MFx – Macroeconomic Forecasting

IMFx

Cointegration and Vector Error Correction Models

Introduction

Main Objectives of the Module

- Introduce the concept of cointegration
- Study the dynamics of cointegrated variables
- Explain the different methods used to test for cointegration

Main Objectives of the Module

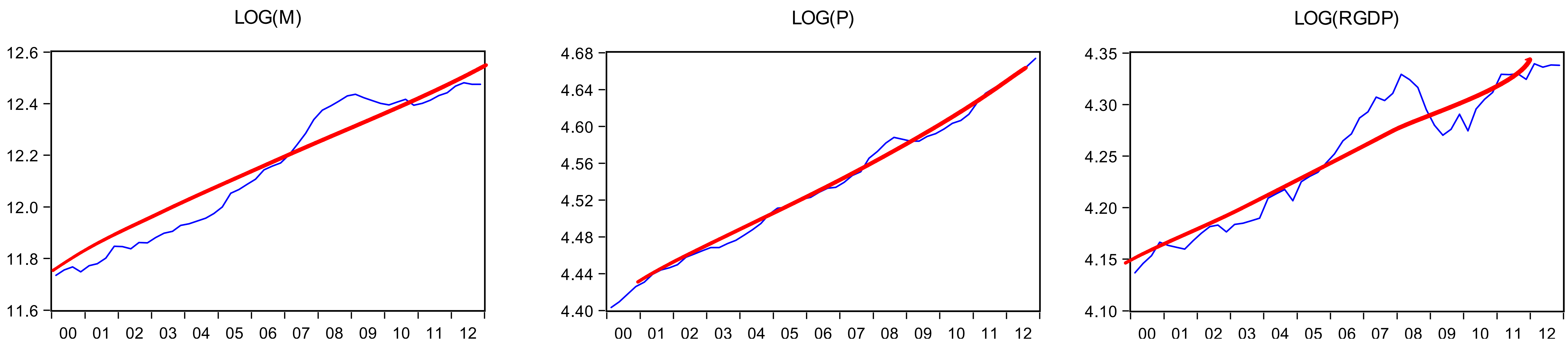
- Illustrate how to properly estimate a system of cointegrated variables

- Perform tests on the cointegration relationship

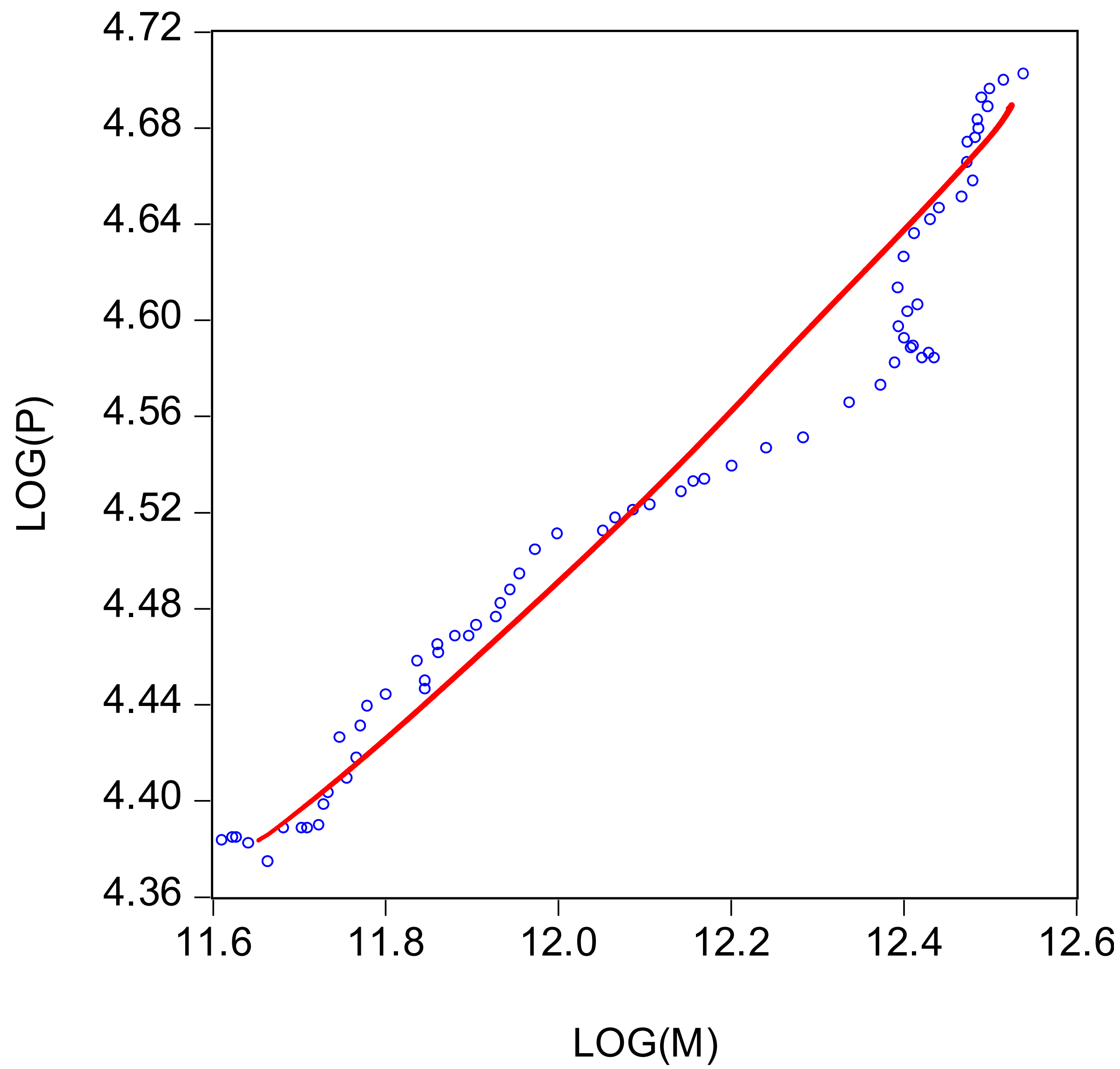
Money, Prices and GDP in Austria

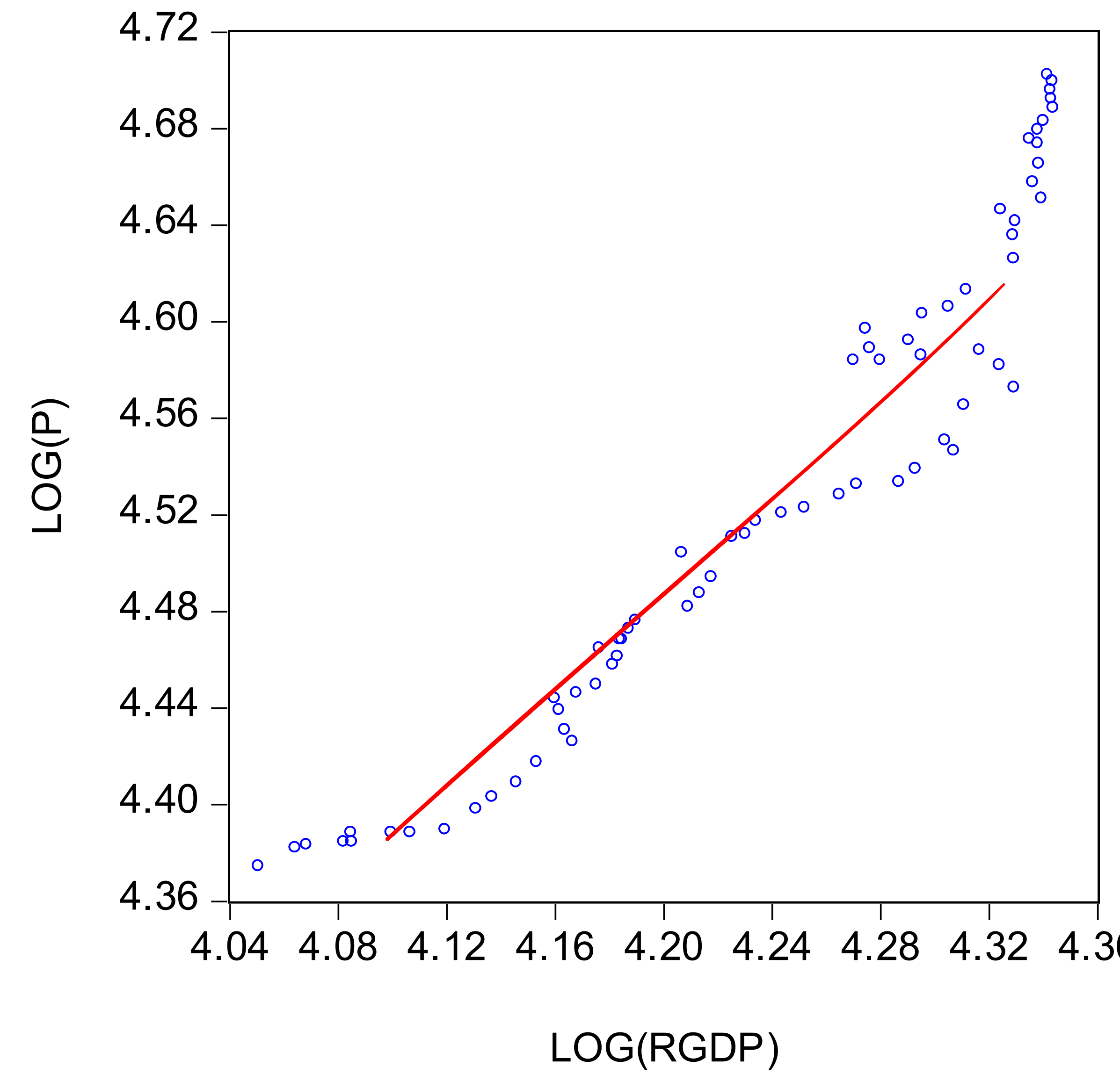
- In the data file you are given quarterly series of M2, CPI and real GDP of Austria between 1997Q3 and 2014Q4
- All series are seasonally adjusted
- Our goal is to forecast these variables over a period of 8 quarters based on a VAR

Let Us Look at the Data



Let Us Look at the Data More Closely





Linear Combinations of I(1) series

- In general a linear combination of I(1) series is also I(1).
- However, in some special cases there could be a linear combination which is I(0).
- So something very special has to happen for a linear combination to become stationary.

Intuition based on Theory

- Economic theories could give the intuition as to why some linear combinations of I(1) series could be I(0).
- For example: Uncovered Interest Parity, Permanent Income Hypothesis, Unbiased Forward Rate Hypothesis, Commodity Markey Arbitrage and Purchasing Power Parity.
- Theory guides us but we can also test it.

What is Cointegration?

- I(1) series are cointegrated if there exists at least one linear combination of these variables that is stationary
- In our case it means that there exist b_1, b_2 and b_3 such that:

$b_1m+b_2p+b_3y$ is stationary or I(0) NOT I(1)

What Kind of VAR?

- We could estimate a VAR in levels…
- …but as it contains I(1) variables this could be problematic

less lag ← VAR in differences

- How about a VAR in first differences?

less parameters to estimate ← VAR in levels

I(0)

- As we will see, this too could be highly problematic without proper testing

What's Next?

- Why does it matter if M, P and RGDP are cointegrated?
- If it makes the VAR in differences "inappropriate," what should we do instead?
- How do we test for cointegration in the first place?

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Cointegration and Vector Error Correction Models

Error Correction

From Session 1: What is Cointegration?

- A number of I(1) series are cointegrated if there exists a linear combination that would be stationary
- In our case it means that there exist b_1, b_2 and b_3 such that:

  $b_1 m + b_2 p + b_3 y$ is I(0)

Does Theory Indicate Cointegration?

- The money market equilibrium could be:

$$m^s{}_t = m^d{}_t = \beta_0 + \beta_1 p_t + \beta_2 y_t + \beta_3 r_t \quad \underline{\underline{+e_t}}$$

- However, what we observe includes an error…
- …which should not be very persistent and its variance should not rise over time

Long-Run Relationships

- If $(b_1m+b_2p+b_3y)$ is stationary…
- …although each series is non-stationary…
- …there must be some adjustment made by m, p and y such that they move together
- such that deviations from $b_1m+b_2p+b_3y=0$ remains bounded

Money and Price

- Suppose that in the long run:

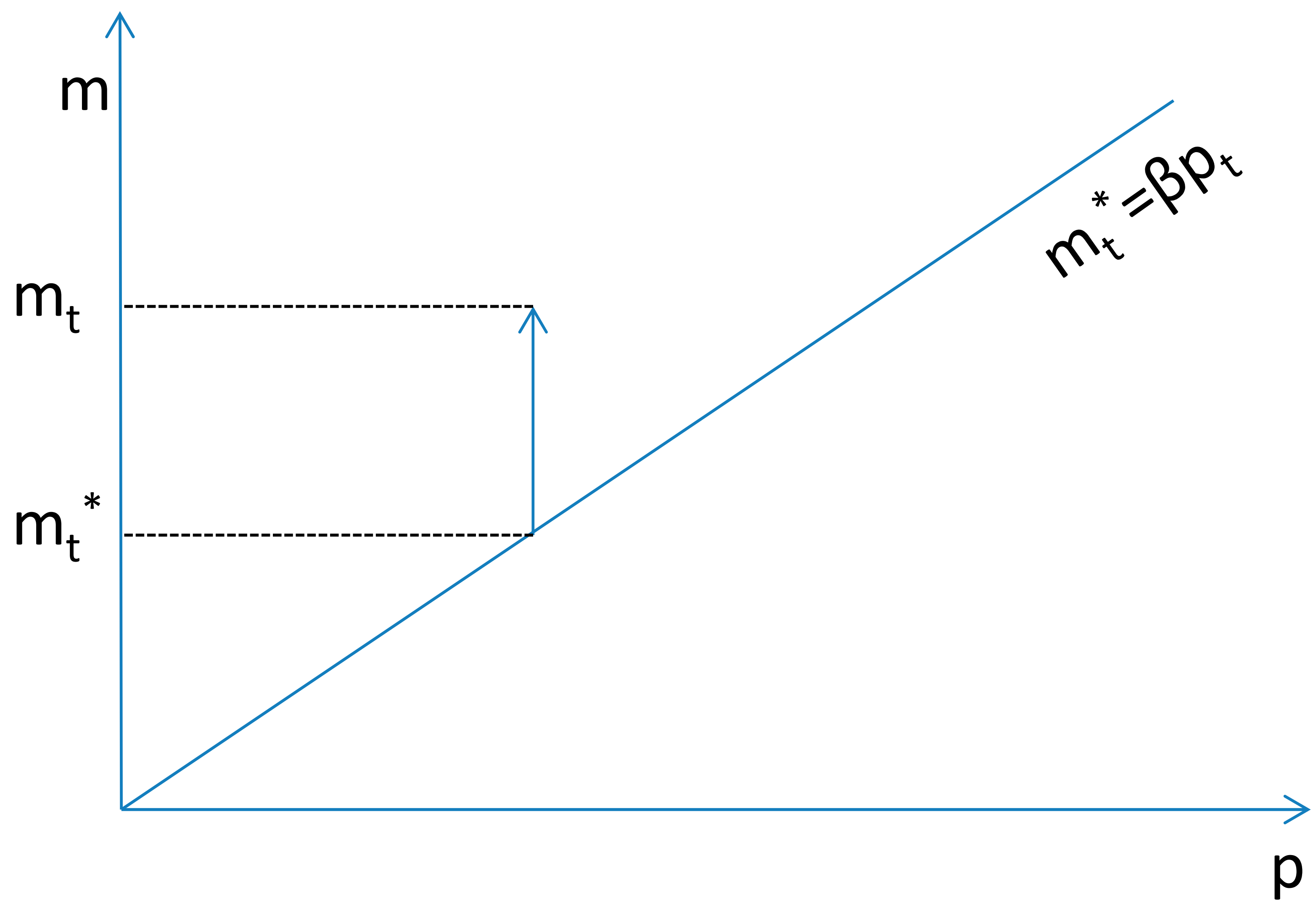
$$m_t = \beta p_t + e_t, \text{ where } \beta > 0$$

$\to I(0)$

$m_t - \beta p_t = e_t$

- That is, m and p are cointegrated. Note that Money Neutrality would imply $\beta = 1$
- Suppose m↑ such that $m_t - \beta p_t > 0$
- What would be the dynamics?

Deviation and Now What?



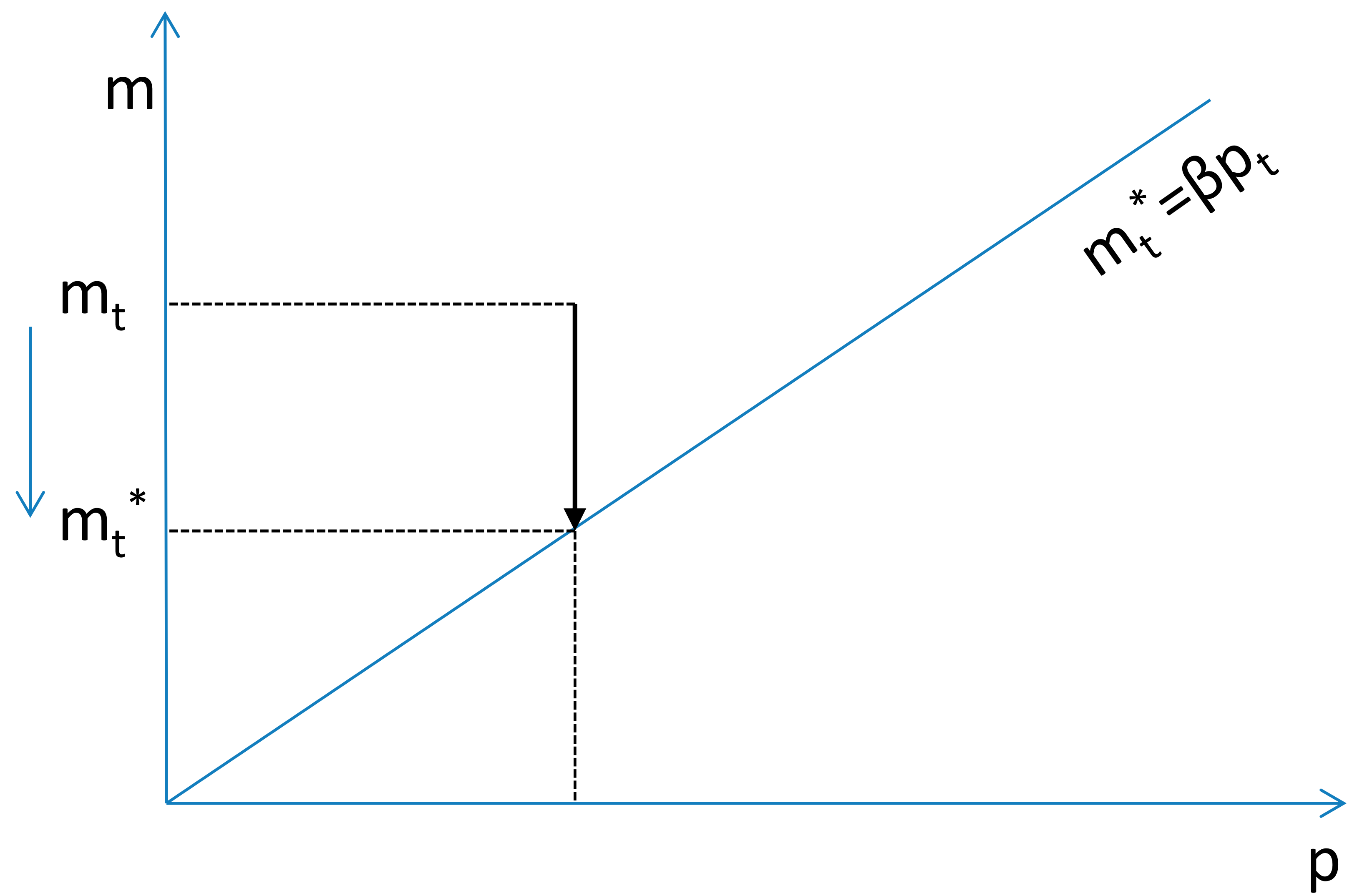
m_t is doing all the adjustment

- m_t* is unchanged and $m_t \downarrow$

$$\Delta m_t = \alpha_m(m_{t-1} - m_{t-1}^*), \text{ where } \alpha_m < 0$$

- Short run change in m_t is a linear function of the deviation from the long run equilibrium

m_t is doing all the adjustment



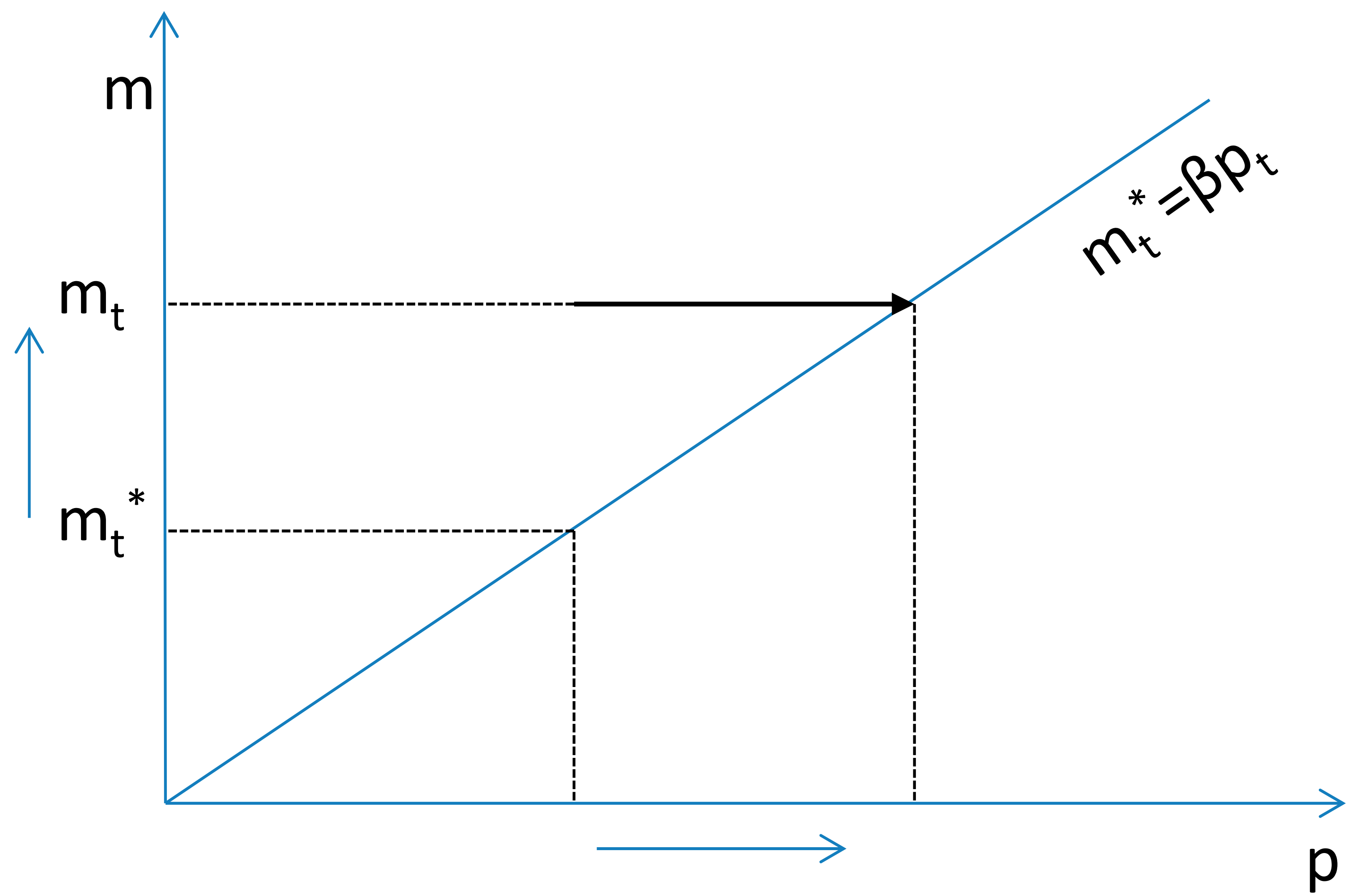
m_t* is doing all the adjustment

- m_t is unchanged, and p_t and m_t* ↑

$$\Delta p_t = \alpha_p(m_{t-1} - m_{t-1}^*), \text{ where } \alpha_p > 0$$

- Short run change in p_t is a linear function of the deviation from money neutrality

m_t* is doing all the adjustment



Both m_t and p_t are Adjusting

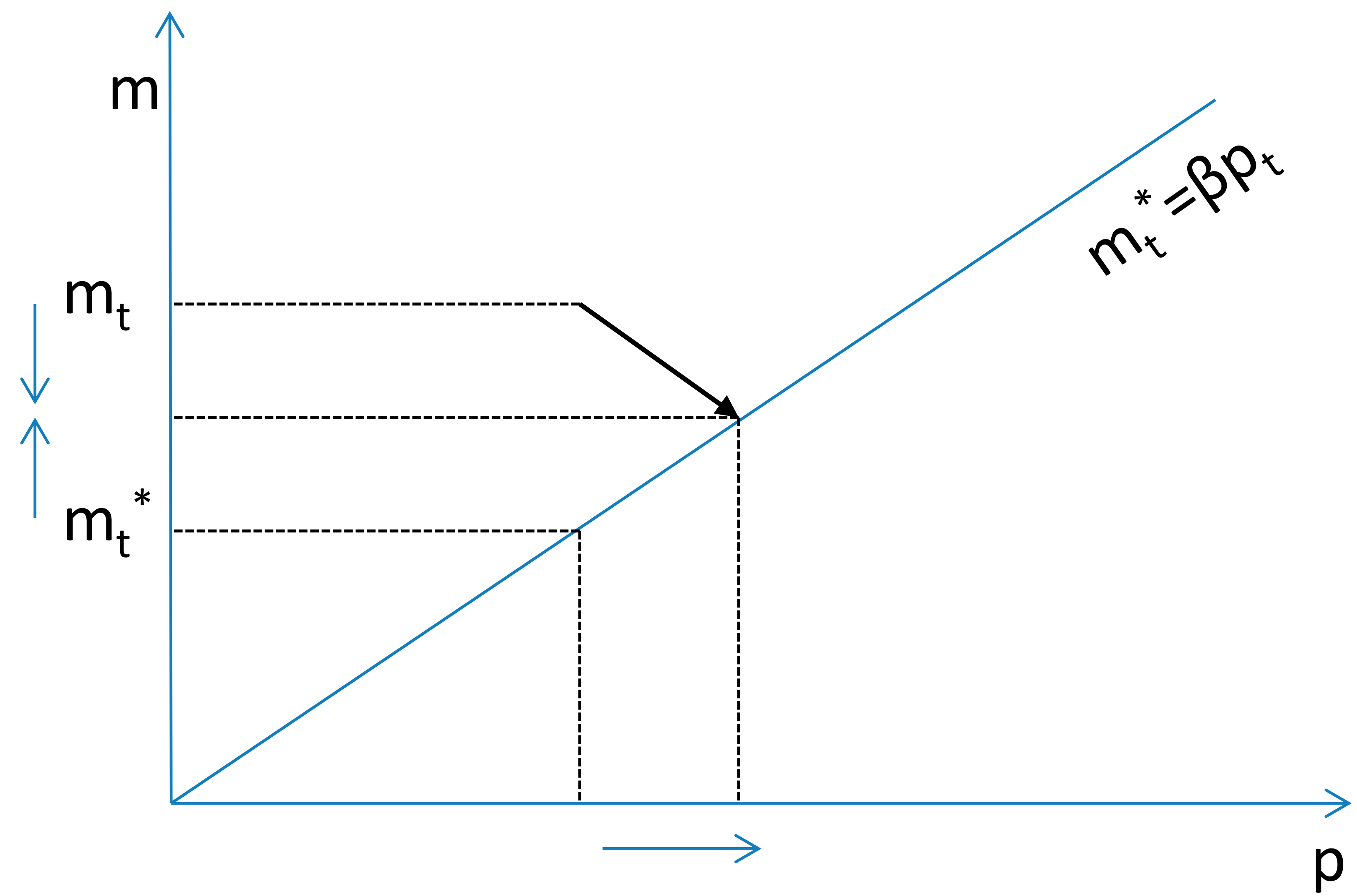
- m_t and p_t are adjusting simultaneously:

$$\Delta m_t = \alpha_m(m_{t-1} - m_{t-1}^*), \text{ where } \alpha_m < 0$$

$$\Delta p_t = \alpha_p(m_{t-1} - m_{t-1}^*), \text{ where } \alpha_p > 0$$

which is a basic error correction model

Simultaneous Adjustment



Taking Stock and What's Next?

- Error correction is a mechanism ensuring cointegration between m and p
- How about m, p and y…or more variables?
- …and we still don't know why it is important to take into account cointegration (and how!)

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Cointegration and Vector Error Correction Models

VECM Specification

From Session 2: Simple ECM

- m_t and p_t are cointegrated/follow the ECM:

$$\Delta m_t = \alpha_m(m_{t-1}-\beta p_{t-1})+\nu_t$$
$$\Delta p_t = \alpha_P(m_{t-1}-\beta p_{t-1})+u_t$$

- A VAR in differences would be misspecified

ECM as a Special VAR

- The previous ECM could be rewritten:

$$m_t = (1+\alpha_m)m_{t-1} - \alpha_m \beta p_{t-1} + \nu_t$$
$$p_t = \alpha_P m_{t-1} + (1-\alpha_P \beta)p_{t-1} + u_t$$

- This is clearly a VAR in levels with non-linear constraints on its coefficients

Back to Our Problem

- We believe that m_t, p_t and y_t are cointegrated
- In the long run: $E[b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4] = 0$
- Let us write a VECM and estimate it at the same time

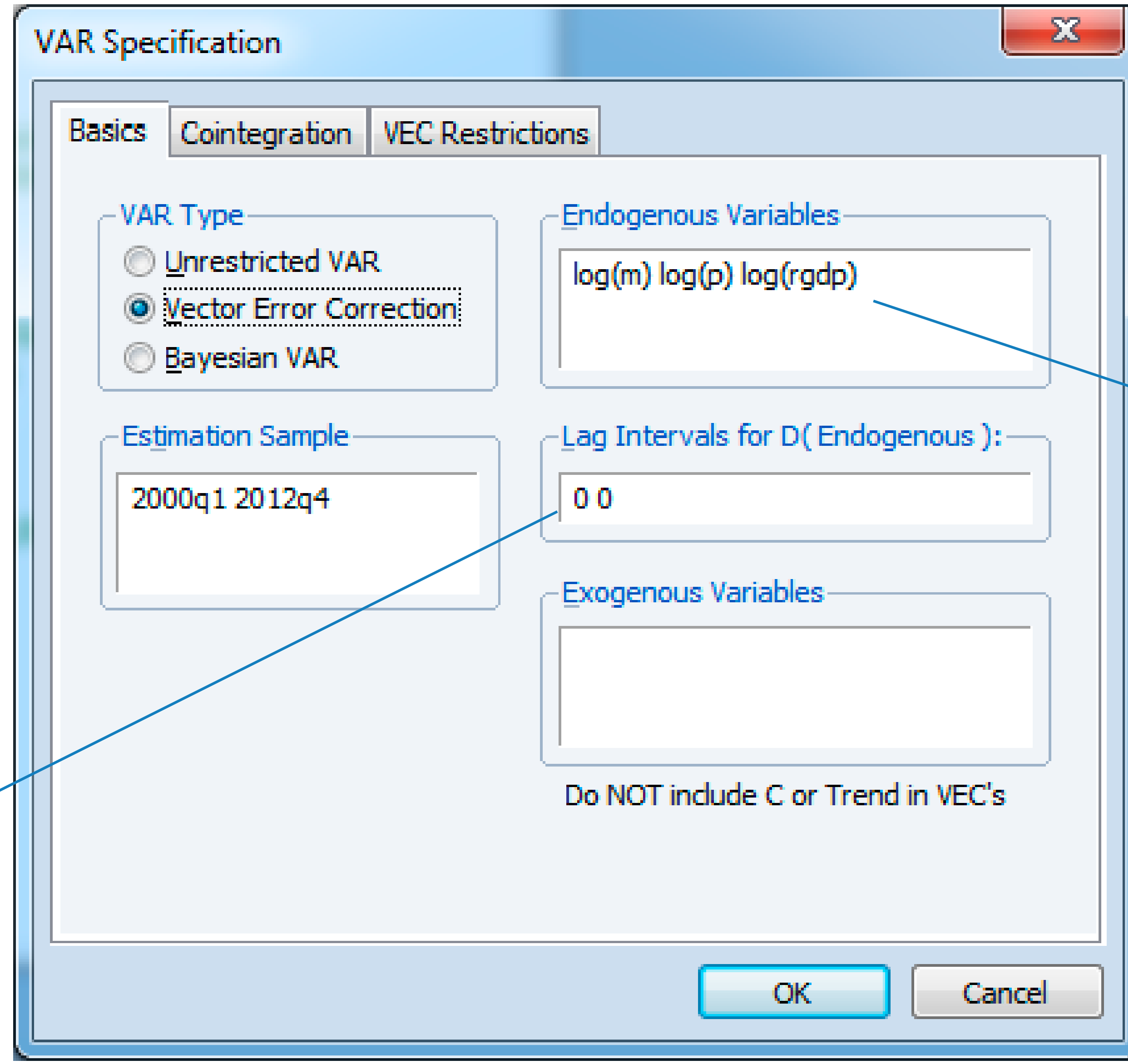
VECM of Lag Order 0

- VECM of m, p and y with no lags of Δm_t, Δp_t, Δy_t

$$\Delta m_t=\alpha_m (b_1 m_{t-1}+b_2 p_{t-1}+b_3 y_{t-1}+b_4)+\nu_t$$

$$\Delta p_t=\alpha_P(b_1 m_{t-1}+b_2 p_{t-1}+b_3 y_{t-1}+b_4)+u_t$$

$$\Delta y_t =\alpha_Y(b_1 m_{t-1}+b_2 p_{t-1}+b_3 y_{t-1}+b_4)+\eta_t$$



Endogenous variables: m,p and y

No lags of Δm, Δp and Δy which could be changed into 1 1 or 1 2

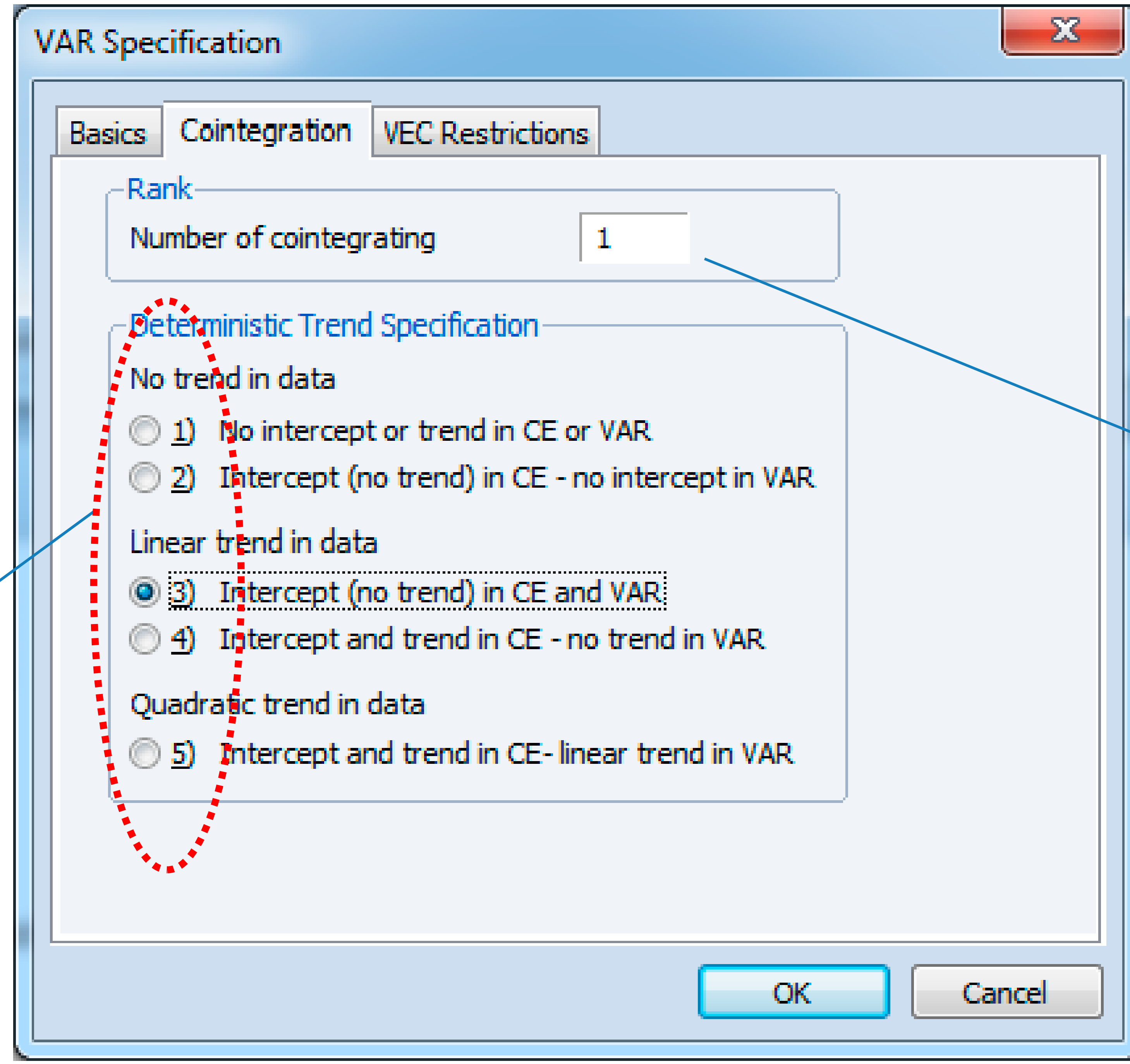
VECM of Order 1

$$\Delta m_t=\alpha_M(b_1m_{t-1}+b_2p_{t-1}+b_3y_{t-1}+b_4)+\lambda_{MM}\Delta m_{t-1}+\lambda_{MP}\Delta p_{t-1}+\lambda_{MY}\Delta y_{t-1}+\nu_t$$

$$\Delta p_t=\alpha_P(b_1m_{t-1}+b_2p_{t-1}+b_3y_{t-1}+b_4)+\lambda_{PM}\Delta m_{t-1}+\lambda_{PP}\Delta p_{t-1}+\lambda_{PY}\Delta y_{t-1}+u_t$$

$$\Delta y_t=\alpha_Y(b_1m_{t-1}+b_2p_{t-1}+b_3y_{t-1}+b_4)+\lambda_{YM}\Delta m_{t-1}+\lambda_{YP}\Delta p_{t-1}+\lambda_{YY}\Delta y_{t-1}+\eta_t$$

- How to decide the number of lags?



Specification of deterministic components

The default is one cointegration relationship but there could be more…

Deterministic Components

Case 1:

$\Delta m_t = \alpha_M(b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1}) + \lambda_{MM}\Delta m_{t-1} + \lambda_{MP}\Delta p_{t-1} + \lambda_{MY}\Delta y_{t-1} + \nu_t$

Case 2:

$\Delta m_t = \alpha_M(b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{MM}\Delta m_{t-1} + \lambda_{MP}\Delta p_{t-1} + \lambda_{MY}\Delta y_{t-1} + \nu_t$

Case 3:

$\Delta m_t = \mu_M + \alpha_M(b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{MM}\Delta m_{t-1} + \lambda_{MP}\Delta p_{t-1} + \lambda_{MY}\Delta y_{t-1} + \nu_t$

VECM of Order 1 with Constants

$\Delta m_t = \mu_M + \alpha_M(b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{MM}\Delta m_{t-1} + \lambda_{MP}\Delta p_{t-1} + \lambda_{MY}\Delta y_{t-1} + \nu_t$

$\Delta p_t = \mu_P + \alpha_P(b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{PM}\Delta m_{t-1} + \lambda_{PP}\Delta p_{t-1} + \lambda_{PY}\Delta y_{t-1} + u_t$

$\Delta y_t = \mu_Y + \alpha_Y(b_1 n_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{YM}\Delta m_{t-1} + \lambda_{YP}\Delta p_{t-1} + \lambda_{YY}\Delta y_{t-1} + \eta_t$

- Under option 3

Once the specification is set EViews will use Maximum Likelihood to estimate the parameters and produce the output:

Vector Error Correction Estimates
Date: 03/14/15 Time: 15:15
Sample (adjusted): 1998Q1 2014Q4
Included observations: 68 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1		
LOG(M(-1))	1.000000		
LOG(P(-1))	1.422433		
	(0.65845)		
	[2.16027]		
LOG(RGDP(-1))	-5.586962		
	(0.76354)		
	[-7.31718]		
C	5.122412		
Error Correction:	**D(LOG(M))**	**D(LOG(P))**	**D(LOG(RGD...**
CointEq1	-0.062230	-0.011734	0.022170
	(0.02113)	(0.00397)	(0.01208)
	[-2.94539]	[-2.95576]	[1.83515]
D(LOG(M(-1)))	0.208293	-0.026039	0.067627
	(0.11837)	(0.02224)	(0.06768)
	[1.75971]	[-1.17075]	[0.99917]
D(LOG(P(-1)))	-0.122835	0.214317	0.416759
	(0.65459)	(0.12300)	(0.37429)
	[-0.18765]	[1.74244]	[1.11345]

Taking Stock and What's Next?

- We now know why it is important to take into account cointegration if it is there…
- …but is there cointegration?
- …and if so how many cointegrating vectors are there?
- …and what are those cointegrating vectors?

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Cointegration and Vector Error Correction Models

Cointegration Test

From Session 3: VECM

$\Delta m_t = \mu_m + \alpha_m(b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_3) + \lambda_{mm}\Delta m_{t-1} + \lambda_{mp}\Delta p_{t-1} + \lambda_{my}\Delta y_{t-1} + \nu_t$

$\Delta p_t = \mu_p + \alpha_p(b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_3) + \lambda_{pm}\Delta m_{t-1} + \lambda_{pp}\Delta p_{t-1} + \lambda_{py}\Delta y_{t-1} + u_t$

$\Delta y_t = \mu_y + \alpha_y(b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_3) + \lambda_{ym}\Delta m_{t-1} + \lambda_{yp}\Delta p_{t-1} + \lambda_{yy}\Delta y_{t-1} + \eta_t$

- We have only one lag for illustration

VECM in Matrix Form

- We can stack the equations and variables into a vector X_t to obtain:

$$\Delta X_t = C + \Pi X_{t-1} + \Lambda \Delta X_{t-1} + e_t$$

- ΠX_{t-1} represents the error correction terms

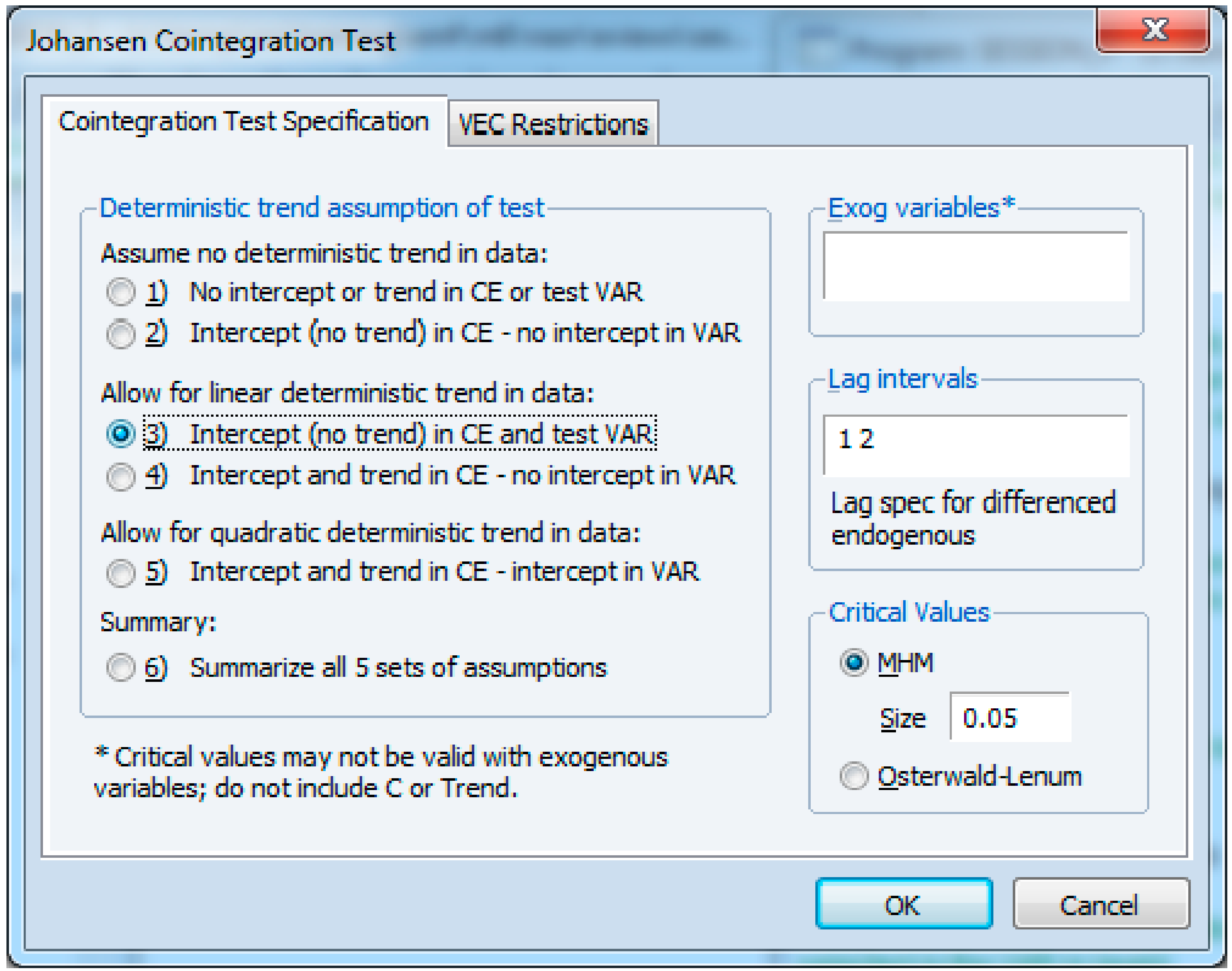
Johansen’s Methodology

- Johansen’s methodology rests on estimating the rank of Π
- The rank of Π is the maximum number of independent vectors it contains
- **It cannot exceed n, the number of variables we have in the system (in our case 3)**

The Rank Informs Us about Cointegration

- If rank(Π)=0: no cointegration
- If rank(Π)=n (full rank): all variables are I(0)
- If 0<rank(Π)=r<n (less than full rank): there are r independent cointegration relationships

Cointegration Test



Johansen's Statistics

- **Trace statistic for r:** the null hypothesis is that the rank is at most r vs. the rank is strictly greater

- **Maximum eigenvalue statistic for r:** the null hypothesis is that the rank is r vs. the rank is r+1

Sample: 2000Q1 2012Q4
Included observations: 52
Trend assumption: Linear deterministic trend
Series: LOG(M) LOG(P) LOG(RGDP)
Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.443819	35.02620	29.79707	0.0114
At most 1	0.067420	4.519818	15.49471	0.8575
At most 2	0.016974	0.890212	3.841466	0.3454

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.443819	30.50639	21.13162	0.0018
At most 1	0.067420	3.629605	14.26460	0.8962
At most 2	0.016974	0.890212	3.841466	0.3454

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Taking Stock and What's Next?

- We now know how to test for cointegration…
- ….we determined that there exists a single cointegrating relationship between m, p and y
- So we can go back and estimate our VECM. But how do we test whether y adjusts to long-run relationship?

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Cointegration and Vector Error Correction Models

Restricted VECM

From Session 4

- We established that there is one cointegration relationship…
- …we can estimate the VECM specified in Session 3
- But we would like to know whether y adjusts to deviations from the long-run relationship

From Session 3: VECM

$\Delta m_t = \mu_m + \alpha_m (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{mm} \Delta m_{t-1} + \lambda_{mp} \Delta p_{t-1} + \lambda_{my} \Delta y_{t-1} + \nu_t$

$\Delta p_t = \mu_p + \alpha_p (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{pm} \Delta m_{t-1} + \lambda_{pp} \Delta p_{t-1} + \lambda_{py} \Delta y_{t-1} + u_t$

$\Delta y_t = \mu_y + \alpha_y (b_1 m_{t-1} + b_2 p_{t-1} + b_3 y_{t-1} + b_4) + \lambda_{ym} \Delta m_{t-1} + \lambda_{yp} \Delta p_{t-1} + \lambda_{yy} \Delta y_{t-1} + \eta_t$

- We would like to test whether $\alpha_y = 0$. This is a test of weak exogeneity of RGDP

Let’s First Look at the Output of Our Unrestricted VECM More Closely

Estimation proc:

==================================

EC(C,1) 1 2 LOG(M) LOG(p) LOG(RGDp) @ C

VAR Model:

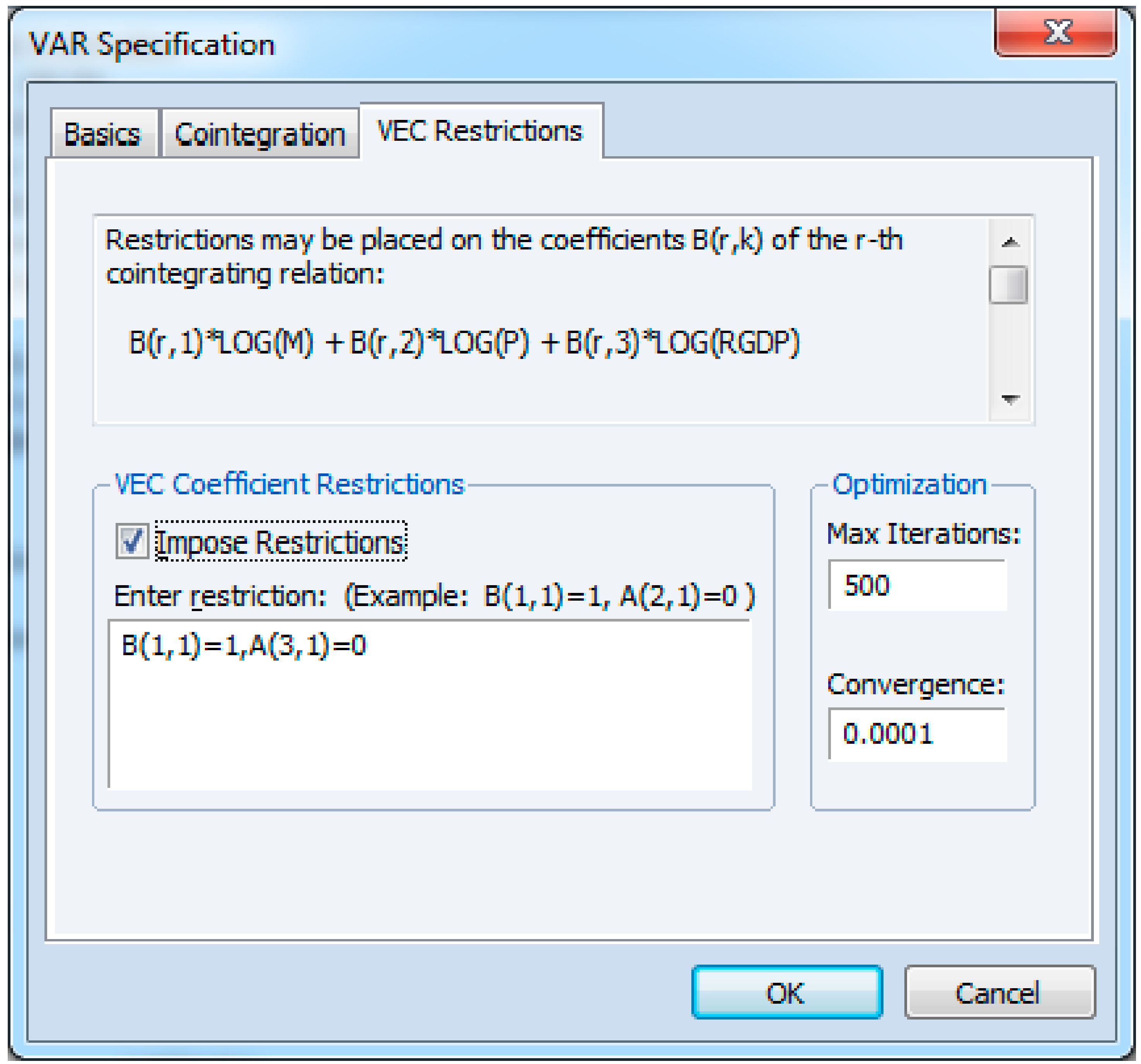
==================================

D(LOG(M)) = A(1,1)*(B(1,1)*LOG(M(-1)) + B(1,2)*LOG(p(-1)) + B(1,3)*LOG(RGDp(-1)) + B(1,4)) + C(1,1)*D(LOG(M(-1))) + C(1,2)*D(LOG(M(-2))) + C(1,3)*D(LOG(p(-1))) + C(1,4)*D(LOG(p(-2))) + C(1,5)*D(LOG(RGDp(-1))) + C(1,6)*D(LOG(RGDp(-2))) + C(1,7)

D(LOG(p)) = A(2,1)*(B(1,1)*LOG(M(-1)) + B(1,2)*LOG(p(-1)) + B(1,3)*LOG(RGDp(-1)) + B(1,4)) + C(2,1)*D(LOG(M(-1))) + C(2,2)*D(LOG(M(-2))) + C(2,3)*D(LOG(p(-1))) + C(2,4)*D(LOG(p(-2))) + C(2,5)*D(LOG(RGDp(-1))) + C(2,6)*D(LOG(RGDp(-2))) + C(2,7)

D(LOG(RGDp)) = **A(3,1)***(B(1,1)*LOG(M(-1)) + B(1,2)*LOG(p(-1)) + B(1,3)*LOG(RGDp(-1)) + B(1,4)) + C(3,1)*D(LOG(M(-1))) + C(3,2)*D(LOG(M(-2))) + C(3,3)*D(LOG(p(-1))) + C(3,4)*D(LOG(p(-2))) + C(3,5)*D(LOG(RGDp(-1))) + C(3,6)*D(LOG(RGDp(-2))) + C(3,7)

Restricted VECM Estimation



Why also impose B(1,1)=1?

Weak Exogeneity Test of RGDP

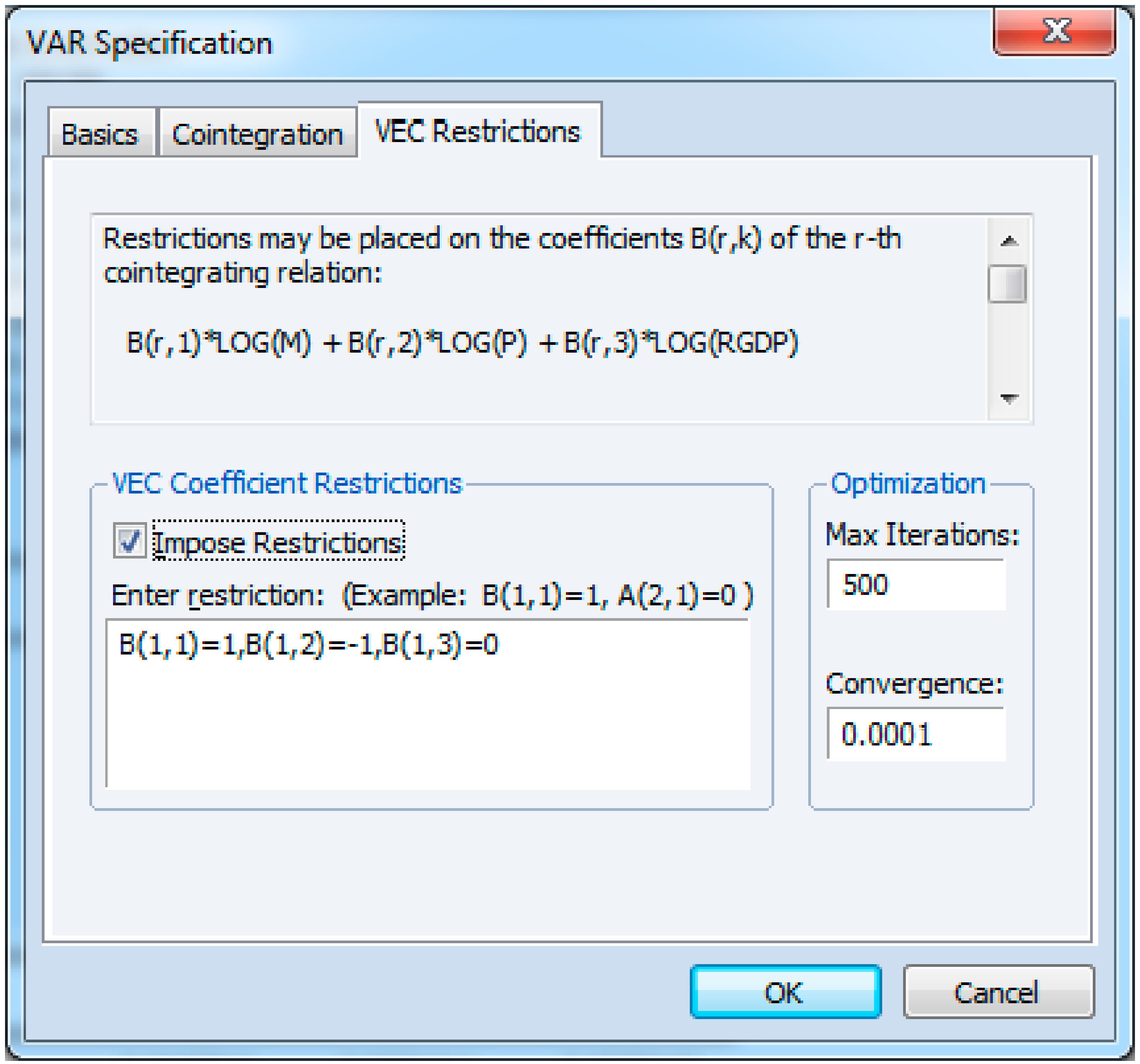
Sample: 2000Q1 2012Q4
Included observations: 52
Standard errors in ( ) & t-statistics in [ ]

Cointegration Restrictions:
B(1,1)=1,A(3,1)=0
Convergence achieved after 11 iterations.
Restrictions identify all cointegrating vectors
LR test for binding restrictions (rank = 1):

Chi-square(1)	3.503393
Probability	0.061243

Cointegrating Eq:	CointEq1
LOG(M(-1))	1.000000
LOG(P(-1))	1.489042 (0.66084) [2.25326]
LOG(RGDP(-1))	-5.591641 (0.75494) [-7.40675]
C	4.873434

Multiple Restrictions



One could test instead
B(1,1)=1,B(1,2)=-1,B(1,3)=0
which means in the LR:
$b_1 m_{t-1} + b_4 = p_{t-1}$

Sample: 2000Q1 2012Q4
Included observations: 52
Standard errors in ( ) & t-statistics in [ ]

Cointegration Restrictions:
B(1,1)=1,B(1,2)=-1,B(1,3)=0
Convergence achieved after 1 iterations.
Restrictions identify all cointegrating vectors
LR test for binding restrictions (rank = 1):

Chi-square(2)	26.87158
Probability	0.000001

Restricted VECM Then What?

- If the test of weak exogeneity indicates that we can’t reject it…
- …then we can estimate the restricted VECM (it is already done as a byproduct of the test!)
- **The restricted model will produce different forecasts, tests and impulse responses**

Taking Stock and What's Next?

- We now know how to test restrictions…
- …we determined that RGDP is weakly exogenous
- Is the restricted VECM the best for forecasting? How does it compare to VAR in differences?

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Forecasting with VECM

From Session 5

- We determined that RGDP was weakly exogenous and we estimated the model…
- …but we were not sure which model is best for forecasting
- We will build different forecasts and compare them

Forecasting with Three Models

- VECM
- Constrained VECM where RGDP is weakly exogenous
- VAR in differences

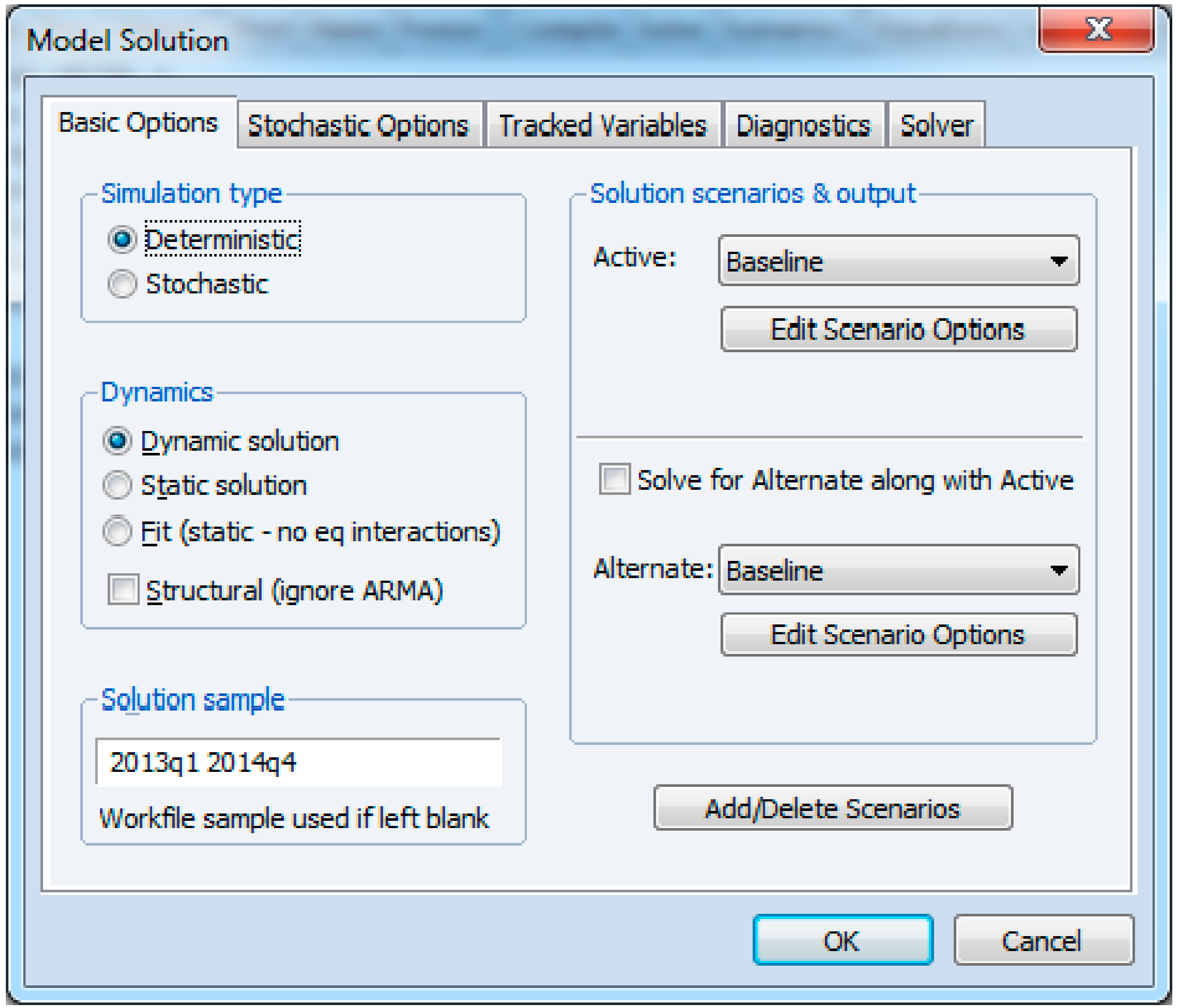
Forecasting: Steps for Each Model

- Estimate over 2000Q1-2012Q4
- Dynamic forecast over 2013Q1-2014Q4
- Compare to outcomes over the forecasting period and calculate RMSE's

Specification of the VECM: Summary

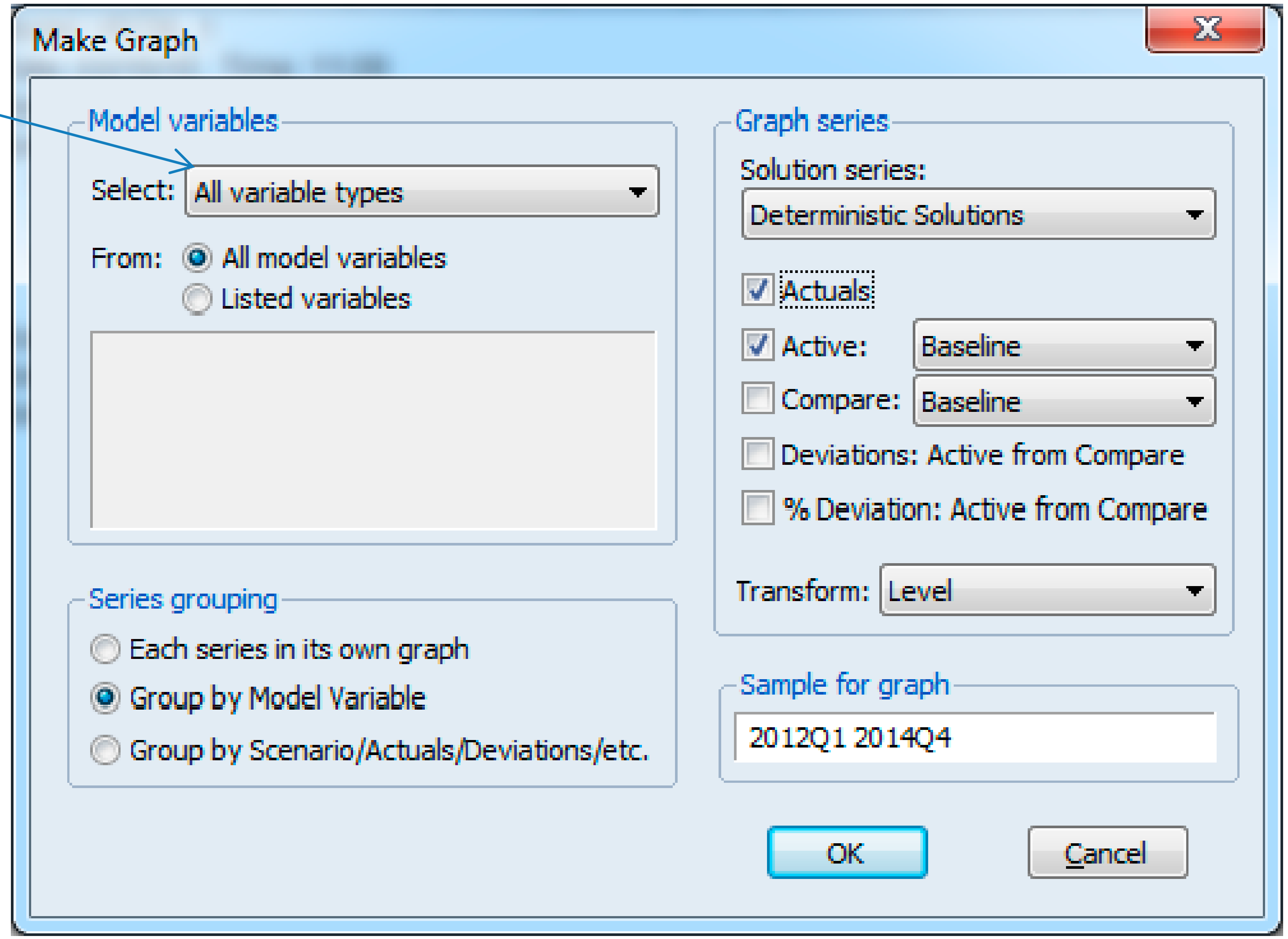
- Lags: 2
- Deterministic components: Option 3
- Number of cointegration relationships: 1
- Sample: 2000Q1-2012Q4
- Once estimated: **Proc/Make Model**

Solve the Model: Dynamic Forecast

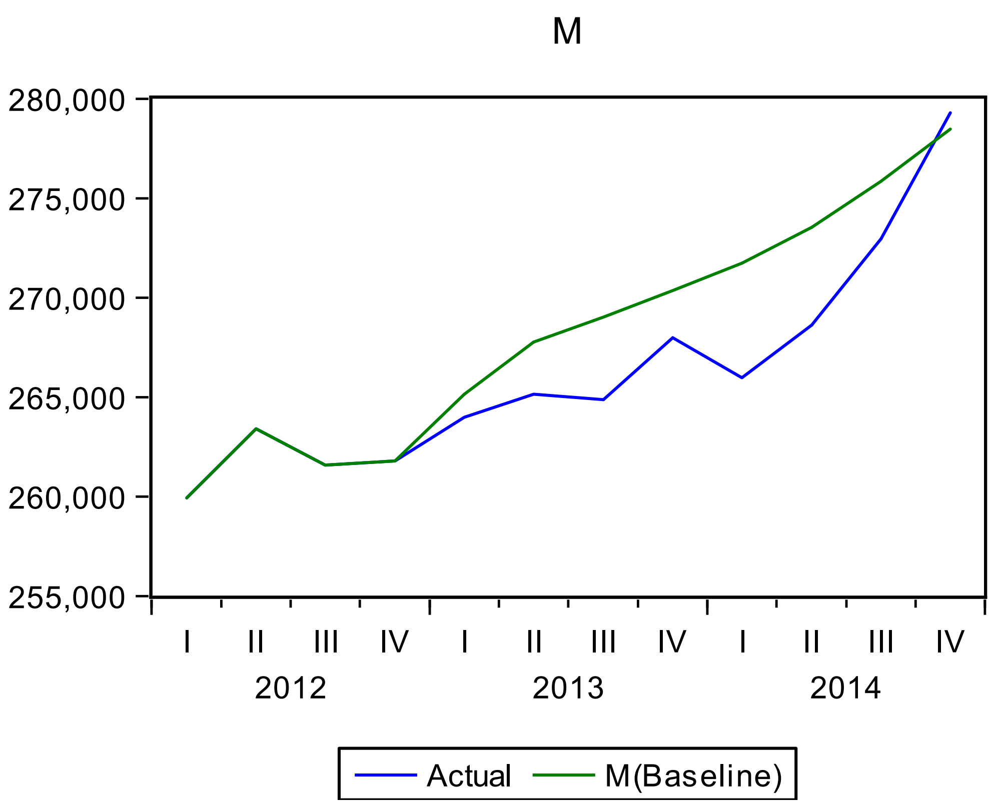


Proc/Make Graph

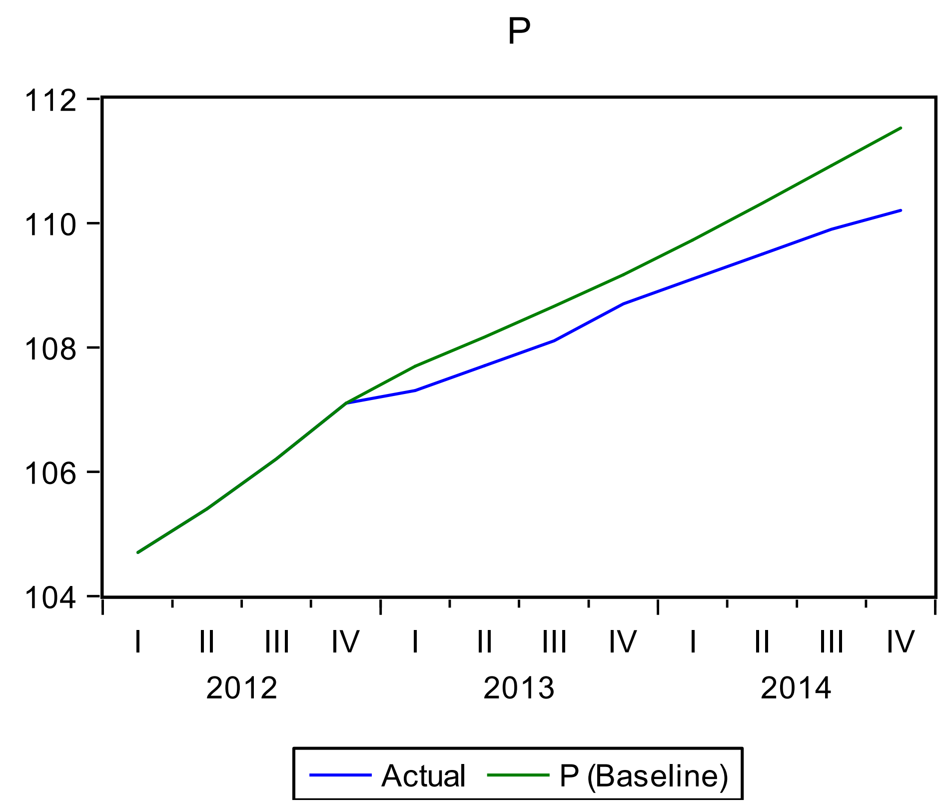
Or select "Endogenous variables" which is the same in our case.



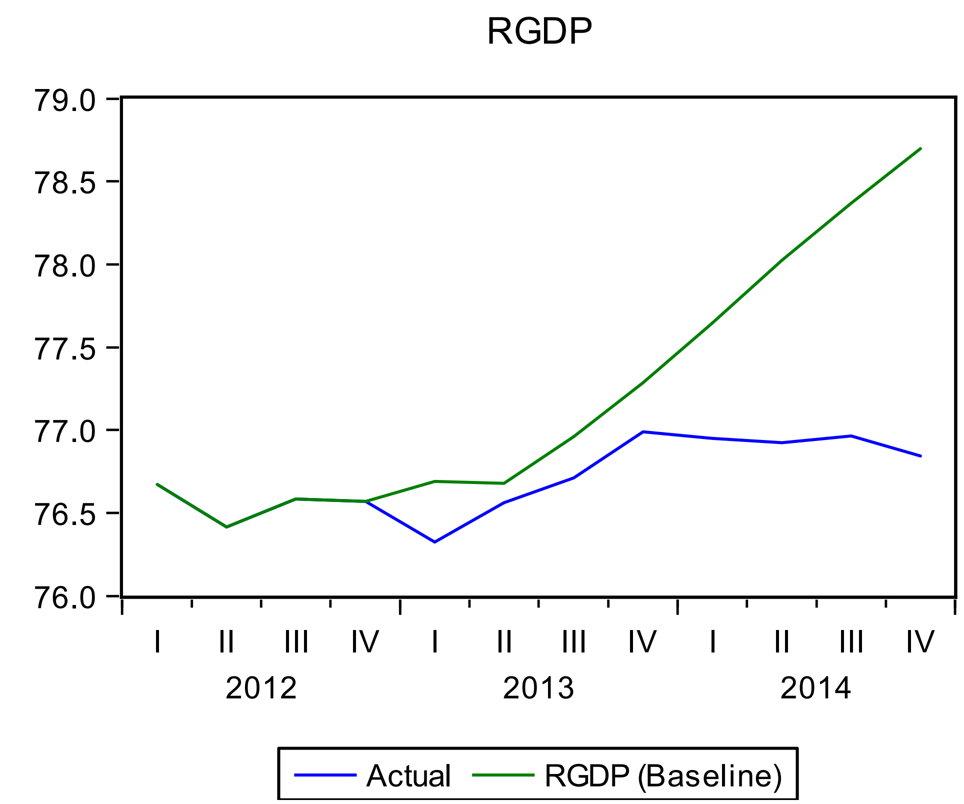
Forecasting Performance



RMSE=2851



RMSE=0.63



RMSE=0.78

Summary: Comparison of RMSEs

	VECM	Constrained VECM	VAR in Differences
P	0.6	**0.3**	0.5
M	**2851.4**	5446.8	8609.8
RGDP	0.8	0.7	**0.5**

Taking Stock and What's Next?

- Taking into account the long term relationship yielded superior forecasts of M and P.
- …imposing restrictions (pre-tested!) does not necessarily improve the forecast
- What should we do if there are more than one cointegration relationships?

MFx – Macroeconomic Forecasting

IMFx

Cointegration and Vector Error Correction Models

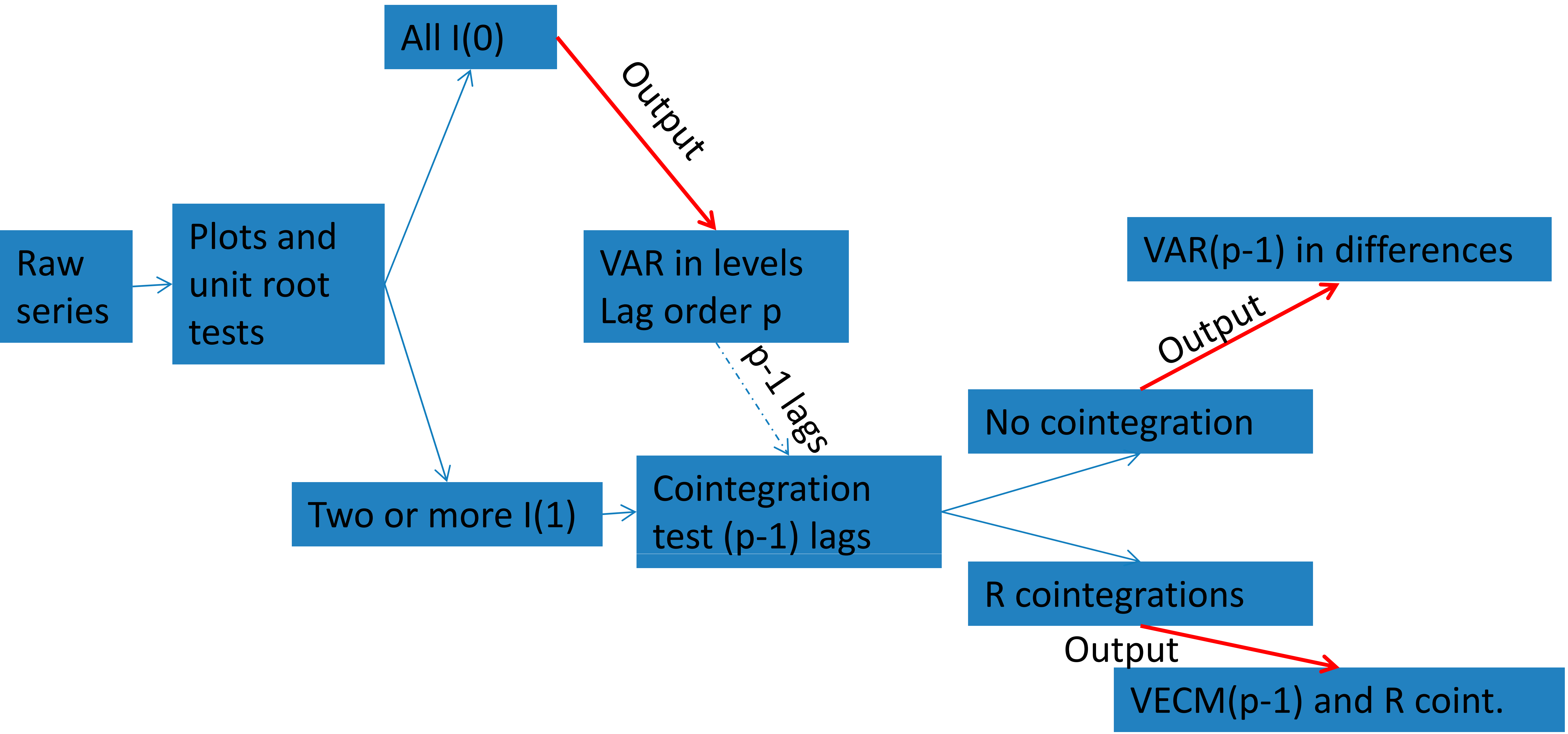
Multiple Cointegrations

From Session 6

- We saw from the forecasts that cointegration should be taken into account…
- …but we found that there was only one cointegration relationship
- How to deal with multiple relationships?

Let Us Study a Similar Problem

- We use Germany instead of Austria
- We have M, P, RGDP and i (interest rate)
- Data are over the same period


Raw series
Plots and unit root tests
All I(0)
Output
VAR in levels Lag order p
p-1 lags
Two or more I(1)
Cointegration test (p-1) lags
No cointegration
Output
VAR(p-1) in differences
R cointegrations
Output
VECM(p-1) and R coint.

Tests show that there are two cointegration relationships at 0.05 level too.

Sample: 2000Q1 2012Q4
Included observations: 52
Trend assumption: Linear deterministic trend
Series: LOG(M) LOG(P) LOG(RGDP) I
Lags interval (in first differences): No lags

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.1 Critical Value	Prob.**
None *	0.466975	67.17082	44.49359	0.0003
At most 1 *	0.380054	34.45313	27.06695	0.0135
At most 2	0.167375	9.590746	13.42878	0.3135
At most 3	0.001265	0.065822	2.705545	0.7975

Trace test indicates 2 cointegrating eqn(s) at the 0.1 level
* denotes rejection of the hypothesis at the 0.1 level
**MacKinnon-Haug-Michelis (1999) p-values

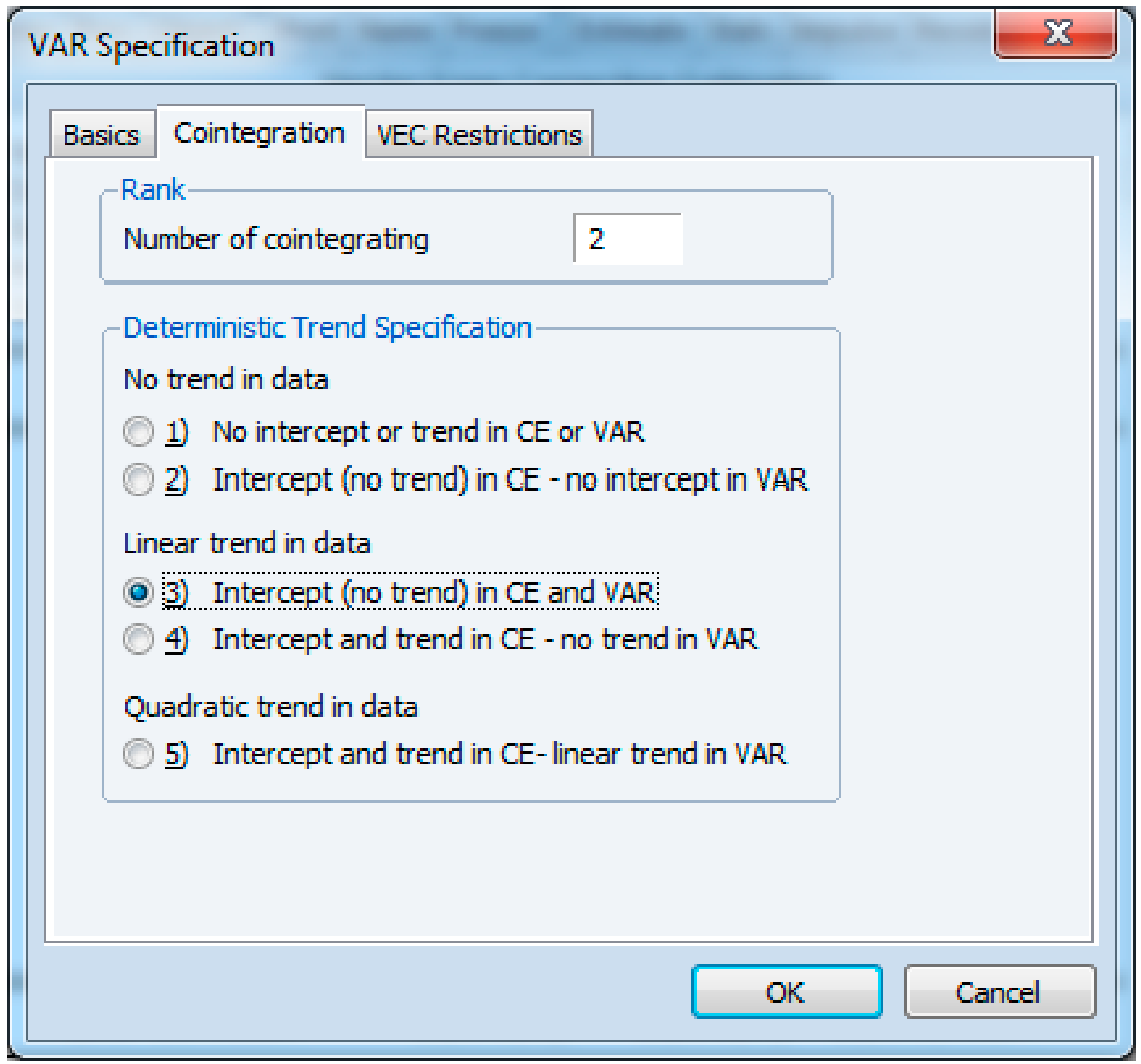
Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.1 Critical Value	Prob.**
None *	0.466975	32.71769	25.12408	0.0100
At most 1 *	0.380054	24.86239	18.89282	0.0142
At most 2	0.167375	9.524925	12.29652	0.2450
At most 3	0.001265	0.065822	2.705545	0.7975

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.1 level
* denotes rejection of the hypothesis at the 0.1 level
**MacKinnon-Haug-Michelis (1999) p-values

Estimate the VECM

Make sure you change the number of cointegrating vectors to 2



EViews Default Restrictions

Now we have two cointegrating equations.

Notice the default restrictions imposed by EViews to achieve identification.

Date: 03/15/15 Time: 17:21
Sample: 2000Q1 2012Q4
Included observations: 52
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	CointEq2
LOG(M(-1))	1.000000	0.000000
LOG(P(-1))	0.000000	1.000000
LOG(RGDP(-1))	-4.742465	-1.239572
	(0.36276)	(0.06852)
	[-13.0732]	[-18.0907]
I(-1)	0.011253	0.012565
	(0.01241)	(0.00234)
	[0.90689]	[5.36103]
C	23.11359	3.406518

Alternative Restrictions

- It may not be natural to exclude money or prices from the cointegrating vectors
- How about excluding RGDP or interest rate?
- To do so run the VECM with the correct restrictions (check View/Representations)

Note the restrictions we used.

Note that in this case P has a negative coefficient in the first cointegrating equation where only nominal variables appear.

Vector Error Correction Estimates
Date: 03/15/15 Time: 17:31
Sample: 2000Q1 2012Q4
Included observations: 52
Standard errors in ( ) & t-statistics in [ ]

Cointegration Restrictions:
B(1,1)=1,B(1,3)=0,B(2,2)=1,B(2,4)=0
Convergence achieved after 1 iterations.
Restrictions identify all cointegrating vectors
Restrictions are not binding (LR test not available)

Cointegrating Eq:	CointEq1	CointEq2
LOG(M(-1))	1.000000	5.935724
		(1.19487)
		[4.96766]
LOG(P(-1))	-3.721503	1.000000
	(0.23347)	
	[-15.9399]	
LOG(RGDP(-1))	0.000000	-29.97341
		(4.70017)
		[-6.37709]
I(-1)	-0.033515	0.000000
	(0.00825)	
	[-4.06160]	
C	9.600450	144.5020

Weak Exogeneity of RGDP

- Let us add one additional restriction in each version, that is EViews default restrictions and the alternative ones
- Recall that we found that weak exogeneity of RGDP was important in Session 6
- Now we compare the RMSEs of each model

Comparing RMSEs

EViews default restrictions

	VECM	Constrained VECM	VAR in Differences
P	**0.55**	0.60	9.70
M	**31.08**	36.83	580.39
RGDP	3.48	**1.87**	43.45
i	**0.20**	0.30	1.85

Alternative restrictions

	VECM	Constrained VECM	VAR in Differences
P	**0.55**	0.59	9.70
M	**31.08**	37.20	580.39
RGDP	3.48	**2.30**	43.45
i	**0.20**	0.27	1.85

Taking Stock and What's Next?

- When there are multiple cointegrating vectors
- …one should be careful in interpreting the results of over-identifying tests and associated forecasts…
- Use theory or intuition if possible to impose alternative restrictions