Mathematical Biology

Data-Enabled Modeling of an Epidemic (Smallpox/COVID-19)

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November 2024

Introduction

Infectious diseases pose a major threat to public health, and mathematical models help us understand how these diseases spread. In this project, we use the SIR (Susceptible-Infectious-Recovered) model to study the dynamics of two different epidemics: smallpox in Abakaliki, Nigeria, in 1967, and the COVID-19 pandemic in the United States during 2020. By estimating key parameters, we aim to understand the transmission dynamics and predict effective intervention strategies.

Part I (small pox)

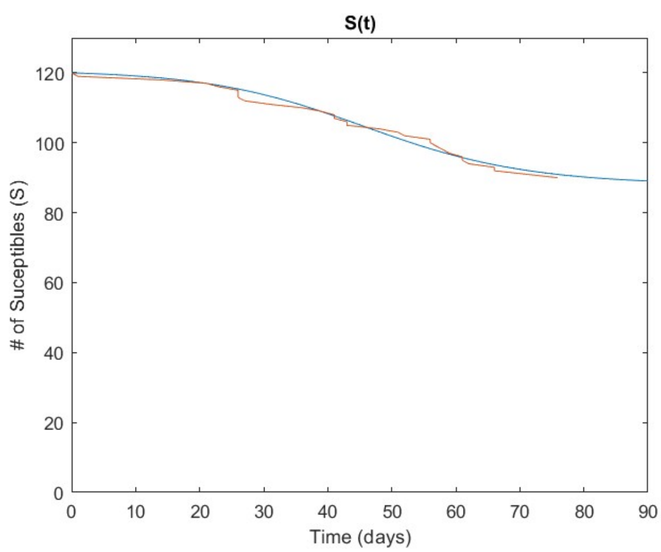

Figure 1: Susceptible population during the given time

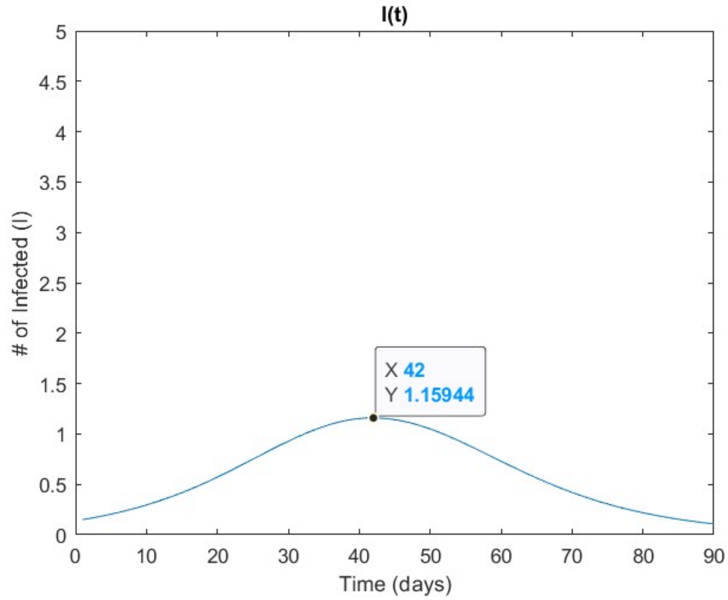

Figure 2: Infected population over time, with the maximum infection occurring at approximately 1 on day 42.

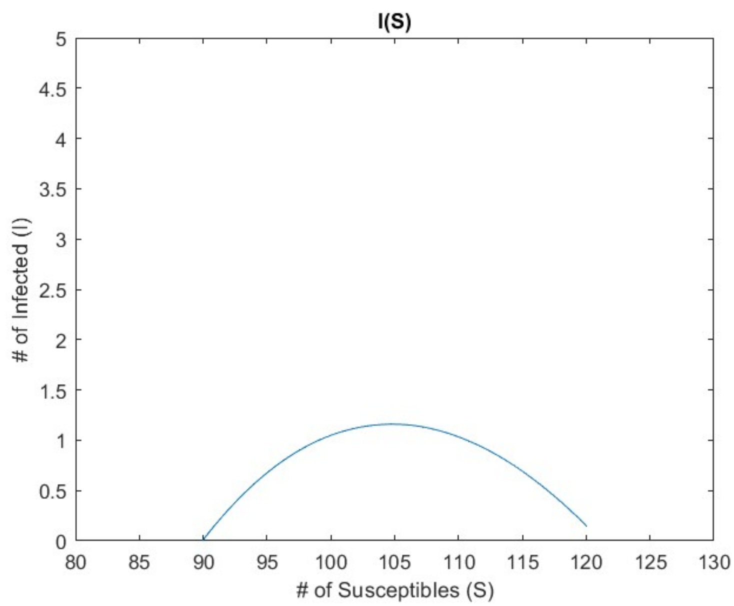

Figure 3: Susceptible and infected populations over time.

Note that the estimated parameters after the fitting is given in the table below:

Fitted Parameter	
Beta	0.512
Nu	53.645

Now, the epidemic threshold, which determines when the disease becomes extinct, is given by $\frac{\nu}{\beta}$.

For the given values of ν and β in the above table, the threshold is

$$\frac{\nu}{\beta} = \frac{53.645}{0.512} \approx 105$$

Reasons why the disease did not wipe out the entire population:

- **Cultural and religious practices:** The community's religious beliefs may have led them to interpret the outbreak as divine punishment for certain disobedient individuals. These individuals could have been quarantined, reducing contact and thereby limiting further spread of the disease.
- **Threshold effects:** The disease affected 30 individuals within a total population of 120, leaving only 90 susceptible individuals. This number is below the epidemic threshold of 105. As a result, the outbreak did not spread to levels that could have kill the entire population.

Let the fraction of the population to be vaccinated be p, so that the minimum number of individuals to be vaccinated is pN. The fraction p is given by:

$$p > 1 - \frac{1}{R_0}, \quad \text{where} \quad R_0 = \frac{\beta N}{\nu}$$

Substituting the values:

$$p > 1 - \frac{\nu}{\beta N} = 1 - \frac{53.645}{0.512 \times 120} \approx 0.1269$$

Thus, the minimum number of individuals to be vaccinated is:

$$pN > 0.1269 \times 120 \approx 15$$

Therefore, 15 is the minimum number of people in the village that should have been vaccinated to stop the epidemic. Thus, the susceptible population would have decreased to below the threshold (105), causing the epidemic to stop.

Part II (COVID-19)

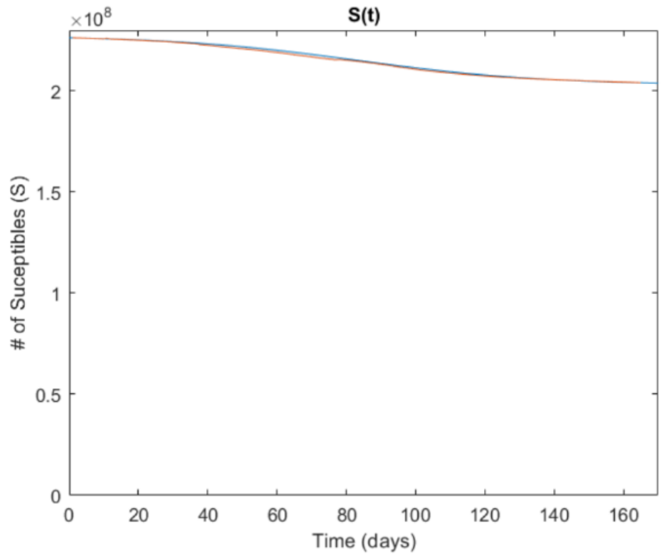

Figure 4: Susceptible population during the given time.

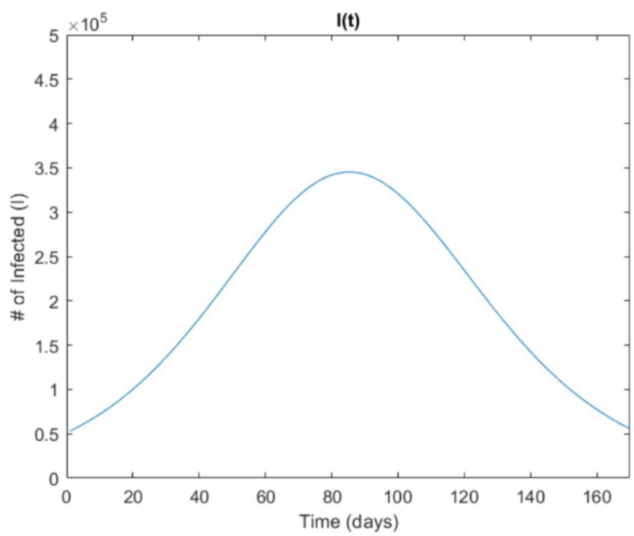

Figure 5: Infected population during the given time

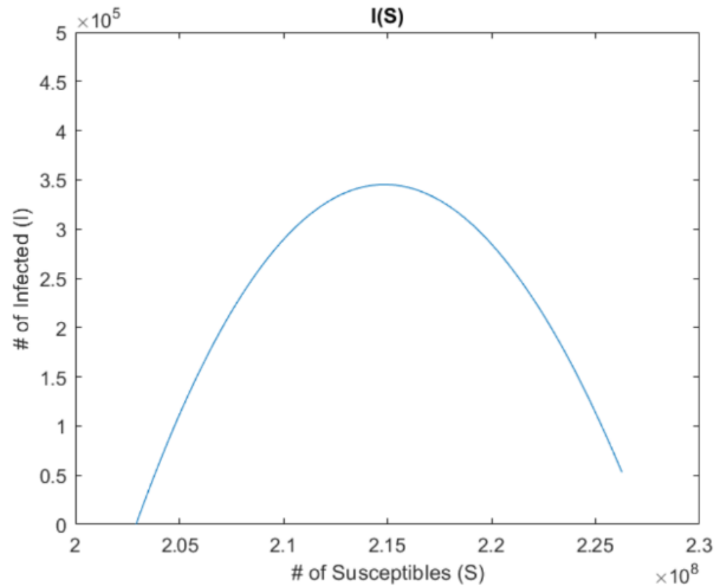

Figure 6: Susceptible and infected populations over time.

Note that the estimated parameters after the fitting is given in the table below:

Fitted Parameter	
Beta	0.000000153
Nu	32.875

Thus, the estimated basic reproduction number, R_0, is calculated as:

$$R_0 = 226266983\left(\frac{\beta}{\nu}\right)$$
$$= \frac{226266983 \times 0.000000153}{32.875} \approx 1.05304.$$

Note that for this estimated R_0, the fraction of the U.S. population that should have been vaccinated or quarantined to stop the pandemic is:

$$p > 1 - \frac{1}{R_0} = 1 - \frac{1}{1.05304} = \frac{663}{13163} \approx 0.05037.$$

Thus, the minimum number of people in the U.S. who should have been vaccinated or quarantined to stop the pandemic is:

$$pN > 0.05037 \times 226320000 \approx 11,399,389.$$

Therefore, if a minimum of $11,399,390$ individuals in the U.S. had agreed to be vaccinated or quarantined, the epidemic could have been prevented.

Reasons for Unrealistic Predictions in our Model:

- **Limitations of data and estimation techniques:** The method used to estimate the infection and recovery rates did not incorporate any scientific methods or uncertainty quantification, leading to potential inaccuracies in our parameter estimation.
- **Impact of human behavior:** The model overlooks the influence of human actions, such as compliance with quarantine or vaccination strategies, which play a critical role in shaping the trajectory of an epidemic.