Mathematical Modeling

Forecasting Growth Trajectories Using Phenomenological Growth Models

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Introduction

The objective of this assignment is to obtain hands-on experience in using the QuantDiffForecast toolbox for fitting and forecasting time-series data. Through this assignment, we will gain practical skills in applying phenomenological dynamic growth models to real-world data (a publicly available dataset of daily monkeypox cases in the USA), understand model fitting techniques, and evaluate forecasting performance.

1. Model Fitting

We fit the Generalized Logistic Growth Model (GLM) to the dataset using the `Run_Fit_ODEModel.m` function for the first 10 weeks of data. This task involves exploring the model parameters, including the growth rate (r), scaling parameter (p), and final epidemic size (K).We applied the Nonlinear Least Squares (NLS) method together with two distinct error structures: the normal error structure and the Poisson error structure.

The GLM Differential equation for the model is given below:

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \left(\frac{N(t)}{K}\right)^p\right)$$

where:

$N(t)$ is the number of infected individual,
r is the growth rat,
K is the carrying capacity or final epidemic size, and
p is the scaling parameter.

The diagrams below shows the fit model for the GLM method.

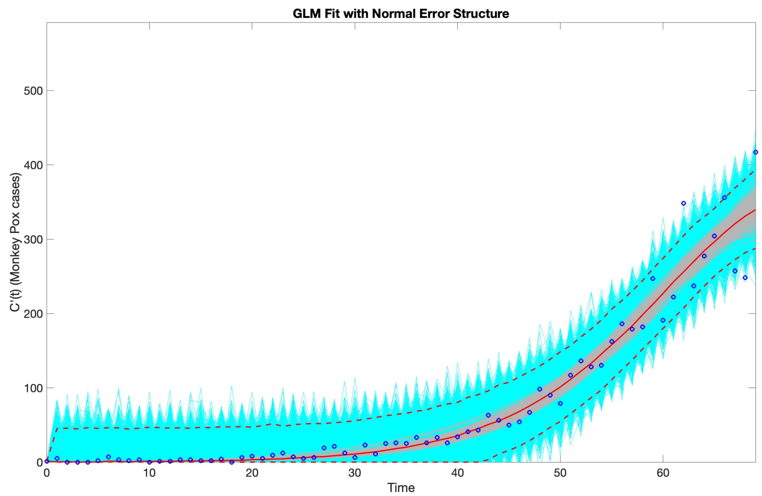

Figure 1: GLM fit with Normal error structure

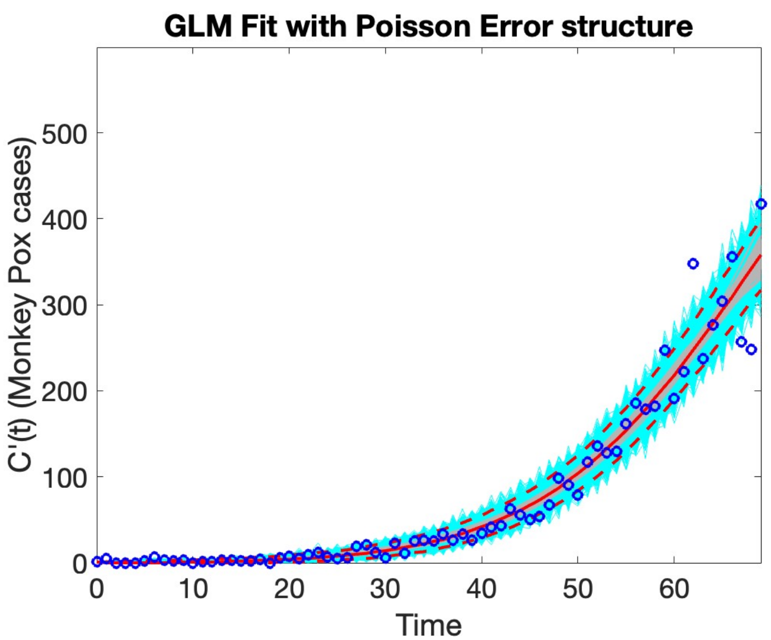

Figure 2: GLM fit with Poisson error structure

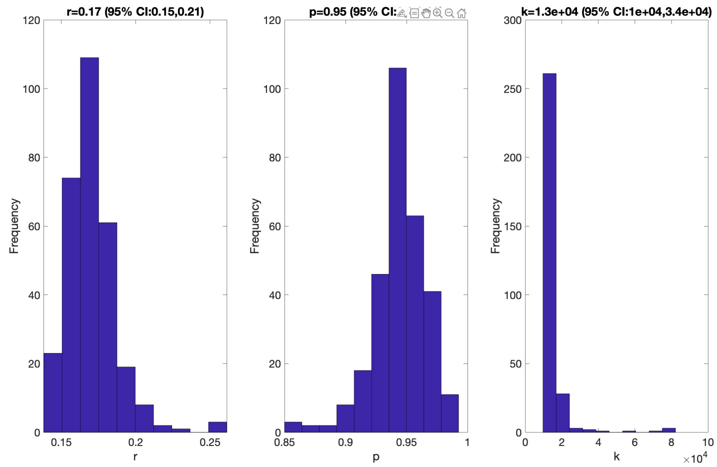

Figure 3: Normal error structure

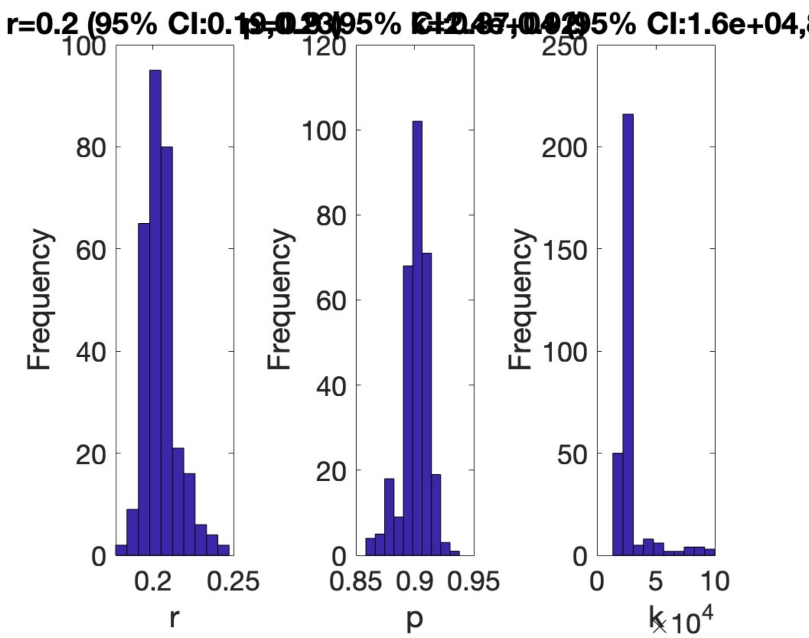

Figure 4: Poisson error structure

2. Generate Short-term Forecasts

We utilize the `Run_Forecasting_ODEModel.m` function to forecast the most recent 30 days of data using the GLM model calibrated with historical data from 2024. The results was plotted using `plotForecast_ODEModel.m` and the forecast was analyzed in comparison to the actual data.

The diagrams below shows the forecast the most recent 30 days for the GLM method.

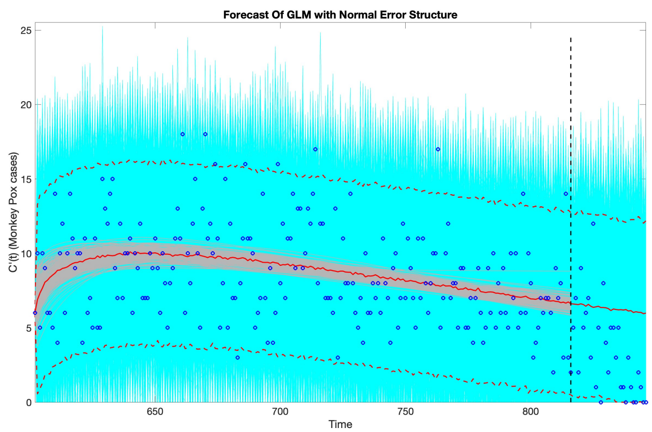

Figure 5: GLM forecast with Normal error structure

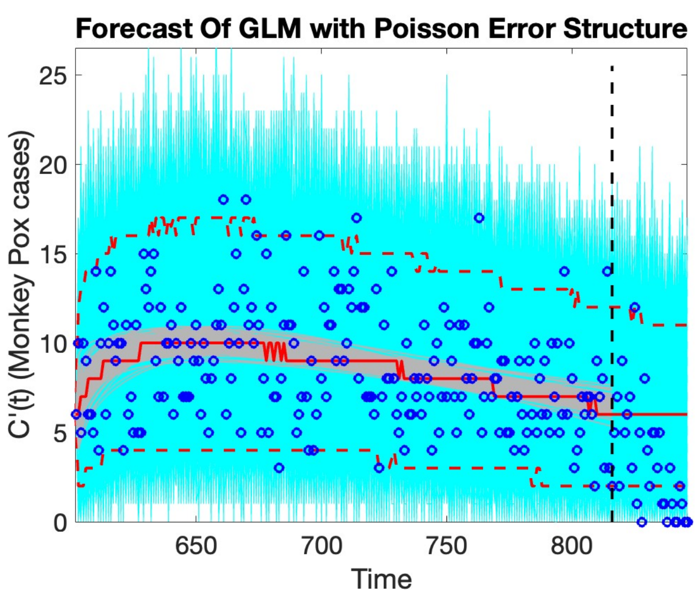

Figure 6: GLM forecast with Poisson error structure

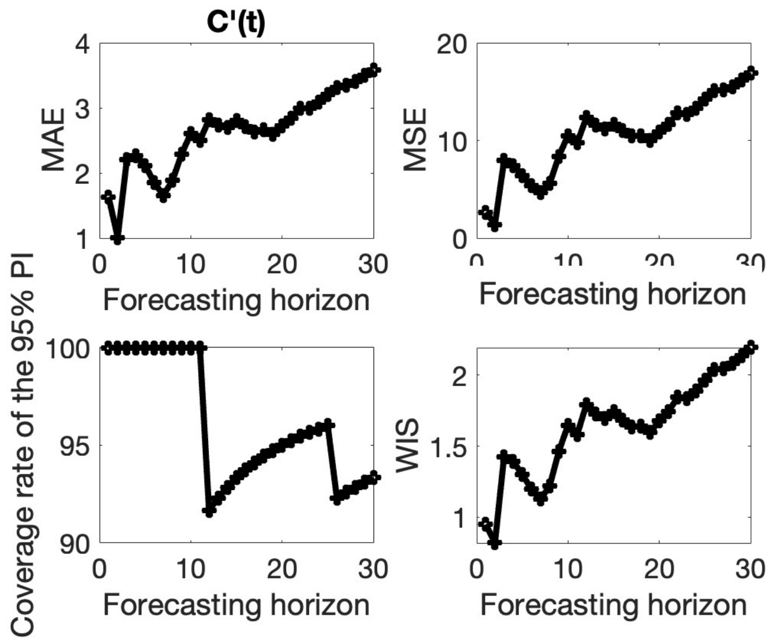

Figure 7: Normal error performance metric

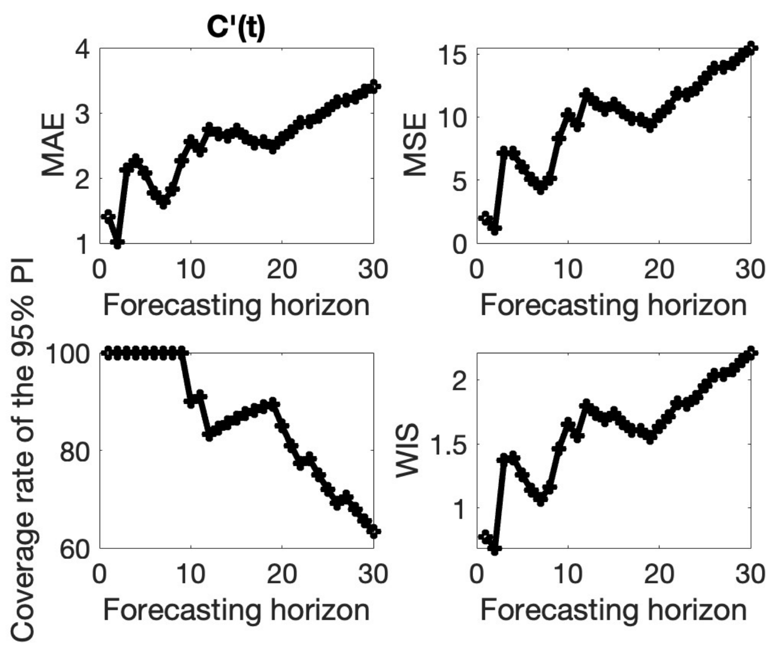

Figure 8: Poisson error performance metric

Comparison of Forecasts to Actual Data.

For the GLM forecast with Normal error structure, the actual data points represented by blue dots are scattered around the forecast line. This shows that there is some deviations at the upper and lower limits. The model successfully identifies the overall downward trend but occasionally fails to precisely capture the peaks.
For the GLM forecast with Poisson error structure, the red forecast line is compared against the actual data in blue dots. Unlike the Normal error model, the Poisson model exhibits a more ”step-like” progression, reflecting changes in a more segmented way. This approach tends to fit the actual data more effectively.

3. Performance Metrics

The evaluation of forecast performance metrics involving: Mean Absolute Error (MAE), Mean Squared Error (MSE), and Weighted Interval Score (WIS), are presented in the tables below. The interpretation of the model's forecasting performance is also reported.

Metric	**Normal Error**	**Poisson Error**
Mean Absolute Error (MAE)	3.5789	3.412
Mean Squared Error (MSE)	16.8913	15.497
Coverage of the 95% PI	93.333%	63.333%
Weighted Interval Score (WIS)	2.194	2.210

Comparison of performance forecast of Normal Error structure and Poisson Error structure

Metric	**Normal Error**	**Poisson Error**
Mean Absolute Error (MAE)	11.683	11.517
Mean Squared Error (MSE)	500.524	516.879
Coverage of the 95% PI	93%	80%
Weighted Interval Score (WIS)	8.941	8.264

Comparison of calibration Performance of Normal Error structure and Poisson Error structure

Interpretation of the model's forecasting performance

1. The Poisson error structure has a lower MAE (3.412) compared to the Normal error structure (3.5789). This indicates that, on average, the Poisson model's forecasts are slightly closer to the actual values, making it marginally more accurate in terms of absolute errors.

2. The Poisson error structure also shows a lower MSE (15.497) than the Normal error structure (16.8913). This suggests that the Poisson model not only has fewer errors overall but also fewer large deviations from the actual data, indicating a better fit in capturing extreme values.

3. The Normal error structure achieves a much higher coverage at 93.333%, while the Poisson error structure only covers 63.333%. This means that the Normal model is far more reliable in capturing the actual data within its forecasted range, indicating superior uncertainty estimation and more consistent prediction intervals.

4. The Weighted Interval Score (WIS) are very close, with the Normal error structure scoring 2.194 and the Poisson error structure at 2.210. Although the difference is small, the slightly lower WIS for the Normal error structure suggests that it provides marginally more accurate and sharper interval forecasts.

4. Discussion Questions

This section involves comparing the fitted models and forecast results using the normal error structure and Poisson error structure.

1. Comparison of Fitted Models Using Different Error Structures.
The analysis includes a comparison of the fitted models and their forecast results by employing two distinct error structure assumptions: the normal error structure and the Poisson error structure. This comparison allows for an evaluation of how these differing assumptions impact the model performance and the accuracy of predictions.

2. Key Benefits and Limitations of Phenomenological Growth Models.

Phenomenological growth models are easy to use, efficient, and provide quick short-term forecasts based on real data. However, they don't explain why epidemics spread, struggle with long-term predictions, and can be affected by poor-quality data.

3. Importance of Uncertainty Quantification in Epidemic Forecasting.

Quantifying uncertainty helps us understand how reliable our model predictions are. It shows that forecasts can be wrong, which helps public health officials make better decisions. Forecasts typically depend on parameters, such as infection and recovery rates, which may carry inherent errors. Incorporating uncertainty quantification helps models account for data variability and inaccuracies.

4. Role of Prediction Intervals in Public Health Policy.

Role of Prediction Intervals in Public Health Policy Prediction intervals give a range of possible outcomes instead of a single prediction, showing uncertainty in forecasts. This helps public health officials plan for different scenarios and adjust strategies, like social distancing or vaccinations, based on what might happen.