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Solution and estimation of RE macromodels with optimal policy

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Abstract

Macro models of monetary policy typically involve forward looking behavior. Except in rare circumstances, we have to apply some numerical method to find the optimal policy and the rational expectations equilibrium. This paper summarizes a few useful methods, and shows how they can be combined with a Kalman filter to estimate the deep model parameters with maximum likelihood. Simulations of a macro model with staggered price setting, interest rate elastic output, and optimal monetary policy illustrate the properties of this estimation approach. © 1999 Elsevier Science B.V. All rights reserved.

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1. Optimal policy in linear rational expectation models

1.1. A class of models

The economies discussed in this paper evolve according to

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + Bu_t + \begin{bmatrix} \varepsilon_{t+1} \\ \mathbf{0}_{n_2 \times 1} \end{bmatrix}, \tag{1.1}$$

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where x_{1t} is an $n_1 \times 1$ vector of predetermined variables (backward looking) with x_{10} given, x_{2t} an $n_2 \times 1$ vector of nonpredetermined (forward looking) variables, u_t a $k \times 1$ vector of policy instruments, and ε_{t+1} an $n_1 \times 1$ vector of innovations to x_{1t} with covariance matrix Σ. Some of the elements in x_{1t} can be exogenous variables like a serially correlated productivity level. For notational convenience, define the vector $x_t = (x_{1t}, x_{2t})$, which is $n \times 1$, where $n = n_1 + n_2$. The matrices A and B, and the properties of the shocks ε_{t+1} are assumed to be constant functions of some structural (deep) model parameters.

The policy maker has the loss function

$$J_0 = E_0 \sum_{t=0}^{\infty} \beta^t (x_t' Q x_t + 2x_t' U u_t + u_t' R u_t). \tag{1.2}$$

The matrices Q and R are (without loss of generality) symmetric. They, as well as the matrix U and the discount factor β are also constant functions of structural model parameters.

1.2. Optimal policy under commitment

The policy maker is here assumed to be able to commit to a constant policy rule. Form the Lagrangian from Eqs. (1.2) and (1.1):

$$\begin{aligned} L_0 = E_0 \sum_{t=0}^{\infty} \beta^t [& x_t' Q x_t + 2x_t' U u_t + u_t' R u_t \\ & + 2\rho_{t+1}' (A x_t + B u_t + \xi_{t+1} - x_{t+1})], \end{aligned} \tag{1.3}$$

where $\xi_{t+1} = (\varepsilon_{t+1}, x_{2t+1} - E x_{2t+1})$, and x_{10} is given. The first-order conditions with respect to ρ_{t+1}, x_t, and u_t are

$$\begin{aligned} & \begin{bmatrix} I_n & \mathbf{0}_{n \times k} & \mathbf{0}_{n \times n} \\ \mathbf{0}_{n \times n} & \mathbf{0}_{n \times k} & \beta A' \\ \mathbf{0}_{k \times n} & \mathbf{0}_{k \times k} & -B' \end{bmatrix} \begin{bmatrix} x_{t+1} \\ u_{t+1} \\ E_t \rho_{t+1} \end{bmatrix} \\ & = \begin{bmatrix} A & B & \mathbf{0}_{n \times n} \\ -\beta Q & -\beta U & I_n \\ U' & R & \mathbf{0}_{k \times n} \end{bmatrix} \begin{bmatrix} x_t \\ u_t \\ \rho_t \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ \mathbf{0}_{n \times 1} \\ \mathbf{0}_{k \times 1} \end{bmatrix}. \end{aligned} \tag{1.4}$$

Take expectations of Eq. (1.4) conditional on the information set in t. Then, expand x_t and ρ_t as (x_{1t}, x_{2t}) and (ρ_{1t}, ρ_{2t}), respectively, and reorder the rows by placing ρ_{2t} after x_{1t}. Write the result as

$$GE_t \begin{bmatrix} k_{t+1} \\ \lambda_{t+1} \end{bmatrix} = D \begin{bmatrix} k_t \\ \lambda_t \end{bmatrix} \quad \text{where } k_t = \begin{bmatrix} x_{1t} \\ \rho_{2t} \end{bmatrix} \quad \text{and } \lambda_t = \begin{bmatrix} x_{2t} \\ u_t \\ \rho_{1t} \end{bmatrix}. \tag{1.5}$$

The reason for this ordering is that we want to exploit the initial conditions for the vector k_t. First, the initial state vector, x_{10}, is given. Second, forward looking variables can be chosen freely in the initial period so their shadow prices, ρ_{20}, are zero (see Currie and Levine, 1993).

The matrix G in Eq. (1.5) is singular. Sims (1995) and Klein (1997) show how the *generalized Schur decomposition* can be used in this situation. In contrast to the algorithms in Levine and Currie (1987) and Backus and Driffill (1986), this approach allows a singular R matrix and available software libraries give very quick solutions. The latter is particularly important in estimations, where the equilibrium has to be calculated for every iteration on the vector of structural parameters.

The following is an application of Klein's method: Given the square matrices G and D, the decomposition gives the square complex matrices Q, S, T, and Z such that

$$G = QSZ^{\mathrm{H}} \quad \text{and} \quad D = QTZ^{\mathrm{H}}, \tag{1.6}$$

where Z^{H} denotes the transpose of the complex conjugate of Z.[1] Q and Z are unitary ($Q^{\mathrm{H}}Q = Z^{\mathrm{H}}Z = I$), and S and T are upper triangular (see Golub and van Loan, 1989). The decomposition can be reordered so the block corresponding to the stable generalized eigenvalues (the ith diagonal element of T divided by the corresponding element in S) comes first.[2] In most cases, stability requires a modulus of the generalized eigenvalue less than one. With unit root exogenous variables, the cutoff point could often be chosen slightly above one, provided the unstable eigenvalues of the endogenous part are sufficiently above unity.

Define the auxiliary variables

$$\begin{bmatrix} \theta_t \\ \delta_t \end{bmatrix} = Z^{\mathrm{H}} \begin{bmatrix} k_t \\ \lambda_t \end{bmatrix}. \tag{1.7}$$

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[1] Not the same Q as in (1.2); "recycling" notation here.

[2] This decomposition is used to solve the generalized eigenvalue problem $Dx = \lambda Gx$. The results in the text hold also for the real generalized Schur decomposition where Q and Z are real orthogonal matrices, S is real and upper triangular, and T is real and upper quasi-triangular. Lapack 3.0 will contain Fortran code for calculating and reordering – a Gauss implementation is found at http://www.hhs.se/personal/psoderlind/. Matlab code is found at http://www.iies.su.se/data/home/kleinp/.

Premultiply Eq. (1.5) with the non-singular matrix Q^{H}, use Eq. (1.7), and partition S and T conformably with θ_t and δ_t

$$\begin{bmatrix} S_{\theta\theta} & S_{\theta\delta} \\ \mathbf{0} & S_{\delta\delta} \end{bmatrix} E_t \begin{bmatrix} \theta_{t+1} \\ \delta_{t+1} \end{bmatrix} = \begin{bmatrix} T_{\theta\theta} & T_{\theta\delta} \\ \mathbf{0} & T_{\delta\delta} \end{bmatrix} \begin{bmatrix} \theta_t \\ \delta_t \end{bmatrix}. \tag{1.8}$$

Since the lower right block contains the unstable roots, it must be the case that $\delta_t = \mathbf{0}$ for all t in order to get a stable solution. The remaining equations are then $S_{\theta\theta} E_t \theta_{t+1} = T_{\theta\theta}\theta_t$, which we solve as

$$E_t\theta_{t+1} = S_{\theta\theta}^{-1} T_{\theta\theta}\theta_t, \tag{1.9}$$

since $S_{\theta\theta}$ is invertible (the determinant of the triangular $S_{\theta\theta}$ equals the product of the diagonal elements, which are all non-zero as a consequence of how the matrices were reordered).

Premultiply Eq. (1.7) with Z and partition conformably with k_t, λ_t, θ_t and δ_t,

$$\begin{bmatrix} k_t \\ \lambda_t \end{bmatrix} = \begin{bmatrix} Z_{k\theta} & Z_{k\delta} \\ Z_{\lambda\theta} & Z_{\lambda\delta} \end{bmatrix} \begin{bmatrix} \theta_t \\ \delta_t \end{bmatrix} = \begin{bmatrix} Z_{k\theta} \\ Z_{\lambda\rho} \end{bmatrix} \theta_t, \tag{1.10}$$

since $\delta_t = \mathbf{0}$. We know that $k_0 = (x_{10}, \mathbf{0}_{n_2\times 1})$, so we can solve for θ_0 from Eq. (1.10) if $Z_{k\theta}$ is invertible. A necessary condition is that Eq. (1.5) has the saddle path property in Proposition 1 of Blanchard and Kahn (1980), that is, that the number of predetermined variables (number of rows in $Z_{k\theta}$) equals the number of stable roots (number of columns in $Z_{k\theta}$). Suppose$Z_{k\theta}^{-1}$ exists; then

$$\theta_0 = Z_{k\theta}^{-1} k_0. \tag{1.11}$$

From Eq. (1.1) we have $x_{1t+1} - E_t x_{1t+1} = \varepsilon_{t+1}$, and Backus and Driffill (1986) show that $\rho_{2t+1} - E_t\rho_{2t+1} = \mathbf{0}_{n_2\times 1}$. Stack these expressions as $k_{t+1} - E_t k_{t+1}$ (recall the definition of k_t in Eq. (1.5)), and use $k_t = Z_{k\theta}\theta_t$ from Eq. (1.10) to express it as

$$Z_{k\theta}(\theta_{t+1} - E_t\theta_{t+1}) = \begin{bmatrix} \varepsilon_{t+1} \\ \mathbf{0}_{n_2\times 1} \end{bmatrix}. \tag{1.12}$$

Invert this expression and substitute for $E_t\theta_{t+1}$ from Eq. (1.9)

$$\theta_{t+1} = S_{\theta\theta}^{-1} T_{\theta\theta}\theta_t + Z_{k\theta}^{-1} \begin{bmatrix} \varepsilon_{t+1} \\ \mathbf{0}_{n_2\times 1} \end{bmatrix}, \tag{1.13}$$

which together with Eq. (1.11) summarizes the dynamics of the model. This can be expressed in terms of the original variables by recalling Eqs. (1.5) and (1.10)

$$\begin{bmatrix} x_{1t+1} \\ \rho_{2t+1} \end{bmatrix} = Z_{k\theta} S_{\theta\theta}^{-1} T_{\theta\theta} Z_{k\theta}^{-1} \begin{bmatrix} x_{1t} \\ \rho_{2t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ \mathbf{0}_{n_2\times 1} \end{bmatrix}, \tag{1.14}$$

with x_{10} given and $\rho_{20} = \mathbf{0}_{n_2 \times 1}$. The same equations also give

$$\begin{bmatrix} x_{2t} \\ u_t \\ \rho_{1t} \end{bmatrix} = Z_{\lambda\theta} Z_{k\theta}^{-1} \begin{bmatrix} x_{1t} \\ \rho_{2t} \end{bmatrix}. \tag{1.15}$$

1.3. An optimal simple rule

The policy maker could alternatively *commit* to a *simple decision rule* of the form $u_t = -Fx_t$, as discussed in Levine and Currie (1987). There may be restrictions on the elements in F, and the vector x_t could be augmented compared to what is necessary to solve for the unrestricted optimal policy above.

Substituting for $u_t = -Fx_t$ in Eq. (1.1) gives

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = (A - BF) \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ \mathbf{0}_{n_2 \times 1} \end{bmatrix}. \tag{1.16}$$

Taking expectations conditional on the information set in t, and letting $k_t = x_{1t}$ and $\lambda_t = x_{2t}$ give the same type of problem as in Eq. (1.5), but with $G = I$ and $D = A - BF$. This means that we can set S and Q in Eq. (1.6) equal to I and Z, respectively. This is the original, not generalized, *Schur decomposition.*[3] In contrast to the Jordan decomposition used by Blanchard and Kahn (1980), it is relatively easy to calculate with good numerical precision (see Golub and van Loan, 1989) and is frequently available in software libraries.

Provided the F matrix implies a unique equilibrium ($Z_{k\theta}$ is invertible) the solution is given by the analogies to Eqs. (1.14) and (1.15),

$$x_{1t+1} = Mx_{1t} + \varepsilon_{t+1} \qquad \text{where } M = Z_{k\theta} T_{\theta\theta} Z_{k\theta}^{-1} \tag{1.17}$$

and

$$x_{2t} = Cx_{1t} \quad \text{where } C = Z_{\lambda\theta} Z_{k\theta}^{-1} \tag{1.18}$$

It is straightforward to show that the loss function value, for a given F matrix, is

$$J_0 = x_{10}' V x_{10} + \frac{\beta}{1-\beta} \operatorname{tr}(V\Sigma), \tag{1.19}$$

[3] This decomposition is used to solve the eigenvalue problem $Dx = \lambda x$. The results in the text hold also for the real Schur decomposition, where Z is a real orthogonal matrix, and T a real upper quasi-triangular matrix. If $A - BF$ has linearly independent eigenvectors, then a spectral decomposition can be used.

where the matrix V is the fixed point in the iteration (backwards in time) on

$$V_s = P' \begin{bmatrix} Q & U \\ U' & R \end{bmatrix} P + \beta M' V_{s+1} M$$

where

$$P = \begin{bmatrix} I_{n_1} \\ C \\ -F \begin{bmatrix} I_{n_1} \\ C \end{bmatrix} \end{bmatrix}. \tag{1.20}$$

The *optimal simple rule* minimizes the loss function Eq. (1.19) by choosing the optimal elements in the decision rule F, subject to the restrictions that the decision rule should give a unique equilibrium and that x_{10} is given. This rule must typically be found by a non-linear optimization algorithm. It is not certainty equivalent – it depends on the covariance matrix, Σ, and the initial state vector, x_{10} (see Currie and Levine, 1993).

1.4. Optimal policy under discretion

In the discretionary case, the policy maker reoptimizes every period by taking the process by which private agents form their expectations as given – and where the expectations are consistent with actual policy. Since the model is linear-quadratic, the solution in $t+1$ gives a value function which is quadratic in the state variables, $x'_{1t+1} V_{t+1} x_{1t+1} + v_{t+1}$, and a linear relation between the forward looking variables and the state variables, $x_{2t+1} = C_{t+1} x_{1t+1}$. Private agents form expectations about x_{2t+1} accordingly. The value function of the policy maker in t will then satisfy the Bellman equation

$$\begin{aligned} x'_{1t} V_t x_{1t} + v_t = \min_{u_t} \, & [x'_t Q x_t + 2x'_t U u_t + u'_t R u_t \\ & + \beta E_t (x'_{1t+1} V_{t+1} x_{1t+1} + v_{t+1})] \end{aligned}$$

$$\text{s.t.} \quad E_t x_{2t+1} = C_{t+1} E_t x_{1t+1}, \quad \text{Eq. (1.1)}, \quad \text{and } x_{1t} \text{ given.} \tag{1.21}$$

Combine the two restrictions to rewrite the problem. Partition the matrices A and B in Eq. (1.1), and Q and U in Eq. (1.21) conformably with x_{1t} and x_{2t}. Rewrite Eq. (1.21) as

$$\begin{aligned} x'_{1t} V_t x_{1t} + v_t = \min_{u_t} \, & [x'_{1t} Q^*_t x_{1t} + 2x'_{1t} U^*_t u_t + u'_t R^*_t u_t \\ & + \beta \mathrm{E}_t (x'_{1t+1} V_{t+1} x_{1t+1} + v_{t+1})] \end{aligned}$$

$$\text{s.t.} \quad x_{1t+1} = A^*_t x_{1t} + B^*_t u_t + \varepsilon_{t+1} \quad \text{and } x_{1t} \text{ given,} \tag{1.22}$$

where the starred matrices are defined as

$$D_t = (A_{22} - C_{t+1}A_{12})^{-1}(C_{t+1}A_{11} - A_{21}),$$

$$G_t = (A_{22} - C_{t+1}A_{12})^{-1}(C_{t+1}B_1 - B_2),$$

$$A_t^* = A_{11} + A_{12}D_t,$$

$$B_t^* = B_1 + A_{12}G_t,$$

$$Q_t^* = Q_{11} + Q_{12}D_t + D_t'Q_{21} + D_t'Q_{22}D_t,$$

$$U_t^* = Q_{12}G_t + D_t'Q_{22}G_t + U_1 + D_t'U_2,$$

$$R_t^* = R + G_t'Q_{22}G_t + G_t'U_2 + U_2'G_t. \tag{1.23}$$

(The D_t and G_t matrices are used to substitute for x_{2t} according to $x_{2t} = D_t x_{1t} + G_t u_t$.)

The first-order condition of Eq. (1.22) with respect to u_t are

$$u_t = -F_{1t}x_{1t}, \quad \mathrm{F}_{1t} = (R_t^* + \beta B_t^{*\prime}V_{t+1}B_t^*)^{-1}(U_t^{*\prime} + \beta B_t^{*\prime}V_{t+1}A_t^*). \tag{1.24}$$

Combining with Eq. (1.22) gives

$$x_{2t} = C_t x_{1t}, \quad \text{with } C_t = D_t - G_t F_{1t}, \tag{1.25}$$

and

$$V_t = Q_t^* - U_t^* F_{1t} - F_{1t}' U_t^{*\prime} + F_{1t}' R_t^* F_{1t} + \beta(A_t^* - B_t^* F_{1t})' V_{t+1}(A_t^* - B_t^* F_{1t}). \tag{1.26}$$

Iterating until convergence ('backwards in time') on Eqs. (1.23)–(1.26) is the algorithm used by Oudiz and Sachs (1985) as well as Backus and Driffill (1986). It should be started with a symmetric positive-definite V_{t+1} and some C_{t+1} (a matrix with zeros seems to work in most cases).

In many cases F_{1t} and C_t converge to constants F_1 and C, even if the iterations may be time consuming.[4] The first n_1 equations in Eq. (1.1) can then be written

$$x_{1t+1} = (A_{11} + A_{12}C - B_1F_1)x_{1t} + \varepsilon_{t+1}, \tag{1.27}$$

and the other variables are calculated as $x_{2t} = Cx_{1t}$ and $u_t = -F_1x_{1t}$.

[4] The general properties of this algorithm are unknown. Practical experience suggests that it is often harder to find the discretionary equilibrium than the commitment equilibrium. It is unclear if this is due to the algorithm.

2. Maximum likelihood estimation of the model

In the models discussed in the previous section, the structural parameters are mapped into the evolution of the economy Eq. (1.1) and loss function Eq. (1.2). These give equilibrium laws of motion of the relevant state variables: VAR(1) systems for x_{1t} in the discretionary case, Eq. (1.27), and in the simple rule, Eq. (1.17), and a VAR(1) for x_{1t} and ρ_{2t} in the commitment case, Eq. (1.14). We write this as

$$\alpha_t = \mu + \Phi\alpha_{t-1} + \eta_t. \tag{2.1}$$

We typically have data on a vector y_t, which is a non-invertible linear combination of the state variables

$$y_t = \lambda + \Gamma\alpha_t + \varepsilon_t, \tag{2.2}$$

where ε_t are some measurement errors. In many cases, Γ is known and the measurement errors are zero; in other cases, they need to be estimated. In practice, the measurement errors are often used to translate the model definitions to something corresponding to data and to avoid stochastic singularity (fewer shocks than data series), which makes maximum likelihood estimation very tricky (singular covariance matrix).

Eqs. (2.1) and (2.2) are the starting point for a time-invariant Kalman filter. With normally distributed shocks, it is ideal for building up the likelihood function for sample of y_t recursively.[5]

This likelihood function can be maximized (with a non-linear optimization algorithm) by choosing the structural parameters, which give the matrices Φ and Γ as well as the covariance matrices of the shocks. In the estimation, the filter needs to be initialized with an estimate of the state vector and a covariance matrix of the associated estimation errors. For a model with stationary variables, the natural starting point is the uninformed forecast, that is, the unconditional mean and the unconditional covariance matrix. For a given transition matrix Φ and a covariance matrix of η_t, these are $(I-\Phi)^{-1}\mu$, and the matrix which solves $\Omega = \Phi\Omega\Phi' + \mathrm{Cov}(\eta_t)$ (iterate by starting from $\Omega = \mathbf{0}$), respectively.

3. An example

To illustrate the properties of this estimation approach, I do a Monte Carlo experiment with the model estimated in Söderlind (1997). This is a simplified version of the model in Fuhrer and Moore (1995). The IS curve for detrended log

[5] See, for instance, Harvey (1989), Lütkepohl (1993), or Hamilton (1994).

output, y_t, is

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_r r_{t-1} + \varepsilon_{yt},$$

where ε_{yt} is an output shock. The long ex ante real interest rate, r_t, obeys an approximate and risk neutral arbitrage condition for a ten year real coupon bond

$$r_t = \frac{1}{41}\sum_{s=0}^{\infty}\left(\frac{40}{41}\right)^s E_t\,(i_{t+s} - \pi_{t+1+s}),$$

where i_t is the annualized one quarter nominal interest rate (the policy instrument), and π_t the annualized one quarter inflation rate, $\pi_t = 4(p_t - p_{t-1})$. Wage contracts negotiated in t specify a flat nominal wage, w_t, for three quarters, t to $t+2$. A fraction θ_1/θ_0 of these contracts 'survive' until in $t+1$, and a fraction $(1-\theta_0-\theta_1)/\theta_0$ until $t+2$. The log price level is the average of the wage contracts still in effect

$$p_t = \theta_0 w_t + \theta_1 w_{t-1} + (1-\theta_0-\theta_1) w_{t-2}.$$

Nominal wage contracts are set so the current real contract wage equals the average real contract price index, v_t, expected to hold over the contract period, adjusted for demand pressure and wage shocks, ε_{pt},

$$\begin{aligned} w_t = p_t &+ \theta_0(v_t + \gamma y_t) + \theta_1 E_t(v_{t+1} + \gamma y_{t+1}) \\ &+ (1-\theta_0-\theta_1)E_t(v_{t+2} + \gamma y_{t+2}) + \varepsilon_{pt}, \\ v_t = \theta_0&(w_t - p_t) + \theta_1(w_{t-1} - p_{t-1}) + (1-\theta_0-\theta_1)(w_{t-2} - p_{t-2}). \end{aligned}$$

I assume that the Fed sets short interest rates to minimize the loss function

$$L_t = E_t \sum_{t=0}^{\infty} \beta^t [q_y y_t^2 + (1-q_y)\pi_t^2 + q_i i_t^2],$$

and that the Fed can commit to a policy rule. It is then straightforward to rewrite the model in the form (1.1) and (1.2), with $x_{1t} = (\varepsilon_{pt}, y_t, y_{t-1}, \Delta w_{t-1}, \Delta w_{t-2}, \Delta w_{t-3})$, $x_{2t} = (r_t, \Delta w_t, E_t\,\Delta w_{t+1})$, and $u_t = i_t$.

Söderlind (1997) estimates the model on a sample of quarterly US data from the mid-1960s to the mid-1990s (123 observations): y_t is taken to be log real GNP per capita detrended with a linear trend, π_t the quarterly changes in the consumer price index for urban workers, and i_t is assumed to differ from the three month T-bill rate ($i_{1,t}$) by an error term only, $i_{1,t} = i_t + \varepsilon_{it}$.

The Monte Carlo experiment generates 3000 samples of 'data' by drawing the initial state vector from its unconditional distribution, and then feeding the dynamics with random numbers for the three shocks. The model is then estimated on each of these samples – using either 123 observations, or a smaller sample of 61 observations (which may be more relevant for many data sets).

The results from the 3000 Monte Carlo simulations are summarized in Table 1. The means of the estimates are generally close to the true values, even if the short sample exaggerates the effectiveness of monetary policy by making output too responsive to real interest rates (α_r too low), and wage setting too responsive to output (γ too high). At the same time, it underestimates the willingness to use monetary policy (q_i too high). These parameters, and also the weight on output in the loss function, q_y, have considerably higher standard deviations than predicted by asymptotic theory. This is particularly clear for the two weights in the loss function (q_y and q_i) in the short sample: the standard deviations are more than three times larger than predicted by asymptotic theory – and twice as large as expected from the results for the longer sample (the factor $\sqrt{2}$ should, in principle, adjust for the sample length).

The correlations of the parameters estimates (not shown) also show some interesting features, all which of work in the direction of 'stabilizing' the properties of the model. First, α_1 and α_2 are strongly negatively correlated (-0.85), which keeps the autocorrelation of output relatively stable across simulations. Second, a_r and q_i are negatively correlated ($-$ 0.65), which keeps the effect of monetary policy on output and inflation relatively constant. Third and finally, the average contract length ($\theta_0 + 2\theta_1 + 3(1 - \theta_0 - \theta_1)$) and γ are

Table 1
Results from maximum likelihood estimation, 3000 repetitions

	Point estimates			Standard deviations		
	Asymptotic	Simulations		Asymptotic	Simulations	
	$T = 123$	$T = 123$	$T = 61$	$T = 123$	$T = 123$	$T = 61$
α_1	1.39	1.38	1.38	0.07	0.07	$0.08\sqrt{2}$
α_2	$-$ 0.50	$-$ 0.50	$-$ 0.49	0.06	0.07	$0.07\sqrt{2}$
α_r	$-$ 0.55	$-$ 0.58	$-$ 0.63	0.13	0.17	$0.22\sqrt{2}$
Std(ε_{yt})	0.84	0.84	0.84	0.05	0.06	$0.06\sqrt{2}$
θ_0	0.62	0.66	0.65	0.10	0.12	$0.10\sqrt{2}$
θ_1	0.29	0.26	0.26	0.05	0.07	$0.06\sqrt{2}$
$\gamma \times 100$	0.19	0.21	0.26	0.12	0.16	$0.19\sqrt{2}$
Std(ε_{pt})	0.19	0.18	0.18	0.09	0.09	$0.08\sqrt{2}$
q_y	0.82	0.83	0.84	0.03	0.05	$0.12\sqrt{2}$
q_i	0.35	0.39	0.47	0.12	0.17	$0.34\sqrt{2}$
Std(ε_{it})	1.41	1.39	1.37	0.09	0.09	$0.09\sqrt{2}$

The parameter values used for simulating the model (the asymptotic point estimates) are taken from Söderlind (1997). The asymptotic standard deviations are calculated from the theoretical information matrix implied by these parameter values. The table also reports the means and the standard deviations of the 3000 estimates on samples with 123 and 61 observations.

strongly positively correlated (0.83), which keeps the effect of a price shock on inflation and output relatively unchanged across simulations.

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