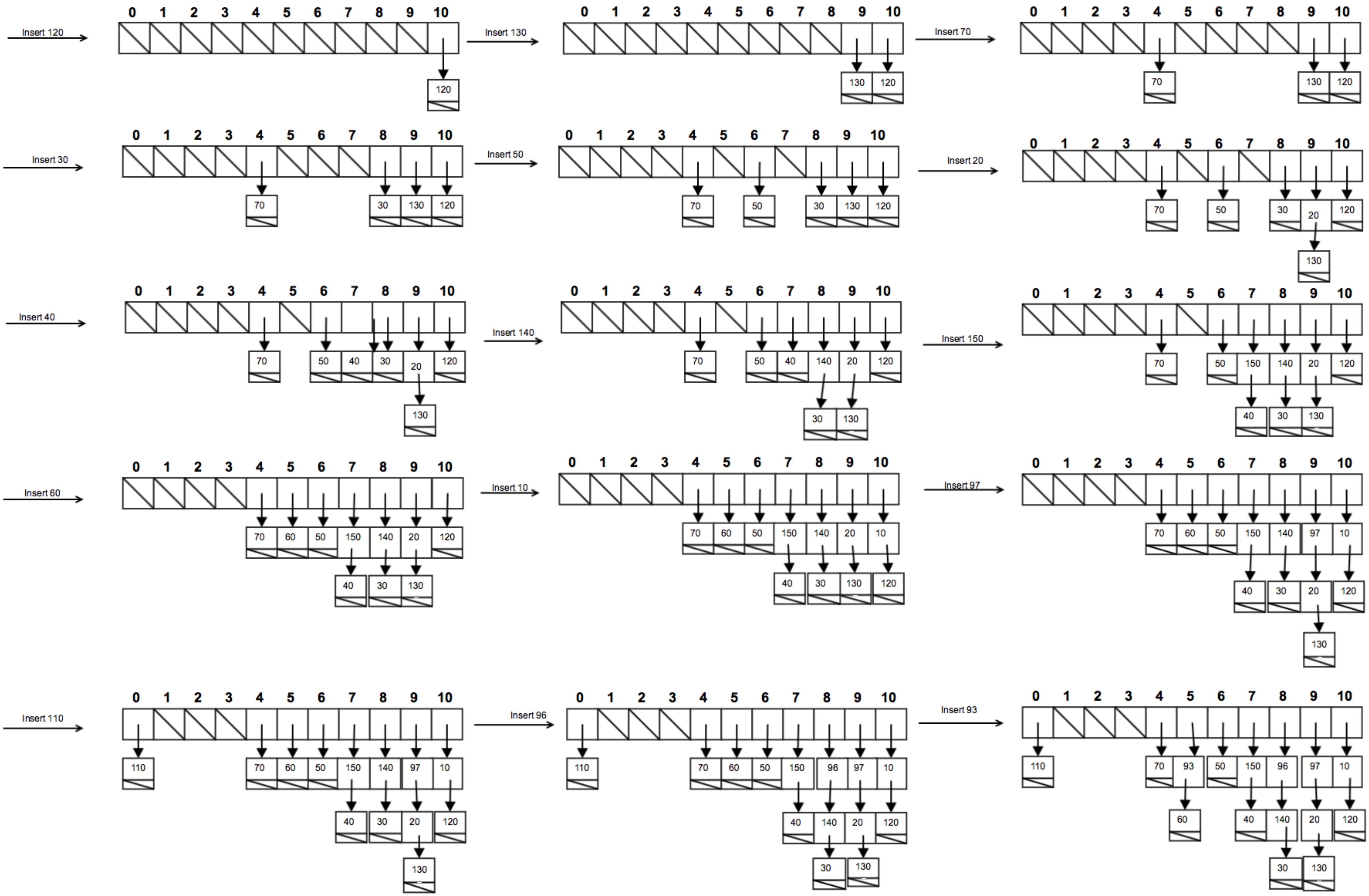
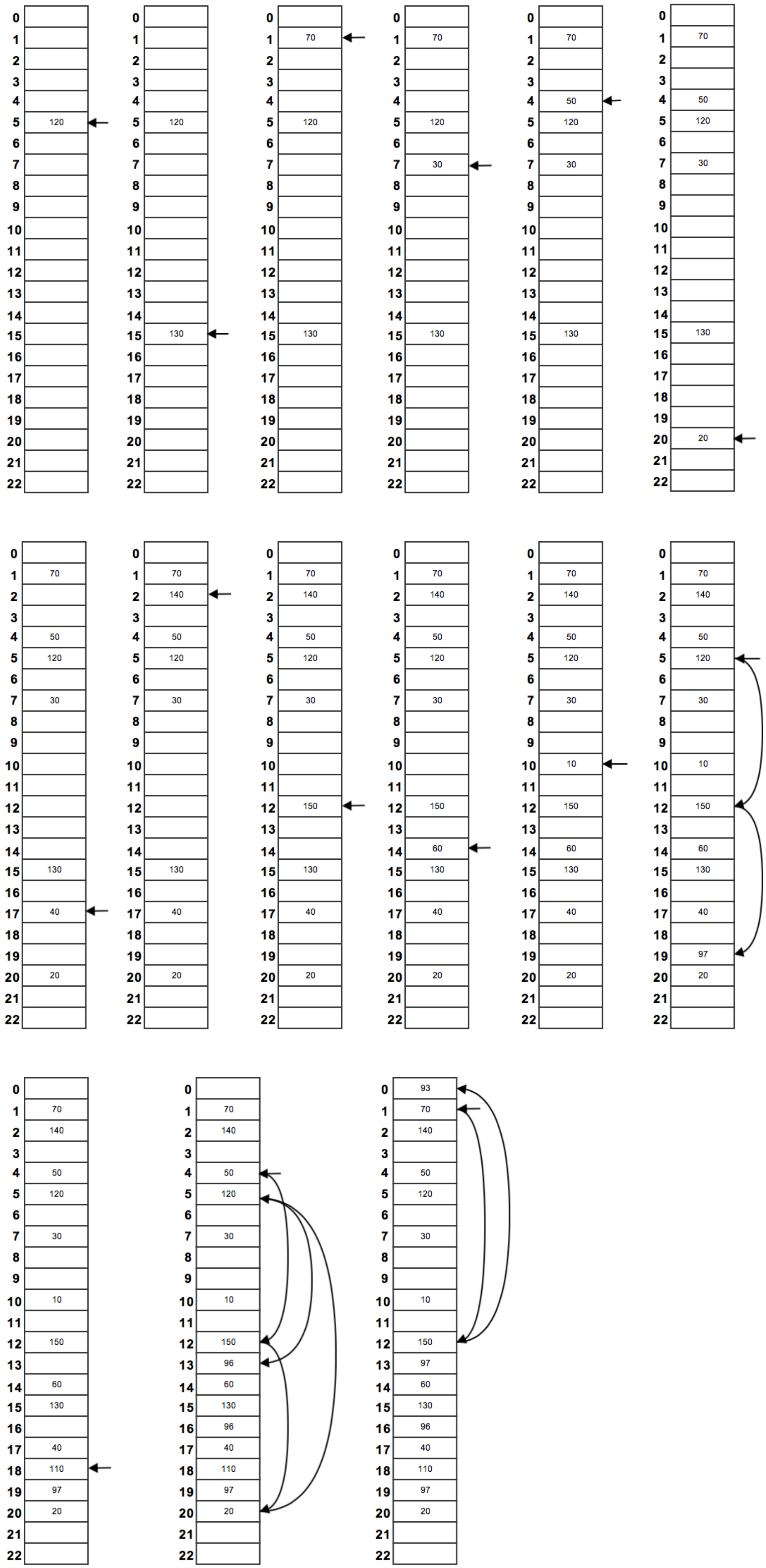
Question#1 (a)

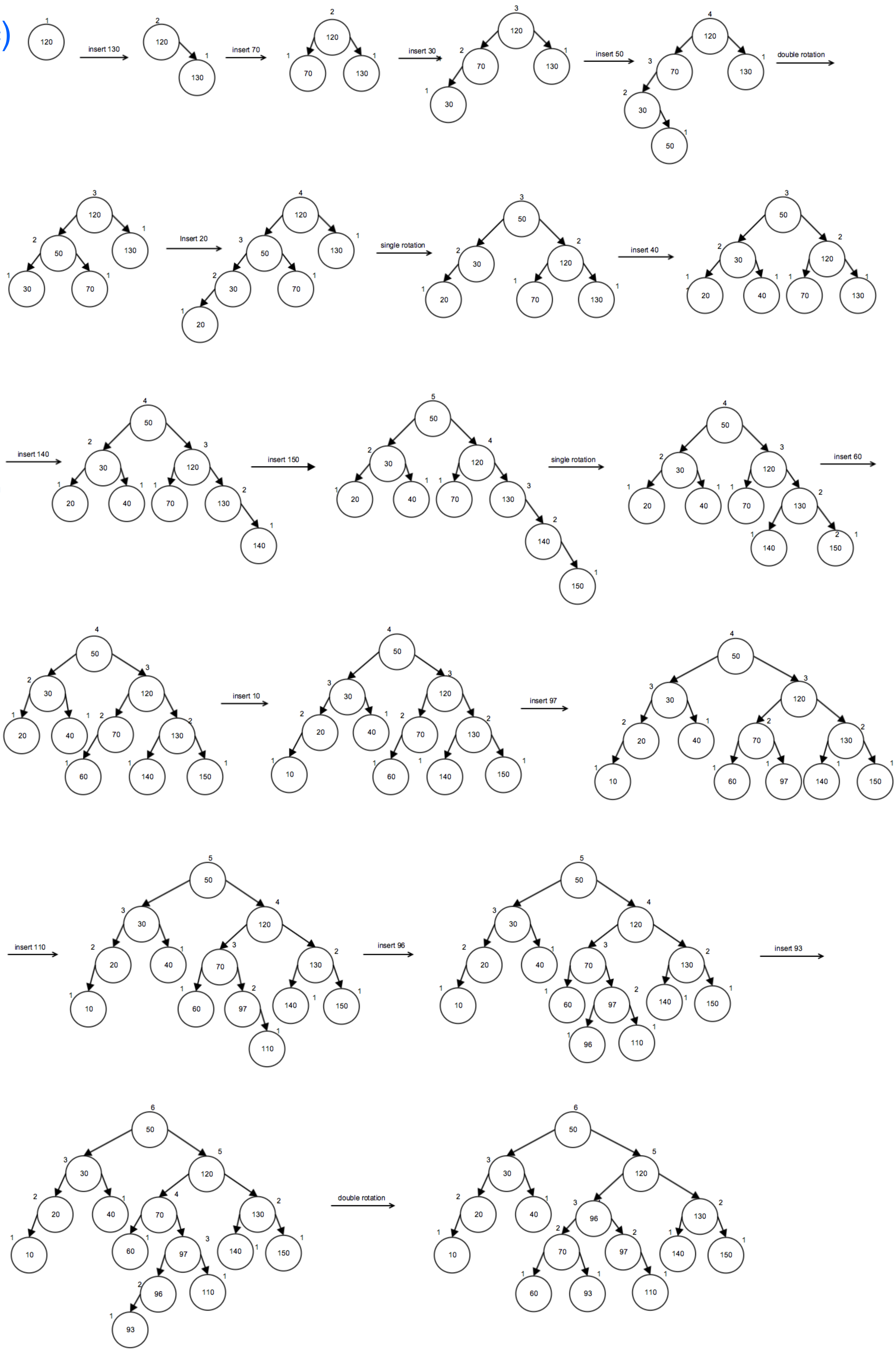


Insert 120
Insert 130
Insert 70
Insert 30
Insert 50
Insert 20
Insert 40
Insert 140
Insert 150
Insert 60
Insert 10
Insert 97
Insert 110
Insert 96
Insert 93
0 1 2 3 4 5 6 7 8 9 10

(b)

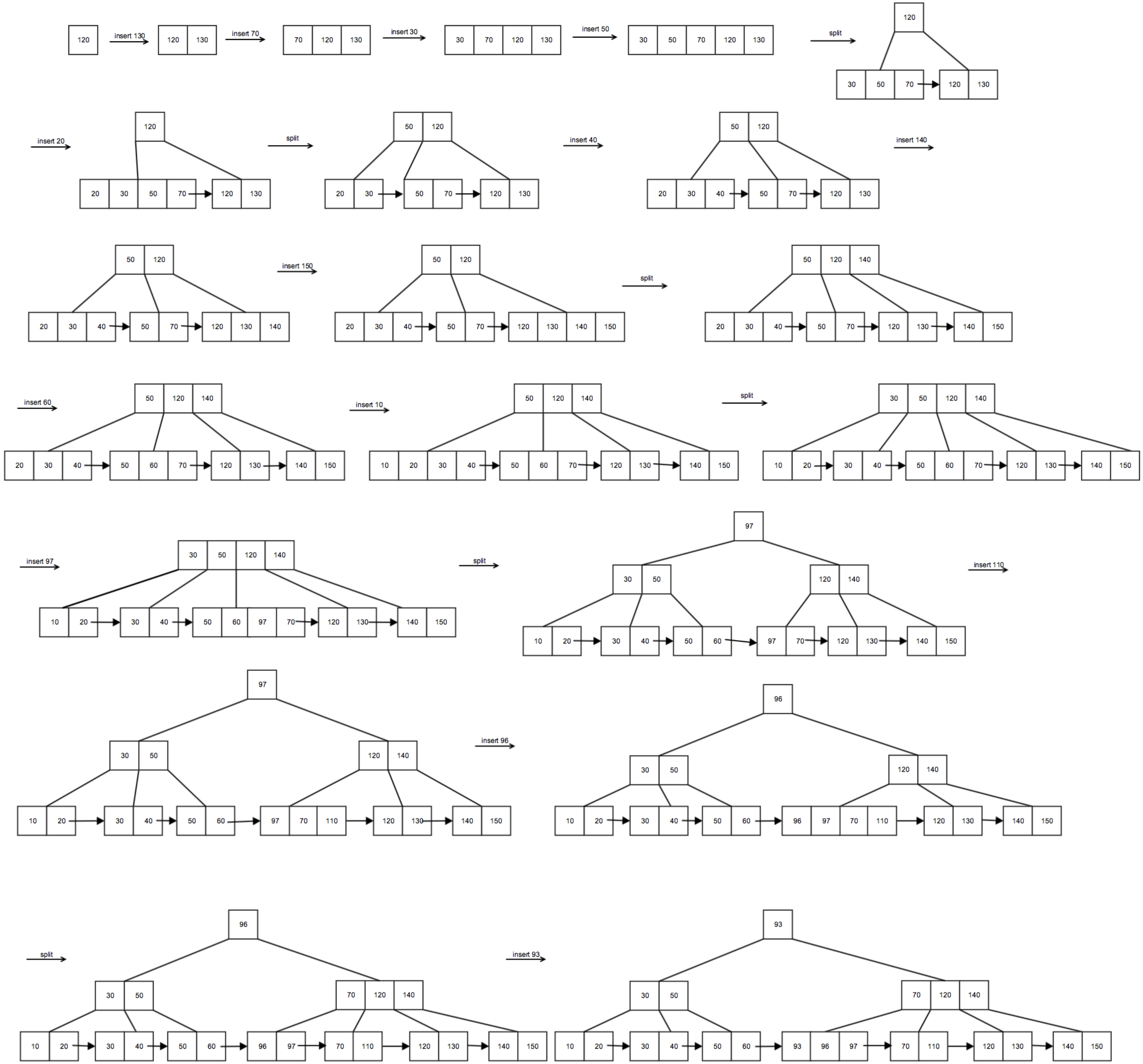


(c)



insert 130
insert 70
insert 30
insert 50
double rotation
Insert 20
single rotation
insert 40
insert 140
insert 150
single rotation
insert 60
insert 10
insert 97
insert 110
insert 96
insert 93
double rotation

(d)


insert 130
insert 70
insert 30
insert 50
split
insert 20
split
insert 40
insert 140
insert 150
split
insert 60
insert 10
split
insert 97
split
insert 110
insert 96
split
insert 93

Q2:

Each computer can connect to 0-5 computers. However, let's say there is a computer that is not connected to any other computers, then the max number of connections among all six computers is 4, so

$\#$ of connections $\in \{0, 1, 2, 3, 4\}$, 5 choices.

Similar things happen when we assume that there is a computer connecting to 1, 2, 3, 4, 5 respectively. There are always 5 possible choices for computer connections and we have 6 computers in total. According to the pigeonhole principle, two computers must have the same number of connections.

Q3: In an AVL tree of height h, at least one of the left and right subtrees is an AVL tree of height $h-1$. Since the height of left and right subtrees can differ by at most 1, then the other subtree must have height of $h-2$. Thus, we the relation

$$N(h) = N(h-1) + N(h-2) + 1$$

Given the base case: $N(0) = 1$ and $N(1) = 2$

Base case:

- $F(3) = F_1 + F_2 = 2$

  $\therefore N(0) = F(3) - 1 = 2 - 1 = 1$, $N(h) = F_{h+3} - 1$ applies
- $F(4) = F_3 + F_4 = 2 + 1 = 3$

  $\therefore N(1) = F(4) - 1 = 3 - 1 = 2$, $N(h) = F_{h+3} - 1$ applies

Induction hyprothesis:

Assume that the hypothesis $N(h) = F(h+3) - 1$ is true for $h \geq 1$

Inductive Step: assume $h = k+1$, $k \in \mathbb{Z}$

$$\begin{aligned} N(k+1) &= N(k) + N(k-1) + 1 \\ &= F(k+3) - 1 + F(k+2) - 1 + 1 \\ &= F(k+4) - 1 \end{aligned}$$

replace $h = k+1$, so we have $N(h) = F(h+3) - 1$

Thus, it is true for all h.