STAT 341: Assignment 1 - My Summary of the Sensitivity Section

Name

Due: Today, at sometime

Suppose we have a population of size N-1, if we add variate of with the value y, the **sensitivity cruve** of an attribute as

$\mathcal{P} = \{y_1, \ldots, y_{N-1}, y\}$ $\mathcal{P}^* = \{y_1, \ldots, y_{N-1}, y\}$

$$\begin{aligned} SC(y\ ;\ a(\mathcal{P})) &= \frac{a(y_1,\ldots,y_{N-1},y) - a(y_1,\ldots,y_{N-1})}{\frac{1}{N}} \\ &= N\,[\ a(y_1,\ldots,y_{N-1},y) - a(y_1,\ldots,y_{N-1})\ ] \end{aligned}$$

The sensitivity curve gives a scaled measure of the effect that a single variate value y has on the value of a population attribute $a(\mathcal{P})$.

- We can explore the sensitivity curve mathematically and computationally for any particular population and any particular attribute.
- A single observation can change the average by a huge (even infinite) amount.

Example: Minimum

$$a(y_1, \ldots, y_N) = \max\{y_1, \ldots, y_N\} = y_{(N)}$$

- Derive the sensitivity curve.

Let $\mathcal{P} = \{y_1, \ldots, y_N\} = y_{(N)}$, $\mathcal{P}^* \{y_1, \ldots, y_N\} = y_{(N)}$ Then

$$a(\mathcal{P}) = \min\{y_1, \ldots, y_N\} = y_1$$

When $y < y_{(N-1)}$

$$\begin{aligned} a(\mathcal{P}^*) &= \min\{y_1, \ldots, y_{N-1}, y\} \\ &= y \end{aligned}$$

$$\begin{aligned} SC &= N[a(\mathcal{P}^*) - a(\mathcal{P})] \\ &= N[a(y_1, \ldots, y_{N-1}, y) - a(y_1, \ldots, y_{N-1})] \\ &= 0 \end{aligned}$$

When $y >= y_{(N-1)}$

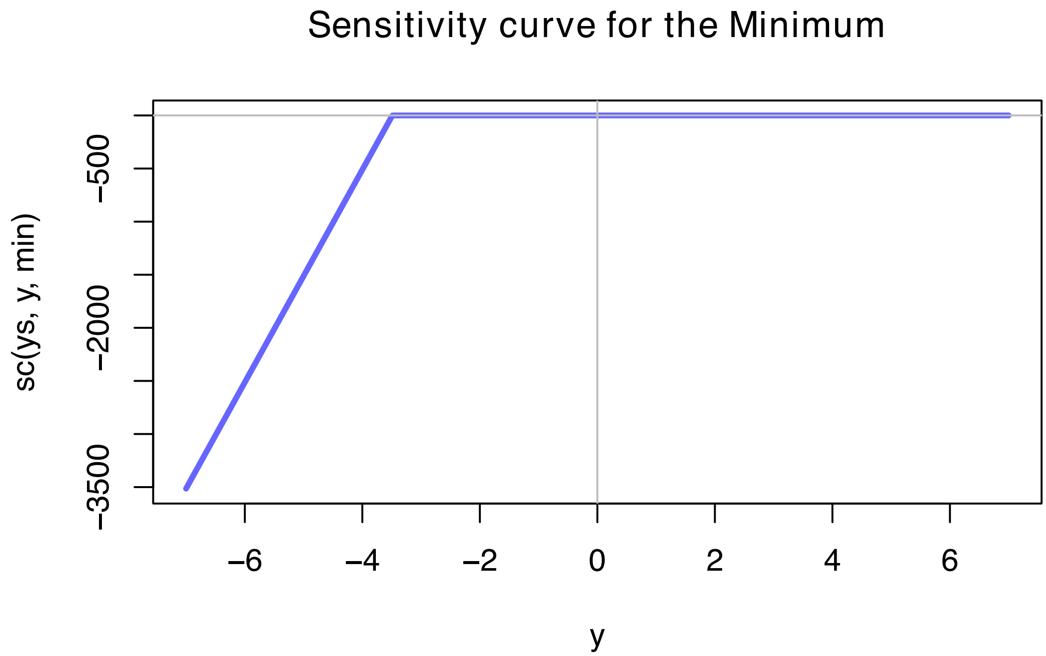
$$\begin{aligned} a(\mathcal{P}^*) &= \min\{y_1, \ldots, y_{N-1}, y\} \\ &= y_{N-1} \end{aligned}$$

$$SC = N[a(\mathcal{P}^*) - a(\mathcal{P})]$$
$$= N[a(y_1, \ldots, y_{N-1}, y) - a(y_1, \ldots, y_{N-1})]$$
$$= N[y - y_{N-1}]$$

- The sensitivity curve for the minimum is

```r
sc = function(y.pop, y, attr, ...) {
  N <- length(y.pop) +1
  sapply( y, function(y.new) {  N*(attr(c(y.new, y.pop),...) - attr(y.pop,...))  } )
}
set.seed(341)
ys <- rnorm(1000)
y <- seq(-7,7, length.out=1000)

plot(y, sc(ys, y, min), type="l", lwd = 3, col=adjustcolor("Blue", 0.6),  main="Sensitivity curve for t
abline(h=0, v=0, col="grey")
```



The sensitivity curve is unbounded for small y means that the minimum is sensitive to small values.