MFx – Macroeconomic Forecasting

Structural Vector Autoregressive Models
Part II

IMFx

Objectives

1. Why a structural VAR
2. VAR identification: Identifying **A**
3. Impulse responses
4. Variance decomposition
5. Two identification strategies
    - Recursive identification: Sims (1992)
    - Non Recursive: Blanchard and Perotti (2002)

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1. Why a structural VAR

Effects of monetary policy. Consider these events:

1. Anticipating a rise in inflation…
2. The central bank increases the monetary policy interest rate…But inflation still rises, as anticipated.

- *One could* ***Wrongly*** conclude that the interest rate hike led to a rise in inflation.
- *It was an endogenous* reaction to *expected* inflation.

1. Why a structural VAR

Impact of fiscal spending. Consider this new set of events:

1. Anticipating a reduction in private demand…, the government…
2. increases public spending, and the deficit…but output keeps declining for some time.

- ***Wrong* conclusion:** public spending caused the output decline.

  No, fiscal policy reacted ***endogenously*** to ***expected*** output.

2. Challenge: SVAR Identification

- We must identify **purely exogenous** (policy or other type of) shocks to be able to trace out its dynamic effects. Identify the structural VAR.

- **Uncovering the structural model is called identification**

- According to *Sims (1986):*

  *"identification is the interpretation of historically observed variation in data in a way that allows the variation to be used to **predict** the consequences of an action not yet undertaken."*

2. SVAR Identification

- Suppose we have the following **Structural VAR(1):**

$$AX_t = \beta_0 + \beta_1 X_{t-1} + u_t$$ Structural shocks *u* are independent

- If **X** has 2 variables GDP gap (***y***) and the interest rate (***r***): $X_t = \begin{bmatrix} y \\ r \end{bmatrix}$
  the system will be:

$$y_t + a_{12} r_t = \beta_{10} + \beta_{11} y_{t-1} + \beta_{12} r_{t-1} + u_{yt}$$

$$a_{21} y_t + r_t = \beta_{20} + \beta_{21} y_{t-1} + \beta_{22} r_{t-1} + u_{rt}$$

- In matrix form:

$$\underbrace{\begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}}_{A} \begin{bmatrix} y_t \\ r_t \end{bmatrix} = \begin{bmatrix} \beta_{10} \\ \beta_{20} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{rt} \end{bmatrix}$$

2. *e* combinations of *u*

- If we pre multiply this VAR by A^{-1} we get the reduced-form VAR (appendix 1):

$$\mathrm{A}^{-1}\mathrm{A}\mathrm{X}_t = \overbrace{\mathrm{A}^{-1}\beta_0}^{G_0} + \overbrace{\mathrm{A}^{-1}\beta_1}^{G_1}\mathrm{X}_{t-1} + \mathrm{A}^{-1}u_t$$

$$\mathrm{X}_t = \mathrm{G}_0 + \mathrm{G}_1\mathrm{X}_{t-1} + e_t$$

$$\mathrm{A}^{-1}\mathrm{A}=\mathrm{I}$$

- Matrix **A** also relates the forecast errors of the reduced-form VAR, ***e***, and the structural shocks ***u***: $\mathrm{e}_t=\mathrm{A}^{-1}u_t$

2. *e* combinations of *u*

- The forecast errors ***e*** are linear combinations of the structural shocks ***u***.

$$\mathrm{e}_t = \mathrm{A}^{-1} u_t$$

$$\mathrm{e}_{yt} = (u_{yt} - a_{12} u_{rt}) / (1 - a_{12} a_{21})$$

$$\mathrm{e}_{rt} = (-a_{21} u_{yt} + u_{rt}) / (1 - a_{12} a_{21})$$

2. Identification, getting **A**

- The structural VAR cannot be estimated directly.

- Start from the reduced-form VAR: $\mathrm{X}_t = \mathrm{G}_0 + \mathrm{G}_1\mathrm{X}_{t-1} + e_t$

  and get to the structural one: $\mathrm{A}\mathrm{X}_t = \beta_0 + \beta_1\mathrm{X}_{t-1} + u_t$

- To do that, we need to restrict matrix A: **IDENTIFICATION**

2. Identification, finding **A**

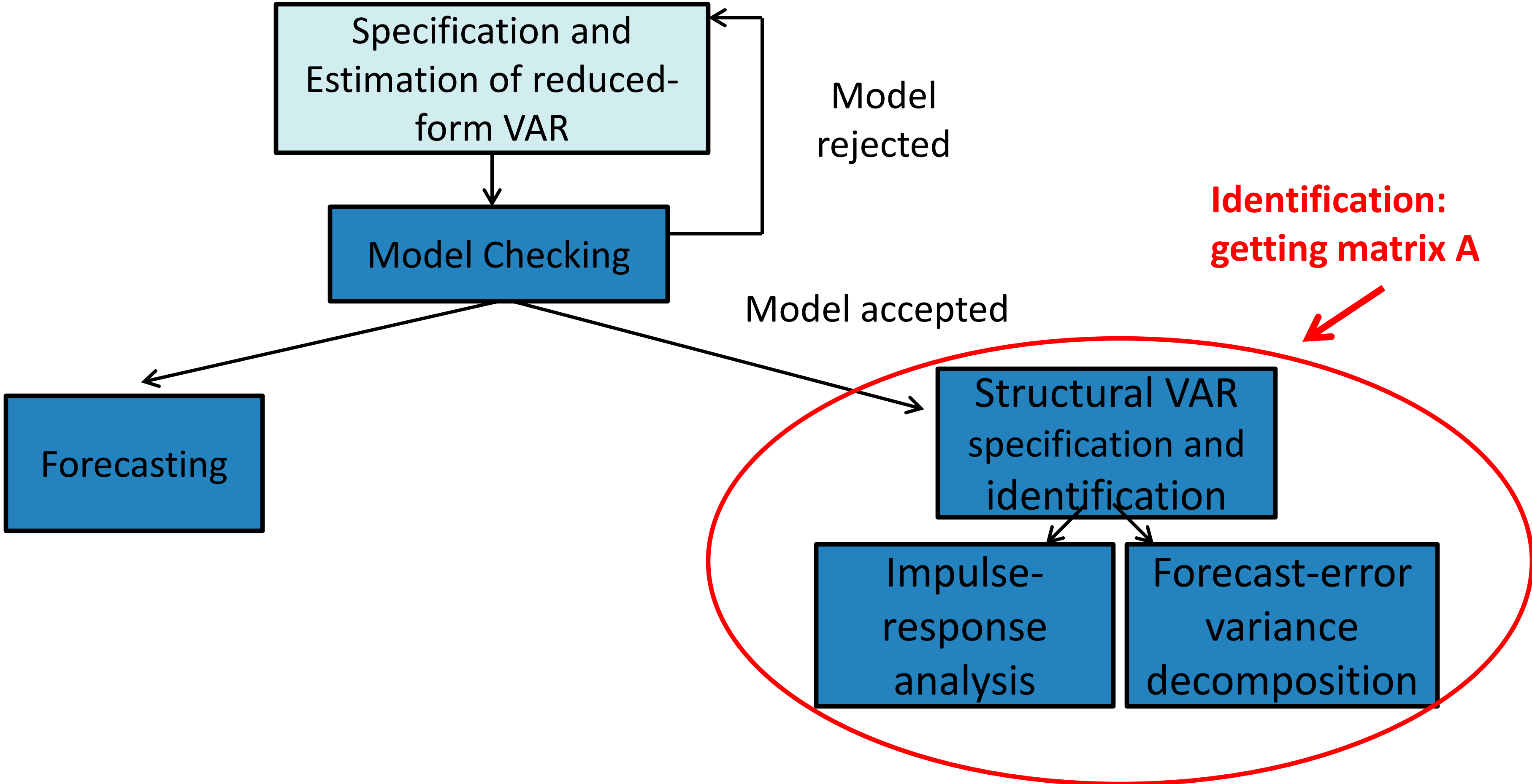
- Get **A** and multiply the reduced-form VAR by **A** to get the structural model, shocks, and contemporaneous relations among variables.

$$\mathrm{AX}_t = \mathrm{A}\,\overbrace{\mathrm{G}_0}^{\mathrm{A}^{-1}\beta_0} + \mathrm{A}\,\overbrace{\mathrm{G}_1}^{\mathrm{A}^{-1}\beta_1}\,\mathrm{X}_{t-1} + \mathrm{A}\,\overbrace{e_t}^{\mathrm{A}^{-1}u_t}$$

$$\mathrm{AX}_t = \beta_0 + \beta_1 \mathrm{X}_{t-1} + u_t$$

$$\mathrm{AA}^{-1}\beta_0 = \beta_0$$

Lecture Plan



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2. SVAR Identification: getting **A**

- Start estimating the reduced-form VAR: $\mathrm{X}_t = \mathrm{G}_0 + \mathrm{G}_1 \mathrm{X}_{t-1} + e_t$

$$\begin{bmatrix} y_t \\ r_t \end{bmatrix} = \begin{bmatrix} g_{10} \\ g_{20} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} e_{yt} \\ e_{rt} \end{bmatrix} = \qquad \begin{aligned} y_t &= g_{10} + g_{11} y_{t-1} + g_{12} r_{t-1} + e_{yt} \\ r_t &= g_{20} + g_{21} y_{t-1} + g_{22} r_{t-1} + e_{rt} \end{aligned}$$

- With the estimation **we obtain 6 coefficients (g)** and the (symmetric) **variance-covariance** matrix of the residuals.

- That is **9 parameters**: 6 coefficients**,** 2 variances and 1 covariance.

2. SVAR Identification: getting **A**

- The structural VAR has 10 unknowns: 8 parameters and 2 variances.
- Note that A has ones (1s) on the diagonal.

$$\begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix} \begin{bmatrix} y_t \\ r_t \end{bmatrix} = \begin{bmatrix} \beta_{10} \\ \beta_{20} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{rt} \end{bmatrix}$$

- **Imposing restrictions (crucial)**

We must **impose 1 restriction** on the structural parameters, since all we have is the 9 parameters from the reduced-form VAR estimation.

2. SVAR Identification: getting A

- Usual approach: impose restrictions on matrix A which is equivalent to:

- Imposing restrictions on the contemporaneous relations among the endogenous variables of the structural model (based on economics).

$$\underbrace{\begin{bmatrix} 1 & \mathbf{0} \\ a_{21} & 1 \end{bmatrix}}_{\mathrm{A}} \begin{bmatrix} y_t \\ r_t \end{bmatrix} = \begin{bmatrix} \beta_{10} \\ \beta_{20} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{rt} \end{bmatrix}$$

- That is what identification is about.

2. SVAR Identification: getting A

- If we impose $a_{12} = 0$, we impose that ***y*** is only affected with a lag by a shock to ***r*** but that a shock to ***y*** affects ***r*** contemporaneously.

$$y_t = \beta_{10} + \beta_{11} y_{t-1} + \beta_{12} r_{t-1} + u_{yt}$$

$$r_t + a_{21} y_t = \beta_{20} + \beta_{21} y_{t-1} + \beta_{22} r_{t-1} + u_{rt}$$

2. SVAR Identification: getting A

- Be aware that by imposing restrictions on A you also impose restrictions on A^{-1}. Again, if we look at the reduced form,

$$\underbrace{\mathrm{A}^{-1}\mathrm{A}}_{\mathrm{I}}\mathrm{X}_t = \underbrace{\mathrm{A}^{-1}\beta_0}_{G_0} + \underbrace{\mathrm{A}^{-1}\beta_1}_{G_1}\mathrm{X}_{t-1} + \mathrm{A}^{-1}u_t$$

$$\begin{bmatrix} y_t \\ r_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \mathbf{0} \\ -a_{21} & 1 \end{bmatrix}}_{\mathrm{A}^{-1}} \begin{bmatrix} \beta_{10} \\ \beta_{20} \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & \mathbf{0} \\ -a_{21} & 1 \end{bmatrix}}_{\mathrm{A}^{-1}} \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & \mathbf{0} \\ -a_{21} & 1 \end{bmatrix}}_{\mathrm{A}^{-1}} \begin{bmatrix} u_{yt} \\ u_{rt} \end{bmatrix}$$

- It is also a restriction on the relation between the forecast errors and the structural shocks. Forecast errors of ***y*** = structural shocks to ***y:*** $e_{yt} = u_{yt}$

2. SVAR Identification: restrictions

- This matrices product makes the reduced-form VAR:

$$\begin{bmatrix} y_t \\ r_t \end{bmatrix} = \begin{bmatrix} \beta_{10} \\ -a_{21}\beta_{10} + \beta_{20} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ -a_{21}\beta_{11} + \beta_{12} & -a_{21}\beta_{12} + \beta_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ -a_{21}u_{yt} + u_{rt} \end{bmatrix}$$

- The OLS VAR estimation gives:

$$\begin{bmatrix} y_t \\ r_t \end{bmatrix} = \begin{bmatrix} g_{10} \\ g_{20} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} e_{yt} \\ e_{rt} \end{bmatrix}$$

2. SVAR Identification: restrictions

- With the restriction, the number of unknown parameters in the structural model is equal to the number of equations known from the estimated VAR.

- Substitute the results from the estimation for these 9 parameters g_{10}, g_{11}, g_{12}, g_{20}, g_{21}, g_{22}, σ^2_{ey}, σ^2_{er} and $\sigma^2_{ey\text{-}er}$ into 9 equations, to solve for: a_{21}, β_{10}, β_{20}, β_{11}, β_{12}, β_{21}, β_{22}, $\sigma^2_{\upsilon y}$, and $\sigma^2_{\upsilon r}$ (see appendix 2).

- The structural shocks may also be recovered.

2. SVAR Identification: restrictions

Summary of SVAR Identification up to now:

- Get the structural model,
- to do that, we need to get A, so
- impose restrictions on A, based on economics, to recover the structural shocks and the structural parameters, using the reduced-form estimation.
- **Therefore, SVAR identification is about imposing restrictions on matrix A.**

2. SVAR Identification: **how many restrictions?**

- The minimum number of restrictions required: **difference** between the number of unknown and known elements. Let ***n*** be the number of variables in the VAR.

- **Unknown**: as the elements of the diagonal of **A** are 1,s, **A** has n^2-n unknown elements. There are also *n* unknown variances of *u*. The total is: $n^2 - n + n = n^2$

- **Known** elements: the estimation allows us to get $(n^2 + n)/2$ distinct elements contained in the **symmetric** var-cov matrix of the errors: $Ee_t e_t' = \Sigma_e$

2. SVAR Identification: how many restrictions?

- **Why the Known** elements are $(n^2+n)/2$? The distinct elements contained in the symmetric Variance-Covariance matrix of the forecast errors $Ee_te_t' = \Sigma_e$

- Because there are ***n*** distinct elements from the diagonal plus $(n^2-n)/2$ elements off the diagonal. The total number of known elements is: $n+(n^2-n)/2=(n^2+n)/2$

- Thus, impose $n^2-(n^2+n)/2$ restrictions. That is $(n^2-n)/2$. A VAR with 3 variables needs: $(3^2-3)/2=3$ restrictions; A VAR with 4 variables needs 6 restrictions: $(4^2-4)/2=6$

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3. Impulse-responses

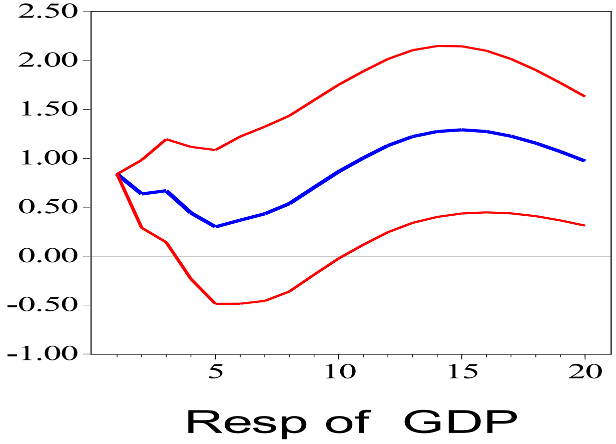
- Impulse-responses trace the effects of structural shocks on the endogenous variables.

- Each response includes the effect of a specific shock on one of the variables of the system at impact **t**, then on **t+1**, and so on.

- Transform our structural autoregressive vector $\mathrm{AX}_t = \beta_0 + \beta_1 \mathrm{X}_{t-1} + u_t$ into a sum of shocks or Wold representation: $\mathrm{X}_t = \mu + \sum_{i=0}^{\infty} \mathrm{C}_i u_{t-i}$ (see appendix 3).

3. Impulse-responses

- EViews does it automatically when impulse-responses are requested.



Dynamic response of GDP to an exogenous government-spending shock that hits the US economy (Blanchard & Perotti , 2002).

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4. Variance Decomposition

- Forecast errors and the relationships among the variables.

- Proportion of the movements of a variable due to shocks to itself and to shocks to other variable(s).

- This forecast error variance decomposition and the forces behind business cycles:

4. Variance Decomposition

- Are all business cycles alike? Are GDP surprises mostly due to technology shocks?
- Are demand shocks the main source of GDP forecast errors at long horizons?

- In our recursive ordering for ***y*** and ***r,*** all of the one period forecast-error variance of ***y*** is due to u_{yt} . At longer horizons the explanatory share of shocks to u_{yt} will diminish.

- The variance of the forecast error increases with the horizon.

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5.1 SVAR Identification: recursive ordering

- In our example the residuals take on a **recursive ordering**:

Matrix A^{-1} only has zeros above the diagonal (lower triangular)

$$e_{yt} = u_{yt}$$

$$e_{rt} = u_{rt} - a_{21} u_{yt}$$

$$\begin{bmatrix} e_{yt} \\ e_{rt} \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 \\ -a_{21} & 1 \end{bmatrix}}^{\mathrm{A}^{-1}} \begin{bmatrix} u_{yt} \\ u_{rt} \end{bmatrix}$$

- Recursive ordering means that there is a sequential chain.

5.1 SVAR Identification: recursive ordering

- Shocks to ***r*** only affect output gap ***y*** with a lag:

$$y_t = \beta_{10} + \beta_{11} y_{t-1} + \beta_{12} r_{t-1} + u_{yt}$$

- Both shocks have a contemporaneous impact on ***r:***

$$r_t + a_{21} y_t = \beta_{20} + \beta_{21} y_{t-1} + \beta_{22} r_{t-1} + u_{rt}$$

- If ***r*** is the monetary-policy interest rate, authorities observe ***y*** and react to it within the period. In other words, shocks to ***y*** affect ***r*** within the period.

5.1 SVAR Identification: recursive ordering

- Another example: a three-variable recursive ordering with ***y***, π (inflation) and ***r*** :

$$e_{yt} = u_{yt}$$
$$e_{\pi t} = -a_{21} u_{yt} + u_{\pi t}$$
$$e_{rt} = (-a_{31} + a_{21} a_{32}) u_{yt} - a_{32} u_{\pi t} + u_{rt}$$

$$\begin{bmatrix} e_{yt} \\ e_{\pi t} \\ e_{rt} \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & \mathbf{0} & \mathbf{0} \\ -a_{21} & 1 & \mathbf{0} \\ -a_{31} + a_{21} a_{32} & -a_{32} & 1 \end{bmatrix}}^{\mathrm{A}^{-1}} \begin{bmatrix} u_{yt} \\ u_{\pi t} \\ u_{rt} \end{bmatrix}$$

- If movements of ***r*** are monetary policy (MP) decisions, policy is reacting to output and inflation within the period. All shocks affect ***r*** within the period.

- The policy interest rate only affects the other variables (***y,*** π) with a lag.

5.1 SVAR Identification: recursive ordering

- The good news is that it is easy to find matrix A^{-1} and **A** when one has a recursive ordering. And with A^{-1} we get the impulse-responses.

- Any invertible matrix may be broken into two lower triangular factors (Choleski factors). It is a numerical technique (**not economics**) to easily estimate a recursive ordering. EViews does it.

- You must be aware of the economic restrictions you are implicitly imposing. **The zeros** on the upper side **are RESTRICTIONS.**

5.1 SVAR Identification: recursive ordering

- Indeed, when one has a recursive ordering,

- A^{-1} is the **Choleski** factor of Σ_e, the variance-covariance matrix:

- Why? substitute $e_t = A^{-1}u_t$ into the var-cov: $Ee_t e_t' = \Sigma_e$ to get $E(\underbrace{A^{-1}u_t}_{e_t}\underbrace{u_t' A^{-1}{}'}_{e_t'}) = \Sigma_e$

- Assuming that $Eu_t u_t' = \Sigma_u = I$, we get $(A^{-1} I A^{-1}{}') = \Sigma_e$ same as $\Sigma_e = (A^{-1}A^{-1}{}')$

  The inverse of **A** is lower triangular consistent with a recursive ordering.

  Note that assuming $\Sigma_u = I$ implies that **A** will not have 1s on the diagonal.

5.1 SVAR Identification: recursive ordering

- If correlations between *the errors* are low, the order is irrelevant. But usually, correlations are strong and the order matters.

- Deciding the ordering (restrictions) to be imposed is crucial. Use economic intuition to decide it.

- It is impractical to try all possible orderings. With 4 variables there are 4!=24 possible orderings. Too many.

5.1 SVAR Identification: recursive ordering

- If you are sure that there is a recursive ordering, or sequential chain, you must then know in what order the variables are recursive.

- It is often difficult to justify the economics of a recursive ordering.

Example: Sims (1992)

"Interpreting the macroeconomic time series facts" European Economic Review

- Sims estimated a VAR to trace the effects of a shock to monetary policy (Fed Funds Interest Rate) using a specific ordering.

- To assess the empirical support for alternative views (models) about monetary policy: VAR agenda.

- He included:

Federal funds rate (*ff*) and the **logs** of **M1** (*lm*), **CPI** (*lp*), and **Industrial production** (ly).

The VAR was estimated using 14 lags during the sample period: 1958:04 - 1991:02.

Example: Sims (1992)

"Interpreting the macroeconomic time series facts" European Economic Review

- The policy variable (***ff***) was ordered first. So, the ordering is equivalent to assuming that:

1. Shocks to the nominal interest rate ***ff***, represent monetary policy shocks.
2. A shock to the policy variable affects all other variables contemporaneously.
3. The variable that is affected by all the others, within the period, is ordered last.
4. Authorities only observe non-policy variables with a lag.

Example: Sims (1992)

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- Now Eviews: by default EViews uses a recursive ordering.

- The estimation can be performed either writing a simple code or using the menus.

```
sample 1958:04    1991:02
var sims0.ls 1:14  ff lm lp ly
sims0.impulse (48,m) ff lm lp ly @ ff lm lp ly
```

Example: Sims (1992)

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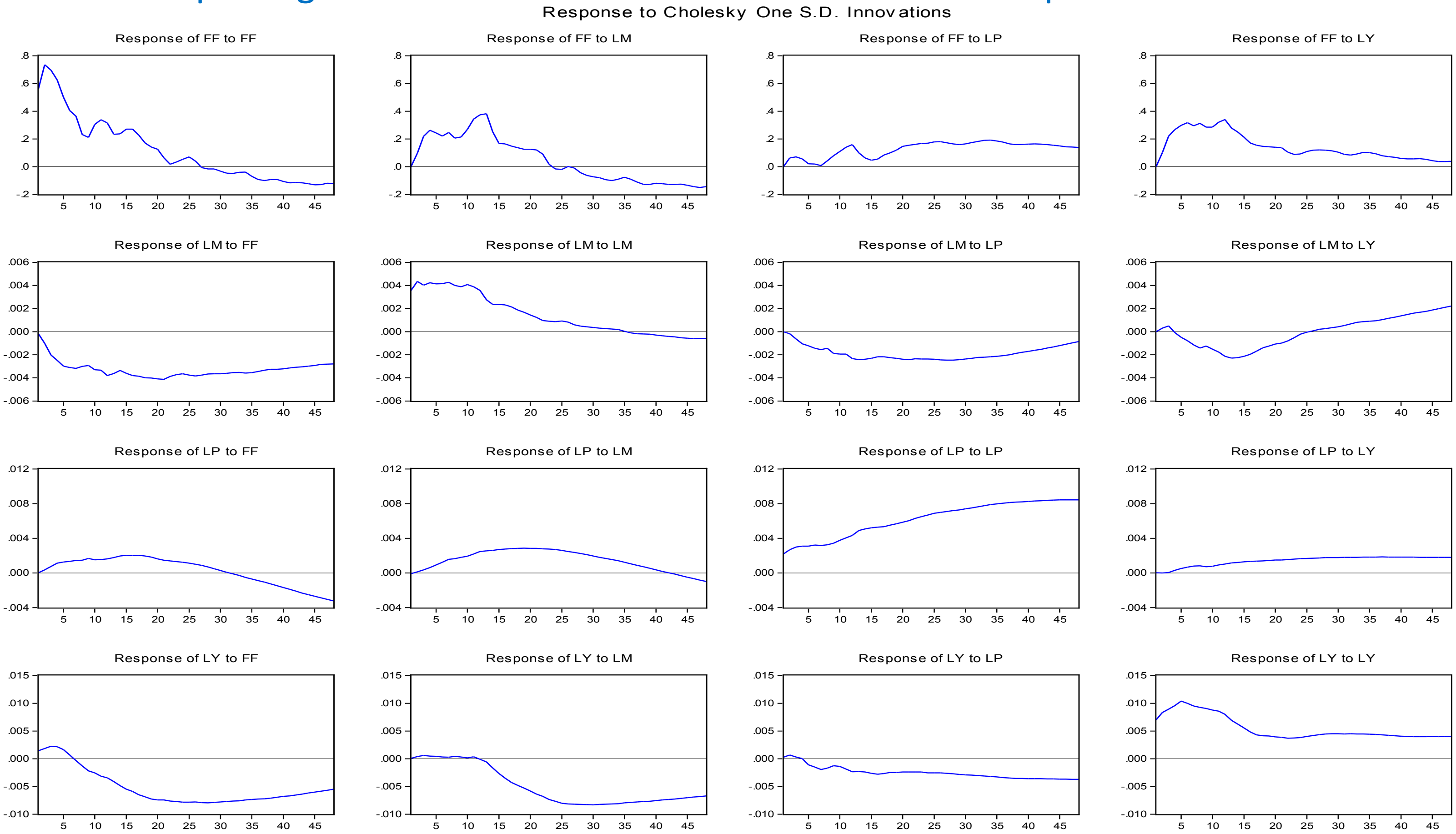

Example: Sims (1992)

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According to Sims:

- There is a puzzling result: prices go up after a funds rate hike.
- If a monetary contraction reduces aggregate demand, lowering output, it cannot be associated with inflation.
- The VAR could be miss-specified. For instance there could be a leading indicator for inflation to which the FED reacts.

Example: Sims (1992)

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Sims continues:

- Thus, the central bank could know that inflationary pressures are about to arrive and counteract them by raising the interest rate.
- And, the “exogenous” movements of the policy interest rate were not totally exogenous in the previous estimation (see example in the introduction).
- Therefore, the exogenous shocks were not properly identified.

Example: Sims (1992)

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- Sims included commodity prices to solve the puzzle.

- The new VAR includes, in the same order:

  Federal funds rate (*ff*) and the **logs** of the **exchange rate** (*lxr*), **commodity prices** (*lcp*) **M1** (*lm*), **CPI** (*lp*), **Industrial production** (ly). Estimated with 14 lags 1958:04 - 1991:02.

Now Eviews:

sample 1958:04 1991:02

var sims1.ls 1:14 ff lxr lcp lm lp ly

freeze(fig5sims) sims1.impulse (48,m) ff lxr lcp lm lp ly @ ff lxr lcp lm lp ly

Example: Sims (1992)

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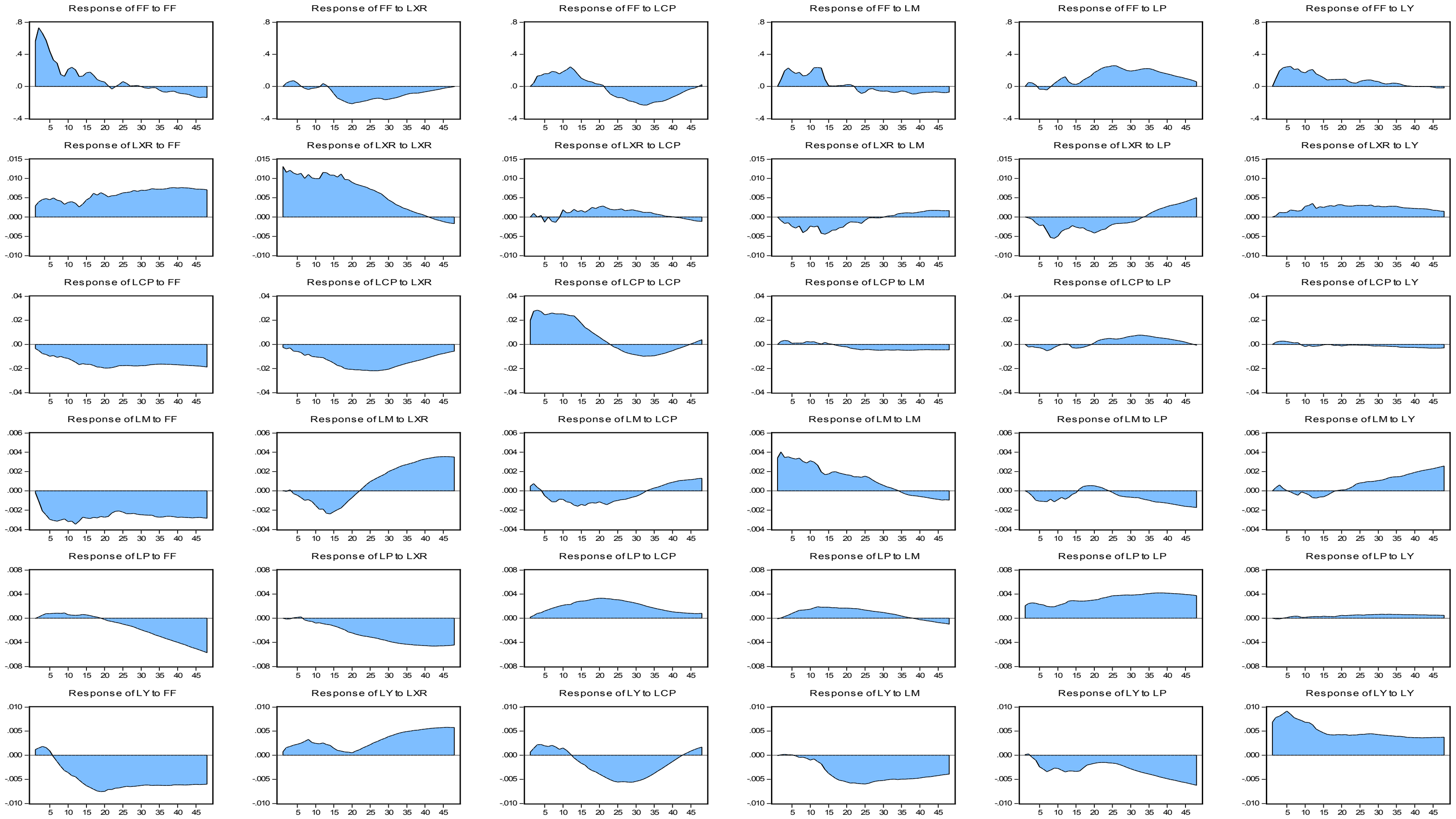
Fig 5 C. Sims, Interpreting…



Example: Sims (1992)

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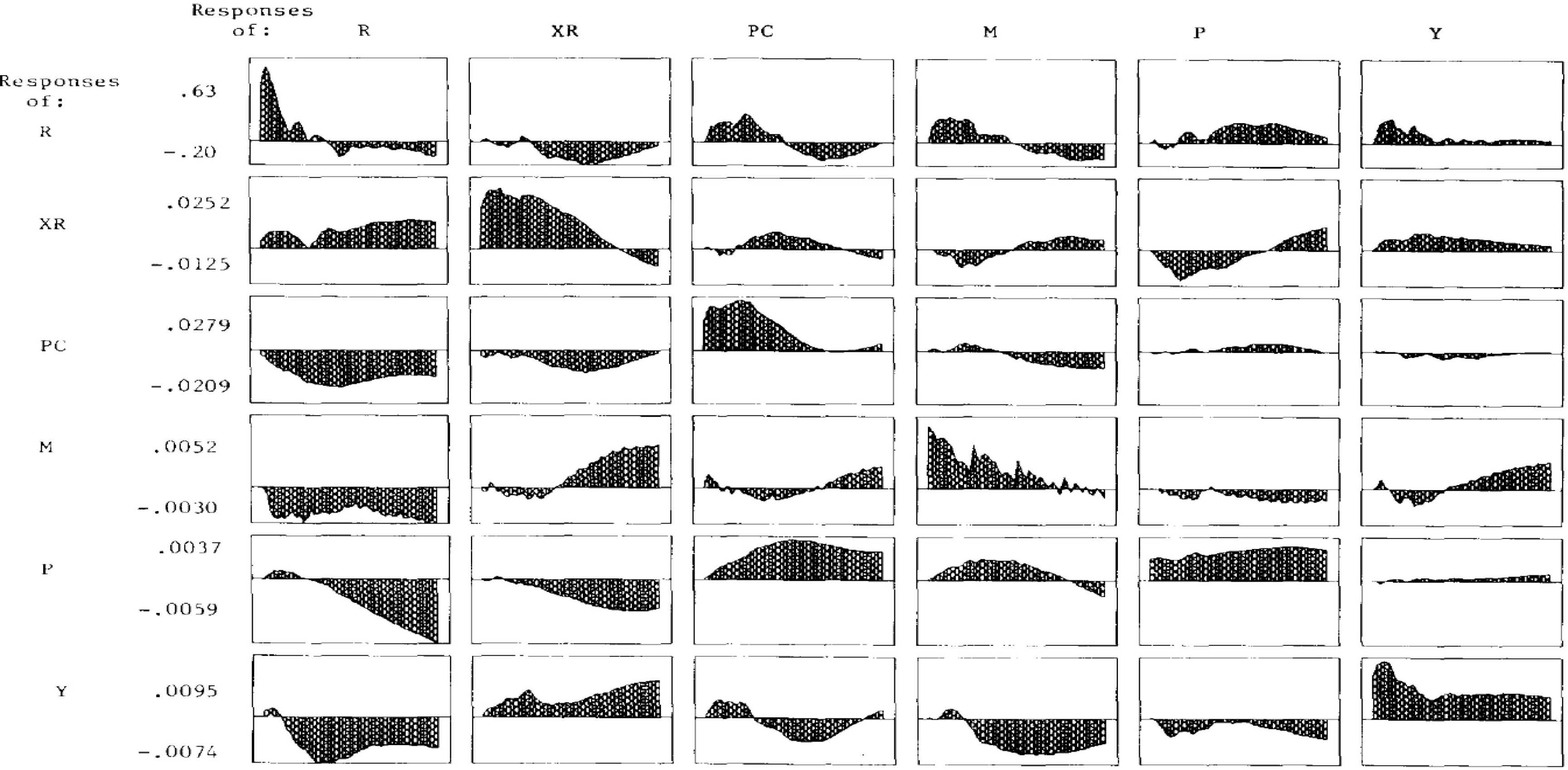

Fig. 5. United States, 1958:4–1991:2.

Example: Sims (1992)

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Variance Decomposition of FF:

Perio...	S.E.	FF	LXR	LCP	LM	LP	LY
1	0.563054	100.0000	0.000000	0.000000	0.000000	0.000000	0.000000
2	0.933237	97.44659	0.239536	0.162813	0.814080	0.316072	1.020911
3	1.185011	91.51034	0.469955	1.221260	3.194231	0.341790	3.262429
4	1.366006	86.60645	0.656302	1.910366	5.208990	0.270176	5.347715
5	1.476977	82.97008	0.602669	2.691273	6.087878	0.299501	7.348602
6	1.550334	79.73776	0.560139	3.444523	6.651684	0.343038	9.262858
7	1.612568	76.91323	0.553971	4.471025	7.368722	0.402639	10.29042
8	1.650126	74.26787	0.581002	5.464787	7.710291	0.385218	11.59083
9	1.679155	72.29028	0.579310	6.173520	8.159543	0.411274	12.38607
10	1.722413	70.27719	0.568590	7.022111	8.808690	0.544019	12.77940

- The forecast error variance of the ***funds rate*** at short horizons is due to itself. This results from the ordering.
- At longer horizons (10 months), the contribution of Commodity Prices and GDP shocks to the movements (forecast-error variance) of ***ff*** increases to 7% and 13% respectively.

Example: Sims (1992)

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Variance Decomposition of LY:

Perio...	S.E.	FF	LXR	LCP	LM	LP	LY
1	0.007019	2.782835	1.124441	0.730435	0.018932	0.064364	95.27899
2	0.010858	3.122677	3.090803	2.143189	0.011190	0.107192	91.52495
3	0.013973	3.565232	3.487431	3.706969	0.015396	0.162892	89.06208
4	0.016791	3.352609	3.802897	4.351516	0.011528	0.452451	88.02900
5	0.019523	2.687458	4.106326	4.217791	0.008929	1.815962	87.16353
6	0.021715	2.193284	4.602728	4.139879	0.014214	3.164212	85.88568
7	0.023624	2.161902	5.330600	4.235448	0.060260	4.735773	83.47602
8	0.025349	2.719657	6.291715	4.158488	0.088494	5.568808	81.17284
9	0.026854	3.850188	6.536904	3.930169	0.142386	5.923964	79.61639
10	0.028243	5.116641	6.638516	3.851522	0.262929	6.254128	77.87626

Cholesky Ordering: FF LXR LCP LM LP LY

- The forecast error variance of **GDP** (***y***) is mostly the result of shocks to itself at short horizons. The longer the horizon, the larger the contribution to the movements of ***y*** of shocks to ***ff,*** the exchange rate, and prices.

Example: Sims (1992)

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- Sims concludes saying that the results are mixed for both monetarist/IS-LM and RBC interpretation of the data:

1. The prize puzzle was partially solved but prices do not fall with the policy shock;

2. The fall of output and money after an interest rate shock is not present in RBC models – No liquidity effect, no real effects of a monetary policy shock.

Objectives

1. Why a structural VAR
2. VAR identification: Identifying **A**
3. Impulse responses
4. Variance decomposition
5. Two identification strategies
   - Recursive identification: Sims (1992)
   - Non Recursive: Blanchard and Perotti (2002)

5.2. SVAR Identification: non recursive ordering

- The system does not need to be recursive.

- You just need to have a minimum number of restrictions to solve for the parameters of the structural model with no specific order.

- The number of **unknown** parameters in the structural model should be equal to the number of **known** parameters from the estimated VAR.

5.2. SVAR Identification: non recursive ordering

- Until now we have used: $e_t = \mathrm{A}^{-1} u_t$ to show the relation between forecast errors and structural shocks.

- Bernanke and Mihov (1998), Blanchard and Perotti (2002), other authors, and also EViews use a more general way of relating errors and shocks in structural VARs: $\mathrm{A} e_t = \mathrm{B} u_t$.

- The specification of these equations can have both errors and shocks on the right hand side.

To get the system responses to shocks one needs to have $e_t = \mathrm{A}^{-1} \mathrm{B} u_t$ or $e_t = \mathrm{F} u_t$. So we use the estimated A and B to compute $\mathrm{F} = \mathrm{A}^{-1} \mathrm{B}$.

Example: Blanchard and Perotti (2002)

“An empirical characterization of the dynamic effects of changes in government spending and taxes on output.” Quarterly Journal of Economics

- Use a semi-recursive VAR to *econometrically* identify fiscal policy shocks; i.e. **exogenous** movements in public expenditure/taxes.

- They estimate a VAR:
  - Using contemporaneous identification restrictions on variables.
  - Imposing, on matrix A, coefficients estimated outside the VAR.

- They pay careful attention to institutional procedures and periods of tax collection and public spending.

Example: Blanchard and Perotti (2002)

"An empirical characterization of the dynamic effects…

- The VAR is $X_t = G(L)X_{t-1} + e_t$ where $X_t = [S_t, T_t, Y_t]'$ includes the logs of quarterly real per capita gov. spending, taxes, and GDP, respectively.

- And $e_t = \left[e_t^S, e_t^T, e_t^Y \right]'$ is the vector of reduced-form residuals or forecast errors of spending, taxes, and GDP, respectively.

- **Spending**: government **consumption** and **investment**.

- **Revenue**: total **taxes** minus **transfers** and minus interest payments.

Example: Blanchard and Perotti (2002)

"An empirical characterization of the dynamic effects…

- The whole system is:

$$\begin{aligned} e_t^S &= b_1 e_t^y + b_2 u_t^T + u_t^S \\ e_t^T &= a_1 e_t^y + a_2 u_t^S + u_t^T \\ e_t^y &= c_1 e_t^T + c_2 e_t^S + u_t^y \end{aligned} \quad = \quad \overbrace{\begin{bmatrix} 1 & 0 & -b_1 \\ 0 & 1 & -a_1 \\ -c_2 & -c_1 & 1 \end{bmatrix}}^{\text{A}} \begin{bmatrix} e_t^S \\ e_t^T \\ e_t^y \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & b_2 & 0 \\ a_2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\text{B}} \begin{bmatrix} u_t^S \\ u_t^T \\ u_t^y \end{bmatrix}$$

- These are matrices A and B to be introduced in EViews.

- $u_t = \left[u_t^S , u_t^T , u_t^Y \right]'$ is the vector of structural shocks to spending, taxes, and GDP.

Example: Blanchard and Perotti (2002)

"An empirical characterization of the dynamic effects...

- Forecast errors are a mix of errors ***e*** and structural shocks ***u***:

1. $$e_t^S = b_1 e_t^y + b_2 u_t^T + u_t^S$$

Unexpected movements in spending (e_t^S) are due to GDP forecast errors ($b_1 e_t^Y$) structural shocks to taxes ($b_2 u_t^T$), and to structural shocks to gov. spending (u_t^S).

– Key: At high frequencies (quarterly), there is **no** discretionary, within the period, response of government spending to unexpected movements in ***y:*** *b1* is always 0.

– Are taxes responding to spending decisions? Or the reverse? Here government spending decisions go before tax decisions so *b2* = 0.

Example: Blanchard and Perotti (2002)

“An empirical characterization of the dynamic effects…

2. $$e_t^T = a_1 e_t^y + a_2 u_t^S + u_t^T$$

- Forecast errors in taxes are due to surprise movements (forecast errors) in GDP ($a_1 e_t^y$), structural shocks to spending ($a_2 u_t^S$), and structural shocks to taxes (u_t^T).
- They constructed the elasticity to output of taxes minus transfers (a_1) to get the automatic effect of GDP on taxes within the quarter. On average $a_1 = 2.08$
- Assumed that spending decisions go first, so ***a2*** was estimated with the SVAR.

Example: Blanchard and Perotti (2002)

"An empirical characterization of the dynamic effects…

3. $e_t^y = c_2 e_t^S + c_1 e_t^T + u_t^y$

- Forecast errors in GDP (e_t^y) are due to surprise movements (forecast errors) in spending ($c_2 e_t^S$), taxes ($c_1 e_t^T$), and structural shocks to GDP (u_t^Y).

- They also estimated the elasticities of GDP to government spending and to taxes (c_2, c_1) outside the VAR. Indeed they performed a separate estimation using instrumental variables (IV) to get *c1* and *c2*.

- One could have obtained *c1* and *c2* directly from the VAR estimation.

Example: Blanchard and Perotti (2002)

"An empirical characterization of the dynamic effects…

SPENDING SHOCK in Eviews: VAR 4 lags 1960:1 1997:4 ($a_2 \neq 0, b_2 = 0$)

scalar a1 = 2.08
scalar c1 = @coef(2) = -0.15115
scalar c2 = @coef(3) = 0.200726

$$\overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -a_1 \\ -c_2 & -c_1 & 1 \end{bmatrix}}^{\mathrm{A}} \begin{bmatrix} e_t^S \\ e_t^T \\ e_t^y \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ a_2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\mathrm{B}} \begin{bmatrix} u_t^S \\ u_t^T \\ u_t^y \end{bmatrix}$$

- *a1, c1,* and *c2* above are elements of matrix A obtained outside the VAR. The VAR estimation will provide an estimated value for $\boldsymbol{a_2}$.
- Recall that *b1* is always zero, and *b2* is zero when it is assumed that spending decisions are taken before increasing taxes, as it is the case here.

Example: Blanchard and Perotti (2002)

"An empirical characterization of the dynamic effects…

- Commands to **declare** and **fill** matrices A and B and **estimate** the VAR.

```
matrix(3,3) pata
matrix(3,3) patb

pata.fill(by=row) 1,0,0,0,1,-a1, -c2,-c1,1
patb.fill(by=row) 1,0,0,na,1,0,0,0,1
```

$$\overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2.08 \\ -0.2 & 0.15 & 1 \end{bmatrix}}^{\text{A}} \begin{bmatrix} e_t^S \\ e_t^T \\ e_t^y \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ na & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\text{B}} \begin{bmatrix} u_t^S \\ u_t^T \\ u_t^y \end{bmatrix}$$

var Bl_PEvar1.ls(noconstant) 1 4 ss tt yy @

- EViews estimates elements with ***na***. Only $\boldsymbol{a_2}$ is being estimated here.

Example: Blanchard and Perotti (2002)

“An empirical characterization of the dynamic effects…

- ‘Now use estimated matrices A, B to compute $F = A^{-1}B$ and get impulse responses:

```
matrix mata1 = BL_PEvar1.@svaramat

matrix matb1 = BL_PEvar1.@svarbmat
```

- ‘mata1=A and matb1=B were estimated in BL_PEvar1 (actually, only element a2).

- ‘compute matrix F:

```
matrix fact1 = @inverse(mata1)*matb1
```

Example: Blanchard and Perotti (2002)

“An empirical characterization of the dynamic effects…

SPENDING SHOCK

- “'Here we get impulse responses with the command *impulse* using the F matrix we just computed (fact1) on the VAR we saved as *BL_PEvar1:*

- **freeze**(fig1) BL_PEvar1**.impulse(**20, imp=user, fname=**fact1**, se=a, m**)** ss tt yy @ 1

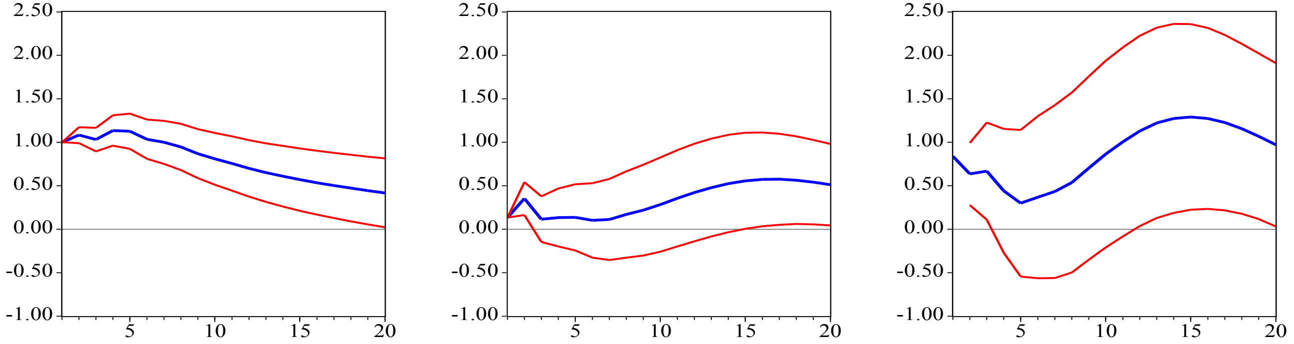
- ‘Generates impulse responses. Also creates a figure with called fig1, using the command freeze.

- **GDP and tax revenues increase after the gov. spending shock (see figure).**
  GDP increases on impact 0.84 then declines and reaches 1.29 after 4 years.

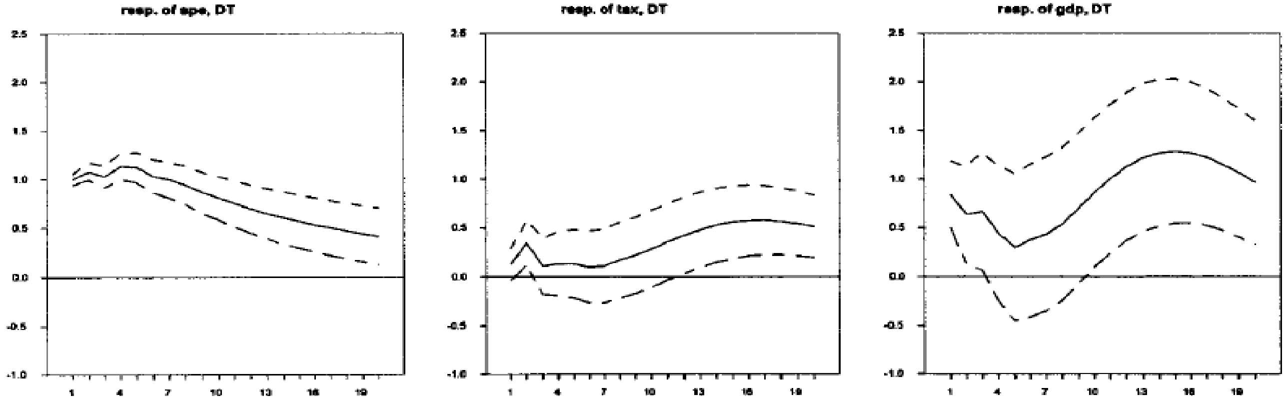
Example: Blanchard and Perotti (2002)

“An empirical characterization of the dynamic effects…

Fig V Response to a Spending Shock



Resp of SPE Resp of TAX Resp of GDP



Example: Blanchard and Perotti (2002)

"An empirical characterization of the dynamic effects…

Variance Decomposition of YY:

Perio...	S.E.	Shock1	Shock2	Shock3
1	0.798610	4.874778	1.899081	93.22614
2	1.230637	3.231791	1.807872	94.96034
3	1.609220	2.651643	1.446138	95.90222
4	1.912451	2.113209	1.425887	96.46090
5	2.137422	1.778216	1.510957	96.71083
6	2.288811	1.665021	1.636879	96.69810
7	2.384403	1.679825	1.793290	96.52689
8	2.438515	1.819793	1.957388	96.22282
15	2.559616	7.251208	2.341387	90.40741
20	2.633874	11.01452	2.286783	86.69870

- The forecast error variance of **GDP** (***y***) is mostly the result of shocks to itself one quarter ahead (93%). At longer the horizons (20 quarters), the contribution of shocks to **Spending** (***S***) increases to (11%).

MFx – Macroeconomic Forecasting

Structural Vector Autoregressive Models
Part II

IMFx

Example: Blanchard and Perotti (2002)

"An empirical characterization of the dynamic effects…

Variance Decomposition of SS:

Perio...	S.E.	Shock1	Shock2	Shock3
1	1.000000	100.0000	0.000000	0.000000
2	1.476181	99.65791	0.067645	0.274448
3	1.805553	99.23467	0.273676	0.491651
4	2.135346	99.23007	0.254158	0.515770
5	2.419537	98.98964	0.332287	0.678075
6	2.640862	98.46854	0.526405	1.005051
7	2.833431	97.97077	0.725335	1.303899
8	2.998815	97.41701	0.946562	1.636427
15	3.608232	94.89032	2.451833	2.657852
20	3.775795	94.62551	2.899531	2.474964

- The forecast error variance of **government spending (S)** is due exclusively to exogenous shocks to itself one quarter ahead (100%). This is due to the identification strategy. Twenty quarters ahead still (95%) is explained by shocks to **spending**.

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1. Inverse Matrix

- Recall that the inverse of A is equal to: $\frac{1}{|\mathrm{A}|}\times \text{Adjugate of A}$

- Further recall that the adjugate is equal to the transpose of the cofactor matrix.
- Find the inverse of the following matrix:

$$\mathrm{A}=\begin{bmatrix} 4 & 1 & -1 \\ 0 & 3 & 2 \\ 3 & 0 & 7 \end{bmatrix}$$

determinant:

$$\begin{aligned} &4[(3\cdot 7)-(0\cdot 2)] \\ &-1[(0\cdot 7)-(3\cdot 2)] \\ &+(-1)\cdot[(0\cdot 0)-(3\cdot 3)] \\ &=84+6+9=99 \end{aligned}$$

1. Inverse Matrix

- The cofactor matrix of $A = \begin{bmatrix} 4 & 1 & -1 \\ 0 & 3 & 2 \\ 3 & 0 & 7 \end{bmatrix}$ is:

$$\begin{bmatrix} \begin{vmatrix} 3 & 2 \\ 0 & +7 \end{vmatrix} & -\begin{vmatrix} 0 & 2 \\ 3 & 7 \end{vmatrix} & \begin{vmatrix} 0 & 3 \\ 3 & 0 \end{vmatrix} \\ -\begin{vmatrix} 1 & -1 \\ 0 & 7 \end{vmatrix} & +\begin{vmatrix} 4 & -1 \\ 3 & 7 \end{vmatrix} & -\begin{vmatrix} 4 & 1 \\ 3 & 0 \end{vmatrix} \\ \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 4 & -1 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} 4 & 1 \\ 0 & 3 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 21 & 6 & -9 \\ -7 & 31 & 3 \\ 5 & -8 & 12 \end{bmatrix}$$

1. Inverse Matrix

- Cofactor matrix

$$\begin{bmatrix} 21 & 6 & -9 \\ -7 & 31 & 3 \\ 5 & -8 & 12 \end{bmatrix}$$

- Transpose of cofactors:

$$\begin{bmatrix} 21 & -7 & 5 \\ 6 & 31 & -8 \\ -9 & 3 & 12 \end{bmatrix}$$

- Inverse of A:

$$\frac{1}{|\mathrm{A}|} adj\,\mathrm{A} = \frac{1}{99}\begin{bmatrix} 21 & -7 & 5 \\ 6 & 31 & -8 \\ -9 & 3 & 12 \end{bmatrix}$$

2. SVAR Identification: restrictions

- Imposing the restriction makes the number of unknown parameters in the structural model equal to the number of parameters known from the standard VAR estimation:

1. $g_{10} = \beta_{10}$
2. $g_{20} = -a_{21}\beta_{10} + \beta_{20}$
3. $g_{11} = \beta_{11}$
4. $g_{21} = -a_{21}\beta_{11} + \beta_{12}$
5. $g_{12} = \beta_{12}$
6. $g_{22} = -a_{21}\beta_{12} + \beta_{22}$

Since $e_{yt} = u_{yt}$ then,

7. $\mathrm{var}(e_{yt}) = \sigma^2_{uy}$
8. $\mathrm{var}(e_{rt}) = \sigma^2_{ur} + a^2_{21}\sigma^2_{uy}$
9. $cov(e_{yt}e_{rt}) = E(u_{yt})(u_{rt} - a_{21}u_{yt}) = -a_{21}\sigma^2_{uy}$

- Substitute the estimated values for these 9 parameters $g_{10}, g_{11}, g_{12}, g_{20}, g_{21}, g_{22}, \sigma^2_{ey}, \sigma^2_{er}$ and $\sigma_{ey\text{-}er}$ in the 9 equations, to solve for: $a_{21}, \beta_{10}, \beta_{20}, \beta_{11}, \beta_{12}, \beta_{21}, \beta_{22}, \sigma^2_{\upsilon y}$, and $\sigma^2_{\upsilon r}$.

3. Impulse-responses

- In a first order autoregressive model:

$$y_t = g_0 + g_1 y_{t-1} + \varepsilon_t$$

- The stability condition is that $|g_1|<1$

- In the case of a VAR, an equivalent condition is required. It becomes clear when the standard VAR is iterated backwards. For a simple VAR(1):

$$X_t = G_0 + G_1 X_{t-1} + e_t \quad \longrightarrow \quad X_{t-1} = G_0 + G_1 X_{t-2} + e_{t-1}$$

$$X_t = G_0 + G_1 (G_0 + G_1 X_{t-2} + e_{t-1}) + e_t$$

3. Impulse-responses

- Successive iterations generate:

$$X_t = G_0 + G_1 (G_0 + G_1 \underbrace{(G_0 + G_1 X_{t-3} + e_{t-2})}_{X_{t-2}} + e_{t-1}) + e_t$$

$$X_t = (I + G_1 + G_1^2 + \cdots + G_1^{\infty}) G_0 + e_t + \sum_{i=1}^{\infty} G_1^i e_{t-i} + G_1^i X_{t-n-i} \rightarrow 0$$

As i → +∞ **X** reduces to a sum of errors or Wold representation of **X**:

$$X_t = \mu + \sum_{i=1}^{+\infty} G_1^i e_{t-i} + e_t \quad \text{assume} \quad \psi_i = G_1^i \quad \text{then} \quad X_t = \mu + \sum_{i=1}^{+\infty} \psi_i e_{t-i} + e_t$$

If **X** has a Wold representation then **X** is **stable**. The condition that guarantees it is that **G has eigenvalues smaller than 1 in modulus**. It is a general result, not only valid for a VAR (1) but valid for any VAR(p).

3. Impulse-responses

- Since the VAR is stationary, the estimated reduced-form VAR ha a moving average:

$$X_t = \mu + e_t + \sum_{i=1}^{\infty} \psi_i e_{t-i}$$

- Using the same old relation between forecast errors and structural shocks $e_t = A^{-1} u_t$ we find: $X_t = \mu + A^{-1} u_t + \sum_{i=1}^{\infty} \psi_i A^{-1} u_{t-i}$ or more compactly $X_t = \mu + \sum_{i=0}^{\infty} C_i u_{t-i}$

  $c_{11,i}$ and $c_{12,i}$ are the responses of ***y*** and ***r*** to a change u_{yt} and u_{rt} respectively.

- Note that $c_{11,0}$ is the effect at impact, $c_{11,1}$ is the effect of u_{yt} on ***y*** in t+1, and so on in succession: $\frac{\partial y_{t+k}}{\partial u_{yt}} = C_{11,k}$

- Also, the cumulative effect is $\sum_{i=0}^{\infty} c_{11,i}$.

4. Variance Decomposition

- Knowledge of the prediction errors can be extremely valuable in examining the relationships among the variables.

- Assume that we know the coefficients G_0 and G_1 and wish to project the values of X_{t+1} conditional on the observed values of X_t.

- If the equation $X_t = G_0 + G_1 X_{t-1} + e_t$ is advanced one period, we obtain $X_{t+1} = G_0 + G_1 X_t + e_{t+1}$ and the prediction error will be $X_{t+1} - EX_{t+1} = e_{t+1}$ for innovation in t+1.

$$X_{t+3} = G_0 + G_1 \overbrace{(G_0 + G_1 \underbrace{(G_0 + G_1 X_t + e_{t+1})}_{X_{t+1}} + e_{t+2})}^{X_{t+2}} + e_{t+3}$$

$$X_{t+3} - EX_{t+3} = G_1^2 e_{t+1} + G_1 e_{t+2} + e_{t+3}$$

4. Variance Decomposition

$$\mathrm{EX}_{t+n} = (\mathrm{I} + \mathrm{G}_1 + \mathrm{G}_1^2 + \ldots + \mathrm{G}_1^{n-1})\mathrm{G}_0 + \mathrm{G}_1^n \mathrm{X}_{t+n}$$

$$\mathrm{X}_{t+n} - \mathrm{EX}_{t+n} = e_{t+n} + \mathrm{G}_1 e_{t+n-1} + \mathrm{G}_1^2 e_{t+n-2} + \cdots + \mathrm{G}_1^{n-1} e_{t+1}$$

- This may also be expressed in terms of structural errors:

$$\mathrm{X}_{t+n} = \mu + \sum_{i=0}^{n} \mathrm{G}_1^i \mathrm{A}^{-1} \mathrm{u}_{t+n-i} = \quad \mathrm{X}_{t+n} = \mu + \sum_{i=0}^{n} \psi_i \mathrm{A}^{-1} \mathrm{u}_{t+n-i} = \quad \mathrm{X}_{t+n} = \mu + \sum_{i=0}^{n} C_i \mathrm{u}_{t+n-i}$$

- The prediction error for only GDP gap ***n*** steps forward will be:

$$\mathrm{y}_{t+n} - \mathrm{Ey}_{t+n} = c_{11,0} u_{yt+n} + c_{11,1} u_{yt+n-1} + \cdots + c_{11,n-1} u_{yt+1} +$$
$$c_{12,0} u_{rt+n} + c_{12,1} u_{rt+n-1} + \cdots + c_{12,n-1} u_{rt+n}$$

- The variance of this prediction error is:

$$\sigma_{y,n}^2 = \sigma_y^2 [c_{11,0}^2 + c_{11,1}^2 + \cdots + c_{11,n-1}^2] +$$
$$\sigma_z^2 [c_{12,0}^2 + c_{12,1}^2 + \cdots + c_{12,n-1}^2]$$

4. Variance Decomposition

- As the values of $c_{11,0}^2$ are necessarily positive, the variance of the errors increases with the projection horizon.

- It is possible to decompose the prediction error n periods forward by the contribution each of the two shocks in our example.

- The proportions of $\sigma_{y,n}^2$ attributable to each structural shock are:

$$\frac{\sigma_y^2[c_{11,0}^2 + c_{11,1}^2 + \cdots + c_{11,n-1}^2]}{\sigma_{y,n}^2} \qquad \frac{\sigma_r^2[c_{12,0}^2 + c_{12,1}^2 + \cdots + c_{12,n-1}^2]}{\sigma_{y,n}^2}$$

- This is the proportion of the changes in one variable attributable to shocks to it and to shocks in another variable.

- If u_{rt-i} fails to explain any changes in ***y*** the latter is exogenous.

- The restriction imposed above requires that the entire variance in the prediction error for ***y*** one period forward be attributable to u_{yt-i}