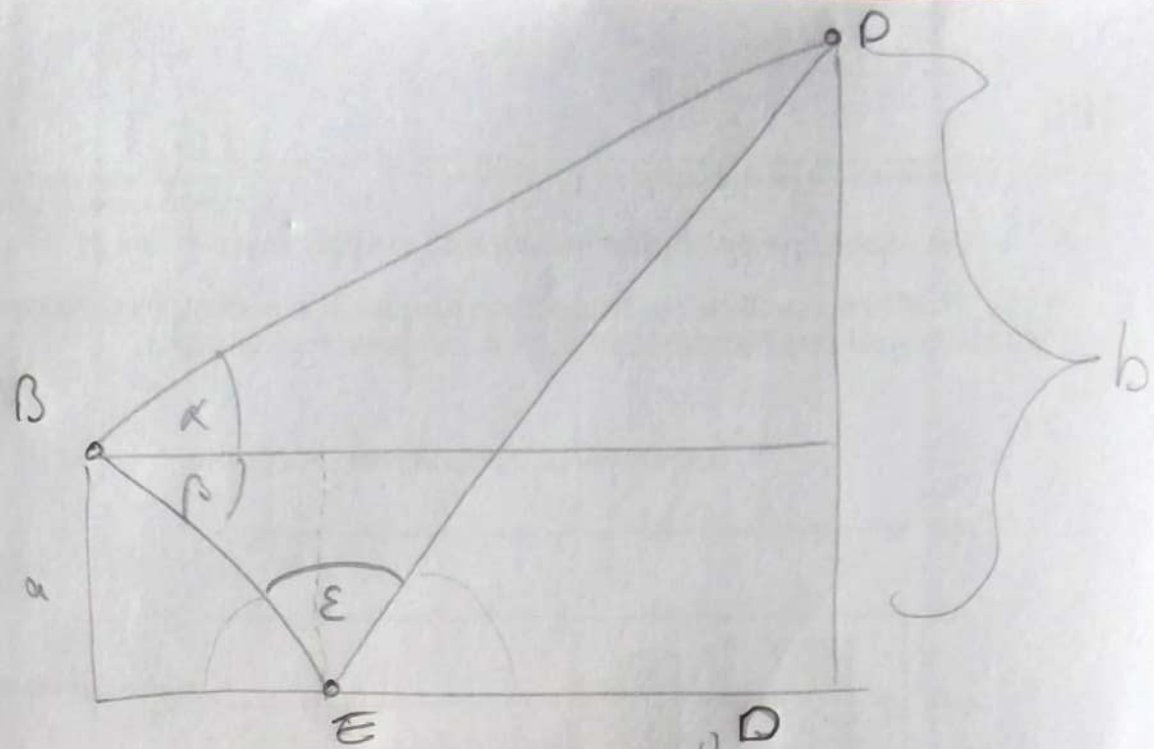
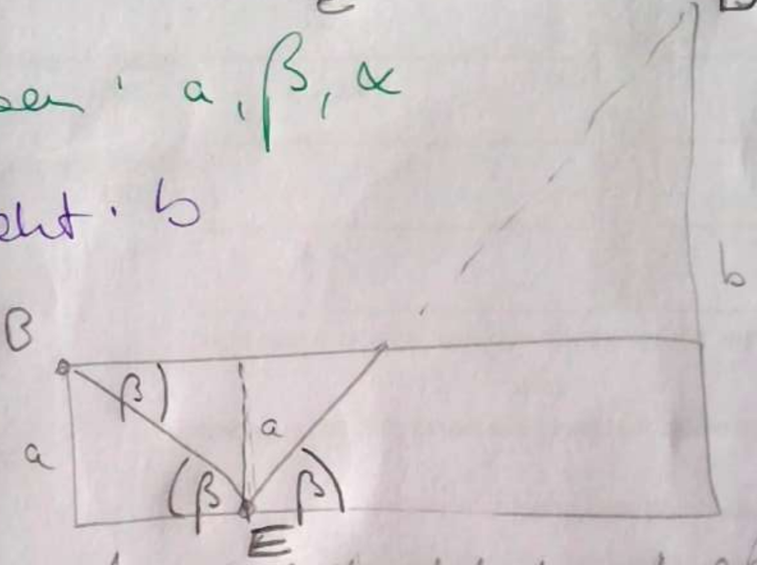


Gegeben: a, β, α

Gesucht: b



Reflexion ≙ Einfallswinkel = Ausfallswinkel

1) $\sin\beta = \frac{a}{\overline{BE}}$ oder auch $\sin\beta = \frac{b}{\overline{ED}}$

2) Sinussatz im ΔBED

$$\frac{\sin(\alpha+\beta)}{\overline{ED}} = \frac{\sin(\overbrace{180^\circ-2\beta}^{=\varepsilon})}{\overline{BD}} = \frac{\sin\left(\overbrace{180^\circ-(\alpha+\beta)-\varepsilon}^{=\beta-\alpha}\right)}{\overline{BE}}$$

⇒ Gleichungen jeweils nach $\overline{BE}$ & $\overline{ED}$ umstellen

1) $\overline{BE} = \frac{a}{\sin\beta}$; $\overline{ED} = \frac{b}{\sin\beta}$

$$\boxed{b = \overline{ED}\cdot\sin\beta}$$

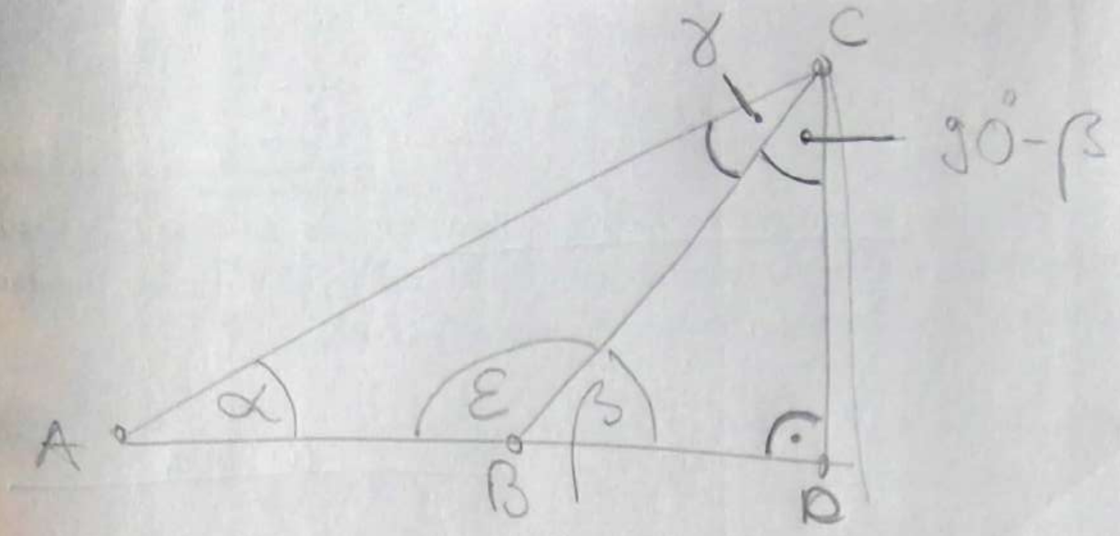
2) $$\overline{ED} = \sin(\alpha+\beta)\cdot\frac{\overline{BE}}{\sin(\beta-\alpha)}$$

$\overline{BE}$ einsetzen

$$\overline{ED} = \sin(\alpha+\beta)\cdot\frac{1}{\sin(\beta-\alpha)}\cdot\frac{a}{\sin\beta}$$

$$\Rightarrow b = \frac{\sin(\alpha+\beta)}{\sin(\beta-\alpha)}\cdot\frac{a}{\cancel{\sin\beta}}\cdot\cancel{\sin\beta}$$

$$= a\cdot\frac{\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$$



Gegeben: $\alpha, \beta, \overline{AB}$

Gesucht: $\overline{CD}$

1) $\sin\beta = \dfrac{\overline{CD}}{\overline{BC}}$; $\sin\alpha = \dfrac{\overline{CD}}{\overline{AC}}$

2) Sinussatz im ΔABC

$$\frac{\overline{BC}}{\sin\alpha} = \frac{\overline{AC}}{\sin(\varepsilon)} = \frac{\overline{AB}}{\sin(\gamma)}$$

$(\varepsilon = 180° - \beta)$ $\quad \gamma = 180° - \alpha - \varepsilon$

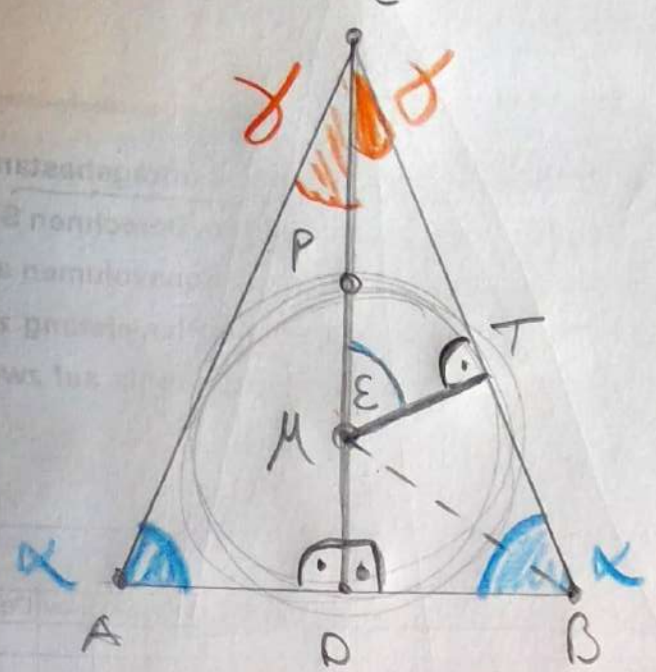
$= \beta - \alpha$

Entweder $\overline{BC}$ oder $\overline{AC}$ ausdrücken:

$\overline{BC} = \dfrac{\overline{AB}}{\sin\gamma} \cdot \sin\alpha \Rightarrow \overline{BC} = \overline{AB}$ $\quad (\gamma = 30° = \alpha)$

einsetzen in 1 $\Rightarrow$ $\overline{CD} = \sin\beta \cdot \overline{BC} = \sin\beta \cdot \overline{AB}$

$= \sin(60°) \cdot 80m = \dfrac{\sqrt{3}}{2} 80m$



Gegeben: $\underbrace{\overline{AD} = \overline{DB}}$, $\overline{MT} = r$, $y = \overline{CP}$

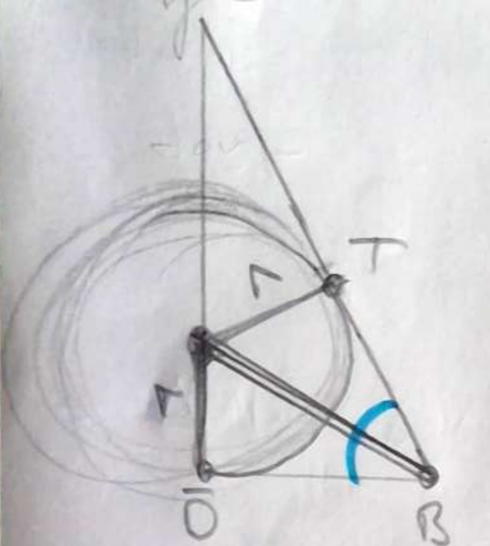
=> Gleichschänkliges Dreieck =>

Gesucht: $x = \overline{CT}$

$\varepsilon = \alpha$ weil $90° + \alpha + \gamma = 180°$ (ΔADC)

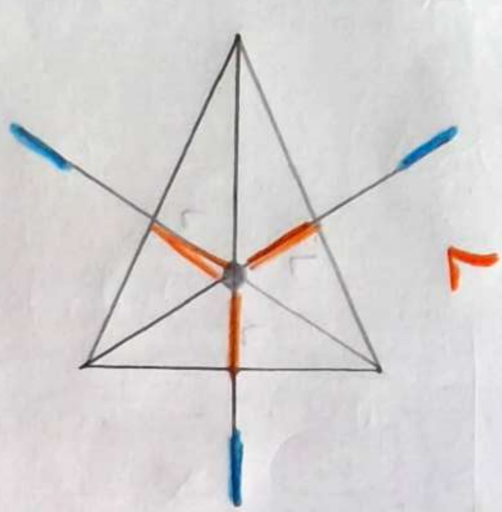
& $90° + \varepsilon + \gamma = 180°$ (ΔMTC)

Aufgabe



$\Rightarrow \overline{BD} = \overline{BT}$

Allgemein



Winkelhalbierenden

radius r

$\Delta \overline{ABC}$ ähnlich zu ΔMTC

$$\frac{\overline{AD}}{\overline{DC}} \overset{!}{=} \frac{\overline{MT}}{\overline{CT}} \Leftrightarrow \frac{3\,cm}{2r+y} = \frac{r}{\overline{CT}}$$

$$\frac{\overline{AD}}{\overline{AC}} \hat{=} \frac{\overline{MT}}{\overline{MC}} \Leftrightarrow \frac{3\,cm}{\overline{AC}} = \frac{r}{r+y}$$

$$\overline{AC} = \overline{BT} + \overline{TC}$$
$$= 3\,cm + \overline{CT}$$

Beide nach $\overline{CT}$ umstellen

$$\overline{CT} = \frac{r(2r+y)}{3cm}$$

$$\frac{3cm}{3cm+\overline{CT}} = \frac{r}{r+y} \Rightarrow$$

$$\frac{3cm(r+y)}{r} = 3cm + \overline{CT}$$

$$\overline{CT} = \frac{3cm(r+y)}{r} - 3cm$$

Die $\overline{CT}$ gleich stellen

$$\frac{r(2r+y)}{3cm} = \frac{3cm(r+y)}{r} - 3cm$$

Werte Einsetzen:

$$\frac{2\,\text{cm}\,(2\cdot 2\,\text{cm} + y)}{3\,\text{cm}} = \frac{3\,\text{cm}\cdot(2\,\text{cm} + y)}{2\,\text{cm}} - 3\,\text{cm}$$

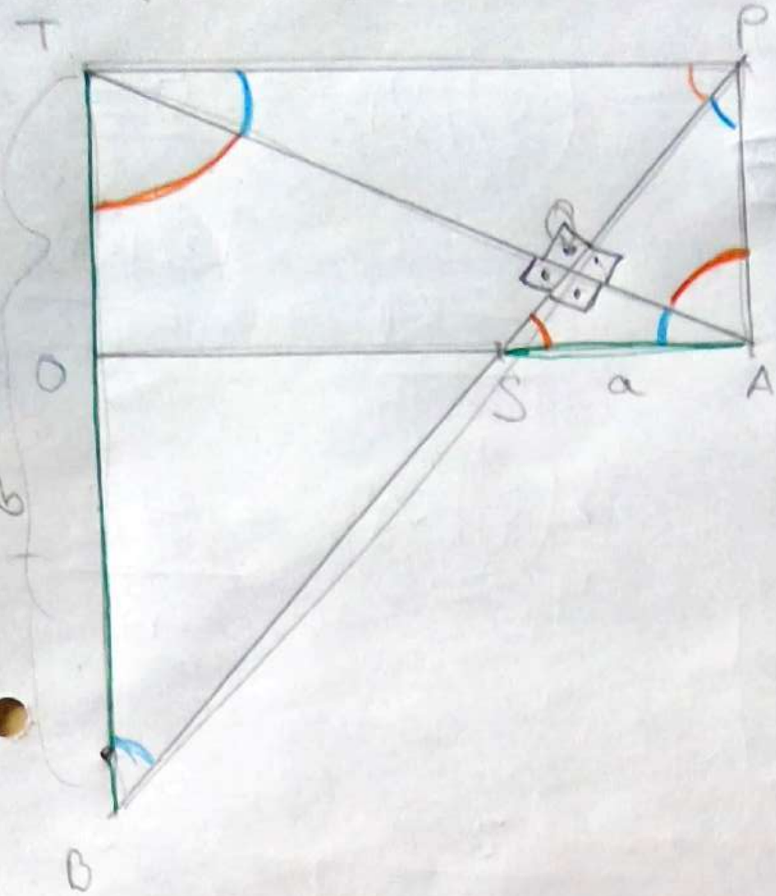
$$\frac{8\,\text{cm}^{\cancel{2}}}{3\,\cancel{\text{cm}}} + \frac{2y\,\cancel{\text{cm}}}{3\,\cancel{\text{cm}}} = \frac{6\,\text{cm}^{\cancel{2}}}{2\,\cancel{\text{cm}}} + \frac{3y\,\cancel{\text{cm}}}{2\,\cancel{\text{cm}}} - 3\,\text{cm}$$

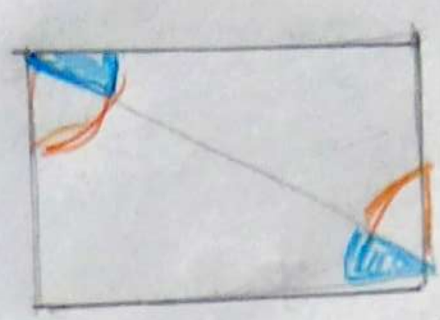
$$\left(\frac{8}{3} + 3 - 3\right)\text{cm} = \left(\frac{3}{2} - \frac{2}{3}\right)y$$

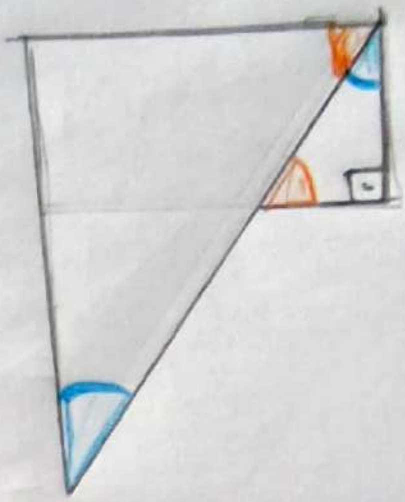
$$\left(\frac{8}{3}\right)\text{cm} = \left(\frac{9}{6} - \frac{4}{6}\right)y$$

$$y = \left(\frac{8}{3}\cdot\frac{6}{5}\right)\text{cm} \quad |y| = \frac{48}{15} = 3 + \frac{3}{15} = 3 + \frac{1}{5}$$

10.9 Skizze







Gegeben: OAPT ist □

$\overline{SA}$, $\overline{BT}$

$\overline{BP} \perp \overline{AT}$

Gesucht: $\overline{OA}$ & $\overline{OT}$

, Welche Dreiecke sind ähnlich?

△AQS zu △QPT zu

△BQT zu △APQ zu

△ASP zu △BQT

$$\frac{\overline{SA}}{\overline{AP}} = \frac{\overline{PT}}{\overline{BT}} \quad ; \quad \frac{\overline{OA}}{\overline{OT}} = \frac{\overline{PA}}{\overline{SA}} \quad ○$$

$$(\overline{OT} = \overline{PA})$$

$$\overline{PT} = \frac{\overline{SA} \cdot \overline{BT}}{\overline{AP}}$$

$$\overline{OA} = \frac{(\overline{PA})^2}{\overline{SA}}$$

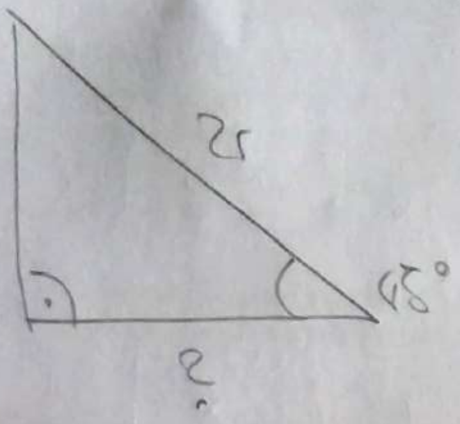
$$☆ \; (\overline{PT} = \overline{OA})$$

$$\frac{\overline{SA} \cdot \overline{BT}}{\overline{AP}} = \frac{(\overline{PA})^2}{\overline{SA}}$$

$$\overline{AP}^3 = (\overline{SA})^2 \cdot \overline{BT} = a^2 \cdot 4a$$

$$\overline{AP} = a \cdot \sqrt[3]{4}$$

$$\Rightarrow \overline{OA} = \cancel{\frac{\left((\overline{SA})^2 \cdot \overline{BT}\right)^{2/3}}{\overline{SA}}} = \cancel{\overline{SA}^{1/3} \cdot \overline{BT}^{2/3}} = \frac{a^2 (4)^{2/3}}{a}$$



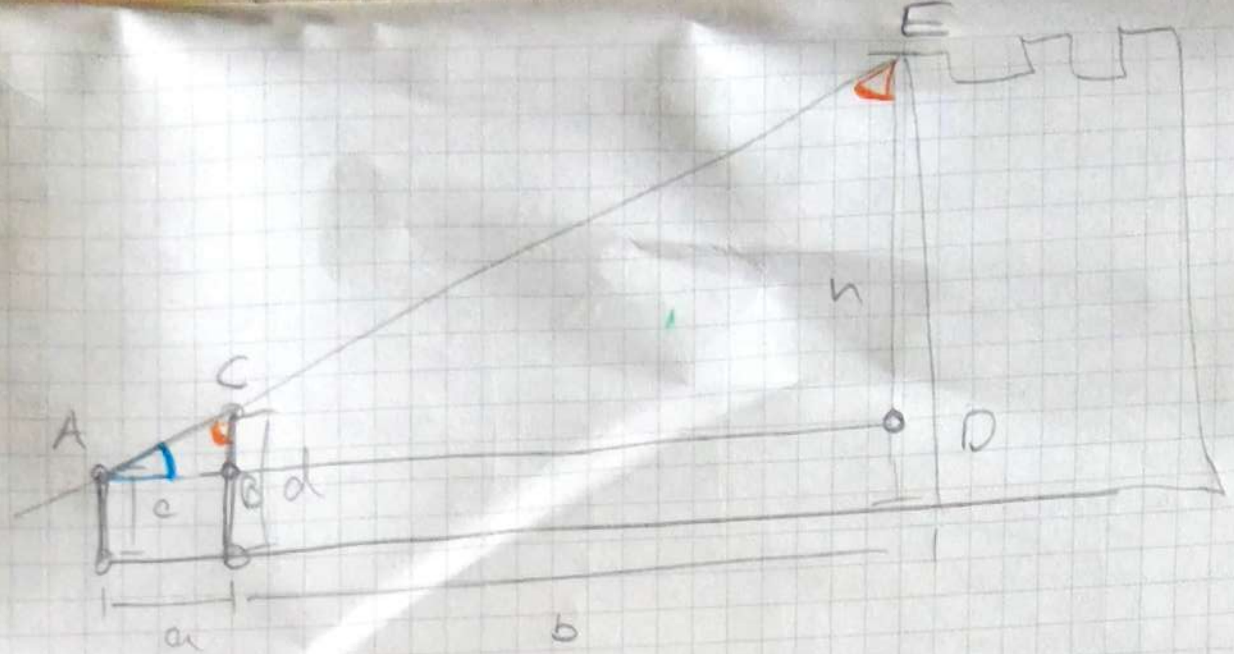
α	0°	30°	45°	60°	90°
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

$$\frac{An}{Hyp} = \cos 45°$$

$$An = \cos 45° \cdot Hyp = \frac{\sqrt{2}}{2} \cdot 2r$$

$$\sqrt{2}r - r = r_{klein}$$

$$= \triangle - \left(\frac{1}{4}\bigcirc_{r} + \frac{1}{4}\odot\right)$$



Gegeben: a, b, c, d

Gesucht: h Höhe Turm

Wo sind ähnliche Dreiecke? · 90° einzeichnen · Winkel einzeichnen

ΔABC ähnlich zu ΔADE

$$\frac{\overline{AB}}{\overline{BC}} = \frac{\overline{AD}}{\overline{DE}} \quad \Bigg| \quad \begin{aligned} \overline{AB} &= a \\ \overline{BC} &= |c-d| \\ \overline{AD} &= a+b \\ \overline{DE} &= h-c \end{aligned}$$

Umstellen:

$$\overline{DE} = \overline{AD} \cdot \frac{\overline{BC}}{\overline{AB}}$$

$$h - c = (a+b) \cdot \frac{|c-d|}{a}$$

$$h = (a+b) \cdot \frac{d-c}{a} + c = \left(108 \cdot \frac{0{,}5}{2} + 1{,}8\right) m$$

$$= \left(\frac{1}{4} \cdot 108 + 1{,}8\right) m = 28{,}8\,m$$