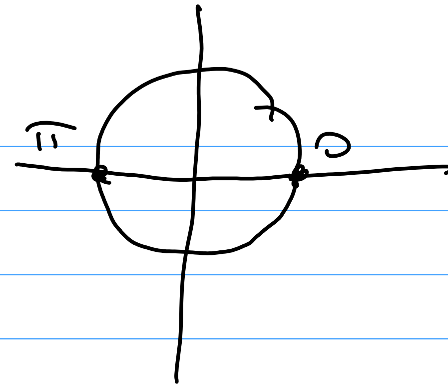
$$\sin x = \sin 2x$$

$$\sin x = 2\sin x \cos x$$

$$2\sin x \cos x - \sin x = 0$$

$$\sin x\,(2\cos x - 1) = 0$$

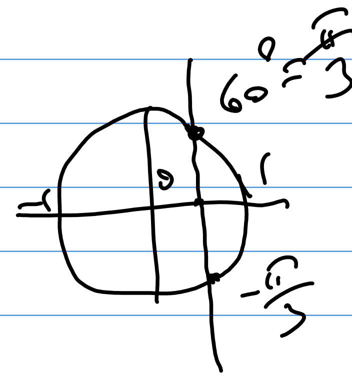



$a = -1, 0, 1$

$\sin x = a$

$$\left\{ \left[ \begin{array}{l} \sin x = 0 \\ 2\cos x - 1 = 0 \end{array} \right. \right.$$

a∂o




$\sin x = 0$ $\qquad$ $\cos x = \frac{1}{2}$

$x = \pi n,\ n \in \mathbb{Z}$

arc

$$x = \pm\frac{\pi}{3} + 2\pi k,\ k \in \mathbb{Z}$$

$$x \in \{\pi n \mid n \in \mathbb{Z}\} \cup \left\{\pm\frac{\pi}{3} + 2\pi k \mid k \in \mathbb{Z}\right\}$$

$$x \in \{180^\circ n \mid n \in \mathbb{Z}\} \cup \{\pm 60^\circ + 360^\circ k \mid k \in \mathbb{Z}\}$$

$$0^\circ \le x < 360^\circ$$

$n = 0, 1$ $\qquad$ $k \ge 0$

$$\{0^\circ, 180^\circ\} \cup \{60^\circ, 300^\circ\}$$

$$= \{0^\circ, 60^\circ, 180^\circ, 300^\circ\}$$

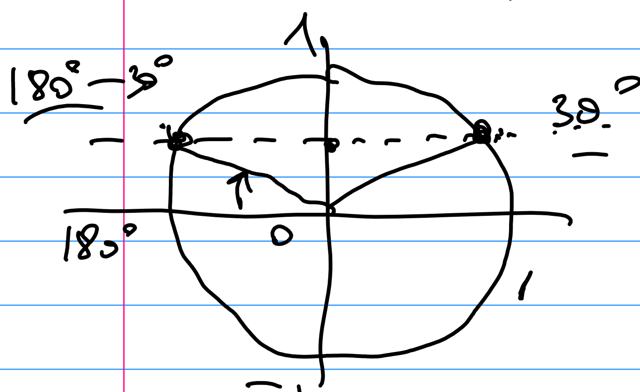
$$\cos^2 x - \sin x = \frac{1}{4}$$

$$1 - \sin^2 x - \sin x = \frac{1}{4} \qquad t = \sin x$$

$$1 - t^2 - t = \frac{1}{4}$$

$$t^2 + t - \frac{3}{4} = 0 \qquad t_1 = +\frac{1}{2} \quad t_2 = -\frac{3}{2}$$

$$\sin x = \frac{1}{2} \qquad \sin x = a$$




$$\left[\begin{array}{l} x = 30° + 360° n \\ x = 180° - 30° + 360° n \end{array}\right.$$

$$x = (-1)^k \cdot 30° + 180° k, \; k \in \mathbb{Z}$$

$$x \in \{30°, 150°\}$$

$$1\sin x + 1\cos x = 1 \quad (:\sqrt{2}) \qquad \sqrt{1^2+1^2}=\sqrt{2}$$

$\sin 45^\circ$ $\qquad$ $\cos 45^\circ$

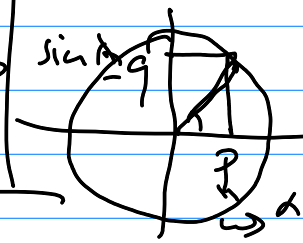
$$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}} \qquad p^2+q^2=1$$

$p=\sin\alpha \quad q=\cos x$

$$\underbrace{\left(\frac{1}{\sqrt{2}}\right)^2}_{\sin\alpha} + \underbrace{\left(\frac{1}{\sqrt{2}}\right)^2}_{\cos\alpha} = \frac{1}{2}+\frac{1}{2} = 1$$

$\alpha = 45^\circ$

$x^2+y^2=1$



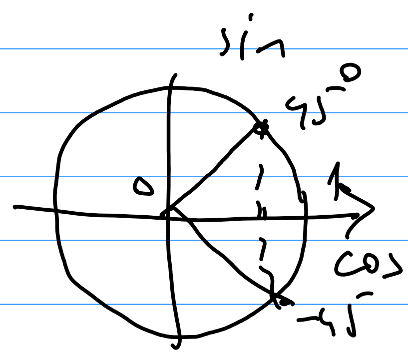

- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
- $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\alpha$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$\sin\overset{\beta}{45^\circ}\sin\overset{\alpha}{x} + \cos\overset{\beta}{45^\circ}\cos\overset{\alpha}{x} = \frac{1}{\sqrt{2}}$$

$$\cos(\boxed{x-45^\circ}) = \frac{1}{\sqrt{2}}$$




$$\boxed{x-45^\circ} = \pm 45^\circ + 360^\circ n$$

$$\left[\begin{array}{l} x-45^\circ = +45^\circ + 360^\circ n \\ x-45^\circ = -45^\circ + 360^\circ n \end{array}\right. \qquad n\in\mathbb{Z}$$

$$\{0^\circ, 90^\circ\} \ni x \quad \left[\begin{array}{l} x = 90^\circ + 360^\circ n \\ x = 360^\circ n \end{array}\right. \qquad \begin{array}{l} x = 90^\circ, \ n=0 \\ x = 0^\circ, \ n=0 \end{array}$$

$$\sin x + \cos x = 1$$

$-2 = 2$

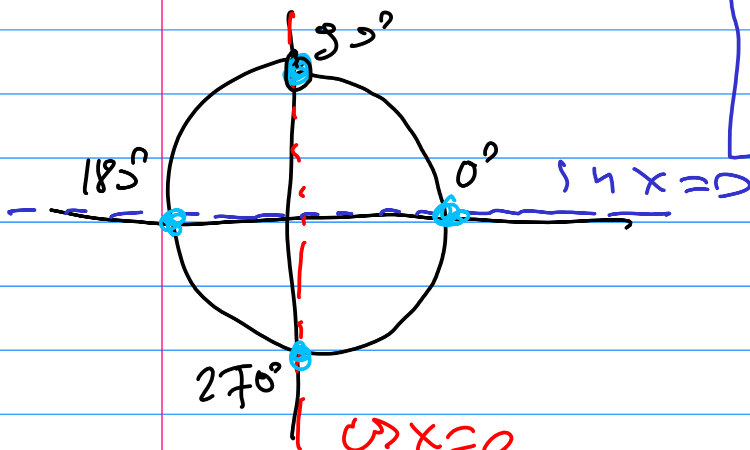
$4 = 4$

$$(\sin x + \cos x)^2 = 1 \leftarrow$$ стор. корені

$$\underbrace{\sin^2 x + 2\sin x\cos x + \cos^2 x}_{=1} = 1$$

$$\cancel{1} + 2\sin x\cos x = \cancel{1}$$

$$2\sin x\cos x = 0$$




$$\left[\begin{array}{l}\sin x = 0\\ \cos x = 0\end{array}\right.$$

$x = 0°$

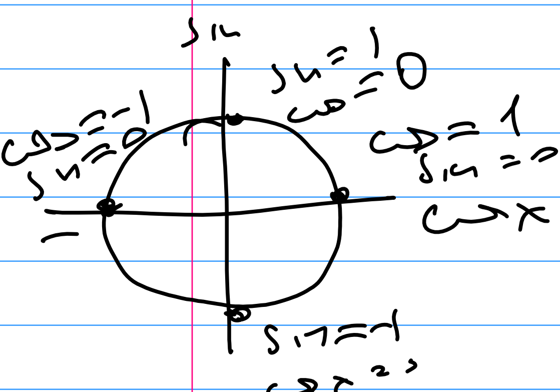
$x = 90°$

$x = 180°$

$x = 270°$

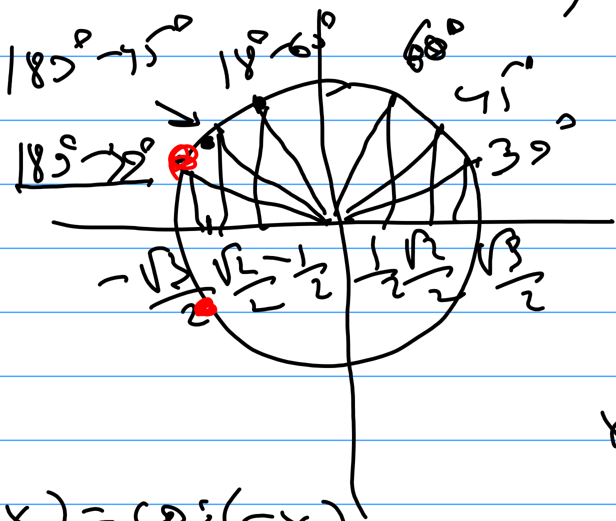
| $\sin x$ | | $\cos x$ | |
|---|---|---|---|
| 0 | + | 1 | = 1 |
| 1 | + | 0 | = 1 |
| 0 | + | (−1) | ≠ 1 |
| −1 | + | 0 | ≠ 1 |

$\{0°, 90°\}$



$\sin$ 30° 45° 60°
$\frac{1}{2}$ $\frac{\sqrt{2}}{2}$ $\frac{\sqrt{3}}{2}$

$$\cos(x-45^\circ) = -\frac{\sqrt{3}}{2}$$



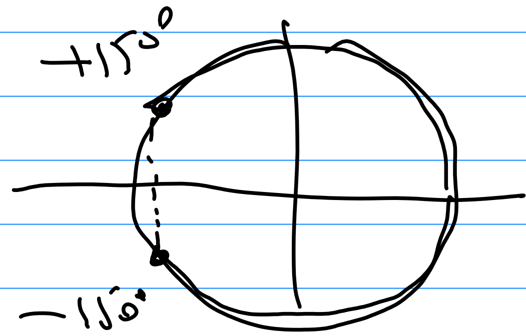

$$\cos(x-45^\circ) = -\frac{\sqrt{3}}{2}$$

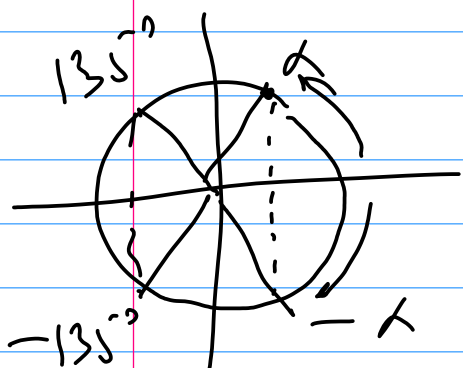
$$x-45^\circ = \pm 150^\circ + 360^\circ n$$

$\cos(x) = \cos(-x)$

$\cos(-\alpha) = \cos\alpha$

$\cos\alpha = -\frac{\sqrt{3}}{2}$
$\cos(-\alpha) = -\frac{\sqrt{3}}{2}$




$$\left[\begin{array}{l} x-45^\circ = -150^\circ + 360^\circ n \\ x-45^\circ = +150^\circ + 360^\circ n \end{array}\right.$$

$$\left[\begin{array}{l} x = -105^\circ + 360^\circ n \\ x = 195^\circ + 360^\circ n \end{array}\right.$$

$x = 255^\circ,\ n = 1$

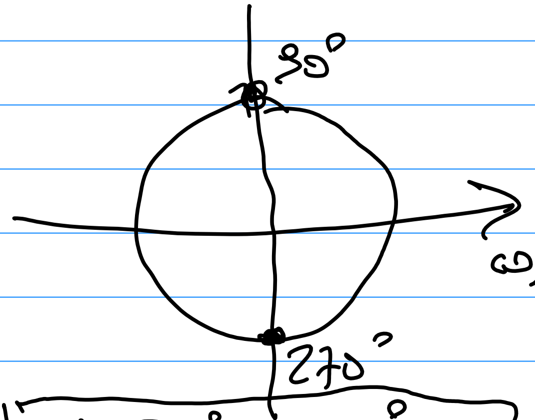
$x = 195^\circ,\ n = 0$

$x \in \{195^\circ, 255^\circ\}$

$1\sin x + 1\cos x = 0 \qquad \sqrt{1^2+1^2} = \sqrt{2}$

$$\underbrace{\frac{1}{\sqrt{2}}}_{\sin 45^\circ}\sin x + \underbrace{\frac{1}{\sqrt{2}}}_{\cos 45^\circ}\cos x = 0$$

$$\cos(\boxed{x-45^\circ}) = 0$$




$$\boxed{x = 90^\circ + 180^\circ n}$$

$$\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta) = \cos(\beta-\alpha)$$

$90^\circ,\ 270^\circ,\ 270^\circ+180^\circ,\ 270^\circ+360^\circ, \ldots$

$\stackrel{!}{=} 90^\circ + 180^\circ n$

$x - 45^\circ = 90^\circ + 180^\circ n \qquad x = 135^\circ + 180^\circ n$

$n = 0, \quad x = 135^\circ$

$n = 1 \quad x = 315^\circ \qquad x \in \{135^\circ, 315^\circ\}$

---

$\sin x + \cos x = 0 \quad :\cos x$

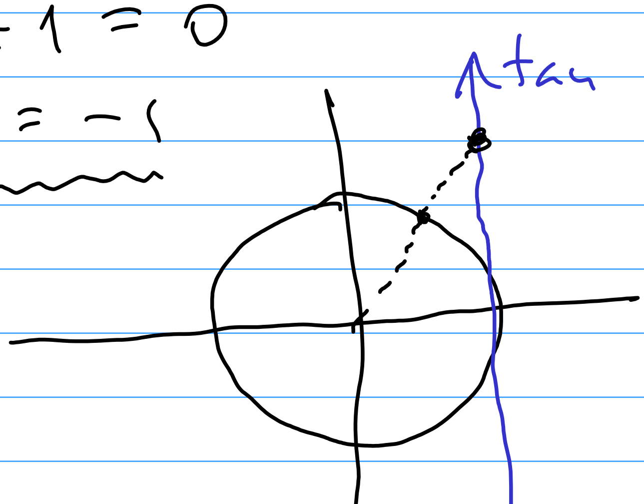
За може $\boxed{\begin{matrix}\cos x = 0 \\ \sin x = 0\end{matrix}}$

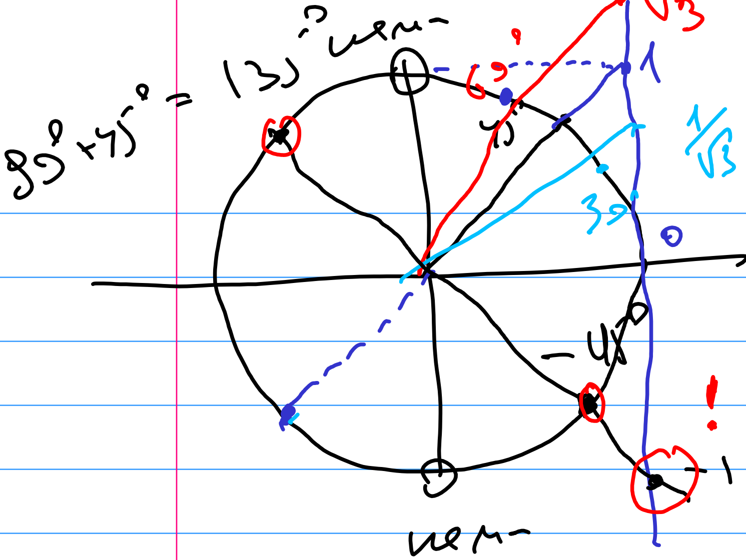
$\sin x + 0 = 0,$

одн. $x$

$\tan x + 1 = 0$

$\tan x = -1$






$\tan 45^\circ = 1$

$\tan(45^\circ + 180^\circ) = 1$

$\tan 135^\circ = -1$

$\tan(-45^\circ) = -1$

~~$x = -45^\circ + 180^\circ n$~~

$x = 135^\circ + 180^\circ n$

$n=0 \quad x = 135^\circ$

$n=1 \quad x = 315^\circ$

$$\sin x - \cos x = 0 \quad : \cos x$$

$$\tan x - 1 = 0 \qquad \tan x = 1$$

$$x = 45^\circ + 180^\circ n \qquad 45^\circ$$

$$285^\circ$$

---

$$\cos^2 x + \frac{\sin^2 x (1 - \cos x)}{1 - \sin x} = \frac{1}{1 - \sin x}$$

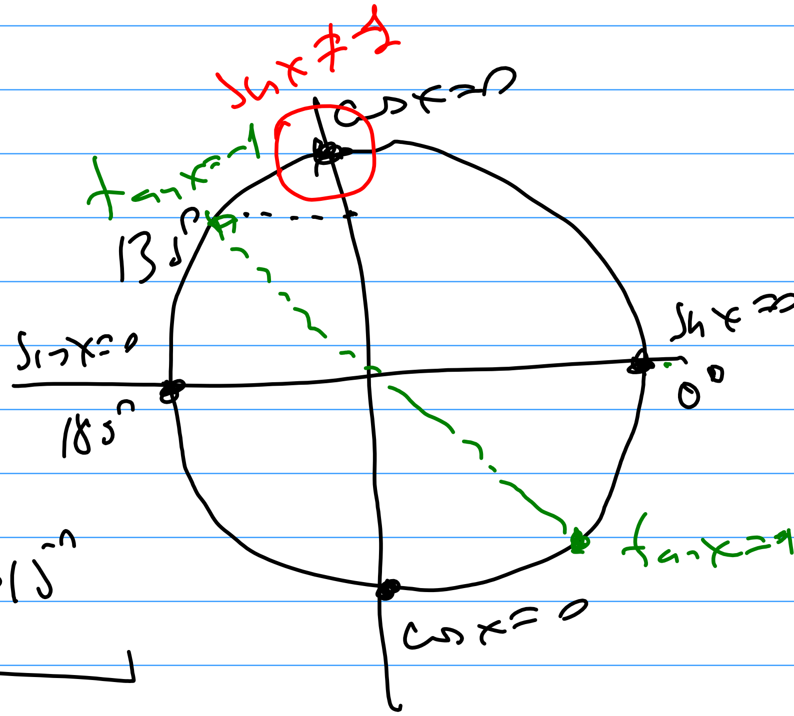
$$0 = \frac{\cos^2 x (1 - \sin x)}{1 - \sin x} + \frac{\sin^2 x - \sin^2 x \cos x}{1 - \sin x} - \frac{1}{1 - \sin x}$$

$$\frac{\cos^2 x - \cos^2 x \sin x + \sin^2 x - \sin^2 x \cos x - 1}{1 - \sin x} = 0$$

$$\frac{-\sin x \cos x (\cos x + \sin x)}{1 - \sin x} = 0$$

$$\begin{cases} -\sin x \cos x (\cos x + \sin x) = 0 \\ 1 - \sin x \neq 0 \end{cases}$$

$$\begin{cases} \left[\begin{array}{l} \sin x = 0 \\ \cos x = 0 \\ \cos x + \sin x = 0 \iff \tan x = -1 \end{array}\right. \\ \sin x \neq 1 \end{cases}$$




$$\underbrace{0°,\ 135°,\ 180°,\ 270°,\ 315°}_{\boxed{+360° n}}$$

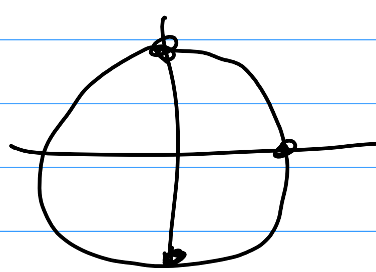
---

$\sin^2 x + \cos x = 1$

$1 - \cos^2 x + \cos x = 1$

$\cos x = 0$

or $\cos x = 1$



$\cos x = t$

$-t^2 + t = 0$

$t = 0$ or $t = 1$

$x = 0°,\ 90°,\ 270°$

$+360°$

$$\sin x + \cos 2x = 1$$

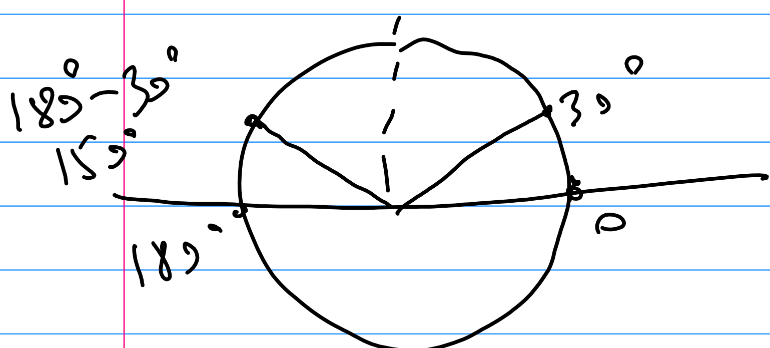
$$\cos 2x = \cos^2 x - \sin^2 x = \underline{2\cos^2 x - 1}$$

$$= 1 - 2\sin^2 x$$

$$\sin x + \cancel{1} - 2\sin^2 x = \cancel{1} \qquad \sin x = t$$

$$t = \frac{1}{2} \text{ or } t = 0 \qquad t - 2t^2 = 0$$

$$\sin x = 0 \text{ or } \underline{\sin x = \frac{1}{2}}$$




$$\{0, 30°, 150°, 180°\}$$

---

$$\sin 2x + 2\cos^2 x = 2$$

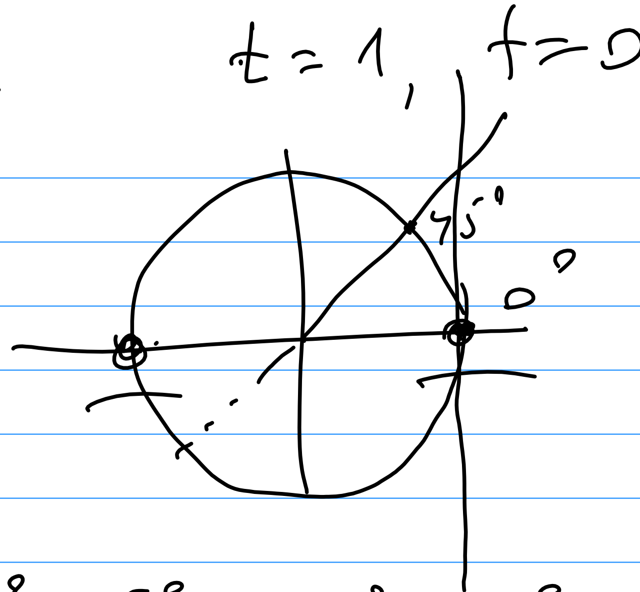
$$2\underline{\sin x}\,\underline{\cos x} + 2\underline{\cos^2} x = 2(\underbrace{\sin^2 x + \cos^2 x}_{1})$$

$$\frac{2\sin x \cancel{\cos x}}{\cos^{\cancel{2}} x} + 2 = 2\frac{\sin^2 x}{\cos^2 x} + 2 \qquad :\cos^2 x$$

$$2\tan x + 2 = 2\tan^2 x + 2 \qquad \tan x = t$$

$2t + 2 = 2t^2 + 2$ $\qquad t = 1,\ t = 0$

$$\left[\begin{array}{l} \tan x = 1 \\ \tan x = 0 \end{array}\right.$$




$$\left[\begin{array}{l} x = 0^\circ + 180^\circ n \\ x = 45^\circ + 180^\circ n \end{array}\right.$$

$0^\circ,\ 45^\circ,\ 180^\circ,\ 225^\circ$

---

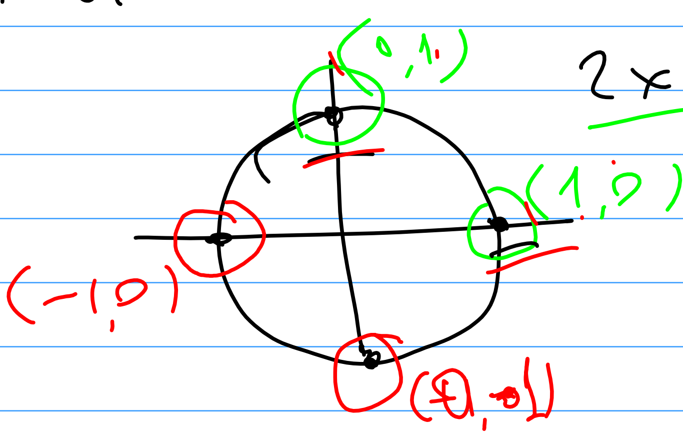
$\sin 2x + 2\cos^2 x = 2 \quad \big|\, -1$

$\sin 2x + \underbrace{2\cos^2 x - 1}_{\cos 2x} = 1$ $\qquad \boxed{\sin x + \cos x = 1}$

$\left(\sin 2x + \cos 2x = 1\right)^2$

$\cancel{\sin^2 2x} + 2\sin 2x \cos 2x + \cancel{\cos^2 2x} = \cancel{1}$

$\sin 2x = 0$ an $\cos 2x = 0$




$2x$

$$\left[\begin{array}{l} 2x = 0^\circ + 360^\circ n \\ 2x = 90^\circ + 360^\circ n \end{array}\right.$$

$$x = 180^\circ n$$
$$x = 45^\circ + 180^\circ n$$

$$\left\{ \begin{array}{l} 0^\circ, 180^\circ \\ 45^\circ, 225^\circ \end{array} \right\}$$

---

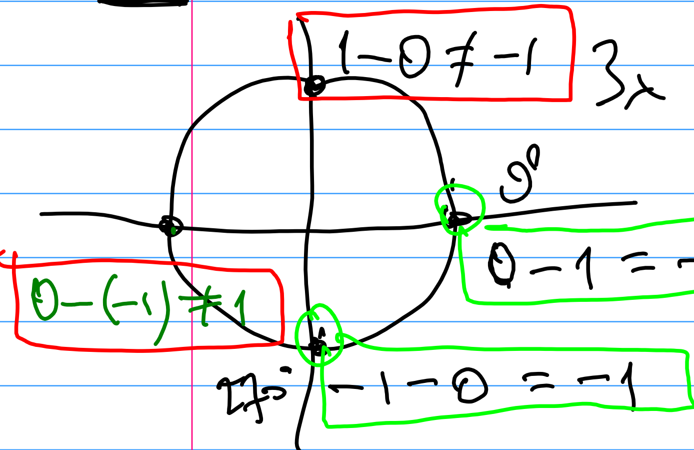
$$\boxed{\sin 3x - \cos 3x = -1}$$

$$(a-b)^2$$
$$a^2 - 2ab + b^2$$

$$\sin^2 3x - 2\sin 3x \cos 3x + \cos^2 3x$$
$$= 1$$

$$-2\sin 3x \cos 3x = 0$$

$$\left[ \begin{array}{l} \sin 3x = 0 \\ \cos 3x = 0 \end{array} \right.$$




$$\left\{ \begin{array}{l} 3x = 0 + 360^\circ n \\ 3x = 270^\circ + 360^\circ n \end{array} \right.$$

$$\left[ \begin{array}{l} x = 120^\circ n \\ x = 90^\circ + 120^\circ n \end{array} \right.$$

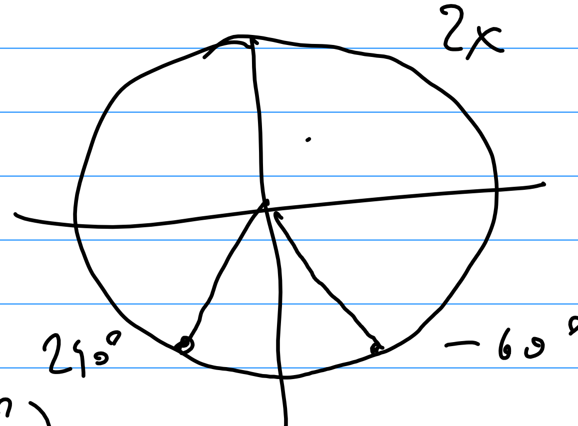
| $n=0$ | $n=1$ | $n=2$ |
|---|---|---|
| $0^\circ$ | $120^\circ$ | $240^\circ$ |
| $90^\circ$, | $210^\circ$, | $330^\circ$ |

$\sin x \cdot \cos x = -\frac{\sqrt{3}}{4}$ $/ \times 2$

$\sin 2x = 2\sin x \cdot \cos x$

$2\sin x \cos x = -\frac{\sqrt{3}}{2}$

$\sin 2x = -\frac{\sqrt{3}}{2}$




$180° - (-60°) = 240°$

$2x = -60° + 360° n$

$2x = 240° + 360° n$

$\left[\begin{array}{l} x = -30° + 180° n \\ x = 120° + 180° n \end{array}\right.$

$\{150°, 330°, 120°, 300°\}$

---

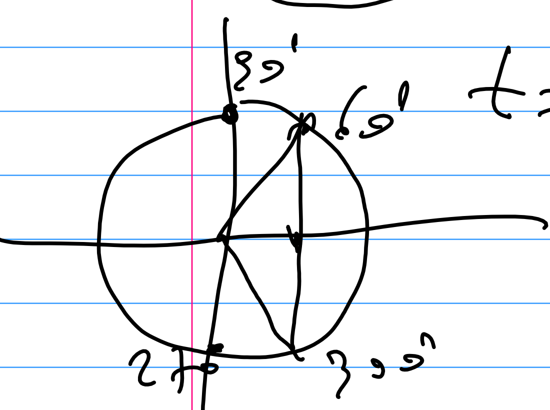
$2\cos^2 x = \cos 2x + 1$

$1 + \underbrace{\cos 2x}_{2\cos^2 x - 1} = \cos x$

$2\cos^2 x = \cos x$

$2t^2 = t$

$t = \frac{1}{2}, \quad t = 0$

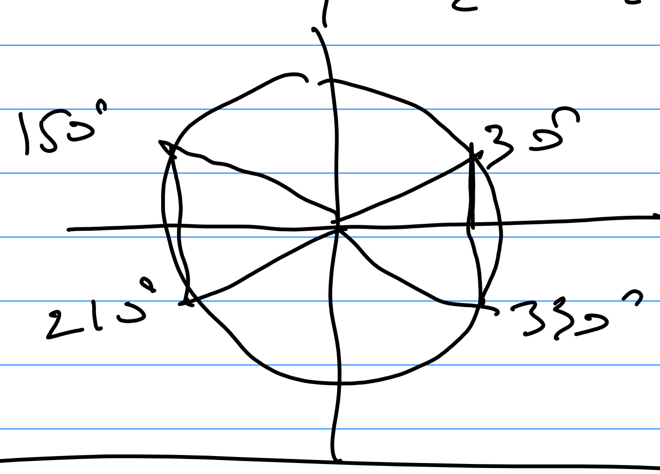


$\cos^2 x = \frac{3}{4}$

$4\cos^2 x - 3 = 0$ $\qquad$ $\cos x = t$

$4t^2 = 3$ $\qquad$ $t_1 = -\frac{\sqrt{3}}{2}$ $\quad$ $t_2 = \frac{\sqrt{3}}{2}$

$$\left[\begin{array}{l} \cos x = -\frac{\sqrt{3}}{2} \\ \cos x = \frac{\sqrt{3}}{2} \end{array}\right.$$

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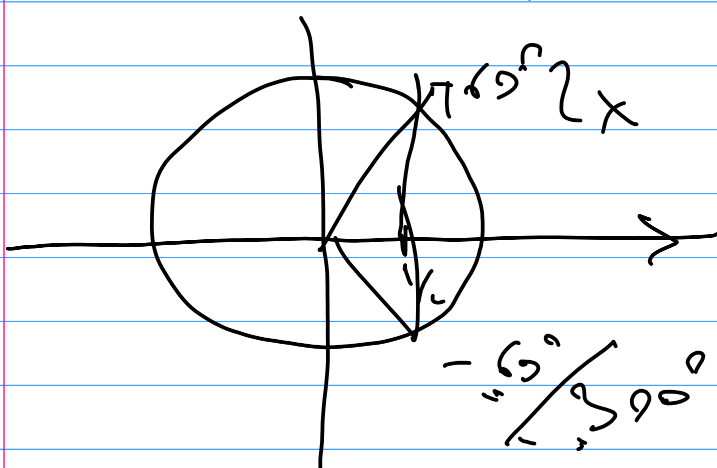
$2(2\cos^2 x) - 3 = 0$

$2(\cos 2x + 1) - 3 = 0$

$2\cos 2x + 2 - 3 = 0$

$2\cos 2x = 1$ $\qquad$ $\cos 2x = \frac{1}{2}$




$$\left[\begin{array}{l} 2x = 60° + 360°n \\ 2x = -60° + 360°n \end{array}\right.$$

$x = 30° + 180°n$

$x = -30° + 180°n$

$\{30°, 210°, 150°, 330°\}$

$\sin^2 x + \cos x = 1{,}25$

$1 - \cos^2 x + \cos x = 1{,}25$

$1 - t^2 + t = 1{,}25$

$t^2 - t + \frac{1}{4} = 0$ $\qquad t = \frac{1}{2}$

$\cos x = \frac{1}{2}$ $\qquad x = 60^\circ$

$x = 300^\circ$

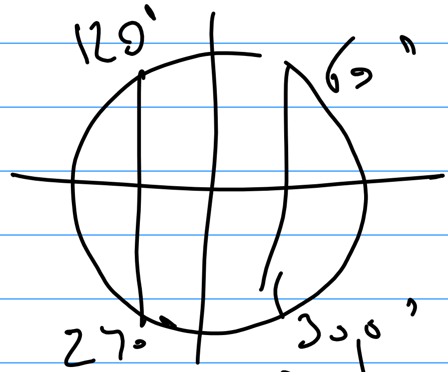
---

$\sin^2 x = 3\cos^2 x$ $\qquad :$

$1 - \cos^2 x = 3\cos^2 x$

$4\cos^2 x = 1$ $\qquad \cos^2 x = \frac{1}{4}$

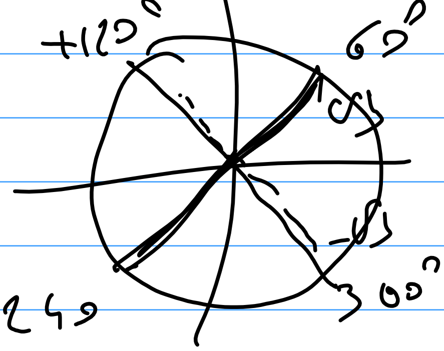
$\left[\begin{array}{l} \cos x = \frac{1}{2} \\ \cos x = -\frac{1}{2} \end{array}\right.$

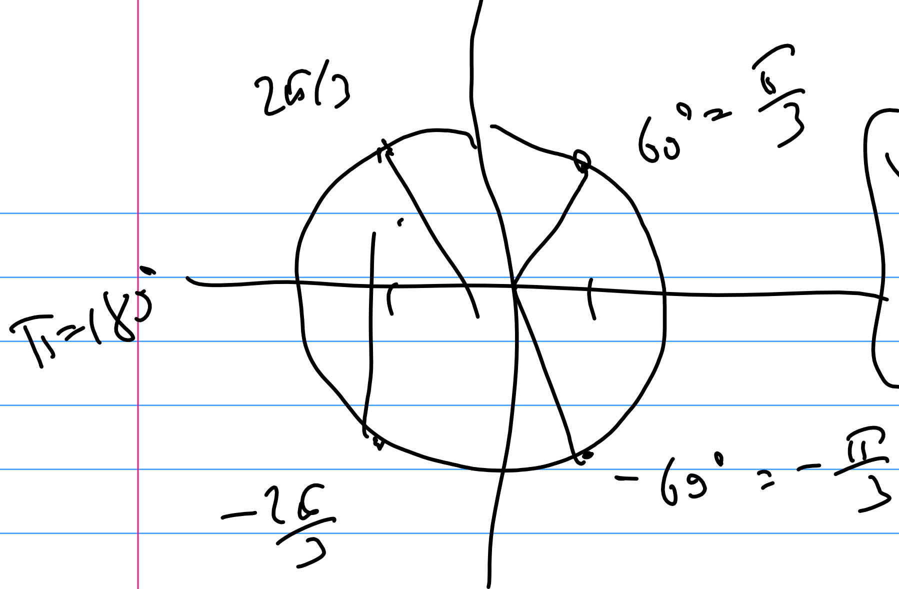



$\tan^2 x = 3$

$\tan x = \sqrt{3}$

$\tan x = -\sqrt{3}$






$$\left[\begin{array}{l} x = \pm\frac{\pi}{3} + 2\pi n \\ x = \pm\frac{2\pi}{3} + 2\pi n \end{array}\right.$$

$$\left[\ x = \pm\frac{\pi}{3} + \pi n\right.$$

$\sin^2 x = 3\cos^2 x$

$\cos 2x = \underbrace{2\cos^2 x}_{} - 1$

$4\cos^2 x = 1$

$\underbrace{4\cos^2 x}_{2\cdot\boxed{2\cos^2 x}} = 1$

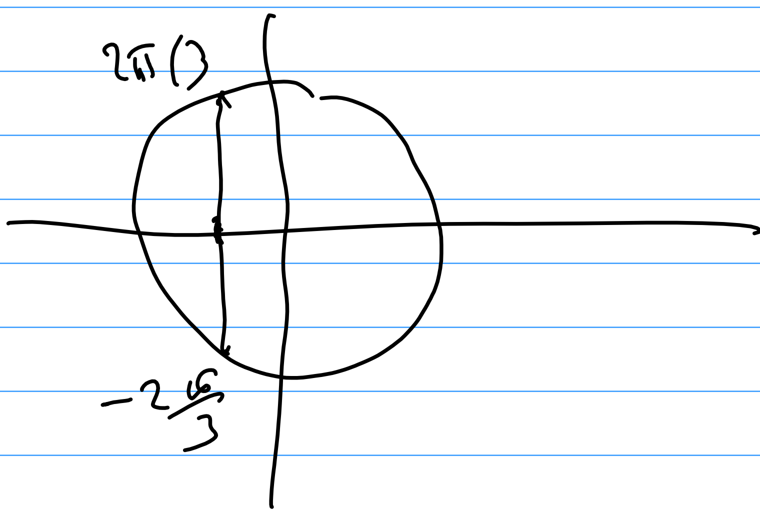
$\boxed{2\cos^2 x} = \cos 2x + 1$

$2(\cos 2x + 1) = 1$

$2\cos 2x + 2 = 1$

$2\cos 2x = -1$

$\cos 2x = -\frac{1}{2}$




$2x = \pm\frac{2\pi}{3} + 2\pi n$

$$\boxed{x = \pm\frac{\pi}{3} + \pi n}$$

$$\boxed{x+y} = \frac{2}{3}\pi$$

$$\sin x + \sin y = \frac{3}{2}$$

$$2\sin\frac{\boxed{x+y}}{2}\cos\frac{x-y}{2} = \frac{3}{2}$$

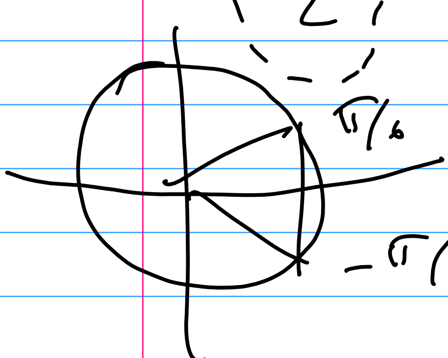
$$2\cdot\sin\frac{\frac{2}{3}\pi}{2}\cos\frac{x-y}{2} = \frac{3}{2}$$

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cancel{2}\cdot\frac{\sqrt{3}}{\cancel{2}}\cos\frac{x-y}{2} = \frac{3}{2}$$

$$\sqrt{3}\cos\frac{x-y}{2} = \frac{3}{2} \qquad :\sqrt{3}$$

$$\cos\frac{x-y}{2} = \frac{\sqrt{3}}{2} \qquad\qquad \frac{\frac{3}{2}}{\sqrt{3}} = \frac{\sqrt{3}}{2}$$




$$\begin{cases} \dfrac{x-y}{2} = \pm\dfrac{\pi}{6} + 2\pi n \\ x+y = \dfrac{2\pi}{3} \end{cases}$$

$$\begin{cases} x-y = \pm\frac{\pi}{3} + 4\pi n \\ x+y = \frac{2\pi}{3} \end{cases}$$

$$x = \frac{2\pi}{3} - y$$

$$\frac{2\pi}{3} - y - y = \pm\frac{\pi}{3} + 4\pi n$$

$$\frac{2\pi}{3} - 2y = \pm\frac{\pi}{3} + 4\pi n$$

$$\frac{2\pi}{3} - \left(\pm\frac{\pi}{3}\right) - 4\pi n = 2y \quad :$$

$$\frac{2\pi}{3} \mp \frac{\pi}{3}$$

$$y = \boxed{\frac{\pi}{3} \mp \frac{\pi}{6} - 2\pi n}$$

$$x = \frac{2\pi}{3} - y = \frac{2\pi}{3} - \frac{\pi}{3} \pm \frac{\pi}{6} + 2\pi n =$$

$$\frac{\pi}{3} + \frac{\pi}{6} = \frac{2\pi+\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$$

$$= \boxed{\frac{\pi}{3} \pm \frac{\pi}{6} + 2\pi n} = x$$

$$\begin{cases} x = \frac{\pi}{3} + \frac{\pi}{6} + 2\pi n = \frac{\pi}{2} + 2\pi n \\ y = \frac{\pi}{3} - \frac{\pi}{6} - 2\pi n = \frac{\pi}{6} - 2\pi n \end{cases}$$

$$x-y=\pm\frac{\pi}{3}+4\pi n$$
$$x+y=\frac{2\pi}{3}$$

$\begin{cases} \dots & + \end{cases}$ или $\begin{cases} & - \end{cases}$

$$\boxed{(x+y)-(x-y)=2y}$$

$$\begin{cases} x-y=\frac{\pi}{3}+4\pi n \\ x+y=\frac{2\pi}{3} \end{cases} \quad \text{или} \quad \pm\begin{cases} x-y=-\frac{\pi}{3}+4\pi n \\ x+y=\frac{2\pi}{3} \end{cases}$$

$$2x=\pi+4\pi n \qquad\qquad 2x=\frac{\pi}{3}+4\pi n$$
$$2y=\frac{\pi}{3}-4\pi n \qquad\qquad 2y=\pi-4\pi n$$

$$\begin{cases} x=\frac{\pi}{2}+2\pi n \\ y=\frac{\pi}{6}-2\pi n \end{cases} \qquad\qquad \begin{cases} x=\frac{\pi}{6}+2\pi n \\ y=\frac{\pi}{2}-2\pi n \end{cases}$$

$$\left(\frac{\pi}{2},\ \frac{\pi}{6}\right) \quad \text{или} \quad \left(\frac{\pi}{6},\ \frac{\pi}{2}\right)$$

$$\boxed{\sin^2\frac{x}{2}}=\frac{1}{2}(1-\cos x)$$

$$\cos 2\alpha=1-2\sin^2\alpha$$
$$2\sin^2\alpha=1-\cos 2\alpha$$

$$\boxed{\sin^2 \alpha} = \frac{1 - \cos 2\alpha}{2} = \boxed{\frac{1}{2}(1 - \cos 2\alpha)}$$

$$\boxed{\sin^2 \frac{x}{2} = \frac{1}{2}(1 - \cos x)}$$

$$\frac{1}{2}(1 - \cos x) = \frac{1}{2}(1 - \cos x)$$

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$$3\cos x + 4\sin x = 2 \quad : 5$$

$$\sqrt{9 + 16} = 5$$

$$\underbrace{\frac{3}{5}}_{= \cos\alpha}\cos x + \underbrace{\frac{4}{5}}_{= \sin\alpha}\sin x = \frac{2}{5}$$

$$\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = 1$$

$$\cos\alpha = \frac{3}{5}$$

$$\alpha = \arccos\frac{3}{5}$$

$$\cos x \cos\alpha + \sin x \sin\alpha = \frac{2}{5}$$

$$\cos(x - \alpha) = \frac{2}{5}$$

$$\cos\left(x - \arccos\frac{3}{5}\right) = \frac{2}{5}$$

$360^\circ n$

$$x - \arccos\frac{3}{5} = \pm\arccos\frac{2}{5} + 2\pi n$$

$$x = \arccos\frac{3}{5} \pm \arccos\frac{2}{5} + 360^\circ n$$