CS 486 — Lecture 23: Game Theory

1 Pareto Dominance

- An outcome o Pareto dominates another outcome o' iff every player is weakly better off in o and at least one player is strictly better off in o. That is:

$$U_i(o) \geq U_i(o'), \forall i$$
$$U_i(0) > U_i(o'), \exists i$$

- A Pareto optimal outcome is an outcome o such that no other outcome o' Pareto dominates o.

2 Prisoner's Dilemma

- Follows a game like so: So, the optimal move for *all* of them is for them to both not say anything.

	Cooperate	Defect
Cooperate	(-1, -1)	(-3, 0)
Defect	(0, -3)	(-2, -2)

- But if they both snitch, they get the objectively *worst* reward for all of them.
- And if only one betrays the other, then one gets the worst possible reward for the player specifically, while one gets the best possible reward.
- By dominant strategy equilibrium, both players defecting is the dominant strategy!
- By pure-strategy Nash equilibria, we have 1 pure outcome — both defecting!
- Now, let's apply Pareto optimality. There are 3 Pareto optimal outcomes — both cooperating, and one defecting (for both players).

3 Mixed Strategy Nash Equilibrium

- Now let's consider a game like:

	heads	tails
heads	(1, 0)	(0, 1)
tails	(0, 1)	(1, 0)

- This game does NOT have a pure-strategy Nash equilibrium! Note this doesn't disprove the claim from before; all finite games have at least one *mixed*-strategy Nash equilibrium.
- Let's assume Bob plays heads with probability q, and Alice with probability p.
- Alice should choose a value for p such that Bob is indifferent between his actions — in this case, $p = 0.5$.
- Likewise, $q = 0.5$ for the same reasoning, but now it's for Bob.
- Each player should, in general, choose their mixing probability to make other players indifferent between their actions.