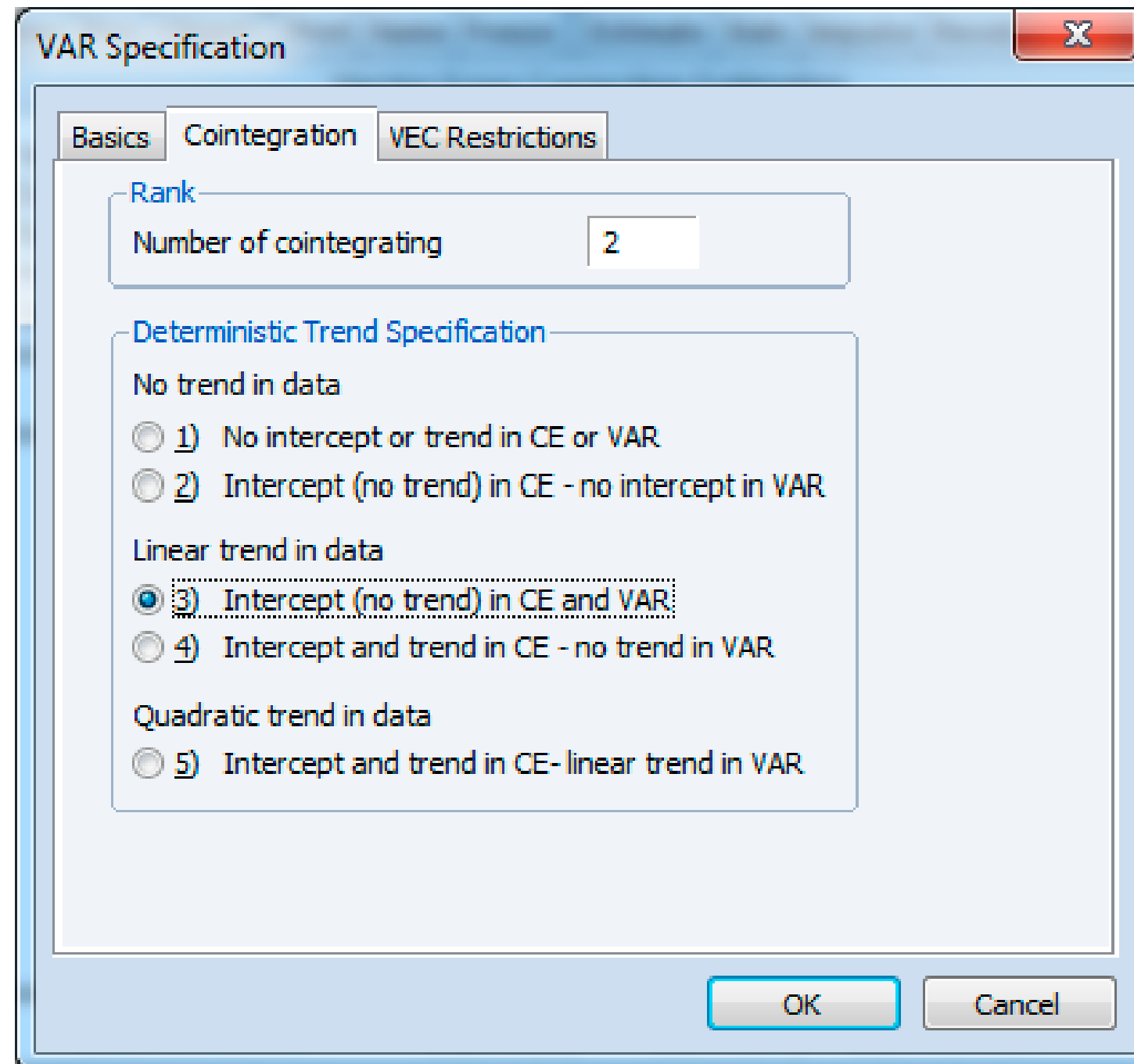
Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.1 Critical Value	Prob.**
None *	0.466975	32.71769	25.12408	0.0100
At most 1 *	0.380054	24.86239	18.89282	0.0142
At most 2	0.167375	9.524925	12.29652	0.2450
At most 3	0.001265	0.065822	2.705545	0.7975

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.1 level
* denotes rejection of the hypothesis at the 0.1 level
**MacKinnon-Haug-Michelis (1999) p-values

Tests show that there are two cointegration relationships at 0.05 level too.

Estimate the VECM

Make sure you
change the
number of
cointegrating
vectors to 2



The screenshot shows the 'VAR Specification' dialog box with the 'Cointegration' tab selected. The 'Rank' section shows 'Number of cointegrating' set to 2. The 'Deterministic Trend Specification' section has five radio button options. Option 3, 'Intercept (no trend) in CE and VAR', is selected. Options 1, 2, 4, and 5 are unselected. The 'OK' and 'Cancel' buttons are at the bottom right.

VAR Specification

Basics Cointegration VEC Restrictions

Rank

Number of cointegrating 2

Deterministic Trend Specification

No trend in data

☐ 1) No intercept or trend in CE or VAR

☐ 2) Intercept (no trend) in CE - no intercept in VAR

Linear trend in data

☒ 3) Intercept (no trend) in CE and VAR

☐ 4) Intercept and trend in CE - no trend in VAR

Quadratic trend in data

☐ 5) Intercept and trend in CE - linear trend in VAR

OK Cancel

EViews Default Restrictions

Now we have two
cointegrating equations.

Notice the default restrictions
imposed by EViews to
achieve identification.

Date: 03/15/15 Time: 17:21
Sample: 2000Q1 2012Q4
Included observations: 52
Standard errors in () & t-statistics in []

Cointegrating Eq:	CointEq1	CointEq2
LOG(M(-1))	1.000000	0.000000
LOG(P(-1))	0.000000	1.000000
LOG(RGDP(-1))	-4.742465 (0.36276) [-13.0732]	-1.239572 (0.06852) [-18.0907]
I(-1)	0.011253 (0.01241) [0.90689]	0.012565 (0.00234) [5.36103]
C	23.11359	3.406518

Alternative Restrictions

- It may not be natural to exclude money or prices from the cointegrating vectors
- How about excluding RGDP or interest rate?
- To do so run the VECM with the correct restrictions (check View/Representations)

Note the restrictions we used.

Note that in this case P has a negative coefficient in the first cointegrating equation where only nominal variables appear.

Vector Error Correction Estimates

Date: 03/15/15 Time: 17:31

Sample: 2000Q1 2012Q4

Included observations: 52

Standard errors in () & t-statistics in []

Cointegration Restrictions:

B(1,1)=1,B(1,3)=0,B(2,2)=1,B(2,4)=0

Convergence achieved after 1 iterations.

Restrictions identify all cointegrating vectors

Restrictions are not binding (LR test not available)

Cointegrating Eq:	CointEq1	CointEq2
LOG(M(-1))	1.000000	5.935724 (1.19487) [4.96766]
LOG(P(-1))	-3.721503 (0.23347) [-15.9399]	1.000000
LOG(RGDP(-1))	0.000000	-29.97341 (4.70017) [-6.37709]
I(-1)	-0.033515 (0.00825) [-4.06160]	0.000000
C	9.600450	144.5020

Weak Exogeneity of RGDP

- Let us add one additional restriction in each version, that is EViews default restrictions and the alternative ones
- Recall that we found that weak exogeneity of RGDP was important in Session 6
- Now we compare the RMSEs of each model

Comparing RMSEs

EViews default restrictions			
	VECM	Constrained VECM	VAR in Differences
P	0.55	0.60	9.70
M	31.08	36.83	580.39
RGDP	3.48	1.87	43.45
i	0.20	0.30	1.85

Alternative restrictions			
	VECM	Constrained VECM	VAR in Differences
P	0.55	0.59	9.70
M	31.08	37.20	580.39
RGDP	3.48	2.30	43.45
i	0.20	0.27	1.85

Taking Stock and What's Next?

- When there are multiple cointegrating vectors
- ...one should be careful in interpreting the results of over-identifying tests and associated forecasts...
- Use theory or intuition if possible to impose alternative restrictions