# CS 486 — Lecture 22: Game Theory

## 1 Intro to Game Theory

- Game theory: given a game, how would agents play it?
- Mechanism design: how should we design the rules of the game so agents behave the way we want them to? Almost like reverse game theory.
- We focus mostly on game theory.
- A game can be:
  - Cooperative where agents have a common goal.
  - Competitive where agents have conflicting goals.
  - Or something in between.

## 2 Dominant Strategy Equilibrium

- Consider a game, where we have two players, Alice and Bob (Alice is rows, Bob is columns):

| | home | dancing |
|---|---|---|
| home | (0, 0) | (0, 1) |
| dancing | (1, 0) | (2, 2) |

- Suppose both cannot communicate with each other. So, they must independently make a decision.
- Furthermore, they must choose at the same time, and cannot observe the other player's action (simultaneous move game).
- We see that the utility is maximized by both of them dancing.
- Let us denote $\sigma_i$ as the strategy of player $i$, and $\sigma_{-i}$ as the strategies of all players except $i$.
- Let us denote $U_i(\sigma) = U_i(\sigma_i, \sigma_{-i})$ as the utility of agent $i$ under the strategy profile $\sigma$.
- We define a strategy, $\sigma_i$, to dominate another one, $\sigma_i'$, if:

$$U_i(\sigma_i, \sigma_{-i}) \geq U_i(\sigma_i', \sigma_{-i}), \forall \sigma_{-i}$$
$$U_i(\sigma_i, \sigma_{-i}) > U_i(\sigma_i', \sigma_{-i}), \exists \sigma_{-i}$$

  Or in other words:

  - The strategy is as good or better than the other strategy for all opposing strategies.
  - The strategy must be strictly better for at least one opposing strategy.
- A dominant strategy dominates *all* other strategies.
- When each player has a dominant strategy, this is called a dominant strategy equilibrium.

## 3 Nash Equilibrium

- Now let's consider another game:

| | dancing | running |
|---|---|---|
| dancing | (2, 2) | (0, 0) |
| running | (0, 0) | (1, 1) |

- So, we know the optimal move is for them to both dance.
- But it's also reasonable for them to both choose to go running.
- This game has *no* dominant strategy equilibrium! Bob's choice would make Alice prefer another action (game is symmetric so WLOG)! So, there is no dominant strategy.
- So what can we do?
- The Nash equilibrium is our answer.
- Given a strategy profile, $(\sigma_i, \sigma_{-i})$, agent $i$'s strategy is a best response to other agents' strategies iff:

$$U_i(\sigma_i, \sigma_{-1}) \geq U_i(\sigma_i', \sigma_{-i}), \forall \sigma_i' \neq \sigma_i$$

  In other words, given what other agents are doing, $\sigma_i$ is the best choice for *me*.
- Nash equilibrium is a strategy profile $\sigma$ iff each agent $i$'s strategy $\sigma_i$ is a best response to the other agents' strategies $\sigma_{-i}$.
- Both (dancing, dancing) and (running, running) are Nash equilibria in the given game.