Why and how to craft a trade-of in a plant functioning model

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This document describes why and how a trade-of has be designed in the model MountGrass. This trade-of has the objective to allow for different strategy based on the level of resources. The same trade-of is used in shoot and roots. Parameters effects over the phenotype determination is explored for a balanced design where area cost is the same for both organs.

Why a trade-of

different conditions = different phenotypes
could be anything, needed one. Explain WUE, nitrogen and others.
Try to keep independences between strategy axis to keep it simple.
Need for plastic driver. Explain the role of plasticity.

How to craft a trade-of

The idea of trade-of suggest that you cannot invest in all strategies at the same time. To be relevant, it must be associated to a range conditions favouring different strategies along this trade-of. In other word, depending on a position on a gradient, the gradient should lead different niches. This can be visualizes as Gaussian's curves (see figure).

The challenge is to go beyond the Gaussian function, and craft these niches from the plant physiology and ecology. Taking as a basis the usual functions used in plant modelling and the theoretical background upon which MountGrass is built, we will try to model different niches.

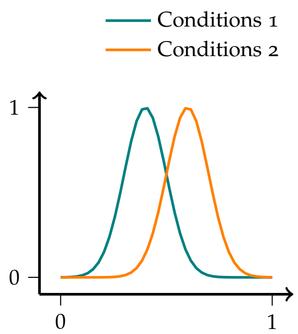

Figure 1: Different niches corresponding to different environmental conditions.

In MountGrass, the Leaf Economic spectrum (LES)[1] is explained by a differential investment between active and structural tissues (supported by analysis of Shipley[2]). This allocation constitutes a major strategic differentiation axis, along which plastic plants can move to optimize their fitness. To keep the approach simple, we hypothesize that such trade-off would also rule the allocation of organic matter in the below-ground compartment[3]. As explained above, plant can modify both the shot:root ratio, and the relative proportion of active tissues in shoot, and in roots. The first dimension is mainly driven by a balance between availability of above-ground versus below-ground resource.[4]

[1]

[2]

[3]

[4] This part does not belong to this section.

Crafting a trade-of for shoot

Let's consider only the shoot dimension for now, the demonstration that the same applies to root is done latter in the document.

The fitness function of the shoot can be described as the sum of the gain function and the cost function as follow:

$$\frac{Net\ Gain}{Biomass} = \frac{Exchange}{Area}.\frac{Area}{Biomass}.(1 - TO\ rate) - \frac{Resp}{Biomass} \tag{1}$$

where $\frac{Exchange}{Area}$ is the resource mass exchanges per area (function of exchange rate and resource availability), $\frac{Area}{Biomass}$ is the SLA (or inverse of leaf construction cost), $\frac{Resp}{Biomass}$ is the normalize biomass respiration and $TO\ rate$ is the turn-over rate. Let's analyse how these variables vary for a constant level of resources with the relative proportion of active tissue. For convenience the exchange rate is assumed to be constant for the following calculations. The SLA is the inverse function of the leaf density. If we consider that active and structural tissue pools are two distinct constant densities respectively noted ρ_{act} and ρ_{str} with $\rho_{act} < \rho_{str}$, the leaf area per carbon unit can be written:

$$SLA = \frac{1}{th.vol_prop}\left(\frac{1}{\rho_{str}} + \left(\frac{1}{\rho_{act}} - \frac{1}{\rho_{str}}\right)p_{act}\right) \tag{2}$$

where th is the thickness of the leaf, vol_prop is the proportion of leaf volume occupied by tissue and p_{act} is the fraction of active tissue. This shows a linear relationship between the gain function ($SLA.exchange_rate$) and the proportion of active tissue (p_{act}).

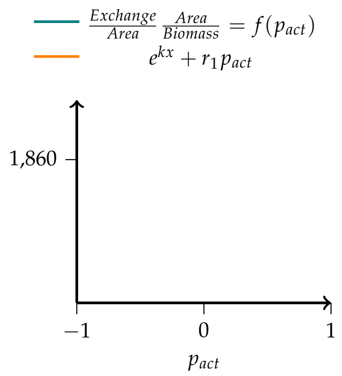

Figure 2: Comparison of "gain" function and "cost" function. Parameter values: $\rho_{act} = 0.05$, $\rho_{str} = 1$, $k = 7$ and $vol_prop = 1$. The area between the two curves is the total gain by the plant.

To avoid convergence toward an extremum (either $p_{act} = 0 or 1$), we need the cost function to have higher signed curvature in order to the difference to have a maximum for $0 < p_{act} < 0$. As respiration and construction cost (area per biomass) are linearly related to p_{act} (null curvature), the turn-over rate must have a positive curvature. While respiration should not change with resource availability (only temperature), the realised exchange rate is related to resource level, leading to an increased slope of the linear gross gain function in case of increasing resource availability. Changes in the slope of the gross gain function, with the convex loss function lead to different optimum for p_{act}.

$$Net\ Gain = R\frac{1}{th.vol_prop}\left(\frac{1}{\rho_{str}} + \left(\frac{1}{\rho_{act}} - \frac{1}{\rho_{str}}\right)p_{act}\right)(1 - *\frac{1}{e^{k_0}}.SLA \quad = R(g_1 + g_2 p_{act})(1 - \frac{1}{e^{k_0}}.(g_1 + g_2 p_{act})^{k_1}) - r_1 p_{act} \tag{3}$$

$$f'(p_{act}) = R.g_2 p_{act} - (k.e^{k.p_{act}} + r_1) \tag{4}$$

but the equation $f'(p_{act}) = 0$ is not numerically solvable, but we can visualize that the net gain function have different maximum for p_{act} in the interval $[0:1]$.

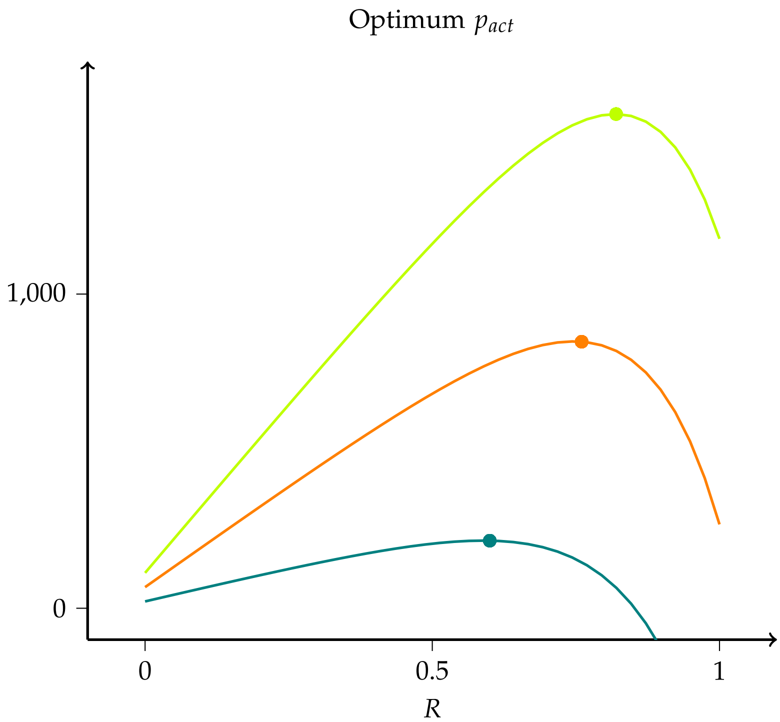

Figure 3: ◂ Comparison of "gain" function and "cost" function. Parameter values: $g_2 =$, $g_2 =$, $k = 7$

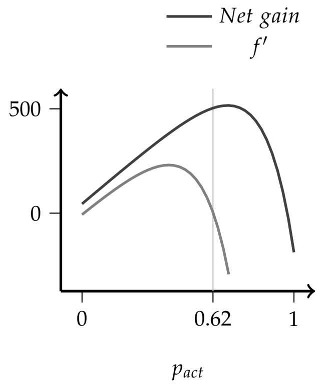

Figure 4: Net gain function and its first derivative.

Looks like there is some kind of mismatch here.

How about root

How do the two trade-of merge ?