CS 241, Lecture 6 - Deterministic Finite Automata

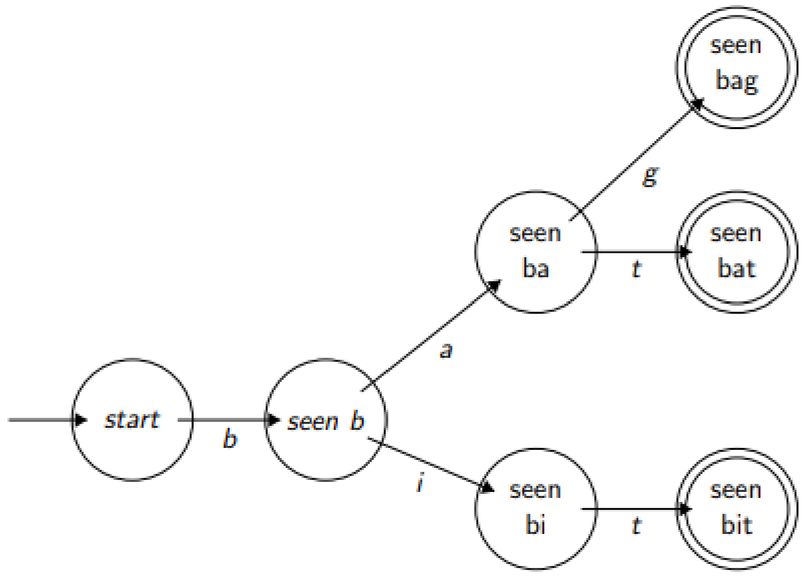
Note that from this point on, I'm using "s" to define "$" when within code blocks.

1 Finite Languages and Membership

- Recall us defining languages and words.
- How can we efficiently determine membership for a finite language? This is the easiest case.
- The naive way of doing it would be to check EVERY single word until we hit it or run through the entire list of words in the language.
- More efficiently, though, we could basically use a structure like a trie, where we determine if a word is in a language based on the previous character.
- For example:

```
if char[0] == 'b' {
    if char[1] == 'a' {
        if char[2] == 'g' {
            accept();
        }
        else if char[2] == 't' {
            accept();
        }
        else if no_more_chars(char*) reject();
    }
    else if char[2] == 'e' {
        if char[3] == 't' {
            accept();
        }
        else if no_more_chars(char*) reject();
    }
}
else reject();
```

- We can represent this pictorially:



- We place an arrow into the inital start state.
- Accepting states are two circles.
- Label your arrows from state to state.
- In CS 241, we do not need to include error states. You'll need them for CS 360 though. If your bubble does not have a valid arrow leaving it, we assume that this means it will go into an error state.

2 Regular Languages

- A **regular language** over an alphabet $\sum$ consists of one of the following:
  1. The empty language and the language consisting of the empty word are regular.
  2. All languages $\{s\}$ for all $s \in \sum$ are regular.
  3. The union, concatenation, or Kleene star of any two reg. lang. are also regular.
  4. Nothing else.
- Let L, L_1, L_2 be two reg. lang. Then the following are regular languages:
  - Union: $L_1 \bigcup L_2 = \{x : x \in L_1 \text{ or } x \in L_2\}$
  - Concatenation: $L_1 \cdot L_2 = L_1L_2 = \{xy : x \in L_1, y \in L_2\}$
  - Kleene star: $L* = \{\epsilon\} \bigcup \{xy : x \in L*, y \in L\} = \bigcup_{n=0}^{\infty} L^n$
    where $L^n = \begin{cases} \{\epsilon\} \text{ if } n = 0 \\ LL^{n-1} \text{ otherwise} \end{cases}$

  - Equivalently, $L*$ is the set of all strings consisting of 0 or more occurences of strings from L concatenated together.

- For example, suppose $L_1 = \{$up, down$\}$, L_2 = $\{$hill, load$\}$, and $L = \{a, b\}$. Then:
  - $L_1 \bigcup L_2 = \{$up, down, hill, load$\}$
  - $L_1 L_2 = \{$uphill, upload, downhill, download$\}$
  - $L* = \{\epsilon, a, b, aa, ab, ba, bb, aaa, aab, aba, \dots\}$

- Another sample question: Let $\sum = \{a, b\}$. Explain why $L = \{ab^n a : n \in \mathbb{N}\}$ is regular.

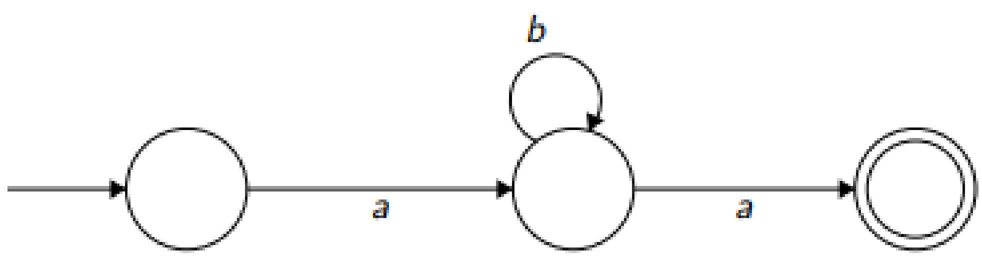
- **Solution:** Since $\{a\}$ is regular and $\{b\}*$ is also regular as $\{b\}$ is regular, then the concatenation of $\{a\} \cdot \{b\} * \cdot \{a\}$ is also regular.

- A regular expression is basically a regular language as well. We just drop all the set notation.
  - Union uses a pipe (|) now.
  - Concationation is still $\cdot$ or just put them together.
  - Kleene star stays the same.
  - Order of operations: * $>$ $\cdot$ $>$ $|$.
  - For example, the previous example would translate to $ab * a$.

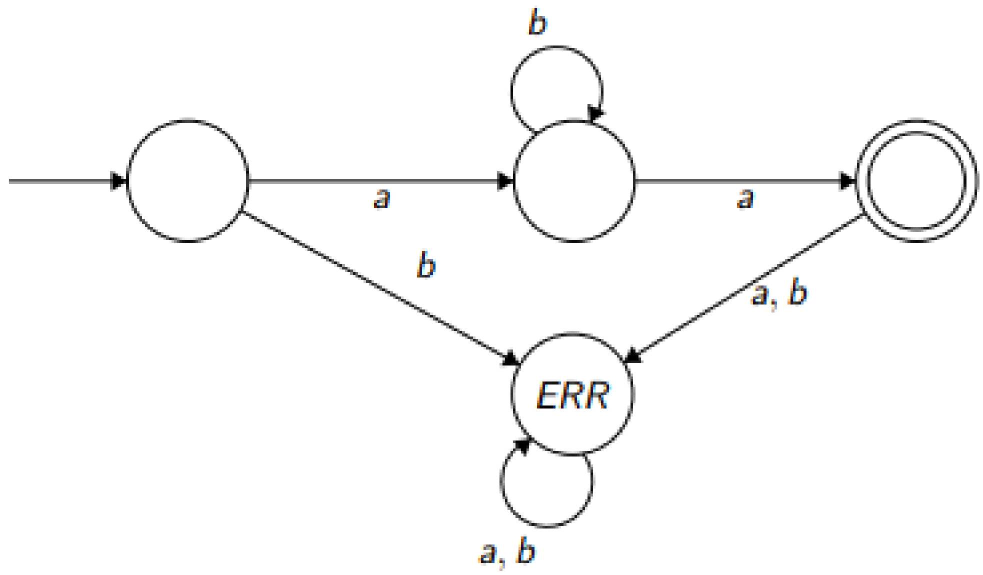
3 Deterministic Finite Automata

- We can use our earlier pictorial representation to represent a regular language IF we allow our picture to include loops!

- This is, of course, to support Kleene stars. For example:



- Or, if you're in CS 360:



- These machines are called **Deterministic Finite Automata (DFA):**
  - A DFA is a 5-tuple $(\sum, Q, q_0, A, \delta)$:
    * $\sum$ is a finite non-empty set (alphabet)
    * Q is a finite non-empty set of states
    * $q_0 \in Q$ is a start state
    * $A \subseteq Q$ is a set of accepting states
    * $\delta : (Q \times \sum) \to Q$ is our transition function - given a state and a symbol of our alphabet, which state do we go to? With no explicit arrows, this means, go to the error state.

- For example, let's consider MIPS labels (Carmen forgot to add the image):
  -¿ [q0] -(a-z. A-Z)-¿ [q1](loops with (a-z, A-Z, 0-9)) -(:)-¿ [[q2]]

- Let's try another DFA example. Write DFAs over $\sum = \{a, b\}$ that:

  1. Accepts only words with an even number of a's
  2. Accepts only words with an odd number of a's and an even number of b's
  3. Accepts only words where the parity of the number of a's is equal to that of the number of b's

- We'll do the third one:

```
--> [even a, even b] --a--> [odd a, even b] <--b--> [odd a
    , odd b]
        ^
        |
        b                                                  ^
        V                                                  |
  [even a, odd b] --b--> [even a, even b] ---------<---a --|
```

¡++¿