MFx – Macroeconomic Forecasting

Module 4: Forecast Uncertainty and Evaluation

IMFx

Session 1: Introduction

Main Objectives

- Learn how to choose between forecasts from competing models or sources
- Learn how to assign a number to forecast performance using various “summary” statistics
  - Interpretation of these statistics

Main Objectives

- Learn about different forecasting strategies
- Learn how to visualize forecast uncertainty using fan charts
- How to do everything above in EViews

Forecast Evaluation - General Idea

- Hard to say how a given strategy will perform in the future
  - Anything can happen?

- Idea: estimate the model on a fraction of the available data and use the remaining part to evaluate out-of-sample forecast performance

Example 1

- Background: 21 Dec 2012 (Friday) - the end of a long cycle according to the Mayan calendar
- Resulting weather forecast:

Example 1

- Is it a good forecast?
- It is very precise, but far from the actual observation
- It has large bias, low uncertainty

Example 2

- Is this a “good” forecast?
- Might be unbiased but uncertainty is high

“If you have to forecast, forecast often”

- Forecasts from the same model need to be repeated

- The properties of a forecaster/model (correct on average, low volatility) are evaluated using various statistical measures

What we learned

- Forecasters/models are evaluated using out-of-sample forecast performance
- There are many dimensions/statistics one could use
  - Average of the forecast errors, degree of uncertainty
- Evaluation has to be repeated
  - “It is often said there are two types of forecasts… lucky or wrong!”

Session 2, Part 1: Sources of Uncertainty

It is often said there are two types of forecasts … lucky or wrong!”

- Economic variables are random processes and therefore each has a probability distribution

- We call that distribution, which is typically unknown, the “data generation process (DGP)” of that variable

It is often said there are two types of forecasts ... lucky or wrong!”

- If the variable can have continuous values, the probability of a single point forecast being equal to the eventual outcome is zero
- Therefore, it is only possible to attach a probability to a range of possible outcomes encompassing the actual outcome
- If a point forecast happens to be equal to the actual outcome, it is purely by chance and quite unlikely to be repeated in the next period

Data generation process

- Every economic variable has an underlying "data generation process (DGP)"
- The DGP is a probability density function, for example,

$$N(\mu, \sigma^2)$$

- For any sequence of data points, π_t drawn from this distribution, we may write

$$y_{t+1} = E_t\left(y_{t+1} \mid Information_t\right) + \varepsilon_{t+1}$$

- where E_t is the "conditional expectation operator" and ε_t is a residual error term, which we expect to have zero mean

Why Conditional Means?

- The density for y_{t+1} is difficult to estimate precisely, requiring a considerable amount of data in practice

- Conditional means can be estimated more readily using a variety of statistical procedures

- For example, it is often assumed that the conditional mean is a linear function of other explanatory variables ("information set"), and to use ordinary least squares to estimate the unknowns in that relationship

Assuming that conditional mean is known

$$y_{t+1} = E_t\left(y_{t+1} \mid Information_t\right) + \varepsilon_{t+1}$$

- We still need an estimate of ε_{t+1} to complete the forecast
- Assuming ε_{t+1} is a white noise process from a normal distribution with zero mean, say, our best guess for all of its future values is zero
- So it is typically impossible to predict the actual outcome exactly

Source of Forecast Uncertainty #1

- Even if we know the conditional mean, our conditional forecast will necessarily differ from the outcome

- Indeed, the observed forecast errors will obviously reflect the distribution of the true error term, which is normally unknown

It follows that…

- The best we can hope for is that, on average, our actual forecast errors are zero. Why?

$$y_{t+1} = E_t\left(y_{t+1} \mid Information_t\right) + \varepsilon_{t+1}$$

- Recall that with as $N \rightarrow \infty,$

$$\frac{\left(\sum_{t=1}^{n} \varepsilon_t\right)}{N} \xrightarrow[p]{} 0$$

Key Implication

- If you want to be a reputable forecaster
  - The average of your forecast errors should be zero
  - Don’t get overly excited if one of your forecasts is exactly correct. Unless you are dealing with discrete outcomes, it is a fluke

Session 2, Part 2: Additional Sources of Uncertainty

L-2b Additional Sources of Uncertainty

Additional sources of forecast uncertainty

- We will continue with our maintained assumption that the conditional mean is known and correctly specified

- However, now assume it depends on a set of variables, X:

$$y_{t+1} = F(X_{t+1};\theta) + \varepsilon_{t+1}$$

- These variables might be jointly determined within the same system (economic model), or be determined outside of the system (exogenous), for example GDP of Thailand

Specifically

- If the forecasting model for the series we are interested in is

$$y_{t+1} = F(X_{t+1};\theta) + \varepsilon_{t+1}$$

- The right-hand side or explanatory variables, X_t, will need to be forecasted using additional equations, namely:

$$X_{t+1} = G(Z_{t+1};\beta) + u_{t+1}$$

Source of forecast uncertainty #2

- Inherent randomness of the explanatory variables
- The uncertainty associated with X_t will also result in a wider forecast confidence interval for y_t

What if parameters of DGP are unknown

- We will continue with our maintained assumption that the conditional mean is known
- However, let's now assume that we don't know the values of the parameters on X, namely θ

$$y_{t+1} = F(X_{t+1};\theta) + \varepsilon_{t+1}$$

Source of forecast uncertainty #3

- The unknown parameters need to be estimated from the available data, which are random
- It follows that the estimates of θ are necessarily random variables and therefore can contribute to your forecast errors
- Why? Estimates of the unknown parameters will typically be different from their true values: $\hat{\theta} \neq \theta$

More sources of forecast uncertainty

- The conditional mean is misspecified. Why? Perhaps
  - The set of variables in X is incomplete
  - Actual functional form used is wrong (linear or non-linear?)
  - Underlying parameters may change over time (i.e., structural break)

- Estimation Issues:
  - Parameters are estimated incorrectly (possibly because of misspecification, measurement error in the explanatory variables and a poor estimation procedure)

What we learned

$$y_{t+1} = F(X_{t+1};\theta) + \varepsilon_{t+1}$$

- Sources of forecast uncertainty
  - Economic variables are inherently random
  - There are unknown parameters in the conditional mean that need to be estimated
  - Explanatory variables also need to be forecast and could be random quantities themselves
  - The working form of the conditional mean may be misspecified

Session 3: Forecast Assessment Statistics

Measures of Forecast Uncertainty

- How can we measure forecast uncertainty?
- How do we use a measure of forecast uncertainty in practice?
- Unfortunately, there is no unique measure of forecast accuracy and precision

Common Statistical Measures

The smaller they are the better the forecast

- Bias: the difference between the forecasts and the correct outcome (on average)

- Variance (“standard forecast error, SE”): A narrow range of outcomes is compatible with the forecast

- Mean Squared Forecast Error (MSFE): A combination of bias and variance that is commonly reported in forecast comparisons

Measures of Forecast Accuracy

- **Bias:** $BIAS = \frac{1}{f}\sum_{t=1}^{f} FE_t \qquad FE_t = \hat{y}_t - y_t$
- **SE:** $SE = \sqrt{\frac{1}{f}\sum_{t=1}^{f}\left(FE_t - BIAS\right)^2}$
- **MSE:** $MSE = \frac{1}{f}\sum_{i=1}^{f} FE_i^2$
- **RMSE:** $RMSE = \sqrt{\frac{1}{f}\sum_{i=1}^{f} FE_t^2}$
- **MAE and MAPE:** $MAE = \frac{1}{f}\sum_{i=1}^{f}\left|FE_t\right| \qquad MAPE = \frac{1}{f}\sum_{i=1}^{f}\left|\frac{FE_t}{y_t}\right|$

Relationship between different measures

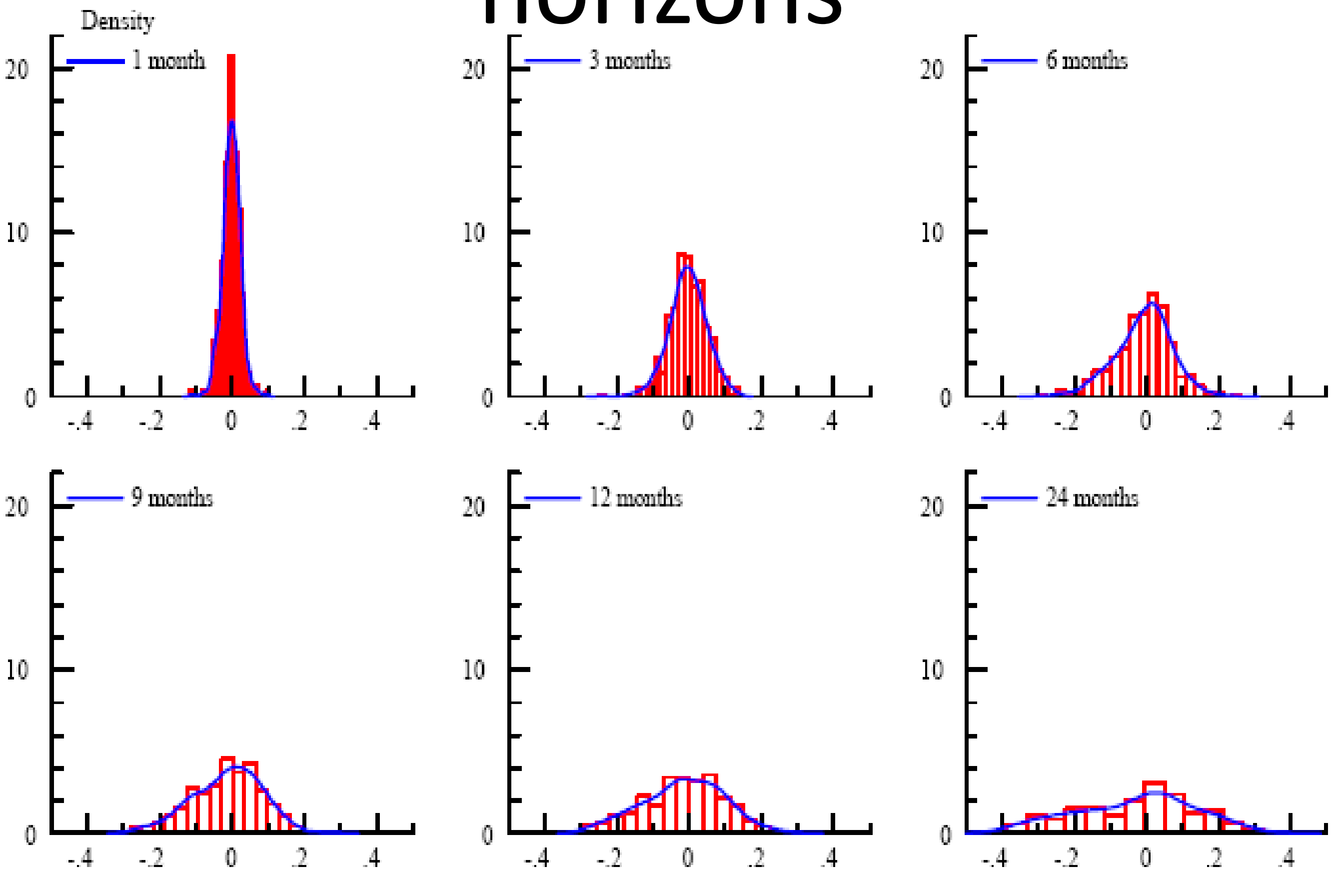
- Bias, Standard forecast error (SE) and MSE:

$$MSE = SE^2 + BIAS^2$$

- That is, mean squared error is a combination of bias and standard forecast error

- See appendix for proof

Forecast uncertainty for different horizons



Symmetric Costs

- The mean square and absolute error assume a symmetric cost associated with positive and negative forecast errors
- However, the cost of forecast errors can be asymmetric
- Examples:
  - Airplane departure
  - Inflation forecast for inflation targeting (deflation often perceived as more costly than inflation)
- Need different criteria for these cases

Comparative Evaluation

- Often useful to compare these measures against those that are obtained from using a benchmark forecast such as the “naïve forecast” or the consensus forecast
- Our economic/behavioral models should be at least as good forecasters as the established benchmark
- Note, however, that there is often a trade-off between forecasting accuracy and the number of parameters one has to estimate in the economic model
- The advantage of economic models is that enable you to assess the reasons behind any forecast error

Naïve Forecasting Model

- Compare the performance of your model against that obtained using a naïve (no change) forecasting model:

- Assumed 1-step ahead forecast is:

$$\hat{y}_t = y_{t-1} \qquad\qquad MAPE_{naive} = 100\frac{1}{f}\sum_{i=1}^{f}\left|\frac{y_t - y_{t-1}}{y_t}\right|$$

- Note that the MAE formula cannot be calculated if $y_t = 0$ for any period

- It also not suitable for variables expressed in log form

Graphical Approaches

- Summary statistics of forecasting performance are useful, but can mask important outcomes (for example, that the higher forecast error for a particular model is due to a single observation error)

- Sensible to back up the statistical analysis with scatter and line plots of actual against fitted, and non-parametric estimates of the relationship between actual and the forecast

Session 3 Workshop: Calculating Forecast Assessment Statistics

Let's evaluate some forecasts! By hand!

- Given: Thailand monthly year-on-year all items CPI inflation data for 2003-2014

$$y_t = \frac{CPI_t - CPI_{t-12}}{CPI_{t-12}}$$

- Source: Haver Analytics
- Objective: Evaluate and compare two 1 month ahead forecasts using actual data for 2014 as a test period
- Let's work in Excel for this exercise
  – So we know what numbers in EViews mean

Step 1

- Calculate observational errors for forecast 1
- Recall that forecast (observational) error is

$$FE_t = \hat{y}_t - y_t$$

- In the spreadsheet forecast errors for forecast 1 are denoted as e1.
- Thus

$$e_{1t} = f_{1t} - \pi_t$$

Step 2

- Let's calculate all of the components for evaluations statistics.
- For t=2014:2..2014:12 for forecast 1 calculate

$$e_{1t}^{2}, \left|e_{1t}\right|, \frac{e_{1t}}{\pi_t}, \left|\frac{e_{1t}}{\pi_t}\right|$$

- Averages will be calculated automatically

Step 3

- Calculate forecast evaluation statistics

- **Bias:** $$BIAS = \frac{1}{f}\sum_{t=1}^{f} FE_t$$
- **SE:** $$SE = \sqrt{\frac{1}{f}\sum_{t=1}^{f}\left(FE_t - BIAS\right)^2} = \sqrt{MSE - BIAS^2}$$
- **MSE:** $$MSE = \frac{1}{f}\sum_{i=1}^{f} FE_i^2$$
- **RMSE:** $$RMSE = \sqrt{\frac{1}{f}\sum_{i=1}^{f} FE_t^2}$$
- **MAE and MAPE:** $$MAE = \frac{1}{f}\sum_{i=1}^{f}\left|FE_t\right| \quad MAPE = \frac{1}{f}\sum_{i=1}^{f}\left|\frac{FE_t}{y_t}\right|$$

Step 4

- Repeat steps 1-3 for the naïve forecast
- It is denoted forecast 2 in the spreadsheet

Step 5

- Let’s interpret the forecast evaluation statistics!

Session 4: Theil’s U Statistics

What we learned

- Statistics for evaluating a forecast model
  - BIAS, MSE, RMSE, MAE, MAPE
  - Properties and advantages/drawbacks
- Appropriate benchmark for forecast evaluation
  - Naïve forecasting model
- Support your numerical analysis with visual cues of actual outcomes against their forecasts

Theil’s U Statistics

- Theil’s U_1 statistic – a measure of forecast accuracy
  - Considers the disproportionate cost of large errors
  - Reported by EViews
  - Has undesirable properties
- Theil’s U_2 statistic – a measure of forecast quality
  - Compares your forecast with a benchmark (naïve) method

Theil's U_1 Statistic

$$U_1 = \frac{\sqrt{\frac{1}{f}\sum_{t=1}^{f}(\hat{y}_t - y_t)^2}}{\sqrt{\frac{1}{f}\sum_{t=1}^{f}\hat{y}_t^{\,2}} + \sqrt{\frac{1}{f}\sum_{t=1}^{f}y_t^{\,2}}}$$

- Intuition: RMSE normalized by the dispersion of actual and forecasted series
- $0 \le U_1 \le 1$
  - $U_1 = 0$ is the best forecast (no obs. error)
- If U_1 statistic is smaller for one model, it generally does not mean that this model is better
- U_1 is reported by EViews

Theil's U_2 Statistic

Assuming 1-step ahead forecast

$$U_2 = \sqrt{\frac{\frac{1}{f}\sum_{t=1}^{f}\left(\frac{\hat{y}_t - y_t}{y_{t-1}}\right)^2}{\frac{1}{f}\sum_{t=1}^{f}\left(\frac{y_{t-1} - y_t}{y_{t-1}}\right)^2}} = \frac{MSE(\text{model})}{MSE(\text{naive model})}$$

- Intuition: root mean squared percentage errors relative to naïve forecast.
- $U_2 \geq 0$
  - $U_2 = 0$ only if the forecasts errors (numerator) are zero.
  - $U_2 = 1$ if $\hat{y}_t = y_{t-1}$ (naïve forecast)
- Can be used to order quality of models: smaller U_2 means better forecast model
- U_2 is NOT reported by EViews (though easy to calculate using EViews)

General Principle

- Note that the U_2 statistic is a ratio of mean square errors.

- We may of course compare the mean square error of the model (i.e., the numerator) to any other method of producing a forecast, such as another model or even the consensus forecast.

- The forecasting model with the smaller mean square error is – in the absence of other (e.g., graphical) metrics - preferred

Theil's U_2 Statistic

When forecasting more than 1 periods ahead

$$U_2 = \sqrt{\frac{\frac{1}{f}\sum_{t=1}^{f}\left(\frac{\hat{y}_t - y_t}{y_{\bar{t}}}\right)^2}{\frac{1}{f}\sum_{t=1}^{f}\left(\frac{y_{\bar{t}} - y_t}{y_{\bar{t}}}\right)^2}}$$

where $y_{\bar{t}}$ is the last known observation

- Intuition: the denominator corresponds to the MSE of a naïve forecast (which is $\hat{y}_t = y_{\bar{t}}$ for $t \geq \bar{t}$)
- The properties of this U_2 stay the same
  - $U_2 = 0$ only if the forecasts errors (numerator) are zero.
  - $U_2 = 1$ if $\hat{y}_t = y_{\bar{t}}$ (naïve, "no change" forecast)
- NOT reported by EViews

Interpretation

- $U_2 = 1$: the naïve method is as good as the forecasting technique being evaluated.
- $U_2 < 1$: the forecasting technique being used is better than the naïve method.
- $U_2 > 1$: there is no point to using the formal forecasting method. Naïve produces better results.

Decomposition of MSE

- From the previous lecture:

$$MSE = SE^2 + BIAS^2$$

- It can be shown that

$$MSE = BIAS^2 + SE^2 = BIAS^2 + (s_y - s_{\hat{y}})^2 + 2(1-r)s_y s_{\hat{y}}$$

where s_y is the standard deviation of actual series, and *r* is the correlation coefficient between actual and the forecast series

- See appendix for proof

Bias Proportion

$$\frac{BIAS^2}{MSE} = \frac{\left(\sum_{t=1}^{f} \hat{y}_t / f - \sum_{t=1}^{f} y_t / f \right)^2}{\sum_{t=1}^{f} (\hat{y}_t - y_t)^2 / f}$$

- The bias proportion tells us how far the mean of the forecast is from the mean of the actual series.
- Reported by EViews

Variance Proportion

$$\frac{(s_y - s_{\hat{y}})^2}{\sum_{t=1}^{f}(\hat{y}_t - y_t)^2 / f}$$

- The variance proportion tells us how far the variation of the forecast is from the variation of the actual series
- Reported by EViews

Covariance Proportion

$$\frac{2(1-r)s_y s_{\hat{y}}}{\sum_{t=1}^{f}(\hat{y}_t - y_t)^2 \Big/ f}$$

- The covariance proportion measures the remaining unsystematic forecasting error.
- Bias proportion + variance proportion + covariance proportion = 1
- Reported by EViews

For a Good Forecast…

- The bias and variance proportions should be small, since this suggests that the model is providing a good estimate of the underlying data generating process.

- Most of the mean square error should be due to the covariance/unsystematic component.

What we learned

- Theil's U_1 and U_2 statistics
  - U_1
    - Has undesirable properties
    - Implemented in EViews
  - U_2
    - Has no undesirable properties
    - Can be used to rank different forecast models
    - NOT implemented in EViews

- Decomposition of MSE into
  - Bias proportion
  - Variance proportion
  - Covariance proportion

- In general, we can simply compare the mean square error of the model against the mean square error of the benchmark model.

Session 4 Workshop: Theil’s U Statistics

Let's calculate Theil's U statistics!

- Given: Thailand monthly core year-on-year CPI inflation data for 2003-2014

$$\pi_t = \frac{CPI_t - CPI_{t-12}}{CPI_{t-12}}$$

- Objective: Evaluate and compare two 1 month ahead forecasts using actual data for 2014 as a test period
- We will start in Excel and then switch to EViews

Step 1: Calculate U1

- Remember that

$$U_1 = \frac{\sqrt{\frac{1}{f}\sum_{t=1}^{f}(\hat{y}_t - y_t)^2}}{\sqrt{\frac{1}{f}\sum_{t=1}^{f}\hat{y}_t^{\,2}} + \sqrt{\frac{1}{f}\sum_{t=1}^{f}y_t^{\,2}}}$$

- The numerator is the RMSE
- Let´s calculate $\sqrt{\frac{1}{f}\sum_{t=1}^{f}y_t^{\,2}}$
- Then let's calculate $\sqrt{\frac{1}{f}\sum_{t=1}^{f}\hat{y}_t^{\,2}}$ for forecasts 1 and 2

Step 2: Calculate U2

- Remember that

$$U_2 = \sqrt{\frac{\frac{1}{f}\sum_{t=1}^{f}\left(\frac{\hat{y}_t - y_t}{y_{t-1}}\right)^2}{\frac{1}{f}\sum_{t=1}^{f}\left(\frac{y_{t-1} - y_t}{y_{t-1}}\right)^2}}$$

- This the formula for 1-period ahead forecast only!
- Let´s calculate the numerator and the denominator separately.

Step 3: Calculate Bias Proportion

- Bias proportion is

$$\frac{BIAS^2}{MSE} = \frac{\left(\sum_{t=1}^{f} \hat{y}_t / f - \sum_{t=1}^{f} y_t / f \right)^2}{\sum_{t=1}^{f} (\hat{y}_t - y_t)^2 / f}$$

Step 4: Calculate Variance Proportion

- Variance proportion is

$$\frac{(s_y - s_{\hat{y}})^2}{\sum_{t=1}^{f} (\hat{y}_t - y_t)^2 / f}$$

Step 5: Calculate Covariance Proportion

- Covariance proportion is

$$\frac{2(1-r)s_y s_{\hat{y}}}{\sum_{t=1}^{f}(\hat{y}_t - y_t)^2 / f}$$

Step 6

- Let’s interpret the statistics!

Assessment hint:

- Remember that when forecasting multiple periods ahead

$$U_2 = \sqrt{\frac{\frac{1}{f}\sum_{t=1}^{f}\left(\frac{\hat{y}_t - y_t}{y_{\bar{t}}}\right)^2}{\frac{1}{f}\sum_{t=1}^{f}\left(\frac{y_{\bar{t}} - y_t}{y_{\bar{t}}}\right)^2}}$$

- The last known data period is $\bar{t} = 2014:1$

How to calculate some evaluation statistics in EViews

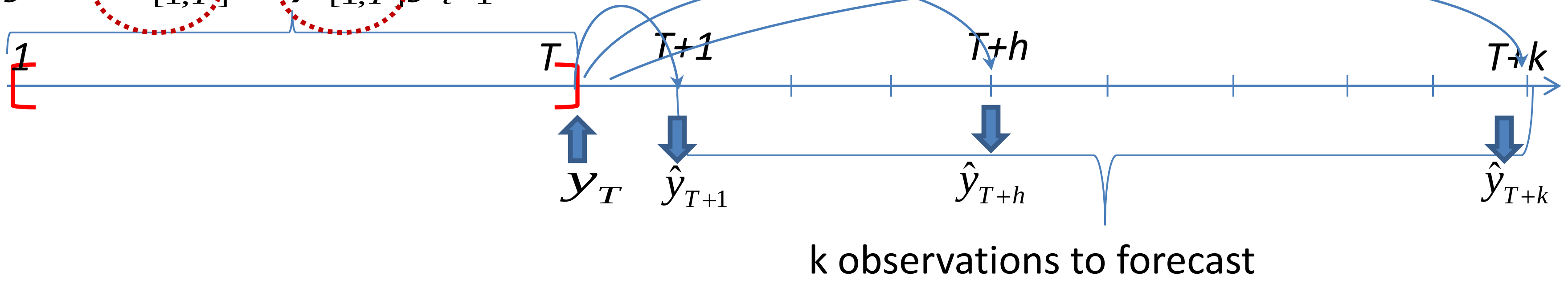
- Open workfile Thailand.wf1
- Run an AR(1) regression for inflation (p)
- Click forecast, choose forecasting period to be 2014m1 2014m12
- Resulting numbers for the statistics should be the same as the assessment exercise.

Session 5: Forecasting Strategies

How EViews calculates forecast evaluation statistics

- Let data for *[1, T]* be available to a forecaster for model estimation
- *Objective: evaluate forecast performance of the model*
- Let *[T+1, … T+k]* be a forecasting period: *k* observations to forecast

$$y_t = \hat{\alpha}_{[1,T]} + \hat{\beta}_{[1,T]} y_{t-1} + \varepsilon_t$$

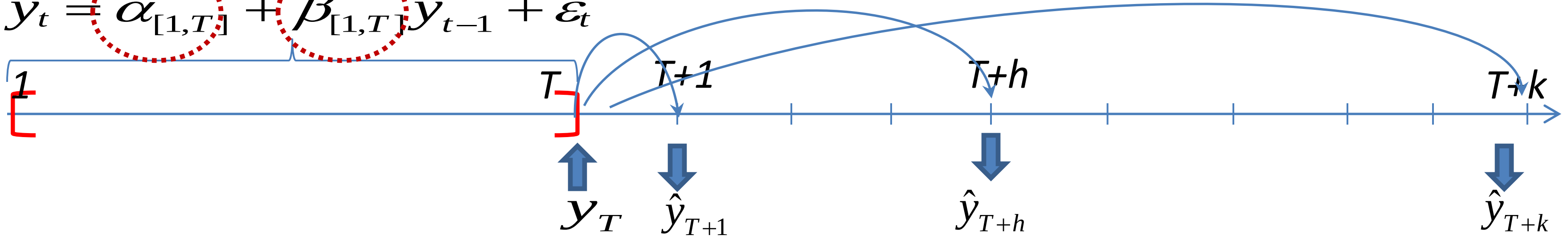


- Fixed Estimation Window (this one is implemented in EViews)
  - A model is estimated <u>once</u> for [*1,T*]
  - Coefficients are <u>fixed</u> at the estimates
  - and are used through the whole forecasting period

How EViews calculates forecast evaluation statistics

- Let data for *[1, T]* be available to a forecaster for model estimation
- *Objective: evaluate forecast performance of the model*
- Let *[T+1, … T+k]* be a forecasting period: *k* observations to forecast

$$y_t = \hat{\alpha}_{[1,T]} + \hat{\beta}_{[1,T]} y_{t-1} + \varepsilon_t$$



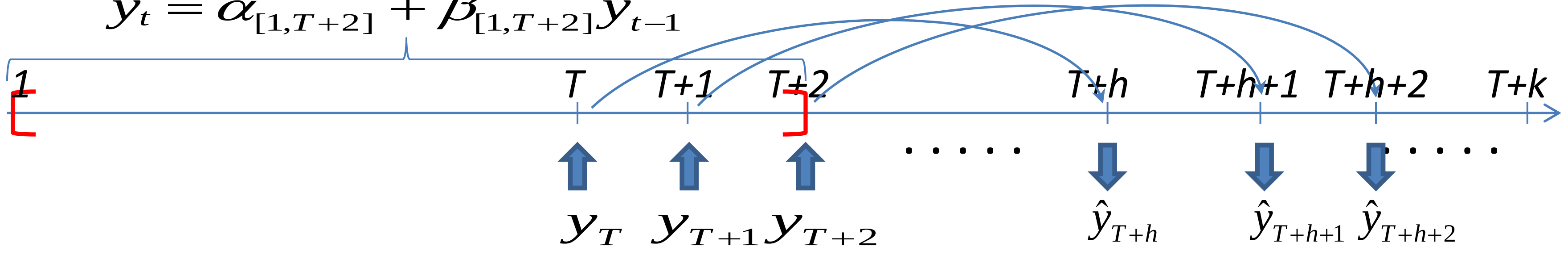
Stats are calculated based on $\hat{y}_{T+1}, \ldots, \hat{y}_{T+k}$ thus averaging forecast errors over different horizons

- What if a model performs very well for short-run forecasts but bad for long-run ones?
- What if a model performs very well for long-run forecasts but bad for short-run ones?
- Question: How to distinguish between these models?
- Answer: Calculate RMSE for every forecast horizon.

A Solution: Expanding Window Strategy

- Let data for *[1, T]* be available to a forecaster for model estimation
- *We are interested in h-steps-*ahead forecast
- Let *[T+h, … T+k]* be a forecasting period: *k-h+1* observations to forecast

$$y_t = \hat{\alpha}_{[1,T+2]} + \hat{\beta}_{[1,T+2]} y_{t-1}$$

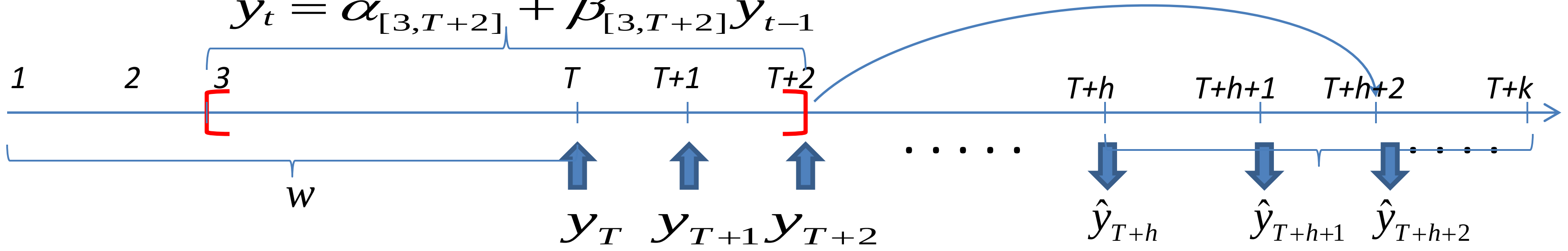


– Expanding Estimation Window (not readily available in EViews – need to program)
  - A model is estimated in [*1,T*] and forecast is made for *T+h*
  - At *T+1* the model is re-estimated on [*1, T+1*] and a forecast is made for *T+h+1*
  - Estimation window expands as we progress into the future: get data, re-estimate, forecast
  - Forecasts are made for the same horizon!

Another Solution: Rolling Window

- Let data for *[1, T]* be available to a forecaster for model estimation
- *We are interested in h-steps-*ahead forecast
- Let *[T+h, … T+k]* be a forecasting period: *h-k+1* observations to forecast

$$y_t = \hat{\alpha}_{[3,T+2]} + \hat{\beta}_{[3,T+2]} y_{t-1}$$



– Rolling Estimation Window (not readily available in EViews – need to program)
  - The size of estimation sample (w) is chosen initially and fixed. w can be less than T.
  - A model is estimated in [*T-w+1,T*] and a forecast is for *T+h*
  - At *T+1* the model is re-estimated on [*T-w+2,T+1*] and a forecast is made for *T+h+1*
  - Estimation sample rolls forward: get data, re-estimate, forecast
  - Again, forecasts are made for the same horizon! Helps you deal w/ structural breaks too!

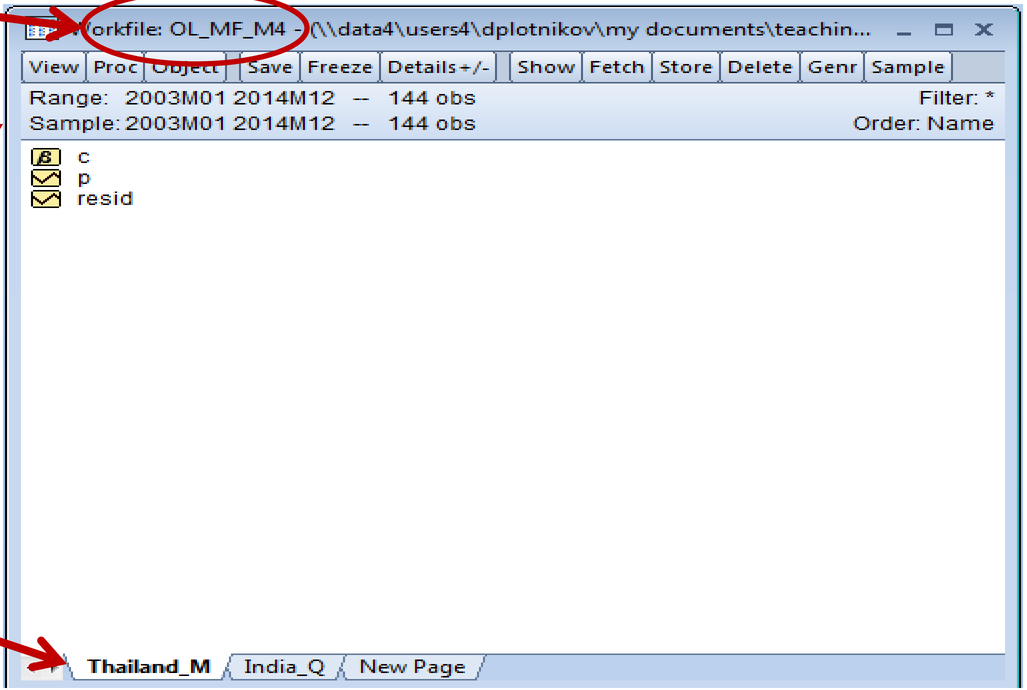
What we learned

- When evaluating a model, important to do it for different forecast horizons.
- Two forecasting strategies
  - Expanding window
  - Rolling window
- Let’s see how it works in EViews in the next video!

Session 5 Workshop: Forecasting Statistics for Different Horizons

Let’s see how it works in EViews!

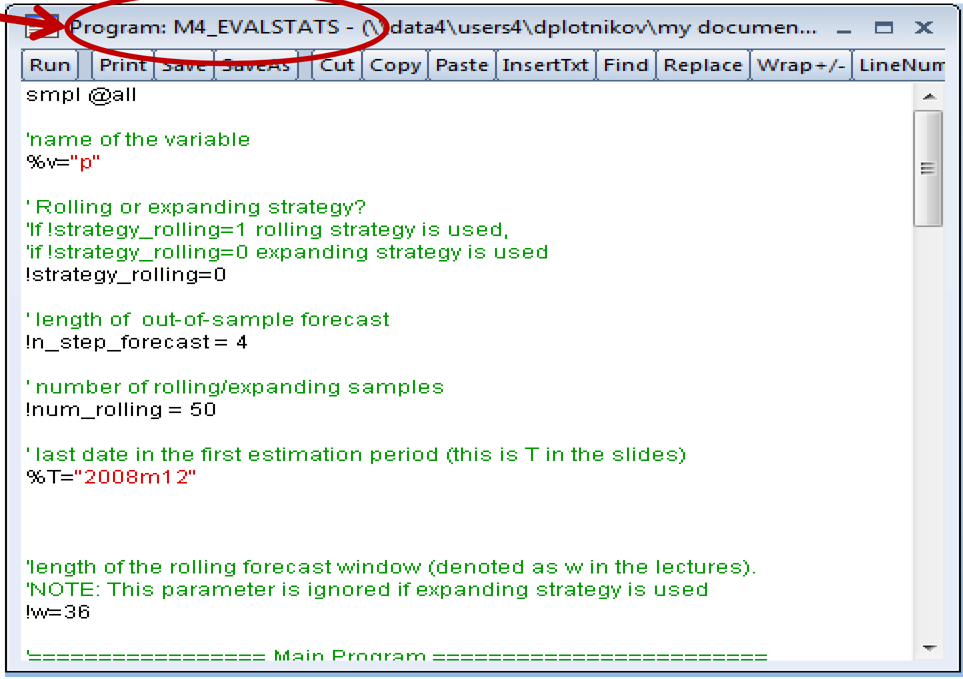
Open the workfile



Check that the sample is set correctly

Check that the correct page is active

Open the program



Variable to be forecasted (p stands for inflation). This will not change throughout this part.

Open the program

Choose forecasting strategy: rolling window or expanding window. This means that the expanding strategy will be used.

Program: M4_EVALSTATS - (\\data4\users4\dplotnikov\my documen...

Run | Print | Save | SaveAs | Cut | Copy | Paste | InsertTxt | Find | Replace | Wrap+/- | LineNum

```
smpl @all

'name of the variable
%v="p"

' Rolling or expanding strategy?
'If !strategy_rolling=1 rolling strategy is used,
'if !strategy_rolling=0 expanding strategy is used
!strategy_rolling=0

' length of  out-of-sample forecast
!n_step_forecast = 4

' number of rolling/expanding samples
!num_rolling = 50

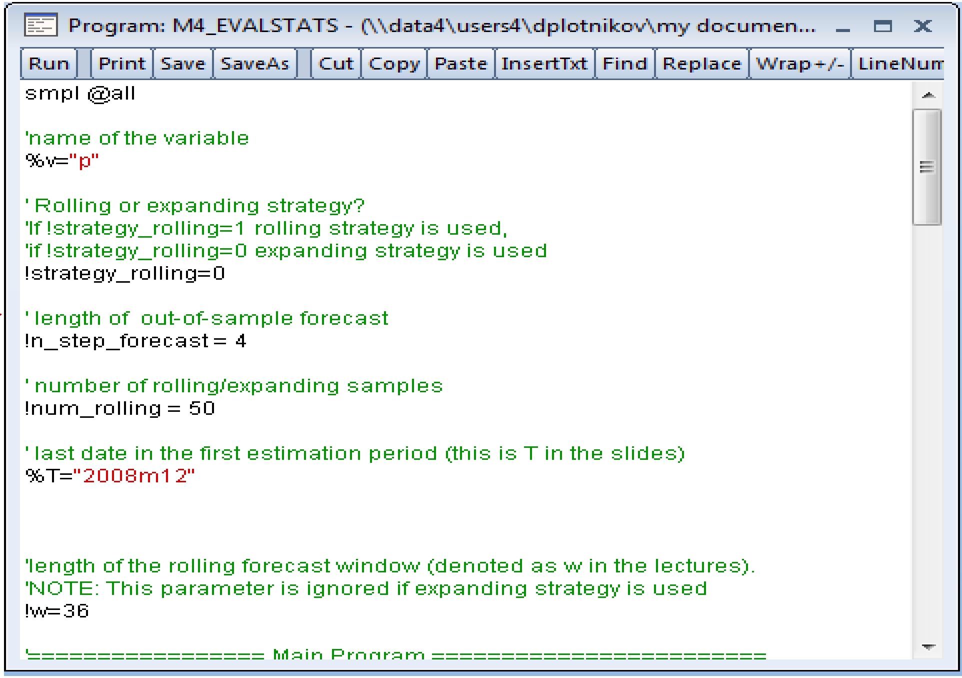
' last date in the first estimation period (this is T in the slides)
%T="2008m12"



'length of the rolling forecast window (denoted as w in the lectures).
'NOTE: This parameter is ignored if expanding strategy is used
!w=36

'================= Main Program ========================
```

Open the program



Expanding/rolling forecasts will be calculated for up to 4 months ahead.

Set the number of rolling/expanding samples

Program: M4_EVALSTATS - (\\data4\users4\dplotnikov\my documen...

Run | Print | Save | SaveAs | Cut | Copy | Paste | InsertTxt | Find | Replace | Wrap+/- | LineNum

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%T="2008m12"

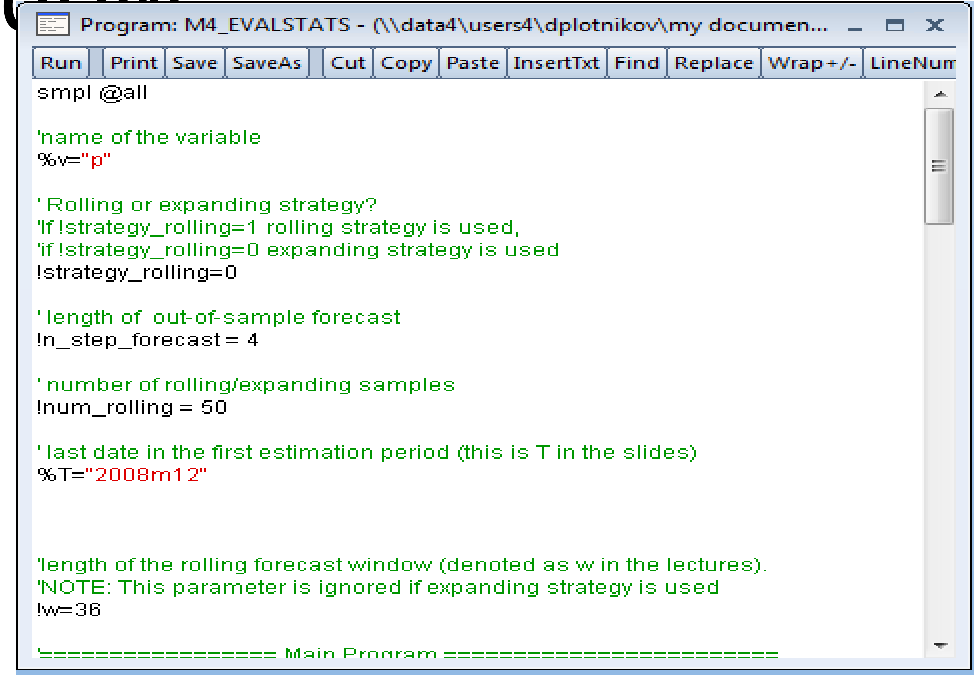


'length of the rolling forecast window (denoted as w in the lectures).
'NOTE: This parameter is ignored if expanding strategy is used
!w=36

'================= Main Program ========================
```

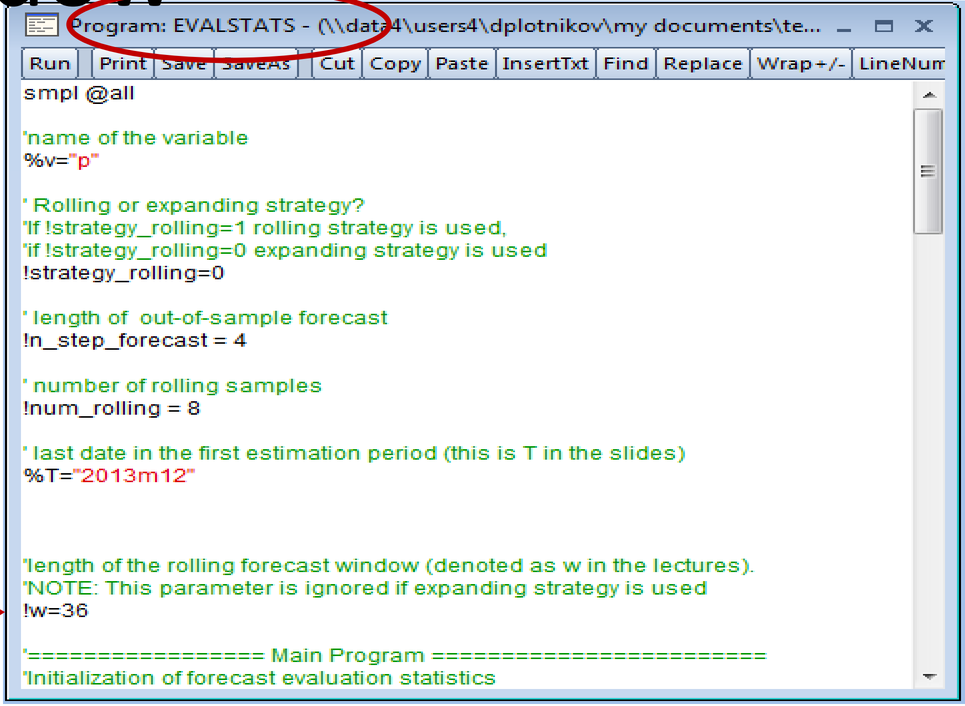
The model will be reestimated on 50 samples.

Set T – the last observation in the first window



If expanding strategy is used The first estimation window is 2003m1-2008m12. The second one will be 2003m1-2009m1 and so on.

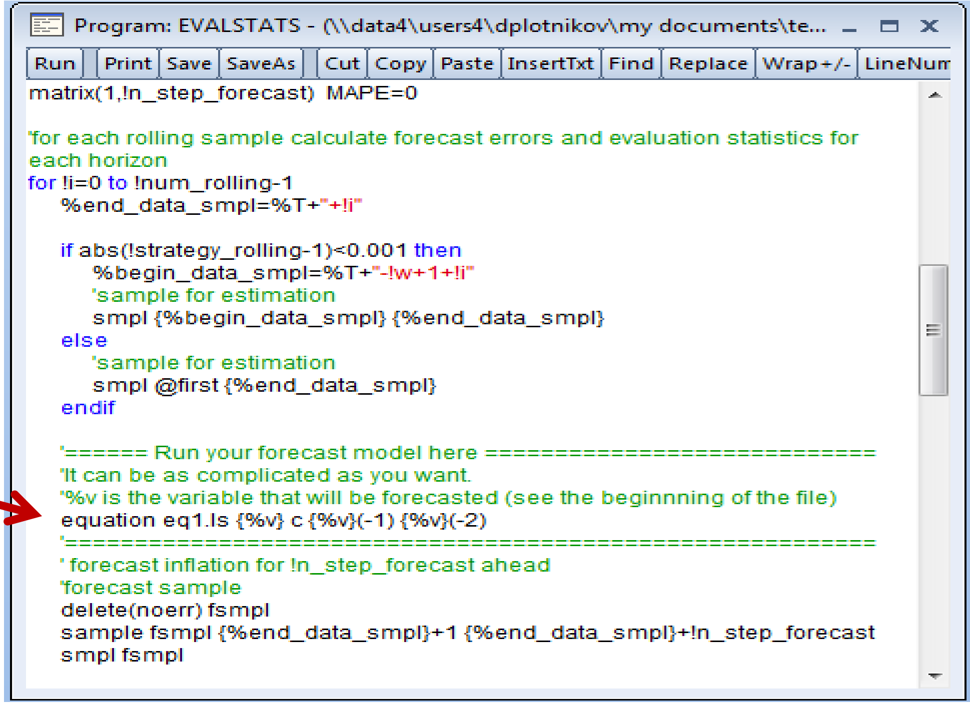
Set w – the length of the rolling window



If rolling strategy is used, you need to set the length of the window. In this case the length of the window is 36 months.

Change the model if needed

The model to be estimated is specified here. By default it is an AR(2) process.



Let’s run the code! Expanding window

Table: RESULTS_AR2_EXP Workfile: OL_MF_M4::Thailand_M\

	A	B	C	D	E
1	HORIZON	h=1	h=2	h=3	h=4
2	Bias	-0.007	-0.019	-0.042	-0.061
3	MSE	0.589	1.360	2.474	3.160
4	RMSE	0.767	1.166	1.573	1.778
5	SE	0.767	1.166	1.572	1.777
6	MAE	0.492	0.769	1.001	1.086
7	MAPE	0.386	0.462	0.610	0.560
8					
9					

- For AR(2) all forecast evaluation get worse if forecast horizon increases.
- However, there is a limit to this process since the forecast of the AR(2) mode will ultimately converge to the mean.
- Important to distinguish between different forecast horizons!
- These numbers become more precise when
  - Initial forecasting window is larger.
  - Number of rolling samples increases.

Let's run the code! Rolling window (36 months)

Table: RESULTS Workfile: OL_MF_M4::Thailand_M\

View | Proc | Object | Print | Name | Edit+/- | CellFmt | Grid+/- | Title | Comments+

	A	B	C	D	E
1	HORIZON	h=1	h=2	h=3	h=4
2	Bias	0.021	0.049	0.066	0.084
3	MSE	0.640	1.569	3.006	3.956
4	RMSE	0.800	1.252	1.734	1.989
5	SE	0.800	1.251	1.733	1.987
6	MAE	0.518	0.833	1.132	1.262
7	MAPE	0.410	0.522	0.671	0.617
8					
9					

- For AR(2) all forecast evaluation statistics worsen as if forecast horizon increases.
- Important to distinguish between different forecast horizons!
- These numbers become more precise when
  - Initial forecasting window is larger.
  - Number of rolling samples increases.

Session 6: Introduction to Structural Breaks

When estimating a model

- The key assumption: structure of the conditional mean does not change

$$\pi_t = F(X_t;\theta) + \varepsilon_t \quad t = 1, \ldots, T$$

  – In particular parameters θ stay constant throughout

- What if they do change? E.g. oil shock, 2008 crisis
- Implications for forecasting?
- More on this in the “Model Evaluation” module

Structural Change

- Abrupt change in parameters of the conditional mean
- Instead of

$$\pi_t = F(X_t;\theta) + \varepsilon_t \quad t = 1,\ldots,T$$

- We have

$$\pi_t = F(X_t;\theta_1) + \varepsilon_t \quad t = 1,\ldots,\bar{t}$$

- and

$$\pi_t = F(X_t;\theta_2) + \varepsilon_t \quad t = \bar{t} + 1,\ldots,T$$

Changing Variance

- It may also be the case that the variance of the residuals changes, which will not affect the conditional mean forecast
- For example, the “Great Moderation” period, during which the volatility of real GDP around potential appeared to decline
- However, our measures of uncertainty of any forecast (confidence intervals) will need to reflect this change, otherwise the forecasting model could be perceived to be incorrect

Motivating Example: Change in the Intercept

- Suppose the true process for inflation is

$$\pi_t = 5 + 0.5\pi_{t-1} + \varepsilon_t - D_L \quad t = 2006:1..2014:12$$

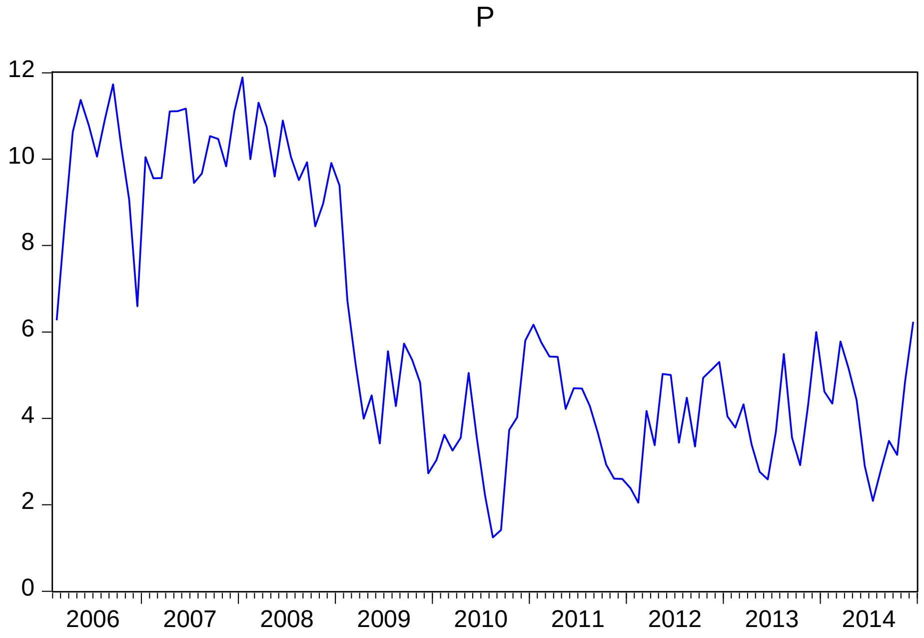
where shocks are i.i.d. from $N(0,1)$ and

D_L is a dummy variable such that

$$D_L = \begin{cases} 3, & if\ t \geq 2009:2 \\ 0, & if\ t < 2009:2 \end{cases}$$

Structural break

$$\pi_t = 5 + 0.5\pi_{t-1} + \varepsilon_t - D_L \quad t = 2006:1..2014:12$$

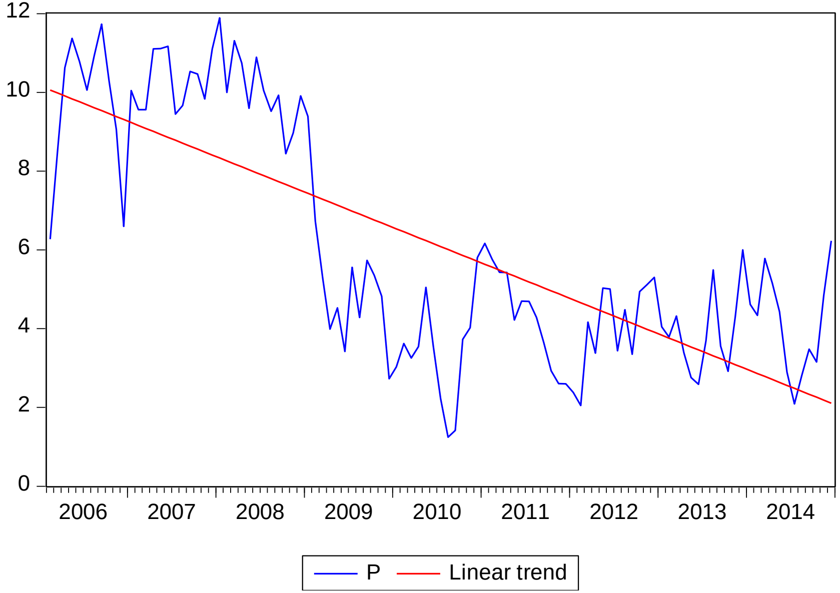


Run struct_break.prg if you want to replicate this graph

Model Misspecification: Linear Trend

$$\underset{}{\pi_t} = \underset{(0.389)}{10.134} - \underset{(0.006)}{0.075}t + u_t$$

- Both coefficients are very significant! (see s.e. in parenthesis)
- But notice the positive serial correlation in the residuals, which acts to inflate the t-statistics
- Clearly not a good model for forecasting



Run struct_break.prg if you want to replicate this graph

Model Misspecification: AR(1) but no break

- The estimated model is

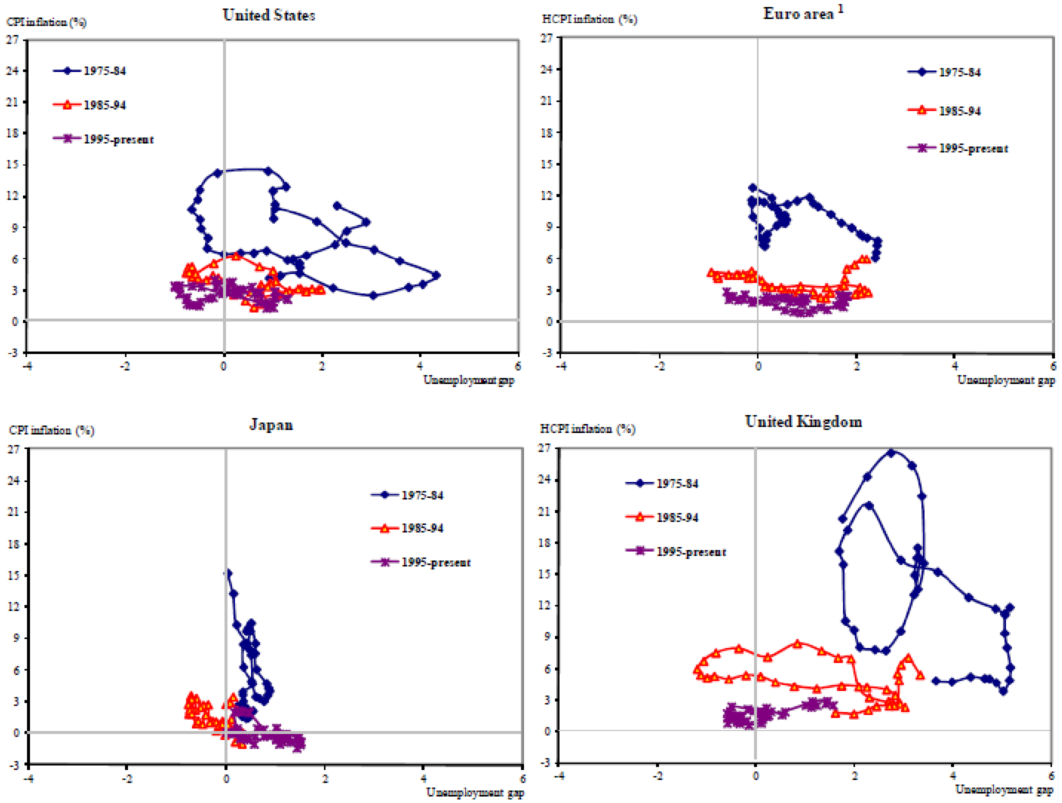
$$\pi_t = \underset{(0.252)}{0.470} + \underset{(0.037)}{0.925}\,\pi_{t-1} + u_t$$

- The AR coefficient is known to be biased towards unity because of the serial correlation
  - A unit root test will likely accept the unit root hypothesis
  - But the process is stationary in 2 sub-periods!

- Using either misspecified model will produce poor forecasts! (see workshop)

Run struct_break.prg if you want to replicate this result

Structural Break in the Phillips Curve



Source: Moccero, D., S. Watanabe and B. Cournède (2011), "What Drives Inflation in the Major OECD Economies?", *OECD Economics Department Working Papers, No. 854*

Accounting for structural changes

- Coefficient stability is very important
  - For forecasting (see workshop)
  - For policymaking
- How to deal them?
  - Detect existing ones in sample = testing
    - This covered in the “Model Evaluation” section
  - Prepare for possible breaks in the future
    - See next lecture

What we learned

- Accounting for structural breaks is important

- Misspecification leads to poor forecasts or wrong conclusions or policy decisions

Session 6 Workshop: Introduction to Structural Breaks

Motivation

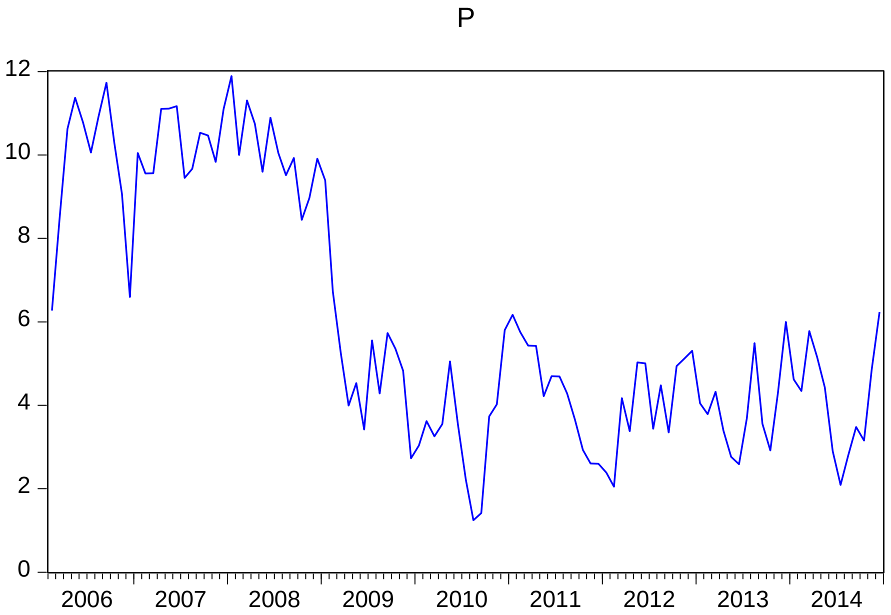
- Objective: see how using rolling forecasting strategy instead of the standard expanding window can improve forecast accuracy
  - What if potential structural breaks in the future are possible?

- I will use simulated data

- For assessment, you will use India GDP growth data and analyze growth slowdown after the global financial crisis

Recall the motivating example

$$\pi_t = 5 + 0.5\pi_{t-1} + \varepsilon_t - D_L \;\; t = 2006:1..2014:12$$



Run struct_break.prg if you want to replicate this graph

Setup

- Assume it is December 2008, and you suspect that the recent decline in inflation might be permanent looking forward

- How do different forecasting strategies affect your forecast?
  – Suppose that you use this strategy for 50 months.

If you knew when the break will happen

- Always use expanding strategy
  - More observations -> higher precision
- The model that needs to be estimated is

```
if @max(dl)<0.5 then
        equation eq1.ls {%v} c {%v}(-1)
else
        equation eq1.ls {%v} c {%v}(-1) dl
endif
```

- Let’s do it in EViews using evalstats.prg

If you knew when the break will happen

- Results

Table: RESULTS Workfile: M4_DATA::StructBreak_M\

View | Proc | Object | Print | Name | Edit+/- | CellFmt | Grid+/- | Title | Comments+/-

	A	B	C	D	E
1	HORIZON	h=1	h=2	h=3	h=4
2	Bias	-0.087	-0.172	-0.231	-0.277
3	MSE	1.127	1.961	2.822	3.072
4	RMSE	1.062	1.400	1.680	1.753
5	SE	1.058	1.390	1.664	1.731
6	MAE	0.887	1.083	1.225	1.308
7	MAPE	0.253	0.322	0.370	0.397
8					

What if the break is NOT accounted for?

- Use standard expanding strategy and AR(1) process

Table: RESULTS_AR1_EXP Workfile: M4_DATA::StructBreak_M\

View | Proc | Object | Print | Name | Edit+/- | CellFmt | Grid+/- | Title | Comments+/-

	A	B	C	D	E
1	HORIZON	h=1	h=2	h=3	h=4
2	Bias	-0.538	-0.974	-1.318	-1.605
3	MSE	1.774	3.688	5.793	7.277
4	RMSE	1.332	1.920	2.407	2.698
5	SE	1.218	1.655	2.014	2.168
6	MAE	1.042	1.517	1.918	2.211
7	MAPE	0.279	0.446	0.588	0.682
8					

What if the break is NOT accounted for?

- BUT use rolling strategy and AR(1) process
- Let' s use a 3 year window (36 months) as before

Table: RESULTS_AR1_ROLL   Workfile: M4_DATA::StructBreak_M\

HORIZON	h=1	h=2	h=3	h=4
Bias	-0.291	-0.503	-0.651	-0.765
MSE	1.661	3.211	4.893	5.704
RMSE	1.289	1.792	2.212	2.388
SE	1.255	1.720	2.114	2.263
MAE	1.000	1.302	1.605	1.789
MAPE	0.261	0.370	0.478	0.523

- Forecast performance is significantly improved!

What is the best window size?

- In general it is difficult to say
  - Especially if no structural breaks have happened in the past
  - Or the break is not similar to the past ones
  - Research suggests that it depends on the actual number of breaks in the data and their magnitude

What is the best window size?

- Size should be at least 30 observations, if possible, to facilitate parameter estimation
- Depends on the series of interest
  - Past behavior, frequency, its persistence, etc
- It is NOT guaranteed that a rolling window strategy will improve results
  - Parameter estimate precision vs. stability

Session 7: Introduction to Fan Charts

The plan for this lecture

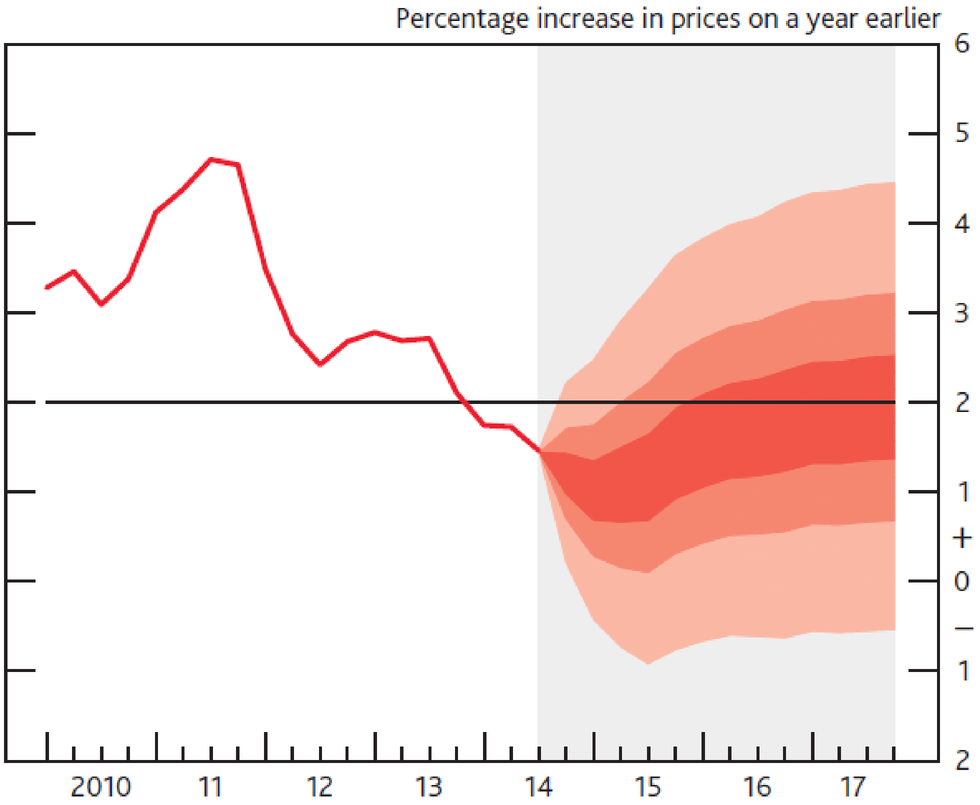
- Definition and intuition of a fan chart
- Properties of a typical fan chart
- Constructing a fan chart in EViews

What is a  chart?

- Forecast + graphical representation of forecast uncertainty around it
- Allows for error, coefficient, and (in some cases) the uncertainty associated with explanatory variables in the model
- Confidence interval of forecast for every period in the forecast horizon
- The term was first was coined by the Bank of England in its Inflation Report in 1997

Typical fan chart

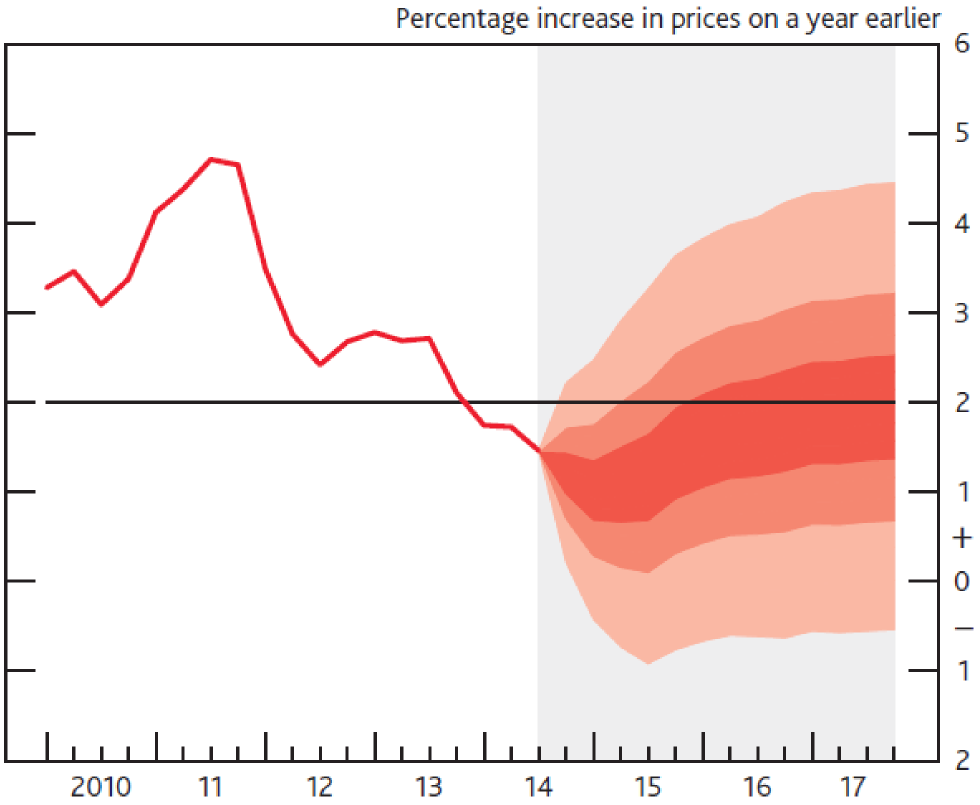
- The darkest area - 30% confidence interval assuming economic conditions stay the same
- The lightest area is 90% conf. interval



Source: Chart 5.3, Bank of England Inflation report, November 2014

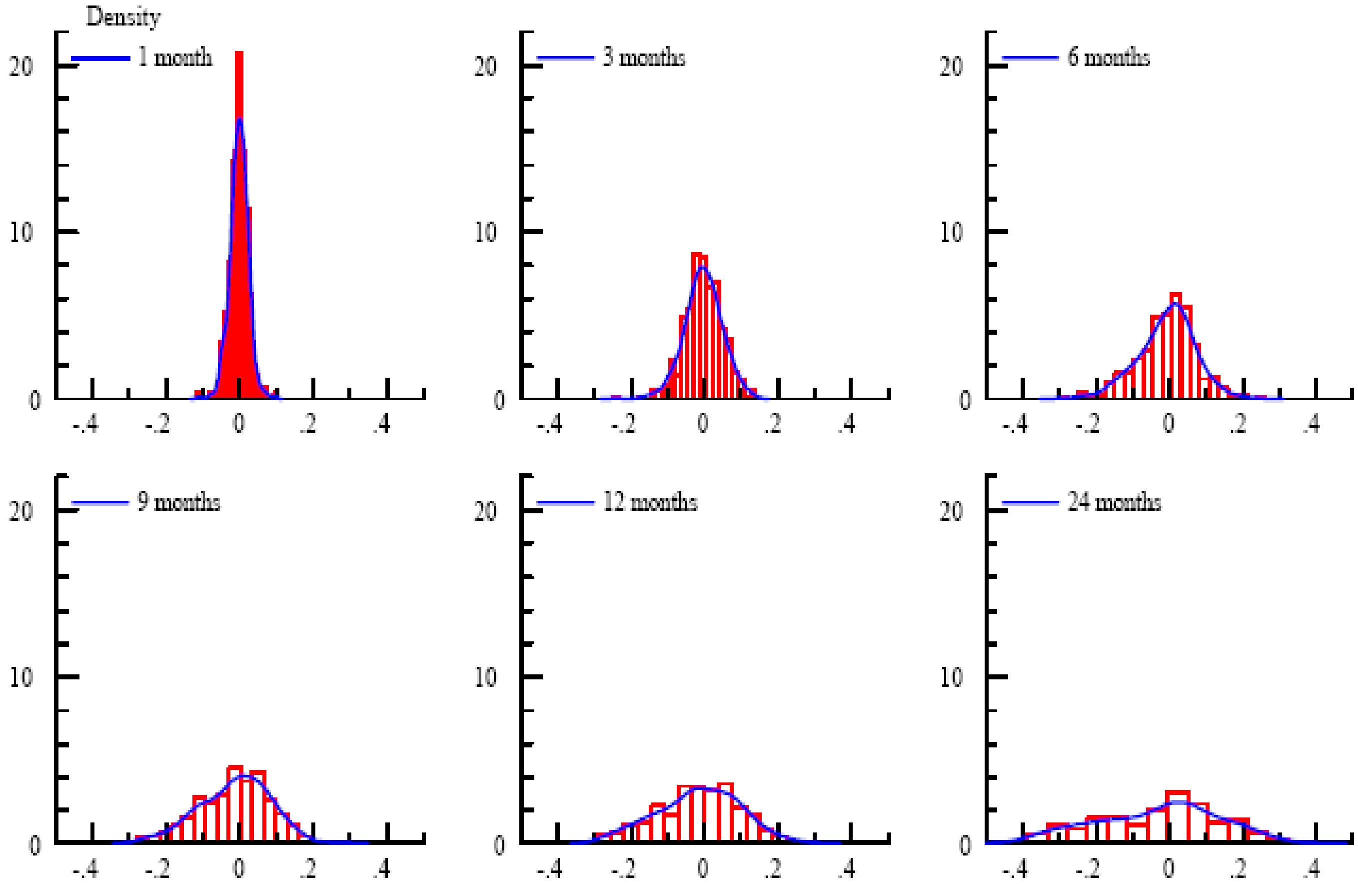
In general, fan chart

- May not be symmetric
  - Depends on type of risks, model you adopt for the variance, and the assumptions regarding exogenous variables
- Bands expand with forecast horizon
  - Stabilize on unconditional confidence interval



Source: Chart 5.3, Bank of England Inflation report, November 2014

Forecast uncertainty for different horizons



Properties of a fan chart

- If errors are normal and the model is linear, the fan chart will be symmetrical around the mean
  - 1 s.e. bounds will correspond to ≈ 60% conf. interval
  - 2 s.e. bounds will correspond to ≈ 95% conf. interval
- If errors are bootstrapped (i.e., selected randomly from the estimated residuals), the fan chart does not have to be symmetrical
- Coefficient uncertainty will expand the bounds

More properties of a fan chart

- As we get more data to calibrate the model, we should expect the confidence interval to shrink

- This is because we are estimating the unknown parameters more precisely. More data usually helps!

- But note that we can't do anything about the variance of the error term, σ_ε

Numerical example of fan chart properties: AR(1) model

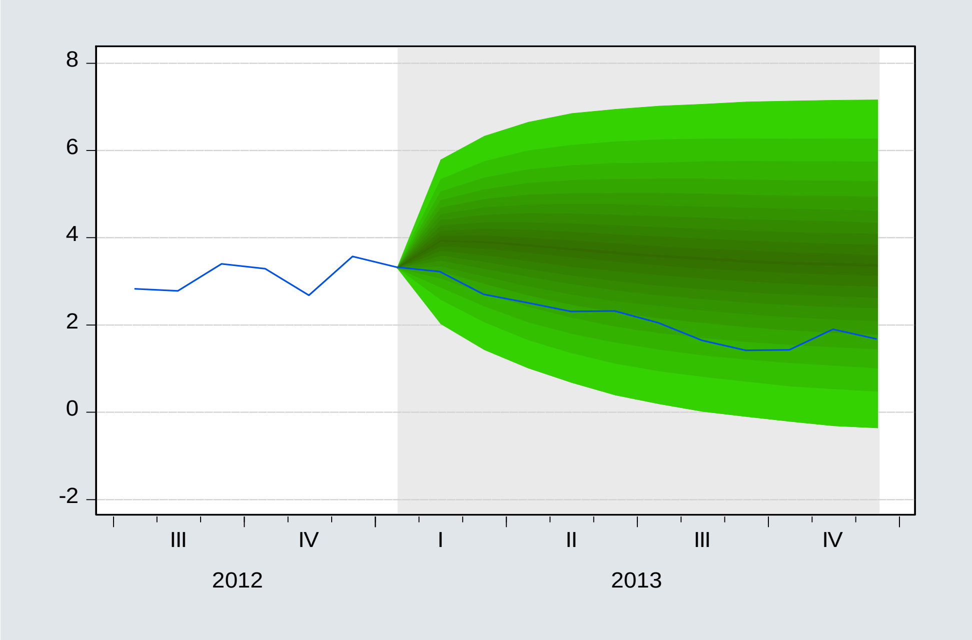
- Consider an AR(1) model
  $y_t = a_0 + a_1 y_{t-1} + \varepsilon_t \;\; \varepsilon_t \sim i.i.d.(0, \sigma^2)$
- Then forecast one period ahead conditional on data up to t $\hat{y}_{t+1} = a_0 + a_1 y_t$
- Forecasting error is $FE_1 = \hat{y}_{t+1} - y_{t+1} = \varepsilon_{t+1}$
- Thus, variance of forecast error is $Var(FE_1) = \sigma^2$
- And 95% confidence bounds are, assuming normality,
  $a_0 + a_1 y_t \pm 1.96\sigma$

Numerical example: AR(1) process

- Forecast <u>two</u> period ahead is
$$\hat{y}_{t+2} = a_0 + a_1(a_0 + a_1 y_t)$$
- Forecasting error is
$$FE_2 = \hat{y}_{t+2} - y_{t+2} = a_1(\hat{y}_{t+1} - y_{t+1}) + \varepsilon_{t+2} = a_1\varepsilon_{t+1} + \varepsilon_{t+2}$$
- Thus, variance of forecast error is
$$Var(FE_2) = (1 + a_1^2)\sigma^2 > Var(FE_1)$$
- And 95% confidence bounds are wider
$$\hat{y}_{t+2} = a_0 + a_1(a_0 + a_1 y_t) \pm 1.96\sigma\sqrt{1 + a_1^2}$$

Fan charts in EViews

- Not explicitly implemented
- BUT: Existing tools let you construct standard errors & confidence interval for a forecast
- Let's learn how to do this!



What we learned

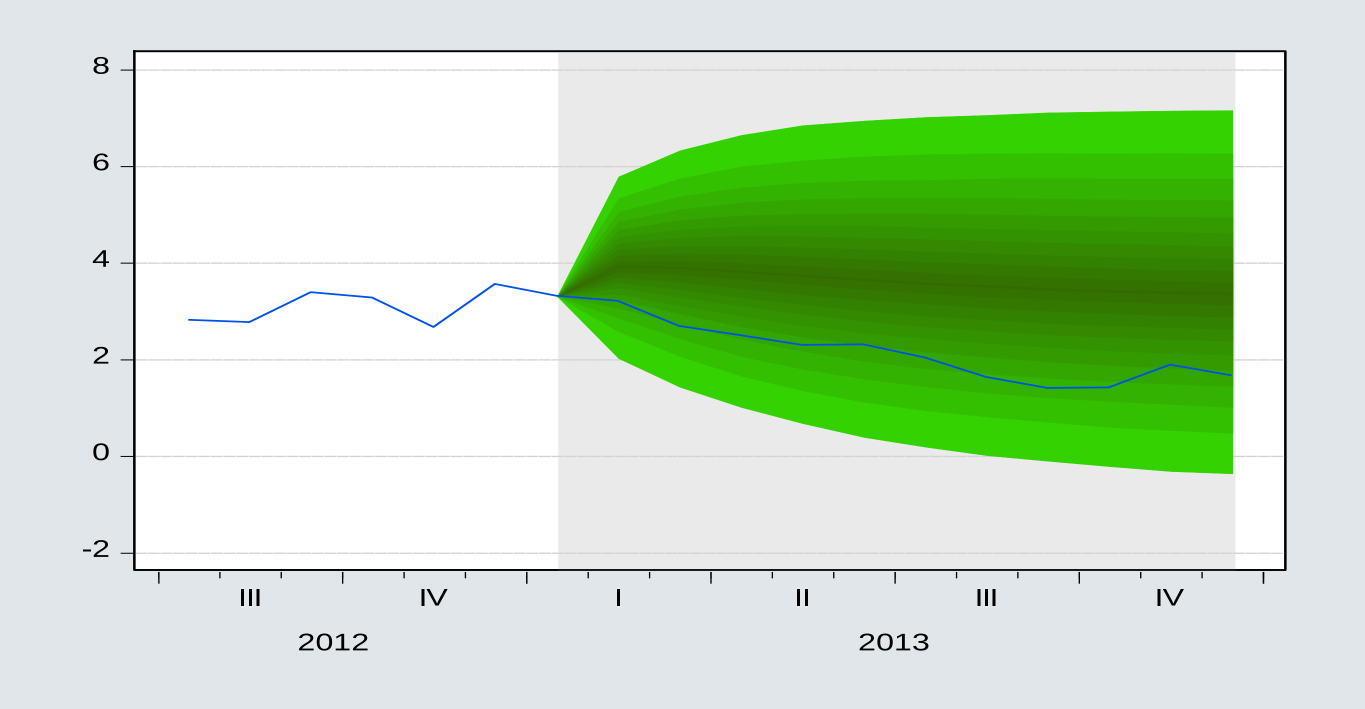
- Definition of a fan chart
- Properties of a fan chart
- Numerical example
- Ready to move to EViews exercises!

Session 7 Workshop: Introduction to Fan Charts

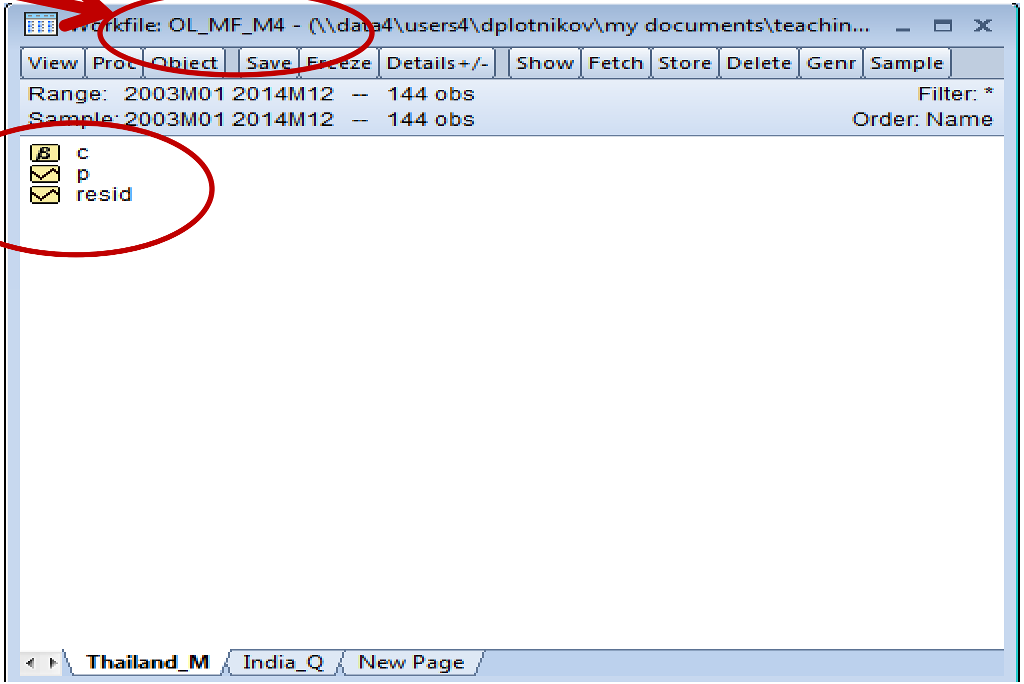
In the lecture

- Definition and intuition of a fan chart
- Properties of a typical fan chart
- Constructing a fan chart in EViews

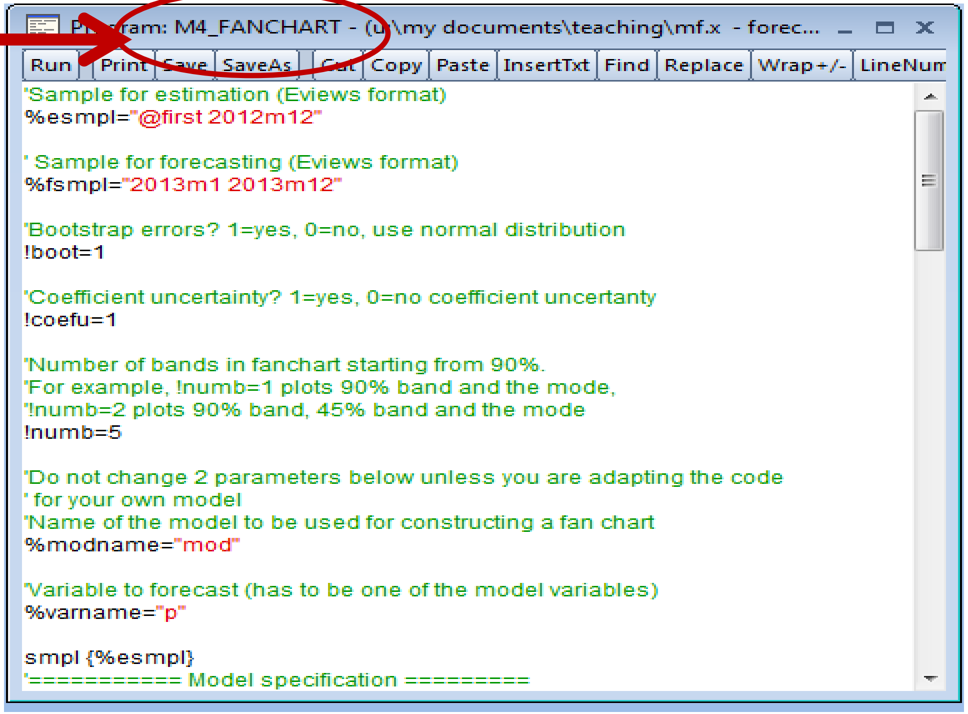
Fan charts in EViews



In EViews open the workfile…



Now open the fan chart program




```
'Sample for estimation (Eviews format)
%esmpl="@first 2012m12"

' Sample for forecasting (Eviews format)
%fsmpl="2013m1 2013m12"

'Bootstrap errors? 1=yes, 0=no, use normal distribution
!boot=1

'Coefficient uncertainty? 1=yes, 0=no coefficient uncertanty
!coefu=1

'Number of bands in fanchart starting from 90%.
'For example, !numb=1 plots 90% band and the mode,
'!numb=2 plots 90% band, 45% band and the mode
!numb=5

'Do not change 2 parameters below unless you are adapting the code
' for your own model
'Name of the model to be used for constructing a fan chart
%modname="mod"

'Variable to forecast (has to be one of the model variables)
%varname="p"

smpl {%esmpl}
'=========== Model specification =========
```

Let’s discuss inputs for the program

The model for inflation (AR(2) by default) will be estimated for the period 2003m1 to 2012m12

Program: M4_FANCHART - (u:\my documents\teaching\mf.x - forec...

Run | Print | Save | SaveAs | Cut | Copy | Paste | InsertTxt | Find | Replace | Wrap+/- | LineNum

```
'Sample for estimation (Eviews format)
%esmpl="@first 2012m12"

' Sample for forecasting (Eviews format)
%fsmpl="2013m1 2013m12"

'Bootstrap errors? 1=yes, 0=no, use normal distribution
!boot=1

'Coefficient uncertainty? 1=yes, 0=no coefficient uncertanty
!coefu=1

'Number of bands in fanchart starting from 90%.
'For example, !numb=1 plots 90% band and the mode,
'!numb=2 plots 90% band, 45% band and the mode
!numb=5

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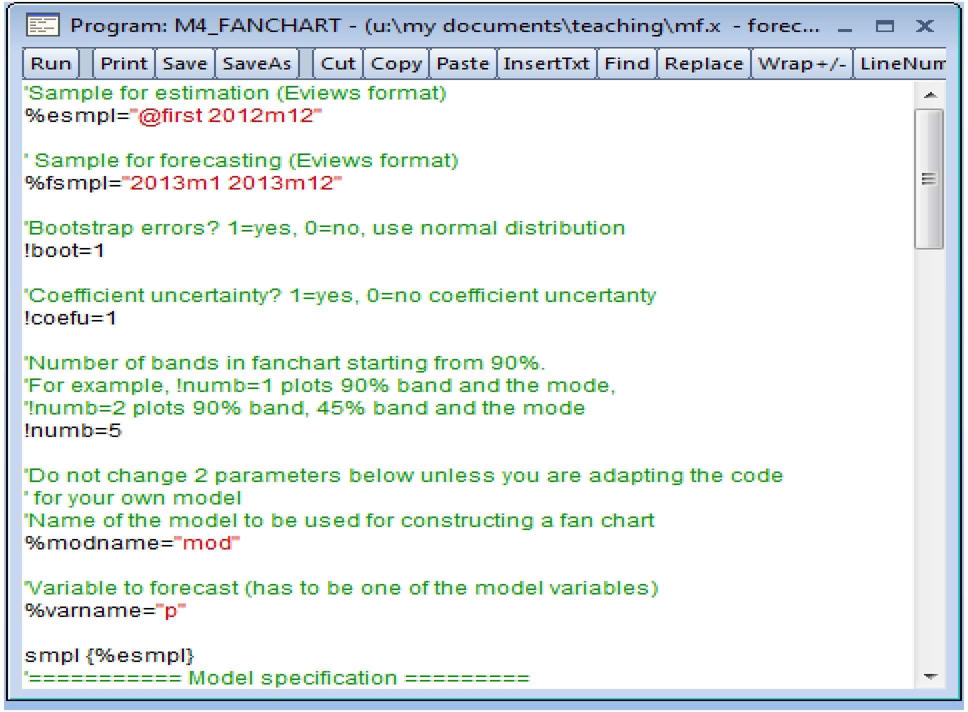
' for your own model
'Name of the model to be used for constructing a fan chart
%modname="mod"

'Variable to forecast (has to be one of the model variables)
%varname="p"

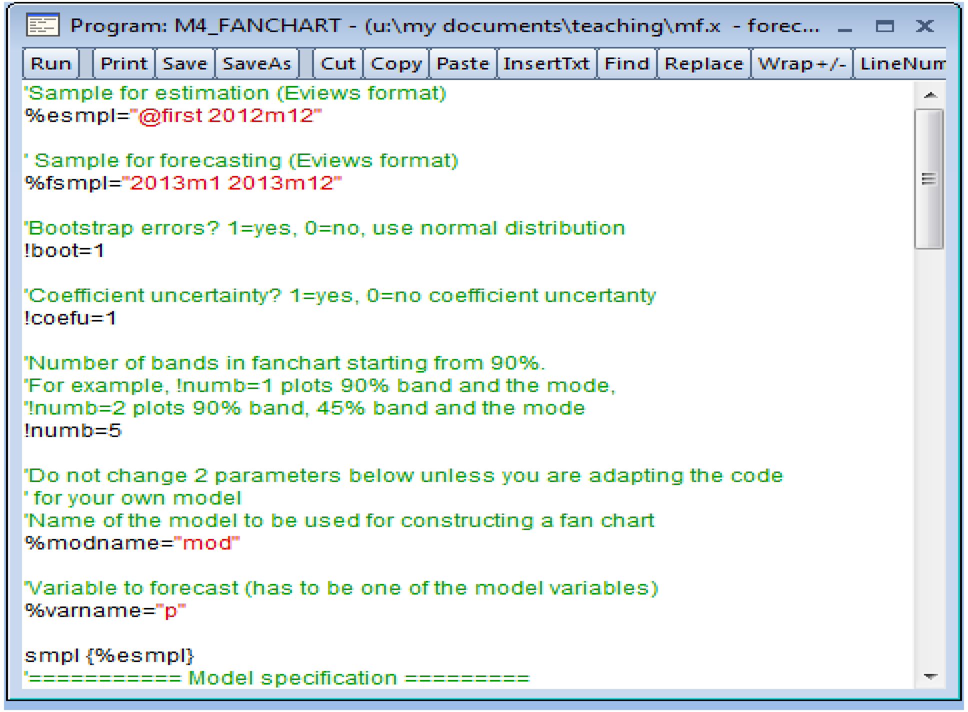
smpl {%esmpl}
'=========== Model specification =========
```

Let’s discuss inputs for the program

The model will be forecasted for the period 2013m1 to 2012m12 using data from %esmpl



Let’s discuss inputs for the program



Errors for the confidence bounds will be bootstrapped. Normality won’t be assumed.

Let’s discuss inputs for the program

Program: M4_FANCHART - (u:\my documents\teaching\mf.x - forec...

Run | Print | Save | SaveAs | Cut | Copy | Paste | InsertTxt | Find | Replace | Wrap+/- | LineNum

```
'Sample for estimation (Eviews format)
%esmpl="@first 2012m12"

' Sample for forecasting (Eviews format)
%fsmpl="2013m1 2013m12"

'Bootstrap errors? 1=yes, 0=no, use normal distribution
!boot=1

'Coefficient uncertainty? 1=yes, 0=no coefficient uncertanty
!coefu=1

'Number of bands in fanchart starting from 90%.
'For example, !numb=1 plots 90% band and the mode,
'!numb=2 plots 90% band, 45% band and the mode
!numb=5

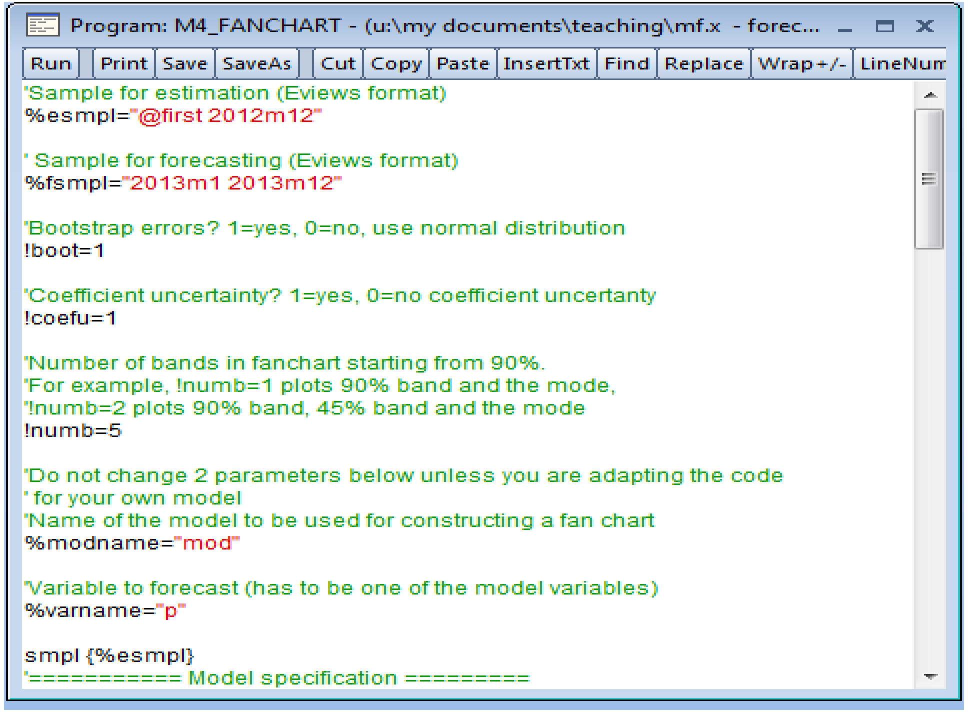
'Do not change 2 parameters below unless you are adapting the code
' for your own model
'Name of the model to be used for constructing a fan chart
%modname="mod"

'Variable to forecast (has to be one of the model variables)
%varname="p"

smpl {%esmpl}
'=========== Model specification =========
```

Confidence bounds will include coefficient uncertainty

Let’s discuss inputs for the program



The fan chart will include 5 different equally spaced confidence bounds: 90%, 72%, 54%, 36%, 18% and the mode.

Let’s discuss inputs for the program

Change this if you use this code for your own forecast model

Program: M4_FANCHART - (u:\my documents\teaching\mf.x - forec...

Run | Print | Save | SaveAs | Cut | Copy | Paste | InsertTxt | Find | Replace | Wrap+/- | LineNum

```
'Sample for estimation (Eviews format)
%esmpl="@first 2012m12"

' Sample for forecasting (Eviews format)
%fsmpl="2013m1 2013m12"

'Bootstrap errors? 1=yes, 0=no, use normal distribution
!boot=1

'Coefficient uncertainty? 1=yes, 0=no coefficient uncertanty
!coefu=1

'Number of bands in fanchart starting from 90%.
'For example, !numb=1 plots 90% band and the mode,
'!numb=2 plots 90% band, 45% band and the mode
!numb=5

'Do not change 2 parameters below unless you are adapting the code
' for your own model
'Name of the model to be used for constructing a fan chart
%modname="mod"

'Variable to forecast (has to be one of the model variables)
%varname="p"

smpl {%esmpl}
'=========== Model specification =========
```

Let’s discuss inputs for the program

Program: M4_FANCHART - (u:\my documents\teaching\mf.x - forec...

Run | Print | Save | SaveAs | Cut | Copy | Paste | InsertTxt | Find | Replace | Wrap+/- | LineNum

```
'Sample for estimation (Eviews format)
%esmpl="@first 2012m12"

' Sample for forecasting (Eviews format)
%fsmpl="2013m1 2013m12"

'Bootstrap errors? 1=yes, 0=no, use normal distribution
!boot=1

'Coefficient uncertainty? 1=yes, 0=no coefficient uncertanty
!coefu=1

'Number of bands in fanchart starting from 90%.
'For example, !numb=1 plots 90% band and the mode,
'!numb=2 plots 90% band, 45% band and the mode
!numb=5

'Do not change 2 parameters below unless you are adapting the code
' for your own model
'Name of the model to be used for constructing a fan chart
%modname="mod"

'Variable to forecast (has to be one of the model variables)
%varname="p"

smpl {%esmpl}
'=========== Model specification =========
```

Variable to be forecasted. In this case it is variable p which corresponds to inflation in Thailand

Let’s discuss inputs for the program

The model is specified here. By default it is AR(2) estimated by OLS

Program: M4_FANCHART - (u:\my documents\teaching\mf.x - forec...

Run | Print | Save | SaveAs | Cut | Copy | Paste | InsertTxt | Find | Replace | Wrap+/- | LineNum

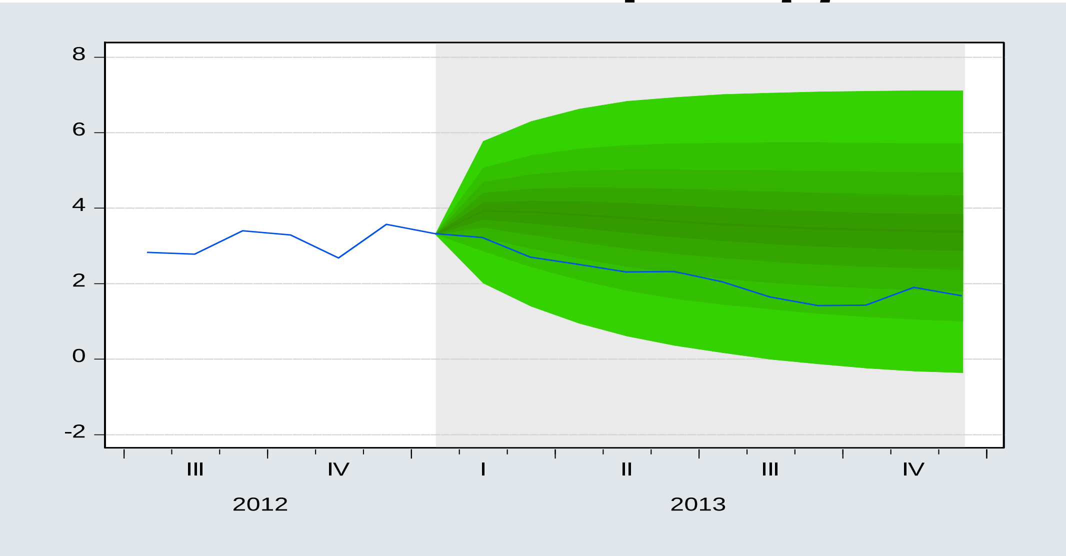
```
'Number of bands in fanchart starting from 90%.
'For example, !numb=1 plots 90% band and the mode,
'!numb=2 plots 90% band, 45% band and the mode
!numb=5

'Do not change 2 parameters below unless you are adapting the code
' for your own model
'Name of the model to be used for constructing a fan chart
%modname="mod"

'Variable to forecast (has to be one of the model variables)
%varname="p"

smpl {%esmpl}
'=========== Model specification =========
'replace the two lines below with your model of inflation if you want
equation eq1.ls p c p(-1) p(-2)
eq1.makemodel({%modname})
'=============== Main Fan Chart Program==============
%quant_array="90 "
!quant_step=90/!numb
!k=90-!quant_step
while !k>5
%quant_array=%quant_array+@str(@round(!k))+" "
!k=!k-!quant_step
wend
%quant_array=%quant_array+"1"
```

Let's run the program!



- You should see something like this
- Blue line corresponds to the actual inflation in 2013

Try to:

- Make errors normal
  - The resulting fan chart should be symmetric

- Remove coefficient uncertainty
  - Confidence bounds should become smaller (~10%)

- Change the number of bands
  - !numb=3 should give you a fan chart with 3 bounds as in the BoE inflation report.

- Enjoy!

Thank you. Good luck in Module 5!