A3Q3

Question (2 marks total):

There is one averaging attribute that widely used for rates and ratios. We called it $'Harmonic\ Mean'$. The *harmonic mean* is defined as:

$$a(x_1, ..., x_N) = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_N}}$$

Derive the sensitivity curve for the *harmonic mean*.

Answers:

We first list the population with and without the added variate.

$$\mathcal{P} = \{y_1, ..., y_{N-1}\} \quad \rightarrow \quad a(\mathcal{P}) = \frac{N-1}{\frac{1}{y_1} + \frac{1}{y_2} + ... + \frac{1}{y_{N-1}}}$$

$$\mathcal{P}^* = \{y_1, ..., y_{N-1}, y\} \quad \rightarrow \quad a(\mathcal{P}^*) = \frac{N}{\frac{1}{y_1} + \frac{1}{y_2} + ... + \frac{1}{y_{N-1}} + \frac{1}{y}} \qquad (0.5\ \ marks)$$

then we calculate the sensitivity curve:

$$\begin{aligned}
SC(y) &= N[a(\mathcal{P}^*) - a(\mathcal{P})] \\
&= N\left(\frac{N}{\frac{1}{y_1} + \frac{1}{y_2} + ... + \frac{1}{y_{N-1}} + \frac{1}{y}} - \frac{N-1}{\frac{1}{y_1} + \frac{1}{y_2} + ... + \frac{1}{y_{N-1}}}\right) \qquad (1\ \ marks) \\
Let\ \ \frac{1}{y_1} + \frac{1}{y_2} + ... + \frac{1}{y_{N-1}} &= A\ \ and\ \ \frac{1}{y_1} + \frac{1}{y_2} + ... + \frac{1}{y_{N-1}} + \frac{1}{y} = A + \frac{1}{y} \\
then\ \ SC(y) &= N\left(\frac{N}{A + \frac{1}{y}} - \frac{N-1}{A}\right) \\
&= \frac{N^2}{A + \frac{1}{y}} - \frac{N^2 - N}{A} \\
&= \frac{N^2 A - (N^2 - N)(A + \frac{1}{y})}{(A + \frac{1}{y})A} \\
&= \frac{N^2 A - N^2 A - \frac{N^2}{y} + NA + \frac{N}{y}}{(A + \frac{1}{y})A} \\
&= \frac{\frac{N^2}{y} + NA + \frac{N}{y}}{(A + \frac{1}{y})A} \\
&= \frac{N^2 + N + ANy}{A(Ay + 1)} \\
&= \frac{N^2 + N + Ny(\frac{1}{y_1} + \frac{1}{y_2} + ... + \frac{1}{y_{N-1}})}{(\frac{1}{y_1} + \frac{1}{y_2} + ... + \frac{1}{y_{N-1}})[y(\frac{1}{y_1} + \frac{1}{y_2} + ... + \frac{1}{y_{N-1}}) + 1]} \qquad (0.5\ \ marks)
\end{aligned}$$