CS 241, Lecture 8 - Non-Deterministic Finite Automata

1 Non-Deterministic Finite Automata

- An **NFA** is a 5-tuple: $(\sigma, Q, q_0, A, \delta)$:
  - Σ is a finite non-empty set (alphabet)
  - Q is a finite non-empty set of states
  - $q_0 \in Q$ is a start state
  - $A \subseteq Q$ is a set of accepting states
  - $\delta : (Q \times \Sigma) \to 2^Q$ is our total transition function, denoting ther *power set* of Q
- We can extend δ to $\delta^* : (2^Q \times \Sigma^*) \to 2^Q$:

$$\begin{aligned}\delta^* : (2^Q \times \Sigma^*) &\to 2^Q \\ (S, \epsilon) &\to S \\ (S, a) &\to \delta^*(\cup_{q \in S} \delta(q, a), w)\end{aligned}$$

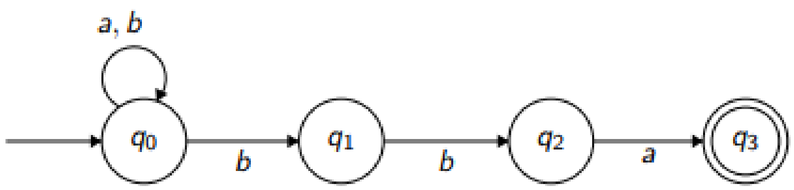
  where $a \in \Sigma$.

- In other words, an NFA given by $M = (\Sigma, Q, q_0, A, \delta)$ **accepts a string** w iff $\delta^*(\{q_0\}, w) \cup A \neq \varnothing$.
- This can be simulated like so with code:

Algorithm 1 Algorithm to Simulate an NFA

$S = \{q_0\}$
while not EOF **do**
    c = read_char()
    $S = \bigcup_{q \in S} \delta(q, c)$
end while
if $S \cap A \neq \emptyset$ **then**
    Accept
else
    Reject
end if

- For example, if $\Sigma = \{a, b\}, L = \{w : w \text{ ends with } bba\}$:
  Example: $\Sigma = \{a, b\}$, $L = \{w : w \text{ ends with } bba\}$

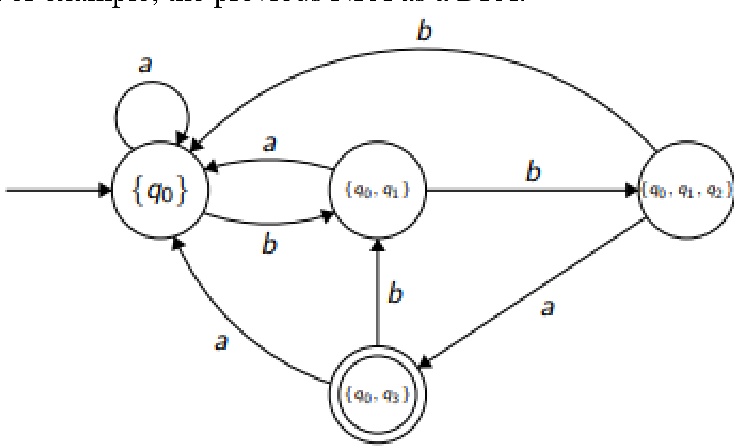


Processed	Remaining	S
ϵ	$abbba$	$\{q_0\}$
a	$bbba$	$\{q_0\}$
ab	bba	$\{q_0, q_1\}$
abb	ba	$\{q_0, q_1, q_2\}$
$abbb$	a	$\{q_0, q_1, q_2\}$
$abbba$	ϵ	$\{q_0, q_3\}$

Since $\{q_0, q_3\} \cap \{q_3\} \neq \emptyset$, accept.

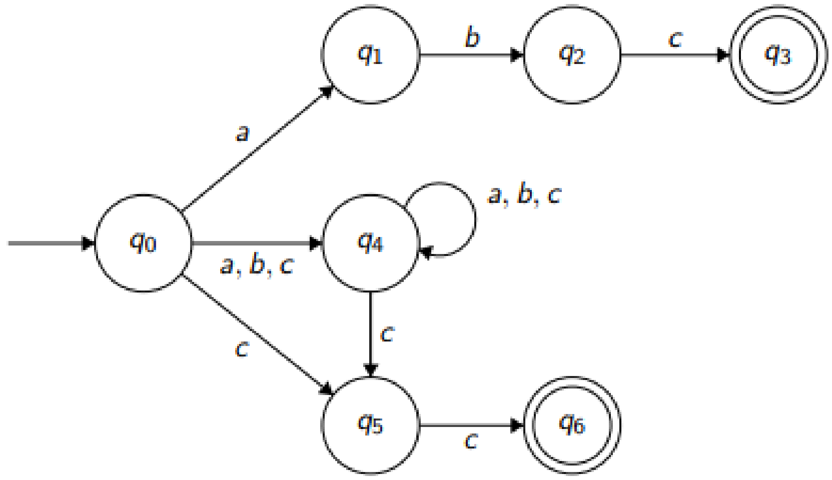
- To convert an NFA to a DFA, we start with state $S = \{q_0\}$. We then go to the NFA and determine what happens on each $a \in \Sigma$ for each $q \in S$. We repeat the previous step until we have every possibility. Accepting states are any states that included an accepting state in the NFA.

- For example, the previous NFA as a DFA:

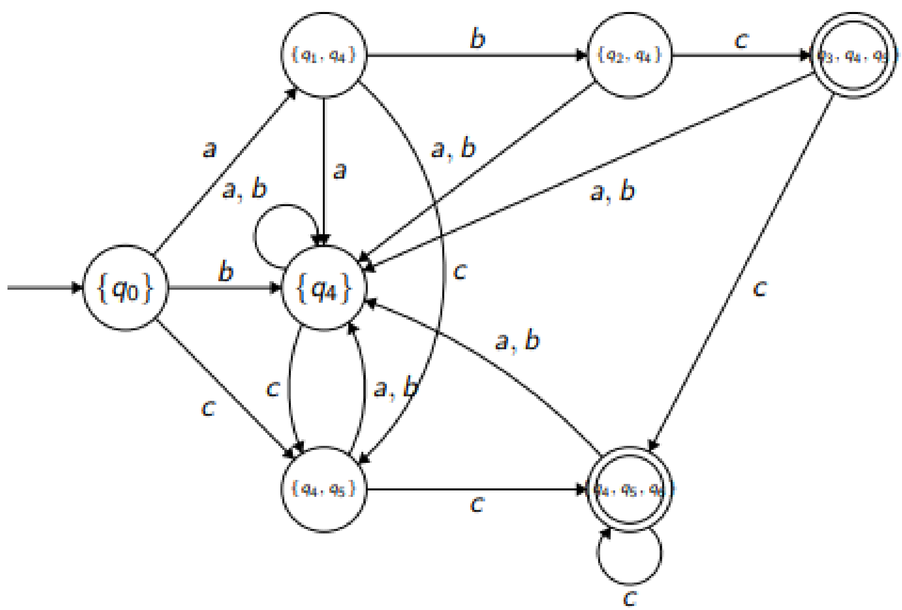


- Let us try another example. Let $\Sigma = \{a, b, c\}$. Write an NFA and DFA for the following examples:
  - $L = \{abc\} \cup \{w : w \text{ ends with } cc\}$
  - $L = \{abc\} \cup \{w : w \text{ contains } cc\}$

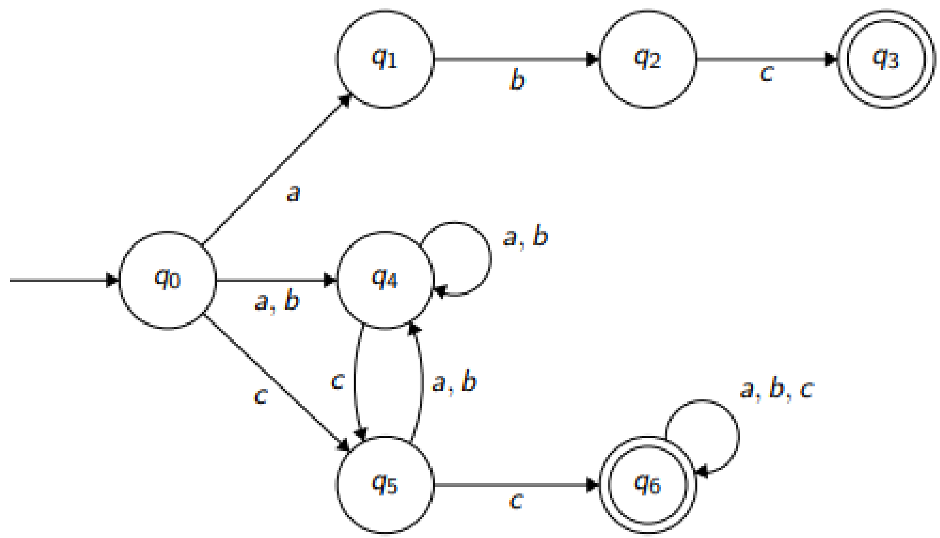
- First example NFA:
  $L = \{abc\} \cup \{w : w \text{ ends with } cc\}$



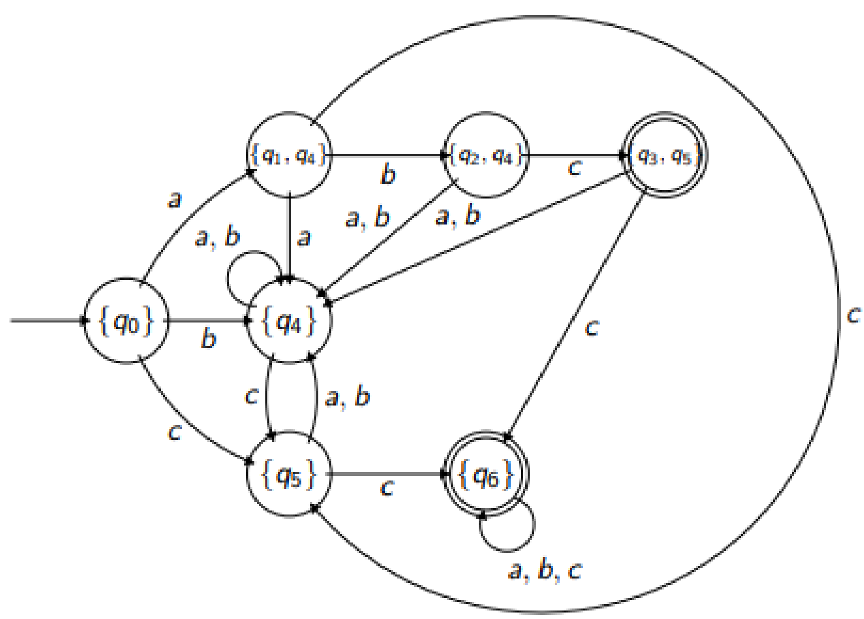
- First example DFA:



- Second example NFA:
  $L = \{abc\} \cup \{w : w \text{ contains a copy of } cc\}$

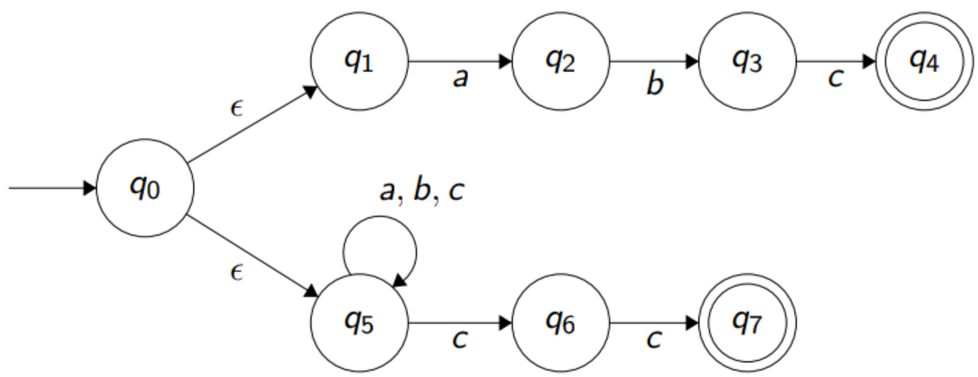


- Second example DFA:



2 ϵ-NFA

- ϵ transitions are state changes without reading a character.
- We define a ϵ-**NFA** is a 5-tuple $(\Sigma, Q, q_0, A, \delta)$:
  - Σ is a finite non-empty set (alphabet) that does **not** contain the symbol ϵ
  - Q is a finite non-empty set of states
  - $q_0 \in Q$ is a start state
  - $A \subseteq Q$ is a set of accepting states
  - $\delta : (Q \times \Sigma \cup \{\epsilon\}) \to 2^Q$ is our total transition function, where 2^Q denotes the power set of Q, the set of all subsets of Q
- ϵ-transitions make it trivial to take the union of two NFAs. For example, for $L = \{abc\} \cup \{w : w \text{ ends with } cc\}$:



Processed	Remaining	S
ϵ	$abcaccc$	$\{q_0, q_1, q_5\}$
a	$bcaccc$	$\{q_2, q_5\}$
ab	$caccc$	$\{q_3, q_5\}$
abc	$accc$	$\{q_4, q_5, q_6\}$
$abca$	ccc	$\{q_5\}$
$abcac$	cc	$\{q_5, q_6\}$
$abcacc$	c	$\{q_5, q_6, q_7\}$
$abcaccc$	ϵ	$\{q_5, q_6, q_7\}$

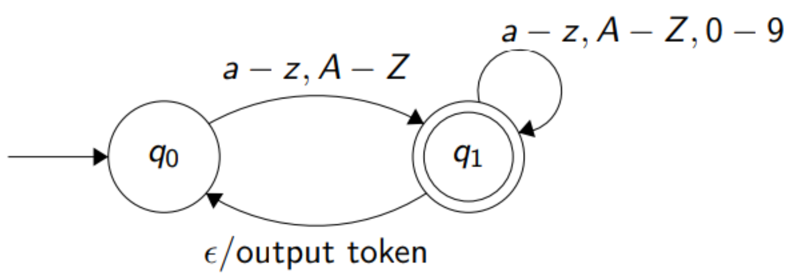
Since $\{q_5, q_6, q_7\} \cap \{q_4, q_7\} \neq \emptyset$, accept.

- If we were to let $E(S)$ to be the epsilon closure of a set of states S (set of all states reachable from S in 0 or more ϵ-transitions. This implies $S \subset E(S)$.
- We can simulate this like so:

Algorithm 2 Algorithm to Simulate an ϵ-NFA

$S = E(\{q_0\})$
while not EOF **do**
    c = read_char()
    $S = E\left(\cup_{q \in S} \delta(q, c)\right)$
end while
if $S \cap A \neq \emptyset$ **then**
    Accept
else
    Reject
end if

- $epsilon$-NFAs that recognize regular languages:
  - $\varnothing$
  - $\{\epsilon\}$
  - $\{a\}$
  - $L_1 \cup L_2$ (that is, given ϵ-NFAs that recognize L_1 and L_2 already, you can point q_0 to the two L_1 and L_2 machines)
  - $L_1 L_2$ (that is, given ϵ-NFAs that recognize L_1 and L_2, you can point an accepting state in the $\epsilon - NFA$ of L_1 to the start state of L_2)
  - L^* (assume we have a ϵ-NFA for L already, then from each accepting state, add an ϵ transition back to the newly created start state
- We can convert every ϵ-NFA to a DFA, following the above technique for normal NFAs.
- By Kleene's Theorem, this implies every language recognized by an ϵ-NFA is regular.
- We can do an example for the language L of ID tokens in C:



- But if we have the input of *abcde*, we could get from 1 to 5 different tokens - what can we do? We introduce maximal and simplified maximal munch

3 Maximal Munch and Simplified Maximal Munch

- Maximal munch consumes characters until we no longer have a valid transition. If we have characters left to consume, backtrack to the *last* valid accepting state, and resume
- Simplified maximal munch consumes characters until we no longer have a valid transition. If we are in an accepting state, produce the token and proceed. Otherwise, go to an error state.