Stevens's Power Law

Visual perception

In this question you are going to write a little bit of Rcode and use the R's base graphics (see the cheat sheet on the course website in the software folder for R plots).

Some of the plotting functions (and their arguments) you will need to know about are the following:

- `plot(...)`
- `lines(...)`
- `abline(...)`
- `seq(...)`
- `extendrange(...)`

See the help description for any of these functions.
For example,

```
# Either
help(plot)
# or
?plot
# will produce help information on the default scatterplot function.
```

15 marks

Stevens's Power Law

a. **(3 marks)** The following (incomplete) function determines how the ratio (`x/y`) of two **actual** stimulus values (`x` and `y`) would be **perceived** according to Stevens's power law. It returns the perceived value of the ratio.

```
stevensRatio <- function(x, y = 1, beta = 1) {
 if(missing(x))  stop("Must supply a non-negative value for x")
 if(any(x < 0))  stop("Must have x >= 0")
 if(any(y <= 0)) stop("Must have y > 0")
 if(beta <= 0)   stop("Must have beta > 0")
 #
 # You need to insert code here to complete the function
 #
}
```

The first few lines provide error checking on argument values. Note that only the value `x` need be supplied; the remaining arguments have default values. Note also either `x`, or `y`, or both could be a vector; `beta` cannot.

i. *(2 marks)* Complete the `stevensRatio()` function above.

```
stevensRatio <- function(x, y = 1, beta = 1) {
    if(missing(x))  stop("Must supply a non-negative value for x")
    if(any(x < 0))  stop("Must have x >= 0")
    if(any(y <= 0)) stop("Must have y > 0")
    if(beta <= 0)   stop("Must have beta > 0")
    #
    # You need to insert code here to complete the function
    #
    return((x/y)^beta)
}
```

ii. *(1 mark)* Execute your code for the ratios 0.5 and 2.0 when $\beta = 0.7$. (Note: these were considered in class. Check that you get the same answers as in class.)

Show the code and the results **for both cases**

```
stevensRatio(x=0.5, beta =0.7)
```

```
## [1] 0.6155722
```

```
stevensRatio(x=2, beta =0.7)
```

```
## [1] 1.624505
```

b. **(7 marks)** One visual representation frequently used to encode magnitude is **length** (i.e. the length of a straight line segment).

According to Stevens's law, the perceived magnitude of length is proportional to a power β of the actual length with $0.9 \le \beta \le 1.1$.

Here you will investigate the perception of length ratios $r = x/y$ for the two extremes $\beta = 0.9$ and $\beta = 1.1$.

To this end, use the ratio

```
r <- seq(from = 0.1, to = 5, by = 0.1)
# from which you can construct the limit
lim <- extendrange(r)
```

and your function `stevensRatio()` for the following.

Construct a scatterplot showing both curves together and make sure that it has **exactly having the following characteristics**.

- the plot limits `xlim` and `ylim` should both have value `lim` (see above)
- the main plot title should be "Stevens's law for lengths"
- the x axis should be labelled "True ratio"
- the y axis should be labelled "Perceived ratio"
- the curve for $\beta = 0.9$ should be of colour `col = "firebrick"` and have line type `lty = 2` and line width `lwd = 3`
- the curve for $\beta = 1.1$ should be of colour `col = "steelblue"` and have line type `lty = 3` and line width `lwd = 4`

- there should be three solid (no dashing) `"grey"` coloured straight lines
  - a vertical one at $x = 1$,
  - a horizontal one at $y = 1$, and
  - a diagonal one at $y = x$
  - all having line width 1
- the following legend (should match the description above)

```
legend("topleft",
       legend = c("0.9", "1.1"),
       lty = c(2, 3), lwd = 3,
       col = c("firebrick", "steelblue"),
       title = expression(paste("power ", beta)))
```

i. *(4 marks)* Show the code (including the legend) used to construct the scatterplot. **Also** show the resulting scatterplot itself.

   NOTE Use the following RMarkdown variable values in the header for your R code that produces the plot.

   {r , fig.align = "center", fig.width = 6, fig.height = 6, out.width = "60%"}

```
r <- seq(from = 0.1, to = 5, by = 0.1)
# from which you can construct the limit
lim <- extendrange(r)
```

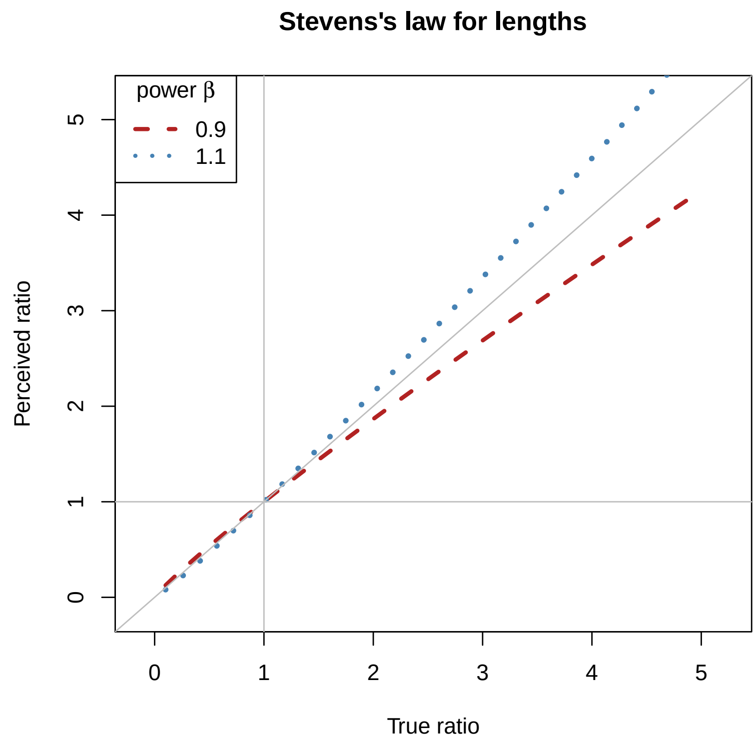
```
plot(
   r,
   stevensRatio(x = r, beta = 0.9),
   main = "Stevens's law for lengths",
   xlim = lim,
   ylim = lim,
   xlab = 'True ratio',
   ylab = 'Perceived ratio',
   col = 'firebrick',
   type = 'l',
   lty = 2,
   lwd = 3
)

lines(
   r,
   stevensRatio(x = r, beta = 1.1),
   col = 'steelblue',
   lty = 3,
   lwd = 4
)

abline(v = 1, col = 'grey', lwd = 1)
abline(h = 1, col = 'grey', lwd = 1)
abline(a = 0, b = 1, col = 'grey', lwd = 1)

legend(
   "topleft",
   legend = c("0.9", "1.1"),
```

```
    lty = c(2, 3),
    lwd = 3,
    col = c("firebrick", "steelblue"),
    title = expression(paste("power ", beta))
)
```



ii. *(3 marks)* What do you conclude about the possible perception of the length ratios when the true ratio is greater than 1? Explain your reasoning.

When the true ratio is greater than 1, for power smaller than 1, the length is perceived to be shorter than it is. For power greater than 1, the length is perceived to be longer than it is. As we can see in the graph, the $\beta = 1.1$ line nearly goes out of bound, but we can see a clear end of $\beta = 0.9$ line. Power plays a magnifying role in the graph.

c. **(5 marks)** Two other ways to visualize magnitude are to use area and volume. As in part (b) above, use your `stevensRatio()` function to produce a scatterplot having the following characteristics:

- the plot limits `xlim` and `ylim` should both have value `lim` (see part b. above)
- the main plot title should be "Stevens's law"
- the x axis should be labelled "True ratio"
- the y axis should be labelled "Perceived ratio"
- the curve for area should be of colour `col = "firebrick"` and have line type `lty = 2` and line width `lwd = 3`
- the curve for volume should be of colour `col = "steelblue"` and have line type `lty = 3` and line width `lwd = 4`
- there should be three solid (no dashing) `"grey"` coloured straight lines
  - a vertical one at $x = 1$,

- a horizontal one at $y = 1$, and
- a diagonal one at $y = x$
- all having line width 1

- there should be a legend appropriately identifying the curves for area (use the word "area") and volume (use the word "volume").
- in each case, use the middle of the range of β values that have been determined experimentally (see slides).

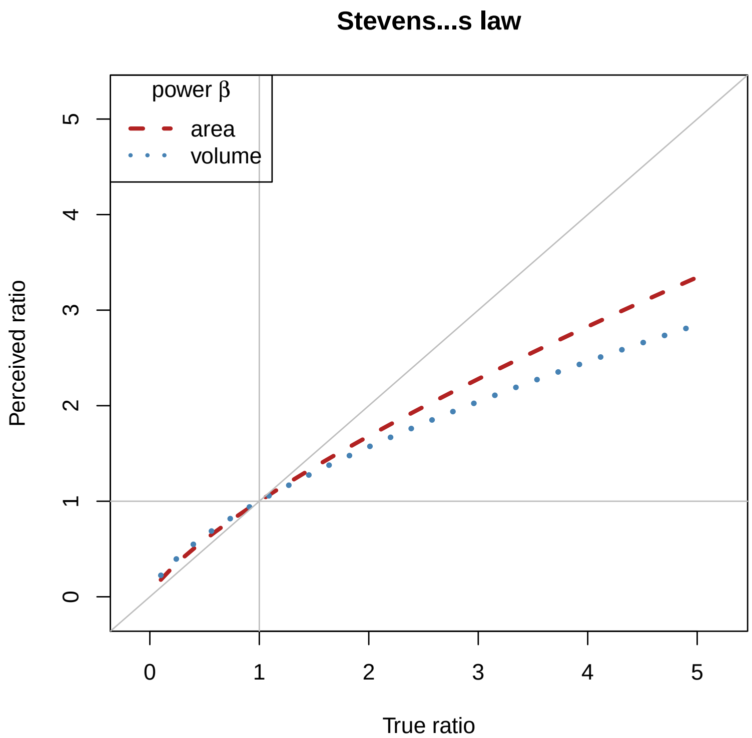
i. *(2 marks)* Produce the scatterplot and show the code used.

```
plot(
   r,
   stevensRatio(x = r, beta = 0.75),
   main = 'Stevens's law',
   xlim = lim,
   ylim = lim,
   xlab = 'True ratio',
   ylab = 'Perceived ratio',
   col = 'firebrick',
   type = 'l',
   lty = 2,
   lwd = 3
)

lines(
   r,
   stevensRatio(x = r, beta = 0.65),
   col = 'steelblue',
   lty = 3,
   lwd = 4
)

abline(v = 1, col = 'grey', lwd = 1)
abline(h = 1, col = 'grey', lwd = 1)
abline(a = 0, b = 1, col = 'grey', lwd = 1)

legend(
   "topleft",
   legend = c("area", "volume"),
   lty = c(2, 3),
   lwd = 3,
   col = c("firebrick", "steelblue"),
   title = expression(paste("power ", beta))
)
```



ii. *(2 marks)* What do you conclude about the use of area or volume when the true ratio is greater than 1? Explain your reasoning.

Large areas and volumes are perceived to be larger than they are. Small areas and columes are perceived to be smaller than they are. As we can see in the graph, when the power becomes smaller, the line becomes lower.

iii. *(1 mark)* How does using area or volume compare to using length to encode magnitude? Explain your reasoning.

We are able to see that lines for length is closer to y=x, this means that using length has smaller bias when encoding a magnitude. Area has smaller bias than the volume.