# Q2

$$\bar{y} = \frac{1}{N}\sum_{u\in\mathcal{P}} y_u$$

$$SD_{\mathcal{P}}(y) = \sqrt{\frac{1}{N}\sum_{u\in\mathcal{P}}(m \times y_u - m \times \bar{y})^2}$$

## a)

$$\begin{aligned}
a(y_1+b, y_2+b, ..., y_N+b) &= \frac{\frac{1}{N}\sum_{u\in\mathcal{P}}(y_u+b-\bar{y}-b)^3}{\left[\sqrt{\frac{1}{N}\sum_{u\in\mathcal{P}}(y_u+b-\bar{y}-b)^2}\right]^3} \\
&= \frac{\frac{1}{N}\sum_{u\in\mathcal{P}}(y_u-\bar{y})^3}{\left[\sqrt{\frac{1}{N}\sum_{u\in\mathcal{P}}(y_u-\bar{y})^2}\right]^3} \\
&= a(y_1, y_2, ..., y_N)
\end{aligned}$$

Since changing in location does not change the attribute, the skewness coefficient is location invariant.

## b)

$$\begin{aligned}
a(m \times y_1, m \times y_2, ..., m \times y_N) &= \frac{\frac{1}{N}\sum_{u\in\mathcal{P}}(m \times y_u - m \times \bar{y})^3}{\left[\sqrt{\frac{1}{N}\sum_{u\in\mathcal{P}}(m \times y_u - m \times \bar{y})^2}\right]^3} \\
&= \frac{\frac{m^3}{N}\sum_{u\in\mathcal{P}}(y_u-\bar{y})^3}{\left[\sqrt{\frac{m^2}{N}\sum_{u\in\mathcal{P}}(y_u-\bar{y})^2}\right]^3} \\
&= \frac{\frac{m^3}{N}\sum_{u\in\mathcal{P}}(y_u-\bar{y})^3}{\left[m \times \sqrt{\frac{1}{N}\sum_{u\in\mathcal{P}}(y_u-\bar{y})^2}\right]^3} \\
&= \frac{\frac{m^3}{N}\sum_{u\in\mathcal{P}}(y_u-\bar{y})^3}{m^3 \times \left[\sqrt{\frac{1}{N}\sum_{u\in\mathcal{P}}(y_u-\bar{y})^2}\right]^3} \\
&= \frac{\frac{1}{N}\sum_{u\in\mathcal{P}}(y_u-\bar{y})^3}{\left[\sqrt{\frac{1}{N}\sum_{u\in\mathcal{P}}(y_u-\bar{y})^2}\right]^3} \\
&= a(y_1, y_2, ..., y_N)
\end{aligned}$$

Therefore, similar to the effect of location, multiplying by m>0 does not change skewness coefficient, it is scale invariant.

## c)

$$
\begin{aligned}
a(m \times y_1 + b, m \times y_2 + b, ..., m \times y_N + b) &= \frac{\frac{1}{N}\sum_{u\in\mathcal{P}}(m \times y_u + b - m \times \bar{y} - b)^3}{\left[\sqrt{\frac{1}{N}\sum_{u\in\mathcal{P}}(m \times y_u + b - m \times \bar{y} - b)^2}\right]^3} \\
&= \frac{\frac{1}{N}\sum_{u\in\mathcal{P}}(m \times y_u - m \times \bar{y})^3}{\left[\sqrt{\frac{1}{N}\sum_{u\in\mathcal{P}}(m \times y_u - m \times \bar{y})^2}\right]^3} \\
&= \frac{\frac{m^3}{N}\sum_{u\in\mathcal{P}}(y_u - \bar{y})^3}{\left[\sqrt{\frac{m^2}{N}\sum_{u\in\mathcal{P}}(y_u - \bar{y})^2}\right]^3} \\
&= \frac{\frac{m^3}{N}\sum_{u\in\mathcal{P}}(y_u - \bar{y})^3}{\left[m \times \sqrt{\frac{1}{N}\sum_{u\in\mathcal{P}}(y_u - \bar{y})^2}\right]^3} \\
&= \frac{\frac{1}{N}\sum_{u\in\mathcal{P}}(y_u - \bar{y})^3}{\left[\sqrt{\frac{1}{N}\sum_{u\in\mathcal{P}}(y_u - \bar{y})^2}\right]^3} \\
&= a(y_1, y_2, ..., y_N)
\end{aligned}
$$

Since changing in location and scaling have no effect on skewness coefficient, it is location-scale invariant.

## d)

$$
\begin{aligned}
a(\mathcal{P}^k) &= \frac{\frac{1}{Nk}\sum_{u\in\mathcal{P}^k}(y_u - \bar{y})^3}{\left[\sqrt{\frac{1}{Nk}\sum_{u\in\mathcal{P}^k}(y_u - \bar{y})^2}\right]^3} \\
&= \frac{\frac{1}{Nk}\sum_{u\in\mathcal{P}} k \times (y_u - \bar{y})^3}{\left[\sqrt{\frac{1}{Nk}\sum_{u\in\mathcal{P}} k \times (y_u - \bar{y})^2}\right]^3} \\
&= \frac{\frac{k}{Nk}\sum_{u\in\mathcal{P}}(y_u - \bar{y})^3}{\left[\sqrt{\frac{k}{Nk}\sum_{u\in\mathcal{P}}(y_u - \bar{y})^2}\right]^3} \\
&= \frac{\frac{1}{N}\sum_{u\in\mathcal{P}}(y_u - \bar{y})^3}{\left[\sqrt{\frac{1}{N}\sum_{u\in\mathcal{P}}(y_u - \bar{y})^2}\right]^3} \\
&= a(\mathcal{P})
\end{aligned}
$$

Therefore, replication does not change the sknewness coefficient. It is replication invariant.

## e)

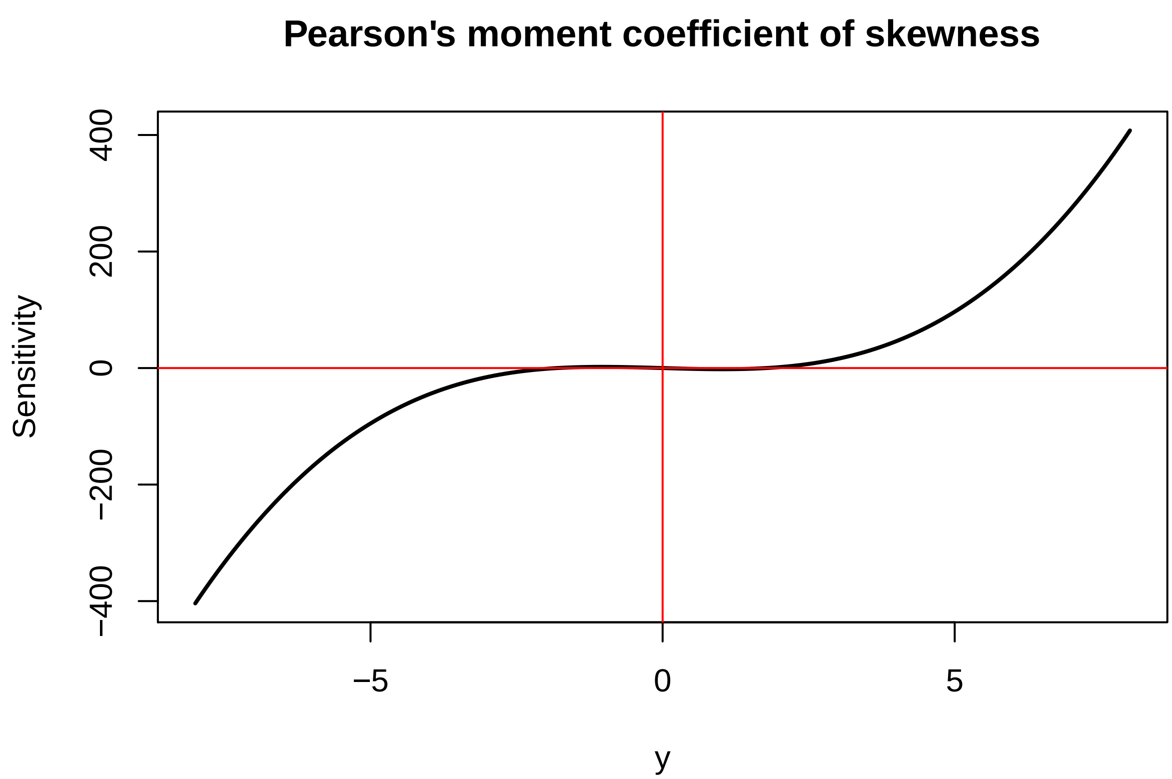
```r
sc = function(y.pop, y, attr, ...) {
  N <- length(y.pop) + 1
  Map(function(y) {  N*(attr(c(y,y.pop),...) - attr(y.pop,...))} ,y)
}

sdn <- function(x) {
  sd(x)*sqrt((length(x)-1)/length(x))
```

```
}

pearson.moment.skewness <- function(x) {
  mean((x-mean(x))^3)/sdn(x)^3
}

set.seed(341)
pop <-rnorm(1001)
y <- seq(-8,8,length.out = 1000)
plot(y, sc(pop,y, pearson.moment.skewness),
     main = "Pearson's moment coefficient of skewness",
     type = 'l', lwd = 2, ylab='Sensitivity')
abline(h=0, v=0, col= 'red')
```




## f)

Good: the skewness coefficient is invariant nom matter how we change the units and dulpicate the population

Bad: the curve is unbounded. The skewness coefficient is sensitive to large values. As y goes to infinity, sensitivity goes to infinity. As y goes to negative infinity, sensitivity goes to negative infinity.