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Idealized Models of Reed Woodwinds. Part I: Analogy with the Bowed String

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Summary
Elementary models of the bowed string and of woodwinds are reviewed in order to underline their analogy. The comparison of the pressure-flow characteristics of a woodwind with the friction characteristics of a bowed string shows that the mouth pressure and the bow velocity on the one hand, and a static flow parameter for the embouchure and the static force applied by the instrumentalist perpendicularly to the motion of the string on the other hand are analogous parameters. Using idealized resonators, conical resonators are found to be analogous to a string bowed elsewhere than at the middle, the position of the bow being analogous to the length of the truncation of the conical tube. A consequence of these analogies is that idealized woodwinds can be modeled with two reflection functions like a bowed string, and that some references dealing with the bowed string can be useful to study reed woodwinds. The behaviour of common solutions, including the Helmholtz motion, is finally detailed within the scope of Raman's model applied to woodwinds.

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1. Introduction

This paper deals with idealized models of woodwinds and their similarity with bowed string models. Some analogies between these families of self-sustained oscillators have been pointed out by McIntyre *et al* [1]. The goal of this study is to show that the analogy can be extended if considering elementary models.

When studying complex systems such as bowed string instruments or wind instruments, many models can be chosen, depending on the simplifications. For the case of models which take many parameters into account, few analytical calculations can be made but the influence of the various parameters can be studied by using time domain simulations. Nevertheless, a difficulty with this approach is the experimental determination of the values of the parameters needed to run the calculations. Consequently a potentially realistic model can become very unrealistic due to erroneous values of some parameters. Another approach consists in using simplified models. Depending on the degree of simplification, these models can differ significantly from real instruments. However, they can be useful in understanding the basics of physical phenomena involved in the oscillation mechanisms. In this paper, attention is focused on the simplest idealized models of bowed strings and woodwinds.

McIntyre *et al*. [1] have pointed out the analogy between the string bowed in the middle and the clarinet. In the present paper, this analogy is extended by comparing basic models of conical-like reed woodwinds to bowed string models. In section 3 we compare elementary models commonly used for the nonlinear elements and find that a static flow parameter which characterizes the embouchure is analogous to the static bow force parameter applied by the instrumentalist perpendicularly to the motion of the string. In section 4 idealized models of a bowed string are briefly reviewed and compared to models of resonators. Two families of resonators, which can be considered as idealized truncated cones, are found to be analogous to a bowed string. Finally, in section 5, common solutions of the coupled system are examined within the scope of dispersionless models. The stability of these solutions is investigated in a companion paper [2].

2. Bowed string and woodwinds: a generic model

Bowed string and wind instruments are both self-sustained oscillators controlled by the instrumentalists throughout

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the duration of the notes. Basic models of such instruments consist in the feedback loop system of Figure 1 resulting from the coupling between a passive resonator and a nonlinear element driven by a static source of energy [1]. The oscillating variables are calculated at the coupling point, where both the equations of the resonator and the excitator must be satisfied. The radiated sound is obtained by using transfer functions between this point and the radiating point. The separation of the systems into two distinct elements implies some assumptions. In simple bowed string models, the contact area between the bow and the string is usually assumed to be reduced to a point (see [3, 4, 5] for the analysis of the differences between point bow and finite width models). In woodwind models, the nonlinear mechanism is assumed to be localized at the entrance of the mouthpiece, which volume is considered as a part of the resonator. With such assumptions, the nonlinear excitator and the resonator can be modelled separately.

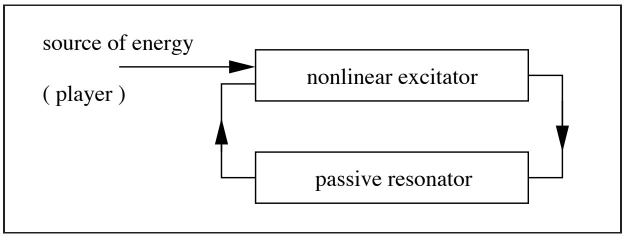

Figure 1. Schematic description of a self-sustained musical instrument.

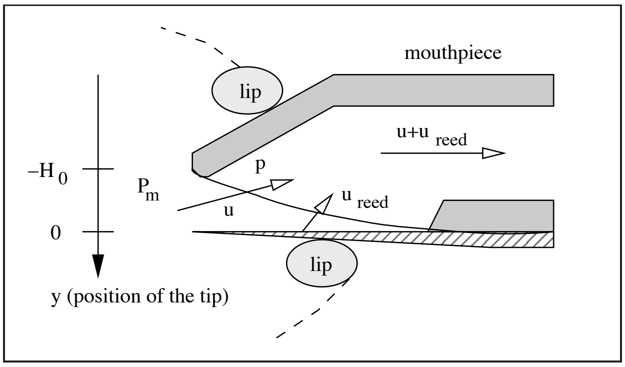

Figure 2. Schematic view of a single reed mouthpiece.

3. The nonlinear excitator

The nonlinear element of self-sustained musical instruments can convert the static energy provided by the player into oscillating motion if it is properly coupled to a resonator. Physical phenomena involved in the nonlinear process are complex in the case of both the woodwinds and the bowed string. In addition, measurement of the state variables and the playing parameters in the dynamical regime is intricate. It is therefore difficult to model the nonlinear excitator in the dynamical regime. In the case of both reed woodwinds and the bowed string, basic models for the nonlinear excitator are derived from the static behaviour, and the models are thought to be valid for the dynamical regime.

3.1. Reed instruments: the pressure-flow characteristics

The acoustic flow in the mouthpiece is the sum of the entering flow u and the flow u_{reed} created by the reed motion (see Figure 2). The flow u_{reed} is a second order contribution and is often neglected or included in the resonator impedance as a length correction if the reed inertia is neglected [6, 7]. The nonlinear mechanism in woodwinds lies in the behaviour of the air flow u entering the instrument through the reed channel when there is a pressure difference between the static mouth pressure and the pressure in the mouthpiece. In the static regime, until the mouth pressure is so high that the reed closes the input channel, the entering flow is assumed to obey to the Bernoulli equation [8]. As a first approximation, it is assumed that the fluid motion is governed by the same law for an upstream and a downstream motion. If ρ is the air density, S_j the cross-sectional area of the jet and

$$\Delta p = P_m - p \tag{1}$$

is the difference between the mouth pressure P_m and the pressure in the mouthpiece p, the application of the Bernoulli equation between the mouth and the jet leads to

$$u = S_j\sqrt{2|\Delta p|/\rho}\,\mathrm{sign}(\Delta p). \tag{2}$$

When the pressure difference Δp is greater than a “closing pressure” P_M, the reed closes the reed channel and the input flow is equal to zero.

It is usually considered that the Bernoulli equation remains valid in the oscillating regime. A major difficulty for an experimental direct determination of the relationship between the pressure and the flow in the mouthpiece is the measurement of the volume velocity. Some authors measured the velocity in dynamical regime by using hot wires [9, 10, 11], but the available data are not sufficiently accurate to question the validity of the Bernoulli equation. When the pressure difference is Δp greater than a “beating pressure”, the input channel is closed. When this occurs during a fraction of the period, the oscillation is called the “beating reed” regime. In the scope of this quasi-static model, the beating pressure is supposed to be equal to the static pressure P_M sufficient to keep the reed closed. Analytical description of the variation of the input section is quite complex, especially for double reeds [6, 12] for whch few reliable experimental data are available. Therefore, interest is focused on single reed woodwinds in this paper, but similar analysis might be done in the case of double reed instruments.

For single reed woodwinds the cross-sectional area of the jet depends on the geometry of the reed channel and of the position of the reed. As discussed in [8], the cross-sectional area S_j of the jet may not be equal to the cross-sectional area of the reed channel because of the contraction of the jet and of the contribution of the lateral opening. Nevertheless, these areas are assumed to be proportional [8] and the cross-sectional area of the jet is written

$$S_j = \sigma w\left[H_0 + y\right], \tag{3}$$

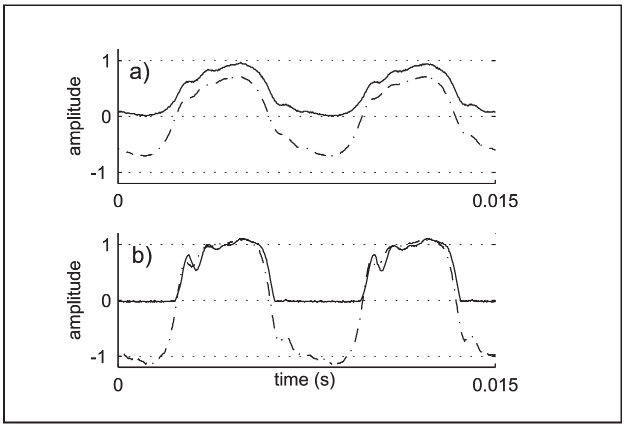

Figure 3. Simultaneous measurement of the pressure p in the mouthpiece (-· — ·—) and of the opening $H_0 + y$ of the reed channel by using an optical method (——). The instrument is a pure cylinder excited by a clarinet mouthpiece playing the first register. The instrument is blown by using an artificial mouth device described in [14]. The arbitrary ordinate scales are the same for the two figures. Figure a: non-beating reed regime, the reed channel is not completely closed. Figure b: beating reed regime, the reed channel is closed during a part of the period. In both cases, p and y are nearly in phase and proportional.

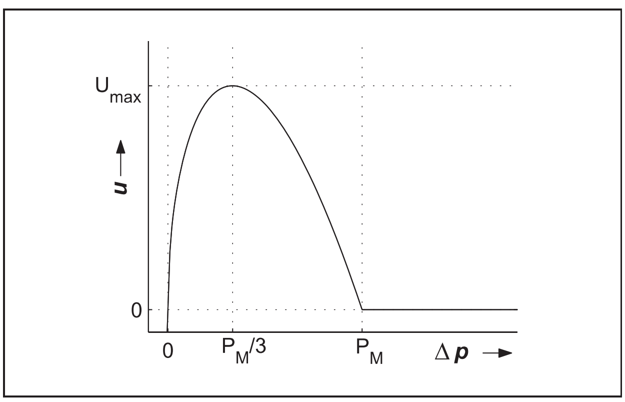

Figure 4. Nonlinear characteristics between Δp and u, from equation (6).

where H_0 is the height at rest of the reed channel, w its width, y is the deviation of the tip of the reed, and σ is a factor that can be greater or smaller than unity. See figure 2 for notations.

The playing frequencies of the first register of single reed instruments are about ten times lower than the first mechanical resonance frequency of the reed. Thus, only the first resonance of the reed influences the oscillations [13] and the motion of the tip of the reed is often described as single degree of freedom valve (a mass/spring damped oscillator) as a first approximation. This assumption leads to a nonlinear relationship between p and u which is time dependent since some hysteresis effect, due to inertia and damping of the reed, appears in the relation between the flow and the pressure [9, 10, 6]. Although this description gives satisfactory results when the signals are computed and compared to measured signals [14, 15, 16, 17], analytical analysis of the mechanisms of oscillation remains difficult.

A simpler description of the relationship between the pressure and the flow can be used if the playing frequency is low compared to the reed first resonance frequency ($\simeq 2000Hz$). With this assumption, the reed is a valve completely characterized by a "stiffness" k_r (units: $\mathrm{Pa/m}$) [18]. The relationship between pressure and volume velocity in the mouthpiece becomes then time independent because, until the reed beats, the variation of the channel height is proportional to the pressure difference Δp:

$$y(t) = -\Delta p/k_r. \tag{4}$$

The static pressure difference sufficient to keep the reed closed is such that the deviation y is equal to the height at rest H_0. Thus, if k_r is assumed to be a constant, the closing pressure is

$$P_M = k_r H_0. \tag{5}$$

The assumption of a linear relationship between the acoustic pressure and the tip displacement is confirmed by experiments: Figure 3 shows that the motion of the tip of the reed and the acoustic pressure in the mouthpiece are almost in phase and proportional when playing the first register. With the previous simplifications, the nonlinear relationship $u = F_{ber}(\Delta p)$ which characterizes the flow in the mouthpiece is finally written

$$\begin{cases} F_{ber}(\Delta p) = U_{rme}\mathrm{sign}(\Delta p)(1 - \frac{\Delta p}{P_M})\sqrt{\frac{|\Delta p|}{P_M}}, \\ \qquad\qquad \text{if } \Delta p < P_M, \\ F_{ber}(\Delta p) = 0, \\ \qquad\qquad \text{if } \Delta p \geq P_M, \end{cases} \tag{6}$$

with

$$U_{rme} = \sigma w (2k_r/\rho)^{1/2} H_0^{3/2}, \tag{7}$$

where ρ is the density of air. This function is plotted on Figure 4. Equation (7) defines a global reed / mouthpiece / embouchure flow U_{rme} which depends on the mouthpiece geometry (H_0, w, σ), the reed stiffness (k_r) and the embouchure (k_r, H_0). The value of U_{rme} can be deduced from the maximum flow since $U_{rme} = 3\sqrt{3}/2\ U_{max}$. Such characterization of the embouchure with a global embouchure parameter was first given by Wilson and Beavers [13], and later by Kergomard [19]. Another model, obtained empirically by Backus [18] has been used up to now by many authors. The shape of the nonlinear characteristics is slightly different but many qualitative results are valid for both models, as explained in [20]. When the reed is thought to be an ideal spring (equation (4)), the volume velocity u_{reed} created by the motion of the reed can be taken into account in the input impedance as an equivalent volume at the input of the resonator [7]. Despite its simplicity, this basic model gives some keys for the understanding of the oscillation mechanisms [19, 21].

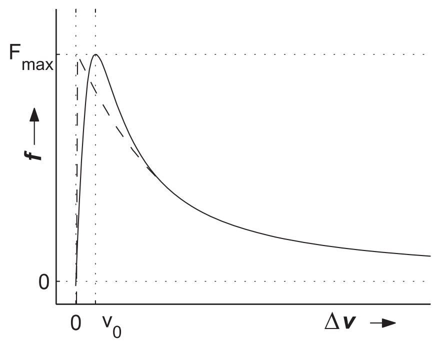

Figure 5. Two force-velocity characteristics. —— with perfect sticking (equation (8)). —— function given by Weinreich and Caussé [25] (equation (9)).

3.2. Bowed string instruments: the stick-slip characteristics

Concerning bowed string instruments, the nonlinear mechanism lies in the stick/slip transition between the string and the rosined hair of the bow. Recent works show that the friction characteristics obtained by studying the changes of the rosin properties during sticking and slipping [22], or the effect of the bow width or the non simultaneous slipping of bow hair must be considered [3, 4, 5]. This characteristics was measured in the dynamical regime, and the results confirm that the nonlinear stick/slip transition is governed by more complex laws than by a simple (force,velocity) relationship [23]. As we are interested in comparing idealized models, these results are not discussed hereafter and only usual idealized friction relationships are considered. Moreover, if the models use too many parameters the comparison between bowed string and woodwinds models is not possible.

In the scope of idealized models, the bow acts at a single point on the string. The friction behaviour is usually modelled with a relationship between the velocity v of the bowed point of the string and the tranverse friction force f in the plane of the motion of the string. A model for the stick/slip process is found by assembling the sticking and the slipping friction behaviours of the string onto the bow hair. When the string sticks on the bow hair, the velocity v of the bowed point is equal to the velocity v_b of the bow and the transverse force f is indeterminate. When the tranverse force is greater than a "maximum bow force" F_{max}, the string slips on the bow hair, and the sliding friction force is a decreasing function of the relative velocity $\Delta v = v_b - v$:

$$f(\Delta v) = F_b\big[\mu_d + (\mu_s - \mu_d)\phi(\Delta v)\big], \qquad (8)$$

where μ_s and μ_d are respectively the static and the dynamic friction coefficients, and $\phi(\Delta v)$ is a function decreasing from 1 when $\Delta v = 0$ to 0 when $\Delta v = -\infty$ [24]. A schematic representation of this function is plotted with dashed lines on Figure 5. As in the case of woodwinds, the derivative of the nonlinear characteristics is not continuous.

The vertical portion of the curve describes the sticking state of the string, when the velocity of the point of the string in contact with the bow is equal to the bow speed v_b. Torsional effects can be taken into account as a modification of the nonlinear characteristics. A consequence is that the derivative of the nonlinear characteristics of the curve during sticking is not infinite but has a high but finite slope related to the impedance of the rotational waves [1]. By using this argument, Weinreich and Caussé [25] proposed a relationship whose derivative is continuous and finite:

$$f(\Delta v) = F_b(\Delta v/v_0)\big[1 + (\Delta v/v_0)^2\big]^{-1}, \qquad (9)$$

where v_0 is a control parameter (instead of μ_s and μ_d). A schematic representation of this function is plotted with a plain line on Figure 5. This kind of model is more convenient if a function with a continuous and finite derivative is needed, for example when driving an experimental "digital bow" [25, 26].

3.3. Analogy

As pointed out by McIntyre *et al.* [1], an analogy can be drawn between reed woodwinds and bowed string models. Within this analogy, the frictional force exerted by the bow on the string is analogous to the flow rate into the clarinet, and the transverse velocity is analogous to the acoustic pressure inside the mouthpiece. The mouth pressure P_m is analogous to the bow velocity v_b since both are continuous sources of energy. This analogy is consistent with the classical admittance electromechanical and impedance electroacoustical analogies, for strings and tubes respectively (see e.g. [27]). In addition, we have pointed out in equation (6) that the amplitude of the nonlinear characteristics of idealized models of single reed instruments is proportional to a global reed-embouchure flow U_{rme}. Similarly, the stick-slip relationship is proportional to the bow force F_b. Thus, the embouchure flow U_{rme} (or similar parameters like those defined in [13, 19]) plays the same role as the bow force F_b. Both nonlinear characteristics are built by assembling two behaviours: the sticking and the sliding friction in the case of the bowed string, the non beating and the beating reed regime in the case of reed woodwinds. In both cases another parameter is needed to determine the transition. For the bow string, this parameter is the maximum force $F_{max} = F_b\mu_s$; in the case of reed woodwinds, it is the pressure $P_M = k_r H_0$ sufficient to keep the reed closed.

It may seem that the embouchure parameter U_{rme} depends on several parameters, whereas the bow force F_b is a well defined physical quantity. Nevertheless, the force applied by the bow on the string depends on many parameters such as the bow width, the non simultaneous stick / slip transition of the bow hair, the mechanical behaviour of the bow, ... etc. Thus the bow force can be considered as a global force equivalent to the force that would be applied at a single point. Therefore, describing the embouchure

Table I. Summary of the formal analogy between the parameters and variables for the nonlinear characteristics of the bowed string and woodwind models

	bowed string	woodwind
variables	string velocity $v(t)$ at the bowing point	acoustic pressure $p(t)$ in the mouthpiece
	transverse force $f(t)$ at the bowing point	volume velocity $u(t)$ in the mouthpiece
playing	bow speed v_b	mouth pressure P_m
param.	bow force F_b	embouchure flow U_{rme}

with a single parameter is an approximation of the same order as the description of bowing with only a "bow force" parameter. Obviously, some aspects of bowing differ if using finite width models instead of point-bow [3, 4, 5]. In the same way some aspects of the behaviour of reed woodwinds models differ when including the details of the reed motion [28]. The formal analogy between the models for the nonlinear element is useful for comparing references on idealized models of bowed strings to papers dealing with idealized woodwinds models. Another interest deals with the control, by using real instruments, of real-time synthesis by physical models. For example if one would control a violin model by a saxophone with sensors for the mouthpressure and the embouchure (see for example [29]), a proper association of the parameters controlled by the musician with the analogous parameters of the models could increase the playability.

4. Resonators

Resonators are usually considered to be linear systems, though nonlinear phenomena can occur both in wind instruments and on bowed strings (see [30, 31, 32] for example). Therefore, they are usually characterized by the input admittance or impedance in the frequency domain, or their impulse response or some reflection function(s) in the time domain. In the case of bowed string instruments, concerning the resonator, the auto-oscillation process is mainly governed by the string, the bridge and the body being considered as boundary conditions. In the case of woodwinds, the resonator is the main part of the instrument. The way those resonators are played is quite different. Even if bowed string instrument players sometimes play "natural" or "artificial" harmonics, strings are usually bowed to get the fundamental vibration. To change the frequency, the string instrument players change the length of the resonator with their fingers, or play another string. Woodwind instrument players change the effective length of their resonator too (by opening toneholes), but they also play several registers.

Both these resonators must have harmonically related resonances in order to produce powerful and stable oscillations at high excitation level, and for having a playing frequency independent of the excitation level [33, 7]. Obtaining this property is one of the difficulties in making woodwinds, mainly because of toneholes and the truncation of conical tubes. This explains why most studies on woodwinds are focused on the resonator in the frequency domain. As a result, woodwind resonators are usually characterized by their input impedance $Z(\omega)$. Contrary to the bowed string, the description of the behaviour of wind instruments in the time domain is quite recent [16] and is linked to the development of computational simulations.

4.1. The string

Many theoretical studies on the oscillations of the bowed string deal with the influence of the reflection functions at the fixed end of the string, and assume that the resonator is a perfect ideal string (no losses, no dispersion). Actually the main causes of dissipation and dispersion are the boundary conditions (the body and the fingers). A hierarchy of bowed string models can be established, depending on the choice of the model for the reflection functions [34]:

- The model presented separately by Keller [35] and Friedlander [36] assumes that the reflection functions at fingers (or nut) and bridge are unit delta functions: $r(t) = -\delta(t-\tau)$, where τ is the travel time from the bow to one boundary (finger or bridge) and back. Within this model, the string is a pure delay line for the transverse waves, rotational waves being not taken into account. Neither losses nor dispersion are taken into account. Friedlander showed that no stable oscillation can be produced by an instrument built with this model for the resonator if sticking is perfect, that is if $v = v_b$ during sticking [36]. Therefore, it is used only when sticking is not perfect (see [25] for example).
- Raman's model [37] is dispersionless too, but losses are introduced by means of a parameter $\lambda < 1$ independent of the frequency. Reflection functions can be written $r(t) = -\lambda\delta(t-\tau)$. The previous model is the lossless limit case of this one. Reflection of torsional waves can be modelled in the same way [38].
- The "rounded corners" model [1] is more realistic. In this model, reflection functions are finite width functions in order to take dispersion into account. This model is convenient to study the influence of many parameters by using time domain simulations, but analytical analysis is intricate.
- A model fo/r which the reflection function at the bridge is the combination of a narrow function and a decreasing oscillating function can be used to study the influence of the oscillation of the board occuring in the "wolf-note" phenomenon [1].

In the frequency domain, strings can be characterized by their input admittance $Y_{st} = v/f$. If only tranverse waves are considered, the input admittance of a lossless string of length L divided by the bow into two parts of lengths L_a and L_b is

$$Y_{st} = \mathrm{j}Y_c\big(\cot(kL_a) + \cot(kL_b)\big)^{-1}, \tag{10}$$

where Y_c is the characteristic admittance of transverse waves ($Y_c = (T\mu)^{-1/2}$, where T is the string tension and μ its mass per unit length), k is the wavenumber and $\mathrm{j} = \sqrt{-1}$.

4.2. Woodwind resonators

Wind instrument resonators have been studied by many authors (see for example [6, 33, 39, 40]) and can now be modelled accurately, at least within the scope of a linear theory. This knowledge leads to the development of softwares assisting instrument makers in improving their instruments or in designing new models [41, 42]. Two basic shapes are used for the bore of reed woodwinds. One is the cylinder, which is used for the clarinet, the other one is the cone, which is used for single and double reed woodwinds. Obviously, a "complete" model of a wind instrument must take into account the influence of the details of the geometry like tone holes and the bell, but when only studying the mechanism of oscillations, resonators can be modelled as pure cones or cylinders.

The clarinet is the most studied wind instrument because cylindrical resonators are simpler than conical ones. Provided that losses are weak, the cylinder is a resonator with almost harmonically related resonance frequencies. Things are more complex for conical instruments. A complete cone has harmonically related resonance frequencies too, but since the conical resonator must be truncated to put in the mouthpiece, resonances are no longer harmonically related. Some corrections to the truncated cone allow this property to be approximately recovered. The main correction is the addition of an input volume which must be nearly equal to the missing part of the cone [33]. This volume is the sum of the volume of the mouthpiece (or of a double reed) with the volume correction which models the volume velocity u_{reed} induced by the reed motion [6, 7]. With such an input volume, the input impedance of a truncated cone of length L_b with a truncature of length L_a is

$$Z_{tc} = \mathrm{j}Z_c\big(1/kL_a - kL_a/3 + \cot kL_b\big)^{-1}, \qquad (11)$$

where Z_c is here the impedance of an infinite cylinder with the same input section as the truncated cone. In relation (11), $1/kL_a - kL_a/3$ can be seen as the first terms of the expansion of $\cot(kL_a)$ with respect to kL_a [43, 44]. This is interesting because the impedance

$$Z_{icw} = \mathrm{j}Z_c\big(\cot(kL_a) + \cot(kL_b)\big)^{-1} \qquad (12)$$

has harmonically related resonance frequencies, provided that losses and dispersion are weak. Harmonicity can be improved by slightly increasing the angle of the cone near the mouthpiece (or the reed in the case of double reed woodwinds). This modification of the bore induces a modification of the input impedance (11) such that an higher order of the expansion of $\cot(kL_a)$ is obtained [7].

Two of the present authors showed that two families of resonators have exactly the input impedance given by (12) [45, 46]. One consists in "stepped cones" of length L_b built with N cylinders, of identical length L_a, with

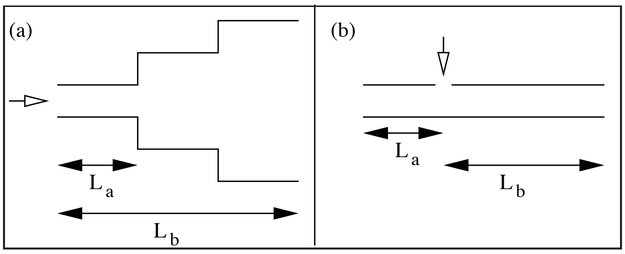

Figure 6. Idealized conical-like resonator. (a): "stepped cone", (b): "cylindrical saxophone". White arrows indicate the location of the mouthpiece.

cross-sections S_i such that $S_i = i(i+1)S_1/2$, where $i = 1, 2, \cdots, N$, and S_1 is the section of the first cylinder (see Figure 6a). For these resonators, $Z_c = 2\rho c/S_1$. Such resonators sound like saxophones when excited with a single reed mouthpiece. To our knowledge, few instrument makers use this kind of bore (see [47] for example). When there is only one cylinder, the impedance of a clarinet $j(\rho c/S_1)\tan(kL_a)$ is found with equation (12).

The other family of resonators whose impedance is also given by equation (12) consists of two cylindrical resonators of section S, and length L_a and L_b, excited by a mouthpiece with a volume reduced to zero (see Figure 6b). These resonators are called "cylindrical saxophones" in this paper and in [2]. For these resonators, the ratio $N = L_b/L_a$ is not necessarily integer and $Z_c = \rho c/S$. If $L_a = L_b$, the impedance $j(\rho c/2S)\tan(kL_a)$ of a clarinet of section $2S$ is found with equation (12). If, contrary to stepped cones, cylindrical saxophones cannot be built easily for experimental observations (because of the finite volume of a real mouthpiece), they are convenient for theoretical analysis and they could be used for sound synthesis by taking advantage of the use of two reflection functions. Because instrument makers tend to obtain an input impedance close to that given by equation (12), the previous resonators are considered here as idealized conical resonators. One consequence is that an input impedance given by equation (12) allows simpler analytical study in the time domain than the impedance of a truncated cone, given by equation (11). Another remarkable property of such resonators is the analogy between impedance (12) and the admittance of a bowed string given by equation (10). Consequently, idealized conical-like resonators can be characterized in the time domain by two reflection functions, like a bowed string [1].

4.3. Analogy

From equations (12) and (10), the formal analogy with the bowed string can be extended to idealized conical-like resonators (see Figure 7). A string bowed so that the ratio L_b/L_a is an integer N is analogous to a stepped cone with N cylinders. For any other ratio, a cylindrical saxophone can be constructed. Changing of the position of the bow is equivalent to the changing the length of the truncation for conical woodwinds. Contrary to bowed string instruments

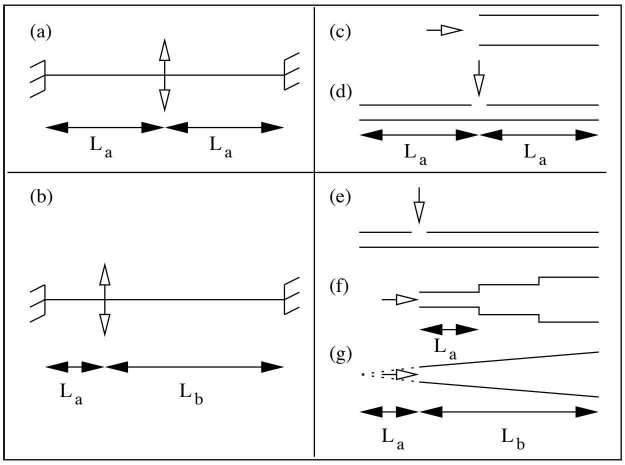

Figure 7. Summary of the formal analogy between the bowed string and woodwind resonators. White arrows indicate the location of the mouthpiece or the bow. A string bowed at the middle (a) is analogous to a clarinet (c) or (d). A string bowed at a location such that $L_a \neq L_b$ (b) is analogous to a cylindrical saxophone (e). If $L_b/L_a = N$ is an integer, it is equivalent to a stepped cone with N cylinders (f). It is approximately analogous to a truncated cone (g) of length L_b, wit h L_a the length of the missing part of the cone (dashed lines).

players, conical woodwind players cannot control this parameter since the length of the truncation is fixed.

Similarly to the bowed string models, a hierarchy of models of resonators can be established:

- The simplest model, with neither dispersion nor losses, was used for the clarinet [19, 48, 49] and for the stepped cone [21]. This model, which is similar to Keller's or Friedlander's model, will be referred to as the "lossless model" both for woodwinds and the bowed string.
- A dispersionless model of a clarinet with a loss parameter independent of the frequency is given by Maganza and others [48, 49]. This is similar to Raman's model. Note that resonators whose losses do not depend on the frequency have particular properties discussed in [50].
- In more realistic models, resonators are modelled within the scope of a linear theory by means of their input impedance, taking dispersion and the influence of tone-holes and the bell into account. This is similar to the "rounded corners" model [1], which has been used since the eighties for the calculation of transient oscillations and physical modelling synthesis [14, 16, 17]. In a simplified version of this model, the details of the geometry are left out, and a clarinet is modelled as a pure cylinder with neither bell nor tone-hole (see [20] for example).
- More complete models can be used in order to take into account all the details of the geometry, nonlinear effects at tone-holes and at the bell [31, 15], mechanical coupling between the fluid and the walls of the pipe [51], temperature gradient, effect of the concentration of CO_2 in the blown air [52], ... etc.
- In addition, a model of a wind instrument resonator close to the one used to study the "wolf-note" on bowed string instruments can be constructed to study the behaviour of some "pathological" organ pipe exhibiting wall vibrations [53].

5. Periodic solutions

In the previous sections we have shown that the analogy is complete between the simplest models for the bowed string (without torsional motion) since there is an analog for each parameter or variable. The similarity between the measured waveforms of the pressure inside the mouthpiece of single reed woodwinds [44] (or in double reeds [54]) and the velocity of the string under the bow [55] is a proof of the relevance of this analogy.

For the bowed string, many studies are based on Raman's model because it allows analytical analysis of many aspects of the oscillation mechanism. Although it is not suited for discussing all the subtleties of real instruments, this model can be used as a first approach to study conical-like woodwinds too. The lossless model ($\lambda = 1$) is not suitable for the bowed string because the periodic solutions are unstable [36]. On the contrary, due to differences in the shape of the nonlinearity, it can be suitable for woodwinds since periodic solutions can be stable, as shown in the companion paper [2].

In this last section we give a method to find the periodic solutions within the scope of Raman's model (after Woodhouse [34]), then we detail the case of "two-step" solutions.

5.1. Periodic solutions in the dispersionless case

The solutions of the coupled system must satisfy both the equation for the resonator and the nonlinear relationship, written

$$u = F(p), \tag{13}$$

which characterizes the flow in the mouthpiece. With the lossless or Raman's model, solutions can be found by use of recurrence equations [36, 34]. The waveforms obtained are somewhat unrealistic since they consist of a succession of constant values, with instantaneous jumps between consecutive values [37]. Nevertheless, these solutions are interesting since they are schematizations of real waveforms [44].

At a given time t, at the entrance of the resonator, the pressure is the sum of the instantaneous contribution $p_i(t)$ due the nonlinear process and the contribution $p_h(t)$ from the past history of the motion:

$$p(t) = p_i(t) + p_h(t). \tag{14}$$

The pressure p_h depends on the values of the flow $u(t')$ and the pressure $p(t')$ with $t' < t$ and can be evaluated by using reflection functions [1]. Solutions must satisfy both equations (13) and (14). In the case of the bowed string, there can be more than one solution to this system. When this occurs, a hysteresis rule must be applied, and with finite width reflection functions a flattening of the pitch is observed [1]. To our knowledge this effect has not been reported in the litterature dealing with woodwinds but this possibility is not excluded. This problem has to be studied but it is not discussed in this paper and we assume the uniqueness of the solution of the system built with equations (13) and (14). Similarly to the case of the bowed string [34], using Raman's model for a cylindrical saxophone or a stepped cone, equation (14) can be written

$$p(t) = \frac{Z_c}{2}\Big\{u(t) - \sum_{j=0}^{\infty}\lambda^{2j}\Big[\lambda u\big(t-2(L_a+jL)/c\big) + \lambda u\big(t-2(L_b+jL)/c\big) - 2\lambda^2 u\big(t-2(j+1)L/c\big)\Big]\Big\}, \tag{15}$$

where $L = L_a + L_b$, and $Z_c = 2\rho c/S_1$ for stepped cones or $Z_c = \rho c/S$ for cylindrical saxophones. In equation (15), the summation is the contribution of the multiple reflections. Equation (15) can be reduced to a difference equation which can be written in a closed form [34]. The following notations are used in order to obtain discrete equations:

$$L_a/L_b = a/b, \tag{16}$$

where a and b are integers such that $a \leq b$,

$$m = a + b, \tag{17}$$
$$\Delta t = 2L/mc, \tag{18}$$
$$2L_a/c = a\Delta t, \tag{19}$$
$$2L_b/c = b\Delta t, \tag{20}$$
$$p_n = p(n\Delta t), \tag{21}$$
$$u_n = u(n\Delta t). \tag{22}$$

For a stepped cone made with N cylinders: $a = 1, b = N$ and $m = N + 1$. With these definitions, equation (15) becomes

$$p_n = \frac{Z_c}{2}u_n - \frac{Z_c}{2}\sum_{j=0}^{\infty}\lambda^{2j}\Big[\lambda u_{n-a-jm} + \lambda u_{n-b-jm} - 2\lambda^2 u_{n-(j+1)m}\Big]. \tag{23}$$

Adding equation (23) and $(-\lambda^2)$ times equation (23) modified by replacing n by $n - m$ leads to:

$$p_n - \lambda^2 p_{n-m} = \frac{Z_c}{2}\Big(u_n - \lambda u_{n-a} - \lambda u_{n-b} + \lambda^2 u_{n-m}\Big). \tag{24}$$

Consider now the case of periodic solutions in which the frequency is the fundamental frequency $c/2L$ or a multiple of this frequency. In this case $p_n = p_{n-m}$ and $u_n = u_{n-m}$. Thus, with $u_n = F(p_n)$, a set of m nonlinear equations which link the m unknown values of the pressure during one period can be deduced from equation (24) [34]:

$$[S_\lambda]\begin{pmatrix} F(p_n) \\ F(p_{n-1}) \\ \vdots \\ F(p_{n-(m-1)}) \end{pmatrix} = \frac{2(1-\lambda^2)}{Z_c}\begin{pmatrix} p_n \\ p_{n-1} \\ \vdots \\ p_{n-(m-1)} \end{pmatrix} \tag{25}$$

where the matrix $[S_\lambda]$ is symmetric. For the clarinet, *i.e* if $a = b$,

$$[S_\lambda] = \begin{pmatrix} 1+\lambda^2 & & -2\lambda & & \\ & \ddots & & \ddots & \\ & & \ddots & & -2\lambda \\ -2\lambda & & & \ddots & \\ & \ddots & & & \ddots \\ & & -2\lambda & & 1+\lambda^2 \end{pmatrix}, \tag{26}$$

where the diagonal line of terms -2λ starts in column $a+1$ of the first row, and non indicated terms are zero. For stepped cones or cylindrical saxophones, *i.e.* if $a \neq b$,

$$[S_\lambda] = \begin{pmatrix} 1+\lambda^2 & & -\lambda & -\lambda & & \\ & \ddots & & \ddots & \ddots & \\ & & \ddots & & \ddots & -\lambda \\ -\lambda & & & \ddots & & \ddots \\ & \ddots & & & \ddots & -\lambda \\ -\lambda & & \ddots & & & \ddots \\ & \ddots & & \ddots & & \ddots \\ & & -\lambda & -\lambda & & 1+\lambda^2 \end{pmatrix} \tag{27}$$

where the diagonal lines of terms $-\lambda$ start in columns $a+1$ and $b+1$ of the first row, and non indicated terms are zero.

Another iteration equation, given in [44], can be useful for the stability calculation [2]. Its validity is less general because it is limited to the case $\lambda = 1$. Its advantage lies in the order of the recurrence, which is $m - 1$, instead of m for equation (24). It is derived below when $a = 1$. Starting from the division of the volume velocity u_n into two parts, $u_{a,n}$ and $u_{b,n}$, and from the reflections at the two extremities, the following equations can be written at each time n:

$$u_n = u_{a,n} + u_{b,n} \tag{28}$$
$$p_n - Z_c u_{a,n} = -\lambda(p_{n-a} + Z_c u_{a,n-a}) \tag{29}$$
$$p_n - Z_c u_{b,n} = -\lambda(p_{n-b} + Z_c u_{b,n-b}). \tag{30}$$

For $\lambda = 1$, combining equations (28) and (29), the following equation is obtained, at each time i:

$$-Z_c u_{b,i} + Z_c u_{b,i-a} = -Z_c u_i + Z_c u_{i-a} + p_i + p_{i-a} \tag{31}$$

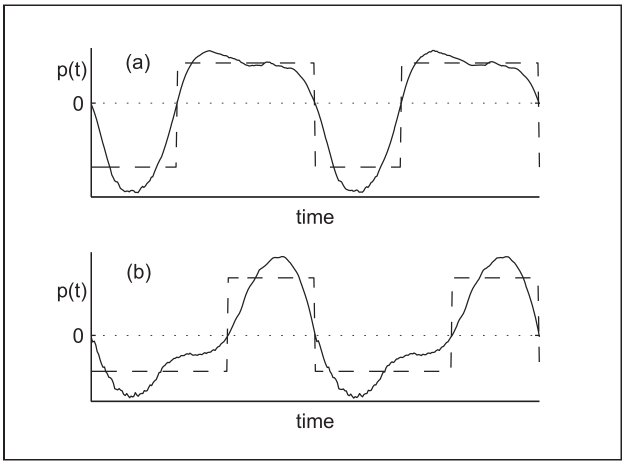

Figure 8. Pressure signals measured in the mouthpiece of a "stepped cone saxophone" made with 2 cylinders. Continuous lines: acoustic pressure in the mouthpiece, discontinuous lines: two-step motion with the same RMS pressure. (a) standard motion; (b) inverted motion.

For $a = 1$, adding equations (31) for $i = n - b + 1$ to $i = n$, the following equation is obtained:

$$Z_c u_{b,n-b} - Z_c u_{b,n} = Z_c u_{n-b} - Z_c u_n + p_{n-b} + p_n + 2 \sum_{i=n-b+1}^{n-1} p_i \quad (32)$$

Using equation (30), the following iteration equation is obtained from equation (32):

$$\frac{Z_c}{2}(u_n - u_{n-b}) = \sum_{i=n-b}^{i=n} p_i. \quad (33)$$

(adding this equation with the same equation with n replaced by $n-1$, leads to equation (24) in the considered case). This result cannot be extended to the case $\lambda \neq 1$, except if $a = b$. For that case, $u_{a,n} = u_{b,n} = u_n/2$, and the following result is obtained from equation (29):

$$p_n - \frac{Z_c}{2} u_n = -\lambda\Big(p_{n-a} + \frac{Z_c}{2} u_{n-a}\Big). \quad (34)$$

5.2. Two-step oscillations

As shown by Raman [37], various waveforms can be observed on the bowed string. Some similar waveforms are observed on woodwinds [44]. Actually, with "standard" playing, both woodwind and bowed instrument players produce a signal which caricature is a "two-step" motion, that is a waveform such that the velocity of the string under the bow or the mouthpiece pressure takes alternatively two values (see Figure 8). Thus, among the many possible solutions, attention is focused on these solutions.

Depending on the values of the playing parameters, different two-step solutions can be generated. Moreover, the pressure can oscillate with the fundamental frequency, with a frequency of higher registers, or with a subharmonic frequency. Interest is focused here on oscillations

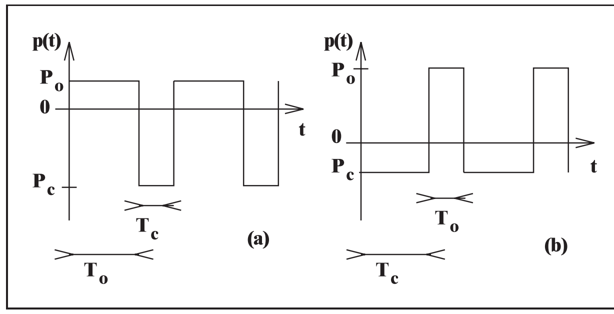

Figure 9. Possible waveforms of two-steps motions: (a) standard motion; (b) inverted motion.

with a frequency corresponding to the first resonance of the resonator, the second register being discussed in section 5.3. With usual nonlinear characteristics, two kinds of two-step motions can be obtained (conditions for having this property are discussed in [21, 50]). One is such that the duration T_o of the opening episode ($p > 0$) is longer than the duration T_c of the closing episode ($p < 0$). This waveform is called "standard motion" (Figure 9a). Note that the opening and closing episodes correspond respectively to the sticking and slipping episodes on the bowed string. The motion such that the closing episode is longer than the opening one is called "inverted motion" (Figure 9b). These motions are "caricatures" of waveforms observed on stepped cones (Figure 8) but, to our knowledge, the inverted motion has not been observed on the bowed string. If T_o/T_c or $T_c/T_o = L_b/L_a$, the oscillation is called "Helmholtz motion", refering to the observations of Helmholtz on the bowed string [56].

In the lossless case ($\lambda = 1$), the set of equation (25) is reduced to:

$$F(P_L) = F(P_S) \quad (35)$$

where P_L and P_S are respectively the values of the pressure in the mouthpiece during the longer and the shorter episodes of respective duration T_L and T_S. Without losses, the mean power over a period must be equal to zero and, due to the zero value of the input frequency at zero value ($k = 0$ in equation (12)), the mean value of the pressure over a period is equal to zero. Using these properties and equation (35), the calculation of the mean power over a period gives a second equation to calculate P_L and P_S:

$$T_S P_S = -T_L P_L. \quad (36)$$

In the case of a lossless clarinet, one solution which satisfies equations (35) and (36) is a "square signal" such that $T_L = T_S = T_o = T_c$, and $P_c = -P_o$. If the nonlinear function F is given by equation (6), from equation (35) the pressure during the opening episode is found to be $P_o = [(1-P_m/P_M)(3P_m/P_M-1)]^{1/2}$ if $\Delta p < P_M$, that is if $P_m < P_M/2$, and $P_o = P_m$ if $\Delta p \geq P_M$ [19]. Condition $\Delta p = P_M$ is the "beating reed" threshold. A convenient way to analyse influence of the playing parameters on the solutions is to draw bifurcation diagrams where the values P_o and P_c of the pressure corresponding to the

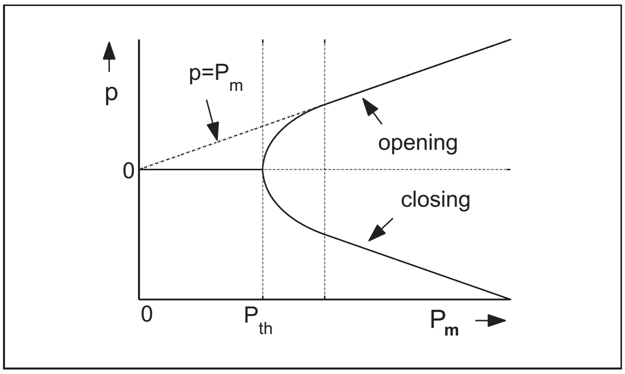

Figure 10. Bifurcation diagram for a lossless clarinet. The pressure in the mouthpiece during the opening and the closing episodes (respectively $p > 0$ and $p < 0$) are plotted versus the blowing pressure P_m

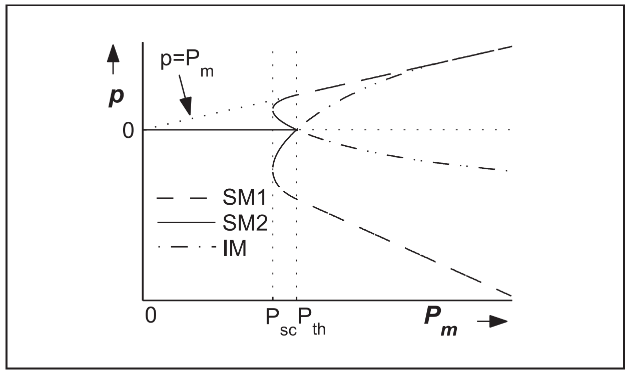

Figure 11. Bifurcation diagram for the two Helmholtz solutions ($N_T = N$). The nonlinear model is F_{ber} (equation (6)), the resonator is lossless and such that $N = 2$. The values of the opening ($p > 0$) and closing ($p < 0$) episodes are plotted versus the blowing pressure.

opening and closing episodes respectively are plotted as a function of the blowing pressure P_m. Such a diagram is given for a lossless clarinet on Figure 10. The bifurcation from the static regime to the square signal is direct, that is there are small oscillations for $P_m \geq P_{th}$, the pressure P_{th} being the threshold of instability for the static solution [50]. The threshold P_{th} is equal to $P_M/3$ with the nonlinear function (6) in the lossless case [19], and higher with losses [2].

In the case of idealized stepped cones or cylindrical saxophones, the pressure P_L corresponding to the longer episode can take two values for a given ratio $N_T = T_L/T_S$. If the resonator is lossless ($\lambda = 1$), when the nonlinear function is the one given by equation (6) and if $\Delta p < P_M$, the possible pressures P_L are [21, 44][1]

$$P_L^{\pm} = \frac{2}{m} \cdot \frac{\alpha(2-3\gamma) \pm \sqrt{\alpha^2 + (1-\gamma)(3\gamma-1)}}{1+3\alpha^2}, \quad (37)$$

with $\gamma = P_m/P_M$, and $\alpha = (T_L - T_S)/(T_L + T_S)$. From equation (36), the pressure corresponding to the shorter episode is $P_S^{\pm} = -N_T P_L^{\pm}$. The solutions given by equation (37) are valid if $P_L^{\pm}$ is real, thus if $P_m \geq P_{sc}$, where P_{sc} is the pressure such that the square root in (37) is equal to zero. The analysis of equation (37) shows that the beating reed regime ($\Delta p \geq P_M$) occurs if $P_m \geq P_M/(N_T + 1)$ with solution P_L^+, and if $P_m \geq P_M N_T/(N_T + 1)$ with solution P_L^-. From equation (6), in the beating reed regime the opening pressure P_o is such that $P_o = P_m$, the closing pressure being $P_c = -N_T P_m$ if the motion is standard ($P_L > 0$), and $P_c = -P_m/N_T$ if the motion is inverted ($P_L < 0$). The two solutions given by equation (37) behave differently. With P_L^+, the longer step is positive, thus the waveform is a standard motion (noted SM1 on the bifurcation diagram of Figure 11). With P_L^-, the longer step is positive if $P_{sc} < P_m < P_{th}$, thus the waveform is a standard motion (noted SM2 on Figure 11), and it is negative if $P_m > P_{th}$, thus the waveform is an inverted motion (noted IM on Figure 11). The bifurcation from the static regime to the considered periodic solution is an inverse bifurcation for the standard motion since there are small oscillation (SM2) for $P_m < P_{th}$ [50].

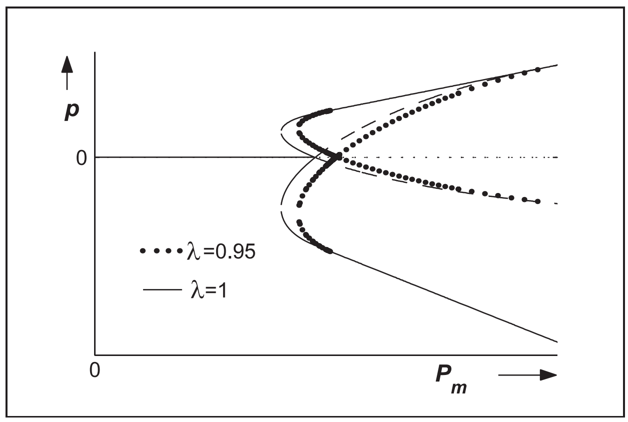

Figure 12. Bifurcation diagram obtained with a given embouchure in the lossless case (see Figure 11) and with Raman's model (dots) with $\lambda = 0.95$

If $\lambda < 1$, solutions are obtained by using equation (25). The bifurcation diagram of Figure 12, calculated in the case of a stepped cone with 2 cylinders, shows that the solutions converge toward the lossless case when $\lambda \to 1$. The main effect of losses is the reduction of the range of the non beating reed regime since P_{sc} increases and the beating reed threshold remains nearly unchanged. For discussions on the behaviour of solutions when dispersion is taken into account, including comparisons with measurements, see [21].

5.3. Second register of conical-like reed woodwinds

When studying the second register, the validity of the low frequency model for the reed can be questioned because the reed resonance influence might not be negligible (see [13] for example). Nevertheless, we suppose that the nonlinear characteristic is the same for the fundamental oscillation (of frequency f_1) and for the octave (of frequency $f_2 = 2f_1$ since dispersion is not taken into account). In

[1] Note that equation (18) in [21] contains a misprint.

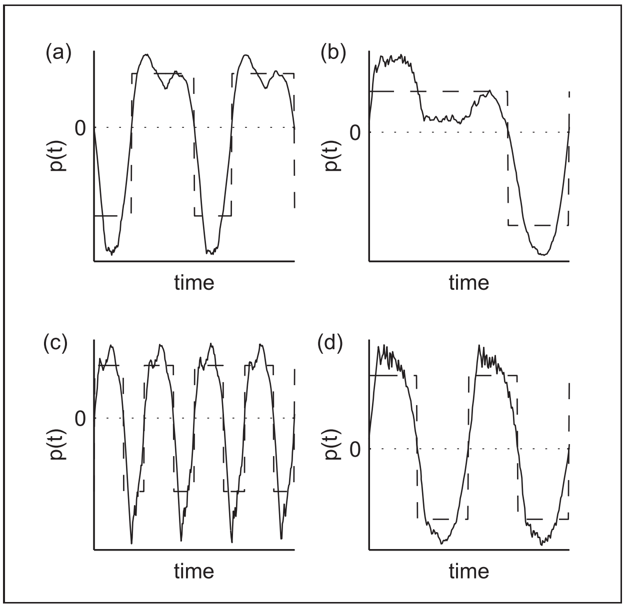

Figure 13. Measured waveforms of the first and the second register of stepped cones. Continuous lines: acoustic pressure in the mouthpiece, discontinuous lines: two-step motion with the same RMS pressure. (a) fundamental vibration of a stepped cone $N = 2$. (c) octave vibration of a stepped cone $N = 2$. (b) fundamental vibration of a stepped cone $N = 3$. (d) octave vibration of a stepped cone $N = 3$.

the case of dispersionless resonators, the waveform of the second register can be deduced from the input impedance of the resonator. If a and b are the smallest integers such that $L_a/L_b = a/b$, the analysis of the input impedance (12) shows that there are no resonances at the frequencies which are multiple of mf_1, where $m = a + b$. Consequently, if m is odd there are no multiples of mf_2 in the spectrum and the second register can have the same waveform as the fundamental. If m is even there are no frequency multiple of $mf_2/2$ and the waveform of the second register differs from the waveform of the fundamental. For example, the pressure signal of the second register of a stepped cone made with three cylinders ($a = 1$, $b = 3$, $m = 4$) is a square wave, like in a clarinet. Measurements of the pressure in the mouthpiece of stepped cones made with 2 and 3 cylinders confirm this analysis (see Figure 13).

6. Conclusion

From the analysis of idealized models of bowed string and reed woodwinds, we showed that the analogy between these models is more complete than mentioned in [1] if considering an elementary model for the reed, since a flow parameter characterizing the embouchure is found to be analogous to the bow force parameter. Moreover, it seems that conical woodwind makers tend to obtain resonators with an input impedance close to the impedance of idealized stepped cone resonators. Since these resonators are analogous to bowed strings, the analogy between bowed string and conical woodwinds models is straightforward. Thus, with this analogy, studies using Raman's model for bowed string can be useful for studies on conical reed woodwinds, and vice versa. This has been used, for example, to investigate the stability of two-step solutions in the companion paper [2]. Nevertheless, a direct transposition of results from one family to the other is not obvious since the nonlinear characteristics differ.

A musical application of this formal analogy can be the use of two reflection functions for time domain synthesis of conical-like reed woodwinds. This possibility is interesting since many numerical instability problems appear when using a single reflection function for conical bores.

Another musical application is the possibility to control a bowed string model with a saxophone, or inversely, with a good playability if analogous parameters are associated. This can lead to strange, but playable, instruments like a virtual saxophone model with a variable length for the truncation, controlled by the position of the bow on a real bowed string instrument.

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