Bayesian Methods for DSGE models
Lecture 5
News, Noise and Structural Economic Models

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News and Noise

The Plan:

- News, Noise and Business Cycles
  - A theoretical framework
  - Empirical challenges
  - Quantifying the effects of noise shocks
  - Using survey data in state space models
- Concluding remarks for the course

Sentiments, Animal Spirits and Undue Optimism/Pessimism

Shocks in macro economics models

"We have not found large, identifiable, exogenous shocks to account for the bulk of of output fluctuations...

...It would be nice to point to recognizable events, of the type that is reported by newspapers, as the source of economic fluctuations, rather than to residuals from some equations."

John Cochrane, 1995

What drives aggregate fluctuations?

"Business cycles are all alike", e.g. Blanchard (1993), Hall (1993), Cochrane (1995), Rebelo (2004)

- Investment and durables fall by more than GDP, non-durable consumption by less
- Consumption, investment and hours worked are strongly pro-cyclical

Some candidates

- Credit, oil price, monetary and fiscal policy shocks are either too small or imply counterfactual correlations

In Hall's and Cochrane's terminology, "consumption shocks" appear to be needed

Optimism, pessimism and aggregate fluctuations

Fluctuations in output due to exogenous variation in expectations is an old idea:

- Pigou cycles (Pigou 1932), animal spirits (Keynes 1936), sunspots (Cass and Shell 1983), news about future productivity (Beaudry and Portier 2004, 2006, 2007, Jaimovich and Rebelo 2009), noisy public signals about common productivity (Lorenzoni 2009), shocks to "sentiments" (Angeletos and La'o 2013)

To different degrees, these theories can match the co-movement of consumption, hours worked, investment etc

An Old Idea I

"The varying expectations of business men ... and nothing else, constitute the immediate cause and direct causes or antecedents of industrial fluctuations."

A.C. Pigou, 1932

An Old Idea II

"Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of animal spirits, a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities."

J.M. Keynes, 1936

Optimism, pessimism and aggregate fluctuations

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To different degrees, these theories can match the co-movement of consumption, hours worked, investment etc

Question: Can we quantify the importance of undue optimism empirically?

Some theory

A Theory of Demand Shocks, Lorenzoni (AER 2009)

Basic set up:

- Standard New-Keynesian model with sticky prices and monopolistic competition
- Aggregate productivity follows a random walk
- Optimal consumption determined by permanent income hypothesis

Noisy signals about productivity makes agents unduly optimistic/pessimistic

Dispersed information

Islands are subject to idiosyncratic productivity shocks

Total island productivity is equal to aggregate + idiosyncratic productivity

Three tricks to prevent full revelation of aggregate state:

- Island inhabitants do not observe the components of productivity directly
- When shopping, inhabitants of an island do not travel to all other islands in the economy
- When selling goods, an island is not visited by inhabitants from all other islands

Neat, and more easily interpretable than pure noise

Production on island j

Firm i on island j produce good i, j using the technology

$$Y_{i,j,t} = \exp\left(a_{j,t}\right) N_{i,j,t}$$

(The log of) productivity $a_{j,t}$ is the sum of a common component x_t and the island specific component $\eta_{j,t}$

$$a_{j,t} = x_t + \eta_{j,t} : \eta_{j,t} \sim N\left(0, \sigma_\eta^2\right) \ \forall \ j, t.$$

The common productivity component x_t follows a random walk

$$x_t = x_{t-1} + \varepsilon_t$$

Dispersed information

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A public noisy signal

In addition to the island specific source of information, all islands also observe a public signal about aggregate productivity

$$s_t = x_t + e_t$$

Whatever interpretation we give to the public signal, that it does not perfectly reveal the state of the economy seems plausible

- MSNBC, CNN, Wall St Journal etc
- Revisions to statistical estimates etc
- Debate on Solow residual

Public signals improve average estimates, but at times must induce agents to respond to “false alarms” (i.e. $\Rightarrow$ ”demand shocks”)

Solving the model

Lucas (1973) assumed that the aggregate state can be observed between periods

- Makes dynamics due to information short lived

In Lorenzoni, the state is never perfectly revealed

- Solving model is harder as one need to worry about the "infinite regress of expectations" as in Townsend (1983) and the associated infinite dimensional state
- Lorenzoni takes similar approach to Townsend and assumes that variables lagged more than 50 periods are uninformative

The effects of noise shocks

shocks, variables

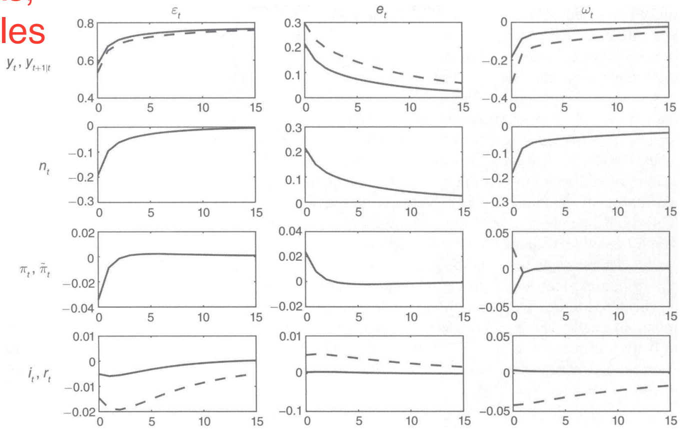


FIGURE 1. IMPULSE RESPONSES OF OUTPUT, EMPLOYMENT, INFLATION, AND THE INTEREST RATE

The effects of noise shocks

Noise shocks thus look like demand shocks (hence the title)

- A better "theory" than shocks to preferences (which we would like to think are pretty stable)

Lorenzoni reports that

- "Noise" shocks can explain a substantial fraction of output volatility

But:

- What happens when $\sigma_e^2 \to 0$?
- What happens when $\sigma_e^2 \to \infty$?

The non-monotonic effect on dynamics of variances

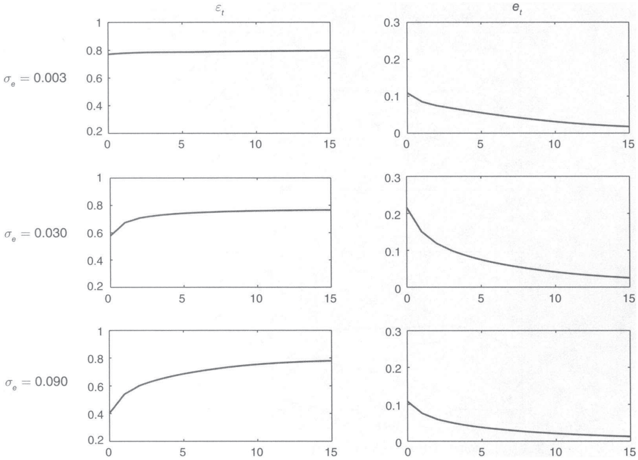


FIGURE 4. OUTPUT RESPONSES FOR DIFFERENT VALUES OF σ_e

Summing up *A Theory of Demand Shocks*

Noisy public information can explain what looks like demand shocks in the data

- Agents become unduly optimistic after a positive noise shock and unduly pessimistic after a negative noise shock
- There is an upper bound on how much aggregate volatility that can be explained by noise shocks

But how can we empirically quantify the importance of undue optimism/ pessimism?

Quantifying the importance of optimism shocks empirically

How can we quantify the role of optimism shocks empirically?

Questions:

- Do we have direct empirical measures of sentiments or undue optimism/pessimism?
- What type of identification strategies can be used to quantify the effect of noise shocks?

Potential strategies:

- Long-run restrictions a la Blanchard and Quah (1989)
- Using structural models, e.g. Barsky and Sims, AER 2012 and Blanchard, L'Huillier and Lorenzoni, AER 2014.

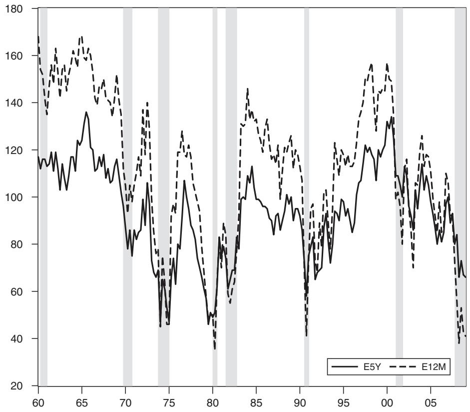
Empirical measures of sentiments, animal spirits and/or undue optimism/pessimism

Barksy and Sims (2014) argue that survey data on *Consumer Confidence* may be helpful.

Main Survey Question:
Turning to economic conditions in the country as a whole, do you expect that over the next five years we will have mostly good times, or periods of widespread unemployment and depression, or what?

Other questions ask about personal economic conditions.

Consumer Confidence and Recessions



Direction of causation

Clearly, the confidence index dips during recessions.

But:

- Is confidence low because of the recessions?
- Is there a recessions because confidence is low?

There are some interesting challenges involved in identifying the direction of causality that are particular to confidence shocks.

Identifying the Effect of Noise Shocks using SVARs

SVARs will generally not work:

- If there is a response to a pure noise shock, then this is a "mistake" from the perspective of the agents
- We cannot identify a mistake made by rational agents from variables that the agents also observe
- If we run a VAR in variables that are publicly available, then if we can back out the shocks, so can the agents
- If agents form expectations rationally, VAR shocks must be orthogonal to expectational errors

Identifying Noise and News Shocks using DSGEs

DSGEs could potentially work:

- Use ML to estimate "deep" parameters and quantify the contribution to overall variance made by noise shocks
- Kalman smoother can be used to back out time series of noise shocks

This strategy is pursued by Blanchard, L'Hullier and Lorenzoni (AER 2013)

News, Noise and Fluctuations: An empirical Investigation (BLL 2013)

Sets up a simplified representative agent version of the model Lorenzoni (AER 2009)

- Standard New-Keynesian model with sticky prices and monopolistic competition
- Permanent and transitory productivity shocks
- Optimal consumption determined by permanent income hypothesis
- Estimate model using standard macro time series

A noisy signal about permanent productivity generates undue optimism and pessimism

A noisy signal about productivity

The representative household observes a noisy signal about permanent productivity x_t

$$s_t = x_t + e_t$$

Observing the signal improves productivity estimates on average, but at times induce agents to respond to "false alarms" (i.e. $\Rightarrow$ "demand shocks")

Prior and posterior estimates

TABLE 5—FULL DSGE: ESTIMATED PARAMETERS

	Parameter	Prior	Posterior	Conf. bands		Distribution	Prior SD
h	Habit	0.5	0.5262	0.4894	0.5787	Beta	0.1
α	Production function	0.3	0.1859	0.1748	0.1933	Normal	0.05
ζ	Inv. Frisch elasticity	2	2.0871	1.0571	3.3012	Gamma	0.75
ξ	Capacity cost	5	3.4919	2.8912	4.3021	Normal	1
χ	Adjustment cost	4	4.3311	3.6751	5.5079	Gamma	1
θ	Calvo prices	0.66	0.8770	0.8545	0.8998	Beta	0.1
θ_w	Calvo wages	0.66	0.8690	0.8227	0.9183	Beta	0.1
γ_π	Taylor rule inflation	1.5	1.0137	1.0102	1.0568	Normal	0.3
γ_y	Taylor rule output	0.005	0.0050	0.0037	0.0061	Normal	0.05
Shock processes							
	Neutral technology and noise						
ρ		0.6	0.9426	0.9230	0.9618	Beta	0.2
σ_u		0.5	1.1977	1.0960	1.2975	Inv. Gamma	1
σ_ν		1	1.4738	0.7908	2.3176	Inv. Gamma	1
	Investment-specific						
ρ_d		0.6	0.4641	0.3263	0.5743	Beta	0.2
σ_d		0.15	11.098	8.4323	14.910	Inv. Gamma	1.5
	Markups						
ρ_p		0.6	0.7722	0.6991	0.8461	Beta	0.2
ϕ_p		0.5	0.4953	0.3749	0.6557	Beta	0.2
σ_p		0.15	0.1778	0.1508	0.2027	Inv. Gamma	1
ρ_w		0.6	0.9530	0.9534	0.9650	Beta	0.2
ϕ_w		0.5	0.9683	0.9700	0.9739	Beta	0.2
σ_w		0.15	0.3057	0.2847	0.3264	Inv. Gamma	1
	Policy						
ρ_r		0.5	0.5583	0.5125	0.6224	Beta	0.2
ρ_q		0.4	0.0413	0.0024	0.0807	Beta	0.2
σ_q		0.15	0.3500	0.3148	0.3782	Inv. Gamma	1
ρ_g		0.6	0.9972	0.9938	0.9998	Beta	0.2
σ_g		0.5	0.2877	0.2680	0.3078	Inv. Gamma	1

Impulse Response Functions (Quantities)

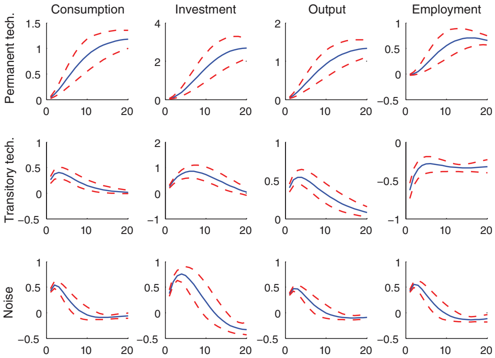


FIGURE 5. IMPULSE RESPONSES, BAYESIAN DSGE, QUANTITIES

Impulse Response Functions (Prices)

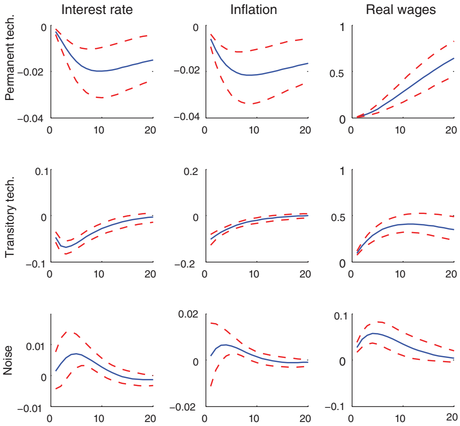


Figure 6. Impulse Responses, Bayesian DSGE, Prices

Variance decompositions

Table 6—Variance Decomposition

Quarter	Perm. tech.	Trans. tech.	Noise	Inv. specific	Price markup	Wage markup	Monetary	Fiscal
Consumption								
1	0.004	0.186	0.512	0.001	0.205	0.037	0.001	0.055
4	0.064	0.246	0.430	0.002	0.117	0.039	0.006	0.095
8	0.331	0.198	0.245	0.003	0.063	0.024	0.015	0.121
12	0.577	0.117	0.134	0.003	0.034	0.013	0.017	0.106
Investment								
1	0.000	0.005	0.011	0.971	0.006	0.006	0.000	0.000
4	0.003	0.017	0.021	0.936	0.008	0.016	0.000	0.000
8	0.031	0.036	0.027	0.869	0.009	0.027	0.000	0.001
12	0.120	0.046	0.025	0.769	0.009	0.029	0.000	0.003
Output								
1	0.003	0.249	0.200	0.372	0.083	0.026	0.001	0.066
4	0.040	0.272	0.198	0.363	0.057	0.039	0.003	0.028
8	0.228	0.270	0.134	0.267	0.036	0.035	0.006	0.024
12	0.477	0.200	0.083	0.167	0.023	0.023	0.008	0.020

Variance decompositions

TABLE 6—VARIANCE DECOMPOSITION

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Estimating the agents' mistakes using the Kalman Smoother

The standard Kalman filter gives an optimal *real time* estimate of the latent state

- Sometimes we are interested in the best estimate given the complete sample, i.e. $X_{t|T}$

$$X_{t|T} = E\left[X_t \mid Z^T, X_{0|0}\right]$$

The *Kalman smoother* can be used to find $X_{t|T}$

Because we use more information than what is available to agents in *real time* we can quantify their mistakes

Estimated true and expected productivity

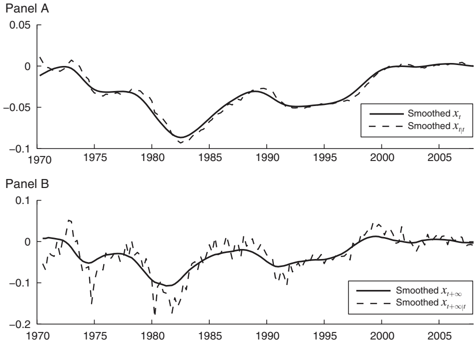


FIGURE 3. SMOOTHED ESTIMATES OF THE PERMANENT COMPONENT OF PRODUCTIVITY, OF LONG-RUN PRODUCTIVITY, AND OF CONSUMERS' REAL TIME EXPECTATIONS

Estimated history of exogenous shocks

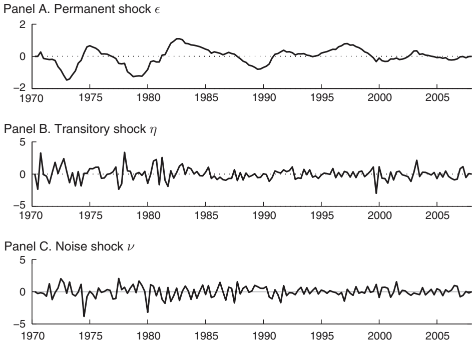


FIGURE 4. SMOOTHED ESTIMATES OF THE SHOCKS

Using survey data in estimation

Using survey data when estimating DSGE models

Blanchard et al use only macro economic time series to estimate their model

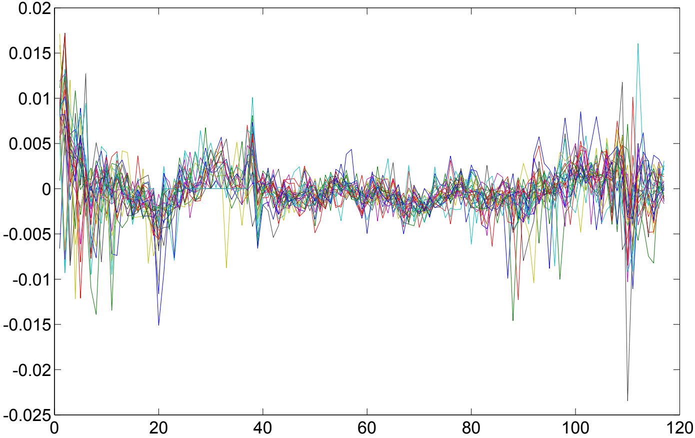
- Given the importance of expectations for the theory, is there any data that directly measures expectations?

One option is to use survey data such as the *Survey of Professional Forecasters*. We have two choices:

1. Use mean or median forecast as a measure of the representative agent's expectations
2. Use individual forecasts as being representative of the forecasts of agents with heterogeneous expectations

We will pursue the second approach.

Individual SPF inflation forecasts



Using individual survey responses

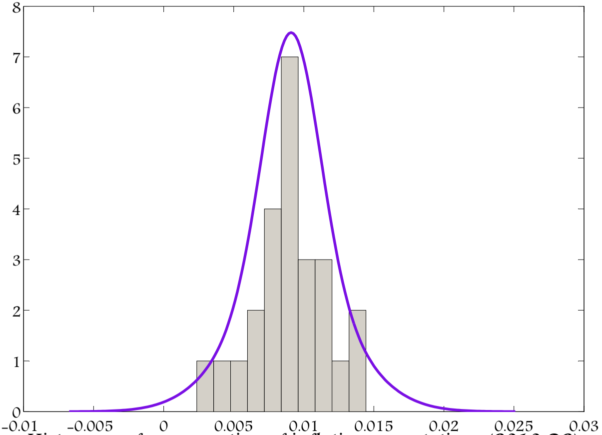
If we have a model where agents disagree about future variables, each survey response can be treated as an independent draw from the model implied cross-sectional distribution of expectations

$$f_{t,\pi} \sim N\left(\int E\left[\pi_{t+1} \mid \Omega_{j,t}\right] dj, \sigma^2_{f\pi}\right)$$

If we observe 25 forecasts, we will have 25 lines corresponding to each forecast in the measurement equation.

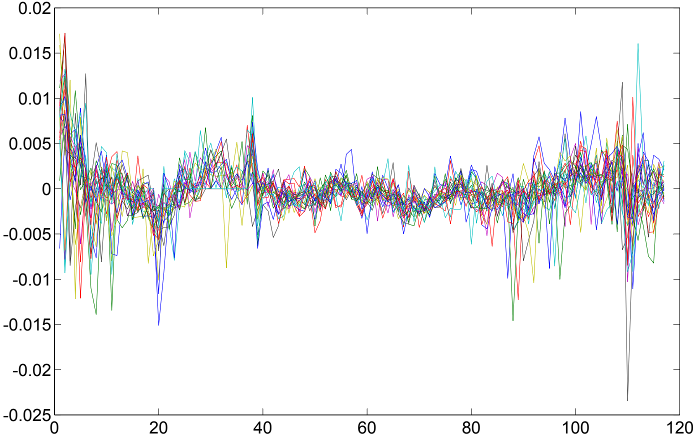
- The "measurement error" variance depends on deep parameters.

Using individual survey responses



Histogram of cross-section of inflation expectations (2010:Q2)

Individual SPF inflation forecasts: Time-varying dispersion



Using Survey Data to Study the Role of News Media in the Business Cycle

Man-bites-dog business cycles, Nimark (AER 2014)

"The phrase **man bites dog** *describes a phenomenon in journalism in which an unusual, infrequent event is more likely to be reported as news than an ordinary, everyday occurrence (such as Dog bites man). The result is that rare events often appear in headlines while common events rarely do."*
from Wikipedia

This paper investigates the implications of this aspect of news reporting for the business cycle

Man-bites-dog business cycles

Three different sources of information:

- Economic interactions in markets
- Statistics: scheduled release dates, reported regardless of content
- News (or man-bites-dog signals): Signals that are more likely to be available about unusual events

That unusual events are more likely to be reported as news may suggest that we should be better informed about unusual events.

- But flip side of that argument is that conditional on that a signal is available, the likelihood of an unusual event has increased and uncertainty may then go up

Man-bites-dog business cycles

Man-bites-dog signals can help us understand why we observe

- Large changes in macro aggregates without a correspondingly large change in fundamentals
- Persistent periods of high macro economic volatility
- A positive correlation between the volatility of macro aggregates and the cross-sectional dispersion of forecasts

A simple business cycle model

Follows closely Lorenzoni (2009) but with a man-bites-dog information structure

- Island economy with common and island specific productivity shocks
  - A large innovation to productivity is more likely to generate a public signal
- Each island consume only a subset of goods and sell their own good only to a subset of islands
  - Prevents local interactions from revealing aggregate state
- Strategic complementarities in consumption and prices
- Nominal interest rate follows a Taylor rule

What are the dynamic effects of man-bites-dog signals?

Time-varying information structure

Number of observables and conditional distribution of productivity depends on whether public signal about productivity is observed or not

In every period agents observe

- Local productivity, prices and demand
- The nominal interest rate

State dynamics in period t depend on entire history of signal availability

Estimating the model

Model is conditionally linear and straightforward to estimate using likelihood based methods

- The data (1981:Q3 - 2010:Q3):
  - Fed funds rate, CPI inflation and detrended real GDP
  - Individual survey responses on one quarter ahead forecasts of CPI inflation and nominal GDP from Survey of Professional Forecasters
  - Quarterly time series of Total Factor Productivity constructed by Fernald (2010)
- Use Multiple-Block Metropolis Algorithm (Chib 2001) to construct posterior estimates of
  - Parameters of preferences and technology
  - Parameters governing the man-bites-dog information structure
  - The history of s_t

Using individual survey responses

Each survey response is treated as an independent draw from the model implied cross-sectional distribution of one quarter ahead inflation expectations

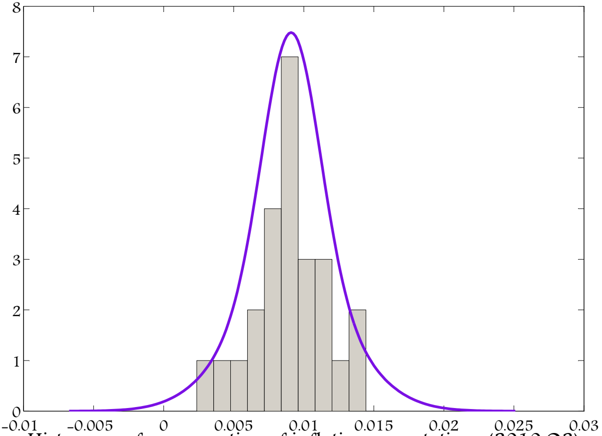
$$f_{t,\pi} \sim N\left(\int E\left[\pi_{t+1} \mid \Omega_{j,t}\right] dj, \sigma^2_{f\pi}(s^t)\right)$$

and nominal GDP growth expectations

$$f_{t,\pi+\Delta y} \sim N\left(\int E\left[\Delta y_{t+1} + \pi_{t+1} \mid \Omega_{j,t}\right] dj, \sigma^2_{f\pi+\Delta y}(s^t)\right)$$

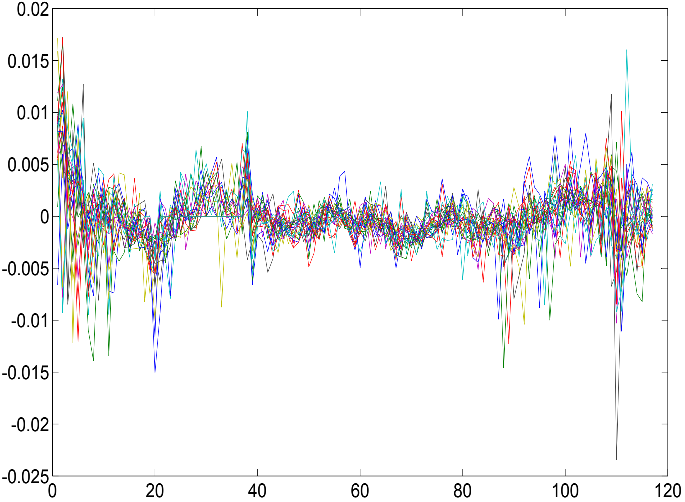
- Note that cross-sectional dispersion depends on s^t

Using individual survey responses



Histogram of cross-section of inflation expectations (2010:Q2)

Time-varying dispersion in survey data



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  - Parameters of preferences and technology
  - Parameters governing the man-bites-dog information structure
  - The history of s_t
- Uniform truncated priors for all parameters

Multiple-Block Metropolis-Hastings algorithm

1. Specify initial values Θ_0 and s_0^T.
2. Repeat for $j = 1, 2, ..., J$
   2.1 Block 1: Draw Θ_j from $p\left(\Theta \mid s_{j-1}^T, Z^T\right)$
      2.1.1 Generate the candidate parameter vector Θ^* from $q_\Theta\left(\Theta^* \mid \Theta_{j-1}\right)$
      2.1.2 Calculate $\alpha_j^\Theta = \min\left\{\frac{L\left(Z^T|s_{j-1}^T,\Theta^*\right)p\left(s^T|\Theta^*\right)p\left(\Theta^*\right)q_\Theta\left(\Theta_{j-1}|\Theta^*\right)}{L\left(Z^T|s_{j-1}^T,\Theta_{j-1}\right)p\left(s^T|\Theta_{j-1}\right)p\left(\Theta_{j-1}\right)q_\Theta\left(\Theta^*|\Theta_{j-1}\right)}, 1\right\}$
      2.1.3 Set $\Theta_j = \Theta^*$ if $U(0,1) \leq \alpha_j^\Theta$ and $\Theta_j = \Theta_{j-1}$ otherwise.
   2.2 Block 2: Draw s_j^T from $p\left(s^T \mid \Theta_j, Z^T\right)$
      2.2.1 Generate s^{*T} from the proposal density $q_S\left(s^{*T} \mid s_{j-1}^T\right)$
      2.2.2 Calculate $\alpha_j^s = \min\left\{\frac{L\left(Z^T|s^{*T},\Theta_j\right)p\left(\Theta_j|s^{*T}\right)q_S\left(s_{j-1}^T|s^{*T}\right)}{L\left(Z^T|s_{j-1}^T,\Theta_j\right)p\left(\Theta^*|s_{j-1}^T\right)q_S\left(s^{*T}|s_{j-1}^T\right)}, 1\right\}$
      2.2.3 Set $s_j^T = s^{*T}$ if $U(0,1) \leq \alpha_j^s$ and $s_j^T = s_{j-1}^T$ otherwise.
3. Return values $\{\Theta_0, \Theta_1, ..., \Theta_J\}$ and $\{s_0^T, s_1^T, ..., s_J^T\}$

Empirical results

Posterior IRFs

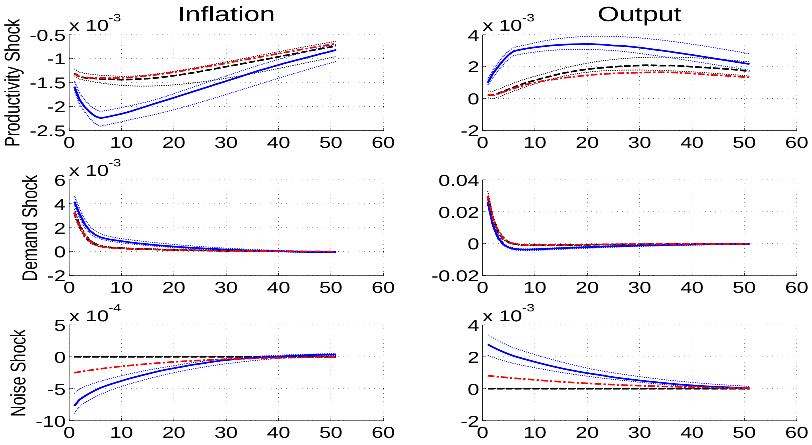
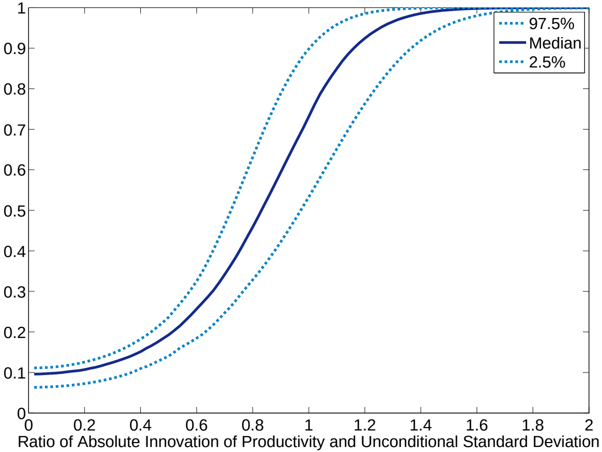
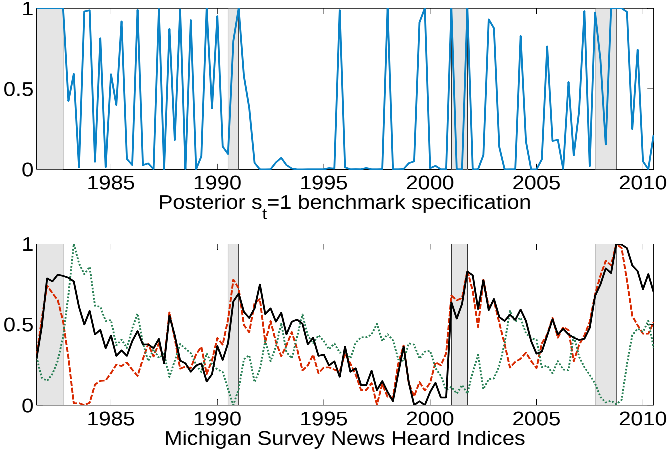


Figure: Blue line: $s^t = 1, 0, 0, \ldots$ Black line: $s^t = 0, 0, 0, \ldots$

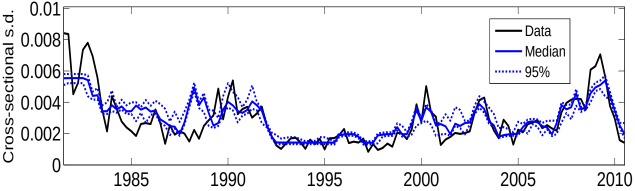
The conditional probability of observing a man-bites-dog signal



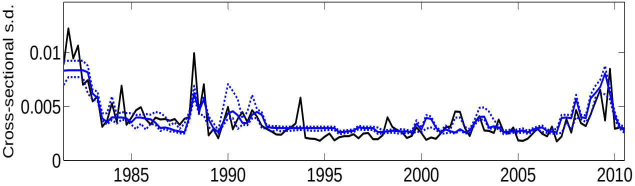
Historical probabilities of man-bites-dog episodes



Actual and model implied dispersion of expectations



Cross-sectional Dispersion of Inflation Forecasts



Cross-sectional Dispersion of Nominal GDP growth Forecasts

Solving Models with Privately Informed Agents

Dynamic Models with Private Information

Many economic settings feature **private information, strategic interaction and dynamic choices**

- Individual agents have private information about variable(s) of common interest
- Individual pay-offs depend on actions taken by others
- Agents optimize intertemporally

Examples

- Price-setting decisions in macro models
- Investment in productive capacity decisions in oligopolistic markets
- Asset pricing

Dynamic Models with Private Information

The principal modeling difficulty is the **infinite regress of expectations** problem

- "Forecasting the forecasts of others" - Townsend (1983)
- Natural state representations tend to be infinite-dimensional

Dynamic Higher Order Expectations, Nimark (2017)

- Shows that there exists a unique equilibrium in a linear class of models with privately informed agents
- Proposes an algorithm to approximate equilibrium to an arbitrary accuracy with a finite dimensional state and explicit approximation error bounds

Results hold if agents discount the future and the exogenous processes are stationary

Notation and definitions

Denote agent j's first order expectation of θ_t as

$$\theta_{t,j}^{(1)} \equiv E\left[\theta_t \mid \Omega_{t,j}\right].$$

The average first order expectation $\theta_t^{(1)}$ is defined as

$$\theta_t^{(1)} \equiv \int E\left[\theta_t \mid \Omega_{t,j}\right] dj.$$

The k^{th} order expectation of θ_t is defined recursively as

$$\theta_t^{(k)} \equiv \int E\left[\theta_t^{(k-1)} \mid \Omega_{t,j}\right] dj$$

Full information implies $\theta_t = \theta_t^{(k)} : k = 1, 2, \ldots$ for all t.

A simple model

An Archetypal Dynamic Model

Simplified version of Singleton (1987) asset pricing model.

Price of asset determined by Euler-type equation

$$p_t = \beta \int E\left[p_{t+1} \mid \Omega_{t,j}\right] \, dj - (\theta_t + \varepsilon_t)$$

where $\Omega_{t,j}$ is the information set of agent $j \in (0,1)$ and $0 \leq \beta < 1$.

Asset supply $\theta_t + \varepsilon_t$ is stochastic and has a transitory and persistent component

$$\begin{aligned} \theta_t &= \rho\theta_{t-1} + u_t : u_t \sim N\left(0, \sigma_u^2\right) \\ \varepsilon_t &\sim N\left(0, \sigma_\epsilon^2\right) \end{aligned}$$

where $0 \leq |\rho| < 1$.

Information sets

Agent j observes the signal vector $\mathbf{s}_{t,j}$

$$\mathbf{s}_{t,j} \equiv \begin{bmatrix} z_{t,j} \\ p_t \end{bmatrix}$$

where

$$z_{t,j} = \theta_t + \eta_{t,j} : \eta_{t,j} \sim N\left(0, \sigma_\eta^2\right).$$

The full information solution

If all agents observe θ_t we can iterate

$$p_t = \beta E\left[p_{t+1} \mid \Omega_t\right] - (\theta_t + \varepsilon_t)$$

forward to get

$$p_t = -\frac{1}{1-\beta\rho}\theta_t - \varepsilon_t$$

if $|\beta\rho| < 1$.

Private information and a complication

With privately informed agents, we can still substitute the price Euler equation forward. After the first step we get

$$p_t = -(\varepsilon_t + \theta_t) - \beta \int E\left[\theta_{t+1} \mid \Omega_t^j\right] dj - \beta^2 \int E\left[\int E\left[p_{t+2} \mid \Omega_{t+1}^{j'}\right] \, dj' \mid \Omega_t^j\right] \, dj.$$

Continued recursive substitution gives

$$\begin{aligned} p_t \quad = \quad & -(\varepsilon_t + \theta_t) + \beta \int E\left[\theta_{t+1} \mid \Omega_t^j\right] dj \\ & -\beta^2 \int E\left[\int E\left[\theta_{t+2} \mid \Omega_{t+1}^{j'}\right] \, dj' \mid \Omega_t^j\right] \, dj - \ldots \\ & \ldots - \beta^k \int E\left[\cdots \int E\left[\theta_{t+k} \mid \Omega_{t+k-1}^{j''}\right] \, dj'' \cdots \mid \Omega_t^j\right] \, dj - \ldots : k \to \infty \end{aligned}$$

Price depends on higher order expectations about θ_{t+k}.

Algorithm:
Iterating on the Euler equation while increasing the orders of expectations

The filtering problem of a naive agent

A naive agent that engages only in first-order reasoning believes that p_t follows

$$p_t^0 = -(\theta_t + \varepsilon_t)$$

Agent j's expectation of θ_t is then given by the Kalman filter update equation

$$\theta_{t,j}^{(1)} = \rho\theta_{t-1,j}^{(1)} + K_0 \left[\mathbf{s}_{t,j} - D_0 \rho \theta_{t-1,j}^{(1)} \right]$$

associated with the state space system

$$\begin{aligned} \theta_t &= \rho\theta_{t-1} + u_t \\ \mathbf{s}_{t,j} &= D_0\theta_t + e_1\eta_{t,j} + e_2\varepsilon_t \end{aligned}$$

where $D_0 = \begin{bmatrix} 1 & -1 \end{bmatrix}'$ and K_0 is the Kalman gain.

The prediction problem of a naive agent

The naive agent's expectation about the next period price is given by

$$E[p^0_{t+1} \mid \Omega^0_{t,j}] \quad = \quad \rho\theta^{(1)}_{t,j}$$

Substitute $\int \rho\theta^{(1)}_{t,j} dj$ into the Euler equation

$$p_t = \beta \int E\left[p_{t+1} \mid \Omega_{t,j}\right] \, dj - (\theta_t + \varepsilon_t)$$

to get the new price process p^1_t as

$$p^1_t = -\left[\begin{array}{cc} 1 & \beta\rho \end{array}\right] \left[\begin{array}{c} \theta_t \\ \theta^{(1)}_t \end{array}\right] - \varepsilon_t$$

where

$$\left[\begin{array}{c} \theta_t \\ \theta^{(1)}_t \end{array}\right] = \left[\begin{array}{cc} \rho & 0 \\ K_0 D_0 \rho & (I - K_0 D_0)\,\rho \end{array}\right] \left[\begin{array}{c} \theta_{t-1} \\ \theta^{(1)}_{t-1} \end{array}\right] + \left[\begin{array}{cc} 1 & 0 \\ K_0 D_0 & K_0 \end{array}\right] \left[\begin{array}{c} u_t \\ \varepsilon_t \end{array}\right]$$

Filtering with $k+1$-order reasoning

The price function with $k+1$-order reasoning

$$p_t^k = \mathbf{g}_k' \theta_t^{(0:k)} - \varepsilon_t$$

Agent j's optimal update equation

$$\theta_{t,j}^{(1:k+1)} = M_k \theta_{t-1,j}^{(1:k+1)} + K_k \left[\mathbf{s}_{t,j} - D_k M_k \theta_{t-1,j}^{(1:k+1)} \right]$$

associated with the state space system

$$\begin{aligned} \theta_t^{(0:k)} &= M_k \theta_{t-1}^{(0:k)} + N_k \mathbf{w}_t : \mathbf{w}_t \sim N(0, I) \\ \mathbf{s}_{t,j} &= D_k \theta_t^{(0:k)} + R_w \mathbf{w}_t + R_\eta w_{t,j} : w_{t,j} \sim N(0,1) . \end{aligned}$$

The hierarchy of $k+1$ orders of expectations

Take the cross-sectional average update equation and amend to the actual process for θ_t to get

$$\begin{bmatrix} \theta_t \\ \theta_t^{(1:k+1)} \end{bmatrix} = M_{k+1} \begin{bmatrix} \theta_{t-1} \\ \theta_{t-1}^{(1:k+1)} \end{bmatrix} + N_{k+1} \mathbf{w}_t$$

where the matrices M_{k+1} and N_{k+1} are given by

$$\begin{aligned} M_{k+1} &= \begin{bmatrix} \rho & \mathbf{0}_{1\times k} \\ \mathbf{0}_{k\times 1} & \mathbf{0}_{k\times k} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{1\times k} & 0 \\ K_k D_k M_k & \mathbf{0}_{k\times 1} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & \mathbf{0}_{1\times k} \\ \mathbf{0}_{k\times 1} & (I - K_k D_k) M_k \end{bmatrix} \\ N_{k+1} &= \begin{bmatrix} \sigma_v e_1' \\ (K_k D_k N_k + K_k R_w) \end{bmatrix}. \end{aligned}$$

The $k+1$ order price function

The expectation of the next period price given k-order reasoning is given by

$$\int E\left[p_{t+1}^{k} \mid \Omega_{t,j}^{k}\right]\, dj = \mathbf{g}_k' M_k H_{k+1} \theta_t^{(0:k+1)}$$

where the *higher-order expectations operator* $H_k : \mathbb{R}^{k+1} \to \mathbb{R}^k$ is defined so that

$$\theta_t^{(1:k)} = H_k \theta_t^{(0:k)}.$$

The new price function can thus be expressed as

$$p_t^{k+1} = \mathbf{g}_{k+1}' \theta_t^{(0:k+1)} - \varepsilon_t$$

where

$$\mathbf{g}_{k+1}' = e_1' + \beta \mathbf{g}_k' M_k H_{k+1}.$$

Computing the dispersion of expectations

The agent specific covariance Σ_j of agent j's state estimate

$$\Sigma_j \equiv E\left(\theta_{t,j}^{(1:\overline{k})} - \int \theta_{t,j'}^{(1:\overline{k})} dj'\right)\left(\theta_{t,j}^{(1:\overline{k})} - \int \theta_{t,j'}^{(1:\overline{k})} dj'\right)'.$$

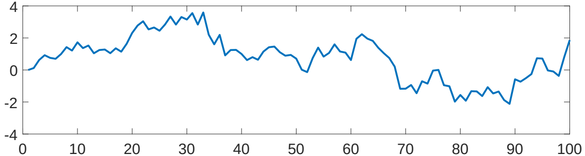
can then be computed by the solving the Lyaponov equation

$$\Sigma_j = \left(I - K_{\overline{k}} L_{\overline{k}}\right) M_{\overline{k}} \Sigma_j M_{\overline{k}}' \left(I - K_{\overline{k}} D_{\overline{k}}\right)' + K_{\overline{k}} R_\eta R_\eta' K_{\overline{k}}'.$$

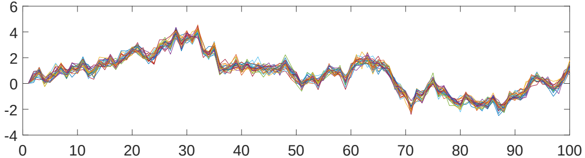
implying that the cross-sectional variance $\sigma_{p,j}^2$ of expectations about the next period price p_{t+1} is given by

$$\sigma_{p,j}^2 = \mathbf{g}_{\overline{k}}' M_{\overline{k}} \Sigma_j M_{\overline{k}}' \mathbf{g}_{\overline{k}}.$$

Simulated price and forecast data from simple model



Price of asset



One period ahead price forecasts

Final remarks

What have we learned?

- Solving a DSGE model
  - Mapping f from parameters θ to a state space system : $f(\theta) : \Theta \rightarrow A, C, D, \Sigma_v$
- The Kalman filter
  - Form recursive estimate of latent state
  - Evaluate the likelihood function
- M-H algorithm
  - Numerically approximate target distribution by MCMC methods
- Make probabilistic statement about functions of θ

Some caveats

1. The Lucas Critique and policy change
2. Representative agent and aggregation
3. Limitations of using historical data

The Lucas Critique, e.g. Lucas (1976)

Historical correlations are not a reliable guide for making predictions when policy changes

- Inflation-unemployment correlation in the data dos not imply an exploitable trade off

The Lucas Critique has made many people sceptical of using SVAR and other more statistically oriented models in favour of micro founded models

Sims' (Chris, not Eric) Critique of the Lucas Critique

Sims (1998) argues that people often unthinkingly perform counterfactual experiments using micro founded models

- If we as researchers want to use a structural model to study the implications of a counterfactual policy change, does it make sense to think of the agents in the model as believing that the considered policy change is impossible?

Maybe not.

Who is the representative agent representing?

One advantage of DSGE models is that we can easily interpret many model parameters

But caution is needed when using priors based on micro studies when using aggregate models

- Naive use of priors based on studies of individual behaviour may be inappropriate if aggregation is not straightforward

Example: Aggregate elasticity of labour supply may be very different from the elasticity of an individual's labour supply

Can historical data help when something happens for the first time?

Uniformity of nature assumption

- The laws of the universe are the same everywhere and at all times

Economists using statistical models make a similar assumption

- The past can help predict the present/the future
- The behaviour of one group of people can help us predict the behavior of others

But what about when something happens for the first time?

- We need to ask ourselves if our models have measured something that can reasonable be thought of as being constant and still relevant

That's it for this week.

Thank you!